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Name:
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MORE Calculus Practice

18th February 2015

Possible useful formulae:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Total: 40 marks**Time:** 45 minutes

If you hand this in at the start of the first maths lesson after half-term I will mark it and you will have the option of including it in your term mark!

I recommend that you do it anyway to consolidate everything we have done so far this term!

1. Determine the following limit: $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{3x - 6}$ (3)

2. Determine the derivative of $f(x) = 2x^2 - 3$ by first principles. (5)

3. Determine the derivatives of the following paying careful attention to notation:

a. $y = (4x+3)(5x-1)$ (2)

b. $f(x) = \frac{4}{x^3} - \sqrt[5]{x^6}$ (3)

c. $g(x) = \frac{x^2 + \sqrt{x}}{3x^3}$ (5)

4. Determine the gradient of $y = -x^3 + 2x^2 - 6$ when $x = 1$. What does this tell you? (3)

5. Consider the function: $y = 2x^3 + 5x^2 - 4x - 2$

a. Determine the intercepts with both axes.

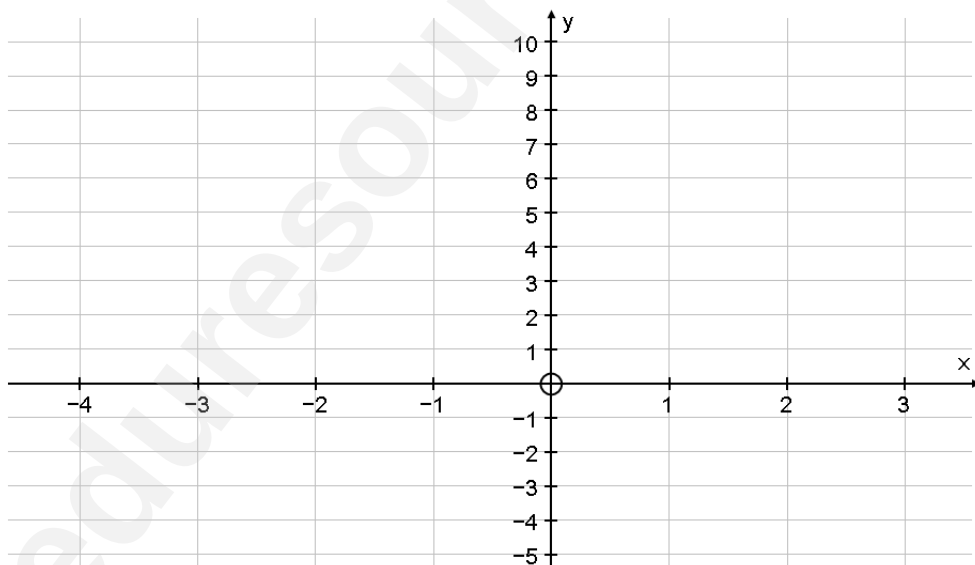
(4)

b. Determine the location and the nature of any stationary points

(5)

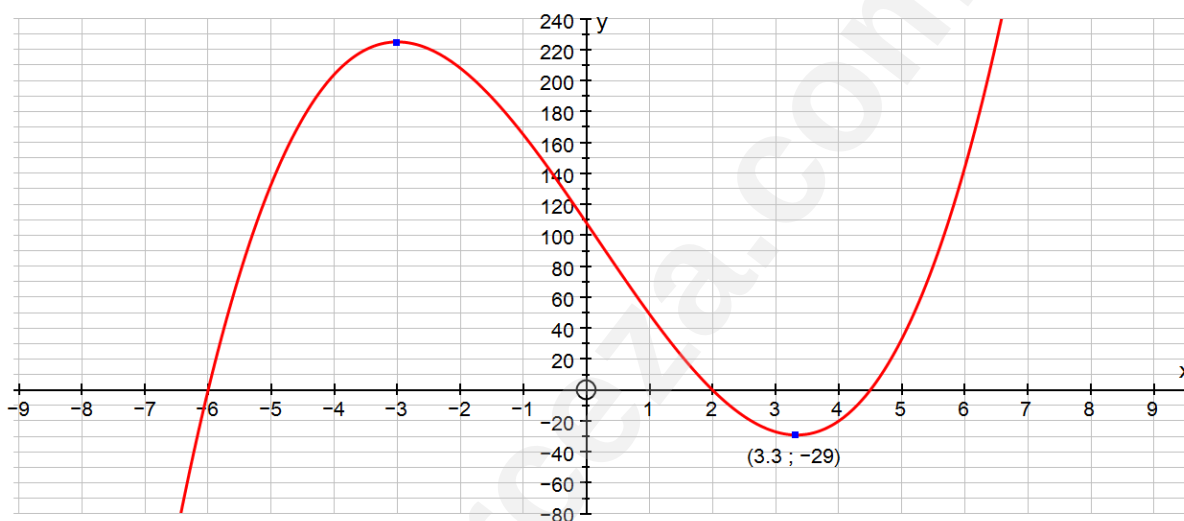
c. Use the above information to draw the graph of the function.

