Physical Quantities and Units

Definition:

Any quantity that can be measured or has units is called a physical quantity. There are two categories of Physical Quantities:-

1) Base Quantity :- Any quantity whose units cannot be furthur simplified. For example:-

Quantity	Unit
Length	meters (m)
Mass	kilograms (kg)
Time	seconds (s)
Temperature	Kelvin (K)
Current	Amphere (A)
Amount of substance	Moles (mol)
Light Intensity	Candela (cd)

2) Derived Quantity:- Any quantity whose units are defined using the units of base quantities. For example:-

Quantity	Unit
Area	m^2
Volume	m^3
Density	kgm^{-3}
Speed	ms^{-1}
Acceleration	ms^{-2}
Gravity	ms^{-2}

- Units of gravity will be the same as acceleration because it is also known as acceleration due to gravity
- kgm^{-3}, ms^{-1}, ms^{-2} is just another way of writing kg/m^3, m/s, m/s^2

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Other Derived Quantities:

1) Force (N) = mass x acceleration

$F = m \times a$

$(kg) \times (ms^{-2})$

∴ Force = $kgms^{-2}$

∴ weight, Tension = $kgms^{-2}$

2) Work done / Energy (J) = Force x distance

$(kgms^{-2}) \times (m)$

∴ Work done / Energy = kgm^2s^{-2}

3) Pressure (Pa) = $\frac{\text{Force}}{\text{Area}}$

$\frac{kgms^{-2}}{m^2}$

$(kgms^{-2})(m^{-2})$

∴ Pressure = $kgm^{-1}s^{-2}$

4) Power (W) = $\frac{\text{Energy}}{\text{Time}}$

$\frac{kgm^2s^{-2}}{s}$

$(kgm^2s^{-2})(s^{-1})$

∴ Power = kgm^2s^{-3}

5) Spring constant (k): $F = ke$

∴ $k = \frac{\text{Force}}{\text{extension}}$

$\frac{kgms^{-2}}{m}$

∴ Spring constant = kgs^{-2}

6) Specific Heat Capacity (c): $Q = mc\Delta\theta$

∴ $c = \frac{Q}{m\Delta\theta}$ where Q = Energy

$\frac{kgm^2s^{-2}}{kg(K)}$

∴ Specific Heat Capacity = $m^2s^{-2}K^{-1}$

7) Charge (C): $Q = It$

where Q = charge

$Q = (A)(s)$

∴ Charge = As

8) Voltage (V): $P = IV$

∴ $V = \frac{P}{I}$

$\frac{kgm^2s^{-3}}{A}$

∴ Voltage = $kgm^2s^{-3}A^{-1}$

9) Resistance (Ω): $V = IR$

∴ $R = \frac{V}{I}$

$\frac{kgm^2s^{-3}A^{-1}}{A}$

∴ Resistance = $kgm^2s^{-3}A^{-2}$

Example 1:- $F = \frac{Q^2}{4\pi\varepsilon r^2}$ and $F = \frac{yI^2}{2\pi d}$, where F = Force, Q = Charge, r = radius, I = current, d = diameter. Find the base units of $\frac{1}{xy}$

Step 1:- Equate both equations together

$$\frac{Q^2}{4\pi\varepsilon r^2} = \frac{yI^2}{2\pi d}$$

Step 2:- make $\frac{1}{xy}$ the subject

$$\frac{1}{xy} = \frac{4\pi\varepsilon r^2 I^2}{Q^2 2\pi d}$$

Step 3:- Solve by adding units

$$\frac{1}{xy} = \frac{(m)(A^2)}{(A^2s^2)(m)^2}$$

$$\frac{1}{xy} = ms^{-2} \text{ (Acceleration)}$$

- 2π and 4π are not taken into account while solving the equation because they are dimensionless quantities i.e. they don't have units. Trignometric ratios are another example of dimensionless constants.

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Example2 $x = kr^3(P_1 - P_2)\sqrt{\frac{M}{RT}}$, where r = radius (m), P = Pressure ($kgm^{-1}s^{-2}$), M = mass per mol ($kg\,mol^{-1}$), R = Energy per temperature per mol ($kgm^2s^{-2}K^{-1}mol^{-1}$), T = temperature (K), and k is a dimensionless constant

$x = (m)^3(kgm^{-1}s^{-2})\sqrt{\frac{kg\,mol^{-1}}{kgm^2s^{-2}K^{-1}mol^{-1}(K)}}$

$x = kgm^2s^{-2}(m^{-1}s^{1})^{1/2}$

$x = kgm^2s^{-2}(m^{-1}s)$

$x = kgms^{-1}$

- The units of $(P_1 - P_2)$ are the same as that of Pressure because two values of Pressure being subtracted has no affect on the units.

Homogeniety of an Equation:

Base Units (LHS) = Base Units (RHS)

1) $KE = \frac{1}{2}mv^2$

$kgm^2s^{-2} = (kg)(ms^{-1})^2$

$kgm^2s^{-2} = kgm^2s^{-2}$

∴ Equation is homogenous

2) $v = f\lambda$

$(ms^{-1}) = (s^{-1})(m)$

$ms^{-1} = ms^{-1}$

∴ Equation is homogenous

3) $P = \rho gh$

$kgm^{-1}s^{-2} = (kgm^{-3})(ms^{-2})(m)$

$kgm^{-1}s^{-2} = kgm^{-1}s^{-2}$

∴ Equation is homogenous

4) $v = \sqrt{g\lambda}$

$ms^{-1} = \sqrt{(ms^{-2})(m)}$

$ms^{-1} = [m^1s^{-1}]^{1/2}$

$ms^{-1} = ms^{-1}$

∴ Equation is homogenous

5) $m/t = kv\sqrt{P \cdot P}$ (speed) where k = dimensionless constant

$kgs^{-1} = (ms^{-1})[(kgm^{-3})(kgm^{-1}s^{-2})]^{1/2}$

$kgs^{-1} = (ms^{-1})[kg^2m^{-4}s^{-3}]^{1/2}$

$kgs^{-1} = (ms^{-1})(kgm^{-2}s^{-1})$

$kgs^{-1} = kgm^{-1}s^{-2}$

∴ Equation is not homogenous

6) $T = 2\pi\sqrt{\frac{m}{k}}$ (Time Period); k → spring constant

$s = \left[\frac{kg}{kgs^{-2}}\right]^{1/2}$

$s = (s^1)^{1/2}$

$s = s$

∴ Equation is homogenous

7) $E = hf$ (Power)

$kgm^2s^{-2} = (kgm^2s^{-3})(s^{-1})$

$kgm^2s^{-2} = kgm^2s^{-4}$

∴ Equation is not homogenous

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How to check for Homogeniety if an equation contains multiple terms?

Example 1: $v = u + at$

- Compare v with u

$ms^{-1} = ms^{-1}$

- Compare u with at

$ms^{-1} = (ms^{-2})(s)$

$ms^{-1} = ms^{-1}$

∴ Equation is homogenous

Example 2: $P = \frac{\frac{1}{2}mv^2}{V} + \frac{mgh}{V}$ (speed, volume)

- Compare P with $\frac{\frac{1}{2}mv^2}{V}$

$kgm^{-1}s^{-2} = (kg)(ms^{-1})^2(m^{-3})$

$kgm^{-1}s^{-2} = kgm^{-1}s^{-2}$

- Compare P with mgh/V

$kgm^{-1}s^{-2} = (kg)(ms^{-2})(m)(m^{-3})$

$kgm^{-1}s^{-2} = kgm^{-1}s^{-2}$

∴ Equation is homogenous

Example 3: $L\left[3 + \frac{r^2}{P}\right] = \theta T^2 \sin\theta$. Given that the equation is homogenous, find the base units of P and θ.

$3L + \frac{Lr^2}{P} = \theta T^2 \sin\theta$

- Compare $\theta T^2 \sin\theta$ with 3L

$3L = \theta T^2 \sin\theta$

$(m) = \theta (s)^2$

$\theta = ms^{-2}$ (acceleration)

- Compare 3L with Lr^2/P

$3L = \frac{Lr^2}{P}$

$P\,m = m^3$

$P = m^2$ (Area)

Example 4: $\frac{1}{T} = \frac{b}{\sqrt{A}} + c$

- Compare $\frac{1}{T}$ with $\frac{b}{\sqrt{A}}$

$\frac{1}{T} = \frac{b}{\sqrt{A}}$

$s^{-1} = b(m^{-1})^{1/2}$

$b = ms^{-1}$ (speed / velocity)

- Compare $\frac{1}{T}$ with c

$\frac{1}{T} = c$

$c = s^{-1}$ (frequency)

How to find the value of unknown constants using Homogeniety?

Example 1. $T = l^x g^y$. Find x and y.

$s = (m)^x (ms^{-2})^y$

- Compare 's' on both sides

$s^1 = s^{-2y}$

$1 = -2y$

$y = -1/2$

- Compare 'm' on both sides

$m^0 = (m)^x . (m)^y$

$m^0 = m^{x+y}$

$0 = x - 1/2$

$x = 1/2$

Example 2. $V = T^x u^y$ where v = speed, T = tension, u = mass per unit length

$(ms^{-1}) = (kgms^{-2})^x (kgm^{-1})^y$

- Compare 's'

$s^{-1} = s^{-2x}$

$-1 = -2x$

$x = 1/2$

- Compare 'm'

$m^1 = (m)^x . (m)^{-y}$

$1 = x - y$

$1 = 1/2 - y$

$y = -1/2$

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Example 3. $I = nAqv^x$ where I = current, A = Area, q = Charge, v = speed, n = number of e^- per unit volume.

$(A) = (m^{-3})(m^2)(As)(ms^{-1})^x$

- Compare 's'

$s^0 = s^1 . s^{-x}$

$0 = 1 - x$

$x = 1$

Example 4. $F = 6\pi r n v$. Find base units of n

$kgms^{-2} = (m)(n)(ms^{-1})$

$kgms^{-2} = m^2 s^{-1}(n)$

$n = kgm^{-1}s^{-1}$

(iii) $\frac{v}{t} = L^x \left(\frac{P}{r}\right)^y (n)^2$, v = volume, t = time, P = pressure, r = radius

$m^3 s^{-1} = (m)^x (kgm^{-2}s^{-2})^y (kgm^{-1}s^{-1})^2$

- Compare 's'

$s^{-1} = s^{-2y} . s^{-2}$

$-1 = -2y - 2$

$-1 = -2(-2) - 2$

$z = -1$

- Compare 'kg'

$kg^0 = kg^y . kg^z$

$0 = y + z$

$y = -z$

$y = 1$

- Compare 'm'

$m^3 = (m)^x . (m^{-2y}) . (m^{-z})$

$m^3 = m^{x-2y-z}$

$3 = x - 2y - z$

$3 = x - 2(1) - (-1)$

$3 = x - 1$

$x = 4$

(ii) $m = \frac{\pi(P_1 - P_2) r^x \rho t}{8nL}$ Show that $x = 4$

$kg = \frac{(kgm^{-1}s^{-2})(m)^x (kgm^{-3})(s)}{(kgm^{-1}s^{-1})(m)}$

$kg = (kgm^{-4})(m)^x$

- Compare 'm'

$m^0 = m^{-4} . m^x$

$0 = -4 + x$

$x = 4$ shown!

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Prefixes used in Physics:

Prefixes	Symbols	Order
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
kilo	k	10^{3}
Mega	M	10^{6}
Giga	G	10^{9}
Tera	T	10^{12}

Example 1: Ratio of $\frac{10^{-3}\,Tm}{10^{3}\,km}$

$$\frac{(10^{-3})(10^{12})}{(10^{3})(10^{3})} = \frac{(10^{9})}{(10^{6})} = \frac{10^{3}}{1} = \frac{1000}{1}$$

Example 2. Distance between the Earth and Sun is 150 million km. Calculate time taken for light to travel from Sun to Earth in Ms.

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{Time} = \frac{(150\times10^{6})\times10^{3}}{3.0\times10^{8}}$$

$$\text{Time} = 500\ s = \frac{500}{1\times10^{6}} = 5.0\times10^{-4}\ Ms$$

Example 3. Red light wavelength = 600nm. Calculate frequency of red light in THz.

$v = f\lambda$

$3.0\times10^{8} = f\,(600\times10^{-9})$

$f = 5\times10^{14}\ Hz$

$$\frac{5\times10^{14}}{1\times10^{12}} = 500\ THz$$

Example 4. $C_D = \frac{2F}{\rho(v)^{x}A}$. C_D is a dimensionless constant

$\rho(v)^{x}A\,C_D = 2F$

$(kgm^{-3})(ms^{-1})^{x}(m^{2}) = kgms^{-2}$

- Compare 's'

$s^{-x} = s^{-2}$

$x = 2$

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Errors and Uncertainties

These can occur due to:

(i) limitation of the observer

(ii) limitation of the instrument used

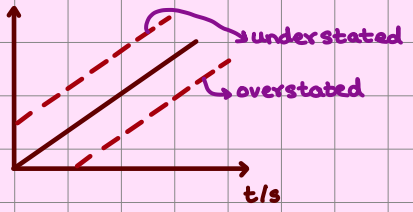
(iii) limitation in the external environment

Systematic Errors :-

- constant sign [all overstated or all understated results]

Examples:- zero error , faulty stopwatch (fast / slow) , Presence of Background Radiation, incorrect caliberation.

- Cannot be reduced by averaging repeated measurements
- Graphical [non-zero intercept]

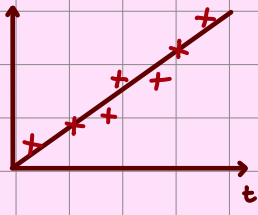


Random Errors :-

- varying sign [some overstated and some understated values]

Examples:- human reaction time, speed of sound [wind direction], volume of a liquid [measuring cylinder] when temperature changes, diameter of a wire [micrometer] when pressure changes.

- Can be reduced by averaging repeated measurements
- Graphical → Scatter of points about a mean value [best-fit line]
  ↓
  Impacts slope / gradient



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Precision and Accuracy:-

- Precision refers to the degree of agreement between the observed values.
- Accuracy refers to the degree of agreement between the observed values when compared with the Actual/True value

Micrometer	vs	Meter Rule
• smallest division = 0.01mm		• smallest division = 1mm
• Hence, values obtained by using a micrometer will be in close proximity to each other. Hence, more precise.		• Hence, values obtained using a meter rule will not be in close proximity to each other when compared to a micrometer. Hence, less precise.

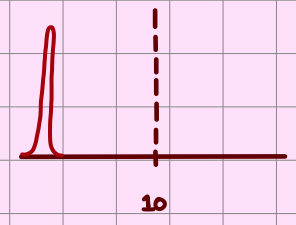
Example 1:- An experiment is conducted on 4 occassions

$g(\text{actual}) = 10\ ms^{-2}$

Experiment 1	Experiment 2	Experiment 3	Experiment 4.
2.95	6.0	9.8	1.0
2.96	8.0	9.9	3.0
2.95	10.0	10.1	7.0
2.97	12.0	10.0	9.0
2.96	14.0	10.2	5.0

Experiment 1

Precise but not Accurate

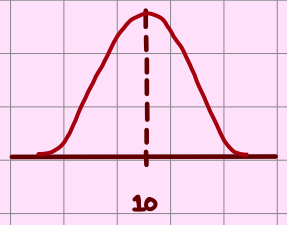


Narrow width means the values are more precise.

The graph is away from the mean which means that these values are less accurate

Experiment 2

Accurate but not Precise

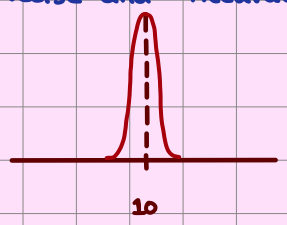


Greater width means the values are less precise.

The graph is placed around the mean which means that these values are more accurate

Experiment 3

Precise and Accurate

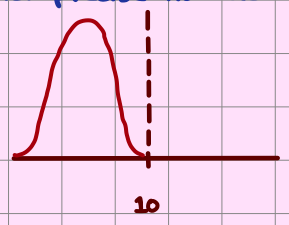


Narrow width means the values are more precise.

The graph is placed around the mean which means that these values are more accurate

Experiment 4.

Neither precise nor accurate



Greater width means the values are less precise.

The graph is away from the mean which means that these values are less accurate

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How can we quote a physical quantity?

$L = 25.0 \pm 0.5\,cm$

$\theta = 40.0 \pm 0.5\,^{\circ}C$

General formula:- $x \pm \Delta x$

x → True Value

Δx → Absolute Uncertainity

Fractional Error:- $\Delta x / x$

↳ Error expressed as a fraction.

Percentage Error:- $\frac{\Delta x}{x} \times 100$

Rules for quoting a physical quantity?

- Uncertainities must always be quoted to one significant figure
- Uncertainities and true values must be consistent
- Uncertainities in their units / tens / hundreths / thousandths must be rounded off consistently with the true value.

1) 16.00 ± 0.38

16.0 ± 0.4

2) 25.0 ± 0.043

25.00 ± 0.04

3) 163.42 ± 0.243

163.4 ± 0.2

4) 327.66 ± 9.63

330 ± 10

5) 334.78 ± 7.68

335 ± 8

6) 327.66 ± 9.4

328 ± 9

7) $12,378 \pm 374$

$12,400 \pm 400$

8) $324,864 \pm 3287$

$325,000 \pm 3000$

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Rules for Addition, Subtraction, Multiplication, and Division:

ADDITION:-

$x = 18.5 \pm 0.5$ => Actual values add

$y = 12.5 \pm 0.5$ => Uncertainities also add

$x + y = 31 \pm 1$

Fractional Uncertainity = $1/31$

Percentage Uncertainity = $1/31 \times 100 = 3.2\%$

SUBTRACTION:-

$x = 18.5 \pm 0.5$ => Actual values subtract

$y = 12.5 \pm 0.5$ => Uncertainities also add

$x + y = 6 \pm 1$

Fractional Uncertainity = $1/6$

Percentage Uncertainity = $1/6 \times 100 = 16.7\%$

e.g. 1 $\theta_1 = 40.0 \pm 0.5^\circ C$

$\theta_2 = 100.0 \pm 0.5^\circ C$

$\Delta\theta = 60 \pm 1\ ^\circ C$

Percentage Error = $1/60 \times 100 = 1.7\%$

MULTIPLICATION:-

$L = 6.0 \pm 0.5$ => Actual values multiply

$B = 5.0 \pm 0.5$ => Fractional Uncertainities to be added

Area = Length x Breadth

Area = 6×5

Area = $30\ cm^2$

$$\frac{\Delta x}{30} = \frac{0.5}{6} + \frac{0.5}{5}$$

$\Delta x = 6$

$30 \pm 6\ cm^2$

DIVISION

$d = 40.0 \pm 0.1\ m$ => Actual values Divide

$t = 2.50 \pm 0.05\ s$ => Fractional Uncertainities to be added

$s = d/t$

$s = 40/2.5$

$s = 16$

$$\frac{\Delta S}{16} = \frac{0.1}{40} + \frac{0.05}{2.5}$$

$\Delta S = 0.4$

$S = 16.0 \pm 0.4$

Example:- $x = 20.0 \pm 0.5$

$y = 40.0 \pm 0.5$ Find z along with its uncertainity.

$$z = \frac{x^2}{y^3}$$

$z = (6.3 \pm 0.5) \times 10^{-3}$

Actual Value:- $z = \frac{(20)^2}{(40)^3} = 6.25 \times 10^{-3}$

Uncertainity:- $\frac{\Delta z}{6.25 \times 10^{-3}} = 2\left(\frac{0.5}{20}\right) + 3\left(\frac{0.5}{40}\right)$

$\Delta z = 5.47 \times 10^{-4}$

0.547×10^{-3}

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Example 1:- A cylinder has a volume of $15.0 \pm 0.5\ cm^3$ and a height of $20.0 \pm 0.1\ cm$. Find radius with its uncertainity in cm.

$V = \pi r^2 h$

True Value

$15 = \pi (r)^2 (20)$

$\sqrt{\frac{15.0}{20\pi}} = r$

$r = 0.489\ cm$

Absolute Uncertainity

$\frac{0.5}{15} = 2\left(\frac{\Delta r}{0.489}\right) + \frac{0.1}{20}$

$\Delta r = 0.00937$

$\Delta r = 0.009$

$r = 0.489 \pm 0.009\ cm$

Example 2:- A cylinder has a mass of $9.6 \pm 0.5 g$, thickness of $2.00 \pm 0.01\ mm$, and a diameter of $22.1 \pm 0.1\ mm$. Calculate the density in kg/m^3 or SI Units.

$\rho = \frac{m}{V}$

$\rho = \frac{M}{\pi r^2 h}$

$\rho = \frac{M}{\pi\left[\frac{d^2}{4}\right]h}$

$\rho = \frac{4M}{\pi d^2 h}$

Actual Value

$\rho = \frac{4(9.6 \times 10^{-3})}{\pi (22.1 \times 10^{-3})^2 (2 \times 10^{-3})}$

$\rho = 12500\ kg/m^3$

Absolute Uncertainity.

$\frac{\Delta\rho}{12513} = \frac{0.5}{9.6} + 2\left(\frac{0.1}{22.1}\right) + \left(\frac{0.01}{2}\right)$

$\Delta\rho = 827.5 \approx 800$

$\rho = 12500 \pm 800\ kg/m^3$

Example 3:- A sphere has density $8100\ kg/m^3 \pm 5\%$ and mass $7.5 kg \pm 4\%$. Calculate the diameter along with its uncertainity.

$\rho = M/V$

$\rho = \frac{M}{\frac{4}{3}\pi\left(\frac{d^3}{8}\right)}$

$\rho = \frac{24M}{4\pi d^3}$

$\rho = \frac{6M/\pi}{d^3}$

$d = \left(\frac{6M}{\pi\rho}\right)^{1/3}$

Actual Value

$d = \left[\frac{6(7.5)}{\pi(8100)}\right]^{1/3}$

$d = 0.121 \approx 0.12$

Absolute Uncertainity

$\frac{\Delta d}{d} \times 100 = \frac{1}{3}\left[\frac{\Delta\rho}{\rho} \times 100\right] + \frac{1}{3}\left[\frac{\Delta m}{m} \times 100\right]$

$\frac{\Delta d}{d} \times 100 = \frac{1}{3}(4) + \frac{1}{3}(5)$

3% [Percentage error in density amounts to 3%]

$\Delta d = \frac{3}{100} \times 0.12 = 0.0036$

$d = 0.121 \pm 0.004\ m.$

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Example 4:- The resistivity (ρ) of a wire is given by the formula:-

$\rho = \frac{RA}{L}$ R= Resistance, A= Area, L= Length

$R = 7.5 \pm 0.2\ \Omega$, $L = 1750 \pm 3mm$, $D = 0.38 \pm 0.1mm$. Find the ρ along with its uncertainity giving your answer in SI Units

$\rho = \frac{R(\pi r^2)}{L}$

$\rho = \frac{R\pi d^2}{4L}$

Actual Value

$\rho = \frac{7.5\ \pi\ (0.38 \times 10^{-3})}{4\ (1750 \times 10^{-3})}$

$\rho = 4.861 \times 10^{-7}$

Absolute Uncertainity

$\frac{\Delta\rho}{4.861 \times 10^{-7}} = \frac{0.2}{7.5} + 2\left(\frac{0.01}{0.38}\right) + \left(\frac{3}{1750}\right)$

$\Delta\rho = 3.94 \times 10^{-8}$

$0.394 \times 10^{-7} \approx 0.4 \times 10^{-7}$

$\rho = (4.9 \pm 0.4) \times 10^{-7}$

Example 5 Voltage = $5.0 \pm 0.1V$

Resistance = $125 \pm 3\%\ \Omega$ Calculate Power along with its uncertainity

$P = V^2/R$

Actual Value

$P = \frac{(5)^2}{125}$

$P = 0.200$

Absolute Uncertainity

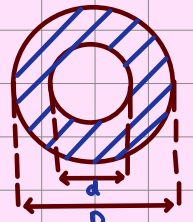
$\frac{\Delta P}{P} \times 100 = \left[2\left(\frac{\Delta V}{V}\right) \times 100\right] + \left[\frac{\Delta R}{R} \times 100\right]$

$\frac{\Delta P}{0.200} = 4 + 3$

$7\% \times 0.200 = 0.014 \approx 0.01$

$P = 0.20 \pm 0.01\ W$

Example 6:-



$D = 16.5 \pm 0.5\ cm$

$d = 7.5 \pm 0.5 cm$

(i) Find D+d along with its uncertainity

$D+d = 24 \pm 1$

(ii) Find D-d along with its uncertainity

$D-d = 9 \pm 1$

(iii) Show that the area of shaded region is

$A = \pi/4\ (D+d)(D-d)$

$A = \pi r^2$

$A = \pi d^2/4$

$\frac{\pi(D)^2}{4} - \frac{\pi(d)^2}{4}$

$\pi/4\ (D^2 - d^2) = \pi/4\ (D+d)(D-d)$

(iv) Hence, calculate the area of the shaded region along with its uncertainity in cm^2

Actual Value

$\frac{\pi}{4}(24)(9) = 169.65\ cm^2$

Uncertainity

$\frac{\Delta A}{169.65} = \frac{1}{24} + \frac{1}{9}$

$\Delta A = 25.9 \approx 30$

$A = 170 \pm 30\ cm^2$

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Example 7 $l = 0.250 \pm 0.001$ m

$r = 0.75 \pm 0.1$ mm

$P = (2.50 \pm 0.05) \times 10^{3}$ Nm^{-2}

$V/t = (1.20 \pm 0.01) \times 10^{-6}$ m^3s^{-1}

r = radius

v = volume

t = time

l = length

P = Pressure

Formula given as $\frac{V}{t} = \frac{\pi P r^4}{8CL}$

(i) Find C

$$1.20 \times 10^{-6} = \frac{\pi (2.50 \times 10^{3})(0.75 \times 10^{-3})^4}{8C\,(0.250)}$$

$C = 1.035 \times 10^{-3} \approx 1.04 \times 10^{-3}$

(ii) Find the uncertainity in C.

$$\frac{\Delta C}{1.035 \times 10^{-3}} = \frac{0.05}{2.50} + 4\left(\frac{0.01}{0.75}\right) + \frac{0.001}{0.250} + \frac{0.01}{1.20}$$

$\Delta C = 8.8665 \times 10^{-5}$

(iii) Write down C along with its uncertainity giving your answer $0.08665 \times 10^{-3} \approx 0.09 \times 10^{-3}$ in SI Units = $(1.04 \pm 0.09) \times 10^{-3}$

Example 8:- Length of the cube = 1.53 ± 0.01 cm

Mass of the cube = 31.3 ± 0.5 g

(i) Calculate the density of the cube in kgm^{-3}

$D = M/V$

$D = M/L^3$

$D = 31.3 \times 10^{-3} / (1.53 \times 10^{-2})^3$

$D = 8739.17\ kgm^{-3} \approx 8740\ kgm^{-3}$

(ii) Calculate percentage uncertainity in the density of the cube.

$$\frac{\Delta D}{D} \times 100 = \left[\frac{\Delta M}{M} \times 100\right] + \left[3\left(\frac{\Delta L}{L}\right) \times 100\right]$$

$$\frac{\Delta D}{8739.17} \times 100 = \left[\frac{0.05}{31.3} \times 100\right] + \left[3\left(\frac{0.01}{1.53}\right) \times 100\right]$$

$\frac{\Delta D}{8739.17} \times 100 = 3.11\%$

(iii) Another cube has the density given as $9200 \pm 6\%$ kg/m^3. Suggest whether or not these cubes are made of the same material

$6\% \times 9200 = 552 \approx 600$

Range = 8600 - 9800

3.11% of $8740 = 271 \approx 300$

Range = 8440 - 9040

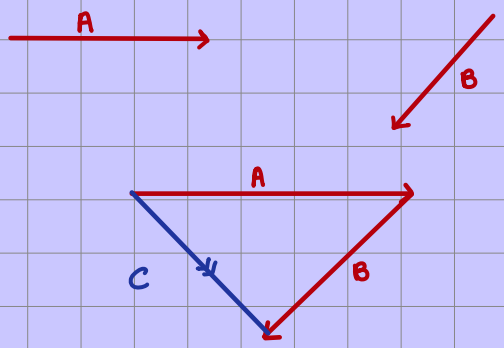
Since overlapping occurs ∴ same material.

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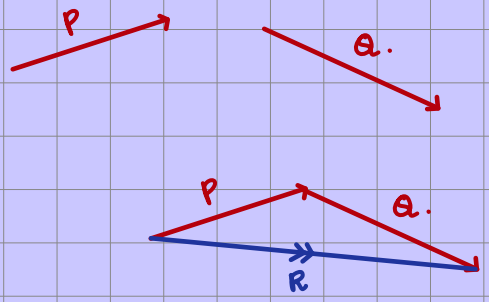
Vectors

Addition of Vectors

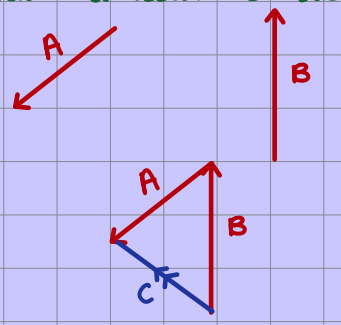
1) Draw a vector C such that C = A + B



2) Draw a vector R such that R = P + Q.

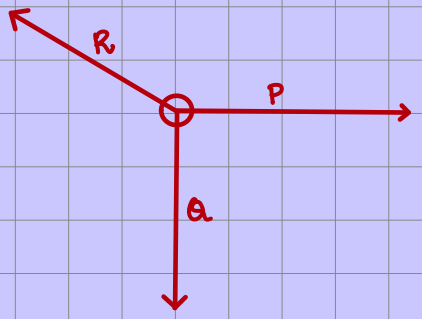


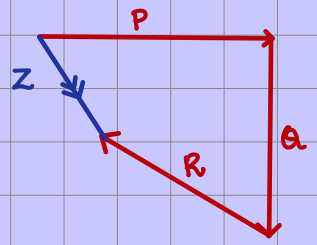
3) Draw a vector C such that C = A + B



- Individual vectors follow head-to-tail rule
- Resultant vectors follow head-to-head rule

4) Draw a vector Z such that Z = P + Q + R

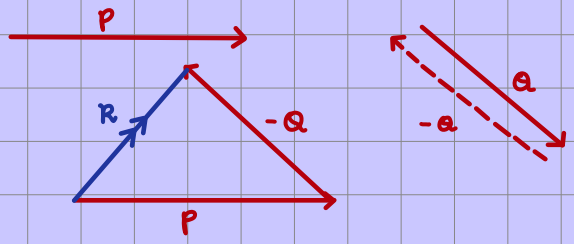




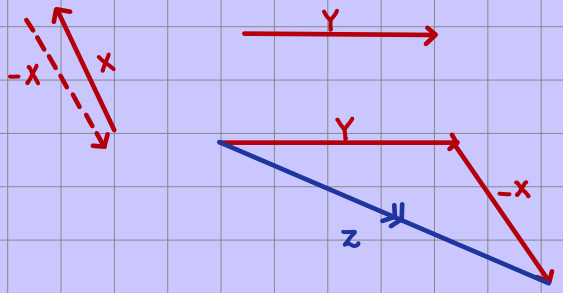
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Subtraction of Vectors

1) Draw a vector R such that $R = P - Q$.

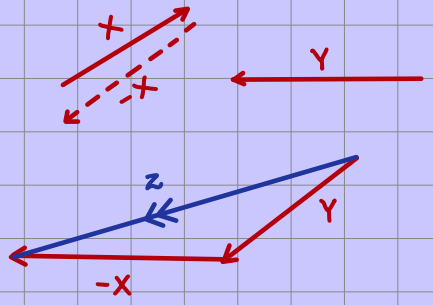


2) Draw a vector Z such that $Z = Y - X$



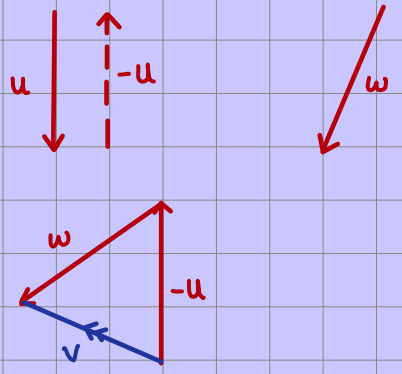
3) Draw a vector Z such that $X = Y - Z$

$Y - X = Z$

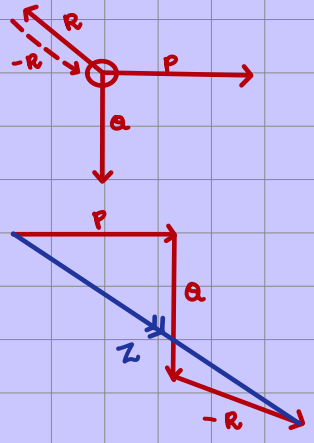


4) Draw a vector V such that $W = U + V$

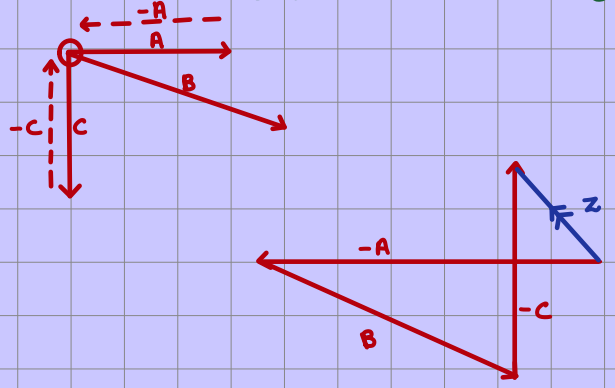
$V = W - U$



5) Draw a vector Z such that $Z = P + Q - R$



6) Draw a vector Z such that $Z = -A + B - C$



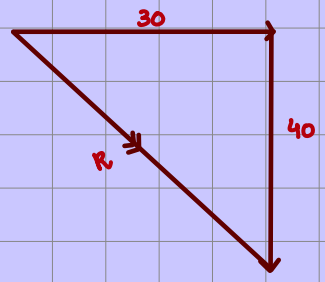
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How to calculate the Resultant of vectors?

① $u = 30\,ms^{-1}$ due East

$v = 40\,ms^{-1}$ due South

Calculate the magnitude of resultant and the angle which the resultant makes with the vertical axis.



$R^2 = \sqrt{(30)^2 + (40)^2}$

$R = 50\,ms^{-1}$

$\tan\theta = 30/40$

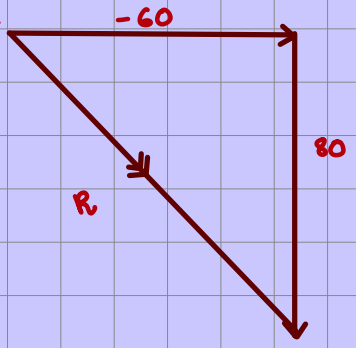
$\tan^{-1}\left[\frac{3}{4}\right] = 37^\circ$

② $u = 60\,ms^{-1}$ due West

$v = 80\,ms^{-1}$ due South.

Calculate magnitude for change in velocity (Δv)

$\Delta v = v - u$



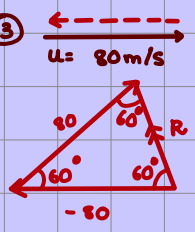
$R^2 = \sqrt{(60)^2 + (80)^2}$

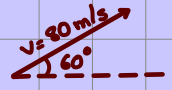
$R = 100\,ms^{-1}$

$\tan\theta = 80/60$

$\tan^{-1}\left[\frac{4}{3}\right] = 53^\circ$

③



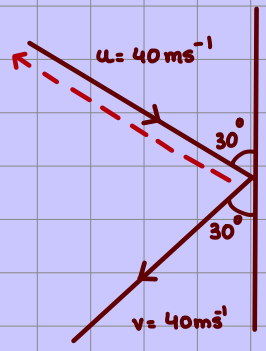


Calculate ΔV $\Delta v = v - u$

$R^2 = 80^2 + 80^2 + 2(80)(80)\cos 60$

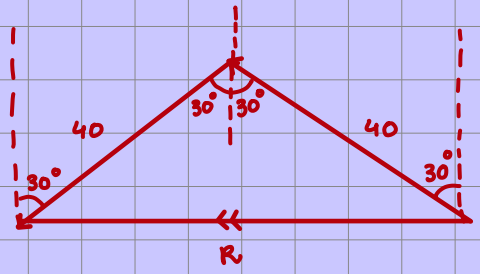
$R = 80\,N$

④



Calculate ΔV

$\Delta v = v - u$.



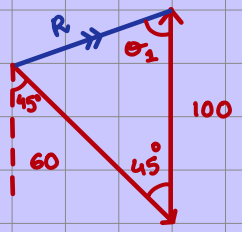
$R^2 = (40)^2 + (40)^2 - 2(40)(40)\cos 60$

$R = 40\,ms^{-1}$

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⑤ The initial velocity of an aircraft is 100 km/h due North and its final velocity is 60 km/h south-east. Calculate the total angle between resultant velocity and change in velocity.

Resultant Velocity:-



$R^2 = (100)^2 + (60)^2 - 2(100)(60)\cos 45$

$R = 72$ km/h

$\frac{\sin 45}{72} = \frac{\sin \theta_1}{60}$

$\theta_1 = 36°$

Change in velocity:-

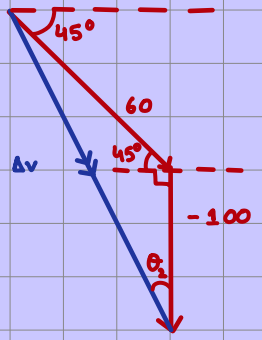
$\Delta v = v - u$

$\Delta v^2 = (100)^2 + (60)^2 - 2(100)(60)\cos 135$

$\Delta v = 150$ km/h

$\frac{\sin 135}{150} = \frac{\sin \theta_2}{60}$

$\theta_2 = 17°$



Total angle = 36 + 17

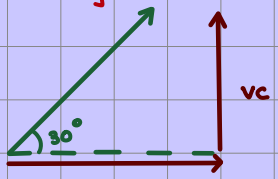
Total angle = 53°

Resolution of Vectors

The term 'resolution' refers to breaking down of any vectors into two perpendicular components. For simplicity purposes, we refer to these components as:-

(i) Horizontal Component (HC)

(ii) Vertical Component (VC)



$\cos 30 = \frac{HC}{20}$

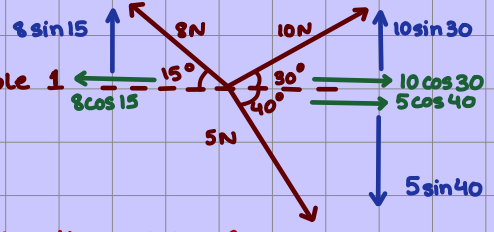
$HC = 20\cos 30$

$\sin 30 = \frac{VC}{20}$

$VC = 20\sin 30$

- Component adjacent to angle will be cosθ.
- Component opposite to angle will always be sinθ.

Example 1



Step 1:- Horizontal Component

$\rightarrow$ $10\cos 30 + 5\cos 40 - 8\sin 15$

4.8N

Step 2:- Vertical Component

$\uparrow$ $10\sin 30 + 8\sin 15 - 5\sin 40$

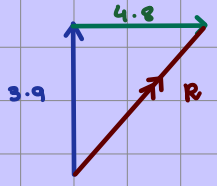
3.9N

Step 3:- Calculate

$R^2 = \sqrt{(4.8)^2 + (3.9)^2}$

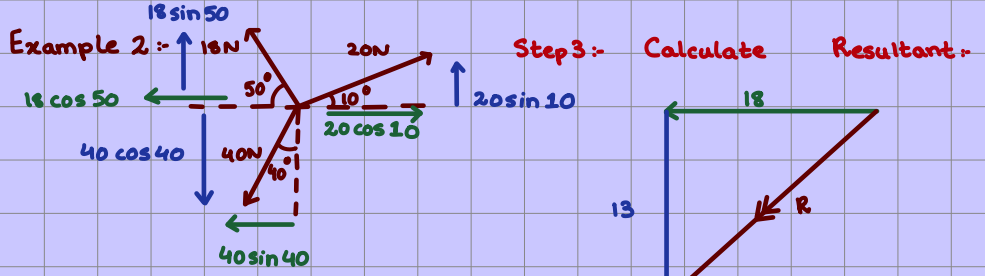
$R = 6.2$N

Resultant:-



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Example 2 :-



Step 1 :- Horizontal Component

$\xrightarrow{+}$ $20\cos 10 - 40\sin 40 - 18\cos 50$

$-18N$

Step 2 :- Vertical Component

$+\uparrow$ $20\sin 10 + 18\sin 50 - 40\cos 40$

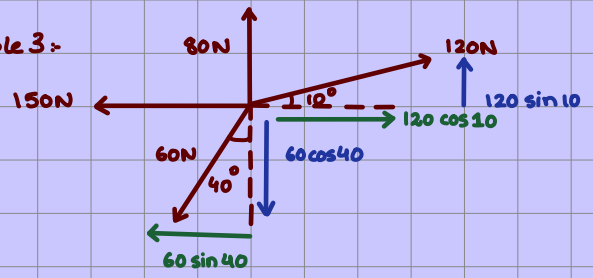
$-13\,N$

Step 3 :- Calculate Resultant :-

$R^2 = \sqrt{(13)^2 + (18)^2}$

$R = 22\,N$

Example 3 :-



Step 1 :- Horizontal Component

$\xrightarrow{+}$ $120\cos 10 - 150 - 60\sin 40$

$-70N$

Step 2 :- Vertical Component

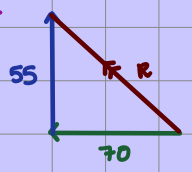
$+\uparrow$ $80 + 120\sin 10 - 60\cos 40$

$55N$

Step 3 :- Calculate Resultant :-

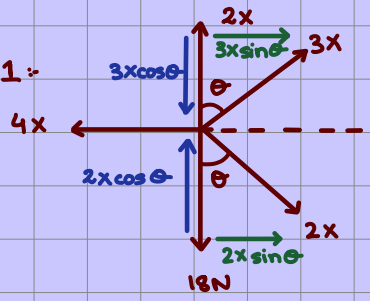
$R^2 = \sqrt{(70)^2 + (55)^2}$

$R = 89\,N$



Concept of Equilibrium and how to construct a Vector Triangle for Equilibrium

Example 1 :-



Given that the object is in equilibrium, find X and θ.

$3X\sin\theta + 2X\sin\theta = 4X$

$5X\sin\theta = 4X$

$\sin\theta = 4/5$

$\theta = 53^\circ$

$2X + 3X\cos\theta = 2X\cos\theta + 18$

$2X + X\cos\theta = 18$

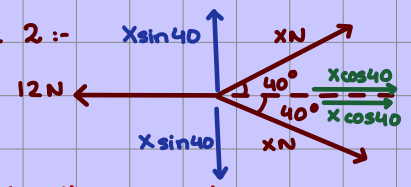
$2X + 0{\cdot}6X = 18$

$X = 6{\cdot}9N$

Vector triangles can only be made if only 3 forces are given.

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Example 2 :-



1) Ignore one force
2) Connect the rest head-to-tail
3) Place the last force, also head-to-tail

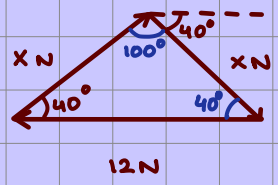
Method 1 :- Using resolution

$2X \cos 40 = 12$

$X \cos 40 = 6$

$X = 7.8\,N$

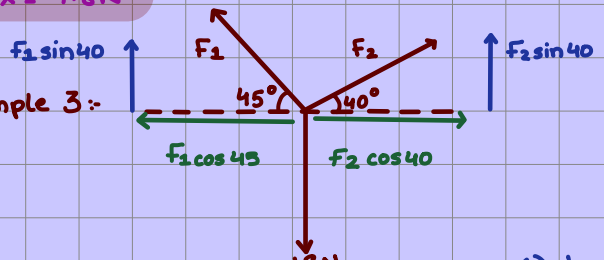
Method 2 :- Vector Triangle for Equilibrium.



$\frac{12}{\sin 100} = \frac{X}{\sin 40}$

$X = 7.8\,N$

Example 3 :-



1) Ignore one force
2) Connect the rest head-to-tail
3) Place the last force, also head-to-tail

Method 1 :- Using resolution

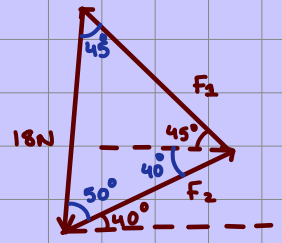
$f_1 \cos 45 = f_2 \cos 40$ → $F_2 = 13\,N$

$f_1 = 1.08 f_2$ → $F_1 = 14\,N$

$f_1 \sin 45 + f_2 \sin 40 = 18$

$0.764 f_2 + 0.643 f_2 = 18$

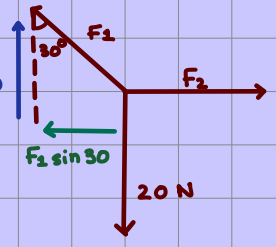
Method 2 :- Vector Triangle for Equilibrium.



