

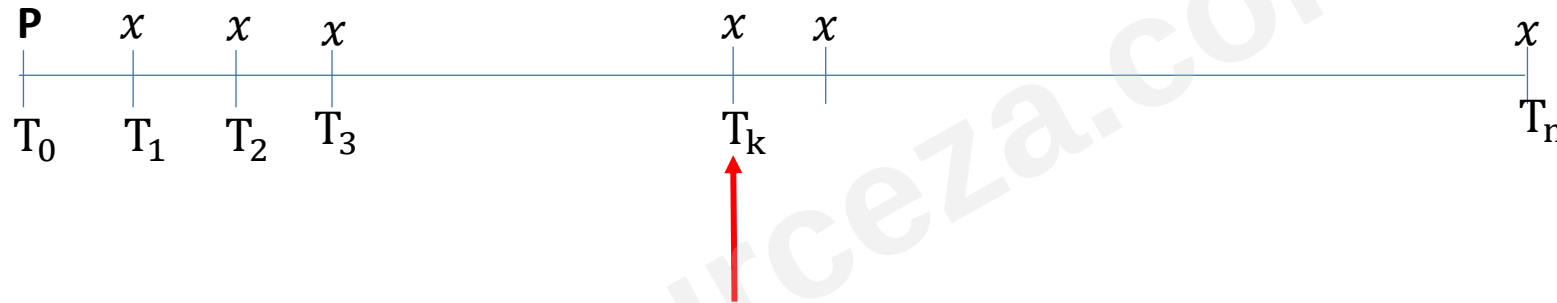


FINANCIAL MATHS

BALANCE
OUTSTANDING
AND
LAST PAYMENT

BALANCE OUTSTANDING

The balance outstanding on a loan (P) after the k th payment has been made is the amount still owed at the stage.



Balance Outstanding (B.O)

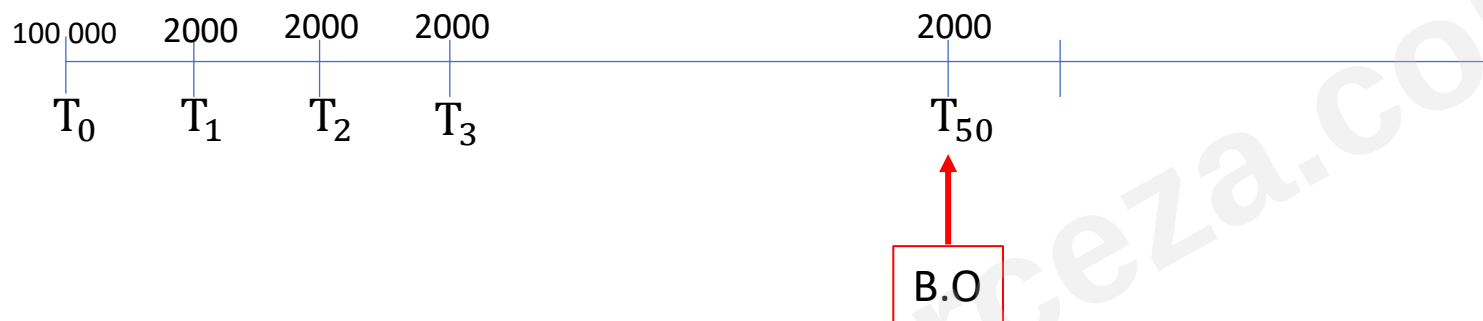
In order to calculate the balance outstanding after the k th payment has been made is to determine the future value of P at T_k minus the future value of all k payments of x at T_k

$$\text{B.O} = P(1 + i)^k - \frac{x[(1+i)^k - 1]}{i}$$

Example 1:

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A twenty year loan of R100 000 is to be repaid by means of monthly payments of R2000 at an interest rate of 18% per annum compounded monthly. Find the balance outstanding after the 50th payment has been made.