(2)



6. The graph of $y = ax^2 + bx + c$ has a minimum at the point $(2, -13)$. Determine a and b . (5)

7. Given the function f below. Use the graph to determine for which values of x , $f'(x)f(x) \geq 0$. (3)



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1. Determine the following limit: $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{3x - 6}$ (3)

$$= \lim_{x \rightarrow 2} \frac{(x+3)(x-2)}{3(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{(x+3)}{3}$$

$$= \frac{5}{3}$$

2. Determine the derivative of $f(x) = 2x^2 - 3$ by first principles. (5)

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3 - 2x^2 + 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h}$$

$$= \lim_{h \rightarrow 0} (4x + 2h)$$

$$= 4x$$

3. Determine the derivatives of the following paying careful attention to notation:

a. $y = (4x+3)(5x-1)$

(2)

$$y = 20x^2 + 11x - 3$$

$$\therefore \frac{dy}{dx} = 40x + 11$$

b. $f(x) = \frac{4}{x^3} - \sqrt[5]{x^6}$

(3)

$$f(x) = 4x^{-3} - x^{\frac{6}{5}}$$

$$\therefore f'(x) = -12x^{-4} - \frac{6}{5}x^{\frac{1}{5}}$$

4. Determine the gradient of $y = -x^3 + 2x^2 - 6$ when $x = 1$. What does this tell you?

(3)

$$\frac{dy}{dx} = -3x^2 + 4x$$

$$\text{when } x = 1, \frac{dy}{dx} = 1$$

this tells us that the function is increasing

5. Consider the function: $y = 2x^3 + 5x^2 - 4x - 3$ Downloaded from eduresourceza.com

a. Determine the intercepts with both axes. (4)

$$y = 2x^3 + 5x^2 - 4x - 3$$

$$y\text{-int } (0; -3)$$

x -intercepts: by inspection, $x = 1$, so $(x - 1)$ is a factor

$$(x - 1)(2x^2 + 7x + 3) = 0$$

$$(x - 1)(2x + 1)(x + 3) = 0$$

$$\therefore x = 1 \text{ or } -\frac{1}{2} \text{ or } -3$$

b. Determine the location and the nature of any stationary points (5)

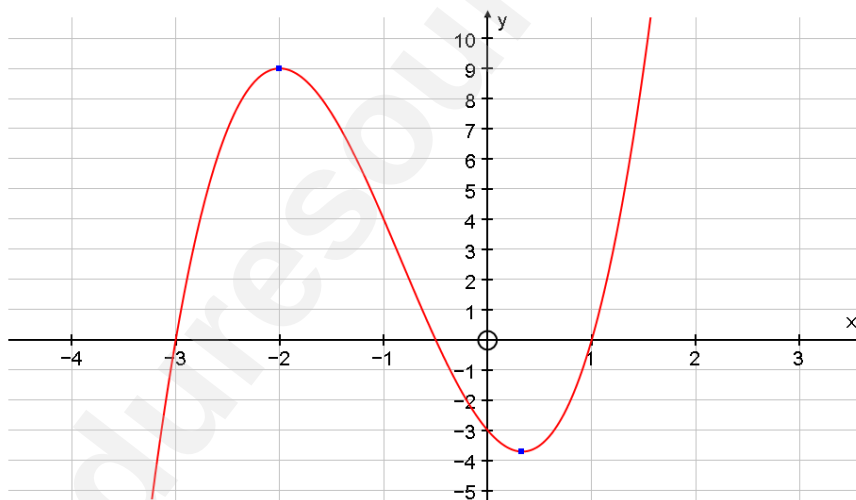
$$\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$$

