



Calculus

Derivatives using Rules

So we have done many derivatives already, but some long steps to get to a general form of the gradient of a curve. Let us investigate the relationship and see if we can discover an easier way to do derivatives.

$$f(x) = 1x^2 + 1$$

$$f'(x) = 2x$$

$$f(x) = -x^2 + 3x - 2$$

$$\longrightarrow f'(x) = -2x + 3$$

$$g(x) = 1x^3$$

$$g'(x) = 3x^2$$

$$h(x) = \frac{1}{x} = x^{-1}$$

$$h'(x) = -1x^{-2} = -\frac{1}{x^2}$$

$$p(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$p'(x) = \frac{1}{2} x^{\frac{-1}{2}}$$

$$f(x) = ax^n \longrightarrow f'(x) = anx^{n-1}$$

Example 1: Determine the following

(a) $D_x[2x^3] = 2 \times 3 x^{3-1}$
 $= 6x^2$

(b) $f'(x)$ if $f(x) = 3x^2$
 $f'(x) = 6x$

(c) $\frac{dy}{dx}$ if $y = x^4$
 $\frac{dy}{dx} = 4x^3$

(d) $D_x[5x] = 5$

(e) $\frac{d}{dx}[12] = 0$

(f) $g'(x)$ if $g(x) = \frac{3x^5}{5}$
 $g(x) = \frac{3}{5} x^5$
 $g'(x) = 3x^4$

$$\frac{3}{5} \times 5 = 3$$

If the **variables** are in the **denominator** of within a root sign, we use the **LAWS OF EXPONENTS** to help us find the derivative.

Example 2: Determine $f'(x)$

$$(a) \quad f(x) = \frac{1}{x^2} = x^{-2} \quad f'(x) = -2x^{-3} = -\frac{2}{x^3}$$

$$(b) \quad f(x) = \frac{3}{x^4} = 3x^{-4} \quad f'(x) = -12x^{-5} = -\frac{12}{x^5}$$

$$(c) \quad f(x) = \frac{1}{2x^3} = \frac{1}{2} x^{-3} \quad f'(x) = -\frac{3}{2} x^{-4} = -\frac{3}{2x^4}$$

$$(d) \quad f(x) = \frac{2x^6}{x^3} = 2x^3 \quad f'(x) = 6x^2$$

Exponent Rule	Examples
$x^a \cdot x^b = x^{a+b}$	$c^3 \cdot c^5 = c^8$ $3^5 \cdot 3^8 = 3^{13}$ $5(5^n) = 5^1(5^n) = 5^{n+1}$
$a^x \cdot b^x = (ab)^x$	$2^4 \cdot 3^4 = 6^4$ $12^5 = 2^{10} \cdot 3^5$
$\frac{x^a}{x^b} = x^{(a-b)}$	$\frac{2^5}{2^{11}} = \frac{1}{2^6} = 2^{-6}$ $\frac{x^{10}}{x^3} = x^7$
$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$\left(\frac{10}{2}\right)^6 = \frac{10^6}{2^6} = 5^6$ $\frac{3^5}{9^5} = \left(\frac{3}{9}\right)^5 = \left(\frac{1}{3}\right)^5$
$(a^x)^y = a^{xy} = (a^y)^x$	$(3^2)^4 = 3^{2 \cdot 4} = 3^8 = 3^{4 \cdot 2} = (3^4)^2$
$x^{-a} = \frac{1}{x^a}$	$\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ $2x^{-4} = \frac{2}{x^4}$
$x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$	$27^{4/3} = \sqrt[3]{27^4} = (\sqrt[3]{27})^4 = 3^4 = 81$ $\sqrt[5]{x^{15}} = x^{15/5} = x^3$
$a^x + a^x + a^x = 3a^x$	$3^4 + 3^4 + 3^4 = 3 \cdot 3^4 = 3^5$ $3^x + 3^x + 3^x = 3 \cdot 3^x = 3^{x+1}$

$$(e) f(x) = \frac{x^2}{3x^5} = \frac{1}{3} x^{-3}$$

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$$\begin{aligned} f'(x) &= -1x^{-4} \\ &= \frac{-1}{x^4} \end{aligned}$$

$$(f) f(x) = \frac{5x^5}{4x^{11}} = \frac{5}{4} x^{-6}$$

$$\begin{aligned} f'(x) &= -\frac{5}{2} x^{-7} \\ &= -\frac{5}{2x^7} \end{aligned}$$

$$(g) f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\begin{aligned} f'(x) &= \frac{1}{3} x^{\frac{-2}{3}} \\ &= \frac{1}{3x^{\frac{2}{3}}} \end{aligned}$$

$$\begin{aligned} (h) f(x) &= \frac{3}{\sqrt[3]{x^4}} \\ &= \frac{3}{x^{\frac{4}{3}}} \\ &= 3x^{-\frac{4}{3}} \end{aligned}$$

$$\begin{aligned} f'(x) &= -4x^{-\frac{7}{3}} \\ &= \frac{-4}{x^{\frac{7}{3}}} \end{aligned}$$

Always
Write final
answer as a
**POSITIVE
EXPONENT**

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Example 3 : Determine $\frac{dy}{dx}$ if

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(a) $y = 3x^2 + 2x - 5$

$$\frac{dy}{dx} = 6x + 2$$

(b) $y = x^3 - \frac{5}{x^2} + \frac{2}{3\sqrt[3]{x}}$

$$= x^3 - 5x^{-2} + \frac{2}{3x^{\frac{1}{3}}}$$

$$= x^3 - 5x^{-2} + \frac{2x^{\frac{-1}{3}}}{3}$$

$$\frac{dy}{dx} = 3x^2 + 10x^{-3} - \frac{2}{9}x^{\frac{-4}{3}}$$

$$= 3x^2 + \frac{10}{x^3} - \frac{2}{9x^{\frac{4}{3}}}$$

(c) $y = \pi^2x + \pi^3 - \sqrt{5}$

$$\begin{aligned}\frac{dy}{dx} &= \pi^2 + 0 + 0 \\ &= \pi^2\end{aligned}$$

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HOMEWORK:

Exercise 3 page 94
(a) & (b)