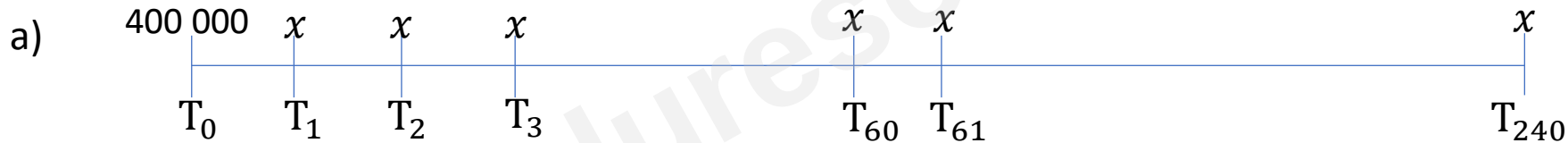
$$\begin{aligned} \text{B.O} &= P(1+i)^k - \frac{x[(1+i)^k - 1]}{i} \\ &= 100\,000\left(1 + \frac{18\%}{12}\right)^{50} - \frac{2000\left[\left(1 + \frac{18\%}{12}\right)^{50} - 1\right]}{\frac{18\%}{12}} \\ &= R\,63\,158,59 \end{aligned}$$

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Example 2:

Mr. Brown has just finished paying off his 20 year home loan of R400 000. During the first 5 years the interest rate was 24% per annum compounded monthly. Thereafter, and for the rest of the term, the interest rate decreased to 18% per annum compounded monthly.

- Calculate his initial monthly payment.
- Calculate his balance outstanding at the end of December in the 5th year.
- When the interest rate changed after 5 years, Mr. Brown was able to pay a decreased monthly payment starting at the end of January in the 6th year. Calculate what this new repayment was.
- How much did Mr. Brown actually pay for his home?
- Suppose that Mr. Brown decided to rather pay R9000 per month starting from the end of January in the 6th. How long would it have taken him to pay off the loan?



$$P = \frac{x[1-(1+i)^{-n}]}{i}$$
$$400\,000 = \frac{x[1-(1+\frac{24\%}{12})^{-240}]}{\frac{24\%}{12}}$$

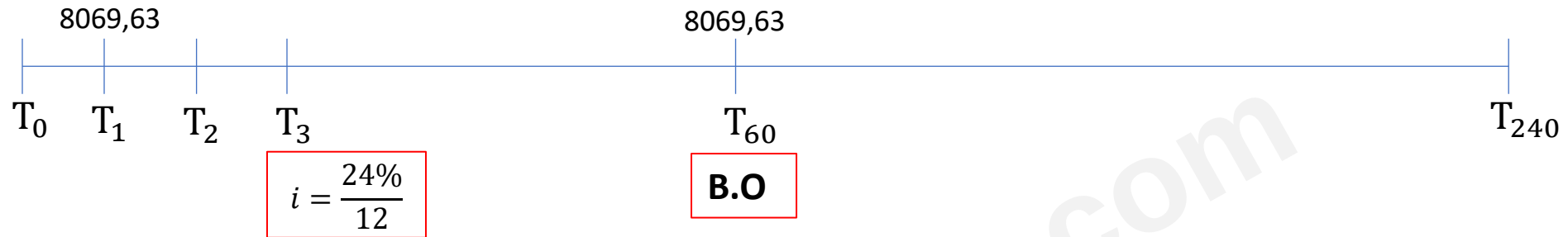
$$\frac{400\,000 \times \frac{24\%}{12}}{[1-(1+\frac{24\%}{12})^{-240}]} = x$$

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$$\therefore x = R\,8069,83$$

b) Calculate his **balance outstanding** at the end of December in the **5th** year.

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$$\text{B.O} = P(1 + i)^k - \frac{x[(1+i)^k - 1]}{i}$$

