



Calculus

Limits & Derivative from First Principles

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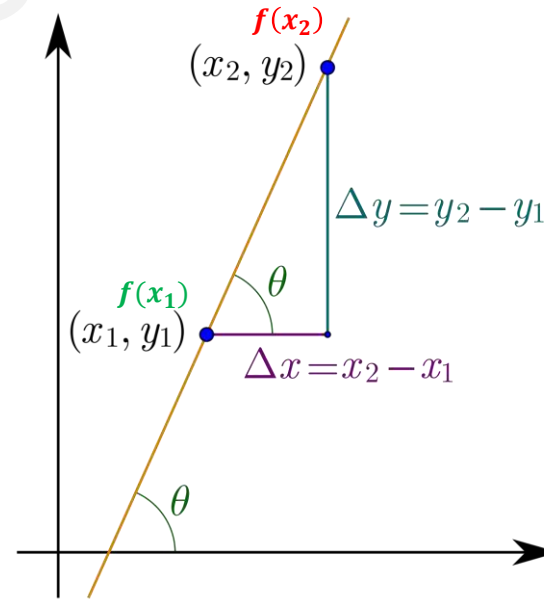
Calculus is a branch of maths that works mainly with **gradients of tangents** and **areas enclosed by curve**. An important mathematical thought that relates to these concepts are finding ways of dealing with **quantities approaching the infinitely small or infinitely large**. Calculus was originally developed to solve problems in Physics, mechanics in particular. Later it developed to solve many other problems that limited modern day advancements.

Average Gradient between 2 points

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

We can write this average gradient as:

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



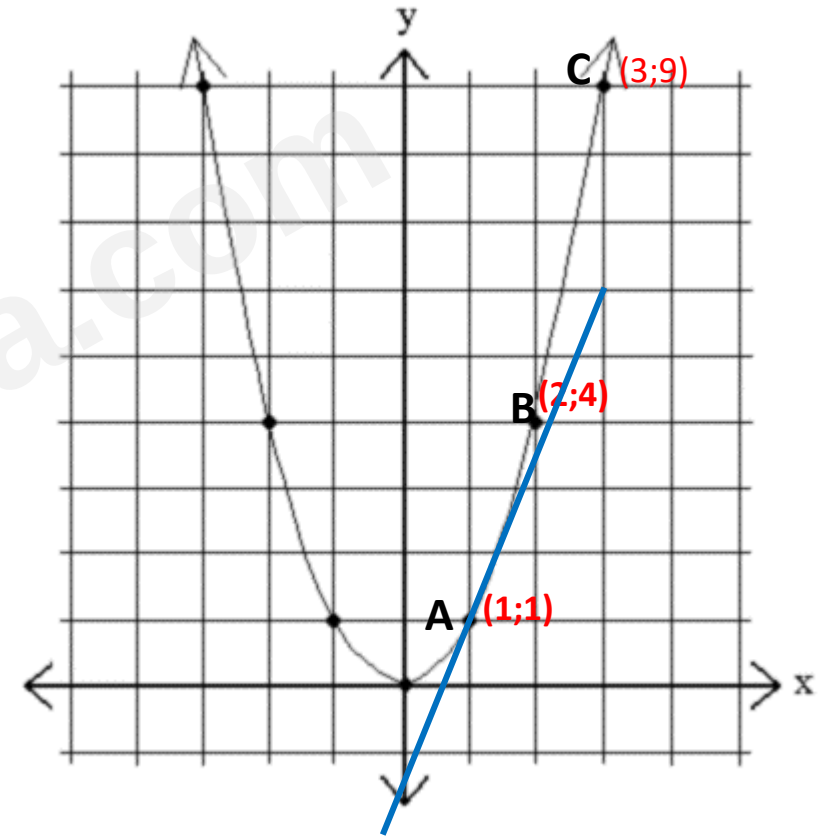
Gradient on a Curve

Lets consider $f(x) = x^2$

$$m_{AB} = \frac{4-1}{2-1} = 3$$

$$m_{AC} = \frac{9-1}{3-1} = 4$$

We can see that this average gradient value doesn't stay constant on a curve, it changes continuously. This is where we prefer working with the **gradient of a tangent at a point**.



So, we use the average gradient concept to help us to get the gradient at a particular point on the curve, i.e. the gradient of the tangent at that point.

