

IDEAL GASES

Ideal gases :

$$1 \text{ mole} = 6.02 \times 10^{23} \text{ particles} = N_A$$

Note :-

$$N = n \times N_A$$

$$^{\circ}\text{C} + 273.15 = \text{K}$$

N = No. of particles

n = moles

N_A = Avogadro's constant

ΔT in $^{\circ}\text{C}$ & K is
same (no conversion required)

$$n = \frac{\text{mass}}{M_r} \rightarrow \text{in g}$$

Some constants :

$$K = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \quad \checkmark$$

$$N_A = \text{Avogadro's number} = 6.023 \times 10^{23} \quad \checkmark$$

$$R = \text{molar gas constant} = (K)(N_A) = 8.31 \quad \checkmark$$

Q. What is an ideal gas?

Ans. An ideal gas is a gas that obeys all the assumptions of the Kinetic theory of gases and follows an equation of the form $PV \propto T$, where P is pressure in Pa, V is volume in m^3 , and T is thermodynamic temperature (in K)

$$PV = nRT$$

or

$$PV = NKT$$

- 1) P is pressure in Pa
- 2) V is volume in m^3
- 3) n is moles
- 4) $R = 8.31$
- 5) T is thermodynamic temperature

- 1) P is pressure in Pa
- 2) V is volume in m^3
- 3) N is no. of particles
- 4) $K = 1.38 \times 10^{-23}$
- 5) T is thermodynamic temperature

Assumptions of Kinetic Model theory :

Particles are always in random motion *

1) No intermolecular forces between particles *

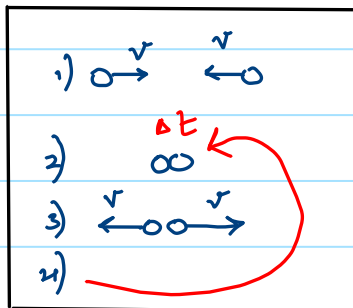
2) The time of a specific particulate collision is comparable to the time between successive collisions

3) The collisions are perfectly elastic * (no loss of KE)

4) The volume of the particles themselves are negligible compared to the volume of gas *

5) There are always a large number of particles present for statistical approximations to be applied on the population.

2)

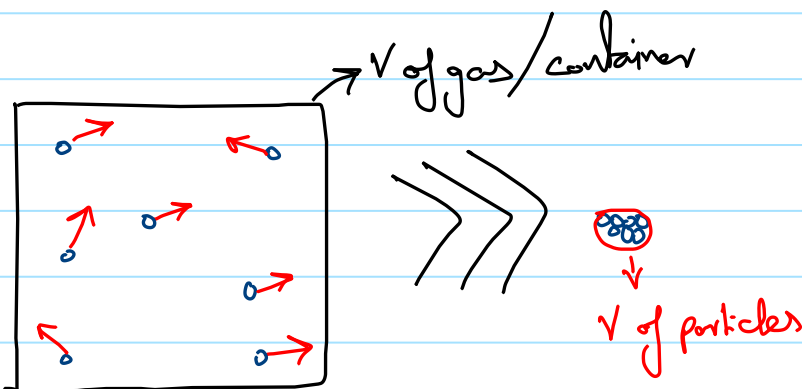


$$\Delta t = \Delta t$$

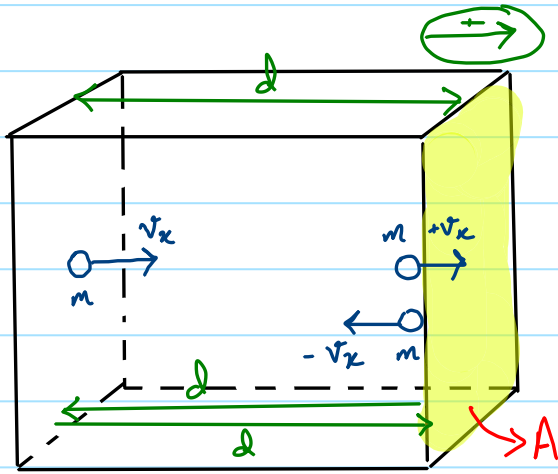
time from
1 to 3 = Δt

The particles travel so fast that once they separated after collision, they travelled so fast to the walls of the container and back that we can assume collision time b/w 2 particles = time it takes them to collide once again

