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EXAM PREPARATION PAPER 2 (D)

Learner Note: In this session you will be given the opportunity to work on a past examination paper (Feb/Mar 2010 DoE Paper 2). The paper consists of 12 questions. Question 10, 11 and 12 will be dealt with in this session.

Structure of Paper 2

TOPIC	APPROXIMATE MARK ALLOCATION
Coordinate Geometry	40
Transformations	25
Trigonometry	60
Data Handling	25
TOTAL	150

Before writing the maths examination, it is essential for you to have at least two blue pens, a pencil, a ruler and a workable scientific calculator (non-programmable and non-graphical). You will be given time to read through the paper prior to starting. It is essential that you read the instructions carefully and ensure that the paper is complete.

Don't waste time on a specific question. If you cannot do that question, rather move on to the next question. Remember to number the questions correctly. You might wish to start with Question 3. This is fine, provided that the question is clearly indicated.

SECTION A: TYPICAL EXAM QUESTIONS (QUESTION 10, 11 AND 12)**Important information regarding Question 10, 11 and 12****Question 10**

This question tests trigonometry. Learners must have mastered the basics before attempting this question.

Question 11

Trig graphs are tested in this question. The four main transformations must be known and learners must be aware that at most two transformations can be applied to one given function.

Question 12

Make sure learners know when to use the sine, cosine and areas rules.

- The **sine-rule** can be used when the following is known in the triangle:
 - more than 1 angle and a side
 - 2 sides and an angle (not included)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- The **cosine-rule** can be used when the following is known of the triangle:
 - 3 sides
 - 2 sides and an included angle

$$a^2 = b^2 + c^2 - 2bc \cos A$$

- The **area** of any triangle can be found when at least two sides and an included angle are known

$$\text{Area of } \triangle ABC = \frac{1}{2} ab \sin C$$

QUESTION 10

10.1 If $\sin 36^\circ \cos 12^\circ = p$ and $\cos 36^\circ \sin 12^\circ = q$, determine in terms of p and q the value of:

10.1.1 $\sin 48^\circ$ (3)

10.1.2 $\sin 24^\circ$ (3)

10.1.3 $\cos 24^\circ$ (3)

10.2 Show that: $\sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ = \frac{3}{2}$ (7)

10.3 10.3.1 Prove: $\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x} = 1 - \cos x$ (4)

10.3.2 For which values of x is $\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x} = 1 - \cos x$ not true? (2)

QUESTION 11

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

11.1 Calculate the points of intersection of the graphs for $x \in [180^\circ; 360^\circ]$ (7)

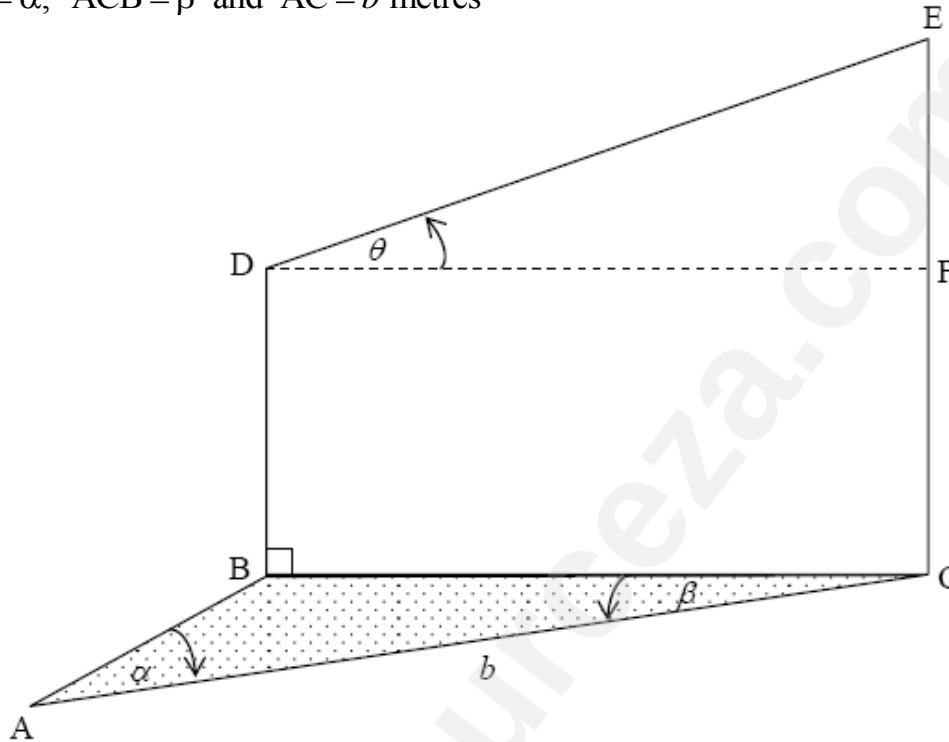
11.2 Draw sketch graphs of f and g for $x \in [180^\circ; 360^\circ]$ on the same set of axes. (4)