$$\therefore (3x - 1)(2x + 4) = 0$$

$$\therefore x = \frac{1}{3} \text{ or } -2$$

$$\therefore \left(\frac{1}{3}; -3.7\right) \text{ MIN or } (-2; 9) \text{ MAX}$$

c. Use the above information to draw the graph of the function. (2)



6. The graph of $y = ax^2 + bx - 1$ has a minimum at the point $(2; -13)$. Determine a and b . (5)

$$y = ax^2 + bx - 1$$

since $(2; -13)$ lies on the curve:

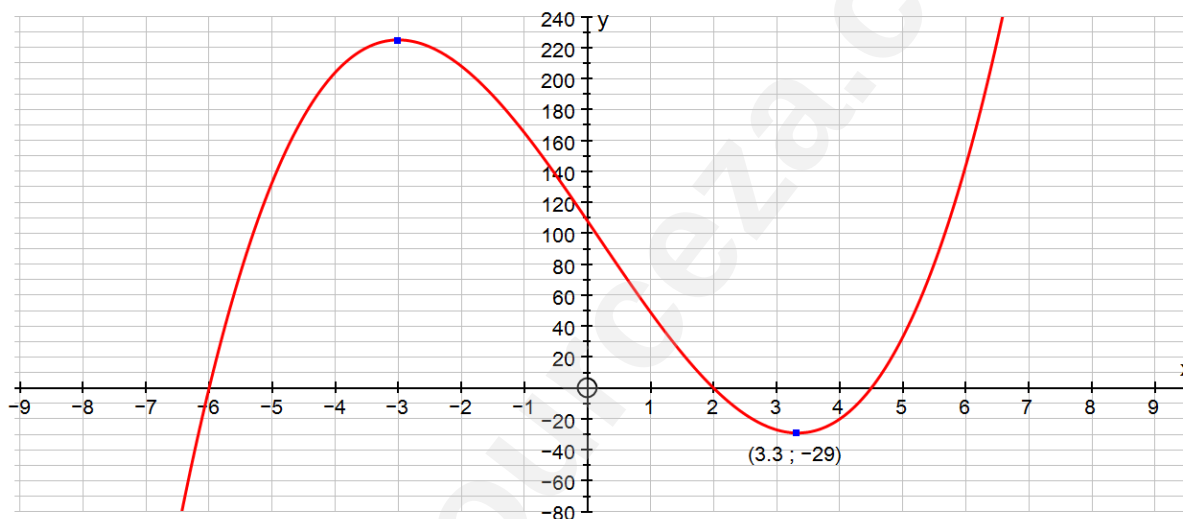
$$-13 = 4a + 2b - 1 \quad \text{or} \quad -12 = 4a + 2b \quad (1)$$

since it is a stationary point, $\frac{dy}{dx} = 2ax + b = 0$ when $x = 2$

$$\text{so, } 4a + b = 0 \quad (2)$$

adding these equations gives $b = -12$ and $a = 3$

7. Given the function f below. Use the graph to determine for which values of x , $f'(x)f(x) \geq 0$. (3)



Must be going uphill with positive y-values or downhill with negative y-values

$$x \in [-6; -3] \quad \text{or} \quad x \in [2; 3.3] \quad \text{or} \quad x \in [4.5; \infty)$$

ALTERNATE NOTATION:

$$-6 \leq x \leq -3 \quad \text{or} \quad 2 \leq x \leq 3.3 \quad \text{or} \quad x \geq 4.5$$