$\frac{18}{\sin 85} = \frac{f_1}{\sin 50}$ → $f_1 = 14\,N$

$\frac{18}{\sin 85} = \frac{f_2}{\sin 45}$ → $f_2 = 13\,N$

Example 4 :-



1) Ignore one force
2) Connect the rest head-to-tail
3) Place the last force, also head-to-tail

Method 1 :- Using resolution

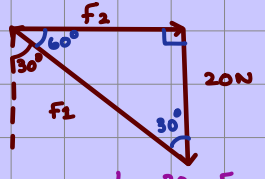
$F_2 = f_1 \sin 30$

$f_1 \cos 30 = 20$

$f_1 = 23\,N$

$F_2 = 12\,N$

Method 2 :- Vector Triangle for Equilibrium.

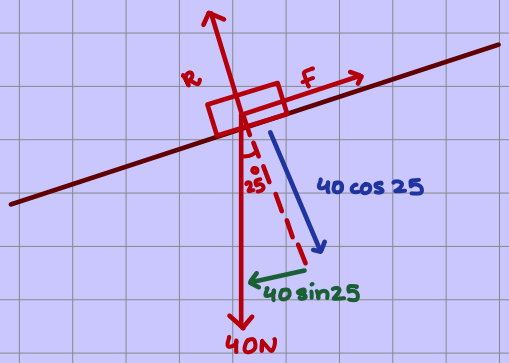


$\sin 60 = \frac{20}{f_1}$ → $f_1 = 23\,N$

$\tan 30 = \frac{F_2}{20}$ → $F_2 = 12\,N$

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How to resolve forces on an Inclined plane?



Given that the object is in equilibrium, find F and R.

- Parallel to the slope (↙↗)

$F = 40 \sin 25$

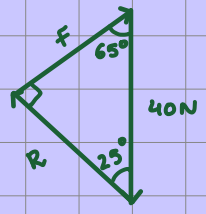
$F = 17\,N$

- Perpendicular to the slope (↑↓)

$R = 40 \cos 25$

$R = 36\,N$

Vector Triangle



$\sin 65 = R/40$

$R = 36\,N$

$\sin 25 = F/40$

$F = 17N$

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Kinematics

(i) If velocity is constant,

displacement = velocity x time $\boxed{s = vt}$

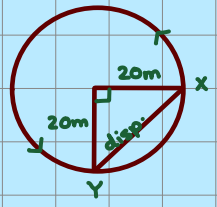
(ii) If velocity keeps changing,

displacement = Average velocity x time $\boxed{s = \left(\frac{u+v}{2}\right)t}$

(iii) Acceleration = rate of change of velocity

$\boxed{a = \frac{v-u}{t}}$ OR $\boxed{v = u+at}$

Example 1



Radius = 20 m

time = 10s (whole journey)

Calculate the average velocity from X to Y

displacement = $\sqrt{(20)^2 + (20)^2} = \sqrt{800}$ m, time = $\frac{3}{4} \times 10 = 7.5$s

$\frac{\text{displacement}}{\text{time}} = \frac{\sqrt{800}}{7.5} = 3.77\ ms^{-1}$

Example 2



Constant speed = $0.5\ ms^{-1}$

Time = 40s (whole journey)

(a) Calculate average velocity for 20s.

Circumference = $0.5 \times 40 = 20$m

Perimeter = πd

$20 = \pi d$

$d = 6.37$m

Average velocity = $\frac{6.37}{20}$

Average velocity = $0.32\ ms^{-1}$

(b) Average velocity for 40 s.

$\frac{0}{40} = 0\ ms^{-1}$

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(iv) $s = ut + \frac{1}{2}at^2$ (Derivation required)

$v = u + at$ $\quad s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+u+at}{2}\right)t$

$s = \frac{2ut+at^2}{2}$

$s = \frac{2ut}{2} + \frac{1}{2}at^2$

$s = ut + \frac{1}{2}at^2$

(v) $v^2 = u^2 + 2as$ (Derivation required)

$v = u + at$ $\quad s = \left(\frac{u+v}{2}\right)t$

$\frac{v-u}{a} = t$

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$

$s = \frac{(v+u)(v-u)}{2a}$

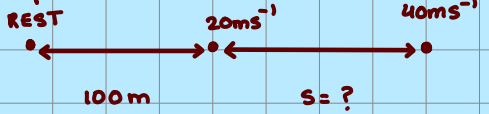
$2as = v^2 - u^2$

$v^2 = u^2 + 2as$

Conditions where Kinematics formulas (iii, iv, v) can be applied :-

- Linear motion
- Acceleration is constant (Non-zero)

Example 1

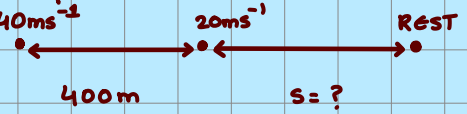


$v^2 = u^2 + 2as$ $\quad v^2 = u^2 + 2as$

$(20)^2 = 2a(100)$ $\quad (40)^2 = (20)^2 + 2(2)(s)$

$a = 2ms^{-2}$ $\quad s = 300m$

Example 2

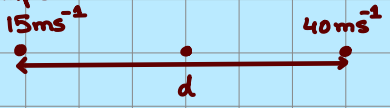


$v^2 = u^2 + 2as$ $\quad v^2 = u^2 + 2as$

$(40)^2 = (20)^2 + 2a(400)$ $\quad (0)^2 = (20)^2 + 2(-1.5)(s)$

$a = -1.5ms^{-2}$ $\quad s = 133.33m$

Example 3



Calculate the speed at the midpoint.

$v^2 = u^2 + 2as$

$(40)^2 = (15)^2 + 2(a)(d)$

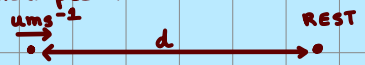
$\frac{687.5}{d} = a$

$x^2 = (15)^2 + 2\left(\frac{687.5}{d}\right)\left(\frac{d}{2}\right)$

$x = 30.21 \approx 30ms^{-1}$

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Example 4.



If u increases by 20%, show that the new distance covered to come to rest will be 1.44d.

Method 1:-

$v^2 = u^2 + 2as$

$-u^2 = 2ad$

$a = \frac{-u^2}{2d}$

$u + 20\%\,u$

$1.2u$

$0^2 = (1.2)^2 + 2\left(\frac{-u^2}{2d}\right)s$

$-1.44u^2 = -\frac{u^2}{d}(s)$

$1.44d = s$ shown!

Method 2:-

$v^2 = u^2 + 2as$

$-u^2 = 2as$

$u^2 \propto s$

$(1.2)^2 \quad 1.44$

Example 5



Find acceleration.

$+\uparrow s = ut + \frac{1}{2}at^2$

$60 = 6u + 18a \rightarrow 1$

$120 = 10u + 50a \rightarrow 2.$

$5(60 = 6u + 18a)$

$-3(120 = 10u + 50a)$

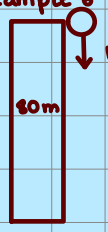
$300 = 30u + 90a$

$-360 = -30u - 150a$

$-60 = -60a$

$a = 1\ ms^{-2}$

Example 6



(i) Find the time taken to reach the ground.

$+\uparrow s = ut + \frac{1}{2}at^2$

$-80 = \frac{1}{2}(-9.81)(t^2)$

$t = 4.04s$

(ii) Find the time taken to cover the last 20 m.

$+\uparrow s = ut + \frac{1}{2}at^2$

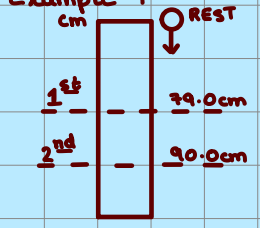
$-60 = \frac{1}{2}(-9.81)t^2$

$t = 3.50s$

time = 4.04 - 3.50

time = 0.54s.

Example 7



Find the time interval between the 1st and 2nd positions.

$\uparrow + s = ut + \frac{1}{2}at^2$

$-0.79 = \frac{1}{2}(-9.81)(t^2)$

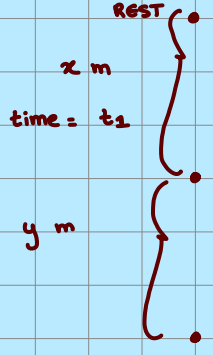
$t = 0.401s$

$-90 = \frac{1}{2}(-9.81)t$

$t = 0.428s$

Time interval = 0.027s

Example 8



The total time is t_2. Show that $a = \frac{2y}{t_2^2 - t_1^2}$

$s = \frac{1}{2}at^2$

$x = \frac{1}{2}at_1^2$

$x + y = \frac{1}{2}at_2^2$

$\frac{1}{2}at_1^2 + y = \frac{1}{2}at_2^2$

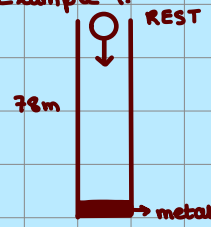
$y = \frac{1}{2}a(t_2^2 - t_1^2)$

$2y = a(t_2^2 - t_1^2)$

$a = \frac{2y}{t_2^2 - t_1^2}$ shown!

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Example 9.



Speed of sound = 340 m/s

Find the time interval between releasing the ball and hearing its echo.

$s = ut + \frac{1}{2}at^2$ ↑ +

$-78 = \frac{1}{2}(-9.81)t^2$

$t = 3.99s$

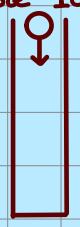
$s = d/t$

$340 = 78/t$

$t = 0.23$

Time interval = 3.99 + 0.23 = 4.22s

Example 10



Show that in a time of $\frac{1}{2}T$, the distance fallen will be $\frac{1}{4}L$.

Method 1:-

$s = ut + \frac{1}{2}at^2$

$L = \frac{1}{2}aT^2$

$\frac{2L}{T^2} = a$

$s = \frac{1}{2}\left[\frac{2L}{T^2}\right]\left[\frac{1}{2}T\right]^2$

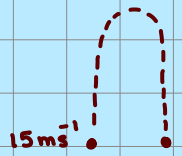
$s = \frac{1}{4}L$

Method 2:-

$L \propto T^2$

$\frac{1}{4}\left(\frac{1}{2}T\right)^2$

Example 11



Find the total time taken for the journey to be concluded.

Method 1:-

$v = u + at$

$0 = 15 + (-9.81)(t)$

$t = 1.53 \times 2 = 3.06s$

Method 2:- $s = ut + \frac{1}{2}at^2$

$0 = 15t + \frac{1}{2}(-9.81)(t^2)$

$t = 3.06s$

Example 12.

An object on Earth falls a distance 'h' in T_E seconds. The same object falls distance 'h' on the moon in T_M seconds. Given that $a_E/a_M = 6$, find T_E/T_M. The initial velocity is zero in both cases.

$s = \frac{1}{2}at^2$

$h = \frac{1}{2}a_E(T_E)^2$

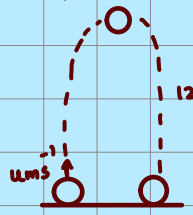
$h = \frac{1}{2}a_M(T_M)^2$

$\frac{h = \frac{1}{2}a_E(T_E)^2}{h = \frac{1}{2}a_M(T_M)^2}$

$\frac{a_M}{a_E} = \frac{(T_E)^2}{(T_M)^2}$

$\frac{T_E}{T_M} = \frac{1}{\sqrt{6}}$

Example 13



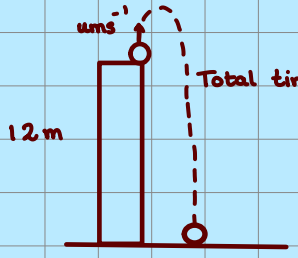
Find the value of 'u'

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2(-9.81)(12.7)$

$u = 15.8\ ms^{-1}$

Example 14.



Find the value of u.

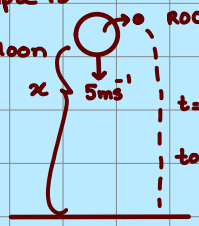
$s = ut + \frac{1}{2}at^2$ ↑ +

$-12 = 3.4u + \frac{1}{2}(-9.81)(3.4)^2$

$u = 13.2\ ms^{-1}$

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Example 15



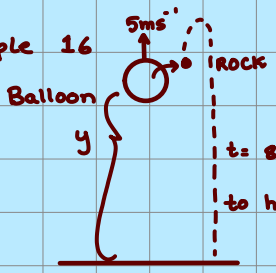
Calculate x.

$s = ut + \frac{1}{2}at^2$

$x = -5(6) + \frac{1}{2}(-9.81)(6)^2$

$x = 207m$

Example 16



Calculate y.

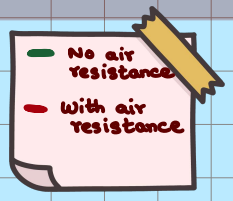
$s = ut + \frac{1}{2}at^2$

$y = 5(8.4) + \frac{1}{2}(-9.81)(8.4)^2$

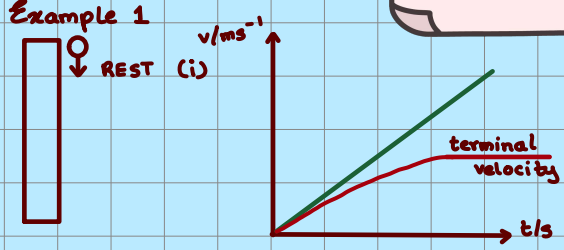
$y = 304m$

Application of Kinematics

1) Graphs:



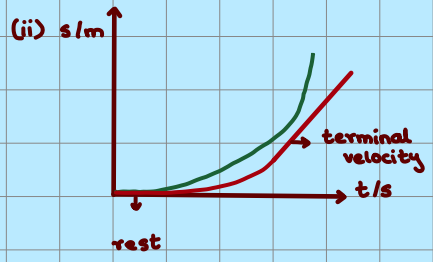
Example 1



Gradient = acceleration

Acceleration = constant (No AR)

Acceleration = decreases (with AR)

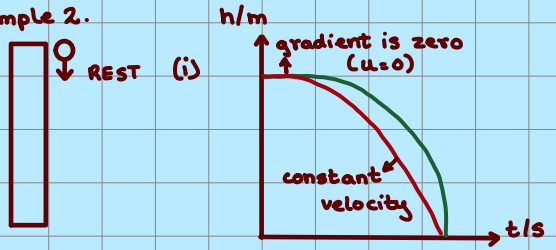


Gradient = velocity

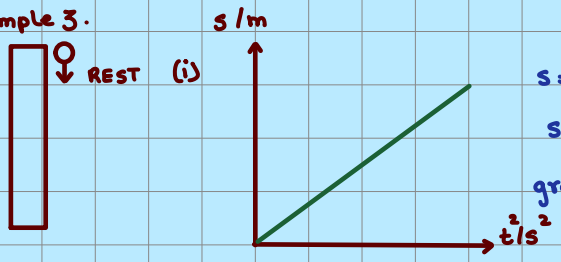
velocity increases (Both)

[gradient = zero]

Example 2.



Example 3.



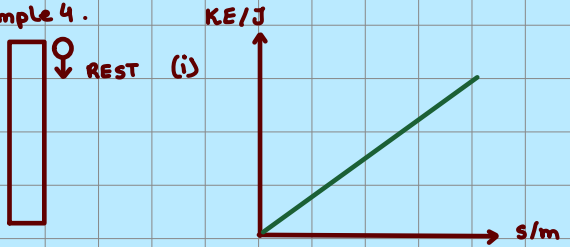
$s = \frac{1}{2}at^2$

$s \propto t^2$

gradient = $\frac{1}{2}a$ OR $a = 2(\text{gradient})$

All graphs with AR must be lower than those without AR. This is because maximum velocity can only be achieved without AR.

Example 4.

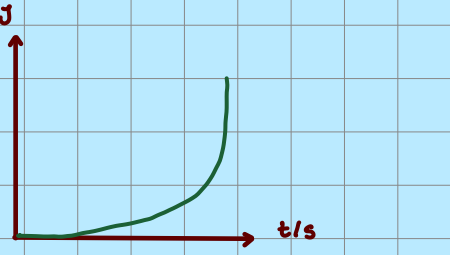


$KE = \frac{1}{2}mv^2$

$v^2 = 0^2 + 2as$ $\quad \therefore KE \propto s$

$KE = \frac{1}{2}m(2as)$

$KE = mas$



$KE = \frac{1}{2}mv^2$

$v = 0 + at$

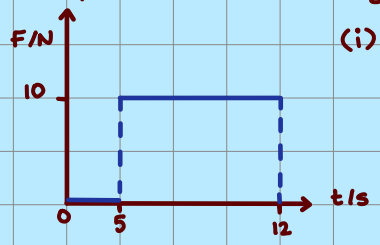
$KE = \frac{1}{2}ma^2t^2$ $\quad y = x^2$

$KE \propto t^2$ (Quadratic Curve)

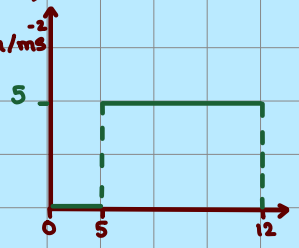
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Example 5.

mass = 2kg , $u = 0$



(i) a/ms^{-2}

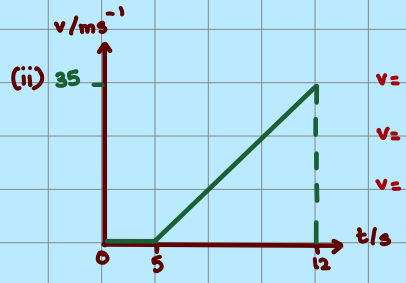


$F = ma$

$10 = 2a$

$a = 5 ms^{-2}$

(ii) v/ms^{-1}



$v = u + at$

$v = 0 + 5(12-5)$

$v = 35\ ms^{-1}$

(iii) s/m



$v^2 = u^2 + 2as$

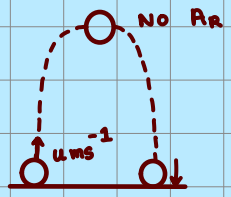
$(35)^2 = 2(5)(s)$

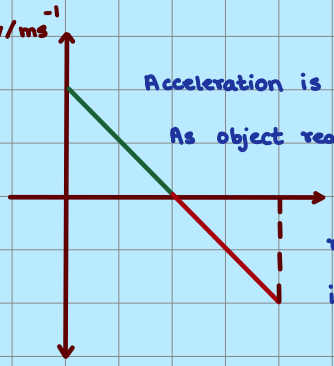
$s = 123\ m$

OR

Area under [v vs t]

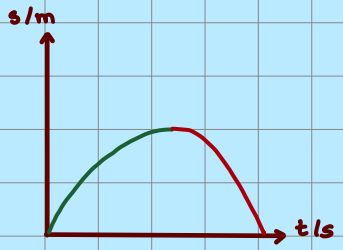
Example 6.

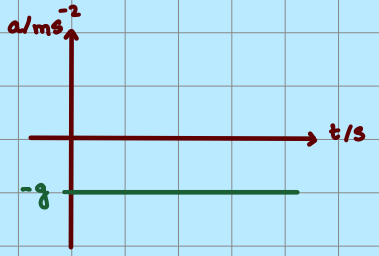




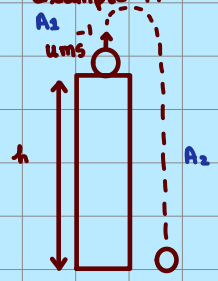
Acceleration is constant

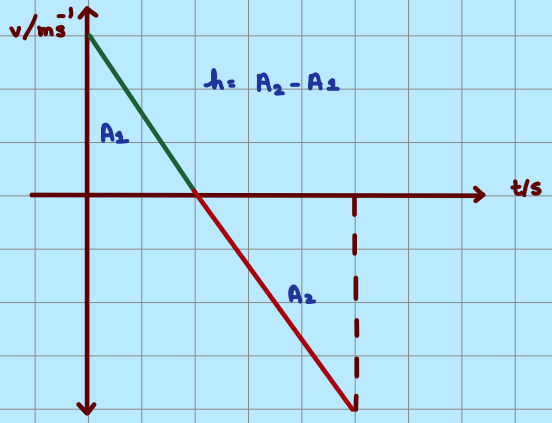
As object reaches max height, velocity decreases till it reaches zero, then increases in the opposite direction.



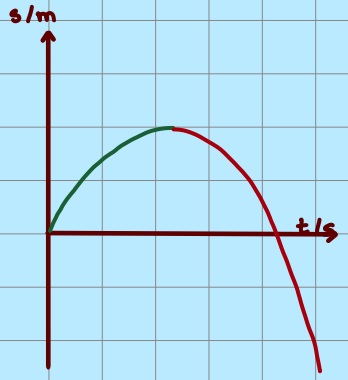


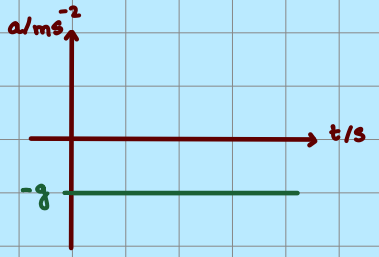
Example 7.





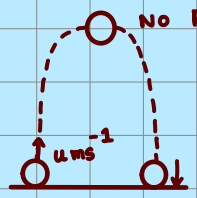
$h = A_2 - A_1$



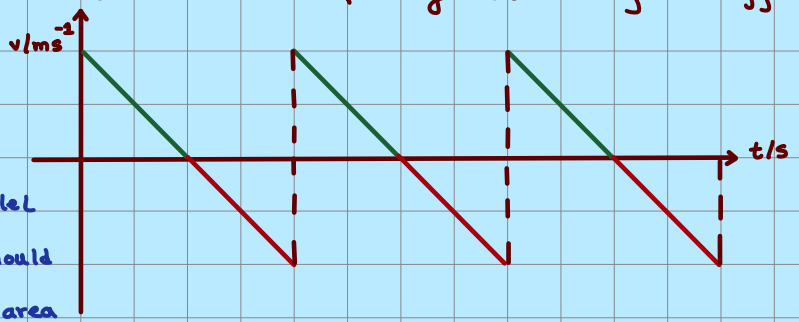


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Example 8.



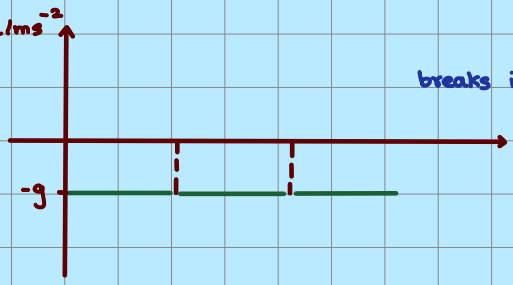
The ball bounces repeatedly without any Energy loss [Elastic Collision]





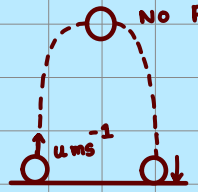
width should be constant

Lines should be parallel and time intervals should be equal along with area under the graph.

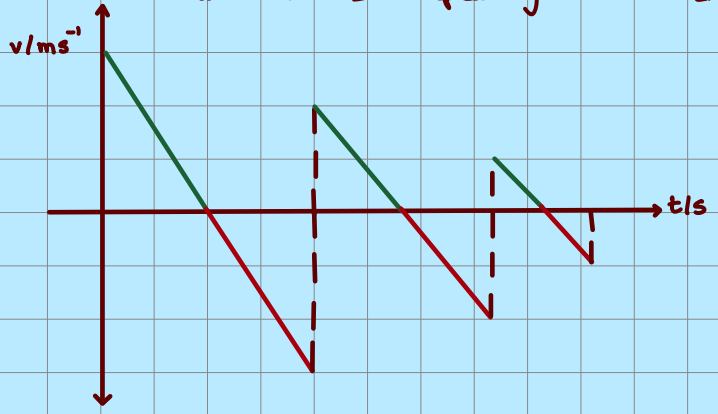


breaks in the line show the ball's impact on the ground.

Example 9



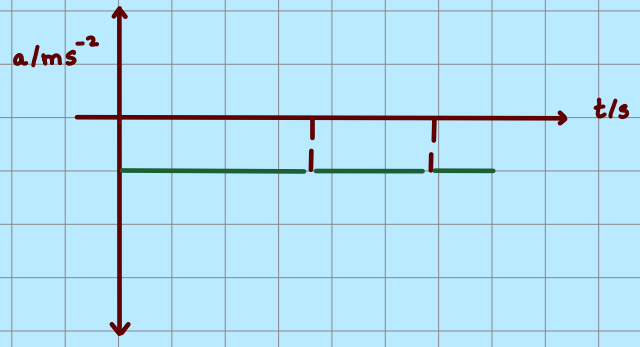
The ball bounces repeatedly with Energy loss [Inelastic collisions]





width decreases

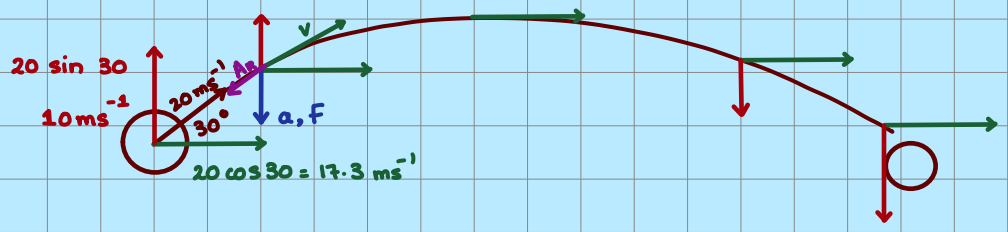
1) Areas decrease along with width
2) Lines should still be parallel

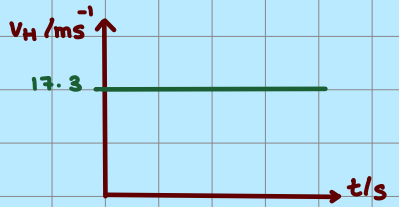


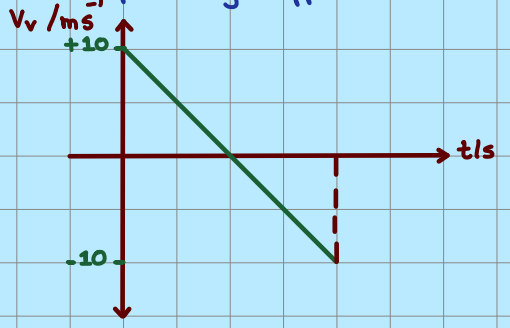
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Projectile Motion

1) Projectile Motion is an example of a 2-dimensional motion i.e. the object travels in the horizontal plane as well as in the vertical plane.
2) Assuming that there is no Air resistance, the horizontal velocity remains constant as shown.
3) Since, acceleration due to gravity acts in the vertical plane. Therefore, the vertical velocity keeps changing continuously.
4) Both, the horizontal and vertical motion must be considered separately and Kinematics formulas can be independantly applied for both the horizontal and vertical motion.







* direction of Force and acceleration will be vertically downward (F=ma)
* At which point does velocity become zero?

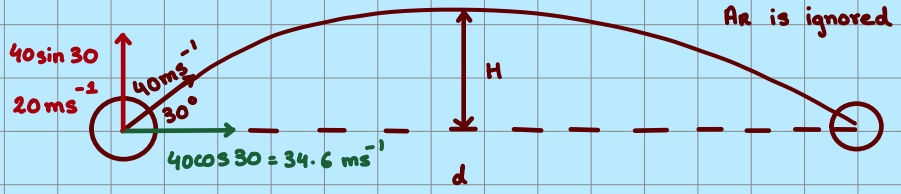
  None.
* How can you mark the direction of velocity?

  Draw a tangent to the curve
* How can you mark the direction of air resistance?

  The direction of air resistance always opposes the air resistance.

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Example 1 :-



(i) Calculate Maximum height (H)

$v^2 = u^2 + 2as$

$0^2 = 20^2 + 2(-9.81)(H)$

H = 20.4m

(ii) Calculate time taken to reach maximum height

+↑

$v = u + at$

$0 = 20 + (-9.81)(t)$

t = 2.045

(iii) Calculate Total time (T)

Method 1 :-

+↑ 2 x 2.04 = **4.08s**

Method 2 :-

$v = u + at$

$-20 = 20 + (-9.81)t$

t = 4.08s

Method 3 :-

$s = ut + \frac{1}{2}at^2$

$0 = 20t + \frac{1}{2}(-9.81)t^2$

t = 4.08s

(iv) Find the horizontal distance travelled (d)

$s = d/t$ (because horizontal velocity is constant)

d = 141m

(v) Find the velocity at t = 1.5s

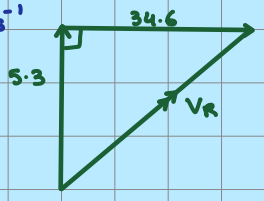
Horizontal Velocity = $34.6\ ms^{-1}$

$v = u + at$

$v = 20 + (-9.81)(1.5)$

$v = 5.3\ ms^{-1}$

Vertical velocity = $5.3\ ms^{-1}$

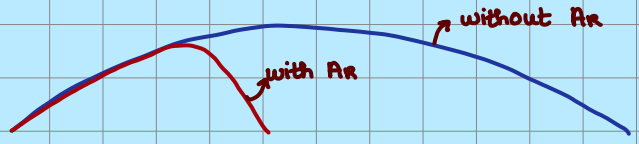


$V_R = \sqrt{(34.6)^2 + (5.3)^2}$

$V_R = 35.0\ ms^{-1}$

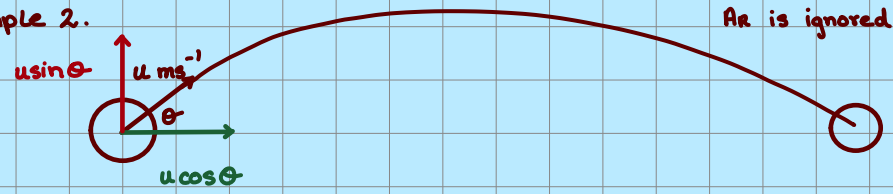
(vi) Sketch the new path if AR is present :-

- The graph rises to a smaller height
- Graphs covers lesser horizontal distance (range)
- Graph will be asymmetrical.



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Example 2.



A_R is ignored

(i) Horizontal distance travelled in time 't'

$v = d/t$

$d = (u\cos\theta)t$

(ii) Vertical Velocity travelled in time 't'

+↑ $s = ut + \frac{1}{2}at^2$

$s = (u\sin\theta)t + \frac{1}{2}(-g)(t)^2$

(iii) Final velocity (vertical) after time 't'

$v = u + at$

$v = u\sin\theta - gt$

(iv) Final velocity (horizontal) after time 't'

$v = u\cos\theta$

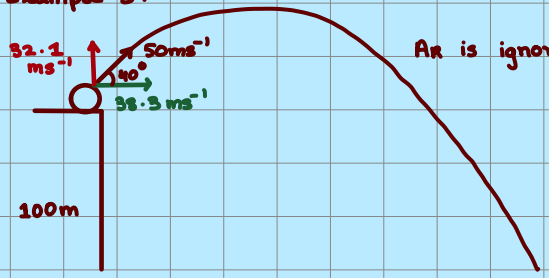
(v) Time taken to reach maximum height

$v = u + at$

$0 = u\sin\theta - gt$

$t = u\sin\theta / g$

Example 3:-



A_R is ignored

(i) Find the total time for the journey.

+↑ $s = ut + \frac{1}{2}at^2$

$-100 = 32.1t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 - 32.1t - 100 = 0$

$t = 8.85s$ $t = -2.30$

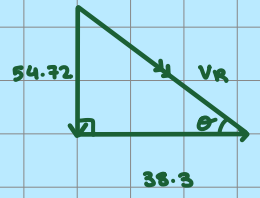
(ii) Find the velocity with which it hits the ground and the angle which it makes with the horizontal.

Horizontal Velocity = $38.3ms^{-1}$

+↑ $v = u + at$

$v = 32.1 + (-9.81)(8.85)$

Vertical Velocity = -54.72

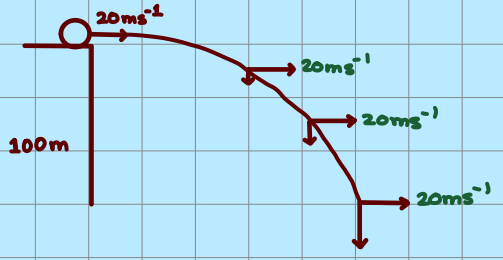


$V_R = 66.8\,ms^{-1}$

$\theta = 55°$

Half - Projectile Motion:-

Example 4:- A particle is projected horizontally at $20ms^{-1}$



(i) Find time taken to reach the ground.

Method 1:- (Theory)

$s = ut + \frac{1}{2}at^2$

$-100 = \frac{1}{2}(-9.81)(t^2)$

$t = 4.52s$

Method 2:- (MCQ's).

$t = \sqrt{\frac{2y}{g}}$ ← vertical axis value

$t = \sqrt{\frac{2(100)}{9.81}}$ $t = 4.52s$

(ii) Find the horizontal distance covered.

Method 1:- (Theory)

$s = d/t$

$d = 4.5 \times 20$

$d = 90m$

Method 2 (MCQ's)

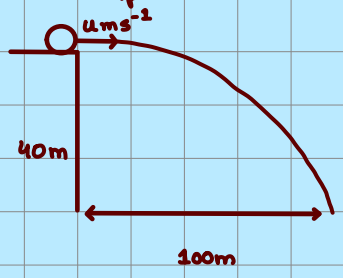
horizontal axis value $x = u\sqrt{\frac{2y}{g}}$

$x = 20\sqrt{\frac{200}{9.81}}$

$\therefore x = 90m$

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Example 5



(i) Find 'u'

Method 1 :- (Theory)

Step 1 :- $+\uparrow \quad s = ut + \frac{1}{2}at^2$

$-40 = 0 + \frac{1}{2}(-9.81)(t)^2$

$t = 2.86s.$

Step 2 :- $s = d/t$

$u = 100/2.86$

$u = 35\,ms^{-1}$

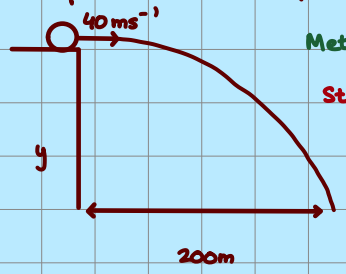
Method 2 (MCQ's)

$x = u\sqrt{\frac{2y}{g}}$

$100 = u\sqrt{\frac{2(40)}{9.81}}$

$u = 35.02\,ms^{-1}$

Example 6 :-



(i) Find 'y'

Method 1 :- (Theory)

Step 1 :- $d = st$

$200 = 40t$

$t = 5s$

Step 2 :- $+\uparrow s = ut + \frac{1}{2}at^2$

$-y = \frac{1}{2}(-9.81)(5)^2$

$y = 122.6\,m$

Method 2 :- (MCQ's)

$x = u\sqrt{\frac{2y}{g}}$

$200 = 40\sqrt{\frac{2y}{9.81}}$

$y = 122.6m$

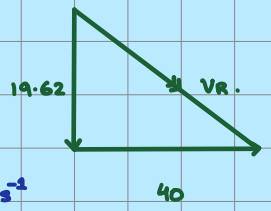
(ii) Find velocity when $t = 2s$

Horizontal Velocity = $40\,ms^{-1}$

$v = u + at$

$v = (-9.81)(2)$

Vertical Velocity = $-19.62\,ms^{-1}$



$V_R = 44.55\,ms^{-1}$

(iii) State what happens to the horizontal distance if :-

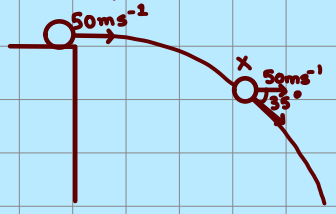
(a) If 'u' is doubled = Horizontal distance doubles

(b) If mass is halved = No change

(c) If A_R is present = Horizontal distance reduces

(d) Height of launch quadrupled = Horizontal distance doubles.

Example 7 :-



(i) Find vertical velocity at X

$X\cos 35 = 50$

$X = 61.04$

$61.04 \sin 35 = 35.01\,ms^{-1}$

(ii) Find the time taken to reach X.

$v = u + at$

$-35 = 0 - 9.81t$

$t = 3.57s$

(iii) Find the vertical distance fallen to reach X.

Method 1 :-

$+\uparrow \quad s = ut + \frac{1}{2}at^2$

$-s = \frac{1}{2}(-9.81)(3.57)^2$

$s = 62.4m$

Method 2 :-

$v^2 = u^2 + 2as$

$-35^2 = 2(-9.81)(s)$

$s = 62.4m$

(iv) Find the horizontal distance covered to reach X.

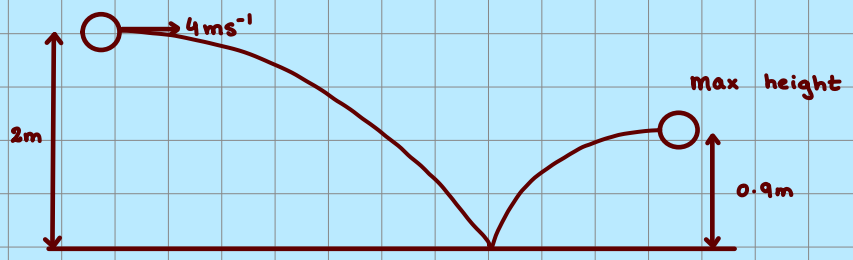
$s = d/t$

$50 = d/3.57$

$d = 178.5m$

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Example 8 :-



(i) Find the vertical velocity as the ball hits the ground.

$+\uparrow v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(-9{\cdot}81)(-2)$

$v = 6{\cdot}26\ ms^{-1}$

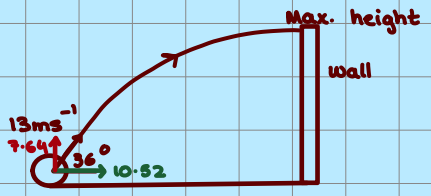
(ii) Find the vertical velocity (x) with which it rebounds from the ground.

$+\uparrow \quad v^2 = u^2 + 2as$

$0^2 = x^2 + 2(-9{\cdot}81)(0{\cdot}9)$

$x = 4{\cdot}20\ ms^{-1}$

Example 9 :-



(i) Find the time taken to reach the wall.

$+\uparrow \quad v = u + at$

$0 = 7{\cdot}64 - 9{\cdot}81t$

$t = 0{\cdot}78s$

(ii) Find the horizontal distance travelled.

$s = d/t$

$10{\cdot}52 = d/0{\cdot}78$

$d = 8{\cdot}20m$

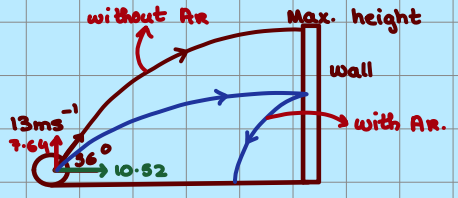
(iii) find height 'h' of the wall

$+\uparrow v^2 = u^2 + 2as$

$0^2 = 7{\cdot}64^2 + 2(-9{\cdot}81)(s)$

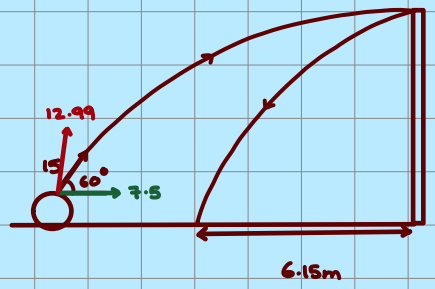
$s = 2{\cdot}98m$

(iv) Assume AR is present, Sketch the new path as the ball strikes the wall and rebounds back from the wall.



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Example 10



(i) Find the time taken to reach the wall.

+↑ $v = u + at$

$0 = 12.99 - 9.81t$

$t = 1.3s$

(ii) Find the height of the wall (h)

+↑ $s = ut + \frac{1}{2}at^2$

$s = 12.99(1.324) + \frac{1}{2}(-9.81)(1.324)^2$

$s = 8.60m$

(iii) Find the time taken for the return journey given that the ball rebounds horizontally.

+↑ $s = ut + \frac{1}{2}at^2$

$-8.6 = \frac{1}{2}(-9.81)(t^2)$

$t = 1.3s$

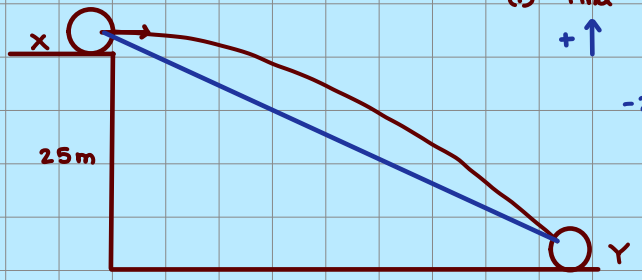
(iv) Given that the ball rebounds horizontally with velocity 'x'. Find 'x'.

$s = d/t$

$x = 6.15/1.324$

$x = 4.645\ ms^{-1}$

Example 11



(i) Find the time taken to reach Y

+↑ $s = ut + \frac{1}{2}at^2$

$-25 = \frac{1}{2}(-9.81)t^2$

$t = 2.3s$

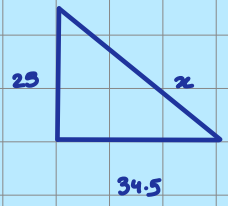
(ii) Find displacement XY

Vertical distance = 25m

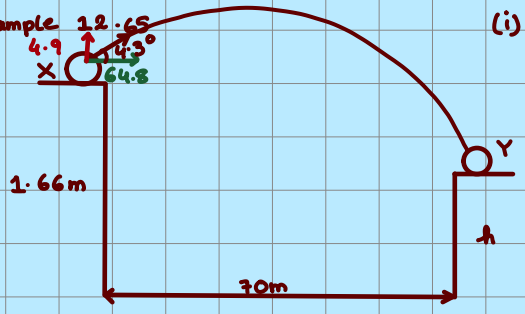
$15 = d/2.3$

Horizontal distance = 34.5m

$x = 42.61m$



Example 12.65



(i) Find the time taken to reach Y

$d = st$

$70 = 64.8t$

$t = 1.08s$

(ii) Find 'h'

$-1.66 + h = 4.9(1.08) + \frac{1}{2}(-9.81)(1.08)^2$

$h = 1.2m$

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Dynamics

- Newton's first law of motion:- An object at rest and an object in motion continues to move at constant velocity as long as no external, unbalanced forces are acting on it.

  Conditions:- Forces are balanced / $F_{resultant}$ is zero.

- Newton's second law of motion:- Force is equal to the rate of change of momentum

  $F = \Delta p / t$

  $F = p_f - p_i / t$

  $F = mv - mu / t$

  $F = m\left[v-u/t\right]$ where m= mass (kg), a = acceleration (ms^{-2}), F= Resultant force (N)

  $F = ma$

  ↳ symbol :- p

  definition :- product of mass and velocity.

  formula :- $p = mv$ Vector quantity

  Units :- $kgms^{-1}$ or Ns.

- Impulse :- Product of force and time.

  Symbol :- I

  formula :- $I = Ft$

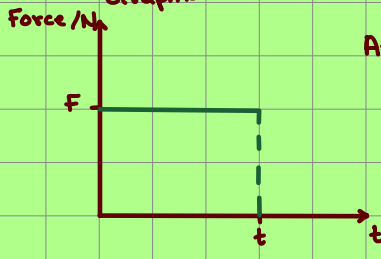
  $I = \Delta p$

  $I = mv - mu$

  Vector quantity

  Units :- $kgms^{-1}$ or Ns

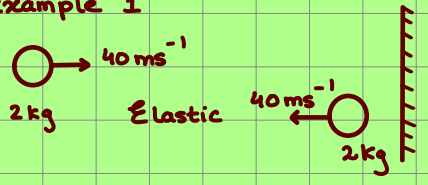
Graph :-



Area under the graph :-

$F \times t = \Delta P$ or I

Example 1



Calculate Δp or I.