4)



Derivation of ideal gas equation :



1) Change in momentum upon collision of 1 particle:-

$$\Delta p = p_f - p_i = m(-v_x) - m(+v_x)$$

$$= -mv_x - mv_x$$

$$\Delta p = -2mv_x$$

↓
direction

$$|\Delta p| = 2mv_x$$

2) Change in momentum of N number of particles:-

$$\Delta p_{\text{of all particles}} = 2Nmv_x$$

3) Force exerted:-

$$F = \frac{\Delta p}{\Delta t}$$

(Assumption 2:- $\Delta t = t$ b/w successive collisions)

$$\text{since } v \text{ is const} \Rightarrow v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{2d}{v_x}$$

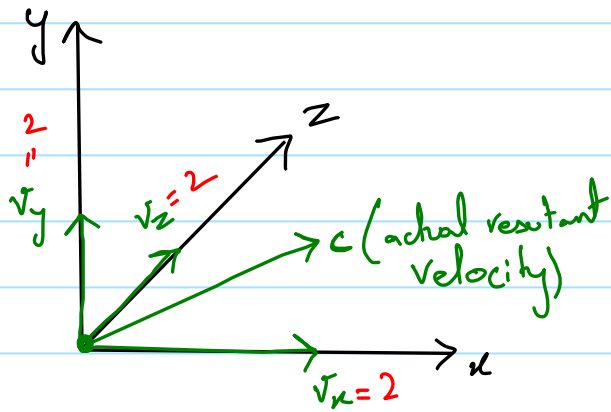
$$F = \frac{2Nmv_x}{\frac{2d}{v_x}} \Rightarrow F = \frac{Nm v_x^2}{d}$$

4) Pressure exerted on the wall:-

$$P = \frac{F}{A} = \frac{Nm v_x^2}{d} \times \frac{1}{A} \Rightarrow P = \frac{Nm v_x^2}{V}$$

$$PV = Nm v_x^2$$

5) In reality, the particle has a component of velocity in all 3 dimensions so it is not correct to take only horizontal velocity



