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EXAM PREPARATION PAPER 1 (D)

Learner Note: In this session you will be given the opportunity to work on a past examination paper (Paper 1). The paper consists of 10 questions. In this session, Question 8, 9 and 10 will be dealt with.

Structure of Paper 1

TOPIC	APPROXIMATE MARK ALLOCATION
Number patterns and Sequences	30
Financial Mathematics	15
Functions and Graphs	35
Algebra	20
Calculus	35
Linear Programming	15
TOTAL	150

Before writing the maths examination, it is essential for you to have at least two blue pens, a pencil, a ruler and a workable scientific calculator (non-programmable and non-graphical). You will be given time to read through the paper prior to starting. It is essential that you read the instructions carefully and ensure that the paper is complete.

Do not waste time on a specific question. If you cannot do that question, rather move on to the next question. Remember to number the questions correctly. You might wish to start with Question 3. This is fine, provided that the question is clearly indicated.

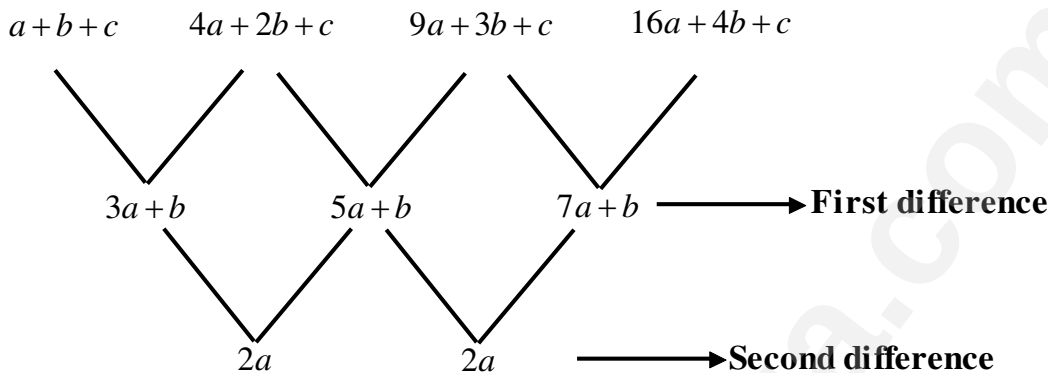
SECTION A: TYPICAL EXAM QUESTIONS (QUESTION 8, 9 AND 10)
Important information regarding Question 8, 9 and 10**Question 8**

This question tests the application of calculus to problems involving surface area and volume of three dimensional objects. It is important for you to know that the maximum or minimum area or volume is obtained by differentiating the expression for area or volume, equating it to zero and solving. The expression for area or volume takes the shape of a quadratic or cubic function which has local maximum or minimum turning points.

Question 9

This question is a problem solving question on quadratic number patterns. It is a variation on the traditional method of determining the general term of a quadratic number pattern.

Remember the following method:



The constant second difference is given by the expression $2a$.

The first term in the first difference row is given by $3a+b$.

The first term is given by $a+b+c$.

Question 10

Linear programming is an exciting part of mathematics. However, in order to master this topic, it is essential that learners know how to sketch linear functions and linear inequalities and understand the concept of gradient of a line. Working with ratios is important.

Master the following before attempting any Linear Programming question.

- (a) The sum of x and y has a minimum value of 2 and a maximum value of 8.

$$x + y \geq 2 \quad (\text{the sum of } x \text{ and } y \text{ is 2 or more})$$

$$x + y \leq 8 \quad (\text{the sum of } x \text{ and } y \text{ is 8 or less})$$

- (b) At least 2 of x and at most 3 of y .

$$x \geq 2 \quad (x \text{ can be 2 or more})$$

$$y \leq 3 \quad (y \text{ can be 3 or less})$$

- (c) The ratio of y to x must not be smaller than 3:5

$$y : x \geq 3 : 5$$

$$\therefore \frac{y}{x} \geq \frac{3}{5}$$

$$\therefore 5y \geq 3x$$

(d) At least 2 of x to 1 of y .

$$x : y \geq 2 : 1$$

$$\therefore \frac{x}{y} \geq \frac{2}{1}$$

$$\therefore x \geq 2y$$

(e) x may not exceed more than 3 times that of y .

$$x : y \leq 3 : 1$$

$$\therefore \frac{x}{y} \leq \frac{3}{1}$$

$$\therefore x \leq 3y$$

(f) y must be at most half that of x .