Option 1

$\xrightarrow{+}$ 2(-40) - 2(40)

-160 Ns

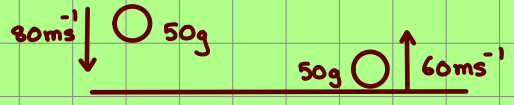
Option 2

$\xleftarrow{+}$ 2(40) - 2(-40)

+160 Ns

Take final direction as (+) to avoid (-)ve answer

Example 2 :- Inelastic



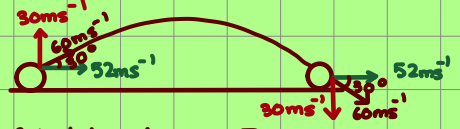
Calculate Δp

$+\uparrow$ m(60) - m(-80)

(0.05)(60) - (0.05)(-80)

+7 Ns.

Example 3 m= 3kg



Calculate Δp or I.

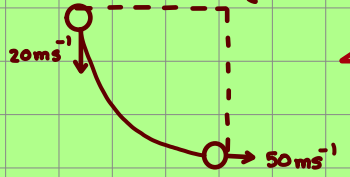
Horizontal momentum= 0

Vertical momentum :-

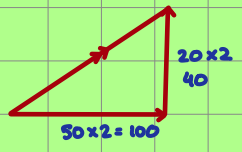
$+\downarrow$ 3(30) - 3(-30)

+180 Ns

Example 4



Calculate Δp or I.

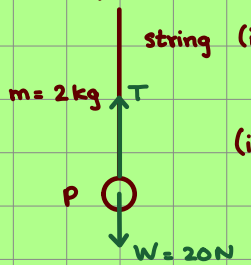


Δp or I = 107.7 ms^{-1}

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Application of $F = ma$ [Assume $g = 10$]

Example 1:- (i) Mark weight (W) and Tension (T)



(ii) Calculate T if the particle is at rest.

Forces balanced $\therefore$ $T = 20N$

(iii) Calculate T if the particle is moving at constant velocity.

Forces balanced $\therefore$ $T = 20N$

(iv) Calculate T if the particle is accelerating upwards at $2ms^{-1}$

$F = ma$ $\quad T - 20 = 2(2)$

$T - W = ma$ $\quad T = 24\ N$

(v) Calculate T if P is accelerating downwards at $3ms^{-2}$:

$W - T = ma$

$20 - T = 2(3)$

$T = 14N$

(vi) Calculate T if P is decelerating downwards at $5ms^{-2}$:

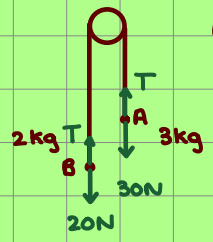
$W - T = ma$

$20 - T = 2(-5)$

$T = 30N$

- Weight is always marked vertically downwards
- Tension is always marked away from the concerned object
- In the same string, tension and acceleration will always remain the same.

Example 2:- How to apply '$F = ma$' in case of connected particles.



(i) Mark weight and tension (ii) Calculate T and acceleration.

$30 - T = 3a$ $\quad T - 20 = 2a$

$3a + T = 30$ → 1 $\quad T - 2a = 20$ → 2

$3a + T = 30$

$-[-2a + T = -20]$

$a = 2ms^{-2}$, $T = 24N$

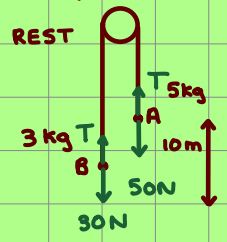
Shortcut for acceleration (MCQ)

$a = \dfrac{\text{Resultant Force}}{\text{Total mass}}$

$a = \dfrac{30 - 20}{3 + 2}$

$a = 2ms^{-2}$

Example 3:- (i) Calculate the speed of A (or B) when the masses cross each other



$a = \dfrac{50 - 30}{5 + 3}$

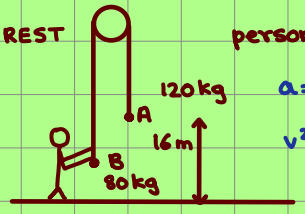
$a = 2.5ms^{-2}$

$v^2 = u^2 + 2as$

$v^2 = 2(2.5)(10)$

$v = 5ms^{-1}$

Example 4:- Calculate speed of the person



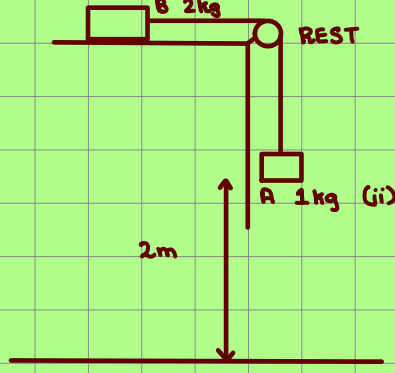
$a = 400/200 = 2ms^{-2}$

$v^2 - u^2 = 2as$

$v^2 = 2(2)(16)$

$v = 5.7ms^{-1}$

Example 5:-



(i) Calculate acceleration

$10 - T = a$ $\quad 10 = 3a$

$-0 + T = 2a$ $\quad a = 3.33ms^{-2}$

(ii) Calculate time taken for A to reach the ground

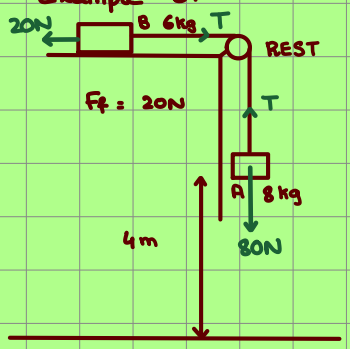
$s = \frac{1}{2}at^2$

$4 = (3.33)(t^2)$

$t = 1.1s$

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Example 6 :-



(i) Calculate acceleration

$80 - T = 8a$

$-20 + T = 6a$

$60 = 14a$

$a = 4.3 ms^{-2}$

(ii) Calculate final speed when particle hits the ground.

$v^2 = u^2 + 2as$

$v^2 = 2(4.3)(4)$

$v = 5.9 ms^{-1}$

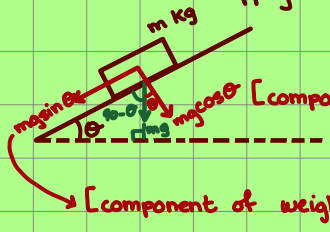
(iii) Find deceleration of B when A hits the ground

$T - 20 = 6a$

$0 - 20 = 6a$

$a = -3.3 ms^{-2}$ ∴ deceleration = $3.3 ms^{-1}$

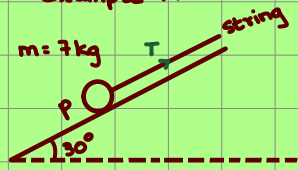
How to apply $F = ma$ in case of an inclined plane.



[component of weight perpendicular to the slope]

[component of weight parallel to the slope]

Example 7 :-



(i) Calculate T if P is at rest or is moving at constant velocity

$T = 70 \sin 30$

$T = 35N$

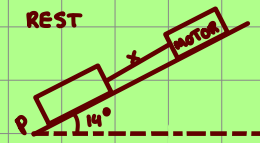
(ii) Calculate T if P accelerates upwards at $4 ms^{-2}$

$F = ma$

$T - 70 \sin 30 = 7(4)$

$T = 63N$

Example 8 Particle P accelerates at $0.13 ms^{-2}$ to travel a distance of 10m to reach Point X.



$m = 452kg$, $g = 9.81 ms^{-2}$

(i) Calculate component of weight down the slope

$4520 \sin 14 = 1074.5N$

(ii) If $F_t = 525N$, calculate tension in the rope.

$T - [1070 + 525] = [452][0.13]$

$T = 1654N$

(iii) Calculate the time taken for P to reach X.

$s = \frac{1}{2}at^2$

$10 = \frac{1}{2}(0.13)t^2$

$t = 12.4s$

(iv) Calculate the speed when P reaches X.

$v = at$

$v = (0.13)(12.4)$ ∴ $v = 1.61 ms^{-1}$

(v) At X, the string breaks, calculate deceleration of P

$T - (1070 + 525) = 452a$

$0 - 1070 - 525 = 452a$

deceleration = $3.53 ms^{-2}$

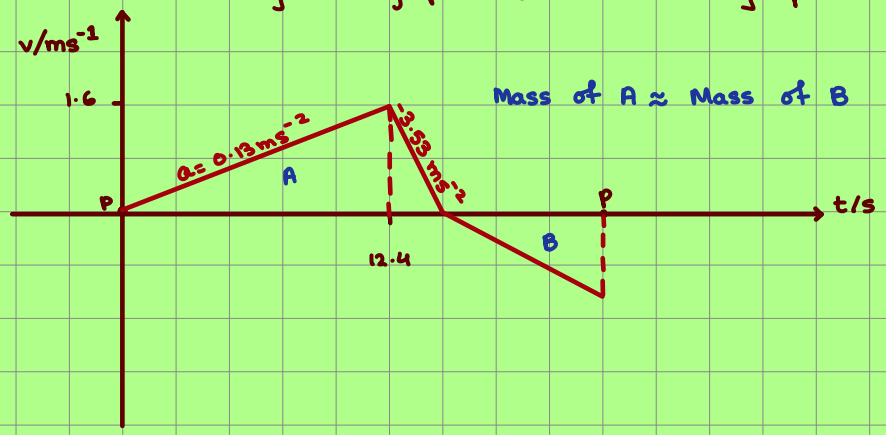
(vi) Eventually P starts to move down the slope, calculate its acceleration as it approaches the starting point.

$1070 - 525 = 452a$

$a = 1.21 ms^{-2}$

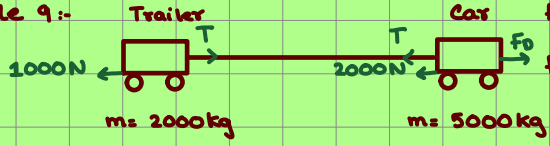
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(vii) Sketch the velocity-time graph from the starting point until when it returns back to the starting point.



Mass of A ≈ Mass of B

Example 9:-



f_f of car = 2000N

f_f of trailer = 1000N

(i) Car pulls the trailer at constant speed. Calculate the driving force [F_D] and tension (T) in the trailer.

$F_D - [T + 2000] = 0$ ∴ forces balanced.

$F_D - [3000] = 0$

$F_D = 3000N$

$T - 1000 = 0$

$T = 1000N$

(ii) Car pulls the trailer with an acceleration of $3ms^{-2}$. Find T and F_D

$F_D - [T + 2000] = 5000(3)$

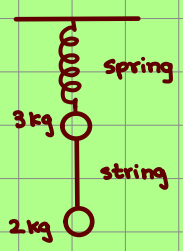
$F_D - 9000 = 5000$

$F_D = 24,000N$

$T - 1000 = 2000(3)$

$T = 7000N$

Example 10:- [Assume $g = 10ms^{-2}$]



(i) System is at rest. Find tension in the string

$T = 30 + 20$

$T = 50\ N$

(ii) The string is cut. Calculate the upward acceleration of 3kg mass.

$T - w = ma$

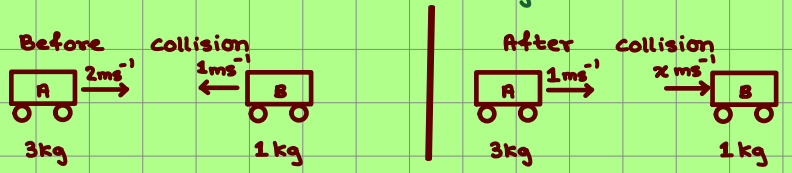
$20 = 3a$ ∴ $a = 6.67ms^{-2}$

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Principal of Conservation of Momentum

According to this principal, for an isolated system / closed system, the total momentum of the system always remains constant or conserved.

This principal can be applied on colliding objects and in case of collision, the total momentum before collision must be equal to the total momentum of the system after collision.



→ (+) Total P (before) = Total P (after)

$3(+2) - 1 = 3(1) + x$

$x = 2ms^{-1}$

Types of collisions:-

1) Elastic Collisions:-

- Total momentum of the system remains conserved
- Total energy of the system remains conserved
- Total Kinetic Energy of the system remains conserved
- The speed of approach (SOA) before collision must be equal to the speed of separation (SOS) after collision.

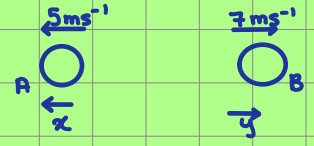
4 cases:-

(i) 

SOA [before collision]

$3 + 6 = 9ms^{-1}$

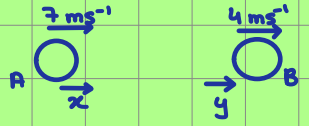
$(x+y)\ ms^{-1}$

(ii) 

SOS [after collision]

$5 + 7 = 12ms^{-1}$

$(x+y)\ ms^{-1}$

(iii) 

SOA [before collision]

$7 - 4 = 3ms^{-1}$

$(x-y)\ ms^{-1}$

(iv) 

SOS [after collision]

$12 - 3 = 9ms^{-1}$

$(y-x)\ ms^{-1}$

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Example 1:- Before Collision | After collision



The objects collide elastically. Find the speed of A and B after collision.

$\xrightarrow{+}$ $(200 \times 5) + [2 \times (-50)]$

$P = +900$

$900 = 5x + 2y$ → (1)

SOA = SOS

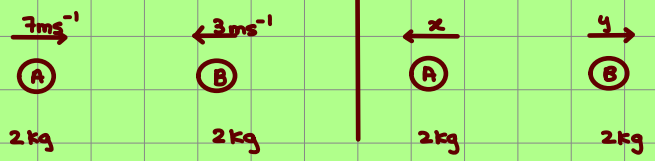
$200 + 50 = y - x$

$250 = y - x$ → (2)

$x = 400/7 \text{ ms}^{-1}$

$y = 307$

Example 2:- Before | After



The objects collide elastically. Find the speed of A and B after collision.

$\xrightarrow{+}$ $(2)(7) + (2)(-3) = -2x + 2y$

SOA = SOS

$2y - 2x = 8$ → (1)

$x + y = 10$ → (2)

$x = 3 \text{ ms}^{-1}$

$y = 7 \text{ ms}^{-1}$

For identical masses performing elastic collision, the speed of particles interchanges with each other. [A initial = B final, B initial = A final]

2) Inelastic Collisions:-

- Total momentum of the system remains conserved
- Total energy of the system remains conserved
- Total Kinetic Energy of the system does not remain conserved
- The speed of approach (SOA) before collision is not equal to the speed of separation (SOS) after collision.

Example 3:- Before | After



$\xrightarrow{+}$ $(1.6 \times 2) + 3(0) = 2v + 3(0.9)$

$v = 0.25 \text{ ms}^{-1}$

SOA ≠ SOS

$1.6 \neq 0.65$

Kinetic Energy [Before] $= \frac{1}{2} \times 2 \times (1.6)^2$

$\therefore 2.56 \text{ J}$

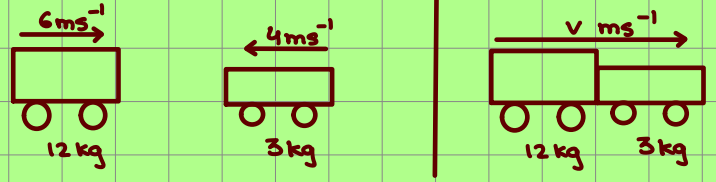
Kinetic Energy [After] $= \frac{1}{2} \times 2 \times (0.25)^2 = 0.0625$

$\frac{1}{2} \times 3 \times (0.9)^2 = 1.215$

$\therefore 1.28 \text{ J}$

$2.56 \text{ J} \neq 1.28 \text{ J}$

Example 4:- Before | After



Whenever particles stick together and move with a common velocity, nature of collision will always be inelastic.

$\xrightarrow{+}$ $12(+6) + 3(-4)$

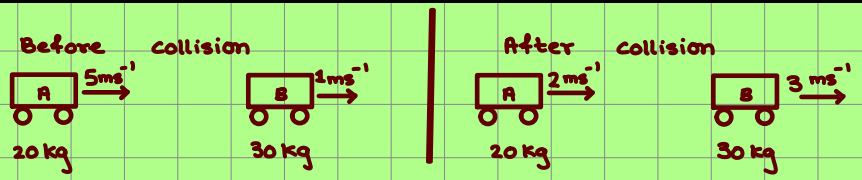
$v = +4 \text{ ms}^{-1}$ [Inelastic] because SOA ≠ SOS

$10 \neq 0$

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How to show that Momentum remains conserved in the case of Inelastic Collisions?

Example 1 :



(i) Show that collision is inelastic.

SOA SOS

5 - 1 = 3 - 2

4 ≠ 1

∴ SOA ≠ SOS hence inelastic

(ii) Calculate ΔP_B

$\xrightarrow{+}$ $P_f - P_i$

90 - 30 = +60Ns

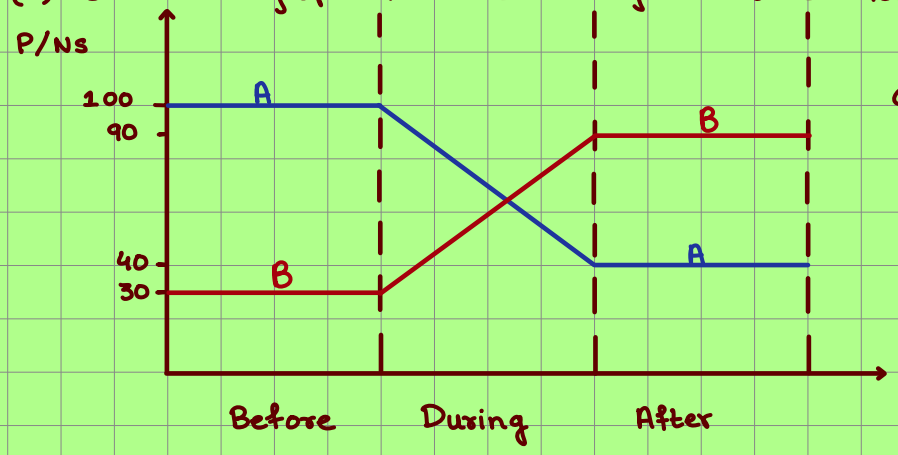
(iii) Calculate ΔP_A

$\xrightarrow{+}$ $P_f - P_i$

40 - 100 = -60Ns

(iv) Conclusion: The loss in momentum of one object is equal to the gain in momentum of the other object. Hence, momentum of the system remains conserved. $\Delta P_A = -\Delta P_B$

(v) Sketch a graph of momentum against time for both A and B.



Gradient = Force

(viii) Use part (iv) to verify Newton's 3rd law.

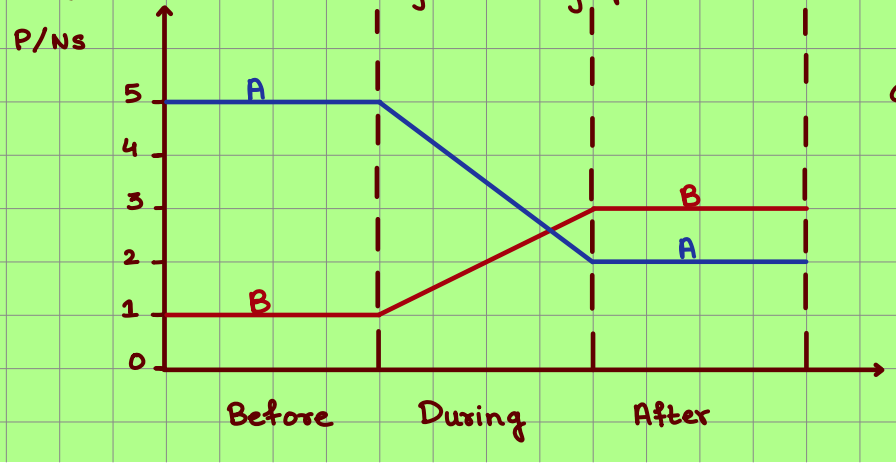
$\Delta P_A = -\Delta P_B$

$\Delta P = Fxt$ ∴ $F_A t_A = -F_B t_B$

Since $t_A = t_B$ $F_A = -F_B$

The force which A exerts on B must be equal and opposite to the force which B exerts on A.

(vi) Sketch a velocity - time graph for both A and B



Gradient = Acceleration

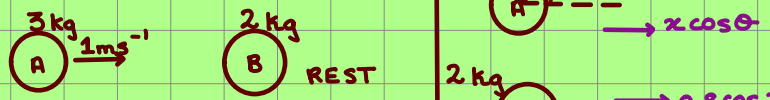
$P = mv$

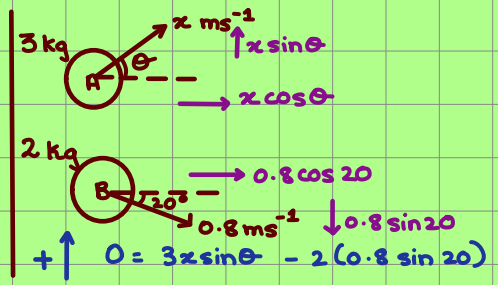
$P \propto v$ [same graph]

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How to apply P.O.C.O.M for 2-dimensional collisions

Example 1





$\rightarrow$ $3(1) + 0 = 3x\cos\theta + 2(0.8\cos 20)$ $\quad$ $+\uparrow$ $0 = 3x\sin\theta - 2(0.8\sin 20)$

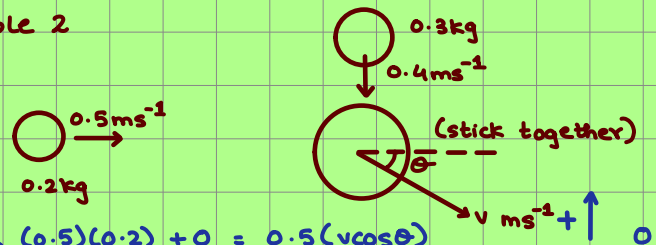
$3x\cos\theta = 1.497$ $\quad$ $3x\sin\theta = 0.574$

$x\cos\theta = 0.499$ $\quad$ $x\sin\theta = 0.182$

$$\frac{x\sin\theta}{x\cos\theta} = \frac{0.182}{0.499}$$

$\theta = 20^\circ$, $x = 0.53\ ms^{-1}$

Example 2



$\rightarrow$ $(0.5)(0.2) + 0 = 0.5(v\cos\theta)$ $\quad$ $+\uparrow$ $0 + (0.3)(0.4) = 0.5(v\sin\theta)$

$v\cos\theta = 0.2$ $\quad$ $v\sin\theta = 0.24$

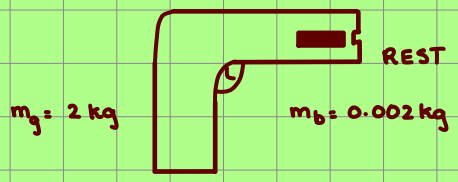
$$\frac{v\sin\theta}{v\cos\theta} = \frac{0.24}{0.2}$$

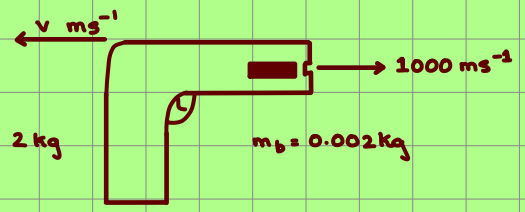
$\theta = 50.2^\circ$, $v = 0.31\ ms^{-1}$

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How to apply P.O.C.O.M in the case of Interacting Bodies

Example 1





(i) Why should the gun recoil backwards?

If the initial momentum is zero, then for the principle to be valid, the final momentum of the interacting bodies must also be zero which is only possible if the 2 particles have equal momentum in the opposite direction.

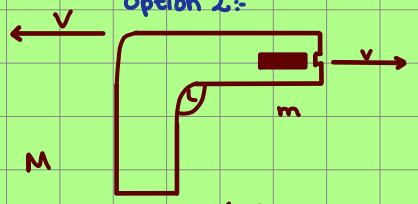
(ii) Calculate the recoil velocity 'v' ms^{-1}

Option 1:- P.O.C.O.M

$\xrightarrow{+}$ $0 = 2(-V) + (0.002)(1000)$

$V = 1\ ms^{-1}$

Option 2:-



$MV = mv$

$(2)(V) = (0.002)(1000)$

$V = 1\ ms^{-1}$

Option 3:- $\frac{M}{m} = \frac{v}{V}$

If mass of the gun is 1000 times greater than mass of the bullet, the velocity of the gun must be 1000 times smaller than the velocity of the bullet. $\therefore V = 1\ ms^{-1}$

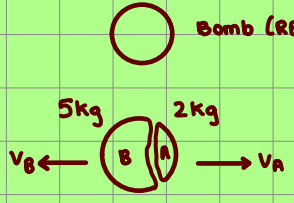
(iii) Show that K.E. depends on velocity rather than mass.

$$\frac{\text{KE of gun}}{\text{KE of bullet}} = \frac{\frac{1}{2}.M.V.V}{\frac{1}{2}.m.v.v}$$

Since $MV = mv$

$$\frac{\text{KE of gun}}{\text{KE of bullet}} = \frac{\frac{1}{2}.\cancel{M}.V.V}{\frac{1}{2}.\cancel{m}.v.v}$$ $\therefore$ proven

Example 2 Total Kinetic Energy = E Joules.



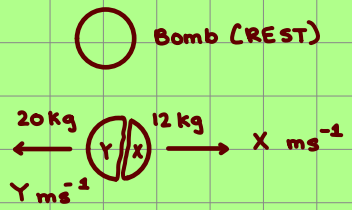
(i) Find KE of A and B.

$\frac{M_A}{M_B} = \frac{2}{5}$ $\therefore$ $\frac{V_A}{V_B} = \frac{5}{2}$ $\therefore$ $\frac{KE_A}{KE_B} = \frac{5}{2}$

$KE_A = \frac{5}{7} \times E$ $KE_B = \frac{2}{7} \times E$

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Example 3



Total K.E. after = 5000 J.

Find x and Y

$\frac{M_X}{M_Y} = \frac{12}{20}$ ∴ $\frac{V_X}{V_Y} = \frac{20}{12}$ ∴ $\frac{KE_X}{KE_Y} = \frac{20}{12}$

$K.E_X = \frac{20}{32} \times 5000$

$K.E_X = 3125\ J$

$3125 = \frac{1}{2}(12)(x^2)$

$x = 22.8\ ms^{-1}$

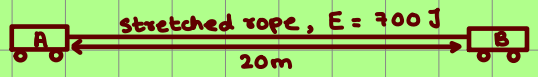
$K.E_Y = \frac{12}{32} \times 5000$

$K.E_Y = 1875\ J$

$1875 = \frac{1}{2}(20)(Y)^2$

$Y = 13.7\ ms^{-1}$

Example 4



410 kg REST

610 kg REST

When released, A and B move towards each other with speeds of x ms⁻¹ and y ms⁻¹ respectively.

(i) Assuming all energy converts into K.E., find x and y. (ii) Calculate the time taken for them to collide.

$\frac{M_A}{M_B} = \frac{410}{610}$ ∴ $\frac{V_A}{V_B} = \frac{610}{410}$ ∴ $\frac{KE_A}{KE_B} = \frac{610}{410}$

$K.E_A = \frac{610}{1020} \times 700$

$K.E_A = 418.6\ J$

$\frac{1}{2}(410)(x)^2 = 418.6\ J$

$\therefore x = 1.43\ ms^{-1}$

$K.E_B = \frac{410}{1020} \times 700$

$K.E_B = 281.4\ J$

$\frac{1}{2}(610)(y)^2 = 281.4$

$\therefore y = 0.96\ ms^{-1}$

$d_A + d_B = 20$

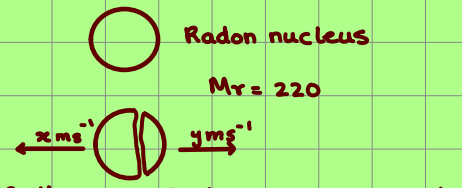
$1.43t + 0.96t = 20$

$t = 8.4\ s$

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Concept of Interacting Bodies when applied to sub-atomic particles

Example 1 :-



(i) Given that the K.E. of Helium nucleus is 1.0×10^{-17} J.
Calculate speed of the Helium nucleus.

$\frac{1}{2}(4u)(y)^2 = 1.0 \times 10^{-17}$ where u = unified atomic mass

$(2)(1.66 \times 10^{-27})(y)^2 = 1.0 \times 10^{-17}$ $u = 1.66 \times 10^{-27}$ Kg

$y = 5.5 \times 10^4 \text{ ms}^{-1}$

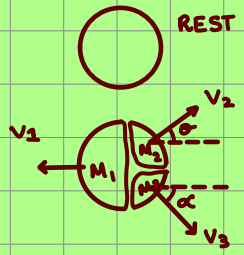
(ii) Hence, calculate the value of x.

$0 = (216u)(-x) + (4u)(y)$

$216(1.66 \times 10^{-27})(x) = 4(1.66 \times 10^{-27})(5.5 \times 10^4)$

$x = 1.02 \times 10^3 \text{ ms}^{-1}$

Concept of Interacting Bodies when applied to two dimensions



$\xrightarrow{+}$ $0 = (M_1)(-V_1) + (M_2)(V\cos\theta) + (M_3)(V_3\cos\alpha)$

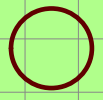
$M_1V_1 = M_2V_2\cos\theta + M_3V_3\cos\alpha$

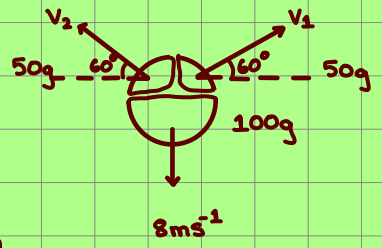
$+\uparrow$ $0 = 0 + (M_2)(V_2\sin\theta) + (M_3)(-V_3\sin\alpha)$

$M_2V_2\sin\theta = M_3V_3\sin\alpha$

For the momentum to be conserved, momentum in one direction must be balanced with momentum in the other direction.

Example 1





Find V_1 and V_2.

$\xrightarrow{+}$ $0 = (0.05)(V_1\cos60) + (0.05)(-V_2\cos60)$

$(0.05)(V_1\cos60) = (0.05)(V_2\cos60)$

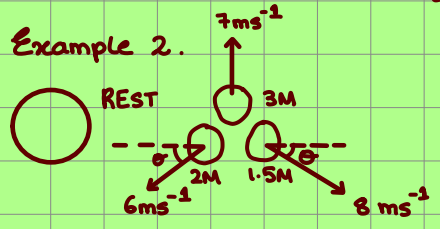
$V_1 = V_2$

$+\uparrow$ $(50)(V_2\sin60) + (50)(V_1\sin60) = 8(100)$

$25\sqrt{3}\,V + 25\sqrt{3}\,V = 800$

$V_1 = V_2 = 9.24 \text{ ms}^{-1}$

Example 2.



Find θ.

$+\uparrow$ $(3M)(7) = (2M)(6\sin\theta) + (1.5M)(8\sin\theta)$

$\theta = 61^\circ$

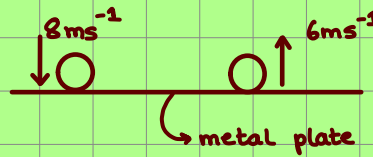
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Example 3.

REST ○ m kg

Explain how P.O.C.O.M is applicable for this collision.

- Ball loses its momentum upon contact
- This loss in momentum is transferred into the metal plate / ground
- Although momentum of the ball changes, momentum of the system (ball + metal plate remains constant or conserved.



Example 4. Relationship between K.E. and momentum.

(i) Show that $K.E. = P^2/2m$

$K.E. = \frac{1}{2}mv^2$

$P = mv$

$K.E. = \frac{1}{2}m\left[\frac{P}{m}\right]^2$

$K.E. = P^2/2m$

(ii) Hence, show that if initial momentum is p_i and final momentum is p_f,

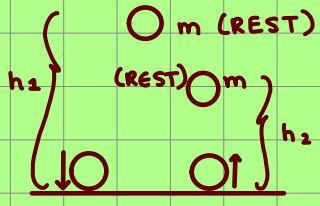
$\Delta K.E.$ is given by:- $\Delta K.E. = P_2^2/2m - P_1^2/2m$

$K.E_f = P_2^2/2m$

$K.E_i = P_1^2/2m$

$\therefore \Delta K.E. = P_2^2/2m - P_1^2/2m$

Example 5. Show that ΔP during collision is given by the formula:-



$\Delta P = m\sqrt{2gh_2} + m\sqrt{2gh_1}$

Gain in K.E. = Loss in P.E.

$\frac{1}{2}mv^2 = mgh_1$

$v = \sqrt{2gh_1}$

Loss in K.E. = Gain in P.E.

$\frac{1}{2}mv^2 = mgh_2$

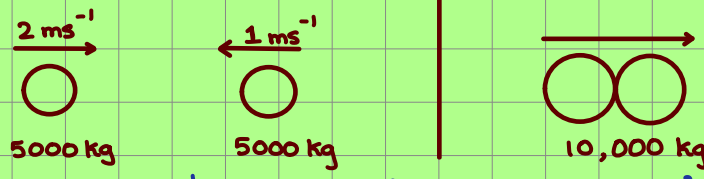
$v = \sqrt{2gh_2}$

$\Delta p = mv - mu$

$\Delta p = m\sqrt{2gh_2} - m\left[-\sqrt{2gh_1}\right]$

$\Delta p = m\sqrt{2gh_2} + m\sqrt{2gh_1}$

Example 6.



Show that the loss in KE of the system is 11,250J.

$\rightarrow$ (5000)(2) + (5000)(−1) = 10,000v

$v = 0.5\ ms^{-1}$

K.E. before collision = $\frac{1}{2}(5000)(2)^2 + \frac{1}{2}(5000)(1)^2 = 12,500\ J$

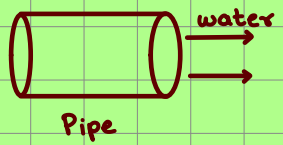
K.E. after collision = $\frac{1}{2}(10,000)(0.5)^2 = 1250\ J$

12,500 − 1,250 = 11,250 J

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Example 7.

Density = ρ , Area = A , Speed of water = v





Water hits the wall and comes to rest. Show that the force exerted by water on the wall is given by:-

$F = A\rho v^2$

$F = \Delta P / t$

$F = \frac{P_i - P_f}{t}$

$F = mu / t$

$m = \rho v$

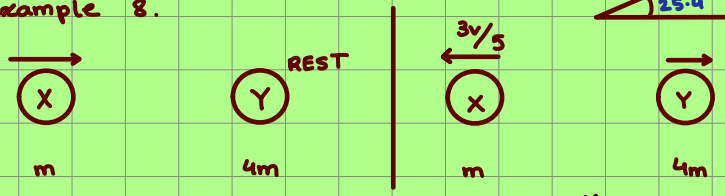
$F = \frac{\rho v . v}{t}$

$F = \frac{\rho (A \times h) . v}{t}$

$F = \rho (Av) v$

$F = A\rho v^2$

Example 8.



Show that KE_y after collision is $^{16}/_{50} mv^2$

P.O.C.O.M

$\rightarrow$ (+)

$mv + 0 = m(-^{3v}/_5) + x$

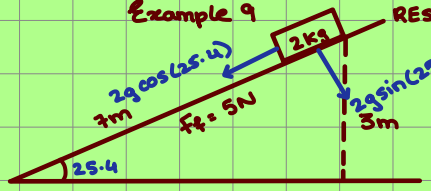
$mv + ^{3mv}/_5 = x$

$x = ^2/_5 v$

K.E. after collision :- $\frac{1}{2}(4m)(^2/_5 v)^2$

$\therefore$ $^{16}/_{50} mv^2$ $\therefore$ shown!

Example 9



Show that the speed of the block at the bottom is approximately $4.9\ ms^{-1}$

$\sin\theta = 3/7$

$\theta = 25.4^\circ$

$F = ma$

$2(9.81)\sin(25.4) = 2a$

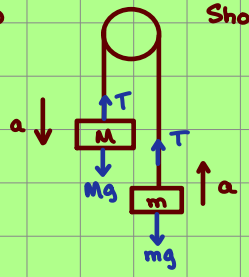
$a = 1.707\ ms^{-2}$

$v^2 = u^2 + 2as$

$v^2 = 2(1.707)(7)$

$v = 4.9 ms^{-1}$ $\therefore$ shown!

Example 10



Show that the acceleration (a) is given by:-

$a = \frac{(M-m)}{(M+m)} g$

$Mg - T = Ma$

$Mg - Ma = T$

$T - mg = ma$

$T = ma + mg$

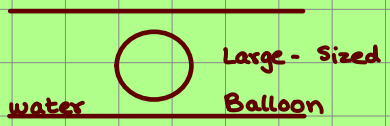
$Mg - Ma = ma + mg$

$Mg - mg = ma + Ma$

$\frac{g(M-m)}{(M+m)} = a$ $\therefore$ shown!

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Example 11.



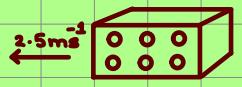
Balloon is at rest in water. Its total mass is 3200kg. If 200 kg of mass is removed from it, calculate upwards acceleration (g = 10)



$32,000 - 30,000 = 3000(a)$

$a = 0.67\ ms^{-2}$

Example 13



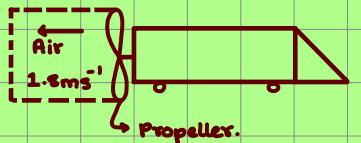
Paint is initially at rest and is ejected through 400 holes at $2.5ms^{-1}$. Each hole has an area of $0.4mm^2$. The density of paint is $900\ kg/m^3$. Calculate the force produced.

$F = \rho A v^2$

$F = (900)(0.4 \times 10^{-6})(2.5)^2$

$F = 0.9N$

Example 12.



The propeller of radius 0.045m pushes air backward with a speed of $1.8ms^{-1}$. Calculate force on the toy car if ρ_{air} is $1.3\ kgm^{-3}$ given that air was initially at REST.

$F = \Delta P / t$

$F = P_f - 0 / t$

$F = mv / t$

$F = \frac{\rho \times V \times v}{t}$

$F = \frac{\rho \times \pi r^2 \times h \times v}{t}$

$F = \rho(\pi r^2)(v)^2$

$F = (1.3)(\pi)(0.045)^2(1.8)^2$

$F = 0.027\ N$

The backward force exerted by the propeller on the air is equal or opposite to the forward force exerted by the air on the propeller.

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Newton's Third Law and its Application

$F_A = -F_B$

- Forces are equal in magnitude
- Forces are opposite in direction
- Forces act on different bodies
- Forces are of the same type / nature

How to calculate the Action / Reaction force?

(1) In the Horizontal Plane

(a) 

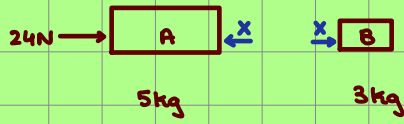
(i) Calculate acceleration

$F = ma$

$24 = 8a$

$a = 3ms^{-2}$

(ii) Calculate force that A exerts on B.



From A:- $24 - X = 5(3)$

$X = 9N$

From B:- $X = 3(3)$

$X = 9N$

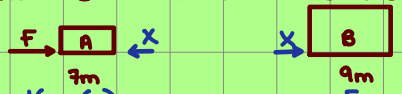
Shortcut:- $\frac{m_B}{m_A + m_B} \times F$

$\frac{3}{8} \times 24$

$X = 9N$

(b) 

(i) Calculate force that A exerts on B



$F = 16m\ (a)$

$a = \frac{F}{16m}$

From A:- $F - X = (7m)\left[\frac{F}{16m}\right]$

$X = \frac{9F}{16}$

From B:- $X = (9m)\left[\frac{F}{16m}\right]$

$X = \frac{9F}{16}$

Shortcut = $\frac{9m}{16m} \times F$

$X = \frac{9F}{16}$

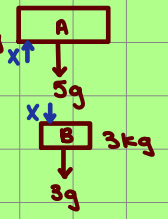
(2) In the Vertical Plane



Free fall

Calculate force that A exerts on B.

$a = 9.81\ ms^{-2}$



$5g - X = 5g$

$X = 0N$

$3g - X = 3g$

$X = 0N$

Statement:- for particles falling freely [in vertical plane], the action / reaction force always amount to zero.

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Forces

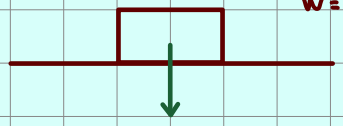
1) Pressure, density, Upthrust
2) Turning effect [Moments]

Pressure

FOR SOLIDS:-

$P = F/A$ or $P = W/A$ or $P = mg/A$

Weight must always be perpendicular to the base area



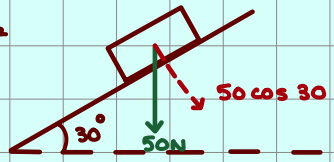
$W = 50N$, $A = 5m^2$

$50N$

$P = W/A$

$P = 50/5$

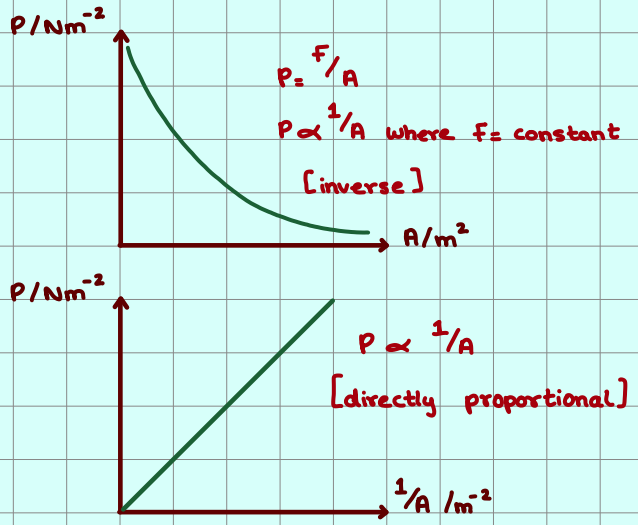
$P = 10\ Nm^{-2}$



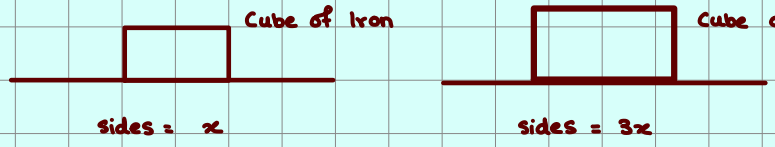
$P = W/A$

$P = 50\cos 30 / 5$

$P = 5\sqrt{3}\ Nm^{-2}$



How to calculate pressure in solids?



Given that $\rho_B = 2\rho_I$, calculate P_B/P_I

$P_I = \frac{mg}{A}$

$P_I = \frac{\rho_I \times V \times g}{A}$ $P_B = \frac{\rho_B \times V \times g}{A}$

$P_I = \frac{\rho_I \times x^3 \times g}{x^2}$ $P_B = \frac{\rho_B \times 27x^3 \times g}{9x^2}$

$P_I = \rho_I . x . g$ $P_B = \rho_B . 3x . g$

$P_B = 2\rho_I . 3x . g$ $P_I = \rho_I . x . g$

$\frac{P_B}{P_I} = \frac{2\rho_I . 3x . g}{\rho_I . x . g}$

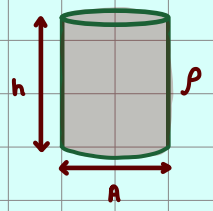
$\frac{P_B}{P_I} = \frac{6}{1}$

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FOR FLUIDS [LIQUIDS AND GASES]

$P = \rho g h$ or $\Delta P = \rho g . \Delta h$ [Any change in Pressure brings about a corresponding change in depth]

Derivation :-



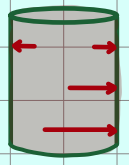
$P = {}^{mg}/_{A}$

$P = \frac{\rho \times V \times g}{A}$ $\therefore P = \rho g h$

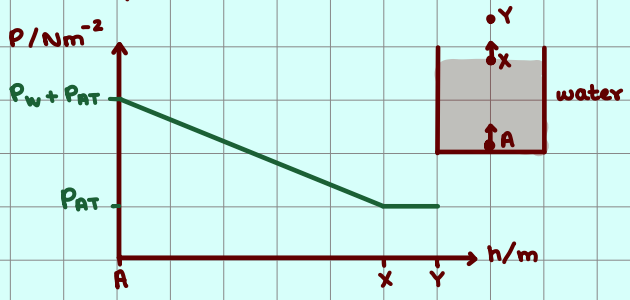
$P = \frac{\rho \times (A \times h) \times g}{A}$

Properties :-

- Pressure increases with depth
- Points at the same depth experience the same pressure
- Pressure exerted by fluid is at right angles to the surface



Graph of Pressure in fluids vs distance.





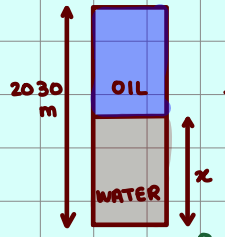
$P = \rho g h$

$Y = m x$

gradient $\propto \rho$

The higher the density, the steeper the gradient and vice versa.