$$c^2 = v_x^2 + v_y^2 + v_z^2$$

$$|v_x| = |v_y| = |v_z|$$

$$c^2 = v_x^2 + v_x^2 + v_x^2$$

$$c^2 = 3v_x^2$$

$$v_x^2 = \frac{1}{3} \langle c^2 \rangle$$

$$PV = Nm v_x^2 \Rightarrow$$

$$PV = \frac{1}{3} Nm \langle c^2 \rangle$$

P = pressure in Pa

V = volume in m^3

N = number of particles

m = mass of 1 particle in kg

$\langle c^2 \rangle$ = mean square speed of all particles in the gas

$\frac{1}{3}$ = showing 3 dimensions

Another form of $PV = \frac{1}{3} Nm \langle c^2 \rangle$:-

$$P = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \Rightarrow$$

$$P = \frac{1}{3} \rho \langle c^2 \rangle$$

Comparing $PV = NkT$ and $PV = \frac{1}{3} Nm \langle c^2 \rangle$

$$\cancel{NkT} = \frac{1}{3} \cancel{Nm} \langle c^2 \rangle$$

$$K.E = \frac{1}{2} mv^2$$

$$\frac{1}{2} \times kT = \frac{1}{3} m \langle c^2 \rangle \times \frac{1}{2}$$

$$\frac{1}{2} kT = \frac{1}{3} \times \frac{1}{2} m \langle c^2 \rangle$$

KE

$$\Rightarrow K.E = \frac{3}{2} kT$$

for one particle

$$\text{Total K.E of all particles in gas} = \frac{3}{2} NkT$$

Internal energy:- (U) $\Rightarrow U = \sum K.E + \sum P.E$

$\downarrow$ random motion $\downarrow$ intermolecular forces

As ideal gases have no intermolecular forces $\Rightarrow \sum P.E = 0$

$$U = \sum K.E$$

$$\Rightarrow U = \frac{3}{2} NkT$$

Comparing $PV = nRT$ and $PV = \frac{1}{3} Nm \langle c^2 \rangle$

$$\underbrace{nRT}_{\text{const}} = \underbrace{\frac{1}{3} Nm \langle c^2 \rangle}_{\text{const}} \Rightarrow T \propto \langle c^2 \rangle$$

$$\frac{T_1}{\langle c^2 \rangle_1} = \frac{T_2}{\langle c^2 \rangle_2}$$

$$\langle c^2 \rangle = \text{mean square speed}$$

$$\sqrt{\langle c^2 \rangle} = c_{r.m.s}$$

- 3 (a) (i) The kinetic theory of gases leads to the equation

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT.$$

Explain the significance of the quantity $\frac{1}{2}m\langle c^2 \rangle$.

Kinetic energy of one particle in the gas

- (ii) Use the equation to suggest what is meant by the absolute zero of temperature.

Absolute zero is the temperature at which particles have no kinetic energy.

[3]

- (b) Two insulated gas cylinders **A** and **B** are connected by a tube of negligible volume, as shown in Fig. 3.1.

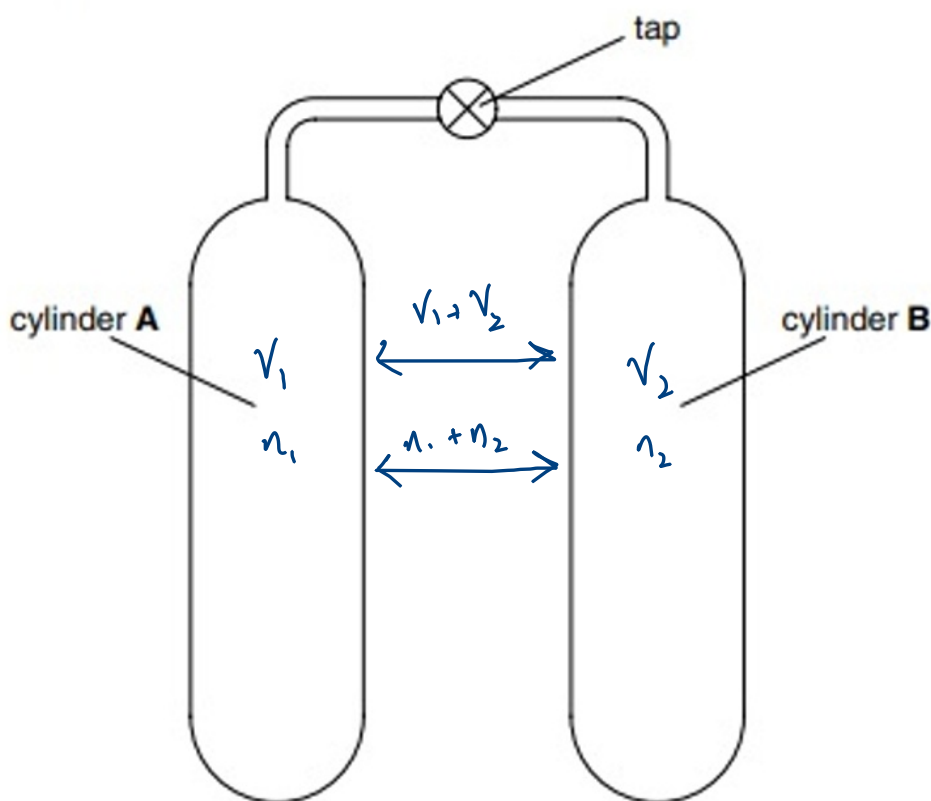


Fig. 3.1

Each cylinder has an internal volume of $2.0 \times 10^{-2} \text{ m}^3$. Initially, the tap is closed and cylinder **A** contains 1.2 mol of an ideal gas at a temperature of 37 °C. Cylinder **B** contains the same ideal gas at pressure 1.2×10^5 Pa and temperature 37 °C.