11.3 For which values of x will $f(x) \leq g(x)$ for $x \in [180^\circ; 360^\circ]$ (3)

QUESTION 12

In the diagram below A, B and C are three points in the same horizontal plane. D is vertically above B and E is vertically above C. The angle of elevation of E from D is θ . F is a point on EC such that $DF \parallel BC$.

$\hat{BAC} = \alpha$, $\hat{ACB} = \beta$ and $AC = b$ metres



12.1 Prove that: $DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$ (6)

12.2 Calculate DE if $b = 2000$ metres, $\alpha = 43^\circ$ and $\theta = 27^\circ$ (3)

SECTION B: SOLUTIONS AND HINTS TO SESSION 40

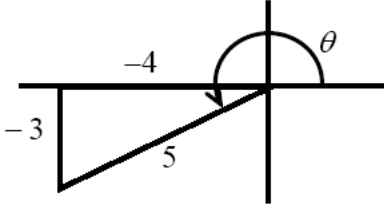
QUESTION 7

7.1.1	$P'(5; -2)$	✓ answer (1)
7.1.2	$P'(5; 2)$	✓ x-coordinate ✓ y-coordinate (2)
7.2.1	Reduction by a scale factor of $\frac{1}{2}$: $(x; y) \rightarrow \left(\frac{1}{2}x; \frac{1}{2}y\right)$ Reflection about the line $y = x$ $\left(\frac{1}{2}x; \frac{1}{2}y\right) \rightarrow \left(\frac{1}{2}y; \frac{1}{2}x\right)$ $\therefore (x; y) \rightarrow \left(\frac{1}{2}y; \frac{1}{2}x\right)$	✓✓ reduction ✓✓ reflection (4)
7.2.2	If the first transformation is the reflection, then: $H'(8; 16)$ If the first transformation is the reduction, then: $H'(8; 4)$	✓ $H'(8; 16)$ ✓ $H'(8; 4)$ (2)
7.2.3	$\frac{\text{Area of original}}{\text{Area of image}} = \frac{1}{k^2}$ $\therefore \frac{\text{Area KUHLE}}{\text{Area } K''U''H''L''E''} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$ $\therefore \text{Area KUHLE} : \text{Area } K''U''H''L''E'' = 4 : 1$	✓✓ answer (2)