Example 1 :- Closed Tube



$\rho_w = 1000\ kgm^{-3}$

$\rho_o = 850\ kgm^{-3}$

Pressure at the bottom = 18MPa

Calculate x.

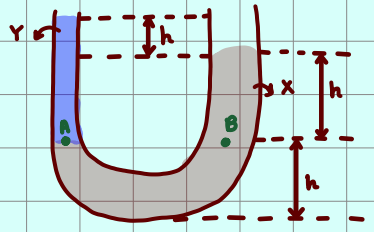
$P_{(bottom)} = P_w + P_o$

$18 \times 10^6 = \rho_w . g . (x) + \rho_o . g . (2030 - x)$

$18 \times 10^6 = (1000)(9.81)(x) + (850)(9.81)(2030 - x)$

$x = 730\ m.$

Example 2 :- U-shaped tube [Open at both ends]



Calculate ratio of ρ_x / ρ_Y

$P_A = P_B$

$P_Y + P_{AT} = P_x + P_{AT}$

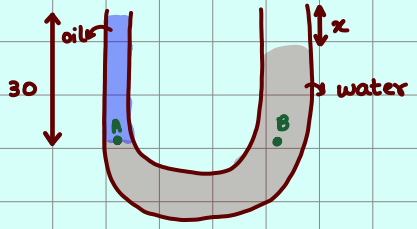
$\rho_Y . g . 2h = \rho_x . g . h$

$\rho_Y / \rho_x = {}^{h}/_{2h}$

$\rho_x / \rho_Y = {}^{2}/_{1}$

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Example 3:- Calculate x given that $\rho_w = 1000$ and $\rho_o = 850$



$P_A = P_B$

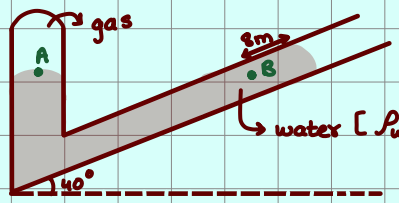
$P_o + P_{AT} = P_w + P_{AT}$

$\rho_o . g . (30) = \rho_w . g . (30 - x)$

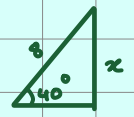
$850\,(30) = 1000\,(30 - x)$

$x = 4.5$ units

Example 4:- U-shaped tube [one open, one closed]



Calculate Pressure exerted by the gas.



$\sin 40 = x/8$

$x = 8 \sin 40$

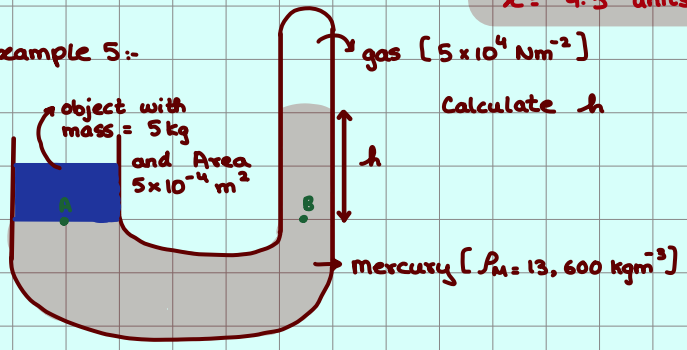
$P_A = P_B$

$P_{gas} = \rho_w . g . h$

$P_{gas} = (1000)(9.81)(8)$

$P_{gas} = 1.5 \times 10^5\ Nm^{-2}$

Example 5:-



Calculate h

$P_A = P_B$

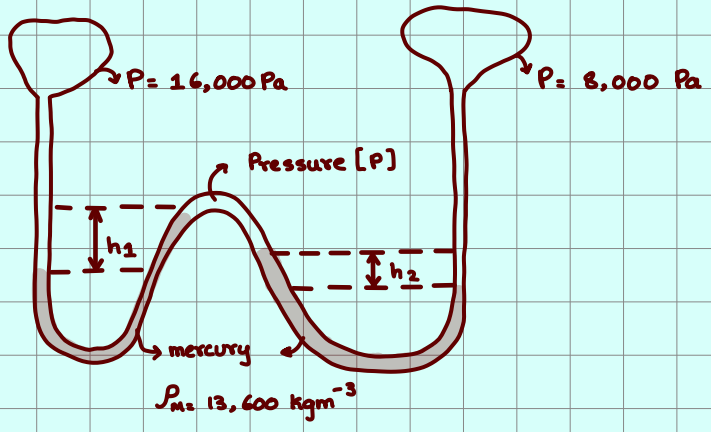
$mg/A = P_g + P_M$

$mg/A = P_g + \rho_M . g . h$

$5(9.81)/5 \times 10^{-4} = 5 \times 10^4 + [(13600)(9.81)(h)]$

$h = 1.12\ m$

Example 6:- U-shaped tube [closed at both ends]



Show that $h_1 - h_2 = 6cm$

$16,000 - P = \rho g h_1$

$-8,000 \overset{+}{-} P = -\rho g h_2$

$8000 = \rho g (h_1 - h_2)$

$\frac{8000}{13,600 \times 9.81} = h_1 - h_2$

$(h_1 - h_2) = 0.06m$

$h_1 - h_2 = 6cm$

OR

$\Delta P = \rho g \Delta h$

$16,000 - 8000 = (13,600)(9.81)(h_1 - h_2)$

$h_1 - h_2 = 0.06m$

$h_1 - h_2 = 6cm$

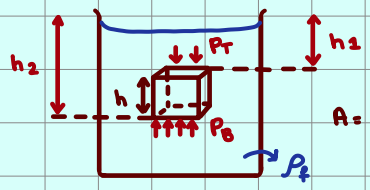
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Upthrust

- Upthrust is defined as an upward force exerted by a fluid when an object is immersed in it.
- Units = N OR $Kgms^{-2}$

Why does Upthrust act?

1) Since pressure increases with depth, therefore pressure exerted on the bottom face [P_B] will always be greater than the pressure exerted on the top face [P_T]
2) The difference in pressure applies a vertically upward force known as Upthrust.



A = base area

- If $W > U$, object sinks
- If $W = U$, object stays at rest or moves with constant speed
- If $U > W$, object rises.

Formulas for Calculation:-

1) $U = [P_B - P_T] . A$ [Only applicable for Uniform bodies]
2) $U = [P_B - P_T] . A$

$U = [\rho_f . g . h_2 - \rho_f . g . h_1] . A$

$U = \rho_f . g . A [h_2 - h_1]$

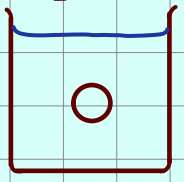
$U = \rho_f . g . A . H$

$U = \rho_f . g . V_o$

where:- ρ_f = density of the fluid
g = acceleration due to gravity
V_o = Volume of the object immersed in the fluid

$U \propto \rho_f$

Example 1:-



mass of ball = 50kg

Volume = 6.0×10^{-2} m^3

ρ_f = 1020 kg/m^3

(i) The ball is held at the start. Suggest what happens when it is released

$W = mg$

$W = 50 \times 9.81$

$W = 491 N$

$U = \rho_f . g . V_o$

$U = 1020 \times 9.81 \times [6.0 \times 10^{-2}]$

$U = 600N$

Since $U > W$, the object starts to rise

(ii) Hence, calculate the initial acceleration as the object begins to rise

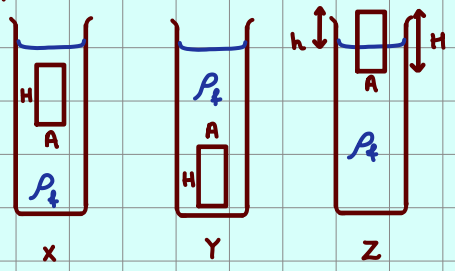
$F = ma$

$600 - 491 = 50(a)$

$a = 2.2\ ms^{-2}$

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Example 2



(i) Compare Upthrust in each case.

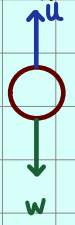
$U_X = \rho_f . g . A . H$ $U_Y = \rho_f . g . A . H$ $U_Z = \rho_f . g . A . [H-h]$

$U_X = U_Y$

U_Z = Least

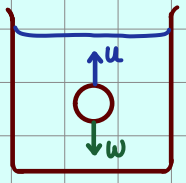
How can we mark relevant forces for an object in a fluid?

(i) A ball falls in AIR at constant speed



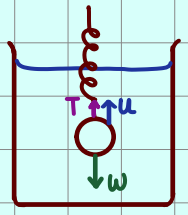
W = U because ball falls at constant speed

(ii) A ball is at rest inside a fluid



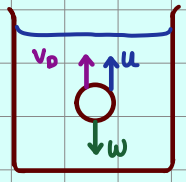
W = U because ball is at rest

(iii) An object at rest in a fluid is held using a spring



W = U + T, where 'W' is the largest because it alone balances two forces.

(iv) A gas molecule falls at constant velocity inside a fluid

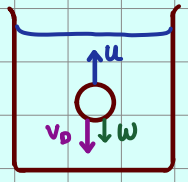


Viscous drag (V_D)

- Direction will be against the motion of the object
- Only acts when the object is moving
- V_D increases as speed increases

W = U + V_D, where 'W' is the largest because it alone balances two forces.

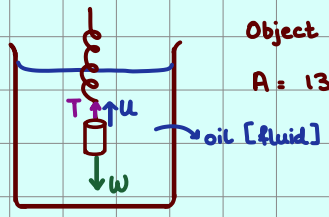
(v) A gas molecule rises at constant speed inside a fluid



U = W + V_D where 'U' is the largest because it alone balances two forces and 'W' is the smallest since we are talking about a gas molecule.

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Example 1



Object is at rest. Given that T = 4.8 N, W = 5.3N, A = 13 cm^2, h = 5cm, calculate density of oil

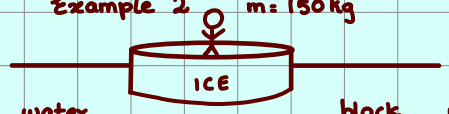
$W = U + T$

$5.3 = \rho_{oil} \cdot g \cdot V_0 + 4.8$

$0.5 = \rho_{oil} \times (9.81) \times [(13 \times 10^{-4})(0.05)]$

$\rho_{oil} = 784\ kg/m^3$

Example 2



water

$\rho_f = 1020\ kg/m^3$

$A_{ice} = 7m^2$

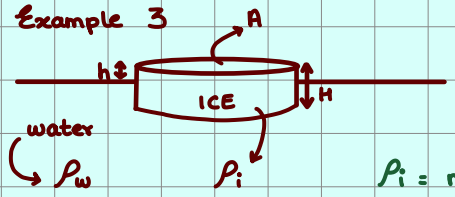
Calculate 'd' in cm through which the block moves before coming to rest

$W = U$

$150 \times 9.81 = 1020 \times 9.81 \times 7 \times d$

$d = 0.021\ m \quad \therefore \quad d = 2.10\ cm$

Example 3



The block is at rest. Show that:-

$$h = \left[1 - \frac{\rho_i}{\rho_w}\right].H$$

$\rho_i = m_i / V$

$m_i = \rho_i . A . H$

$W = U$

$\rho_i . A . H . g = \rho_w . g . A . (H - h)$

$\rho_i . H = \rho_w H - \rho_w h$

$\rho_w h = \rho_w H - \rho_i H$

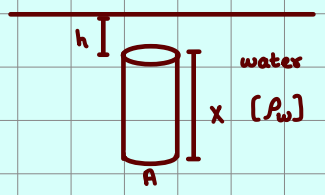
$$h = \frac{\rho_w H}{\rho_w} - \frac{\rho_i H}{\rho_w}$$

$$h = H - \frac{\rho_i H}{\rho_w}$$

$$h = H\left[1 - \frac{\rho_i}{\rho_w}\right]$$

$$h = \left[1 - \frac{\rho_i}{\rho_w}\right].H \quad \therefore \text{ shown !}$$

Example 4



(i) Calculate the Pressure difference between the bottom face and the top face

$P_B - P_T$

$\rho_w . g . [X + h] - \rho_w . g . h$

$\rho_w . g [X + h - h]$

$\rho_w . g . X$

(ii) Hence, show that upthrust equals $\rho_w . g . A . X$

$U = (P_B - P_T) . A$

$U = \rho_w . g . X . A$

$U = \rho_w . g . A . X \quad \therefore$ shown!

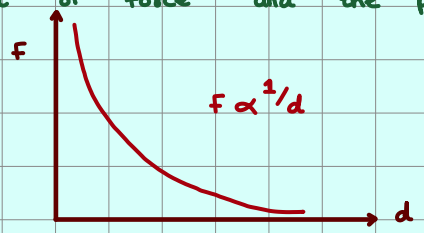
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Turning Effect

- Also known as Moments or Torque
- It is defined as :- " The product of force and the perpendicular distance between the line of action of force and the pivot."

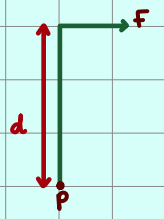
Moments = F x d

Units = Nm or Ncm

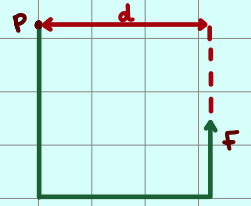


$F \propto 1/d$

Case 1 :-

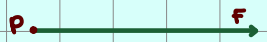


Case 2 :-



the Line of action of force passes through the pivot.

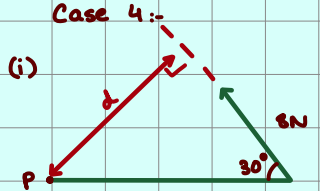
Case 3 :-



M = 0 Nm because

Case 4 :-

(i)



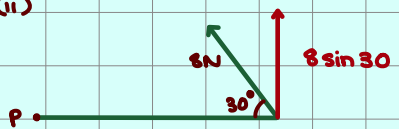
10m

$10 \sin 30 = d$

$M = 8 \times 10 \sin 30$

$M = 40$ Nm

(ii)



10m

$F = 8 \sin 30$

$M = 10 \times 8 \sin 30$

$M = 40$ Nm

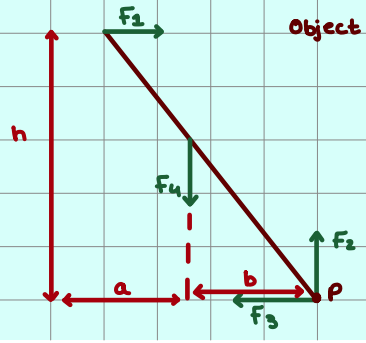
Principle of Moments :-

For an object to remain in equilibrium, sum of clockwise Moments about a point must be equal to the sum of Anti-clockwise moments about the same point.

Conditions for an object to remain in equilibrium :-

- No resultant forces [Balanced forces]
- No resultant torque [Σ CWM = Σ ACWM]

Example 1 :-



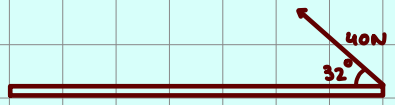
Object is in equilibrium. Apply Principle of Moments using P as the Pivot

$CW = ACWM$

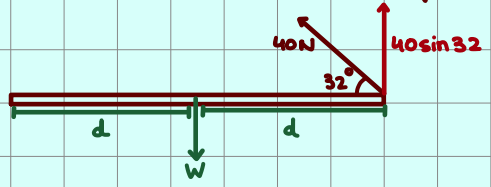
$F_1 . h = F_4 . b$

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Example 2 :-



Given that the rod is uniform and in equilibrium, find its weight.

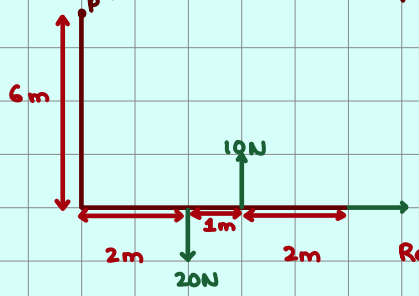


CW = ACW

W.~~d~~ = 40sin32.2~~d~~

W = 42.4N

Example 3 :- Not in equilibrium



CWM = 20x2

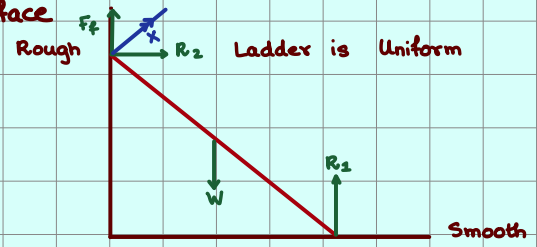
CWM = 40Nm

ACWM = (10x3) + (50x6)

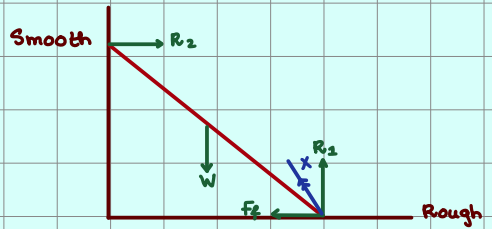
ACWM = 330Nm

Resultant Moment = 290 Nm (ACW)

How to mark forces acting on an object which is at rest and in contact with a smooth and a rough surface

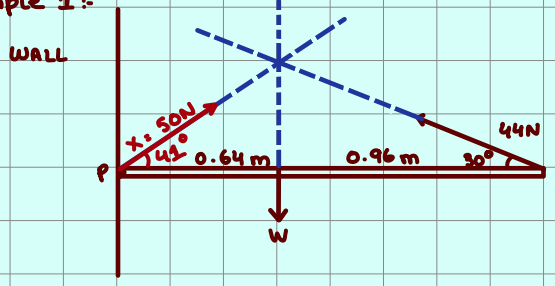


Ladder is Uniform



How to calculate the magnitude and direction of Force X ?

Example 1 :-



(i) The rod is in equilibrium. Calculate its weight.

CW = ACW

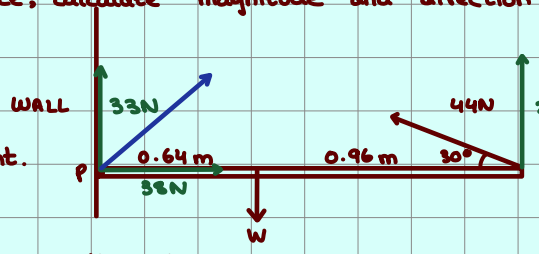
W x 0.64 = 1.6 (44 sin30)

W = 55 N

(ii) Explain why the wall must exert a force X on the rod at Point P ?

To allow equilibrium to be established/ forces to be balanced in both the vertical and horizontal planes

(iii) Hence, calculate magnitude and direction of force X.



$X = \sqrt{33^2 + 38^2}$

X = 50N

$\theta = \tan^{-1}\left[\frac{33}{38}\right]$

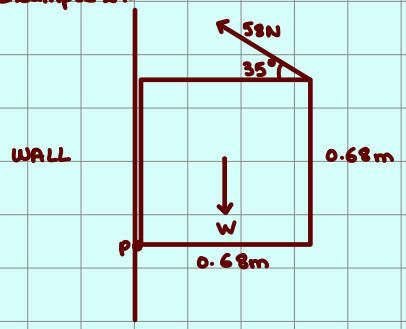
$\theta = 41°$

(iv) Draw Force X on the diagram.

If magnitude and direction is not given, construct X such that all the Lines of Action of forces pass through a common point.

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Example 2:-



(i) Box is in equilibrium. Calculate its weight

CW = ACW

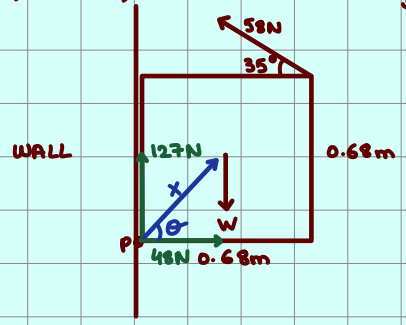
$0.34 \cdot W = [58\sin 35 \times 0.68] + [58\cos 35 \times 0.68]$

$W = 162\ N$

(ii) Explain why the wall must exert a force X on the rod at Point P?

To allow equilibrium to be established/ forces to be balanced in both the vertical and horizontal planes

(iii) Hence, calculate the magnitude and direction of Force X.



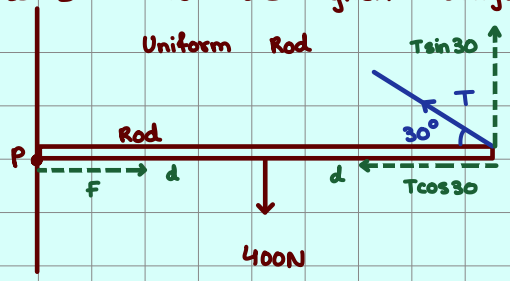
$X = \sqrt{127^2 + 48^2}$

$X = 136N$

$\theta = \tan^{-1}\left[\frac{127}{48}\right]$

$\theta = 69°$

Example 3:- For the given diagram, the object is in equilibrium,



(i) Calculate Tension in the string

$T\sin 30 \times 2d = 400 \times d$

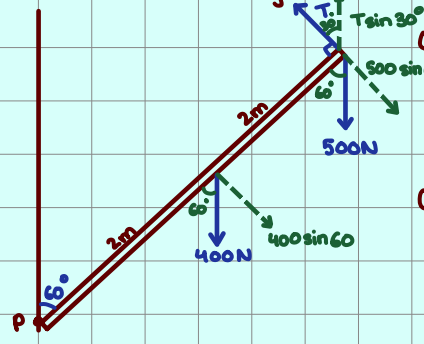
$T = 400N$

(ii) Hence, calculate the horizontal force which the wall exerts on the rod P.

$F = 400\cos 30$

$F = 350N$

Example 4:- For the given diagram, the object is in equilibrium,



(i) Find Tension in the string.

$4T = (400\sin 60)(2) + (500\sin 60)(4)$

$T = 610N$

(ii) Find the vertical component of this Tension

$T\cos 30$

$610\cos 30 = 530\ N$

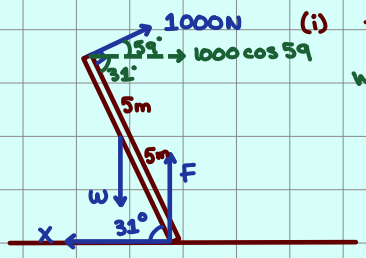
(iii) Hence, calculate the vertical force exerted by the wall on the rod at Point P.

$(400 + 500) - 530N$

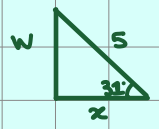
370 N upwards.

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Example 5:- The diagram below shows a rod pivoted on the ground at P. Given that the rod is in equilibrium,



(i) Find the weight of the rod.



$5\cos 31 = x$

$5\cos 31 \times W = 1000 \times 10$

$W = 2300N$

(ii) Calculate the value of X.

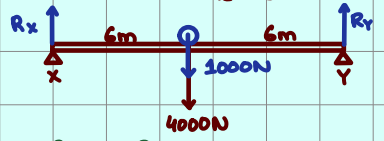
$1000 \cos 59 = X$

$X = 515N$

How to calculate the reaction force(s) exerted at the support?

Example 1:- Calculate R_x and R_Y when the package is at the following positions.

(i)

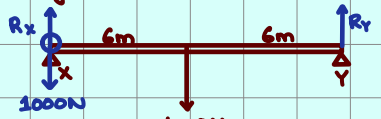


$R_x + R_Y = 5000$

$R_x = R_Y$

$2R = 5000 \quad \therefore R_x = R_Y = 2500N$

(ii)



Assume Y to be pivot

$(1000 \times 12) + (4000 \times 6) = 12 R_x$

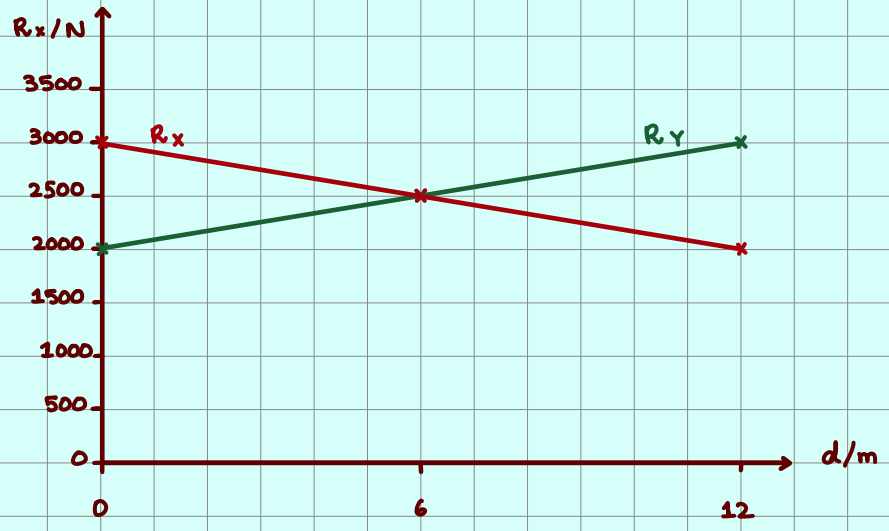
$R_x = 3000N$

$R_Y = 5000 - 3000$

$R_Y = 2000N$

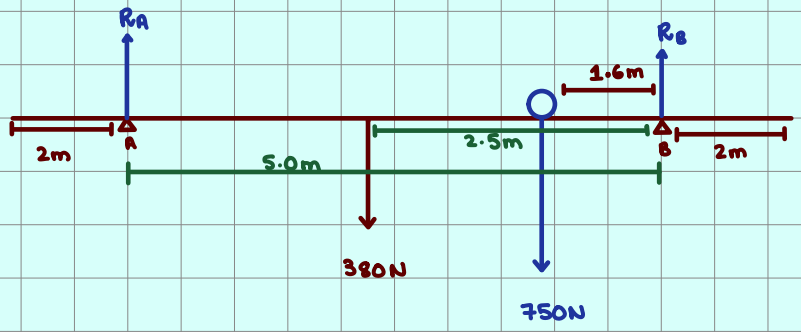
If Package is at Y, the values of R_x and R_Y is going to flip.

(iii) Hence draw a graph for R_x and R_Y as the package moves from X to Y



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Example 2:-



For the given diagram, the uniform rod of length 9m is in equilibrium. The weight of the rod is 380N. A package of weight 750N is positioned in equilibrium

(i) Calculate the reaction force at support A this instant.

$750\ (1.6) + 380\ (2.5) = 5\ (R_A)$

$R_A = 430N$

(ii) The package (ball) begins to roll towards the right. How far away from the Pivot B should the ball reach when the rod begins to tip off?

$F_A = 0N$

$750 \times d = 380\ (2.5)$

$d = 1.3\ m$

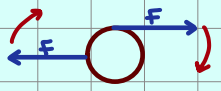
If ball is placed before 1.3m, no tipping takes place. At 1.3m the rod is about to tip off. If ball is placed after 1.3m, the rod tips off (tipping action takes place).

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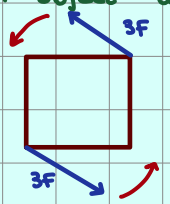
Concept of Couple

- Two equal and anti-parallel forces acting on an object at different points constitutes a couple.
  - Couples allow the object to rotate

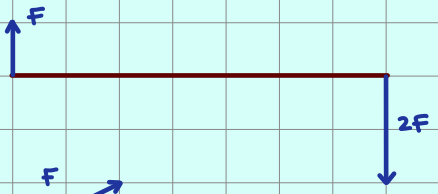








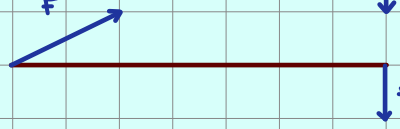
Why are these not couples?



Unequal magnitude



Not acting at 2 different points.



Not antiparallel to each other

Formula of Turning effect / Moment / Torque of a couple:-

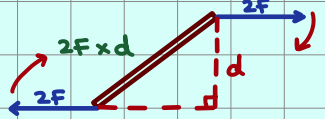
Any one of the 2 forces x Perpendicular distance between them

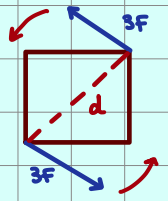
Fxd



Fxr

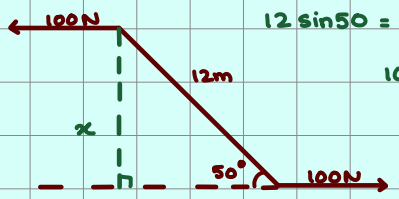






3F x d

Example 1:-

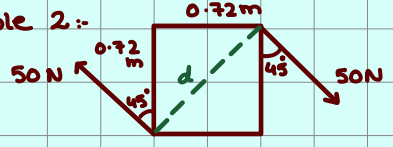


12 sin50 = x

100 x 12sin50

920 Nm

Example 2:-



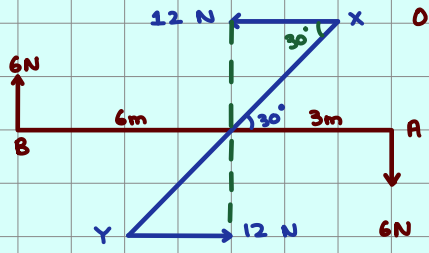
$d = \sqrt{0.72^2 + 0.72^2}$

d = 1.02m

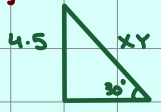
1.02 x 50

51 Nm

Example 3:-



Object is in equilibrium. Calculate XY



6x9 = 12 x d

d = 4.5m

$\sin 30 = 4.5/XY$

XY = 9 m

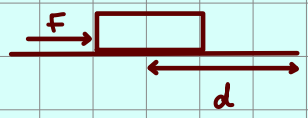
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Work, Energy, and Power

Concept of Work done :-

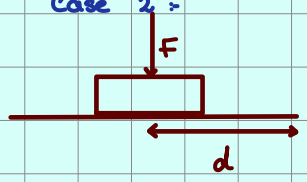
- Symbol = W
- Units = J
- Scalar quantity
- Product of force and the distance moved in the direction of the force.

Case 1 :-



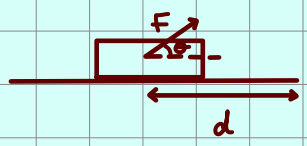
$W = F \times d$

Case 2 :-



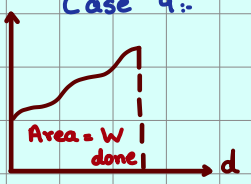
$W = zero$

Case 3 :-



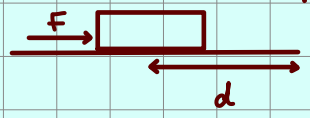
$W = F \cos\theta \times d$

Case 4 :-



Case 5 :-

'F' is not constant



$W = F_{AVG} \times d$

Classification of Work done :-

(a) Work done by driving force / forward force / engine

$W = F_D \times d$

(b) Work done against Frictional force / Opposing force / Resistive force / Backward Force

$W = F_f \times d$

(c) Work done against gravity / Weight / Gravitational force

$GPE = mgh$

Relationship between Work done and Energy :-

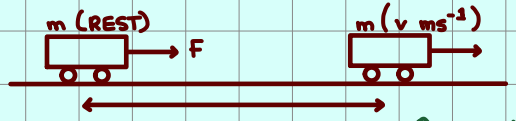
- Whenever work is done on any object, the object will always convert this work done into some form of energy

1) Speed changes → K.E
2) Position / height changes → G.P.E
3) length → EPE
4) Heat produced → Work done against Friction

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Derivation of K.E. formula

Energy possessed by an object by virtue of its motion.



Assumptions:-
- Horizontal motion
- Smooth Surface

Work done = KE

$F \times s = KE$

$ma \times s = KE$

$v^2 = 2as$

$a = v^2/2s$

$\therefore \frac{mv^2}{2s} \times s = KE$

$KE = \frac{1}{2}mv^2$ → 1

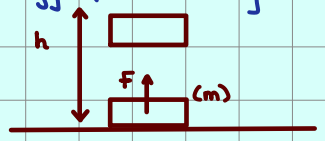
$\Delta K.E. = \frac{1}{2}m(v^2-u^2)$ → 2

If speed = constant

$\Delta KE = zero$ → 3

Derivation of GPE formula:-

Energy possessed by an object by virtue of its position.



Assumptions:-
- Constant Speed
- No A_R

Work done = GPE

$F \times h = GPE$

$mg \times h = GPE$

$GPE = mgh$ → 1

$\Delta GPE = mg(h_2-h_1)$ → 2

If constant height

$\Delta GPE = zero$ → 3

Power:- Rate of Work done OR Rate at which energy is transformed

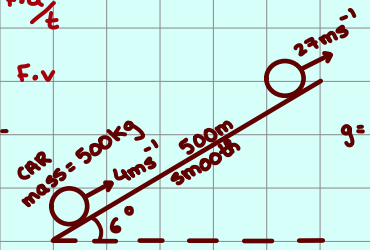
$P = W/t$

$P = F.d/t$

$P = F.v$

$P = KE/t$, $P = GPE/t$, $P = (K.E + GPE)/t$

Example 1:-



$g = 10\ ms^{-2}$

(i) Calculate acceleration.

$v^2 = u^2 + 2as$

$27^2 - 4^2 = 2(a)(500)$

$a = 0.71\ ms^{-2}$

(ii) Calculate time

$v = u + at$

$27 - 4 = 0.71\ t$

$t = 32\ s$

(iii) $\Delta K.E.$

$\frac{1}{2}m(v^2-u^2)$

$\frac{1}{2}(500)(27^2-4^2)$

$1.8 \times 10^5\ J$

(iv) $\Delta G.P.E$

mgh

$500 \times 10 \times 500\sin 6$

$2.6 \times 10^5\ J$

(v) Power supplied

$\frac{(1.8\times10^5) + (2.6\times10^5)}{32}$

$1.4 \times 10^4\ W$

(vi) Why is the actual power greater?

(a) Heat produced by engine

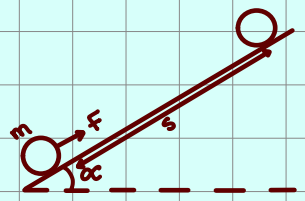
(b) Chemical energy wasted for fuel

(c) Energy lost in overcoming A_R

Formula for Efficiency:-

- $\frac{\text{Useful } P_{OUTPUT}}{\text{Total } P_{INPUT}}$
- $\frac{\text{Useful } E_{OUTPUT}}{\text{Total } E_{INPUT}}$

Example 1:-



$g = 10\ ms^{-2}$

Show that efficiency is given by:- $eff = mg\sin\alpha/F$

Work done = F x s

Work done = GPE

$mgs.\sin\alpha$

$eff = \frac{mgs.\sin\alpha}{F \times s}$

$eff = mg\sin\alpha/F$ ∴ shown!

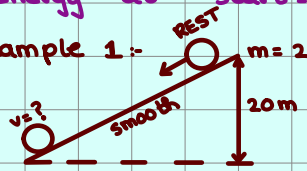
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How to apply Principle of Conservation of Energy in different cases:-

Case 1:- No Friction

Energy at Start = Energy at End

Example 1:-

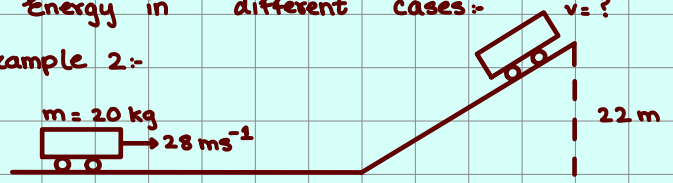


$mgh = \frac{1}{2}mv^2$

$2(10)(20) = \frac{1}{2}(2)(v)^2$

$v = 20\ ms^{-1}$

Example 2:-



$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mv^2$

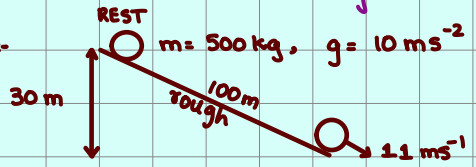
$\frac{1}{2}(20)(28)^2 = (20)(10)(22) + \frac{1}{2}(20)(v)^2$

$v = 19\ ms^{-1}$

Case 2:- If friction is present

Energy at start − Work done against friction = Energy at end

Example 1:-



$mgh - 100(F_f) = \frac{1}{2}mv^2$

$(500)(10)(30) - 100(F_f) = \frac{1}{2}(500)(11)^2$

$F_f = 1.2 \times 10^3\ N$

Example 2:-



Show that $A_R = m\left[g - \frac{v^2}{2h}\right]$

$mgh - A_R.h = \frac{1}{2}mv^2$

$mgh - \frac{1}{2}mv^2 = A_R.h$

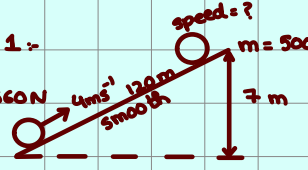
$m\left[\frac{gh}{h} - \frac{v^2}{2h}\right] = A_R.$

$A_R = m\left[g - \frac{v^2}{2h}\right]$ ∴ shown !

Case 3:- If F_D is present instead of F_f

Energy at Start + Work done by F_D = Energy at end

Example 1:-



$\frac{1}{2}mv^2 + 560(120) = mgh + \frac{1}{2}mv^2$

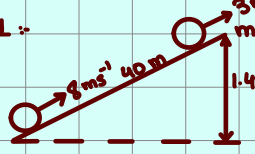
$\frac{1}{2}(500)(4)^2 + 560(120) = 500(10)(7) + \frac{1}{2}(500)(v)^2$

$v = 12.0\ ms^{-1}$

Case 4:- If F_D and F_f both are present

Energy at start − Work done against friction + Work done by F_D = Energy at end

Example 1:-



(i) Calculate Work done by engine

$P = W/t$ ∴ $W = 600J$

(ii) Calculate Work done against Friction $75 \times 8 = W$

$\frac{1}{2}mv^2 + 600 - x = mgh + \frac{1}{2}mv^2$

$\frac{1}{2}(2)(8)^2 + 600 - x = 2(10)(1.4) + \frac{1}{2}(2)(3)^2$

$x = 630\ J$

(iii) Hence, calculate F_f

$630 = F_f \times 40$

$F_f = 16\ N$

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Deformation of Solids

Hooke's Law :-

$F \propto e$, provided that Limit of Proportionality is not exceeded

$F = ke$

where k = spring constant

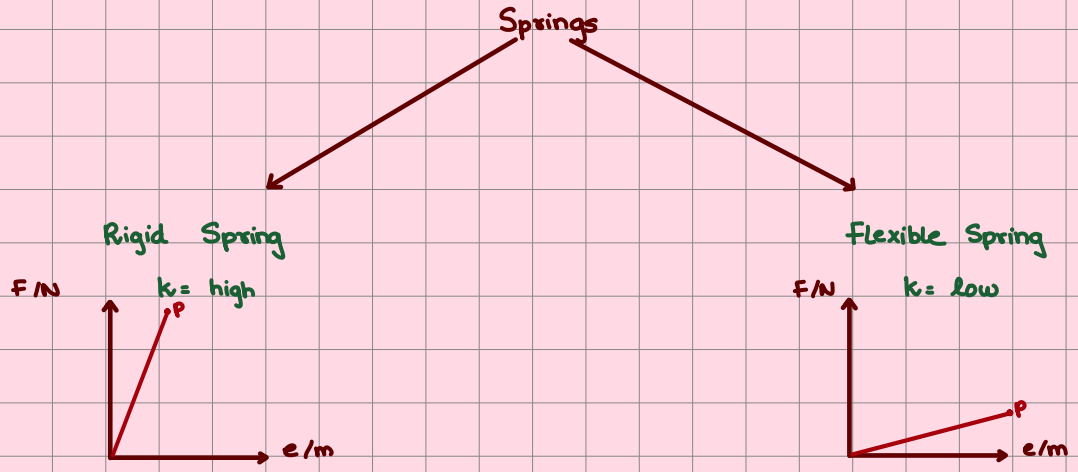
$k = F/e$ Spring constant can be defined as force per unit extension.

Units = Nm^{-1}

$k = 50\ Ncm^{-1}$ to Nm^{-1}

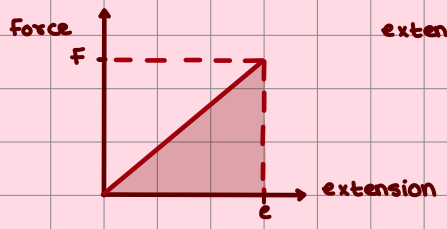
$50 \times 100 = 5000\ Nm^{-1}$

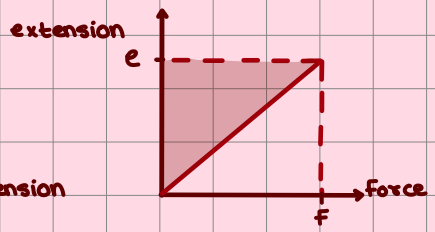




Whenever a spring is stretched or compressed, it stores energy within it. This energy is called Elastic Potential Energy or Strain Energy or Work done on a spring. The value can be worked out by obtaining the area under the Force - extension graph

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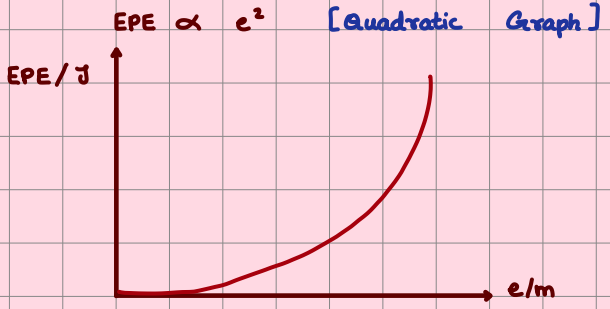


Strain Energy :- $\frac{1}{2}Fe$ OR $\frac{1}{2}ke^2$

In terms of compression :-

Hooke's Law :- $F = kx$

Strain Energy :- $\frac{1}{2}Fx$ OR $\frac{1}{2}kx^2$



Example 1 :-

Spring :-	P	vs	Q
Force :-	F	vs	F
Spring Constant :-	k	vs	4k

Find $\frac{\text{Strain Energy in P}}{\text{Strain Energy in Q}}$

$F = ke_P$ $\quad F = 4ke_Q$

$e_P = F/k$ $\quad e_Q = F/4k$

$$\frac{P}{Q} = \frac{\frac{1}{2}Fe_P}{\frac{1}{2}Fe_Q}$$

$$\frac{P}{Q} = \frac{F/k}{F/4k}$$

$$\frac{\text{Strain Energy in P}}{\text{Strain Energy in Q}} = \frac{4}{1}$$

Example 2 :-

Spring :-	P	vs	Q
Force :-	3F	vs	7F
Spring Constant :-	k	vs	5k

Find $\frac{\text{Strain Energy in P}}{\text{Strain Energy in Q}}$

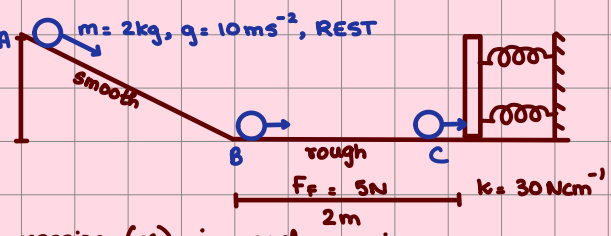
$3F = ke_P$ $\quad 7F = 5ke_Q$

$e_P = 3F/k$ $\quad e_Q = 7F/5k$

$$\frac{P}{Q} = \frac{\frac{1}{2}\cdot k\cdot\left(\frac{3F}{k}\right)^2}{\frac{1}{2}\cdot 5k\cdot\left(\frac{7F}{k}\right)^2}$$

$$\frac{\text{Strain Energy in P}}{\text{Strain Energy in Q}} = \frac{45}{49}$$

Example 3 :-



Calculate compression (x) in each spring.

Step 1 :- GPE at A = KE at B

mgh = KE at B

$(0.2)(10)(20)$ = KE at B

400J = KE at B

Step 2 :- Work done against Friction

$5N \times 2m = 10J$

Step 3 :- KE at C :-

$400 - 10 = 390J$

Step 4 :- KE at C = Strain Energy

390 = Strain Energy

$390/2 = 195J$ → for each spring

$195 = \frac{1}{2}kx^2$

$195 = \frac{1}{2}(3000)(x)^2$

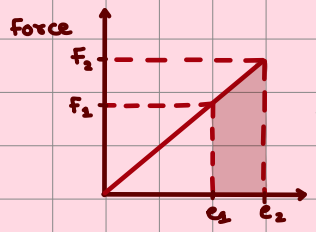
$x = 0.36m.$

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Concept of Additional Strain Energy:-

Consider a spring which undergoes an extension of e_1 when a force of F_1 is applied. When a force increases to F_2, the corresponding extension becomes e_2 as shown in the graph. The extra energy which the spring stores when extended from e_1 to e_2 is called Additional Strain Energy. It is given by the formula:-

$\frac{1}{2} k [e_2^2 - e_1^2]$



$\frac{1}{2} \times [F_1 + F_2] \times [e_2 - e_1]$

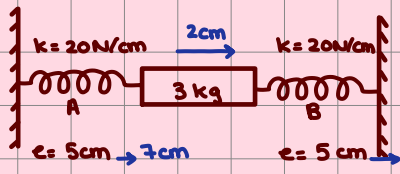
$\frac{1}{2} \times [ke_1 + ke_2] \times [e_2 - e_1]$

$\frac{1}{2} k (e_1 + e_2)(e_2 - e_1)$ $\therefore$ $\frac{1}{2} k [e_2^2 - e_1^2]$

e_1 = initial extension.

e_2 = final extension.

