



FINANCIAL MATHS

PRESENT
VALUE
ANNUITIES

PRESENT VALUE FORMULA

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

P is the present value

x is the payment

i is the interest rate per interval

n is the number of payments

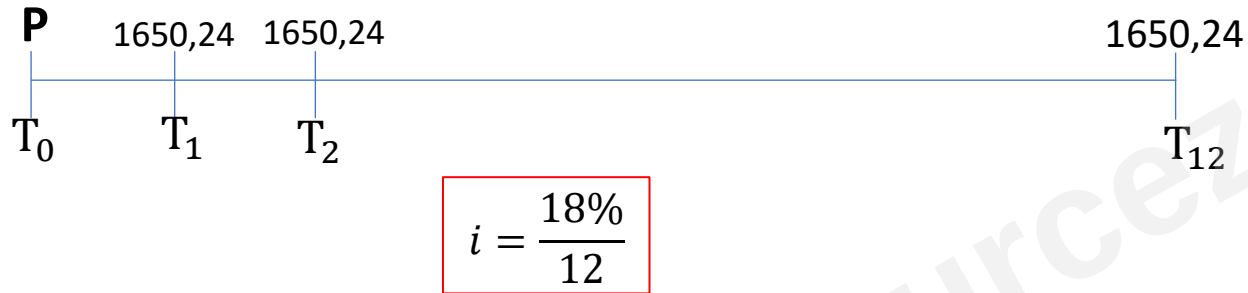
The Present value formula is used whenever the total (large sum of money often as loan) is received first and small payments are made later. This includes all loans. This formula is also used when a sum of money is set aside in order to make regular withdrawals in the future.

A loan is said to be **AMORTISED** (paid off) when it together with the interest charges is paid off.

Example 1:

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James takes out a one – year bank loan to pay for an expensive laptop. The interest rate is 18% per annum compounded monthly and monthly repayments of R1650.24 are made starting one month after the granting of the loan. Show that James borrowed R18 000.00.



$$\begin{aligned} P &= \frac{x[1-(1+i)^{-n}]}{i} \\ &= \frac{1650,24[1-(1+\frac{18\%}{12})^{-12}]}{\frac{18\%}{12}} \\ &= R\ 18\ 000,00 \end{aligned}$$

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Example 2:

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David takes out a bank loan to pay for his new car. He repays the loan by means of monthly payments of R4000.00 for a period of 5 years starting one month after the granting of the loan. The interest rate is 24% per annum compounded monthly. Calculate the purchase price of his new car.

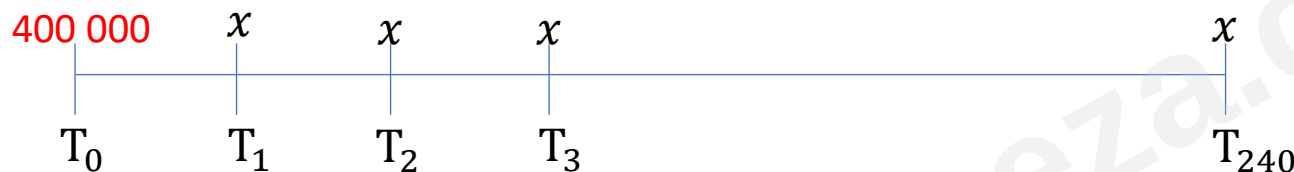
$$\begin{aligned} P &= \frac{x[1-(1+i)^{-n}]}{i} \\ &= \frac{4000[1-(1+\frac{24\%}{12})^{-60}]}{\frac{24\%}{12}} \\ &= R\ 139\ 043,55 \end{aligned}$$

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Example 3: ★

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Moses inherits R400 000 from his father. He invests the money at an interest rate of 12% per annum compounded monthly. He wishes to earn a monthly salary from the investment for a period of twenty years beginning in a month's time. How much will he receive each month?



$$i = \frac{12\%}{12}$$

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$400\,000 = \frac{x[1-(1+\frac{12\%}{12})^{-240}]}{\frac{12\%}{12}}$$

$$\frac{400\,000 \times \frac{12\%}{12}}{[1-(1+\frac{12\%}{12})^{-240}]} = x$$

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$$\therefore x = R\,4\,404,34$$

Example 4:

Peter borrows R500 000 from a bank and repays the loan by means of monthly payments of R8000, starting one month after receiving the loan. Interest is fixed at 18% per annum compounded monthly.

- How many payments of R8000 will be made?
- What will the final payment be?
- How long is the loan period?

a)

