

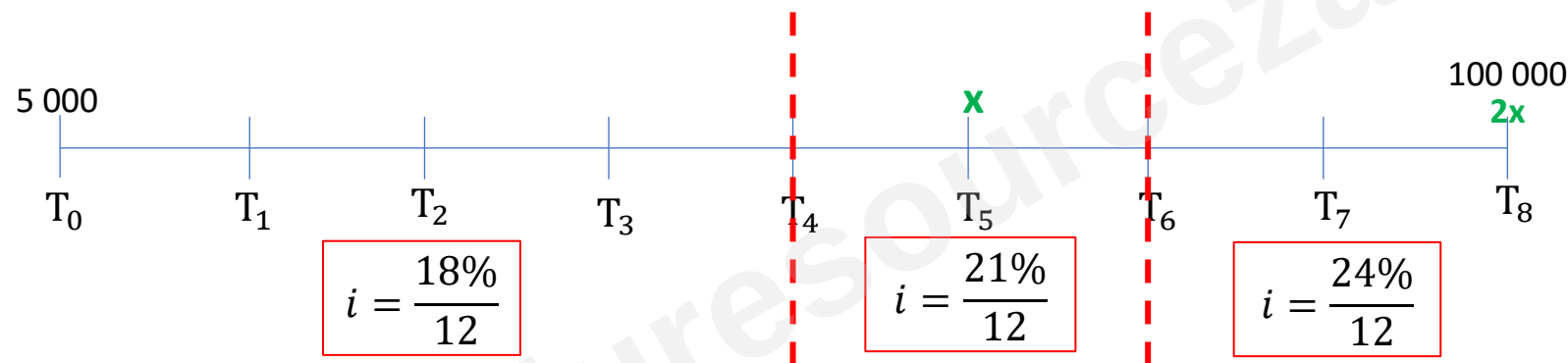


FINANCIAL MATHS

FUTURE
VALUE
ANNUITIES

REVISION OF GR 11 FINANCIAL MATHS – ADVANCED TIMELINES

Example : Sean opens a savings account and deposits R5000.00 into the account immediately. Five months later he invests a further Rx into the account. Three months later, he deposits R2x into the account. The interest rate for the first four months is 18% per annum compounded monthly, 21% per annum compounded monthly for the next two months and 24% per annum compounded monthly for the remaining two months. The investment has a future value of R100 000 at the end of the 8th month. Calculate the value of the final payment at the end of the 8th month.



$$100\,000 = 5000\left(1 + \frac{18\%}{12}\right)^4 \left(1 + \frac{21\%}{12}\right)^2 \left(1 + \frac{24\%}{12}\right)^2 + x\left(1 + \frac{21\%}{12}\right)^1 \left(1 + \frac{24\%}{12}\right)^2 + 2x$$

$$100\,000 - 5000\left(1 + \frac{18\%}{12}\right)^4 \left(1 + \frac{21\%}{12}\right)^2 \left(1 + \frac{24\%}{12}\right)^2 = x\left[\left(1 + \frac{21\%}{12}\right)^1 \left(1 + \frac{24\%}{12}\right)^2 + 2\right]$$

$$\frac{100\,000 - 5000\left(1 + \frac{18\%}{12}\right)^4 \left(1 + \frac{21\%}{12}\right)^2 \left(1 + \frac{24\%}{12}\right)^2}{\left[\left(1 + \frac{21\%}{12}\right)^1 \left(1 + \frac{24\%}{12}\right)^2 + 2\right]} = x$$

$$\therefore x = R\,30\,825,75 \quad \therefore \text{the final payment is } 2x = R\,61\,651,50$$

FUTURE VALUE ANNUITIES

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An **ANNUITY** is a series of **equal payments** made at **regular time interval**. Eg. retirement annuities, home/vehicle loans.

The Annuity formulae that you do this year have the following conditions:

- ❖ All payments are equal
- ❖ Payments are made at regular intervals
- ❖ The interest rate remains fixed
- ❖ The compounding period for interest is the same as the payment intervals

THE FUTURE VALUE FORMULA

$$F = \frac{x[(1 + i)^n - 1]}{i}$$



F is the future value

x is the regular payment

i is the interest rate per interval

n is the number of payments

Generally this formula is used when small regular payments are made into a bank account to have a large amount of money at the end. Examples include savings for an education, holiday, retirement, replacing equipment etc.

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Example 1:

Suppose that at the beginning of the month, R1000 is deposited into a bank. At the end of the month a further R1000 is deposited and a further R1000 at the end of the next month. If the interest rate is 6% per annum compounded monthly, how much will be saved after 8 years?



Immediately

$$\longrightarrow n = 96 + 1 = 97 \longrightarrow$$

Two payments were made in the first months at the beginning and at the end

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$F = \frac{1000 \left[\left(1 + \frac{6\%}{12} \right)^{97} - 1 \right]}{\frac{6\%}{12}}$$

$$= R\ 124\ 442,68$$

Example 2:

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Cindy has just turned twenty years old and has a dream of saving up an amount of R10 000 000 by the time she reaches the age of 50. She starts to pay equal monthly amounts into a retirement annuity which pays 8% per annum compounded monthly. Her first payment starts at the end of her birth month on her 20th birthday and her last payment is made on her 50th birthday. How much will she pay each month?