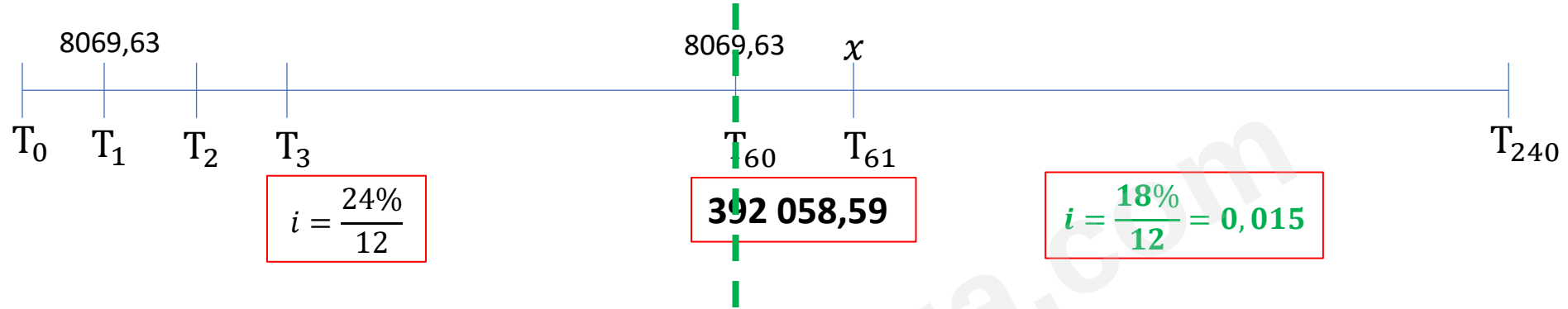
$$= 400\,000(1,02)^{60} - \frac{8069,63[(1,02)^{60} - 1]}{0,02}$$

$$= R\,392\,058,59$$

$$\left(1 + \frac{24\%}{12}\right) = 1,02$$

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- c) When the interest rate changed after 5 years, Mr. Brown was able to pay a decreased monthly payment starting at the end of January in the 6th year. Calculate what this new repayment was.



$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$392\,058,59 = \frac{x[1 - (1,015)^{-180}]}{0,015}$$

$$\frac{392\,058,59 \times 0,015}{[1 - (1,015)^{-180}]} = x$$

$$\therefore x = \therefore R\,6\,313,79$$

d) How much did Mr. Brown actually pay for his home?

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First 5 years	R 8 069,63 x 60	= R 484 177,80
Next 15 years	R 6 313,79 x 180	= R 1 136 482,20
Total		= R 1 620 660,00 !!!! (scary!!!!)

e) Suppose that Mr. Brown decided to rather pay R9000 per month starting from the end of January in the 6th.year How long would it have taken him to pay off the loan?

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$392\,058,59 = \frac{9000[1-(1,015)^{-n}]}{0,015}$$
$$\frac{392\,058,59 \times 0,015}{9000} = 1 - (1,015)^{-n}$$

$$(1,015)^{-n} = 1 - \frac{392\,058,59 \times 0,015}{9000}$$

$$(1,015)^{-n} = 0,346569016$$

$$-n = \log_{1,015} 0,346569016$$

$$\therefore n = 71,17340842$$

So Mr. Brown would have taken

60 months(R8069,63) + **71** months(R9000) + **1** month(last payment less than R9000)
= total of **132** months.

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LAST PAYMENT

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Example 3: George has to repay a loan of R 375 000. The interest is 14% p.a. compounded monthly. He pays back R7 500 monthly

- a) how many payments will George have to make?
- b) What will his last payment be?

a)

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$
$$375\,000 = \frac{7500[1-(1+\frac{14\%}{12})^{-n}]}{\frac{14\%}{12}}$$

$$\frac{375\,000 \times \frac{14\%}{12}}{7500} = 1 - \left(1 + \frac{14\%}{12}\right)^{-n}$$

$$\left(1 + \frac{14\%}{12}\right)^{-n} = 1 - \frac{375\,000 \times \frac{14\%}{12}}{7500}$$

$$\left(\frac{607}{600}\right)^{-n} = \frac{5}{12}$$

$$\therefore -n = \log_{\frac{607}{600}}\left(\frac{5}{12}\right)$$

$$\therefore n = 75,47706563$$

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So 75 payments of R 7500 will be made & 1 lesser payment

For the calculation of the last payment :

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1. calculate the balance outstanding after the last full payment
2. then grow that B.O for one more interest charged payment.

$$b) \text{ B.O} = P(1 + i)^k - \frac{x[(1+i)^k - 1]}{i}$$

$$= 375\,000\left(1 + \frac{14\%}{12}\right)^{75} - \frac{7500\left[\left(1 + \frac{14\%}{12}\right)^{75} - 1\right]}{\frac{14\%}{12}}$$

$$= R\,3547,457499$$

$$\begin{aligned} \text{Last payment} &= R\,3547,457499\left(1 + \frac{14\%}{12}\right)^1 \\ &= R\,3\,588,84 \end{aligned}$$

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Homework:

Exercise 4 page 285

All please!