QUESTION 8

8.1	$A'(x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$ $x' = x \cos \theta - y \sin \theta$ $\therefore x' = 3 \cos 120^\circ - 2 \sin 120^\circ$ $\therefore x' = 3(-\cos 60^\circ) - 2 \sin 60^\circ$ $\therefore x' = -3\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$ $\therefore x' = \frac{-3 - 2\sqrt{3}}{2}$ $y' = y \cos \theta + x \sin \theta$ $\therefore y' = 2 \cos 120^\circ + 3 \sin 120^\circ$ $\therefore y' = 2(-\cos 60^\circ) + 3 \sin 60^\circ$ $\therefore y' = -2\left(\frac{1}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right)$ $\therefore y' = \frac{-2 + 3\sqrt{3}}{2}$	✓ formula ✓ simplification ✓ substitution ✓ answer ✓ simplification ✓ answer (6)
8.2	$Q(x; y) \rightarrow Q'(-2; 0)$ $\therefore Q(x; y) \rightarrow Q'(x \cos 120^\circ - y \sin 120^\circ; y \cos 120^\circ + x \sin 120^\circ)$ $-2 = x \cos 120^\circ - y \sin 120^\circ \text{ and } 0 = y \cos 120^\circ + x \sin 120^\circ$ $-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right) \text{ and } 0 = y\left(-\frac{1}{2}\right) + x\left(\frac{\sqrt{3}}{2}\right)$ $\therefore -4 = -x - \sqrt{3}y \text{ and } 0 = -y + \sqrt{3}x$ $\therefore 4 = x + \sqrt{3}y \text{ and } y = \sqrt{3}x$ $\therefore 4 = x + \sqrt{3}(\sqrt{3}x)$ $\therefore 4 = x + 3x$ $\therefore 4 = 4x$ $\therefore x = 1$ $\therefore y = \sqrt{3}(1) = \sqrt{3}$ $\therefore Q(1; \sqrt{3})$	✓ $-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ ✓ $0 = y\left(-\frac{1}{2}\right) + x\left(\frac{\sqrt{3}}{2}\right)$ ✓ x-coordinate ✓ y-coordinate (4)

QUESTION 9

9.1.1	$\sin \theta = -\frac{3}{5}$ and $\cos \theta = -\frac{4}{5}$ $\therefore \sin \theta + \cos \theta = -\frac{7}{5}$ 	✓ correct quad and values ✓ $\sin \theta = -\frac{3}{5}$ ✓ $\cos \theta = -\frac{4}{5}$ ✓ answer (4)
9.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ $= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2} = \frac{24}{7}$	✓ $\frac{\sin 2\theta}{\cos 2\theta}$ ✓ $2 \sin \theta \cos \theta$ ✓ $\cos^2 \theta - \sin^2 \theta$ ✓ substitution ✓ answer (5)
9.2.1	$\frac{\cos(360^\circ - x) \tan^2 x}{\sin(x - 180^\circ) \cos(90^\circ + x)}$ $= \frac{(\cos x) \tan^2 x}{\sin[-(180^\circ - x)](-\sin x)}$ $= \frac{(\cos x) \tan^2 x}{-\sin(180^\circ - x)(-\sin x)}$ $= \frac{(\cos x) \left(\frac{\sin^2 x}{\cos^2 x}\right)}{(-\sin x)(-\sin x)}$ $= \frac{\sin^2 x}{\cos x \sin^2 x}$ $= \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin^2 x}$ $= \frac{1}{\cos x}$	✓ $(\cos x)$ ✓ $(-\sin x)$ ✓ $(-\sin x)$ ✓ $\frac{\sin^2 x}{\cos^2 x}$ ✓ answer (5)
9.2.2	Let $x = 30^\circ$ $\therefore \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$	✓ $x = 30^\circ$ ✓ answer (2)