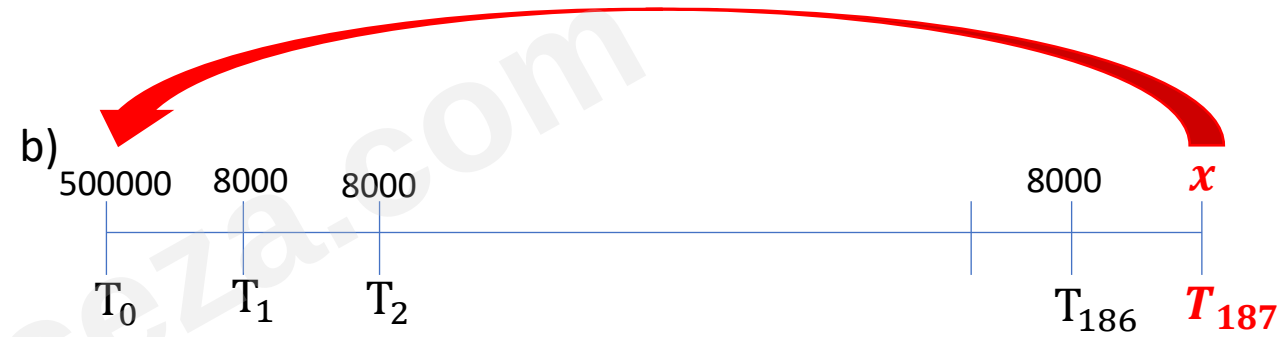
$$P = \frac{x[1-(1+i)^{-n}]}{i}$$
$$500\,000 = \frac{8000 \left[1 - \left(1 + \frac{18\%}{12}\right)^{-n}\right]}{\frac{18\%}{12}}$$

$$\frac{500\,000 \times \frac{18\%}{12}}{8000} = 1 - \left(1 + \frac{18\%}{12}\right)^{-n}$$
$$\left(1 + \frac{18\%}{12}\right)^{-n} = 1 - \frac{500\,000 \times \frac{18\%}{12}}{8000}$$

$$(1,015)^{-n} = 0,0625$$
$$-n = \log_{1,015} 0,0625$$
$$-n = -186,2221025$$

$\therefore$ 186 payments of R 8000 will be made

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$$500\,000 = \frac{8000 \left[1 - \left(1 + \frac{18\%}{12}\right)^{-186}\right]}{\frac{18\%}{12}} + x \left(1 + \frac{18\%}{12}\right)^{-187}$$

$$500\,000 - \frac{8000 \left[1 - \left(1 + \frac{18\%}{12}\right)^{-186}\right]}{\frac{18\%}{12}} = x \left(1 + \frac{18\%}{12}\right)^{-187}$$

$$110,4090631 = x(1,015)^{-187}$$

$$\frac{110,4090631}{(1,015)^{-187}} = x$$

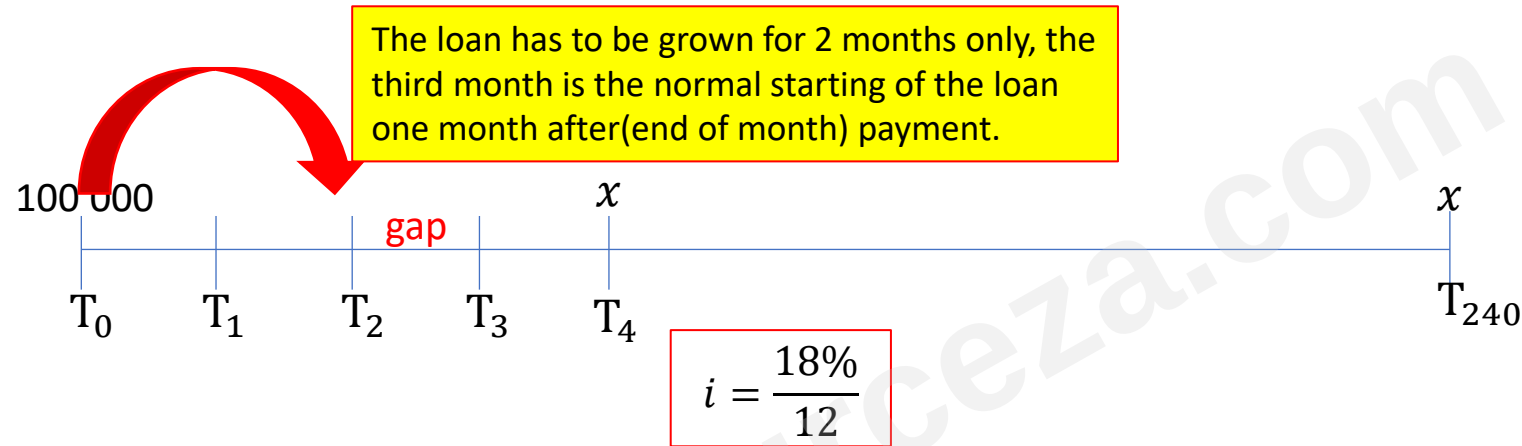
$$\therefore x = R\,1787,12$$

c) The loan will take 187 months to pay off.

Example 5:

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Simphiwe takes out a twenty year loan of R100 000. He repays the loan by equal monthly payments starting three months after the granting of the loan. The interest rate is 18% per annum compounded monthly. Calculate the monthly payments.



$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$100\,000\left(1 + \frac{18\%}{12}\right)^2 = \frac{x\left[1 - \left(1 + \frac{18\%}{12}\right)^{-238}\right]}{\frac{18\%}{12}}$$

$$\frac{100\,000\left(1 + \frac{18\%}{12}\right)^2}{\frac{1 - \left(1 + \frac{18\%}{12}\right)^{-238}}{\frac{18\%}{12}}} = x$$

$$\therefore x = R\,1\,591,35$$

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Homework:

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