Example 1:-



The mass is pulled 2cm to the right and then released. Calculate the speed with which the mass begins to move assuming no losses.

For Spring A:-

$\frac{1}{2} k [e_2^2 - e_1^2]$

$\frac{1}{2} [20 \times 100][7^2 - 5^2]$

2.4 J

For Spring B:-

$\frac{1}{2} k [e_2^2 - e_1^2]$

$\frac{1}{2} [20 \times 100][3^2 - 5^2]$

-1.6 J

In net terms, there is a gain of 0.8 J

$0.8 = \frac{1}{2} mv^2$

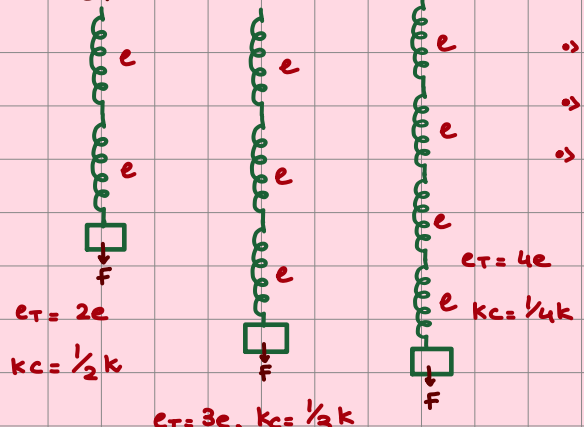
$0.8 = \frac{1}{2} (3)(v)^2$

$v = 0.73 ms^{-1}$

Properties of springs when connected in series and in parallel

Assume all springs are identical. Reference :- k, e, F

Series:-

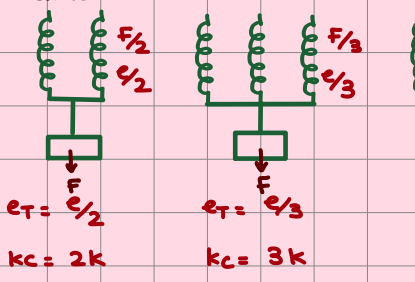


⇒ F remains same throughout

⇒ e_T = sum of indivisual extensions

⇒ $k_C \propto 1/e_T$

Parallel:-



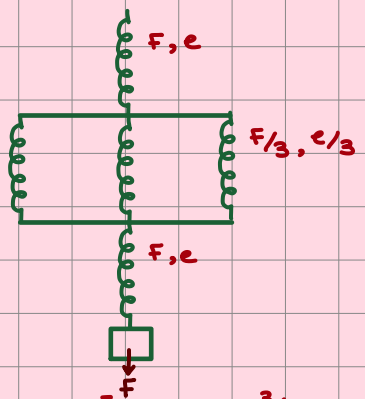
⇒ Force equally divides

⇒ e_T = e of any 1 spring

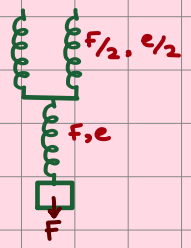
⇒ $k_C \propto 1/e_T$

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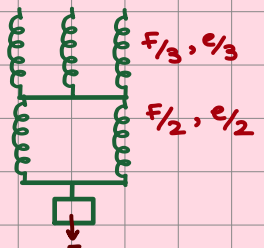
Series and Parallel



$e_T = \frac{7}{3}e$, $k_C = \frac{3}{7}k$

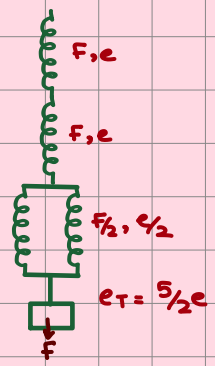


$e_T = \frac{3}{2}e$, $k_C = \frac{2}{3}k$



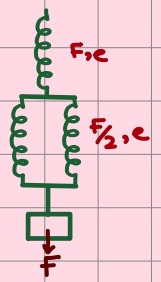
$e_T = \frac{5}{6}e$, $k_C = \frac{6}{5}k$

A



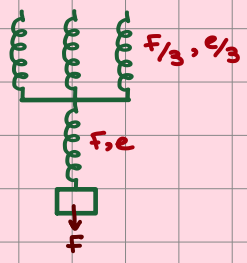
$e_T = \frac{5}{2}e$

B



$e_T = \frac{3}{2}e$

C

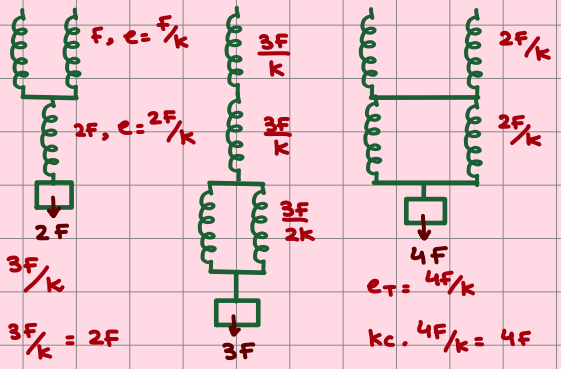


$e_T = \frac{4}{3}e$

Arrange them in ascending order of extension.

C, B, A

Springs are still identical. No reference given. Different forces.



$e_T = \frac{3F}{k}$

$k_C \cdot \frac{3F}{k} = 2F$

$k_C = \frac{2k}{3}$

$e_T = \frac{15F}{2k}$

$k_C \cdot \frac{15F}{2k} = 3F$

$k_C = \frac{2}{5}k$

$e_T = \frac{4F}{k}$

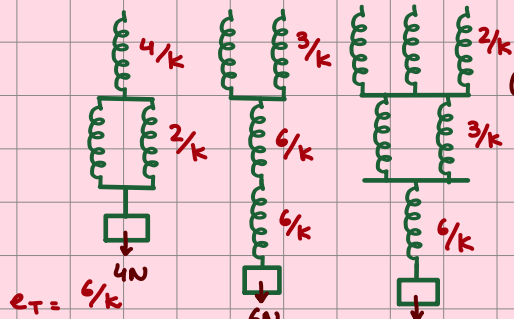
$k_C \cdot \frac{4F}{k} = 4F$

$k_C = k$

(i) Calculate e_{Total} giving your answer in terms of F and k

(ii) Calculate $k_{combined}$ giving your answer in terms of k.

What if forces are given in Absolute terms?



$e_T = \frac{6}{k}$

$k_C \cdot \frac{6}{k} = 4$

$k_C = \frac{2k}{3}$

$e_T = \frac{15}{k}$

$k_C \cdot \frac{15}{k} = 6$

$k_C = \frac{2k}{5}$

$e_T = \frac{11}{k}$

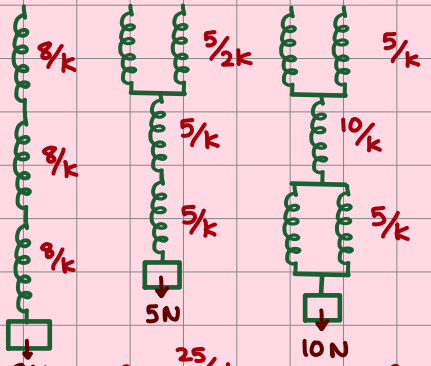
$k_C \cdot \frac{11}{k} = 6$

$k_C = \frac{6k}{11}$

(i) Calculate e_{Total} giving your answer in terms of k

(ii) Calculate $k_{combined}$ giving your answer in terms of k.

Example 1.



$e_T = \frac{24}{k}$

$e_T = \frac{25}{2k}$

$e_T = \frac{20}{k}$

Arrange in ascending order of extension

∴ B, C, A.

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Non - identical springs:-



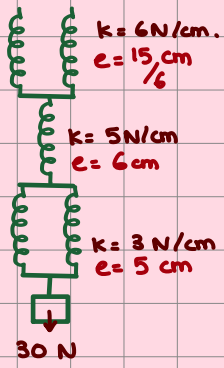
⇒ e_T in cm

12cm

⇒ k_c in Ncm^{-1}

$k_c \cdot 12 = 12$

$k_c = 1\ Ncm^{-1}$



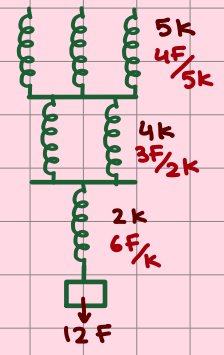
⇒ e_T in cm

$5 + 6 + 15/6 = 35/2$ cm

⇒ k_c in Ncm^{-1}

$F = k_c \cdot e$

$k_c = 12/7\ Ncm^{-1}$



⇒ e_T in terms of F and k.

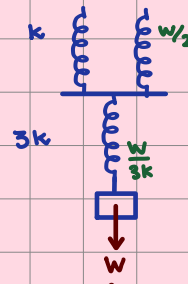
$\frac{83F}{10k}$

⇒ k_c in terms of k.

$12F = \frac{83F}{10k} \cdot k_c$

$k_c = 120k/83$

Example 1:-

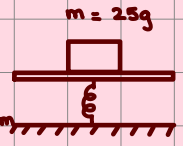


Show that $e_T = 5W/6k$.

$W/2k + W/3k$

$e_T = 5W/6k$.

Example 2:-



(i) Calculate Strain Energy in spring

$\frac{1}{2}kx^2 = \frac{1}{2}(1250)(36 \times 10^{-3})^2 = 0.81\ J$

(ii) Upon release, strain energy converts into K.E., calculate speed (v)

$0.81 = \frac{1}{2}mv^2$

$0.81 = \frac{1}{2}(0.025)(v)^2$

$v = 8.0$ m/s.

(iii) Calculate (x) required to launch the mass with twice the speed.

Option 1:- $\frac{1}{2}(0.025)(16)^2 = 3.24$

$3.24 = \frac{1}{2}(1250)(x)^2$

$\therefore x = 72$mm

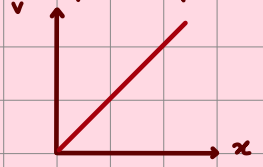
Option 2. $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$x \propto v$

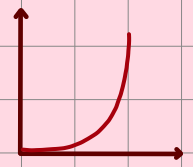
If v doubles, x doubles too.

$\therefore x = 72$mm

(iv) Graph of speed vs compression.



(v) Graph of K.E vs compression.



$KE = \frac{1}{2}kx^2$

$KE \propto x^2$

$y = x^2$

∴ curve with increasing gradient

(vi) Assuming that K.E. converts into GPE as mass rises (no loss), calculate height gained when v = 8 : height gained when v = 16

$0.81 = (0.025)(9.81)(h_1)$

$h_1 = 3.30$

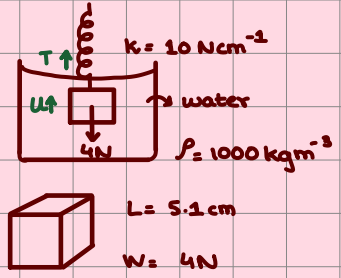
$3.24 = (0.025)(9.81)(h_2)$

$h_2 = 13.2$

$\frac{3.30}{13.2} = \frac{1}{4}$

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Example 3:-



If cube remains stationary, calculate extension in the spring

$W = U + T$

$4 = \rho g V + T$

$4 = 1000(9.81)(5.1 \times 10^{-2})^3 + T$

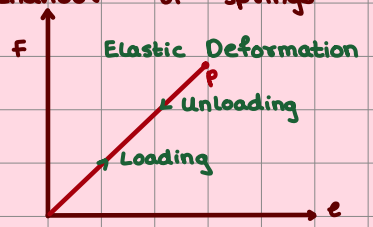
$T = 2.7\ N$

$F = ke$

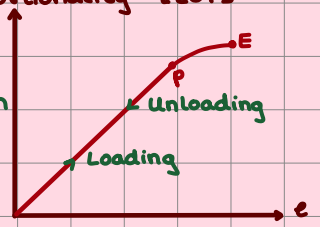
$4 = 10(100)(e)$

$e = 2.7 \times 10^{-3}\ m.$

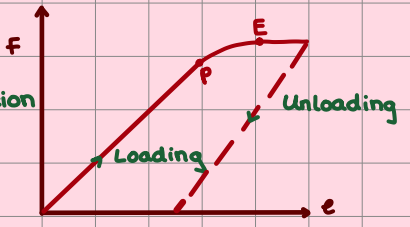
Behaviour of springs beyond Limit of Proportionality [LOP]



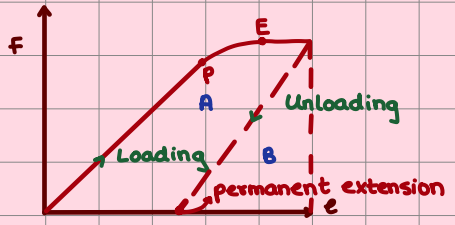
- Hooke's law is applicable
- $f \propto e$
- Loading and Unloading follows same pattern
- No permanent extension.



- Hooke's law is not applicable
- $f \propto e$ [no longer valid]
- Loading and Unloading follows same pattern
- No permanent extension.



- Hooke's law is not applicable
- $f \propto e$ [no longer valid]
- Loading and Unloading follows different pattern
- permanent extension is produced in spring



A + B = Area under graph labelled as loading → Work done on a spring as it undergoes the loading process

B = Area under graph labelled as unloading → Energy recovered / regained from the spring as it unloads.

A = Area between 2 graphs

- work done used by a spring to undergo plastic deformation
- Energy wasted in form of heat
- Energy that cannot be recovered during a complete cycle of loading and unloading

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Concept of Stress, Strain, and Young's Modulus

Spring	Wires
Force [f]	Stress [σ]
Extension [e]	Strain [ε]
Spring Constant [k]	Young's Modulus [E]

- Stress is defined as Force per unit Area

Stress = F/A , Units = Pa or Nm^{-2}

- Strain is defined as :-

(a) Ratio of change in length and original length

Strain = $\Delta L/L$, Units = No units.

(b) Ratio of extension and original length

Strain = e/L , Units = No units.

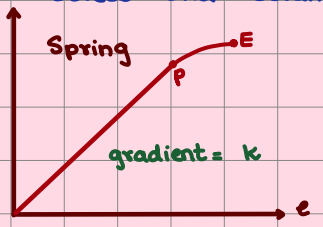
- For springs :-
  - High K → Rigid Spring
  - Low K → Flexible Spring
- For wire
  - High E → stiff wire
  - Low E → flexible wire

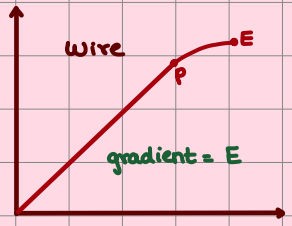
- Young's Modulus is defined as the ratio of Stress and Strain

$E = \text{Stress}/\text{Strain}$, Units = Pa or Nm^{-2}

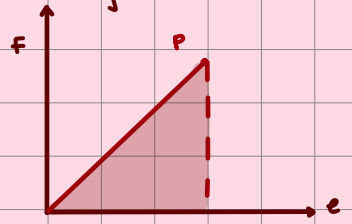
$E = \frac{F/A}{e/L} = F \times L/A \times e = kL/A$

$\therefore E = FL/Ae = kL/A$



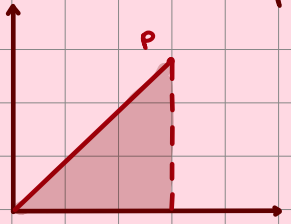


Calculating Area under Stress vs Strain Graph.



Area = $\frac{1}{2}Fe$

Strain Energy



Area = $\frac{1}{2} \times \text{Stress} \times \text{Strain}$

Area = $\frac{1}{2} \times \frac{F}{A} \times \frac{e}{L}$

Area = $\frac{\text{Strain Energy}}{\text{Volume}}$

Area represents Strain Energy per unit volume

Example 1 :- Wire has Stress = S, Young's Modulus = E, Show that Strain Energy per unit volume is given by $S^2/2E$

$\frac{\text{Strain Energy}}{\text{volume}} = \frac{1}{2} \times \frac{F}{A} \times \frac{e}{L}$

$E = FL/Ae$

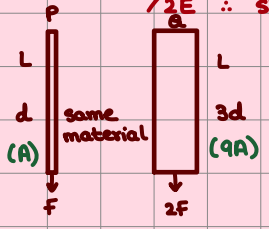
$\frac{e}{L} = F/AE$

$\frac{1}{2} \times S \times F/AE$

$\frac{1}{2} \times S^2/E$

$S^2/2E$ ∴ Shown !

Example 2 :-



(i) Stress in P / Stress in Q.

$A = \pi d^2/4$, $A \propto d^2$

$F/A \times 9A/2F = 9/2$

(ii) Strain in P / Strain in Q.

Strain = Stress / E [constant]

Strain ∝ Stress

∴ 9/2.

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Example 3:-

	P	Q
	2F	F
	L	3L
(A)	d	4d (16A)
	E	2E

(i) e_p/e_q.

$E = FL/Ae$

$e = FL/AE$

$e_p = \frac{2FL}{AE}$

$e_q = \frac{3FL}{32AE}$

$\frac{2FL}{AE} \times \frac{32AE}{3FL}$

$e_p/e_q = 64/3$

Example 4:-

	A	B
	F	F
	2L	3L
(A)	d	3d (9A)
	e	2e

(i) E_A/E_B.

$E = FL/Ae$

$E_A = \frac{2FL}{Ae}$

$E_B = \frac{3FL}{18Ae}$

$\frac{2FL}{Ae} \times \frac{6\ 18Ae}{3FL}$

$E_A/E_B = 12/1$

Example 5:- P: E = 40 GPa, L = 2m, d; Q: E = 2 GPa, L = 2m, d. If e_T = 84 cm, find stress in this arrangement

$e = FL/AE$

$e_p + e_q = 0.84$

$\frac{2F}{A(40\times10^9)} + \frac{2F}{A(2\times10^9)} = 0.84$

$\frac{F}{A}\left[\frac{2}{40\times10^9} + \frac{2}{2\times10^9}\right] = 0.84$

$F/A = 0.8 \times 10^9$ ∴ Stress = 0.8 GPa

Example 6:- Stress in P is 4 times that of Q. Force applied on P is twice that of Q.

Calculate $d_p : d_q$

$4S_p = \frac{2F}{\pi d^2/4}$

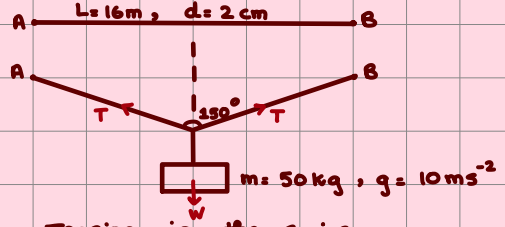
$\pi d^2 = 8F/4S$

$d_p^2 = \frac{8F}{4\pi S}$ $d_q^2 = \frac{4F}{S\pi}$

$\frac{8F}{4\pi S} \times \frac{S\pi}{4F} = \sqrt{1/2} = \sqrt{d_p^2/d_q^2}$

$d_p/d_q = 1/\sqrt{2}$

Example 7:-



(i) Calculate Tension in the spring

$2T\cos 75 = 500$

$T = 970$ N

(ii) Calculation extension [e] in the wire

$\sin 75 = 8/x$

$x = 8.28$

$8.28 + 8.28 = 16.56$

$16.56 - 16 = 0.56$ cm

(iii) Calculate E of the wire

$E = FL/Ae$

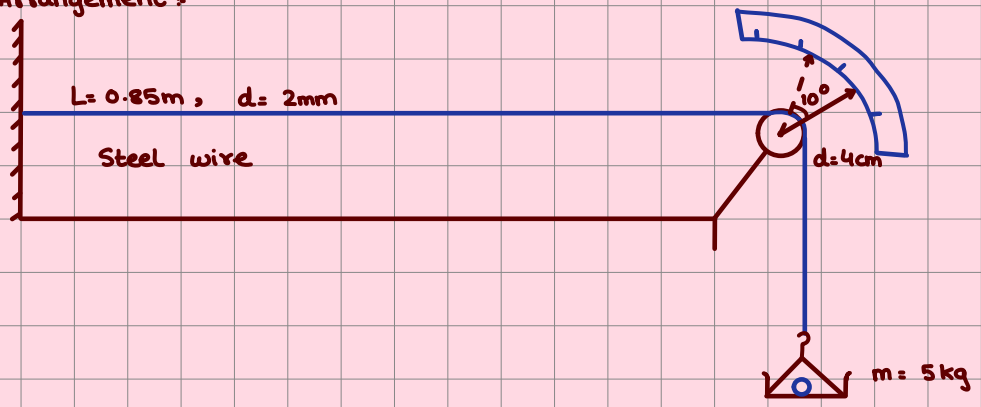
$E = \frac{970 \times 16}{\frac{\pi(0.02)^2}{4} \times 0.56}$

$E = 8.2 \times 10^7$ Pa

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Experiment to determine the Young's Modulus of any material. E.g. Steel wire

(i) Arrangement :-



(ii) Measurements to be taken

$E = FL/Ae$

- $F = mg$
- $F = (5)(9.81)$
- $F = 49.1\,N$
- $L = 0.85\,m$
- $360^\circ \rightarrow 2\pi r$
- $10^\circ \rightarrow x$
- $r = 0.02$
- $x = 3.5 \times 10^{-3}$ = extension.
- $d = 2mm$, $A = ?$
- $A = \pi(2\times10^{-3})^2/4$
- $A = 3.14 \times 10^{-6}\,m^2$
- $E = \dfrac{49.1 \times 0.85}{[3.14\times10^{-6}]\times[3.5\times10^{-3}]}$
- $E = 3.8 \times 10^{9}\,Pa.$

(iii) Measuring Instruments :-

- Length → Meter rule, Measuring tape
- Mass → Electronic Balance
- Diameter → Micrometer
- Extension → Pulley connected to a pointer + protractor.

(iv) Precautions :-

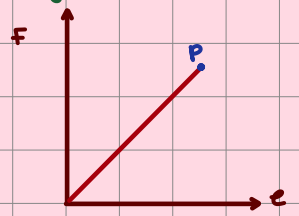
- There must be no kinks in the wire i.e. the wire must remain taut at all times
- The mass hanger attached must be of negligible mass
- The pulley must be properly oiled to minimize friction during its movement
- The wire must not exceed its elastic limit. This can be checked by removing the load. If the pointer returns to its original position, then the above statement is verified.

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Force vs Extension graphs for Brittle, Ductile, and Polymeric Material

(i) Brittle Material

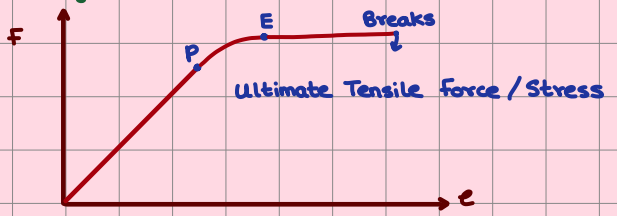
e.g. Glass, Bone



- Obeys Hooke's Law
- Large amount of force provides small extension
- Breaks as it reaches P.

(ii) Ductile Material

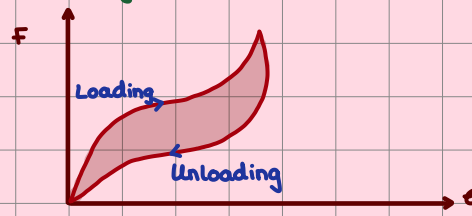
e.g. metal



- Obeys Hooke's Law
- Less amount of force provides large extension
- Can undergo plastic deformation
- Breaking point occurs well beyond the Elastic Limit.
- Ultimate Tensile Stress / Force is defined as the maximum amount of Stress or force a material withstands before it breaks.

(iii) Polymeric Graph

e.g. Rubber



- Does not obey Hooke's Law
- Has no breaking point, Elastic Limit, or Limit of Proportionality
- Never undergoes permanent extension
- Area of shaded region = Energy lost as heat

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Waves

WAVES

Progressive
- wave which travels
- with transfer of energy

Stationary
- no net transfer of energy

Transverse
- e.g. - Electromagnetic waves
  - Water
  - Rope

Longitudinal
- e.g. - Sound waves
  - Spring

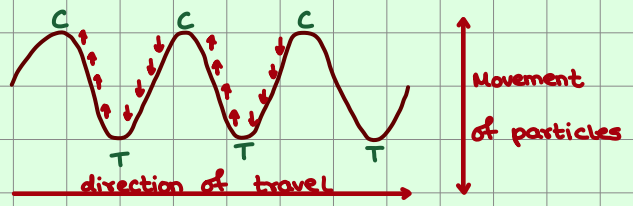


Figure 1

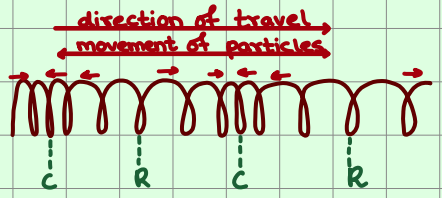


Figure 2

For the direction of travel indicated on Figure 1, particles to the right of the crest move upwards, whereas, particles to the left of the crest move downwards. Since vibration of the particles is perpendicular to the direction of travel, hence, the wave is called a Transverse wave. Points at crest or trough exhibit zero movement.

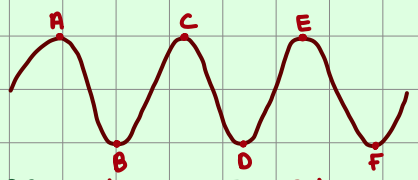
For the direction of travel indicated on Figure 2, particles near the center of compression move towards each other, whereas, particles near the center of rarefaction move away from each other. Since vibration of the particles is parallel to the direction of travel, hence, the wave is called a Longitudinal wave. Points at the center of compression or rarefaction remains in its equilibrium position.

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General terms used to describe a wave motion:-

(i) PATH DIFFERENCE:-

The gap between any 2 points on a wave profile if expressed in terms of λ is called path difference



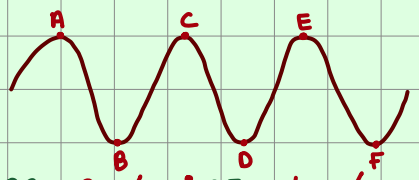
AC = 1λ AE = 2λ

AB = $\frac{1}{2}\lambda$ AD = $\frac{3}{2}\lambda$

Note:- A path difference of 1λ corresponds to a phase difference of 2π

(ii) PHASE DIFFERENCE

The gap between any 2 points on a wave profile if expressed in terms of degrees or radians is called phase difference



AC = $2\pi/360°$ AE = $4\pi/720°$

AB = $\pi/180°$ AD = $3\pi/540°$

(iii) IN-PHASE POINTS:-

Points classified as in-phase with each other must establish a path difference of $\lambda, 2\lambda, 3\lambda, 4\lambda$ ---

and a phase difference of $2\pi, 4\pi, 6\pi, 8\pi$ ---



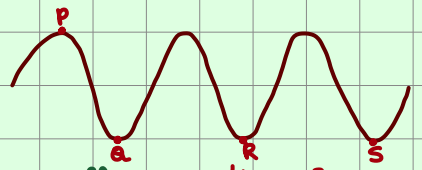
Path Difference:- $1\lambda, 2\lambda, 3\lambda, 4\lambda, 5\lambda, 6\lambda$

Phase Difference:- $2\pi, 4\pi, 6\pi, 8\pi, 10\pi, 12\pi$

(iv) OUT-OF-PHASE POINTS:-

Points classified as out-of-phase with each other must establish a path difference of $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$ ---

and a phase difference of $\pi, 3\pi, 5\pi$ ---



Path Difference:- $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \frac{7}{2}\lambda, \frac{9}{2}\lambda$

Phase Difference:- $\pi, 3\pi, 5\pi, 7\pi, 9\pi$

Example 1:- v = 640 m/s, f = 800Hz. Write phase difference in radians between 2 points 0.2 m apart

$v = f\lambda$

$640 = 800\lambda$

$\lambda = 0.8\,m$

$1\lambda(0.8m) = 2\pi$

$0.2m = x$

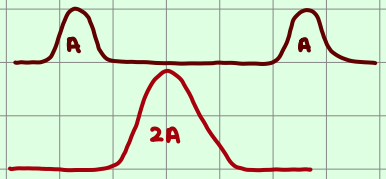
$x = \pi/2$ rad.

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Principle of Superposition and the phenomenon of interference

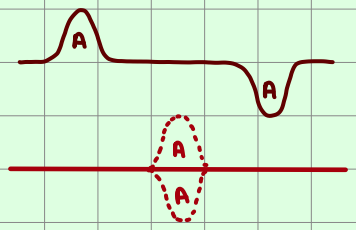
According to the principle of superposition, whenever 2 or more waves meet at a common point, the total displacement due to these waves will always be the sum of their individual displacements.

The principle of superposition results in the phenomenon of interference. As shown in Fig 1, constructive interference (CI) occurs when in-phase points superimpose, hence for CI to occur, there must be a path difference of $1\lambda, 2\lambda, 3\lambda \ldots$ and a phase difference of $2\pi, 4\pi, 6\pi$ ---. As shown in Fig.2, destructive interference (DI) occurs, when out-of-phase points superimpose. Hence, for DI to occur, there must be a path difference of $\lambda/2, 3\lambda/2, 5\lambda/2 \ldots$ and a phase difference of $\pi, 3\pi, 5\pi$ ---.



Amplitude is maximum / increases / doubles

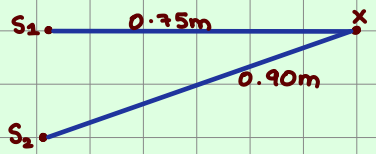
Fig. 1



Amplitude is minimum / decreases / zero

Fig. 2

Example 1:- 2 identical sources operating in-phase produce waves which meet at X. Determine with calculation whether CI or DI occurs.



$v = 3.0 \times 10^8\ ms^{-1}$

$f = 12\ GHz$

Method 1:-

Step 1:-

$v = f\lambda$

$3 \times 10^8 = 12 \times 10^9\ (\lambda)$

$\lambda = 0.025$

Step 2:-

$0.75 / 0.025 = 30\lambda$

$0.90 / 0.025 = 36\lambda$

Step 3:-

$36\lambda - 30\lambda = 6\lambda$

Hence, CI occurs at X

(ii) Suggest what is observed at X.

Amplitude doubles

(iii) Suggest any difference in observation if sources were operating out-of-phase with each other. DI ∴ resultant amplitude is zero.

Method 2:-

Step 1:-

$v = f\lambda$

$3 \times 10^8 = 12 \times 10^9\ (\lambda)$

$\lambda = 0.025$

Step 2:-

$S_2X - S_1X$

$0.9 - 0.75 = 0.15$

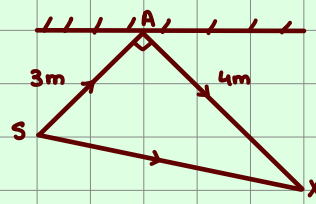
Step 3:-

$0.15 / 0.025 = 6\lambda$

Hence, CI occurs at X

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Example 2:-



Reflective Surface

$f = 400\,Hz$

$v = 320\,ms^{-1}$

Waves from a single source 'S' superimpose at 'X'. Suggest what is observed at X.

Step 1:- $v = f\lambda$ $\quad h^2 = p^2 + b^2$

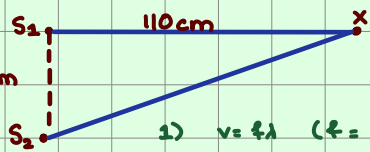
$320 = 400\lambda$ $\quad h = \sqrt{3^2 + 4^2}$

$\lambda = 0.8m$ $\quad SX = 5m$

Step 2:- Path difference = 7 − 5 = 2m ∴ zero amplitude or no movement

Step 3:- $^2/_{0.8} = 2.5\lambda = {}^5/_2\lambda$ → DI occurs at X.

Example 3:- Two sources S_1 and S_2 are producing waves which meet at X. The frequency of both sources is simultaneously varied from 1 kHz to 4 kHz. The speed of the sources remains constant at $320\,ms^{-1}$. Determine on how many occasions would you expect the phenomenon of DI to occur at X.



1) $v = f\lambda$ $(f = 1kHz)$

$320 = 1000\lambda$

$\lambda = 0.32m$

2) $v = f\lambda$ $(f = 4kHz)$

$320 = 4000\lambda$

$\lambda = 0.08m$

3) $h^2 = p^2 + b^2$

$h = \sqrt{110^2 + 80^2}$

$S_2X = 1.36m$

4) Path difference :- $\lambda/2,\ 3\lambda/2,\ 5\lambda/2,\ 7\lambda/2$

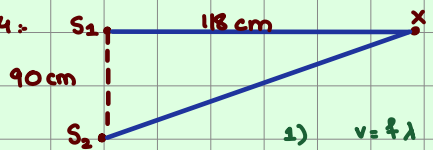
$136 - 110 = 26cm = \lambda/2,\ 3\lambda/2,\ 5\lambda/2,\ 7\lambda/2$

$\lambda = 52cm,\ \boxed{17.3cm},\ \boxed{10.4cm},\ 7.4cm$

Range = 8cm − 32cm

∴ Destructive Interference occurs at X on 2 occassions.

Example 4:-



Determine on how many occassions would you expect the phenomenon of CI to occur at X. use 3 s.f for your answers.

$f = 200\,Hz - 4.5\,kHz$

$v = 250\,ms^{-1}$

1) $v = f\lambda$

$250 = 200\lambda$

$\lambda = 1.25m = 125cm$

2) $v = f\lambda$

$250 = 4500\lambda$

$\lambda = 0.056m = 5.56cm$

3) $h^2 = p^2 + b^2$

$h = \sqrt{118^2 + 90^2}$

$S_2X = 1.48m = 148cm$

4) Path difference = $\lambda, 2\lambda, 3\lambda, 4\lambda, 5\lambda, 6\lambda$

$148 - 118 = 30 = \lambda, 2\lambda, 3\lambda, 4\lambda, 5\lambda, 6\lambda$

$\boxed{30cm}, \boxed{15cm}, \boxed{10cm}, \boxed{7.5cm}, \boxed{6cm}, 5cm$

Range = 5.56cm − 125cm

∴ Constructive Interference occurs at X on 5 occassions

Conditions required for interference to occur:-

- The 2 waves superimpose at a common point
- The 2 waves must be travelling in the same plane
- The 2 waves must be of the same type / nature
- The sources producing the 2 waves must be coherent in nature [sources are said to be coherent if they maintain a constant phase difference between them.

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Concept of 'Intensity of a wave':

- Symbol = I
- Units = Wm^{-2} [Watts per square meter]
- It is defined as "Power of a wave falling into a unit area."
- Formula:-

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}}, \quad I = \frac{P}{A}$$

$$I = \frac{E}{t.A}$$

Factors which affect the intensity of a wave:-

- Amplitude (A) of the wave:- Intensity is known to be directly proportional to the square of the amplitude

$$I \propto A^2 \therefore I = kA^2$$

Amplitude	Intensity
A	I
2A	4I
3A	9I
4A	16I
$\sqrt{2}$ A	2I
$\sqrt{3}$ A	3I
2A	4I
$\frac{1}{\sqrt{2}}$ A	$\frac{1}{2}$ I

(square: Amplitude → Intensity; Root: Intensity → Amplitude)

- Distance (d) of the wave from the source:- Intensity is known to be inversely proportional to the square of the distance from the source.

$$I \propto \frac{1}{d^2} \therefore I = \frac{k}{d^2} \quad \text{[inverse-square law]}$$

Distance	Intensity
d	I
2d	$\frac{1}{4}$ I
3d	$\frac{1}{9}$ I
4d	$\frac{1}{16}$ I

Relationship between amplitude and distance

$$I = kA^2 \qquad I = \frac{k}{d^2}$$

$$kA^2 = \frac{k}{d^2}$$

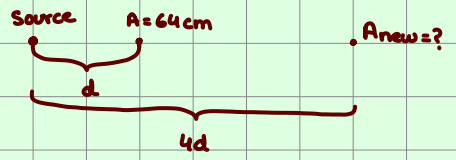
$$A^2 \propto \frac{1}{d^2}$$

$$A \propto \frac{1}{d}$$

Amplitude is inversely proportional to distance.

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Example 1 :



$A \propto \frac{1}{d}$

$\therefore \frac{64}{4} = 16\ cm$

Example 2:- Two waves of intensity I and 2I meet in-phase to undergo constructive interference. Calculate the resultant intensity, giving your answer in terms of I.

Wave 1 :- I Wave 2 = 2I

Assume amplitude = A Amplitude = $\sqrt{2}A$

$1 + \sqrt{2} = 2.41A$

$I = kA^2$ $x = kA^2$

$k = \frac{I}{A^2}$ $x = \frac{I}{A^2}(2.41A)^2$ $\therefore$ $x = 5.81I$

Example 3:- Wave 1 = I, Wave 2 = 3I, out-of-phase, Resultant intensity = ?

I → A 3I → $\sqrt{3}A$

$I = kA^2$ Resultant amplitude = $\sqrt{3} - 1 = 0.73A$

$k = \frac{I}{A^2}$ $x = \frac{I}{A^2}(0.73A)^2$

$x = 0.54I$

Example 4:- Wave 1 = I, Wave 2 = 5I, in-phase, Resultant Intensity = ?

I → A 5I → $\sqrt{5}A$

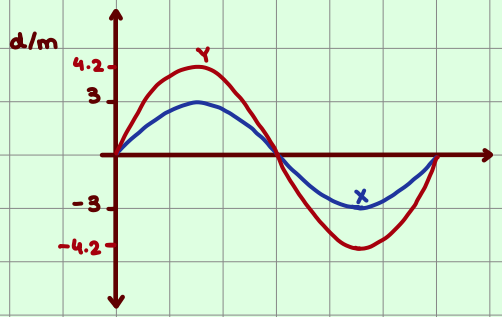
$I = kA^2$ Resultant amplitude = $1 + \sqrt{5} = 3.2A$

$k = \frac{I}{A^2}$ $x = \frac{I}{A^2}(3.2A)^2$ $\therefore$ $x = 10.5I$

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Example 5:- The graph below shows a wave X of Intensity I.

(i) Construct a second wave on the same axis which has twice the intensity and is in-phase with the first one. Label this wave as Y.



$I = kA^2$ $\quad 2I = I/9\,(A)^2$

$k = I/3^2$ $\quad A = \sqrt{18}$

$k = I/9$ $\quad A = 4.2\,m$

(ii) Hence, calculate the resultant intensity if these 2 waves interfere constructively giving your answer in terms of I.

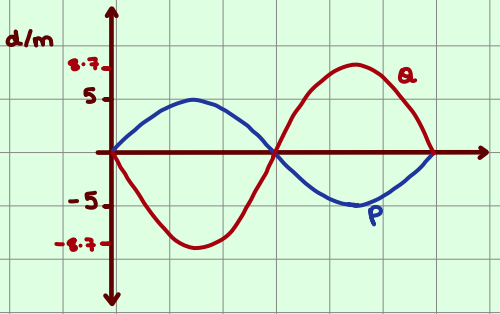
$k = I/9$, Resultant amplitude $= 3 + 4.2 = 7.2\,m$

$z = I/9\,(4.2)^2$

$z = 5.8\,I$

Example 6:- The graph below shows a wave P of Intensity I.

(i) Construct a second wave on the same axis which has thrice the intensity and is out-of-phase with the first one. Label this wave as Q.



$I = kA^2$ $\quad 3I = I/25\,(A)^2$

$k = I/5^2$ $\quad A = \sqrt{75}$

$k = I/25$ $\quad A = 8.7\,m$

(ii) Hence, calculate the resultant intensity if these 2 waves interfere destructively giving your answer in terms of I.

$k = I/25$, Resultant amplitude $= 8.7 - 5 = 3.7\,m$

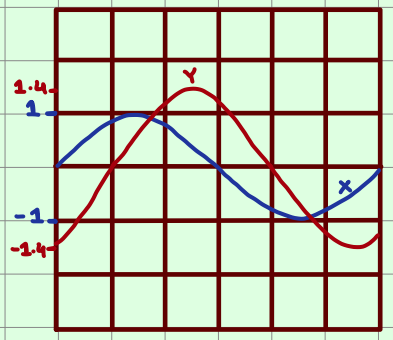
$z = I/25\,(3.7)^2$

$z = 0.55\,I$

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Concept of phase difference / leading / lagging behind

Example 7:- The graph below shows a wave X of intensity I. Construct a second wave Y which has twice the intensity and maintains a phase difference of 60° with the first wave.

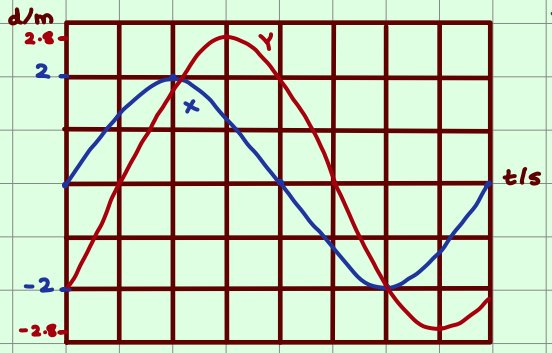


$I = kA^2$ $\quad$ $I = kA^2$ $\quad$ 6 boxes → 360°

$k = I$ $\quad$ $2I = I(A)^2$ $\quad$ x → 60°

$A = 1.4$ $\quad$ x = 1 box

Example 8:- The graph below shows a wave X of intensity I. Construct a second wave Y which has twice the intensity and lags behind the first wave with a phase difference of 45°



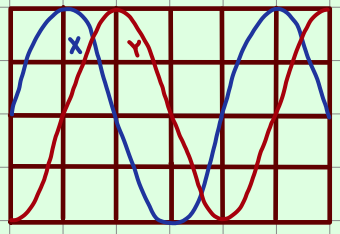
$I = k(2)^2$ $\quad$ $I = kA^2$ $\quad$ 8 boxes → 360°

$k = I/4$ $\quad$ $2I = I/4\,(A)^2$ $\quad$ x → 45°

$A = 2.8$ $\quad$ x = 1 box

> when the question says 'lags behind' and the quantity on the x-axis is time, graph moves ahead because the graph that starts later takes more time and vice versa.

Example 9:- Construct a second wave of the same intensity having a phase difference of 90° with the wave below



4 boxes → 360°

x → 90°

x = 1 box

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Example 10 :- Wave P, A = 1·2 cm, Intensity = I

Wave Q, A = 3.6 cm, Intensity = ?

(i) Calculate the intensity of wave Q in terms of I.

$3.6/1.2 = 3$ times

$I = kA^2$

$\therefore I = 9I$ (MCQ only)

$I = k(1.2)^2$

$k = I/1.44$

$I = I/1.44\,(3.6)^2$

$I = 9I$ (Theory)

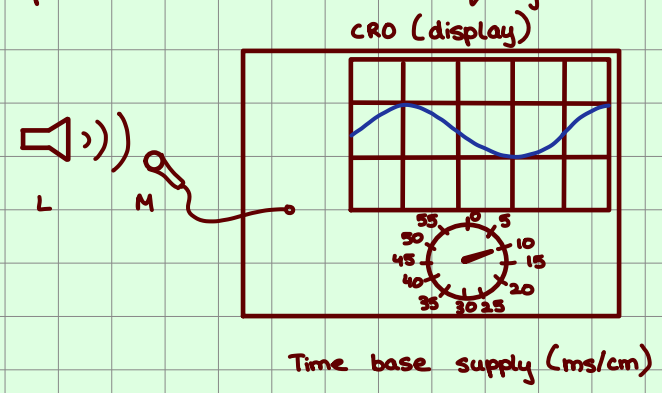
(ii) Calculate resultant intensity if they interfere constructively.