SECTION B: SOLUTIONS AND HINTS TO SESSION 41

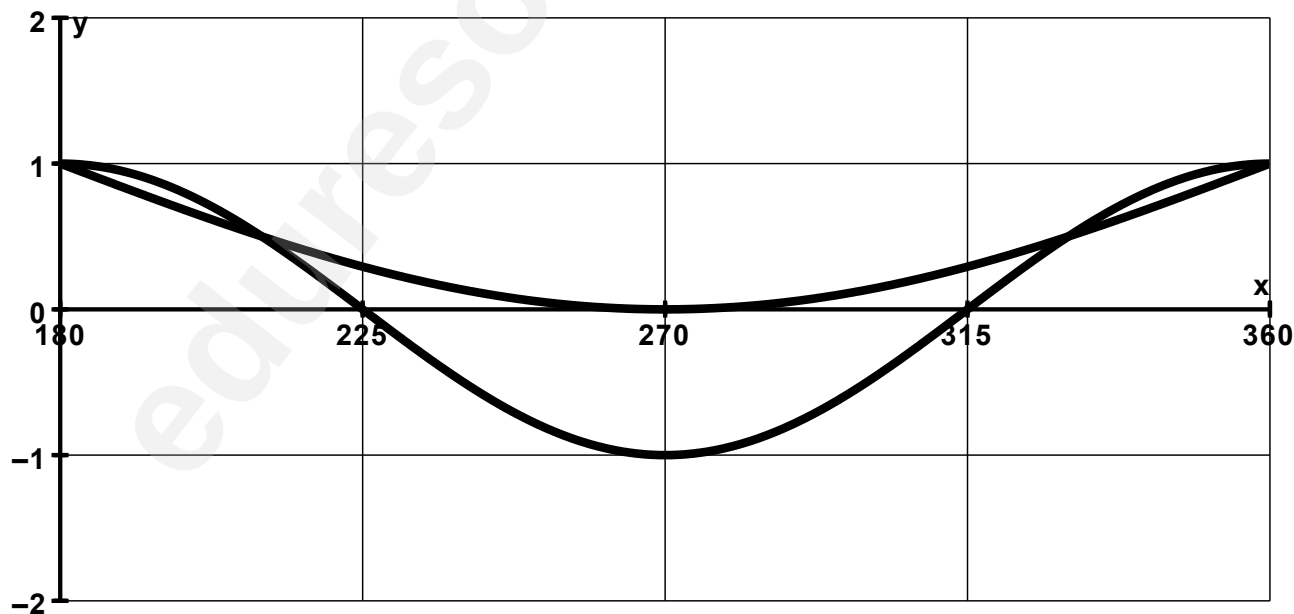
10.1.1	$\sin 48^\circ = \sin(36^\circ + 12^\circ)$ $= \sin 36^\circ \cos 12^\circ + \cos 36^\circ \sin 12^\circ$ $= p + q$	✓ $36^\circ + 12^\circ$ ✓ expansion ✓ answer (3)
10.1.2	$\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$	✓ $36^\circ - 12^\circ$ ✓ expansion ✓ answer (3)
10.1.3	$\sin 48^\circ = 2 \sin 24^\circ \cos 24^\circ$ $\therefore p + q = 2(p - q) \cos 24^\circ$ $\therefore \frac{p + q}{2p - 2q}$ OR $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ $\therefore \cos 24^\circ = \sqrt{1 - \sin^2 24^\circ}$ $\therefore \cos 24^\circ = \sqrt{1 - (p - q)^2}$	✓ identity ✓ substitution ✓ answer (3)
10.2	$\sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ$ $= \sin^2 20^\circ + \sin^2(60^\circ - 20^\circ) + \sin^2(60^\circ + 20^\circ)$ $= \sin^2 20^\circ + [\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ]^2$ $+ [\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ]^2$ $= \sin^2 20^\circ + \left[\left(\frac{\sqrt{3}}{2} \right) \cos 20^\circ - \left(\frac{1}{2} \right) \sin 20^\circ \right]^2$ $+ \left[\left(\frac{\sqrt{3}}{2} \right) \cos 20^\circ + \left(\frac{1}{2} \right) \sin 20^\circ \right]^2$ $= \sin^2 20^\circ + \left[\frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{2} \right]^2$ $+ \left[\frac{\sqrt{3} \cos 20^\circ + \sin 20^\circ}{2} \right]^2$	✓ $\sin^2(60^\circ - 20^\circ)$ ✓ $\sin^2(60^\circ + 20^\circ)$ ✓✓ expansions ✓ substitution ✓ simplification ✓ answer (7)

