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Calculus

Calculus – The derivative function

In this lesson we find an increasingly better approximation of the gradient of the tangent to the graph $y = x^2$ at the point $(1;1)$. We then find a general solution to the problem of finding the gradient of a tangent at a point on a curve.



Lesson Outcomes

By the end of this lesson, you should be able to:

- predict the gradient of the tangent at any point on the function of at least the following functions: $y = x^2$ and $y = x^3$



Curriculum Links

LO 2: Functions and Algebra

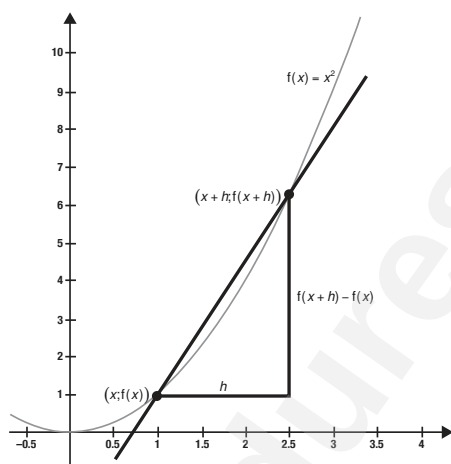
- 12.2.7 (a) Establishing the derivatives of functions from first principles
(c) Determine the equations of the tangents to graphs.



Lesson Notes

We can find an approximation of the gradient of the tangent to a graph at any point on the graph algebraically using the same approach that as we used in the previous lesson where we determined the approximation of the gradient numerically.

Use the diagram below to find the gradient of the tangent to any point on the curve $f(x) = x^2$:



If x -coordinate of the point where we want to find the gradient of the tangent equals x then its y -coordinate will be $f(x)$ and the point will be $(x; f(x))$. If the other point used to determine the gradient is a horizontal distance of h from the point $(x; f(x))$ then the other point will have the coordinates $(x+h; f(x+h))$. The horizontal change between the two points will be h and the vertical change will be $f(x+h) - f(x)$. The gradient for the line between the two points for the function $y = x^2$ will be:

$$\begin{aligned} \text{gradient} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - x^2}{h} \\ &= \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \frac{2xh + h^2}{h} \\ &= \frac{h(2x + h)}{h} \\ &= 2x + h \end{aligned}$$

As h gets closer and closer to zero so the gradient of the tangent will become closer and closer to $2x$.

We can repeat the same process for the function $y = x^3$ and find that the gradient of the tangent is $3x^2$.



Task

Use the formula for the gradient below to determine the gradient in terms of h for any two points on the functions:

- $f(x) = x^2$;
- $f(x) = x^3$;
- $f(x) = x$;
- $f(x) = a$; and
- $f(x) = x^2 + x$

$$\text{Gradient} = f(x+h) - f(x)h$$