$k = I/1.44$, Resultant amplitude = 1·2 + 3·6 = 4·8 cm

$x = I/1.44\,(4.8)^2$

$x = 16\,I$

1) Experiment to calculate frequency of sound waves



Time base supply (ms/cm)

1 cm → 10 ms

4 cm → 40 ms

$f = \frac{1}{T}$

$f = 1/40 \times 10^{-3}$

$f = 25\ Hz$

List of Apparatus:-

1) Loudspeaker
2) Microphone
3) CRO + connecting wires

Precaution:-

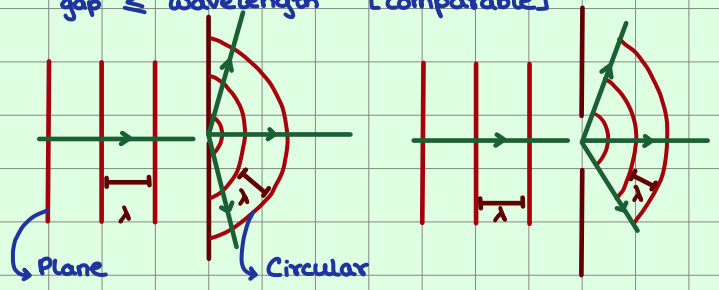
1) Experiment to be conducted in sound-proof room
2) The sound producing source must operate at constant frequency
3) No echo of sound.

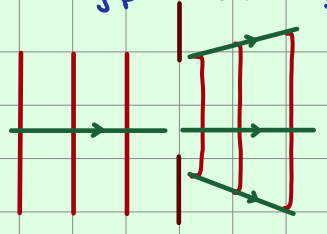
Concept of diffraction:-

Diffraction is defined as the spreading of waves when they pass through a narrow gap, opening, aperture or slit.

Significant diffraction occurs when size of the gap ≤ wavelength [comparable]

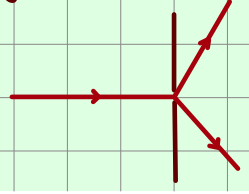
Amount of diffraction becomes negligible when size of the gap > wavelength





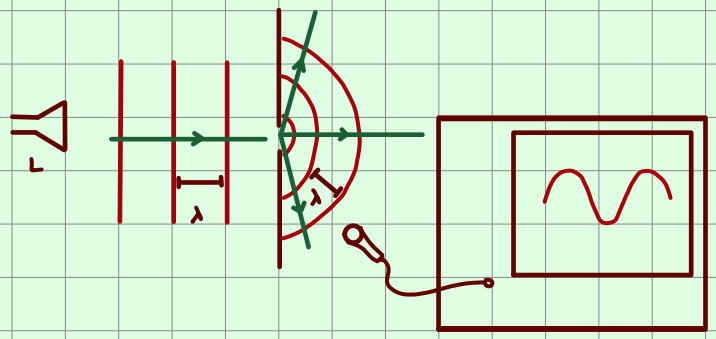
* λ, v, f remain same

If you are asked to show diffraction for 1 mark



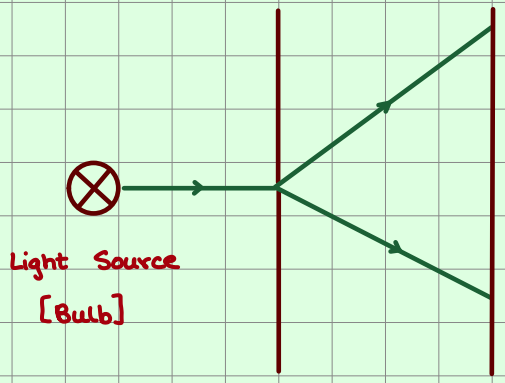
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2) Experiment to show that sound undergoes diffraction:-



- λ sound ≈ 1m
- Size of Aperture = 0.5m
- Observation:- A waveform if displayed on the CRO confirms that sound undergoes diffraction.

3) Experiment to show that light undergoes diffraction:-



- λ light :- 400nm (Violet) – 700nm (Red)
- Size of gap is very small
- Observation:- A large area of the screen is well lit-up (illuminated), hence implying that light undergoes diffraction.

List of Apparatus :-
- Light Source (Bulb)
- Barrier with a much smaller slit size
- Screen

Precaution :-
- Dark Room

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Young's Double - Slit Experiment :-

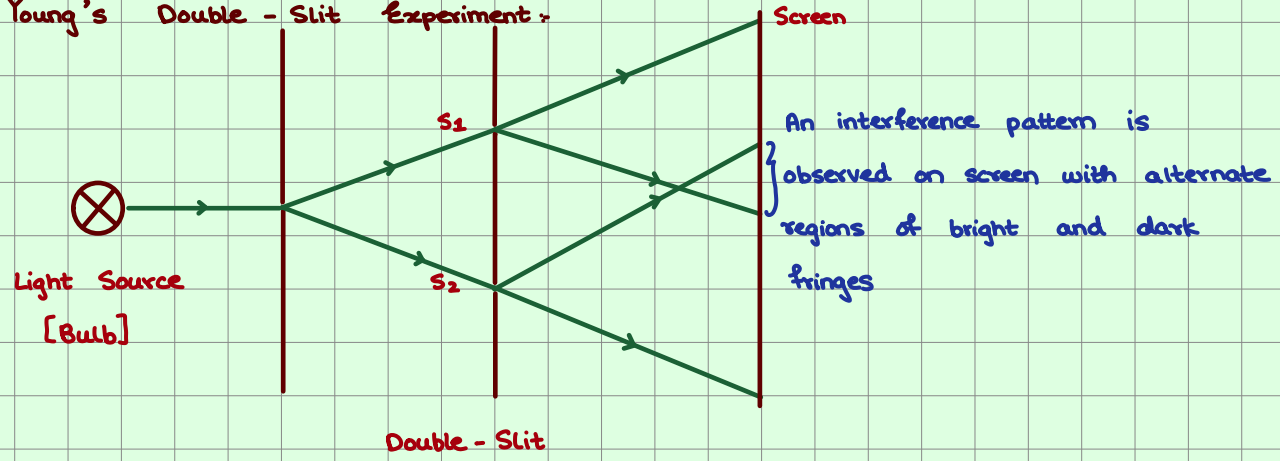


Figure 1 :- Young's Double - Slit Experiment

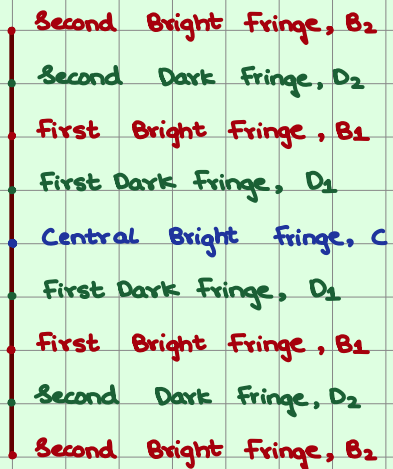
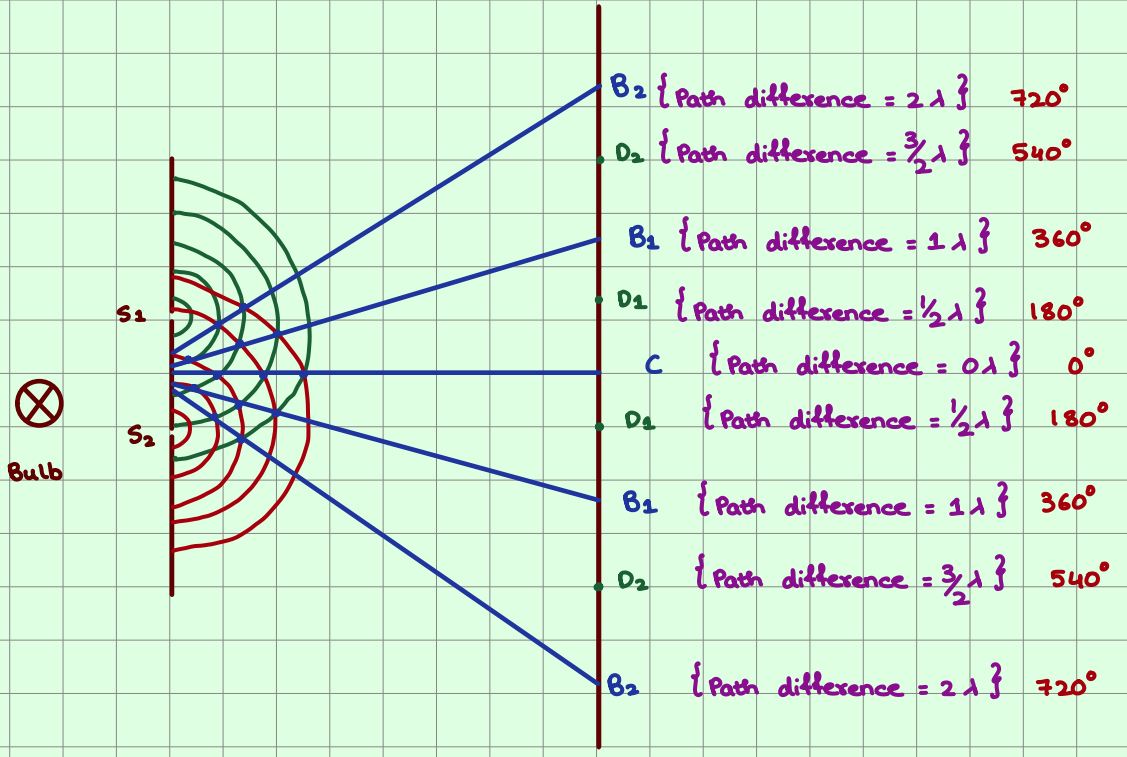


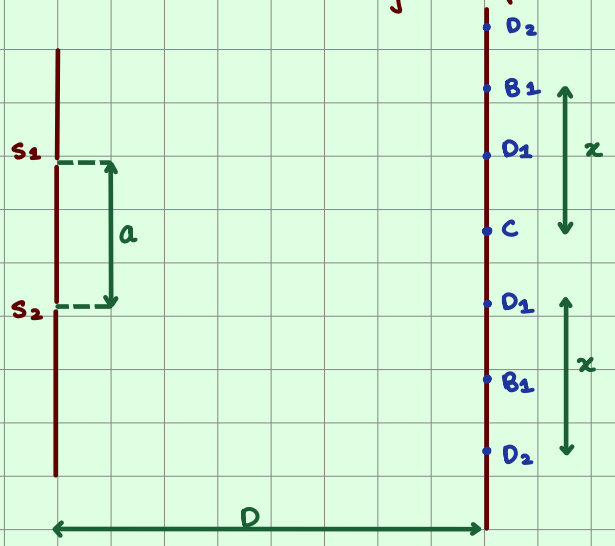
Figure 2 :- A zoom - in version of the interference pattern

1) This experiment was conducted to observe interference pattern using a source of light waves. A single source was preffered so that the points S_1 and S_2 can be classified as examples of coherent sources.
2) Light from the two slits is made to undergo diffraction.
3) An interference pattern is observed in the overlapping regions
4) This pattern consists of alternate regions of bright and dark fringes
5) Bright fringes are observed due to constructive interference between two in-phase points. Example :- A crest from S_1 overlapping with a crest from S_2.
6) Dark fringes are observed due to destructive interference between two out-of-phase points. Example :- A crest from S_1 overlapping with a trough from S_2.
7) This interference pattern can be labelled as shown above in figure 2.

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Formula to calculate "fringe separation" (x)



λ = wavelength

a = slit separation

D = Distance between double slit and screen

x = fringe separation {Distance between 2 Bright fringes and 2 dark fringes}

Formula :- $x = \frac{\lambda D}{a}$

Suggest suitable values for λ, D, a so that an interference pattern can easily be observed through the screen.

λ = 400nm - 700nm

D = 1m - 3m

a = 0.5mm - 1.5mm

} Learn.

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Example 1:- $\lambda = 550\,nm$, $D = 1.8m$, $a = 0.9mm$

(i) Find fringe separation (x) (ii) Find the distance between C and B_2 (iii) Find the distance between C and D_1

$x = \lambda D / a$

$x = \frac{(550 \times 10^{-9})(1.8)}{(0.9 \times 10^{-3})}$

$x = 1.1 \times 10^{-3}\,m$ $(1.1\,mm)$

$1.1 \times 2 = 2.2mm$

$1.1/2 = 0.55mm$

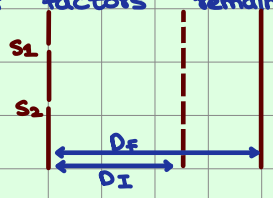
(iv) Find the distance between C and D_3

$C \rightarrow B_1 = 1.1mm$ $B_2 \rightarrow D_3 = 0.55mm$

$B_1 \rightarrow B_2 = 1.1mm$ $\therefore$ $2.75mm$

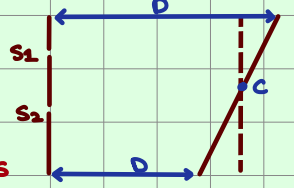
Factors affecting the fringe separation (x) and the brightness of the fringes.

1) The distance (D) is increased while all other factors remain unchanged.



- $\uparrow x = \frac{\lambda D \uparrow}{a}$ $\therefore$ x increases
- $\downarrow I \propto 1/d^2 \uparrow$ $\therefore$ Brightness decreases

2) The screen is tilted in a clockwise direction through an angle of 45°.



Top:-

- $\uparrow x = \frac{\lambda D \uparrow}{a}$ $\therefore$ x increases
- $\downarrow I \propto 1/d^2 \uparrow$ $\therefore$ Brightness decreases

Bottom:-

- $\downarrow x = \frac{\lambda D \downarrow}{a}$ $\therefore$ x decreases
- $\uparrow I \propto 1/d^2 \downarrow$ $\therefore$ Brightness increases

3) The size of each slit is increased while keeping λ, D, a constant

- x is unchanged
- Bright fringes appear to be brighter
- Dark fringes remain unchanged

4) The size of only one slit increased while size of other slit is unchanged keeping λ, D, a constant

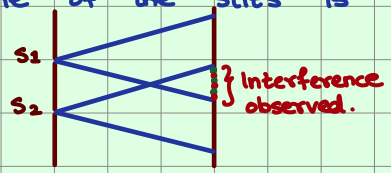
- x is unchanged
- Bright fringes appear to be brighter
- The term 'dark fringes' would now be replaced by 'less bright fringes'

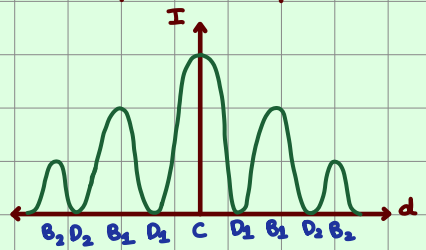
5) The source of light is now replaced with another source of longer wavelength. No other changes were made in this arrangement (violet replaced with red).

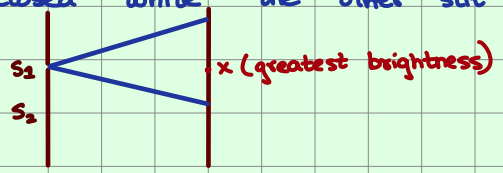
- $\uparrow x = \uparrow \lambda D / a$ $\therefore$ x increases
- Since fringes are now more spaced out, light has to travel a slightly larger distance to reach the screen. Hence, the brightness of the fringes marginally decreases or almost remains unchanged.

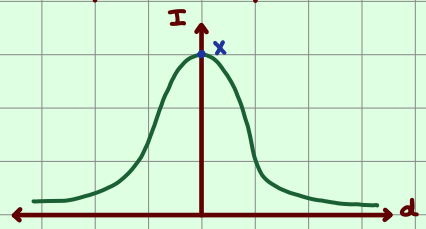
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6) One of the slits is completely closed while the other slit remains unchanged.









Interference will not happen, hence, no fringes are formed on the screen. However, a large area on the screen appears to be illuminated.

7) The slit separation 'a' is reduced while keeping all the other factors unchanged.

- $\uparrow x = \frac{\lambda D}{a\downarrow}$ ∴ x increases
- Since fringes are now spaced out further, therefore, the intensity of light either marginally decreases or remains almost unchanged

8) The light source is now replaced with a sound-producing source

- $\uparrow x = \frac{\lambda D\uparrow}{a}$ ∴ x increases $\lambda_{sound} > \lambda_{light}$
- The term bright and dark fringes will now be replaced by loud and soft sound.
- No appearance is visible on the screen, however, if a microphone connected to a CRO is moved along the screen, maximum intensity (loud) and minimum intensity (soft) is alternately detected on the CRO screen.

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Concept of Diffraction grating :-

- The term "diffraction grating" refers to an optical instrument constructed either using glass or plastic.
- This instrument has microscopic lines also called slits on it, so that when light falls onto this diffraction grating, spreading of light occurs as shown in the diagram below.
- If a screen is positioned in the background, diffraction of light can be observed
- The angle through which light diffracts is denoted by "θ." This angle is measured using the formula :-

$$d \sin\theta = n\lambda$$

where λ = wavelength

θ = angle of spread / diffraction measured from the center.

n = number of orders $n = 1, 2, 3 \ldots\ldots$

d = line spacing / grating spacing* [constant provided by manufacturer]

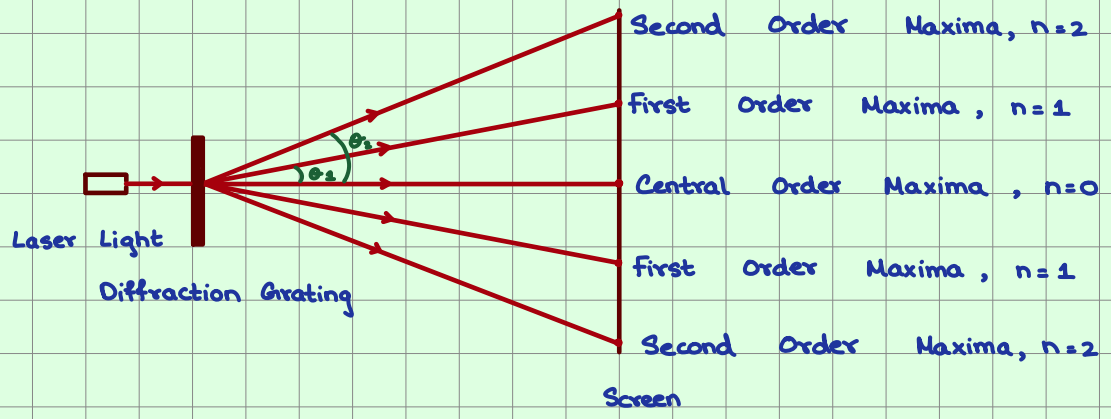


Figure 1 :- Appearance of light after passing through a diffraction grating

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Example 1:- $d = 2.0 \times 10^{-6}\,m$, $\lambda = 450\,nm$

(i) Find angle for First Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_1 = (1)(450\times10^{-9})$

$\theta_1 = 13^\circ$

(ii) Find angle for Second Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_2 = (2)(450\times10^{-9})$

$\theta_2 = 27^\circ \therefore \theta_2 > 2\times\theta_1$

(iii) Find angle for Third Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_3 = (3)(450\times10^{-9})$

$\theta_3 = 42^\circ \therefore \theta_3 > 3\times\theta_1$

(iv) Find angle for Fourth Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_4 = (4)(450\times10^{-9})$

$\theta_4 = 64^\circ \therefore \theta_4 > 4\times\theta_1$

(v) Find angle for Fifth Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_5 = (5)(450\times10^{-9})$

$\theta_5 =$ error.

(vi) What is the maximum / highest order

4^{th} order

(vii) Total number of orders

$2(4) + 1 = 9$

above and below Central Order / Central Maxima

$\therefore$ Formula = $2n + 1$

(viii) Calculate the highest order directly

$\theta < 90^\circ \therefore \sin\theta < 1$

$d\sin\theta = n\lambda$

$n\lambda/d = \sin\theta$ $\quad n < 4.4$

$n\lambda/d < 1$ $\quad \therefore n = 4^{th}$ order

$\dfrac{n(450\times10^{-9})}{(2.0\times10^{-6})} < 1$

If $\lambda = 620\,nm$

(ix) Find angle for First Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_1 = (1)(620\times10^{-9})$

$\theta_1 = 18^\circ$

(x) Find angle for Second Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_2 = (2)(620\times10^{-9})$

$\theta_2 = 38^\circ \therefore \theta_2 > 2\times\theta_1$

(xi) Find angle for Third Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_3 = (3)(620\times10^{-9})$

$\theta_3 = 68^\circ \therefore \theta_3 = 3\times\theta_1$

(xii) Find angle for Fourth Order Maxima

$d\sin\theta = n\lambda$

$(2.0\times10^{-6})\sin\theta_4 = (4)(620\times10^{-9})$

$\theta_4 =$ error

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(xiii) Find angular separation in the following:-

(a) 1st Order :- $\Delta\theta_1 = 18^\circ - 13^\circ = 5^\circ$

(b) 2nd Order :- $\Delta\theta_2 = 38^\circ - 27^\circ = 11^\circ$ $[\Delta\theta_2 > 2 \times \Delta\theta_1]$

(c) 3rd Order :- $\Delta\theta_3 = 68^\circ - 42^\circ = 26^\circ$ $[\Delta\theta_3 > 3 \times \Delta\theta_1]$

Statement :-

» Angular separation increases as number of orders increase

» Hence, maximum angular separation will always arise in the highest order i.e. 3rd order in our case as it is common to both.

Example 1 :- $d = 1.8 \times 10^{-6}$ m, $\lambda_1 = 420$ nm, $\lambda_2 = 590$ nm. Calculate minimum angular separation.

Step 1 :- $\downarrow n \propto \Delta\theta \downarrow$

$d\sin\theta = \downarrow n\lambda \uparrow$ ∴ longer wavelength i.e. 590 nm.

Step 2 :- $n\lambda/d < 1$

$\frac{n(590 \times 10^{-9})}{(1.8 \times 10^{-6})} < 1$

$n < 3.05$ ∴ n = 3rd order

Step 3 :- $d\sin\theta_1 = n\lambda_1$

$(1.8 \times 10^{-6})\sin\theta_1 = (3)(420 \times 10^{-9})$

$\theta_1 = 44^\circ$

$d\sin\theta_2 = n\lambda_2$

$(1.8 \times 10^{-6})\sin\theta_2 = (3)(590 \times 10^{-9})$

$\theta_2 = 80^\circ$

Step 4 :- $\Delta\theta = 80^\circ - 44^\circ = 36^\circ$

Example 2 :- A diffraction grating contains 500 lines per mm. Use this information to show that grating spacing 'd' must be equal to 2.0×10^{-6} m.

500 lines → 1 mm

500×10^3 ← 1 m

↳ N (number of lines per meter)

$N = 500{,}000\ m^{-1}$

$d = 1/500{,}000 = 2.0 \times 10^{-6}$ m

Example 3 :- A diffraction grating contains 550 lines per mm. Find grating spacing 'd'.

550 lines → 1 mm

550×10^3 ← 1 m

↳ N (number of lines per meter)

$N = 550{,}000\ m^{-1}$

$d = 1/550{,}000 = 1.8 \times 10^{-6}$ m

FORMULA :- $d\sin\theta = n\lambda$

$d = 1/N$

∴ $\frac{1}{N}\sin\theta = n\lambda$

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Example 4:- For light of wavelength 'λ', third order appears at an angle of 45°. Calculate highest order in this arrangement.

$d\sin\theta = n\lambda$

$d\sin 45 = 3\lambda \rightarrow (1)$

$\lambda = d\sin 45 / 3$

$n\lambda/d < 1$

$$n\left[\frac{d\sin 45}{3}\right] < 1$$

$$\frac{d}{}$$

$n < 4.24$ ∴ 4th order

Example 5:- Third order for wavelength 'λ_2' appears at the same angle as fourth order for wavelength 'λ_2'. If the difference in these wavelengths is 170 nm, find λ_2.

$d\sin\theta = n\lambda$

$d\sin\theta = 3\lambda_1 \rightarrow 1$

$d\sin\theta = 4\lambda_2 \rightarrow 2$

$3\lambda_1 = 4\lambda_2 \rightarrow 3$

$\lambda_1 = 4\lambda_2/3$

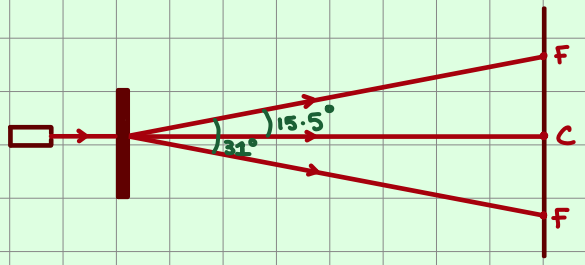
$\lambda_1 - \lambda_2 = 170\text{ nm}$

$4\lambda_2/3 - \lambda_2 = 170$

$\lambda_2 = 510\text{ nm}.$

Example 6:-

(i) Light of wavelength 500nm causes an 31° to be formed between its two first orders. Calculate the number of lines per mm present on the diffraction grating.



$d\sin\theta = n\lambda$

$1/N \sin(15.5) = (1)(500\times10^{-9})$

$N = 534{,}000$

$\frac{534{,}000}{1000} = 534$ lines per mm.

(ii) Advantage and disadvantage for using Second order as opposed to First order

Advantage:-

⇒ Less % error while measuring θ

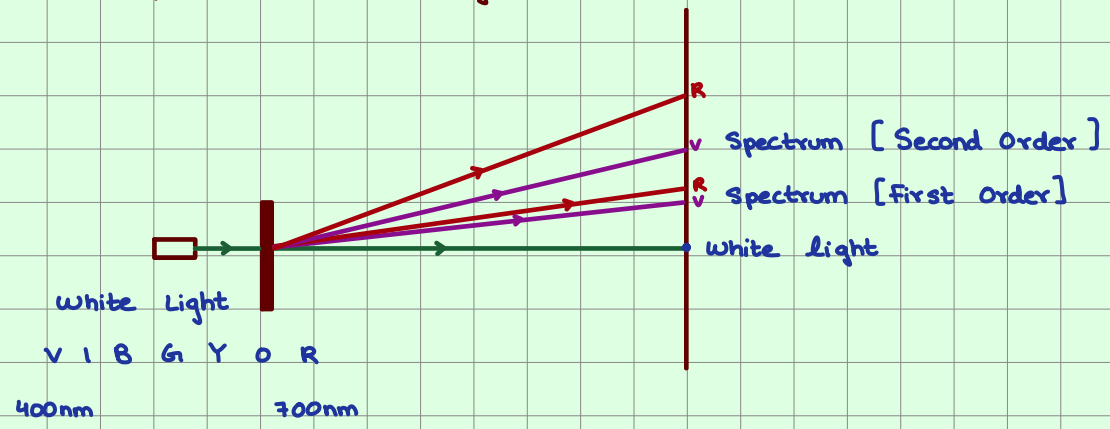
Disadvantage:-

⇒ ↓ $I \propto 1/d^2$ ↑ Less Bright spots ∴ difficulty in observation.

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Example 7: White light is allowed to fall on a diffraction grating.

(i) Draw the diagrammatic appearance for the orders observed on the screen. On one side of the central order upto and including the second order.



(ii) Why does the central order have no spectrum of colours?

No diffraction occurs as path difference is 0λ. Hence, all colours merge / superimpose, therefore, no spectrum is established.

(iii) If grating contains 750 lines per mm. Calculate angular separation in the First Order

$N = 750,000$ lines per meter

$d = {}^{1}/_{N} = 1.33 \times 10^{-6}$ m

for λ_R

$d\sin\theta = n\lambda$

$(1.33\times10^{-6})\sin\theta_1 = (1)(700\times10^{-9})$

$\theta_1 = 32^\circ$

for λ_V

$d\sin\theta = n\lambda$

$(1.33\times10^{-6})\sin\theta_2 = (1)(400\times10^{-9})$

$\theta_2 = 17.5^\circ$

$\Delta\theta = 32^\circ - 17.5^\circ = 14.5^\circ$

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Example 5:

(i) Light of wavelength 625 nm in its second order appears at an angle of 61°. Calculate grating spacing (d).

$d\sin\theta = n\lambda$

$d\sin(61) = (2)(625 \times 10^{-9})$

$d = 1.43 \times 10^{-6}$ m.

(ii) Which other wavelengths from the range of visible spectrum can also be found to appear at the same angle provided that it belongs to a different order.

400 - 700nm → visible spectrum

n = 1, 3, 4, 5 ....

$d\sin\theta = n\lambda$

$(1.43 \times 10^{-6})\sin 61 = (1)\lambda$ $\quad$ $(1.43 \times 10^{-6})\sin 61 = (3)\lambda$ $\quad$ $(1.43 \times 10^{-6})\sin 61 = (5)\lambda$

$\lambda = 1.25 \times 10^{-6}$ m $\quad$ $\lambda = 4.17 \times 10^{-7}$ m $\quad$ $\lambda = 3.13 \times 10^{-7}$ m

$\lambda = 1250$nm. $\quad$ $\lambda = 417$nm $\quad$ $\lambda = 313$nm

Accepted wavelength: 417nm

Doppler's effect and its formulas:

The term 'doppler effect' refers to the apparent change in the frequency or the wavelength when a source of sound or light moves relative to the observer.

1) Consider a source emitting sound waves of constant frequency. If the source is stationary, the observed frequency will always be identical to the actual frequency of the source i.e.

$f_o = f_s$

2) If the source is moving towards the observer (approach), the observed frequency will always be greater than the actual frequency of the source. This observed frequency is given by the formula: $f_o = \dfrac{f_s \times v}{v - v_s}$ f_o = observed frequency, f_s = Actual frequency, v = speed of sound, v_s = speed of source

3) If the source is moving away from the observer (receding), the observed frequency will always be lesser than the actual frequency of the source. This observed frequency is given by the formula: $f_o = \dfrac{f_s \times v}{v + v_s}$

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In case of light waves, consider an observer who is looking at a star which is emitting light waves of constant frequency.

» If the star is receding away from the observer, the observed frequency will be lower than the actual frequency of light emitted by the star. Since lower frequencies are inclined towards the red end of the spectrum, this phenomenon is termed as "Doppler Red Shift."

» If the star is moving towards the observer, the observed frequency will be greater than the actual frequency of light emitted by the star. Since higher frequencies are inclined towards the blue end of the spectrum, this phenomenon is termed as "Doppler Blue Shift."

Example Cases of Doppler Shift

Case 1:

• REST •

Source Observer

$f_s = 500 Hz$

Calculate observed frequency

$f_s = f_o$ [No Doppler Effect]

$\therefore f_o = 500 Hz.$

Case 2 (a):

• $v_s = 25 ms^{-1}$ →

Source Observer

$f_s = 500 Hz$

$v = 340 ms^{-1}$

Calculate observed frequency

$$f_o = \frac{f_s \times v}{v - v_s}$$

$$f_o = \frac{400 \times 340}{340 - 25}$$

$f_o = 430 Hz$

Case 2 (b):

• constant speed → •

Source Observer [Microphone + CRO]

What is observed on the CRO screen?

» Since approach, $f_o > f_s$, therefore, waves appear to be compressed / closer together

» $I \propto 1/d^2$, as 'd' decreases, intensity increases, therefore, amplitude / height of the wave increases.

Case 3 (a): • ← $v_s = 20 ms^{-1}$ •

Source Observer

$f_s = 600 Hz$

$v = 330 ms^{-1}$

Calculate observed frequency

$$f_o = \frac{f_s \times v}{v + v_s}$$

$$f_o = \frac{600 \times 330}{330 + 20}$$

$f_o = 570 Hz$

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Case 3 (b) :- • ← constant speed •

Source Observer [Microphone + CRO]

What is observed on the CRO screen?

⇒ Since recession, $f_o < f_s$, therefore, waves appear to be stretched out

⇒ $I \propto 1/d^2$, as 'd' increases, intensity decreases, therefore, amplitude / height of the wave decreases.

Case 4:- • speeds up → •

Source Observer

What is observed if source keeps speeding up as it approaches?

$$f_o = \frac{f_s \times v}{v - v_s \uparrow\uparrow}$$

⇒ As v_s keeps increasing, $v - v_s$ keeps decreasing

⇒ Therefore, f_o keeps getting higher and higher.

Case 5:- • ← speeds up •

Source Observer

What is observed if source keeps speeding up as it recedes?

$$f_o = \frac{f_s \times v}{v + v_s \uparrow\uparrow}$$

⇒ As v_s keeps increasing, $v + v_s$ keeps increasing

⇒ Therefore, f_o keeps getting lower and lower

Case 5 continued .... • •

Source Observer [Microphone + CRO]

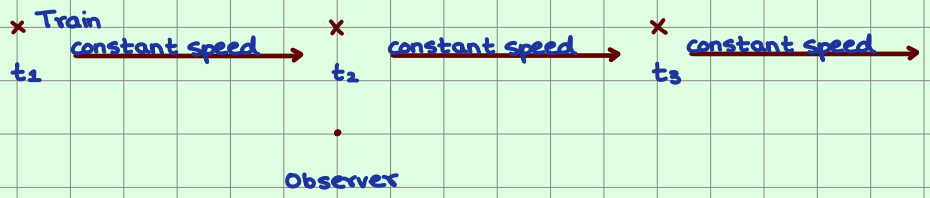
If time period on the CRO increases continuously, describe the motion of the source.

⇒ T keeps increasing continuously, f_o keeps decreasing continuously, hence, source recedes away

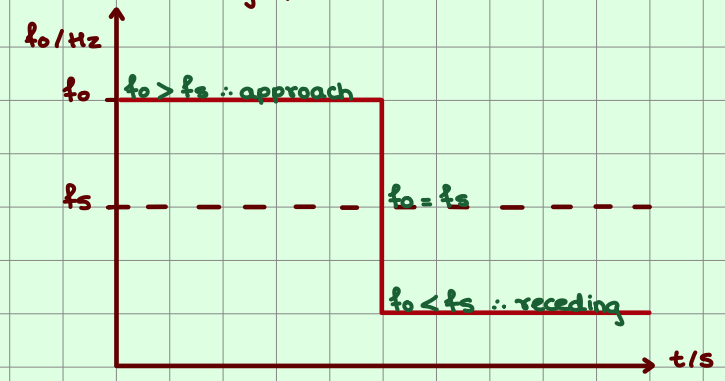
⇒ $f_o = \frac{f_s \times v}{v + v_s}$ Since f_o keeps decreasing, speed of source (v_s) keeps increasing.

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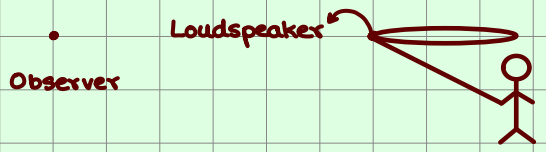
Case 6 :-



Sketch a graph of f_o vs t



Case 7 :-



What is observed as the person holding the loudspeaker makes it move in Circular Motion ?

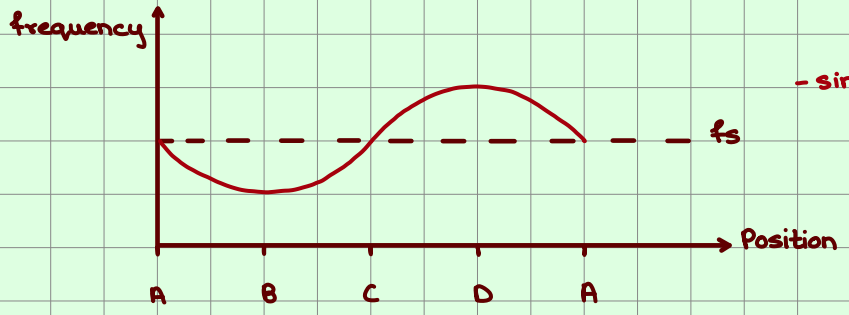
⇒ As the loudspeaker moves away while performing circular motion, f_o lower than f_s.

⇒ As the loudspeaker approaches, f_o higher than f_s.

Case 8 :-

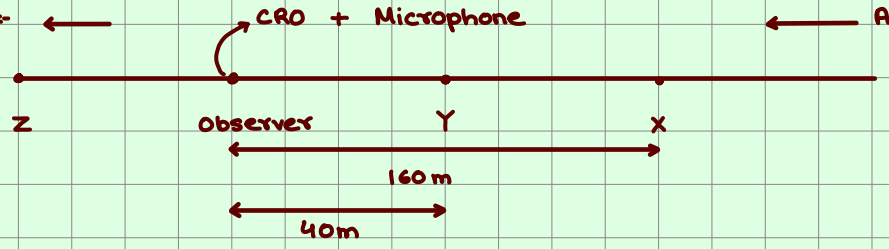


Sketch a graph to show how f_o varies with position of loudspeaker. Assume loudspeaker starts from position A.



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Case 9 :-



Speed of Ambulance :- 20 m/s

Speed of sound :- 340 m/s

f_s :- 780 Hz

(i) A_x/A_y = ??

$I \propto 1/d^2$

$I \propto A^2$

$A \propto 1/d$

$d_x/d_y = 160/40 = 4/1$

$\therefore \frac{A_x}{A_y} = \frac{1}{4}$

(ii) Calculate the difference between 2 wavelengths ($\Delta\lambda$) observed as ambulance travels from Y to Z.

At Y approach

$f_o = \frac{f_s \times v}{v - v_s}$

$f_o = \frac{780 \times 340}{340 - 20}$

$f_o = 830$ Hz

$\lambda_o = \frac{340}{830}$

$\lambda_o = 0.41$ m

At Z receding

$f_o = \frac{f_s \times v}{v + v_s}$

$f_o = \frac{780 \times 340}{340 + 20}$

$f_o = 740$ Hz

$\lambda_o = \frac{340}{740} = 0.46$ m

$0.46 - 0.41 = 0.05$ m

Concept of Stationary Waves OR Standing Waves :-

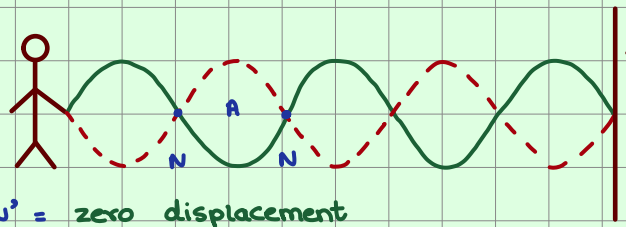
How to produce a Stationary Wave ?

- To produce a stationary wave using a rope, one end of the rope is fixed at a point while the other end is made to vibrate continuously.
- This will initially produce an incident progressive wave which travels towards the fixed end.
- It undergoes reflection from the fixed end causing a reflected progressive wave which travels in the opposite direction.
- Both these waves have the same amplitude, wavelength, frequency, and speed, and they are travelling in opposite directions.
- The superposition of these waves result in the formation of a stationary wave. The wave is called a stationary wave or a standing wave because it neither travels forward nor backwards. In fact, it appears to be standing in one position.

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Note:

- For a given length and a given speed, there are only certain frequencies at which a stationary wave is formed
- The lowest frequency is termed as 'fundamental frequency' (f_0).
- All the higher frequencies are termed as 'overtones.'
- Each overtone frequency is a multiple of the fundamental frequency.



Fixed end

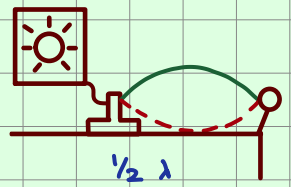
——— incident progressive wave

– – – – reflected progressive wave

Node 'N' = zero displacement

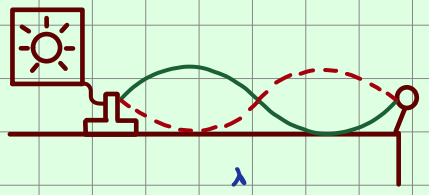
Antinode 'A' = max displacement

N - N = $\frac{1}{2}\lambda$, N - A = $\frac{1}{4}\lambda$, A - A = $\frac{1}{2}\lambda$



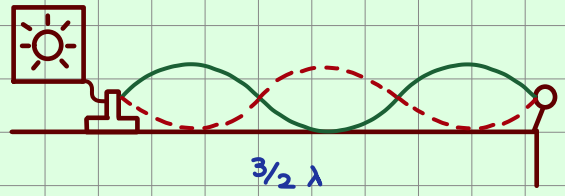
$\frac{1}{2}\lambda = 2m$ $\quad v = f\lambda$

$\lambda = 4m$ $\quad 40 = f(4)$ $\therefore$ $f = 10Hz$



$\lambda = 2m$ $\quad v = f\lambda$

$40 = f(2)$ $\therefore$ $f = 20Hz$



$\frac{3}{2}\lambda = 2m$ $\quad v = f\lambda$

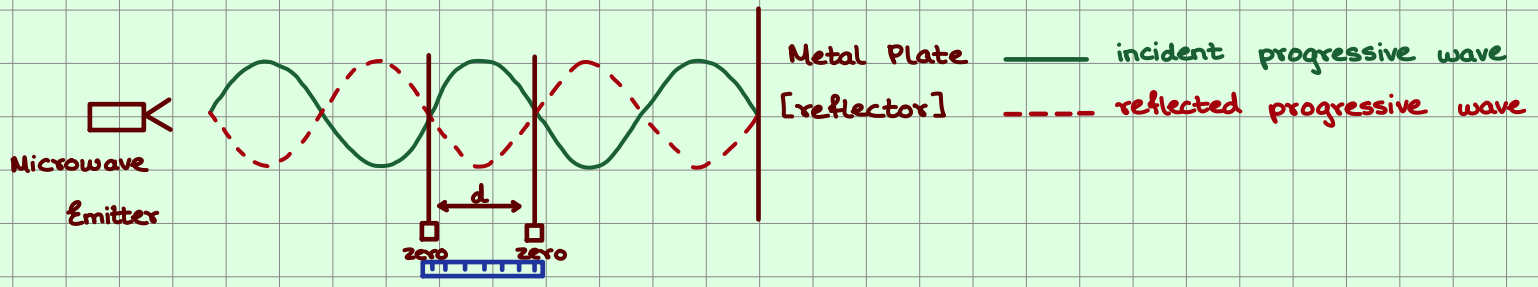
$\lambda = \frac{4}{3}m$ $\quad 40 = f\left[\frac{4}{3}\right]$ $\therefore$ $f = 30Hz$

L = 2m , v = 40 ms^{-1}

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Experiment to calculate wavelength and frequency for electromagnetic waves [using concept of stationary waves]:-

1) As the intensity meter is moved between the source and the metal plate, its reading alternately varies between zero and maximum. We measure the distance between 2 successive zero intensity points [d].
2) Twice this distance provides the wavelength [λ]
3) Apply $f = v/\lambda$ to obtain the corresponding frequency.



$\frac{1}{2}\lambda = d$

$\lambda = 2d$

$v = f\lambda$

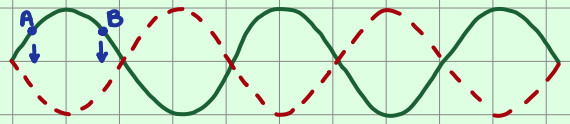
$3.0 \times 10^8 = f(2d)$

$f = \frac{3.0 \times 10^8}{2d}$

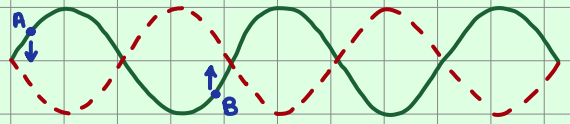
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How to calculate phase difference in a Stationary Wave?

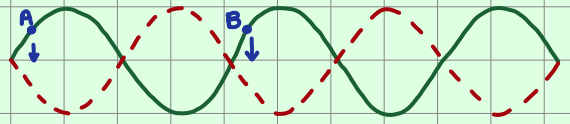
1) Points in the same loop operate in-phase with a phase difference 0π / $0°$



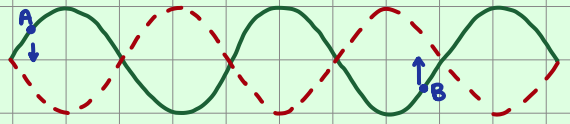
2) Points in the neighbouring loop are always out-of-phase with each other with a phase difference of $\pi/180°$



3) Points in the alternate loop are again in-phase with a phase difference of 2π /$360°$



4) Points A and B which are separated by a gap of 2 complete loops are out-of-phase with a phase difference of 3π / $540°$



How to compare amplitude in a stationary wave



$A_X < A_Y$ $A_P = A_Q$

Different points in a stationary wave have different amplitudes i.e. nodes exhibit zero displacement at all times, antinodes exhibit maximum displacement at all times, and all points in between will have some displacement ranging between zero and maximum. To compare amplitudes, measure the horizontal distance of those points from their respective nodes. The larger the distance, greater the amplitude.