	$= \sin^2 20^\circ + \frac{3 \cos^2 20^\circ - 2\sqrt{3} \cos 20^\circ \sin 20^\circ + \sin^2 20^\circ}{4}$ $+ \frac{3 \cos^2 20^\circ + 2\sqrt{3} \cos 20^\circ \sin 20^\circ + \sin^2 20^\circ}{4}$ $= \sin^2 20^\circ + \frac{6 \cos^2 20^\circ + 2 \sin^2 20^\circ}{4}$ $= \frac{4 \sin^2 20^\circ + 6 \cos^2 20^\circ + 2 \sin^2 20^\circ}{4}$ $= \frac{6(\sin^2 20^\circ + \cos^2 20^\circ)}{4}$ $= \frac{6(1)}{4} = \frac{3}{2}$	
10.3.1	$\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x}$ $= \frac{\sin^2 x (\sin^2 x + \cos^2 x)}{1 + \cos x}$ $= \frac{(\sin^2 x)(1)}{1 + \cos x}$ $= \frac{1 - \cos^2 x}{1 + \cos x}$ $= \frac{(1 + \cos x)(1 - \cos x)}{1 + \cos x}$ $= 1 - \cos x$	✓ factorisation ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $1 - \cos^2 x$ ✓ $(1 + \cos x)(1 - \cos x)$ (4)
10.3.2	$1 + \cos x = 0$ $\therefore \cos x = -1$ $\therefore x = \pm 180^\circ + k.360^\circ$ where $k \in \mathbb{Z}$	✓ $1 + \cos x = 0$ ✓ $x = \pm 180^\circ + k.360^\circ$ (2)

QUESTION 11

11.1	$1 + \sin x = \cos 2x$ $\therefore 1 + \sin x = 1 - 2\sin^2 x$ $\therefore \sin x + 2\sin^2 x = 0$ $\therefore \sin x(1 + 2\sin x) = 0$ $\therefore \sin x = 0 \quad \text{or} \quad \sin x = -\frac{1}{2}$ For $\sin x = 0$: $x = 0^\circ + k360^\circ$ or $x = 180^\circ + k360^\circ$ For $\sin x = -\frac{1}{2}$: $x = -30^\circ + k360^\circ$ or $x = 210^\circ + k360^\circ$ $\therefore x \in \{180^\circ; 210^\circ; 330^\circ; 360^\circ\}$	$\checkmark \quad 1 - 2\sin^2 x$ $\checkmark \quad \sin x(1 + 2\sin x) = 0$ $\checkmark \quad \text{two equations}$ $\checkmark\checkmark \quad \text{general solutions}$ $\checkmark\checkmark \quad \text{answers}$ (7)
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11.2



11.2	see diagram	For $f(x) = 1 + \sin x$ ✓ max and min values ✓ shape For $g(x) = \cos 2x$ ✓ amplitude ✓ intercepts (4)
11.3	$f(x) \leq g(x)$ $\therefore 180^\circ \leq x \leq 210^\circ$ or $330^\circ \leq x \leq 360^\circ$	✓ $180^\circ \leq x \leq 210^\circ$ ✓ $330^\circ \leq x \leq 360^\circ$ ✓ inequality signs correct (3)

QUESTION 12

12.1	$\frac{b}{\sin[180^\circ - (\alpha + \beta)]} = \frac{BC}{\sin \alpha}$ $\therefore \frac{b}{\sin(\alpha + \beta)} = \frac{BC}{\sin \alpha}$ $\therefore BC = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ But $BC = DF$ $\therefore DF = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ Now $\cos \theta = \frac{DF}{DE}$ $\therefore DE = \frac{DF}{\cos \theta}$ $\therefore DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$	✓ sine rule ✓ $180^\circ - (\alpha + \beta)$ ✓ $\sin(\alpha + \beta)$ ✓ $BC = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ ✓ $BC = DF$ ✓ manipulation (4)
12.2	$DE = \frac{2000 \sin 43^\circ}{\sin 79^\circ \cos 27^\circ}$ $\therefore DE = 1559,50m$	✓ substitution numerator ✓ substitution denominator ✓ answer (3)