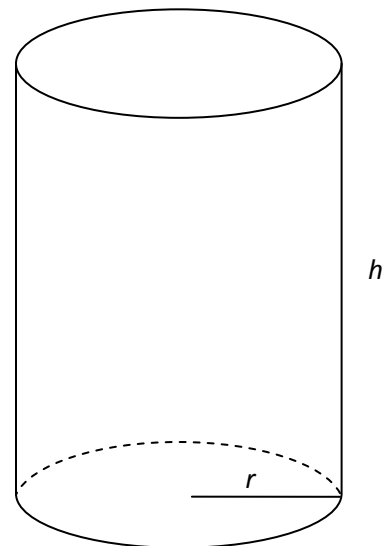
$$y : x \leq 1 : 2$$

$$\therefore \frac{y}{x} \leq \frac{1}{2}$$

$$\therefore y \leq \frac{1}{2}x$$

QUESTION 8

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.



8.1 Show that the height of the glass, h , can be expressed as $h = \frac{200}{\pi r^2}$. (2)

8.2 Show that the surface area of the glass can be expressed as $S(r) = \pi r^2 + \frac{400}{r}$. (2)

8.3 Hence determine the value of r for which the total surface area of the glass is a minimum. (5)

[9]

QUESTION 9

Consider the sequence: $3; a; 10; b; 21; \dots$

The sequence has a constant second difference of 1.

- 9.1 Determine the value of a and b . (4)
- 9.2 Determine the n th term of the sequence. (5)
- 9.3 Hence, prove that the sum of any two consecutive numbers in the sequence equals a square number. (4)
- [13]**

QUESTION 10

A clothing company manufactures white shirts and grey trousers for schools.

- A minimum of 200 shirts must be manufactured daily.
- In total, not more than 600 pieces of clothing can be manufactured daily.
- It takes 50 machine minutes to manufacture a shirt and 100 machine minutes to manufacture a pair of trousers.
- There are at most 45 000 machine minutes available per day.

Let the number of white shirts manufactured in a day be x .

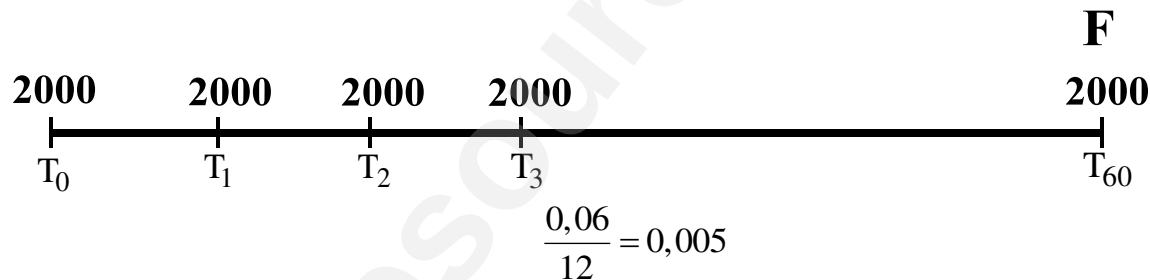
Let the number of pairs of grey trousers manufactured in a day be y .

- 10.1 Write down the constraints, in terms of x and y , to represent the above information.
(You may assume: $x \geq 0$, $y \geq 0$) (3)
- 10.2 Use the attached graph paper to represent the constraints graphically (3)
- 10.3 Clearly indicate the feasible region by shading it. (1)
- 10.4 If the profit is R30 for a shirt and R40 for a pair of trousers, write down the equation indicating the profit in terms of x and y . (2)
- 10.5 Using a search line and your graph, determine the number of shirts and pairs of trousers that will yield a maximum daily profit. (2)
- [11]**