- (i) Calculate the amount, in mol, of the gas in cylinder B.

$$PV = nRT$$

$$(1.2 \times 10^5)(2 \times 10^{-2}) = n(8.31)(37 + 273.15)$$

$$n = 0.93119$$

amount = 0.93 mol

- (ii) The tap is opened and some gas flows from cylinder **A** to cylinder **B**. Using the fact that the total amount of gas is constant, determine the final pressure of the gas in the cylinders.

$$PV = nRT$$

$$P(4 \times 10^{-2}) = (0.93 + 1.2)(8.31)(37 + 273.15)$$

$$P = 137243.7$$

pressure = 1.4×10^5 Pa
[6]

2 The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle.$$

(a) Explain the meaning of the symbol $\langle c^2 \rangle$.

mean square speed of all particles in the gas

[2]

(b) The ideal gas has a density of 2.4 kg m^{-3} at a pressure of $2.0 \times 10^5 \text{ Pa}$ and a temperature of 300 K .

(i) Determine the root-mean-square (r.m.s.) speed of the gas atoms at 300 K .

$$p = \frac{1}{3} \rho \langle c^2 \rangle \Rightarrow 2 \times 10^5 = \frac{1}{3} (2.4) \langle c^2 \rangle$$

$$\langle c^2 \rangle = 250000$$

$$c_{\text{r.m.s.}} = \sqrt{250000} = 500$$

r.m.s. speed = 500 m s^{-1} [3]

(ii) Calculate the temperature of the gas for the atoms to have an r.m.s. speed that is twice that calculated in (i).

$$\sqrt{\frac{\langle c^2 \rangle_1}{T_1}} = \sqrt{\frac{\langle c^2 \rangle_2}{T_2}} \Rightarrow \frac{c_{\text{r.m.s.1}}}{\sqrt{T_1}} = \frac{c_{\text{r.m.s.2}}}{\sqrt{T_2}}$$

$$\frac{500}{\sqrt{300}} = \frac{1000}{\sqrt{T_2}} \Rightarrow (\sqrt{T_2})^2 = \left(\frac{1000 \times \sqrt{300}}{500} \right)^2$$

temperature = 1200 K [3]

2 (a) The equation

$$pV = \text{constant} \times T$$

$\xrightarrow{\text{Amount}}$
 $\xrightarrow{\text{moles}} nR \quad \xrightarrow{\text{number}} /nR$

relates the pressure p and volume V of a gas to its kelvin (thermodynamic) temperature T .

State two conditions for the equation to be valid.

1. It should be an ideal gas

2. Amount of gas must remain constant

[2]

(b) A gas cylinder contains $4.00 \times 10^4 \text{ cm}^3$ of hydrogen at a pressure of $2.50 \times 10^7 \text{ Pa}$ and a temperature of 290 K .

The cylinder is to be used to fill balloons. Each balloon, when filled, contains $7.24 \times 10^3 \text{ cm}^3$ of hydrogen at a pressure of $1.85 \times 10^5 \text{ Pa}$ and a temperature of 290 K .

Calculate, assuming that the hydrogen obeys the equation in (a),

(i) the total amount of hydrogen in the cylinder,

$$PV = nRT$$

$$(2.5 \times 10^7) (4 \times 10^4 \times 10^{-6}) = n (8.31) (290)$$

$$n = 414.9 \text{ mol}$$

amount = 415 mol [3]

(ii) the number of balloons that can be filled from the cylinder.