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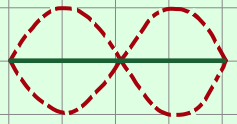
Differences between Progressive wave and Stationary Wave:-

1) Neighbouring points in a progressive wave have some phase difference between them whereas neighbouring points in a stationary wave are always in-phase with a phase difference of zero between them
2) In a progressive wave, every point attains the same amplitude whereas in a stationary wave, different points have different amplitudes.
3) In a progressive wave, energy is transferred along the direction of propogation whereas in a stationary wave, there is no net transfer of energy.
4) Every point gets displaced in a progressive wave whereas in a stationary wave there are points known as nodes which have zero displacement occur at all times.

How to construct a stationary wave at different instances of time?



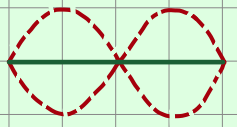
$t = 0$ (start)



$t = \frac{1}{4}T$



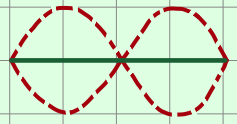
$t = \frac{1}{2}T$



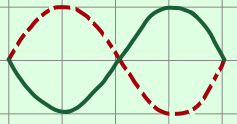
$t = \frac{3}{4}T$



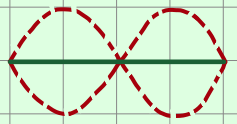
$t = T$



$t = \frac{5}{4}T$



$t = \frac{3}{2}T$



$t = \frac{7}{4}T$



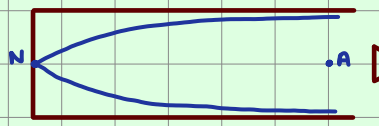
$t = 2T$

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Stationary Waves using a Sound Source:



- $\frac{1}{4}\lambda, \frac{3}{4}\lambda, \frac{5}{4}\lambda, \frac{7}{4}\lambda$ ....
- $f_o, 3f_o, 5f_o, 7f_o$ ....
- fundamental frequency, 1st overtone, 2nd overtone, 3rd overtone ....

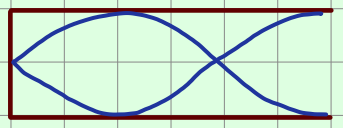


$L = 2m$, $v = 320 ms^{-1}$

$\frac{1}{4}\lambda = 2$ $\quad f = 320/8$

$\lambda = 8m$ $\quad f = 40 Hz$ $[f_o]$

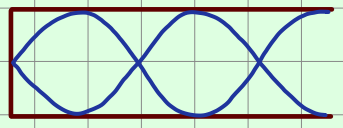
fundamental frequency



$\frac{3}{4}\lambda = 2m$ $\quad f = 320/8 \div 3$

$\lambda = 8/3 m$ $\quad f = 120 Hz$ $[3f_o]$

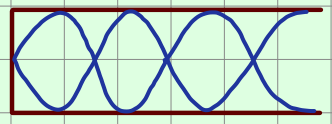
1st overtone



$\frac{5}{4}\lambda = 2m$ $\quad f = 320/8 \div 5$

$\lambda = 8/5 m$ $\quad f = 200 Hz$ $[5f_o]$

2nd overtone



$\frac{7}{4}\lambda = 2m$ $\quad f = 320/8 \div 7$

$\lambda = 8/7 m$ $\quad f = 280 Hz$ $[7f_o]$

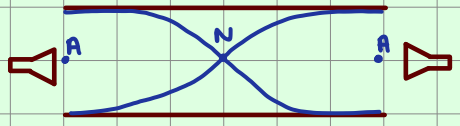
3rd overtone

Nodes formed on closed ends because lesser air molecules present ∴ zero displacement. Antinodes formed on open ends because more air molecules present ∴ maximum displacement.



- $\lambda/2, \lambda, \frac{3}{2}\lambda, 2\lambda$ ....
- $f_o, 2f_o, 3f_o, 4f_o$ ....
- fundamental frequency, 1st overtone, 2nd overtone, 3rd overtone ....

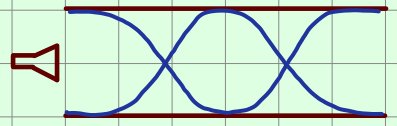
$L = 2m$, $v = 320 ms^{-1}$



$\frac{1}{2}\lambda = 2m$ $\quad f = 320/4 = 80 Hz$ $[f_o]$

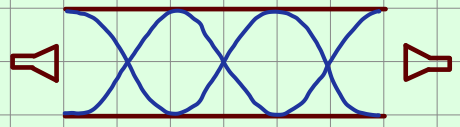
$\lambda = 4m$

fundamental frequency



$\lambda = 2m$ $\quad f = 320/2 = 160 Hz$ $[2f_o]$

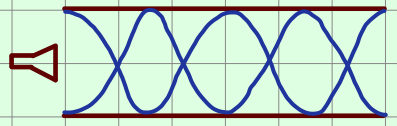
1st overtone



$\frac{3}{2}\lambda = 2m$ $\quad f = 320/4 \div 3 = 240$ $[3f_o]$

$\lambda = 4/3 m$

2nd overtone



$2\lambda = 2$ $\quad f = 320/1 = 320 Hz$ $[4f_o]$

$\lambda = 1m$

3rd overtone

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Polarization :-

1) A wave produced due to oscillations / vibrations restricted in a single plane is called a plane - polarized wave.

2) If these oscillations occur in a vertical plane, the wave is called vertical plane - polarized wave [Fig 1]

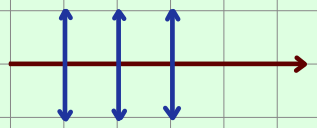


Figure 1 [Vertical - Plane polarization wave]

3) If a wave is produced due to oscillations in the horizontal plane, it is called the horizontal plane - polarized wave [Fig 2]

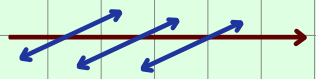


Figure 2 [Horizontal - Plane polarization wave]

4) If a wave is produced due to oscillations in the horizontal plane, vertical plane, and in other planes between these, it is called an unpolarized wave. [Fig 3]

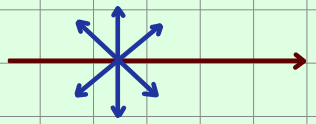


Figure 3 [Unpolarized Wave]

5) An unpolarized wave can be converted into a plane - polarized wave by making it pass through a transmission filter known as polaroid or polarizer and this phenomenon is termed as polarization as shown in Fig 4 and Fig 5.

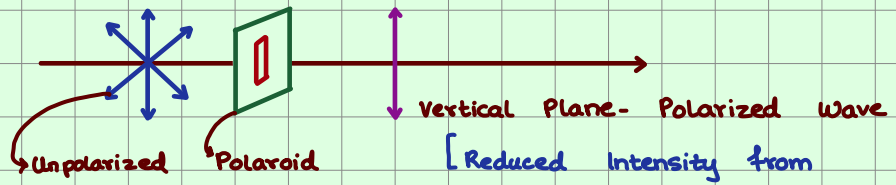


Figure 4

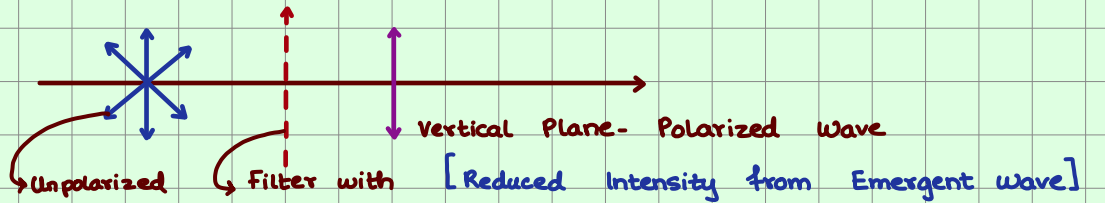


Figure 5

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6) Hence, polarization is defined as: Restrictions in the vibration of a wave so that the vibrations are allowed to occur in a single plane.

7) Polarization is a phenomenon only applicable for transverse waves. This is because their vibrations can occur in all directions perpendicular to their direction of travel. It is, therefore, possible to confine their vibrations to a single plane. This phenomenon is not applicable to longitudinal waves because their vibrations already occur in a single plane which is parallel to their direction of travel. Hence, no possibility of confining them to a single plane.

8) If a vertical plane-polarized wave has its transmission filter also aligned in the vertical axis, no reduction in intensity of the wave is observed [Fig 6].

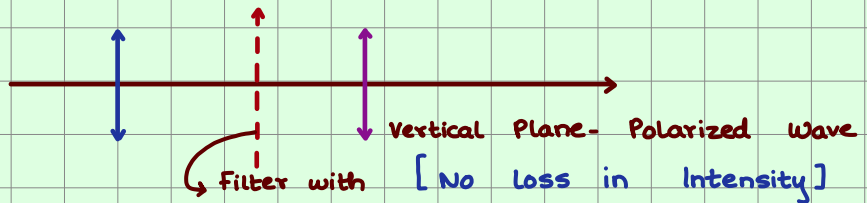


Figure 6

9) If, however, the transmission filter is rotated through 90°, zero intensity of light emerges out through the transmission filter [Fig 7].

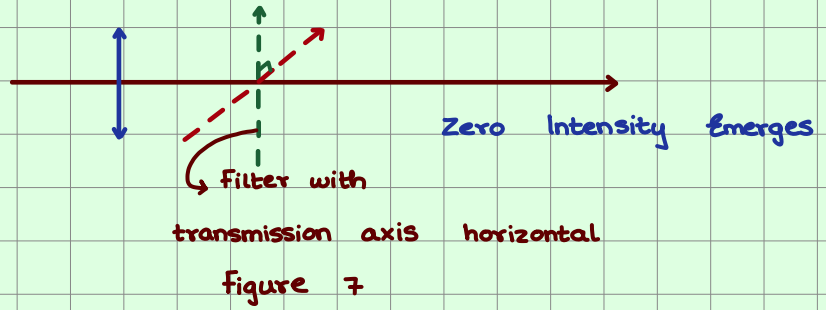
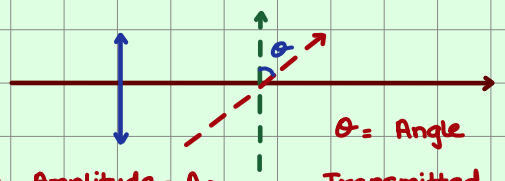


Figure 7

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Malus's Law:

Malus's law provides us with an inter-relationship between the transmitted intensity [I] and the incident intensity [I_o] as light emerges through a polarizing filter whose transmission axis has been rotated through an angle θ.



θ = Angle through which the transmission axis has been rotated.

Incident Amplitude = A_o Transmitted Amplitude = $A_o \cos\theta$

Incident Intensity = I_o Transmitted Intensity = I

Derivation:

$I = kA^2$

$I_o = kA_o^2$

$k = \frac{I_o}{A_o^2}$

$I = kA^2$

$I = \left[\frac{I_o}{A_o^2}\right][A_o^2 \cos^2\theta]$

$I = I_o \cos^2\theta$

Graph for intensity VS Angle θ



$I = kA^2$

$I = I_o \cos^2\theta$

$I \propto \cos^2\theta$

Example 1: Calculate angles θ if $I_o \rightarrow I_o/2$

$I = I_o \cos^2\theta$

$\frac{I_o}{2} = I_o \cos^2\theta$

$\sqrt{\cos^2\theta} = \sqrt{1/2}$

$\theta = \cos^{-1}[\sqrt{1/2}]$

$\theta = 45°$, $\theta = 180 - 45 = 135°$, $\theta = 180 + 45 = 225°$, $\theta = 360 - 45 = 315°$

Example 2: (i) Calculate angles θ if $I_o \rightarrow I_o/4$, given that the filter has already been rotated through 20°.

$\frac{I_o}{4} = I_o \cos^2\theta$

$\theta = \cos^{-1}[\sqrt{1/4}]$

$\theta = 60°$, $\theta = 180 - 60 = 120°$, $\theta = 180 + 60 = 240°$, $\theta = 360 - 60 = 300°$

Ans = $\theta - 20°$, $\theta = 40°, 100°, 220°, 280°$

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(ii) If filter has been rotated through 90°.

$\theta = 60°,\ 120°,\ 240°,\ 300°$

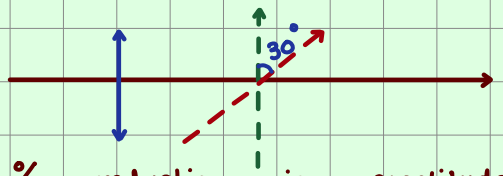
Ans: $\theta - 90°$

$\theta = -30°,\ 30°,\ 150°,\ 210°$

$360 - 30° = 330°$

$\theta = 330°,\ 30°,\ 150°,\ 210°$

Example 3:



(i) Calculate % reduction in amplitude of transmitted wave

Incident amplitude: A_o Transmitted amplitude: $A_o \cos 30°$

$$\% \text{ reduction} = \frac{A_o - A_o\cos 30}{A_o} \times 100$$

$$\frac{\cancel{A_o}[1 - \cos 30]}{\cancel{A_o}} \times 100 = 13.4\ \%$$

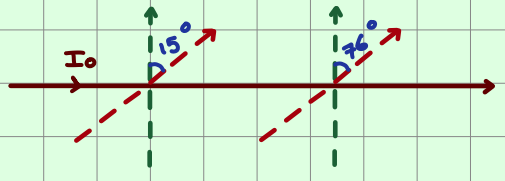
(ii) Calculate % reduction in intensity

Incident intensity = I_o Transmitted intensity: $I_o\cos^2\theta$

$$\frac{I_o - I_o\cos^2 30}{I_o} \times 100$$

$$\frac{\cancel{I_o}[1 - \cos^2 30]}{\cancel{I_o}} \times 100 = 25\ \%$$

Example 4: Calculate transmitted intensity as light passes through this filter combination



Filter 1: $I = I_o\cos^2 15$

$I = 0.93\, I_o$

Filter 2: $I = I_o\cos^2\theta$

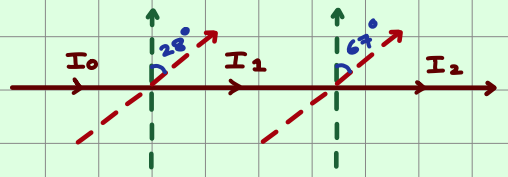
$I = (0.93 I_o)\cos^2(76-15)$

$I = (0.93 I_o)(\cos^2 61)$

$I = 0.219\ I_o$

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Example 5:



(i) Find I_1/I_0

$I_1 = I_0 \cos^2 28$

$I_1 = 0.78 I_0$

$\frac{I_1}{I_0} = \frac{0.78 I_0}{I_0} = 0.78$

(ii) Find I_2/I_0

$I_2 = (0.78 I_0)\cos^2(67-28)$

$I_2 = (0.78 I_0)(\cos^2 39)$

$\frac{I_2}{I_0} = \frac{0.47 I_0}{I_0} = 0.47$

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Current Electricity

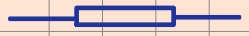
- Ohm's Law:-

  $V \propto I$ [Length, Area, Temperature constant]

  $V = IR$

Define Resistance:- $R = V/I$. Ratio of $\frac{\text{Voltage}}{\text{Current}}$

Ohmic Conductors:-

- Fixed Resistors 

All metals at constant temperature



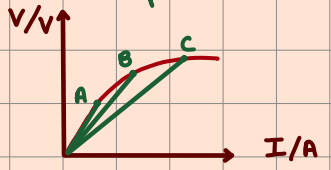
Since ratio of V/I is constant

∴ Resistance is also constant

Non - Ohmic Conductors:-

- Thermistor 

As temperature increases, resistance decreases



- A is the steepest
- C is least steep
- Ratio of V/I decreases
- ∴ Resistance also decreases

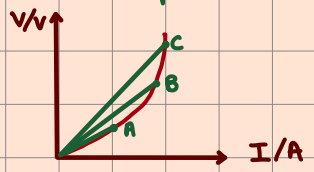


- A is least steep
- C is the steepest
- Ratio of I/V increases
- Ratio of V/I decreases
- ∴ Resistance also decreases

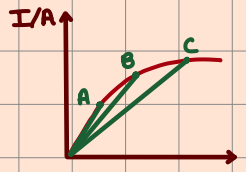
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- Filament Bulb, Filament lamp 

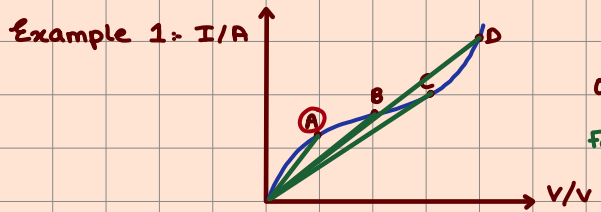
As temperature increases, resistance increases



- A is least steep
- C is the steepest
- Ratio of $^{V}/_{I}$ increases
- ∴ Resistance also increases



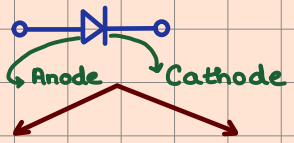
- A is the steepest
- C is least steep
- Ratio of $^{I}/_{V}$ decreases
- Ratio of $^{V}/_{I}$ increases
- ∴ Resistance also increases

Example 1: 

On which point is the resistance lowest?

For A: $^{I}/_{V}$ is the greatest, $^{V}/_{I}$ is lowest ∴ Resistance is also lowest

- Semi-connected Diodes / Diodes

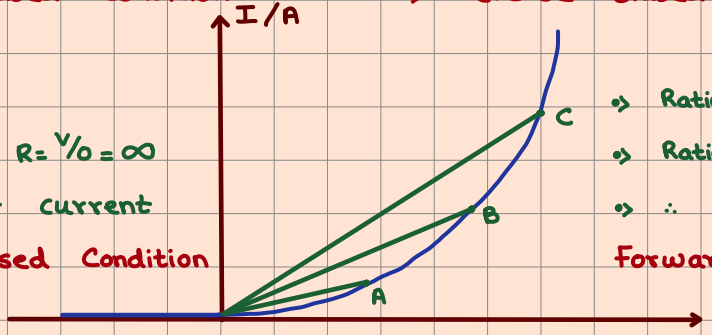




- offers a very low resistance
- Allows large current to flow
- Forward-Biased condition



- offers infinite resistance
- Allows zero current to flow
- Reverse-Biased condition



I= zero $R = {}^{V}/_{0} = \infty$

No flow of current

Reverse-Biased Condition

- Ratio of $^{I}/_{V}$ increases
- Ratio of $^{V}/_{I}$ decreases
- ∴ Resistance also decreases

Forward-Biased Condition

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What are the factors affecting the Resistance ?

$R \propto L$ $\quad R \propto 1/A$

$\therefore R \propto L/A$

$R = \rho L/A$

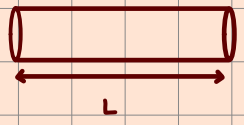
↳ Resistivity of a material [constant]

$\rho = RA/L = \Omega m^2/m = \Omega m$

Definition of resistivity :-

The resistivity of a material is said to be equal to its resistance provided that the material has a unit length and offers a unit cross-sectional area.

Example 1 :-



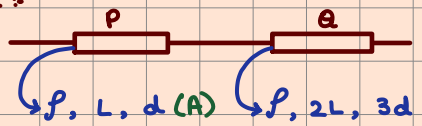
Resistance = 10 Ω

If the length of this material is doubled keeping volume unchanged, calculate the new Resistance

$R = \rho L/A$ $\quad V = A \times L$ (A: halved, L: doubled; V: constant)

$R = \rho L^2/A \times 1/2$ = Quadrupled $\quad \therefore$ New Resistance = 40 Ω

Example 2 :-



(i) $\frac{R_P}{R_Q}$ $\quad A = \pi d^2/4$ $\quad A \propto d^2$

$R_P = \frac{\rho L}{A}$ $\quad R_Q = \frac{\rho \cdot 2L}{9A}$

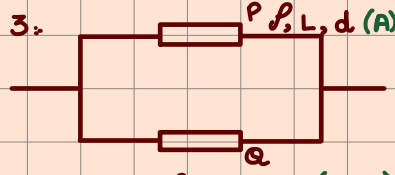
$\frac{R_P}{R_Q} = \frac{\rho L}{A} \times \frac{9A}{\rho \cdot 2L}$ $\quad \therefore \frac{R_P}{R_Q} = \frac{9}{2}$

(ii) $\frac{I_P}{I_Q} = 1$ [Same for Series]

(iii) $\frac{V_P}{V_Q} = \frac{I_P \cdot R}{I_Q \cdot R} = \frac{9}{2}$ $\quad V \propto R$

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Example 3:



2ρ, 4L, 5d (25 A)

(i) $\frac{R_P}{R_Q}$ $R_P = \frac{\rho L}{A}$ $R_Q = \frac{(2\rho)(4L)}{25A}$

$\frac{R_P}{R_Q} = \frac{\rho L}{A} \times \frac{25A}{2\rho . 4L}$ $\therefore \frac{R_P}{R_Q} = \frac{25}{8}$

(ii) $\frac{I_P}{I_Q}$ $I \propto 1/R$ $\therefore \frac{8}{25}$

(iii) $\frac{V_P}{V_Q} = 1$ [Same in parallel]

Example 4: Copper → 5R, 3ρ, 7L

Silver → 7R, 2ρ, 12L

Calculate $\frac{\text{Diameter of Copper}}{\text{Diameter of Silver}}$

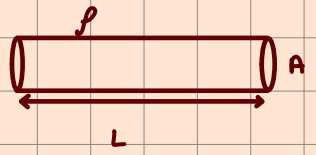
$5R = \frac{3\rho . 7L}{A_{copper}}$ $7R = \frac{(2\rho)(12L)}{A_{silver}}$

$A_{copper} = \frac{21\rho L}{5R}$ $A_{silver} = \frac{24\rho L}{7R}$

$\frac{A_{copper}}{A_{silver}} = \frac{21\rho L}{5R} \times \frac{7R}{24\rho L} = \frac{49}{40}$

$\frac{\text{Diameter of Copper}}{\text{Diameter of Silver}} = \sqrt{\frac{49}{40}} = 1.11$

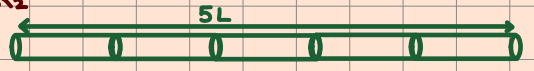
Example 5:



- 5 such wires are connected end-to-end to give resistance R_1
- 5 such wires are stacked up together forming a bundle with resistance R_2

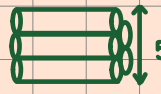
Find $\frac{R_1}{R_2}$

For R_1



$R_1 = \frac{\rho . 5L}{A}$

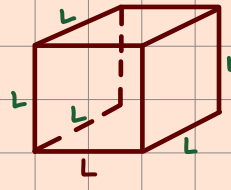
For R_2



$R_2 = \frac{\rho . L}{5A}$

$\frac{R_1}{R_2} = \frac{\rho . 5L}{A} \times \frac{5A}{\rho . L} = \frac{25}{1}$

Example 6:



ρ, V [Volume]. Show that the resistance between 2 opposite faces is $R = \frac{\rho}{[V]^{1/3}}$

$V = A \times L = L^3$ $\therefore [V]^{1/3} = L$

$R = \frac{\rho . L}{L^2} = \frac{\rho}{L}$

$\therefore R = \frac{\rho}{[V]^{1/3}}$

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Example 7:



V [voltage], I, ρ, L, 2d [4A]

2V, x, 2ρ, 3L, d [A]

Find the amount of current (x) in the second wire

$V = IR$

Wire 1: $\frac{V}{I} = \frac{\rho . l}{4A}$ Wire 2: $\frac{2V}{x} = \frac{2\rho . 3L}{A}$

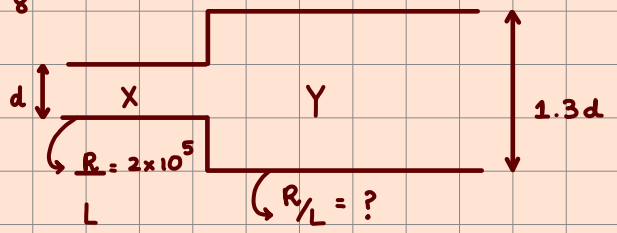
$V = \frac{I . \rho L}{4A}$ $V = \frac{x . 6\rho L}{2A}$

$\frac{I . \rho L}{4A} = \frac{x . 3\rho L}{A}$

$I . \rho LA = x . 12 \rho LA$

$\therefore x = \frac{I}{12}$

Example 8



X and Y are made of the same material

$R = \frac{\rho . l}{A}$

$\frac{R}{L} = \frac{\rho}{A}$

$\therefore \rho = 2 \times 10^5 A$

$R = \frac{\rho . l}{1.69A}$

$\frac{R}{L} = \frac{\rho}{1.69A}$

$\frac{R}{L} = \frac{2 \times 10^5 A}{1.69 A}$

$\therefore \frac{R}{L} = 1.2 \times 10^5 \ \Omega m^{-1}$

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Current is the rate of flow of charge / charge carrier

- $I = \frac{Q}{t}$ OR $Q = It$ [I = constant]
- Units of Current : Cs^{-1}

Example 1 : $I = 10\ A$, $t = 2s$

$Q = It$

$10 \times 2 = 20C$

Example 2 : (i) I varies from 100mA to 20mA in 8s.

$$Q = \left(\frac{[100 + 20] \times 10^{-3}}{2} \right) [8]$$

$Q = 0.48\ C$

(ii) If $1e^-$ carries 1.6×10^{-19} C, calculate number of e^- required for this charge to be produced

$1 \longrightarrow 1.6 \times 10^{-19}\ C$

$x \longrightarrow 0.48C$

$x = 3.0 \times 10^{18}$ electrons

Voltage is defined as the energy transferred per unit charge

$V = \frac{WD}{Q}$ OR $WD = QV$

- Units of Voltage : Volts (V) or JC^{-1}

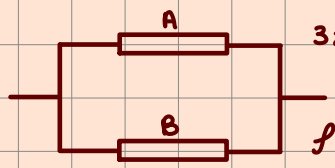
Power is defined as energy per unit time

Power : $\frac{Energy}{Time}$ OR Energy = Power x time

1) $P = IV$
   - Power generated, developed, supplied, produced, transformed
2) $P = I^2R$
   - Power loss, dissipation.
   - $P = V^2/R$
   - No restriction

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Example 1:-



3ρ, L, 2d

ρ, 2L, d

Calculate $\frac{P_A}{P_B}$

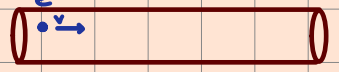
$P = V^2/R$

$P \propto 1/R$

$R_A = \frac{3\rho . L}{4A}$ $\quad R_B = \frac{\rho . 2L}{A}$

$\frac{R_A}{R_B} = \frac{3\rho L}{4A} \times \frac{A}{2\rho L} = \frac{3}{8}$ $\quad \therefore \frac{P_A}{P_B} = \frac{8}{3}$

Concept of Drift Velocity:-



Cross-sectional area (A)

Length (L)

- when no potential difference is applied across a conductor, the e^- within the conductor move randomly, hence, net flow of current is regarded to be zero.
- however, if a potential difference is applied, these e^- now start moving in one direction with a speed known as the drift velocity of the e^-
- The drift velocity 'v' can be worked out using the formula:-

$I = nAve$

where:-

I = current, A = cross-sectional area, e = charge on an electron, v = drift velocity, n = number of e^- present in $1 m^3$, number of e^- per unit volume, number density of e^-

Derivation:-

1) $1 m^3 \rightarrow n$ electrons

$AL\ m^3 \rightarrow nAL$ electrons

2) $1 e^- \rightarrow e$

$nAL\ e^- \rightarrow nALe$

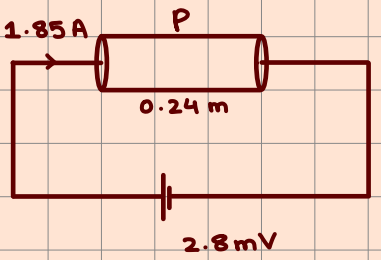
3) $I = \frac{nALe}{t}$

$I = nAve$

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How to apply the formula for drift velocity "$I = nAve$"

Example 1 :



$\rho_P = 1.8 \times 10^{-8}\ \Omega m^{-1}$

Total number of e^- in P: $6.0 \times 10^{22}\ e^-$

(i) Calculate the drift velocity (v)

Step 1 $V = IR$

$(2.8 \times 10^{-3}) = 1.85(R)$

$R = 1.5 \times 10^{-3}\ \Omega$

Step 2 $R = \rho L / A$

$1.5 \times 10^{-3} A = (1.8 \times 10^{-8})(0.24)$

$A = 2.9 \times 10^{-6}\ m^2$

Step 3 Volume: $(0.24)(2.9 \times 10^{-6}) = 6.912 \times 10^{-7}$

$6.0 \times 10^{22} \rightarrow 6.912 \times 10^{-7}\ m^3$

$x \rightarrow 1\ m^3$

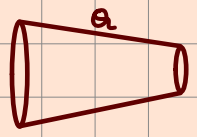
$x = 8.8 \times 10^{28}\ e^-$

Step 4 $I = nAve$

$1.85 = (8.8 \times 10^{28})(2.9 \times 10^{-6})(v)(1.6 \times 10^{-19})$

$v = 4.6 \times 10^{-5}\ ms^{-2}$

(ii)



same material, same volume

Compare the drift velocity of Q with the drift velocity of P.

$v = I / nAe \quad \therefore v \propto 1/A$

Going from left to right, the area of cross-section decreases. Since area is inversely proportional to drift velocity, the drift velocity increases $\therefore v_Q > v_P$

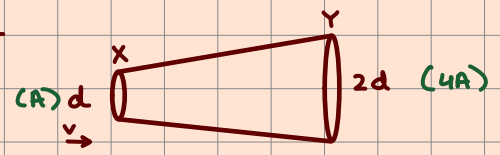
(iii) Compare resistance of Q with resistance of P.

$V = \downarrow A \times L \uparrow$

$\uparrow\uparrow R = \rho L \uparrow / A \downarrow \quad \therefore R_Q > R_P$

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Example

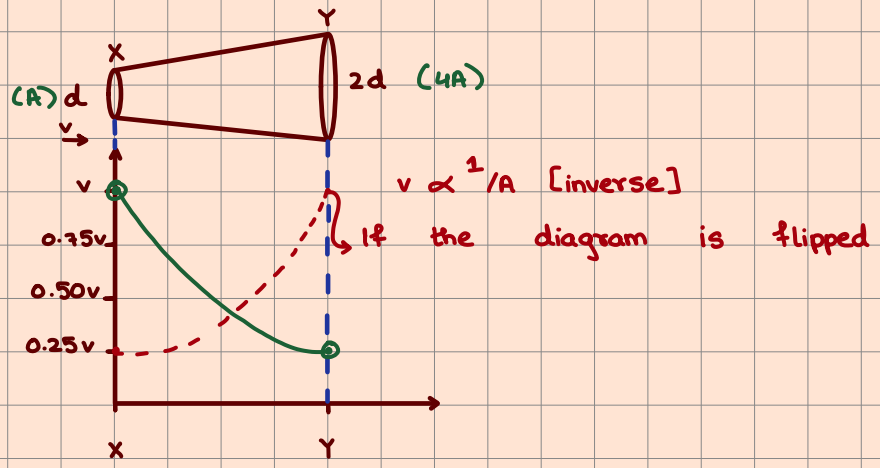


(i) Calculate drift velocity at Y in terms of v.

$v = I/nAe$ ↓ ↑

Since area increases 4 times, $v_Y = 0.25\,v$ or $1/4\,v$

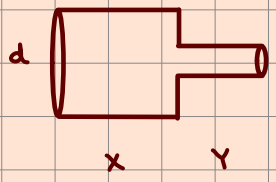
(ii) Draw a sketch for how 'v' varies



$v \propto 1/A$ [inverse]

If the diagram is flipped

Example 3:



X and Y are made of the same material

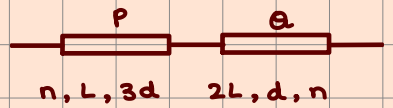
Find v_x/v_y

$v_x = \frac{I}{n(A)e}$ $\quad v_y = \frac{I}{n(0.35A)e}$

$\frac{v_x}{v_y} = \frac{I}{n(A)e} \times \frac{n(0.35A)e}{I} = \frac{0.35}{1}$

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Example 4:- Series Combination

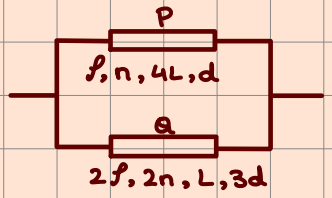


P and Q are made up of the same material

Find ratio of $\frac{V_P}{V_Q}$

$V_P = \frac{I}{n(9A)(e)}$ $V_Q = \frac{I}{n(A)e}$ $\frac{V_P}{V_Q} = \frac{I}{d(9A)e} \times \frac{n(A)e}{I} = \frac{1}{9}$

Example 5:- Parallel Combination



(i) Calculate $\frac{V_P}{V_Q}$

Step 1:- $R = \frac{\ell L}{A}$

$R_P = \frac{\ell . 4L}{A}$ $R_Q = \frac{2\ell . L}{9A}$

$\frac{R_P}{R_Q} = \frac{\ell^2 . 4}{A} \times \frac{9A}{2\ell . L} = \frac{18}{1}$

Step 2:- $V = IR$

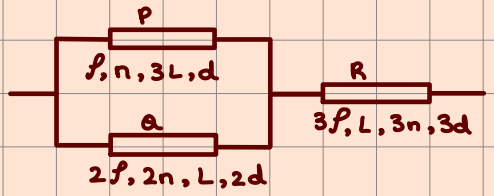
$I \propto 1/R$

$\therefore I = \frac{1}{18}$

Step 3:- $\frac{I}{d(A)e} \times \frac{(2n)(9A)(e)}{18I} = 1$

$\therefore \frac{V_P}{V_Q} = 1$

Example 6:- Series and Parallel



Find $\frac{V_P}{V_R}$

Step 1:- $\frac{R_P}{R_Q} = \frac{\ell . 3L}{A} \times \frac{4A}{2\ell . 2L} = \frac{3}{1}$

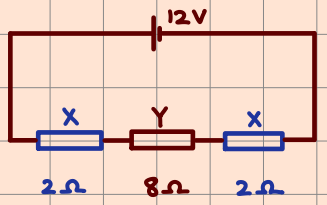
Step 2:- $I \propto 1/R$

$\therefore I = \frac{1}{3}$

Step 3:- $\frac{V_P}{V_R} = \frac{I}{n(A)e} \times \frac{3n . 9A e}{(1+3)I} = \frac{27}{4}$

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Example 7:



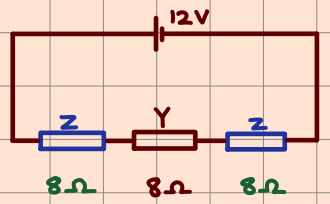
(i) Calculate Current [I]

V = IR

12 = I (2 + 8 + 2)

I = 1A

(ii)



All resistors are made of the same material. The diameter of Z is ½ the diameter of Y

Calculate new Current

Diameter ½ ∴ Area ¼

$R \propto 1/A$

∴ R = 4R

V = IR

12 = I (24)

I = 0.5A

(iii) Calculate $\frac{V_Z}{V_X}$

$$\frac{V_Z}{V_X} = \frac{0.5}{(n)(\frac{1}{4}A)(e)} \times \frac{(n)(A)(e)}{1} = \frac{2}{1}$$

(iv) Compare the drift velocity of Z with drift velocity of X.

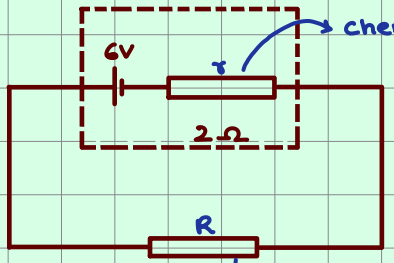
- $v \propto 1/A$
- I = half, A = one-fourth
- Impact of area outweighs the impact of current
- Since $v \propto 1/A$, as area decreases, drift velocity increases

∴ $V_Z > V_X$

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D.C. Circuits

Concept of external resistor [R], internal resistor (r), terminal potential difference (V), lost volts (v), and EMF (E):-



→ chemicals present inside the battery that provide resistance

→ connected by us to the battery

- $R_T = 6\Omega$ $\quad V = IR$
- $I = {}^{6}/_{6} = 1A$ $\quad V = 1(4) = 4V$ → Terminal potential difference which is the amount of voltage required by the external resistor
- $v = Ir$

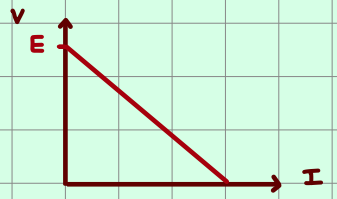
  $v = (1)(2) = 2V$ → Lost volts are wasted across the internal resistance
- $EMF = V + v$

  $E = V + Ir$ $\quad \therefore EMF = 4 + 2 = 6V$

  $E = IR + Ir$

  $E = I[R + r]$

Graph of Terminal potential difference [V] against current [I]



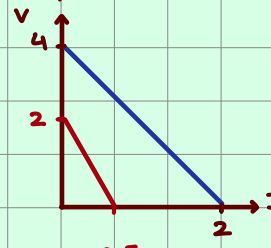
$E = V + Ir$ $\quad \therefore$ straight line graph

$V = (-r).I + E$ → -ve gradient

$y = mx + c$ → +ve y-intercept

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Example 1



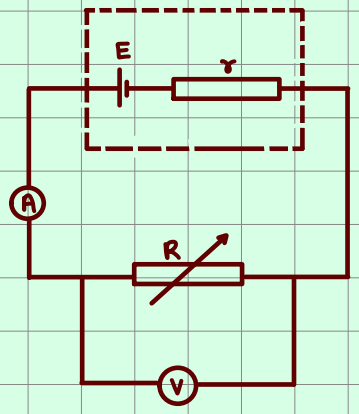
(i) EMF = 4V

(ii) Internal resistance (r): $\frac{\text{Rise}}{\text{Run}} = \frac{4}{2} = 2\,\Omega$

(iii) Sketch a new graph where E is halved and r is doubled.

$r = \frac{\text{Rise}}{\text{Run}}$ ∴ $4 = \frac{2}{x}$ ∴ $x = 0.5$

Finding values of EMF and internal resistance (r) experimentally:



» Set the variable resistor at a particular value and record current and voltage.

E.g. I = 0.25 A and V = 1V

» Repeat the same process again to get one more set of values of current and voltage.

E.g. I = 0.3A and V = 0.9V

$E = V + Ir$

$E = 1 + 0.25r$ → ① $\qquad E = 0.9 + 0.3r$ → ②

$\pm 1 = {}^{-}0.25r \pm E$

$-0.9 = 0.3r - E$

$0.1 = 0.05r$

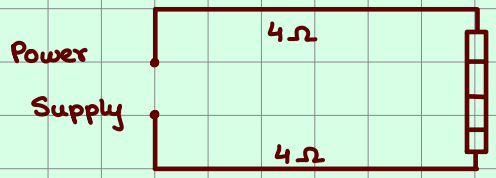
$r = 2\,\Omega$, $E = 1.5V$

Definition of the following terms using energy considerations:

1) Terminal potential difference is defined as the amount of electrical energy converted into other forms e.g. light or heat when a unit charge flows through the external circuit.
2) Lost volts is defined as the energy dissipated in the form of heat when a unit charge flows through the internal circuit
3) Electromotive force (EMF) is defined as the amount of chemical energy in the battery which is converted into electrical form when a unit charge flows through the complete circuit.

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How to calculate the efficiency of a circuit?



(i) Calculate current required by the heater

$P = IV$

$1200 = I(240)$

$I = 5A$

(ii) Calculate voltage supplied by the Power Supply

$V = IR$

$V = 5 \times 4 = 20V$ required by wires

$\therefore 240 + 20 + 20 = 280V$

(iii) Calculate efficiency.