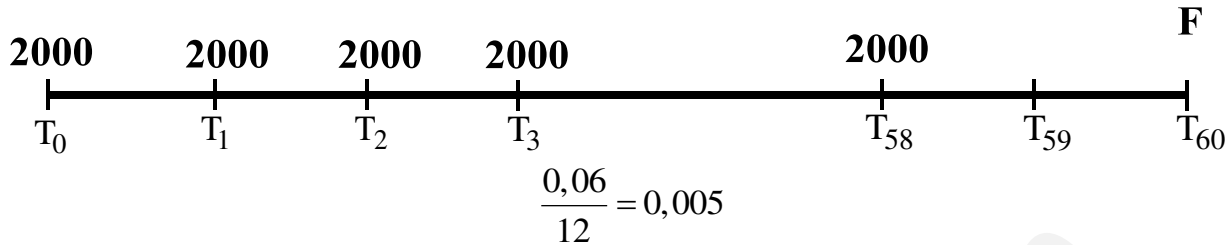
SECTION B: SOLUTIONS AND HINTS TO SESSION 36

QUESTION 5

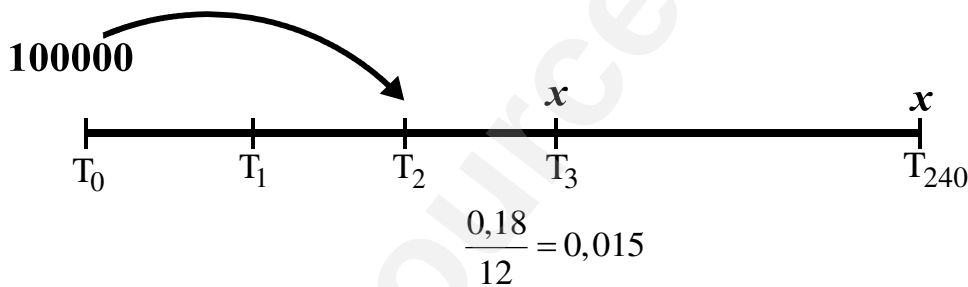
5.1	$A = P(1 - i)^n$ $\therefore 90\,000 = 200\,000(1 - 0,08)^n$ $\therefore 90\,000 = 200\,000(0,92)^n$ $\therefore \frac{90\,000}{200\,000} = (0,92)^n$ $\therefore 0,45 = (0,92)^n$ $\therefore \log_{0,92} 0,45 = n$ $\therefore n = 9,576544593$ It will take approximately 9 years 7 months to depreciate to R90 000.	✓ correct formula ✓ correct substitution ✓ use of logs ✓ value of n (4)
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5.2.1	$F = \frac{x[(1 + i)^n - 1]}{i}$ $\therefore F = \frac{2000 \left[\left(1 + \frac{0,06}{12} \right)^{61} - 1 \right]}{\frac{0,06}{12}}$ $\therefore F = R142\,237,76$	✓ correct formula ✓ $\frac{0,06}{12}$ ✓ value of n ✓ answer (4)
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5.2.2	$F = \frac{x \left[(1+i)^n - 1 \right]}{i}$ $\therefore F = \frac{2000 \left[\left(1 + \frac{0,06}{12} \right)^{59} - 1 \right]}{\frac{0,06}{12}} \cdot \left(1 + \frac{0,06}{12} \right)^2$ $\therefore F = R138\,227,76$	✓ correct annuity formula ✓ $n = 59$ ✓ $\left(1 + \frac{0,06}{12} \right)^2$ ✓ answer (4)
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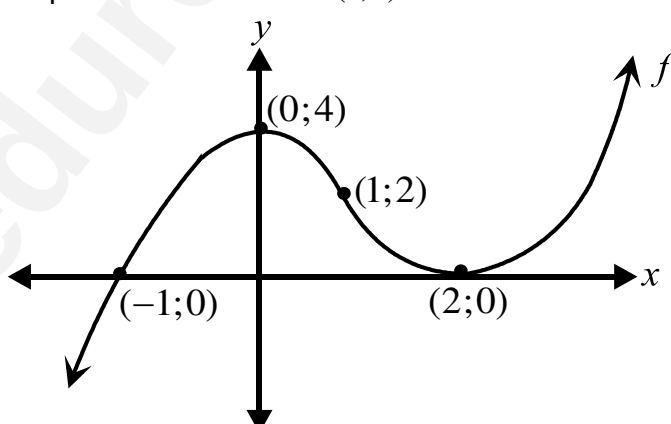
5.3.1	At T_2 : $A = 100\,000(1,015)^2$ $\therefore A = R103\,022,50$	✓ correct formula ✓ answer (2)
5.3.2	$P = \frac{x \left[1 - (1+i)^{-n} \right]}{i}$ $\therefore 103\,022,50 = \frac{x \left[1 - (1,015)^{-238} \right]}{0,015}$ $\therefore x = R1591,35$	✓ correct annuity formula ✓ $n = 238$ ✓ $P = 103\,022,50$ ✓ answer (4)