The investment is for a period of 30 years x 12 months = 360 payments.

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$10\,000\,000 = \frac{x\left[\left(1 + \frac{8\%}{12}\right)^{360} - 1\right]}{\frac{8\%}{12}}$$

$$10\,000\,000 \times \frac{8\%}{12} = x\left[\left(1 + \frac{8\%}{12}\right)^{360} - 1\right]$$

$$\frac{10\,000\,000 \times \frac{8\%}{12}}{\left[\left(1 + \frac{8\%}{12}\right)^{360} - 1\right]} = x$$

$$\therefore x = R\,6\,709,79$$

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Example 3:

Tumelo decides to open a savings account for his baby daughter's future education. Upon opening the account, he immediately deposits R2000 into the account and continues to make monthly payments at the end of each month thereafter for a period of 16 years. The interest rate remains fixed at 15% per annum compounded monthly.

- How much money will he have accumulated at the end of the 16th year?
- At the end of the 16-year period, he leaves the money in the account for a further year. How much money will he then have accumulated?

- a) Immediately $\longrightarrow$ 16 years = 192 months + 1 = 193

$$\begin{aligned} F &= \frac{x[(1+i)^n - 1]}{i} \\ &= \frac{2000 \left[\left(1 + \frac{15\%}{12} \right)^{193} - 1 \right]}{\frac{15\%}{12}} \\ &= R\ 1\ 599\ 386,02 \end{aligned}$$

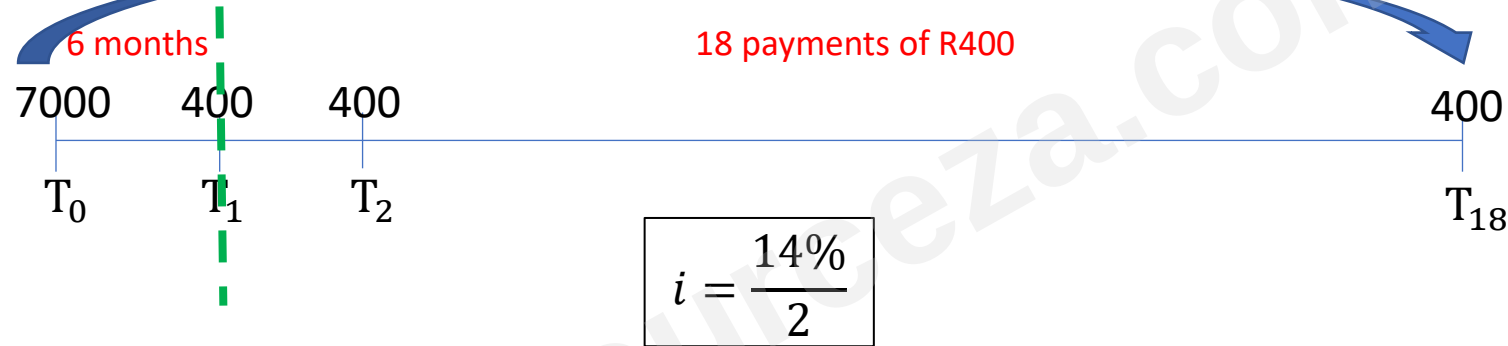


$$\begin{aligned} A &= P(1+i)^n \\ &= 1\ 599\ 386,02 \left(1 + \frac{15\%}{12} \right)^{12} \\ &= R\ 1\ 856\ 494,55 \end{aligned}$$

Example 4:

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Jesse deposits R7000 into an account paying 14% per annum compounded half-yearly. Six months later, she deposits R400 into the account. Six months after this, she deposits a further R400 into the account. She then continues to make half-yearly deposits of R400 into the account for a period of nine years from the deposit of R7000. Calculate the value of her savings at the end of the savings period.



$$F = P(1 + i)^n + \frac{x[(1+i)^n - 1]}{i}$$

$$F = 7000\left(1 + \frac{14\%}{2}\right)^{19} + \frac{400\left[\left(1 + \frac{14\%}{2}\right)^{18} - 1\right]}{\frac{14\%}{2}}$$

$$= R\ 38\ 915,31$$

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Example 5:

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Anna wants to save R300 000 by paying monthly amounts of R4000, starting in one month's time, into a savings account paying 15% p.a. compounded monthly. How long will it take Anna to accumulate the amount of R300 000?

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$300\,000 = \frac{4000 \left[\left(1 + \frac{15\%}{12} \right)^n - 1 \right]}{\frac{15\%}{12}}$$

$$\frac{300\,000 \times \frac{15\%}{12}}{4000} = \left(1 + \frac{15\%}{12} \right)^n - 1$$

$$\frac{300\,000 \times \frac{15\%}{12}}{4000} + 1 = \left(1 + \frac{15\%}{12} \right)^n$$

$$1,9375 = (1,0125)^n$$

$$n = \log_{1,0125} 1,9375$$

$$n = 53,24189314$$

$0,24189314 \times 30 = 7,256...$



$\therefore$ it will take Anna 53 months and approximately 7 – 8 days to get the R300 000.

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HOMEWORK:

Exercise 2 page 279
ALL

