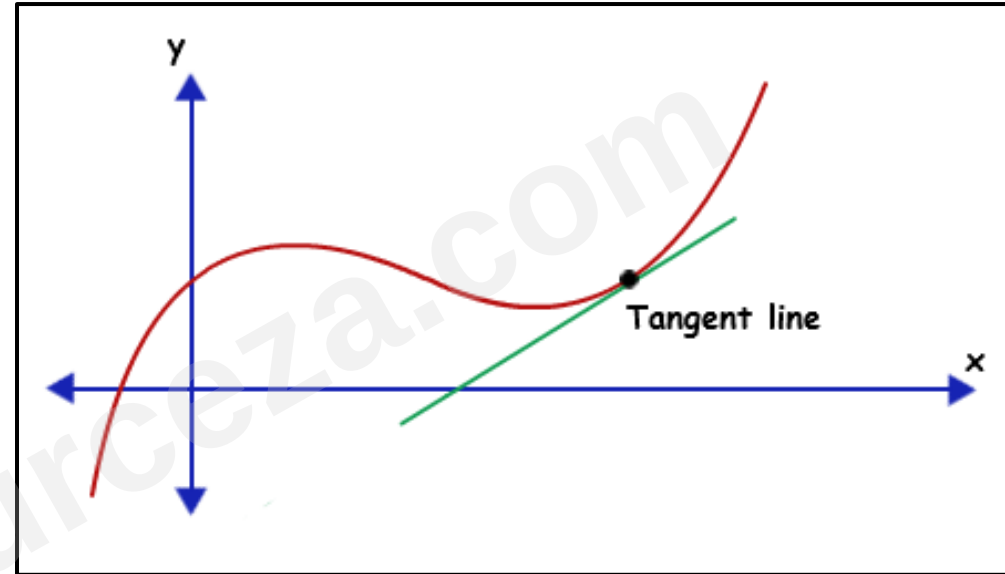
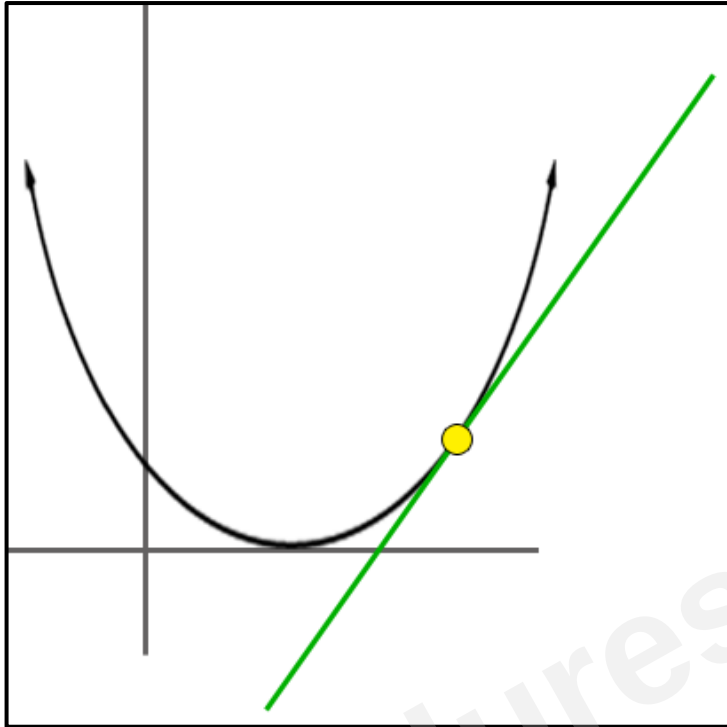




CALCULUS

Equation of Tangents and related applications

Remember: the **DERIVATIVE** is the **GRADIENT** at a particular point.



- ❖ The equation of any line is given by $y - y_1 = m(x - x_1)$
- ❖ m is the gradient of the line which in the case of a tangent is found by finding **the derivative** of the function **at the point** at which the tangent and the function meet.
- ❖ $(x_1; y_1)$ is the coordinates of a point on the tangent.

Example 1 : Determine the equation of the tangent to the graph of $f(x) = x^2 - x$ at $x = 3$

- Calculate the **y – value** , by using the **original** function **f(x)**.

$$\begin{aligned} f(x) &= x^2 - x \\ \therefore y = f(3) &= (3)^2 - (3) \\ &= 6 \end{aligned}$$

So, the point of contact between the tangent and f is **(3; 6)**

- Determine the **gradient** of the tangent at $x = 3$. We use the derivative **f'(x)**

$$\begin{aligned} f'(x) &= 2x - 1 \\ \mathbf{m} = \mathbf{f'(3)} &= 2(3) - 1 \\ &= 5 \end{aligned}$$

- Determine the **equation** of the tangent using **y – y₁ = m(x – x₁)**

$$\begin{aligned} \mathbf{y - 6} &= \mathbf{5(x - 3)} \\ y - 6 &= 5x - 15 \\ y &= 5x - 9 \end{aligned}$$

Example 2: FINDING THE POINT OF CONTACT OR POINT OF TANGENCY

The graph of $f(x) = x - 2x^2$ has a tangent with a gradient of 9.

- Calculate the coordinates of the point of contact between the tangent and the graph.
- Determine the equation of the tangent.

(a) Gradient = 9 $\longrightarrow f'(x) = 9$

$$f(x) = x - 2x^2$$

$$f'(x) = 1 - 4x$$

$$9 = 1 - 4x$$

$$8 = -4x$$

$$-2 = x$$

to find the matching y, substitute into f(x): $f(-2) = (-2) - 2(-2)^2$
 $= -10$

$\therefore$ the point of contact is $(-2; -10)$

(b) $m = 9$ and point is $(-2; -10)$ $\longrightarrow$

$$y - y_1 = m(x - x_1)$$

$$y - (-10) = 9(x - (-2))$$

$$y + 10 = 9x + 18$$

$$y = 9x + 8$$

Example 3: Determine the equation of the tangent to the graph of $f(x) = -2x^2 + 5$, parallel to the line $y = 8x + 7$.

Parallel lines have equal gradients: $f'(x) = 8$

$$-4x = 8$$

$$x = -2$$

→ this gives us the x value of the point of contact

To find the coordinates use $f(x)$: $f(-2) = -2(-2)^2 + 5$
 $= -3$ → $(-2; 3)$

Remember we know the gradient: $m = 8$

$$y + 3 = 8(x - (-2))$$

$$y + 3 = 8x + 16$$

$$y = 8x + 13$$

Example 4: A line with gradient of -3 is a tangent to the graph of $y = ax^2 + bx + 3$ at the point $(2; 5)$. Determine the values of a and b .

Gradient of -3

$$\frac{dy}{dx} = -3$$

at $x = 2$

$$2ax + b = -3$$

$$2a(2) + b = -3$$

$$4a + b = -3$$

$$b = -3 - 4a$$

$(2; 5)$ satisfies $f(x)$

$$y = ax^2 + bx + 3$$

$$5 = a(2)^2 + b(2) + 3$$

$$5 = 4a + 2b + 3$$

$$2 = 4a + 2b$$

$$1 = 2a + b$$

$$1 - 2a = b$$

$$-3 - 4a = 1 - 2a$$

$$-2a = 4$$

$$a = -2$$

$$\begin{aligned}\therefore b &= 1 - 2(-2) \\ &= 5\end{aligned}$$



HOMEWORK:

See Exercise
uploaded onto Google
Classroom.