QUESTION 6

6.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{4x - 4(x+h)}{x(x+h)}}{h} \quad (\text{LCD of numerator is } x(x+h))$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{4x - 4x - 4h}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-4h}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-4h}{x(x+h)} \times \frac{1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-4}{x(x+h)}$ $\therefore f'(x) = \frac{-4}{x(x+0)}$ $\therefore f'(x) = -\frac{4}{x^2}$	$\checkmark \lim_{h \rightarrow 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h}$ $\checkmark f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-4h}{x(x+h)}}{h}$ $\checkmark f'(x) = \lim_{h \rightarrow 0} \frac{-4}{x(x+h)}$ $\checkmark f'(x) = -\frac{4}{x^2} \quad (4)$
6.2	$D_x \left[\frac{1}{\sqrt{x}} (x^3 - 2x^2 + 3x) \right]$ $= D_x \left[\frac{x^3}{x^{\frac{1}{2}}} - \frac{2x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} \right]$ $= D_x \left[x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right]$ $= \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2} x^{-\frac{1}{2}}$ $= \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2x^{\frac{1}{2}}}$	$\checkmark x^{\frac{1}{2}}$ $\checkmark D_x \left[x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right]$ $\checkmark \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2} x^{-\frac{1}{2}}$ $\checkmark \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2x^{\frac{1}{2}}} \quad (4)$

QUESTION 7

7.1	$f(x) = 3x^2 - 5x + 1$ $\therefore f'(x) = 6x - 5$ $y = 7x - 4$ $\therefore 6x - 5 = 7$ $\therefore 6x = 12$ $\therefore x = 2$ $f(2) = 3(2)^2 - 5(2) + 1$ $\therefore f(2) = 3$ $\therefore y - 3 = 7(x - 2)$ $\therefore y - 3 = 7x - 14$ $\therefore y = 7x - 11$	$\checkmark f'(x) = 6x - 5$ $\checkmark 6x - 5 = 7$ $\checkmark x = 2$ $\checkmark f(2) = 3$ $\checkmark y - 3 = 7(x - 2)$ $\checkmark y = 7x - 11$ (6)
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7.2	x-intercepts: $0 = x^3 - 3x^2 + 4$ $\therefore (x+1)(x^2 - 4x + 4) = 0$ $\therefore (x+1)(x-2)(x-2) = 0$ $\therefore x = -1 \text{ or } x = 2$ y-intercept: 4 $f'(x) = 3x^2 - 6x$ $\therefore 0 = 3x^2 - 6x$ $\therefore 0 = x^2 - 2x$ $\therefore 0 = x(x-2)$ $\therefore x = 0 \text{ or } x = 2$ For $x = 0$ $f(0) = (0)^3 - 3(0)^2 + 4 = 4$ Max turning point at $(0; 4)$ For $x = 2$ $f(2) = (2)^3 - 3(2)^2 + 4 = 0$ Min turning point at $(2; 0)$ $f'(x) = 3x^2 - 6x$ $\therefore f''(x) = 6x - 6$ $\therefore 0 = 6x - 6$ $\therefore -6x = -6$ $\therefore x = 1$ $f(1) = (1)^3 - 3(1)^2 + 4$ $f(1) = 2$ The point of inflection is $(1; 2)$ 	✓ $0 = x^3 - 3x^2 + 4$ ✓ $(x+1)(x^2 - 4x + 4) = 0$ ✓ $x = -1 \text{ or } x = 2$ ✓ $f'(x) = 3x^2 - 6x$ ✓ $0 = 3x^2 - 6x$ ✓ $x = 0 \text{ or } x = 2$ ✓ $(0; 4)$ ✓ $(2; 0)$ ✓ $f''(x) = 6x - 6$ ✓ $x = 1$ ✓ $(1; 2)$ ✓ intercepts with the axes ✓ turning points ✓ shape ✓ point of inflection (15)
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