How many moles will be left behind in the cylinder?

$$PV = nRT \Rightarrow (1.85 \times 10^5) (7.24 \times 10^3 \times 10^{-6}) = n (8.31) (290)$$

$$n = 3.071$$

3.071 mol will be left behind in cylinder, meaning we can extract $415 - 3.071 \approx 412 \text{ mol}$ only

$$n_{\text{balloon}}: PV = nRT \Rightarrow (1.85 \times 10^5) (7.24 \times 10^3 \times 10^{-6}) = n (8.31) (290)$$

$$n = 0.5557$$

$$\text{Number of balloons} = \frac{412}{0.557} \approx 741$$

[3/4]

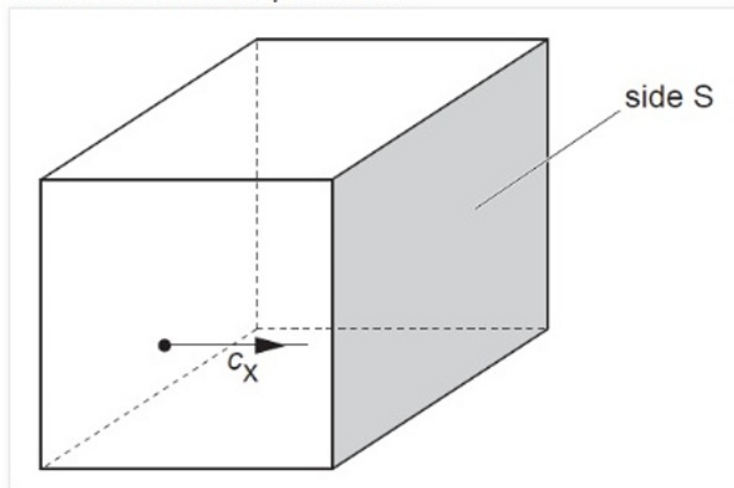
(a) Kinetic theory of gases is based on some simplifying assumptions.

Molecules of the gas are assumed to behave as hard elastic identical spheres.

State assumption about ideal gas molecules based on

- (i) nature of their movement $\rightarrow$ particles always in random motion
(ii) their volume $\rightarrow$ volume of particles is negligible compared to volume of gas

(b) Cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_x of velocity normal to one side S of the cube, as shown.



Pressure p of the gas due to component c_x of velocity is given by expression

$$pV = Nmc_x^2$$

where m is mass of a molecule.

Explain how expression leads to the relation

$$pV = (1/3) Nm \langle c^2 \rangle$$

where $\langle c^2 \rangle$ is mean square speed of the molecules.

the mean speed will be a 3D vector having components c_x, c_y, c_z

$$\langle c^2 \rangle = c_x^2 + c_y^2 + c_z^2$$

$$\text{Assuming } c_x = c_y = c_z \Rightarrow \langle c^2 \rangle = c_x^2 + c_x^2 + c_x^2$$

$$\langle c^2 \rangle = 3c_x^2$$

$$c_x^2 = \frac{1}{3} \langle c^2 \rangle$$

$$\text{substitute in } pV = Nm c_x^2 \text{ to get } pV = \frac{1}{3} Nm \langle c^2 \rangle$$

(c) Molecules of an ideal gas have a root-mean-square (r.m.s.) speed of 520 m s^{-1} at a temperature of 27°C .

Calculate r.m.s. speed of the molecules at a temperature of 100°C .

$$\langle c^2 \rangle \propto T \Rightarrow \text{r.m.s.} \propto \sqrt{T}$$

$$\frac{\text{r.m.s.}_1}{\sqrt{T_1}} = \frac{\text{r.m.s.}_2}{\sqrt{T_2}} \Rightarrow \frac{520}{\sqrt{27+273.15}} = \frac{\text{r.m.s.}_2}{\sqrt{100+273.15}}$$

$$\text{r.m.s.}_2 = \frac{520 \sqrt{100+273.15}}{\sqrt{27+273.15}}$$

$$\text{r.m.s.}_2 = 580 \text{ m s}^{-1}$$

- 2 (a) State what is meant by an *ideal gas*.