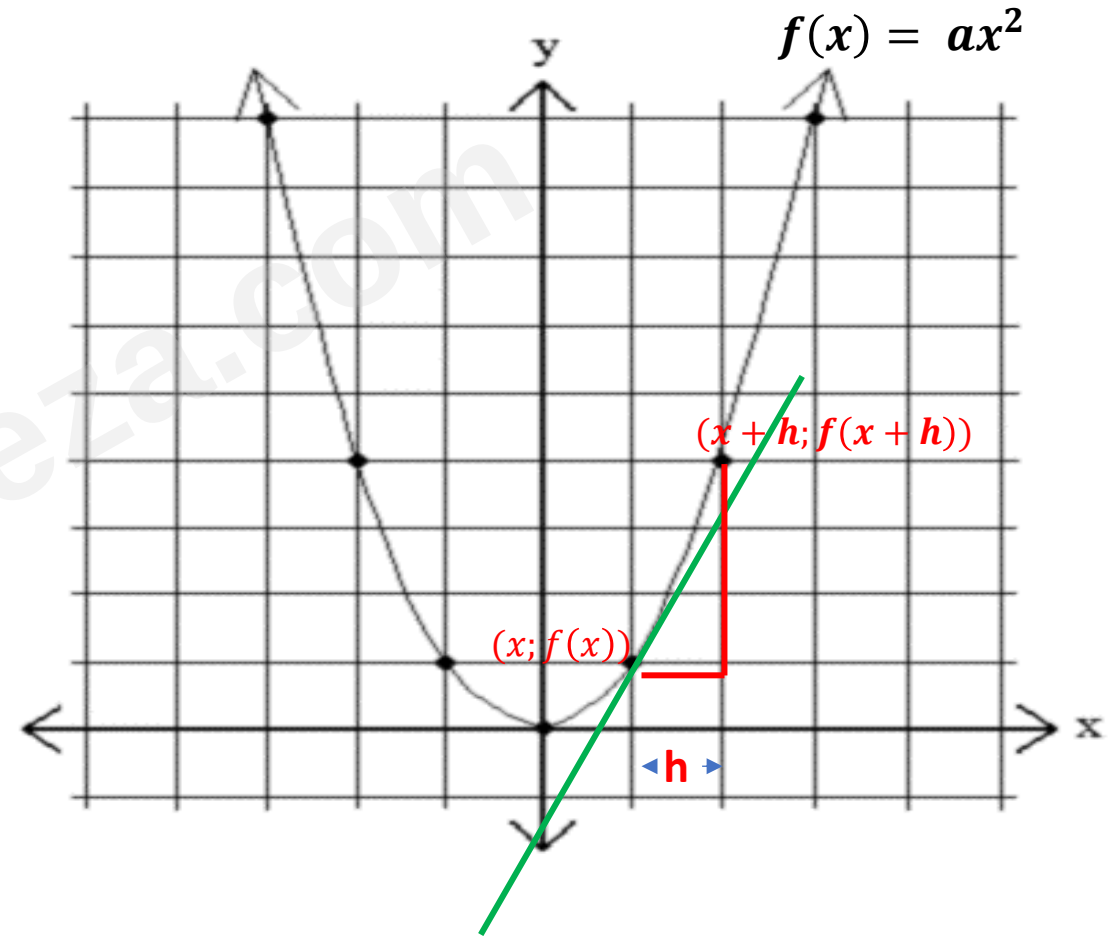
$$\text{Average gradient} = \frac{f(x+h) - f(x)}{x+h - x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

To get the gradient at the point (of the tangent):

$$\text{Gradient at a point} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$



There are a number of different notations for this gradient at a point : $f'(x)$, $\frac{dy}{dx}$, $D_x [\dots]$, $\frac{d}{dr}$

So the **DERIVATIVE** is the gradient/change of a tangent at a particular/specific point.

We can find this gradient by this formula:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

This is referred to as **DERIVATIVE FROM FIRST PRINCIPLES** or by **DEFINITION**

So how do we use this:

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Example 1: (Lets first see how to use limits)

Determine the $\lim_{x \rightarrow 0} (x^2 - 2x - 1)$

This is a simple substitution : $\lim_{x \rightarrow 0} (x^2 - 2x - 1) = (0)^2 - 2(0) - 1$
 $= -1$

Example 2: Determine the average gradient of $f(x) = x^2 - 2x + 2$, between $x = 2$ & $x = 4$

$$\begin{aligned} \text{when } x=2 : f(2) &= (2)^2 - 2(2) + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{when } x = 4 : f(4) &= (4)^2 - 2(4) + 2 \\ &= 10 \end{aligned}$$

$$\text{Average gradient} = \frac{f(4) - f(2)}{4 - 2}$$

$$= \frac{10 - 2}{2}$$

$$= 4$$

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Example 3: Determine the derivative from first principles of $f(x) = x^2 + 1$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} f(x+h) &= (x+h)^2 + 1 \\ &= x^2 + 2xh + h^2 + 1 \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 1 - (x^2 + 1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

→ we always take out only h as a common factor

$$= \lim_{h \rightarrow 0} 2x + h$$

$$= 2x + 0$$

$$= 2x$$

This gives us a general equation to find gradients on the function $f(x) = x^2 + 1$

Example 4: Given $g(x) = -x^2 + 3x - 2$ Downloaded from eduresourceza.com

- a) Determine $g'(x)$ from first principles
- b) Determine the gradient of the tangent to g at $x = -2$
- c) At which value of x is the gradient equal to -3 .

$$\begin{aligned} \text{(a)} \quad g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x^2 - 2xh - h^2 + 3x + 3h - 2 - (-x^2 + 3x - 2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2xh - h^2 + 3h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(-2x - h + 3)}{h} \\ &= \lim_{h \rightarrow 0} -2x - h + 3 \\ &= -2x - 0 + 3 \\ &= -2x + 3 \end{aligned}$$

$$\begin{aligned} g(x+h) &= -(x+h)^2 + 3(x+h) - 2 \\ &= -(x^2 + 2xh + h^2) + 3x + 3h - 2 \\ &= -x^2 - 2xh - h^2 + 3x + 3h - 2 \end{aligned}$$

(b) $g'(x) = -2x + 3$

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$$g'(-2) = -2(-2) + 3$$

$$= 7$$

Note: this is a positive gradient

(c) Gradient = -3 means $g'(x) = -3$

$$g'(x) = -2x + 3$$

$$-3 = -2x + 3$$

$$-6 = -2x$$

$$3 = x$$

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HOMEWORK:

Exercise 2 page 91
(a) 1-6

I Like Pushing
Things to the Limits

$$\frac{d}{dx} f(x) = \lim_{\Delta \rightarrow 0} \frac{f(x + \Delta) - f(x)}{\Delta}$$

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**Q: Why was the parent
function upset
with its child?**

**A: It was stretched
to its limit.**



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