An ideal gas obeys all the assumptions of the kinetic model theory and follows the equation $PV \propto T$, where P is pressure, V is volume, and T is thermodynamic temperature. [3]

- (b) Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 2.1.

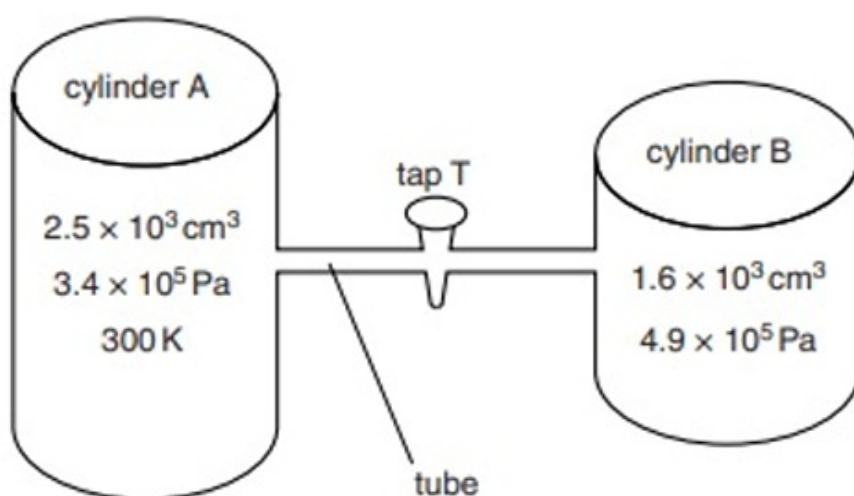


Fig. 2.1

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

- (i) Cylinder A has a constant volume of $2.5 \times 10^3 \text{ cm}^3$ and contains gas at pressure $3.4 \times 10^5 \text{ Pa}$ and temperature 300 K .

Show that cylinder A contains 0.34 mol of gas.

$$PV = nRT$$

$$(3.4 \times 10^5)(2.5 \times 10^3 \times 10^{-6}) = n(8.31)(300)$$

$$n = 0.34 \text{ mol}$$

$$\begin{aligned} \text{cm} \times 10^{-2} &= \text{m} \\ \text{cm}^2 \times (10^{-2})^2 &= \text{m}^2 \\ \text{cm}^3 \times (10^{-2})^3 &= \text{m}^3 \end{aligned}$$

[1]

$$V = 1.6 + 2.5 = 4.1 \times 10^3 \text{ cm}^3$$

- (ii) Cylinder B has a constant volume of $1.6 \times 10^3 \text{ cm}^3$ and contains 0.20 mol of gas.
When tap T is opened, the pressure of the gas in both cylinders is $3.9 \times 10^5 \text{ Pa}$.
 No thermal energy enters or leaves the gas.

Determine the final temperature of the gas.

$$PV = nRT$$

$$(3.9 \times 10^5)(4.1 \times 10^3 \times 10^{-6}) = (0.2 + 0.34)(8.31)T$$

$$T = 356.33$$

temperature = ... 360 K [2]

- (c) By reference to work done and change in internal energy, suggest why the temperature of the gas in cylinder A has changed.

.....

.....

.....

..... [3]

- 2 (a) The kinetic theory of gases is based on a number of assumptions about the molecules of a gas.

State the assumption that is related to the volume of the molecules of the gas.

Volume of the particles of the gas is negligible compared to the volume of the gas

[2]

- (b) An ideal gas occupies a volume of $2.40 \times 10^{-2} \text{ m}^3$ at a pressure of $4.60 \times 10^5 \text{ Pa}$ and a temperature of 23°C .

- (i) Calculate the number of molecules in the gas.

$$PV = NKT$$

$$(4.60 \times 10^5)(2.40 \times 10^{-2}) = N(1.38 \times 10^{-23})(23 + 273.15)$$

$$N = 2.7 \times 10^{24}$$

number = 2.7×10^{24} [3]

- (ii) Each molecule has a diameter of approximately $3 \times 10^{-10} \text{ m}$.

Estimate the total volume of the gas molecules.

$$V_{\text{of sphere}} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \pi \times (1.5 \times 10^{-10})^3$$

$$= 1.41 \times 10^{-29}$$



$$d = 3 \times 10^{-10}$$

$$r = 1.5 \times 10^{-10}$$

$$V_T = (1.41 \times 10^{-29}) \times (2.7 \times 10^{24}) = 3.8 \times 10^{-5}$$

volume = $3.8 \times 10^{-5} \text{ m}^3$ [3]

- (c) By reference to your answer in (b)(ii), suggest why the assumption in (a) is justified.

$3.8 \times 10^{-5} < 2.4 \times 10^{-2}$, so the total volume of the gas molecules is negligible compared to volume of gas [1]

[Total: 9]

- 2 (a) A square box of volume V contains N molecules of an ideal gas. Each molecule has mass m .

Using the kinetic theory of ideal gases, it can be shown that, if all the molecules are moving with speed v at right angles to one face of the box, the pressure p exerted on the face of the box is given by the expression

$$pV = Nm\bar{v}^2 \quad (\text{equation 1})$$

This expression leads to the formula

$$p = \frac{1}{3}\rho\langle c^2 \rangle \quad (\text{equation 2})$$

for the pressure p of an ideal gas, where ρ is the density of the gas and $\langle c^2 \rangle$ is the mean-square speed of the molecules.

Explain how each of the following terms in equation 2 is derived from equation 1:

ρ : $p = \frac{Nm}{V}$ where $Nm = \text{total mass of gas}$
 $V = \text{total volume of gas}$

$\frac{1}{3}$: We are considering 3 dimensional velocity of particles

$\langle c^2 \rangle$: the components of velocity in all 3 dimension are equal so $v_x = v_y = v_z = v$ and so $v^2 = \frac{1}{3} \langle c^2 \rangle$ [4]

- (b) An ideal gas has volume, pressure and temperature as shown in Fig. 2.1.

volume $6.0 \times 10^{-3} \text{ m}^3$
 pressure $3.0 \times 10^5 \text{ Pa}$
 temperature 17°C

Fig. 2.1

The mass of the gas is 20.7 g.

Calculate the mass of one molecule of the gas.

1) $PV = nRT \Rightarrow n = ?$

$n = \frac{\text{mass}}{Mr} \Rightarrow Mr = ?$

2) $PV = NKT \Rightarrow N = ?$

$\frac{\text{total mass}}{N} = \text{mass of 1 particle}$

1) $PV = nRT$
 $(3 \times 10^5)(6 \times 10^{-3}) = n(8.31)(17 + 273.15)$
 $n = 0.7465 \text{ mol}$

$Mr = \frac{\text{mass}}{n} = \frac{20.7}{0.7465} = 27.71 \text{ u}$

mass of 1 particle = $27.71 \times (1.66 \times 10^{-27})$
 $= 4.6 \times 10^{-26} \text{ Kg}$

2) $PV = NKT$
 $(3 \times 10^5)(6 \times 10^{-3}) = N(1.38 \times 10^{-23})(17 + 273.15)$
 $N = 4.495 \times 10^{23}$

mass of 1 particle = $\frac{20.7 \times 10^{-3}}{4.495 \times 10^{23}} \rightarrow \text{in Kg}$
 $= 4.6 \times 10^{-26}$

$4.6 \times 10^{-26} \text{ Kg}$