$\frac{240}{280} \times 100 = 85.7\%$

(iv) How to improve efficiency of this circuit

→ Double the diameter of the wire, hence, area increases 4 times

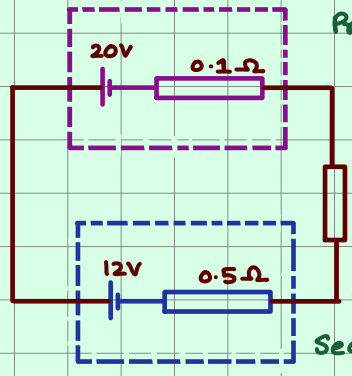
$R \propto 1/A$: R: one-fourth

$1/4 \times 4 = 1\Omega$ $\therefore 1 \times 5 = 5V$ required by wires

$\frac{240}{250} \times 100 = 96\%$ efficiency

Advantage → Less overheating, Disadvantage → Expensive

How to charge a secondary cell using a primary cell



→ Make sure that (+)ve terminal is connected with (+)ve terminal so that the cells work against each other

(i) Calculate charging current

$V = IR_T$

$20 - 12 = I(0.1 + 3.4 + 0.5)$

$I = 2A$

(ii) Calculate power supplied by the primary cell

$P = IV$ $\therefore P = 2 \times 20 = 40W$

(iii) Calculate power dissipated in the circuit

$P = I^2R_T$ $\therefore P = (2)^2(4) = 16W$

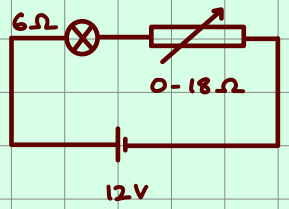
(iv) Calculate effiency

$\frac{40 - 16}{40} \times 100 = 60\%$

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Comparision between a variable resistor and a potential divider circuit

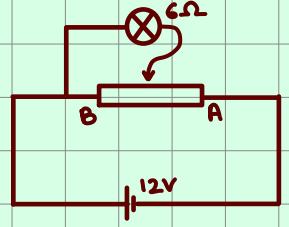
Variable Resistor / Rheostat



- Rmin = 6 + 0 = 6Ω Rmax = 6 + 18 = 24Ω
- I max = $^{12}/_{2}$ = 2A I min = $^{12}/_{24}$ = 0·5A

$0{\cdot}5A \leq I \leq 2A$

Potential Divider / Potentiometer



Scenario 1 :- Connected to A

- Bulb receives all 12V

$I = {}^{12}/_{6} = 2A$

Scenario 2 :- Connected to B

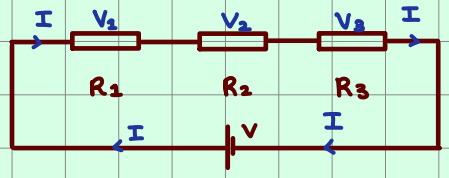
- Bulb receives 0V

$I = {}^{0}/_{6} = 0A$

$0A \leq I \leq 2A$

Advantage :- Provides a larger range of current

Properties for resistors in Series:



- $R_T = R_1 + R_2 + R_3$ ⟶ Derivation
- Current is the same
- $V \propto R$

$V = V_1 + V_2 + V_3$

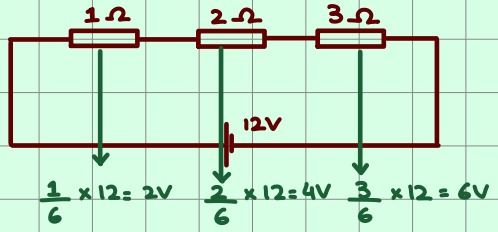
~~I~~ $R_T = {}/R_1 + {}/R_2 + {}/R_3$

$R_T = R_1 + R_2 + R_3$

Ratio method

$V = \frac{R}{R_T} \times V_B$

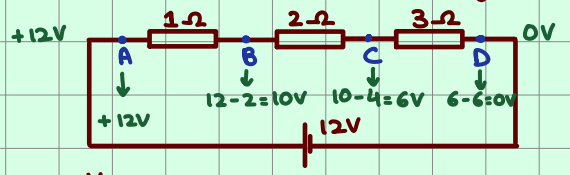
Example 1:



The lower the resistance, lower the voltage drop across the resistor and vice versa

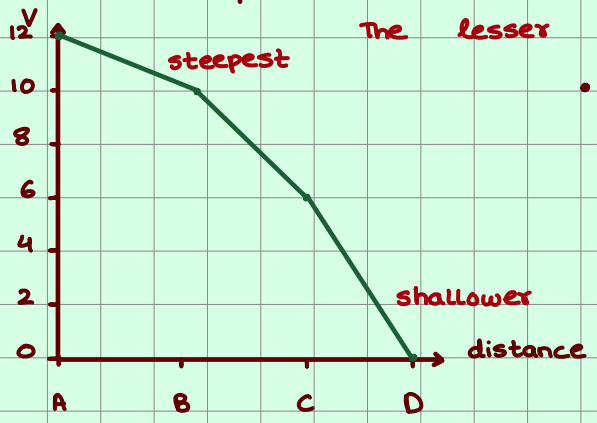
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- How to calculate Voltage / Potential at any point in a series circuit



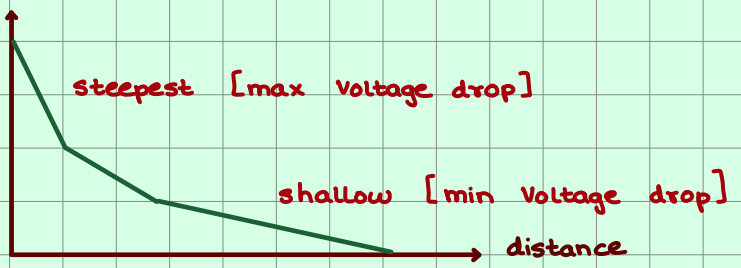
Negative terminal is zero voltage

Positive terminal is maximum voltage



The lesser the voltage drop across resistors, shallower the graph and vice versa

- +V 3R 2R R 0V



- +5V 2R 7R 3R 0V

(i) Find V_A in terms of V

$V = \frac{2R}{12R} \times 5 = \frac{5}{6}V$ → This should be done mentally

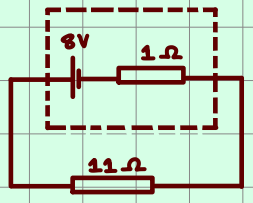
$V_A = 5V - \frac{5}{6}V = \frac{25}{6}V$

(ii) Find V_B in terms of V

$V = \frac{7R}{12R} \times 5 = \frac{35}{12}V$ → This should be done mentally

$V_A = \frac{25}{6}V - \frac{35}{12}V = \frac{5}{4}V$

Example 1:-



(i) Voltage across the external resistor

$\frac{11}{12} \times 8 = \frac{22}{3}V$

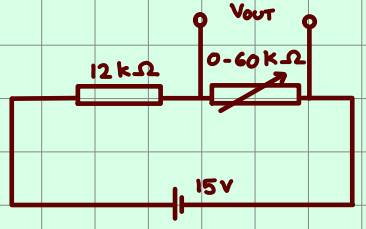
(ii) Power across external resistor

$P = \frac{V^2}{R}$

$P = \frac{\left[\frac{22}{3}\right]^2}{11}$

$P = \frac{44}{9}\ W$

Example 2:-



Find out range of output voltage

If R=0Ω, V=0V

If R= 60kΩ
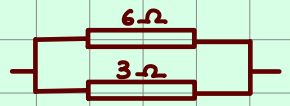
$\frac{60}{72} \times 15 = \frac{25}{2}V$

$0 \leq V \leq \frac{25}{2}$

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Properties of resistors in Parallel:

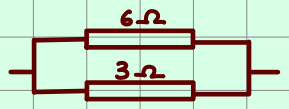
1)



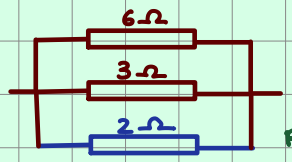
$^1/_{R_T} = {}^1/_6 + {}^1/_3$ vs $R_T = {}^{Product}/_{Sum}$

$R_T = 2\Omega$ $R_T = {}^{18}/_9 = 2\Omega$ [only works if 2 resistors are present]

2)



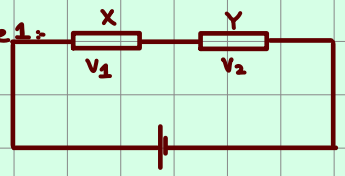
$R_T = 2\Omega$ vs



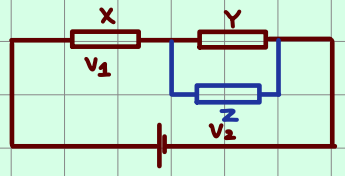
$R_T = 1\Omega$

Unlike in series, increasing the number of resistors in parallel decreases R_T

Example 1:



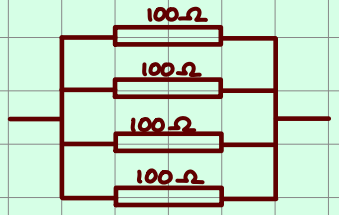
vs



What happens to V_2 when Z is attached in parallel to Y

Resistance across parallel circuit decreases and $V \propto R$ ∴ V_2 decreases and V_1 increases

3) Identical Resistors

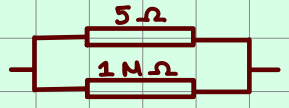


$^1/_{R_T} = {}^1/_{100} + {}^1/_{100} + {}^1/_{100} + {}^1/_{100}$ vs $R_T = {}^R/_N$ [only for identical resistors]

$R_T = 25\Omega$

where R = Resistance of 1 resistor, N = number of resistors

4) 2 resistors with difference of at least 1000 times



$^1/_{R_T} = {}^1/_5 + {}^1/_{1\times10^6}$

$R_T = 5\Omega$ ∴ R_T will be always be equal to the resistance of the smaller resistor

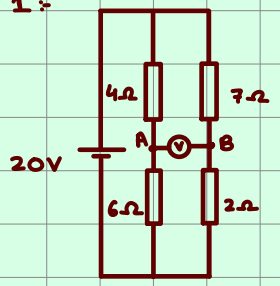
5) Voltage remains the same across parallel circuits

6) Current divides inversely compared to resistance, as $I \propto {}^1/_R$

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How to calculate Potential difference between any 2 points in a parallel circuit?

Example 1:-



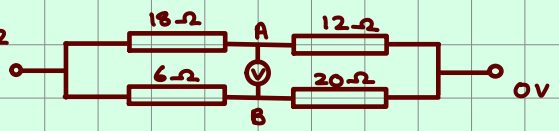
(i) Calculate V_A (ii) Calculate V_B

$V = 8V \therefore 20 - 8 = 12V$ $V = 15.6V \therefore 20 - 15.6 = 4.44V$

(iii) Calculate reading on voltmeter (iv) What is the direction of current in Voltmeter

$12 - 4.4 = 7.6V$ Higher to lower potential $\therefore$ A to B

Example 2



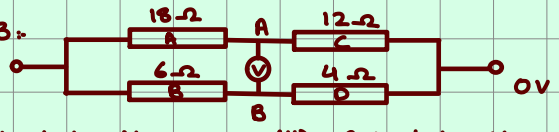
(i) Calculate V_A (ii) Calculate V_B

$V = 36V \therefore 60 - 36 = 24V$ $V = 13.8V \therefore 60 - 13.8 = 46.2V$

(iii) Calculate reading on voltmeter (iv) What is the direction of current in Voltmeter

$46.2 - 24 = 22.2V$ Higher to lower potential $\therefore$ B to A

Example 3:-



(i) Calculate V_A (ii) Calculate V_B

$V = 36V \therefore 60 - 36 = 24V$ $V = 36V \therefore 60 - 36 = 24V$

(iii) Calculate reading on voltmeter (iv) What is the direction of current in Voltmeter

$24 - 24 = 0V$ No flow of current $\therefore$ Null deflection / zero deflection / Null point

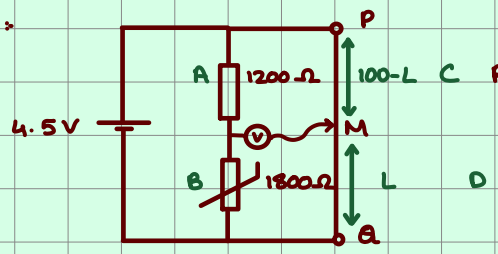
Easier way to detect null deflection:-

$\frac{A}{B} = \frac{C}{D}$ OR $\frac{A}{C} = \frac{B}{D}$

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For Null deflection :- $A/B = C/D$

Example 1 :



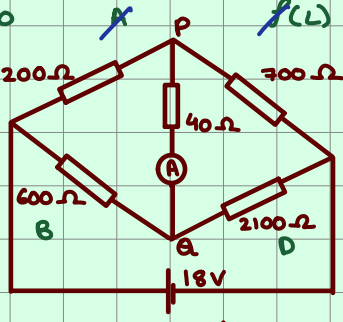
Resistance wire PQ = 100 cm

(i) Find the length MQ for the voltmeter to give a null deflection.

$\frac{A}{B} = \frac{C}{D}$

$\frac{1200}{1800} = \frac{(100-L)}{(L)} \times \frac{A}{A}$ ∴ L = 60 cm

Example 2 :



Calculate the reading on the ammeter.

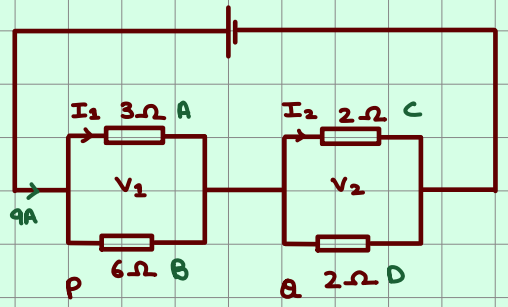
$A/B = C/D$

$200/600 = 700/2100$

$1/3 = 1/3$

$V_P = V_Q$ ∴ Potential difference = zero, ammeter gives zero reading

Example 3 :



Compare V_1 with V_2 and I_1 with I_2

- In P, ratio of A and B = $1/2$ ∴ ratio of current = $2/1$
- In Q, ratio of C and D = $1/1$ ∴ ratio of current = $1/1$
- R_T in P = 2 Ω, R_T in Q = 1 Ω

∴ $V_1 > V_2$

- Assume current flowing is 9A
- Lower resistance receives greater current

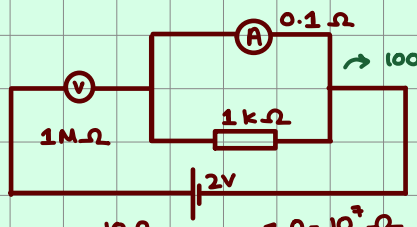
∴ In P, A recieves 6A

∴ In Q, C receives 4.5A

∴ $I_1 > I_2$

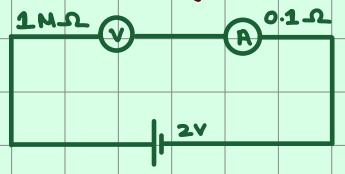
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Example 4:



Calculate the reading on the Voltmeter and Ammeter

difference ∴



On Voltmeter: $\frac{1 \times 10^6}{(10^6) + 0.1} \times 2 = 2V$

On Ammeter: $\frac{2}{(10^6) + 0.1} = 2\mu A$

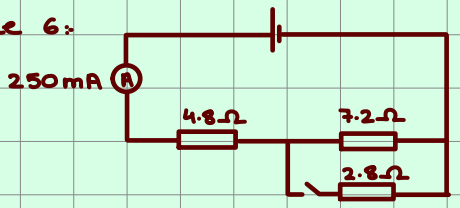
Example 5: P



Q Calculate the range of resistance between points PQ

At zero: $R_T = 10\Omega$, At 10^3: $R_T = 10^4\Omega$ $\therefore 10\Omega \leq R_T \leq 10^4\Omega$

Example 6:



When the switch is open, ammeter records 250 mA. Calculate its reading once the switch is closed.

Step 1: R_T in series = 12 Ω

$V = IR$

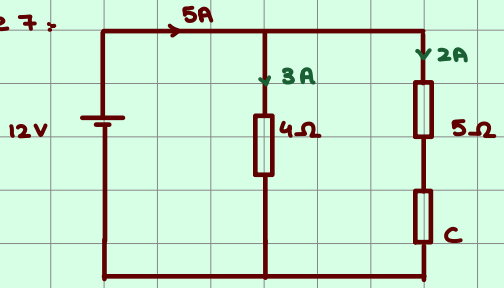
$V = (250 \times 10^{-3})(12) = 3V$

Step 2: Switch closed

$R_T = \frac{7.8 \times 2.8}{7.8 + 2.8} + 4.8 = 6.82\ \Omega$, $3 = I(6.82)$

$I = 440mA$

Example 7:



Calculate value of resistor C

Step 1: $12/4 = 3A$

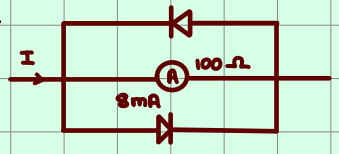
$5 - 3 = 2A$

Step 2: $V = IR$

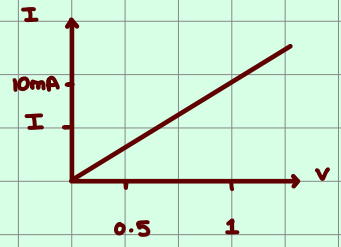
$12 = 2(5 + C)$

$C = 1\Omega$

Example 8:



Use the graph to calculate value of current I.



Step 1: $V = IR$

$V = (8 \times 10^{-3})(100)$

$V = 0.8V$

Step 2: $1 - 0.5 \rightarrow 10$

$0.5 \rightarrow 10$

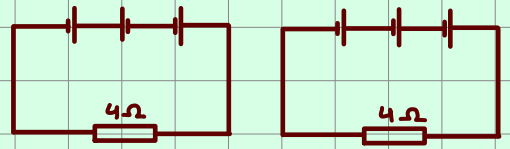
$0.3 \rightarrow x$

$x = 6mA$ $\therefore I_{DIODE} = 6mA$

Step 3: $I = 0 + 8 + 6 = 14mA$

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Example 9:



Circuit 1

Circuit 2

Each cell is of 1.5V with internal resistance of 2Ω. Calculate P_2/P_2.

Circuit 1: 1.5 - 1.5 + 1.5 = 1.5V, R_T = 10Ω

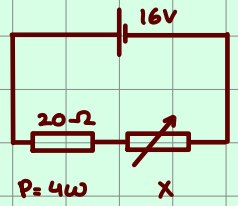
Circuit 2: 1.5 + 1.5 + 1.5 = 4.5V, R_T = 10Ω

$P = I^2R$

$P_1 = [1.5/10]^2(4)$ $P_2 = [4.5/10]^2(4)$ $\therefore P_1/P_2 = 1/9$

$P_1 = 0.09W$ $P_2 = 0.81$

Example 10: If the fixed resistor delivers a power of 4 watts. Calculate resistance of variable resistor X.

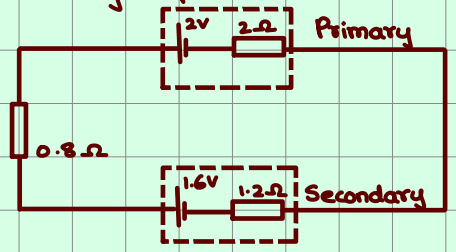


Step 1: $P = I^2R$ $4 = I^2(20)$ $I = 0.45A$

Step 2: $P = IV$ $4 = 0.45.V$ $V = 9V$

Step 3: $V_x = 16 - 9 = 7V$ $R_x = 7/0.45 = 16Ω$

Example 11: Calculate the current supplied by the primary cell to charge the secondary cell and by calculating power obtain the efficiency of the circuit.



Step 1: Net pd = 2 - 1.6 = 0.4V

R_T = 4Ω

$V = IR$

$0.4 = I(4) \therefore I = 0.1A$

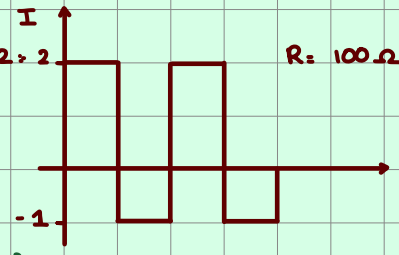
Step 2: $P_{primary} = IV = 0.1 \times 2 = 0.2W$

Step 3: $P_{lost} = I^2R = (0.1)^2(4) = 0.04W$

$0.2 - 0.04 = 0.16W$

Step 4: $0.16/0.2 \times 100 = 80\%$

Example 12:



R = 100Ω, Calculate Pavg

$P = I^2R$

$P = (2)^2(100) = 400W$

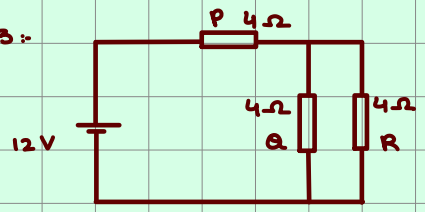
$P = I^2R$

$P = (-1)^2(100) = 100W$

$P_{avg} = \frac{400 + 100}{2} = 250W$

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Example 13:



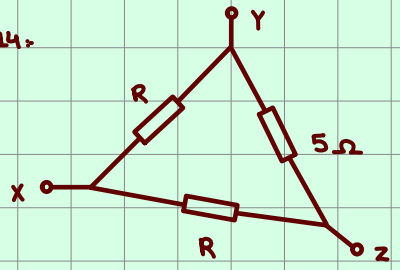
Find Power of Resistor R.

Step 1: R_T of Q & R = 2Ω

$^2/_6 \times 12 = 4V$

Step 2: $P = V^2/R = 4^2/4 = 4$ watts

Example 14:



$R_{YZ} = 2.5\Omega$

$R_{XY} = ?$

Step 1: 2R is parallel with 5Ω = 2.5Ω

$$\frac{2R \times 5}{2R + 5} = 2.5$$

$R = 2.5\Omega$

Step 2: 7.5Ω is parallel to 2.5Ω

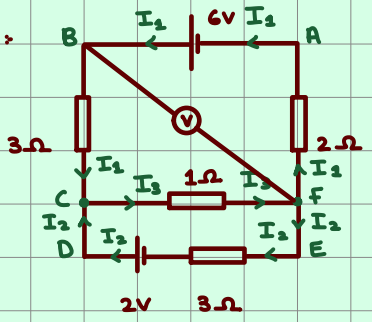
$$\frac{7.5 \times 2.5}{7.5 + 2.5} = 1.875\Omega \approx 2\Omega$$

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Kirchhoff's Law:

Purpose: Find current and voltage with multiple batteries

Example:



Kirchhoff's Current Law (KCL):

1) Label either CW or ACW
2) Mark nodes or junctions
3) Mark the direction of current
4) Form Equation → Current entering node = Current Leaving the node

$\therefore I_1 + I_2 = I_3$

Kirchhoff's Voltage Law (KVL):

1) Consider any closed loop. E.g. ABCFA

When facing a battery:

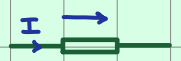


raise ∴ +V

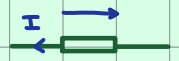


fall ∴ -V

When facing a resistor:



R ∴ - IR



R ∴ + IR

For ABCFA: $+6 - I_1(3) - I_3 - I_1(2) = 0$

For FCDEF: $I_3 - 2 + I_2(3) = 0$

Solve Simultaneously:

$I_1 + I_2 - I_3 = 0$

$-5I_1 + 0 - I_3 = -6$

$0 + 3I_2 + I_3 = 2$

$I_1 = {}^{22}/_{23}$ A, $I_2 = {}^{6}/_{23}$ A, $I_3 = {}^{28}/_{23}$ A

To calculate V_{BF}

Option 1: FEDCB

$V_{BF} = -I_2(3) + 2 + I_1(3) = {}^{94}/_{23}$V

Option 2: FAB

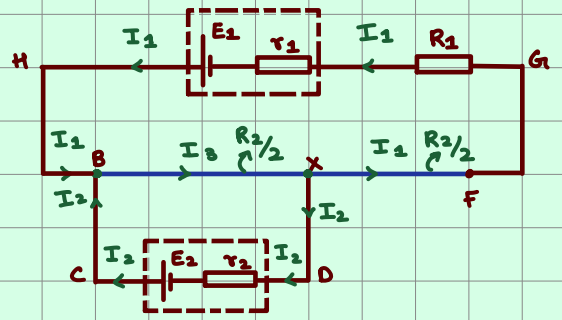
$V_{BF} = -I_1(2) + 6 = {}^{94}/_{23}$ V

Statement: According to Kirchhoff's Current Law, current arriving at a node must be equal to the current leaving the node. This law is based on the Law of Conservation of Charge which states that charges can neither be created nor destroyed.

According to Kirchhoff's Voltage Law, the sum of voltage / EMF around a closed loop must always be zero. This law is based on the Law of Conservation of Energy.

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Example 1:



$\Rightarrow$ X is the midpoint of BF

$\Rightarrow$ $R_{BF} = R_2$

(i) KCL:

$$I_1 + I_2 = I_3$$

(ii) GHBXFG

$$-I_1(R_1) - (I_1)(r_1) + E_1 - I_3[R_2/2] - I_1[R_2/2] = 0$$

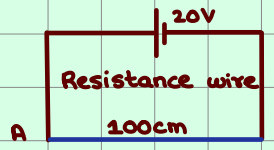
(iii) GHBCDXFG

$$-I_1(R_1) - I_1(r_1) + E_1 - E_2 + I_2(r_2) - I_1[R_2/2] = 0$$

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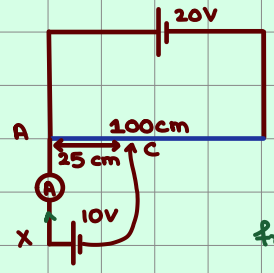
Potentiometer:

Purpose: To help us calculate the voltage [V] of an unknown power supply

1) 

Primary cell

$V_{AB} = 20V$ No extra resistor is present in the primary cell $\therefore V_{AB} = V_{PC}$

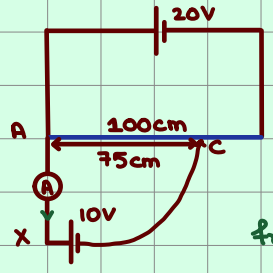
2) 

20V → 100 cm

V_{AC} → 25 cm

$\therefore V_{AC} = 5V$

If $V_{SC} > V_{AC}$, current flows from X to A.

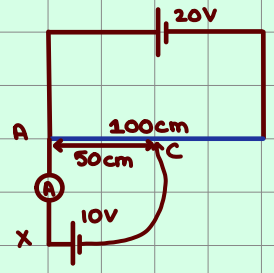
3) 

20V → 100 cm

V_{AC} → 75 cm

$\therefore V_{AC} = 15V$

If $V_{SC} < V_{AC}$, current flows from A to X.

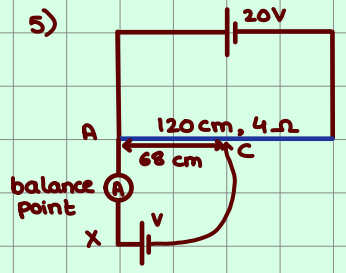
4) 

20V → 100 cm

V_{AC} → 50 cm

$\therefore V_{AC} = 10V$

If $V_{SC} = V_{AC}$, a balance point is reached i.e. ammeter records zero current / null deflection.

5) 

To find V:

1) Unitary method:

120 cm → 20V

68 cm → x

$V_{AC} = 11.3V$

$\therefore V = 11.3V$

2) Ratio Method:

$L_{AC} / L_{AB} \times V_{PC}$

$68/120 \times 20$

$V_{AC} = 11.3V$

$\therefore V = 11.3V$

3) O-Levels Method:

Step 1: $V = IR$

$20 = I(4)$

$I = 5A$

Step 2: 120 cm → 4Ω

68 cm → x

$x = 2.27\Omega$

Step 3: $V_{AC} = IR$

$V_{AC} = 5 \times 2.27$

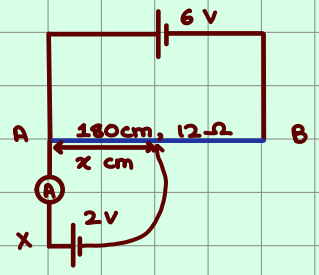
$V_{AC} = 11.3V$

Step 4: Since balance point is achieved, $V = 11.3V$

Using the ratio method, unitary method, or the O-Levels formula, $V=IR$, the unknown voltage of a secondary cell can be determined.

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6)



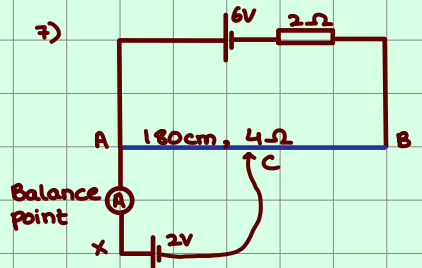
180 cm → 6V

x cm → 2V

x = L_{AC} = 60cm

These formulas can also be used in reverse to help us determine the length required to achieve a balance point.

7)



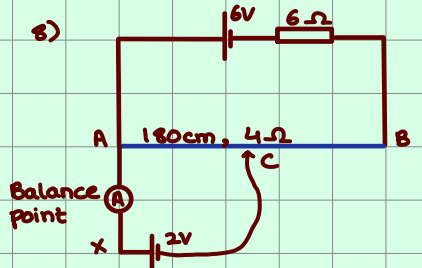
$V_{AB} = {}^{4}/_{6} \times 6 = 4V$

180 cm → 4V

x cm → 2V

x = 90 cm

8)



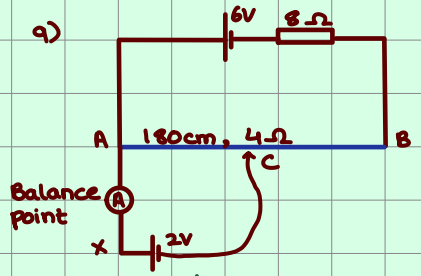
$V_{AB} = {}^{4}/_{10} \times 6 = 2.4V$

180 cm → 2.4V

x cm → 2

x = 150cm

9)



$V_{AB} = {}^{4}/_{12} \times 6 = 2V$

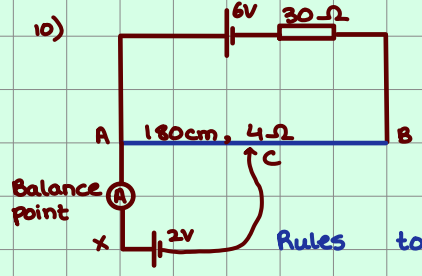
180 cm → 2V

x cm → 2V

x = 180 cm

If an extra resistor, either internal or external is positioned in the primary circuit, V_{AB} will decrease ∴ a longer length will now be needed to achieve a balance point. In fact, the larger the value of the resistor, the greater is the length required for the balance point to be achieved.

10)



${}^{4}/_{34} \times 6 = 0.71V$

180 cm → 0.71V

x cm → 2V

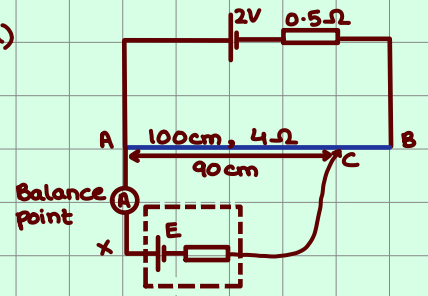
x = 507cm → Not Possible

Rules to make a correct circuit:-

- Orientation of the battery in both circuits must be the same
- Ammeter should be attached in the secondary circuit
- $V_{primary}$ should be greater than $V_{secondary}$
- Voltage across the secondary cell should not surpass the voltage across the resistance wire.

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11) a)



(i) Find the value of E

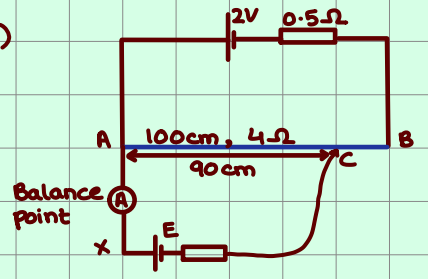
$^4/_{4.5} \times 2 = 1.78\,V = V_{AB}$

100 cm → 1.78V

90cm → V_{AC}

$V_{AC} = 1.6V$ ∴ E = 1.6 V

b)



(i) Find the value of E using O-Levels formula:-

Step 1:- V = IR

2 = I(4.5)

$I = ^4/_9\,A$

Step 2:- 100cm → 4Ω

90cm → x

$x = 3.6\,\Omega$

Step 3:- $V_{AC} = IR$

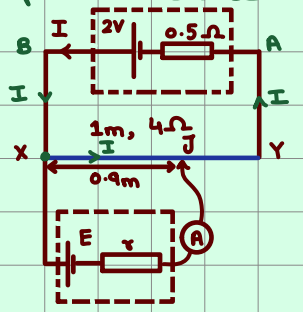
$V_{AC} = [^4/_9] \times (3.6)$

$V_{AC} = 1.6V$

Step 4:- Balance point achieved ∴ E = 1.6 V

In case, if any resistor [R or x] is placed in the secondary circuit, it will have no impact on the balance point because at the balance point, there is zero current through the secondary cell.

Example 1:-



Given that the diagram shows a condition for balance point

(i) Use KVL to calculate current I.

ABXYA:- $-I(0.5) + 2 - I(4) = 0$

$I = ^4/_9\,A$

(ii) Calculate V_{XJ}

$V_{XY} = ^4/_{4.5} \times 2 = 1.78\,V$

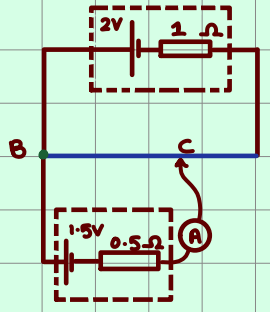
1m → 1.78V

0.9m → x ∴ $x = V_{XJ} = 1.6V$

(iii) Hence, calculate EMF (E) of the cell.

Since balance point achieved, E = 1.6 V

Example 2:-



$L_{BD} = 100cm$, $d_{BD} = 0.38mm$, $\rho_{BD} = 4.5 \times 10^{-7}\,\Omega m$

Given that the diagram represents a balance point, calculate length (L) of the resistance wire BC.

Step 1:- $R_{BD} = \dfrac{(4.5\times10^{-7})(1)}{\pi(0.38\times10^{-3})^2/4} = 4\,\Omega$

Step 2:- $V_{BD} = ^4/_5 \times 2 = 1.6V$

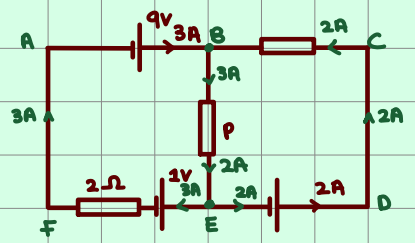
100cm → 1.6V

L_{BC} → 1.5V

∴ $L_{BC} = 93.8\,cm$.

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Example 3:



Apply KVL to calculate the resistance of P.

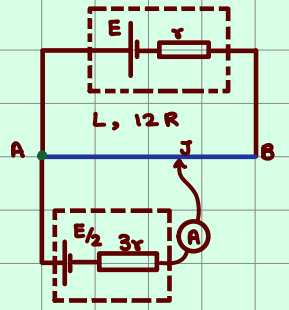
KCL: $3+2 = 5A$

KVL: ABEFA

$+9 - 5(P) - 1 - 3(2) = 0$

$P = 0.4\,\Omega$

Example 4:



For the given diagram, show that for the balance point to be achieved the length AJ of the wire AB must be equal to $^{13}/_{24}\,L$

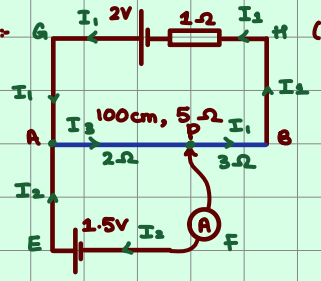
$V_{AB} = {}^{12R}/_{13R} \times E = {}^{12}/_{13}E$

$L \rightarrow {}^{12}/_{13}E$

$L_{AJ} \rightarrow E/2$

$\therefore L_{AJ} = {}^{13}/_{24}\,L$

Example 5:



(i) The diagram shows a condition for balance point. Prove that length of AP required amounts to 90 cm.

$V_{AB} = {}^{5}/_{6} \times 2 = {}^{5}/_{3}V$

$100\,cm \rightarrow {}^{5}/_{3}V$

$L_{AP} \rightarrow 1.5V \quad \therefore L_{AP} = 90\,cm$

(ii) Calculate the reading on the ammeter if the sliding contact is repositioned such that L_{AP} is 40cm.

$100\,cm \rightarrow 5\,\Omega$

$40\,cm \rightarrow x$

$x = 2\,\Omega$

KCL: $I_1 + I_2 = I_3$

$0.125 + I_2 = 0.75$

$I_2 = 0.625\,A$

KVL: APFEA

$-I_3(2) + 1.5 = 0$

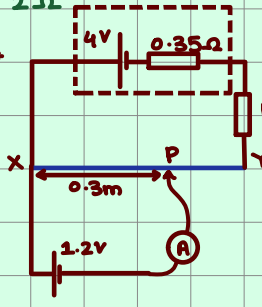
$I_3 = 0.75A$

HGAPBH

$-I_1(1) + 2 - I_3(2) - I_1(3) = 0$

$I_1 = 0.125A$

Example 6:



$R_{XY} = 0.9\,\Omega$, $V_{XY} = 1.8\,\Omega$

Use the given information to calculate the resistance of R. Hence, calculate the total length XY of the wire if the condition represents a balanced point.

$I = {}^{1.8}/_{0.9} = 2A$

$E = I(R+r)$

$4 = 2(0.9 + R + 0.3\Omega)$

$R = 0.75\,\Omega$

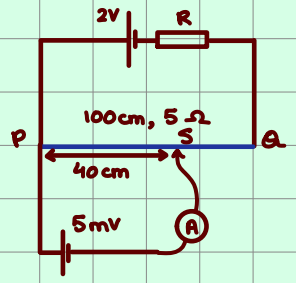
$1.2V \rightarrow 0.3\,m$

$1.8V \rightarrow x$

$L_{XY} = 0.45\,m$

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Example 7: Show that for balance point to be achieved, the value of R must be 795Ω.



40 cm → 5mV

100 cm → V_{PQ}

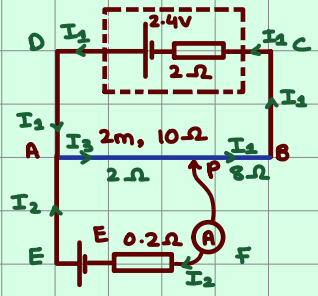
$V_{PQ} = 0.0125V$

$I = {}^{0.0125}/_{5} = 0.0025\ A$

$E = I(R+r)$

$2 = 0.0025(5+R)$ ∴ $R = 795\Omega$

Example 8: (i) For the given circuit, calculate Voltage across AB.



${}^{10}/_{12} \times 2.4 = 2V$

(ii) Hence, calculate EMF of the Secondary cell when $L_{AP} = 1.6m$

2m → 2V

1.6m → E ∴ $E = 1.6\ V$

(iii) Calculate the ammeter reading when $L_{AP} = 0.4m$

2m → 10Ω

0.4 → x

$x = 2\Omega$

$I_1 + I_2 = I_3$

APBCDA

$-I_3(2) - I_1(8) - I_1(2) + 2.4 = 0$

$-10I_1 - 2I_3 = -2.4$

AEFPA

$-1.6 + I_2(0.2) + I_3(2) = 0$

$2I_3 + 0.2I_2 = 1.6$

$I_1 = {}^{13}/_{140}\ A$, $I_2 = {}^{9}/_{14}\ A$, $I_3 = {}^{103}/_{140}\ A$

∴ Ammeter records ${}^{9}/_{14}\ A$.

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Radioactivity

Quantities and formulae required :- [— = not given, — = given in databook]

- $q_n = 0$
- $q_p = +e$ where $e = 1.6 \times 10^{-19}$ C
- $q_e = -e$ where $e = 1.6 \times 10^{-19}$ C
- $m_n \approx m_p \approx 1u$ where $u = 1.67 \times 10^{-27}$ kg
- $m_e = 9.11 \times 10^{-31}$ kg

Units of energy used in radioactivity :-

Electronvolts

$1eV = 1.6 \times 10^{-19}$ J

Nuclear Physics

Radioactivity :- The process of radioactivity is classified as a random and spontaneous process.

The term 'random' refers to fluctuations in the radiations emitted from the source both in terms of magnitude and direction.

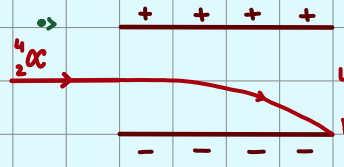
The term 'spontaneous' implies that radioactive emissions are not dependant on any changes in the external environmental conditions.

Type of Radiations :-

Alpha particles :-

- Resembles Helium nucleus
- 2 protons + 2 neutrons
- Charge = +2e
- Mass = 4u
- Approximately 10% speed of light
- Same speed ∴ Same KE

$\frac{1}{2} \times 4u \times [3.0 \times 10^{7}]^2 = 3.0 \times 10^{-12}$ J

- Symbol $^4_2\alpha$ or ^4_2He
- Range of 4-5 cm in air [shortest]
- Least penetration power ∴ can be stopped using paper or cardboard
- 

Larger mass so hard to deflect

- Greatest ionization power

Example 1 :- If energy required to ionise 1 air molecule = 2.0×10^{-21} J and α travels with E = 40eV, How many ions produced over 1m?

Step 1 :- $40 \times (1.6 \times 10^{-19}) = 6.4 \times 10^{-18}$ J

Step 2 :- 1 ion → 2.0×10^{-21} J

x → 6.4×10^{-18} J

$x = 3200$ ions

Step 3 3200 → 4.5 cm

y → 100 cm

$y = 7.1 \times 10^{4}$ ions

Radioactive decay :- $^{220}_{88}Ra$ decays by emitting an α-particle to form Polonium-216.

Represent this decay equation.

$^{220}_{88}Ra \longrightarrow {}^4_2\alpha + {}^{216}_{86}Po$

Mass decreases by 4

Atomic number decreases by 2

Extra :- ↳ proton number remains conserved

↳ Mass number remains conserved

↳ Momentum remains conserved

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Example 2:- A nucleus $^{230}_{90}P$ decays into Q by an α - emission. v_P before decay is $3.2 \times 10^5\ ms^{-1}$. Calculate v_α after decay if Q remains at REST.

$$^{230}_{90}P \longrightarrow {}^{226}_{88}Q + {}^{4}_{2}\alpha$$

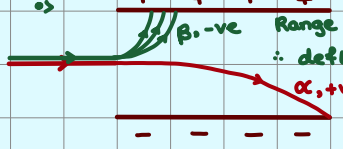
Momentum is conserved.

$$230u\,(3.2 \times 10^5) = 0 + 4u(v_\alpha)$$

$$v_\alpha = 1.84 \times 10^7\ ms^{-1}$$

Beta Particles :-

- Resembles electron
- Charge = $-1e$ $[e = 1.6 \times 10^{-19} C]$
- Mass = $mass_e$ $[m_e = 9.11 \times 10^{-31} kg]$
- Around 90% to 99% speed of light
- SLOW → can be stopped using 1 - 3mm AL sheets
- FAST → stopped using 5mm AL sheets
- Symbol $^{0}_{-1}\beta$ or $^{0}_{-1}e$ or β^-
- Range of 1 - 2 m in air
- Range of K.E.
- Deflect in Electric Fields



- Moderate ionization power because mass and charge are less
- β^- decay:-

$$^{40}_{20}X \longrightarrow {}^{0}_{-1}\beta + {}^{40}_{21}Y$$

↳ Mass number unchanged

$n = 20, p = 20$ $\qquad$ $n = 19, p = 21$ ↳ Proton number increases by 1

Neutron → proton + electron ← Why?

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + {}^{0}_{-1}e$$

Example 1:- A nucleus $^{230}_{90}P$ decays to form $^{206}_{82}R$ by emitting multiple α and β particles. Calculate number of α and β emitted.

$$^{230}_{90}P \longrightarrow {}^{206}_{82}R + x\,{}^{4}_{2}\alpha + y\,{}^{0}_{-1}\beta$$

Since mass number is conserved

$$230 = 206 + 4x$$

$$x = 6\ \alpha\text{- particles.}$$

Since proton number is conserved

$$90 = 82 + 2x - y$$

$$8 = 12 - y$$

$$y = 4\ \beta\text{- particles}$$

To check:-

$$^{230}_{90}P \xrightarrow{6\,\alpha} {}^{206}_{78} \xrightarrow{4\,\beta} {}^{206}_{82}$$ ∴ Correct!

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Comparision of α vs β - particles in an Electric Field based only on:-

(i) Mass:- Strongest influence $[m_\alpha : m_\beta = 7000 : 1]$

$m_\alpha > m_\beta$ ∴ greater inertia ∴ $\text{deflection}_\alpha < \text{deflection}_\beta$

(ii) Charge:- $[q_{v\alpha} : q_{v\beta} = 2 : 1]$

$q_{v\alpha} > q_{v\beta}$ ∴ $\text{deflection}_\alpha > \text{deflection}_\beta$

(iii) Speed:- $[v_\beta : v_\alpha = 9 : 1]$

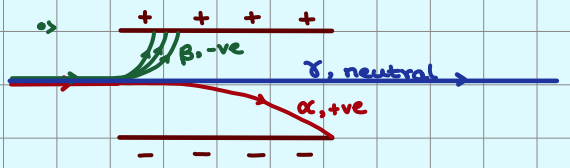
$v_\alpha < v_\beta$ ∴ α travels slower in field ∴ field applies greater force on α ∴ $\text{deflection}_\alpha > \text{deflection}_\beta$

Properties of γ - rays:-

- Highest frequency Electro magnetic radiation
- Shortest wavelength
- No mass, no charge
- 100% speed of light
- f_{HIGH} vs Very f_{HIGH}

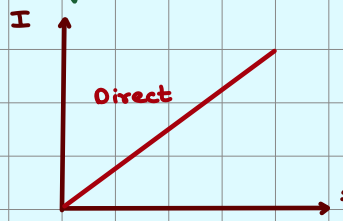
few cm thick Al sheets cannot be stopped by Al sheets. >2cm of lead or 1-2 cm lead sheets required.

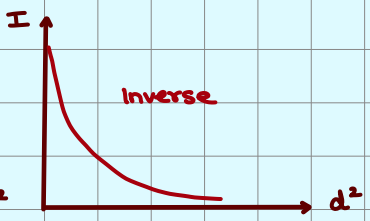
- Symbol $^{0}_{0}\gamma$ or γ
- Infinite Range
- Emitted in all nuclear reactions
- Do not Deflect in Electric Fields



- Zero ionization power
- $I \propto 1/d^2$

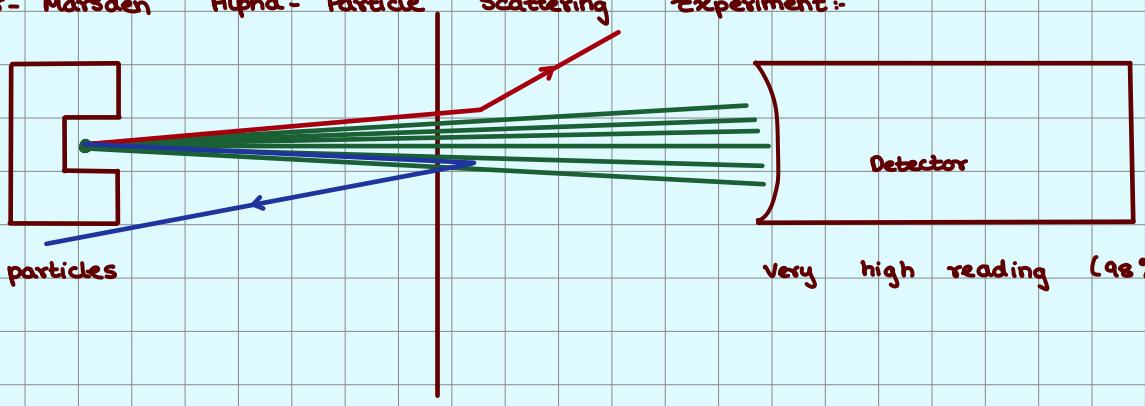
Intensity of γ - rays vary inversely with the square of distance (d)



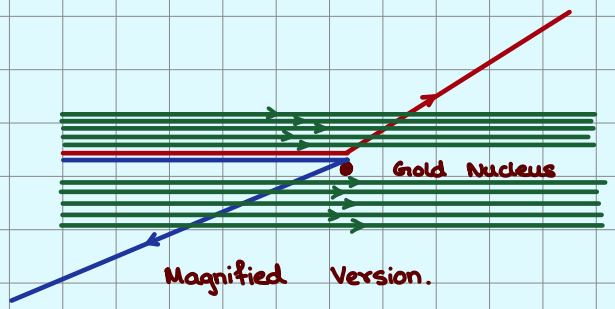


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Geiger- Marsden Alpha- Particle Scattering Experiment :-



Very high reading (98% - 99%)



Magnified Version.

Statements :-

⇒ Experiment was conducted to obtain information regarding structure of an atom.

⇒ The following observations and conclusions were made :-

Observation 1 :- Almost 99% of the α - particles travelled right through the gold foil undergoing very little deflection i.e. less than 10°

Conclusion 1 :- Size of the nucleus is very small compared to size of an atom.

Observation 2 :- Very few (less than 1%) of the α - particles deflected through large angles i.e. greater than 90°

Conclusion 2 :- Nucleus is dense (contains large amount of particles) and positively charged

Suggest how can we infer that the gold nucleus has a very high density whereas the gold atom has very low density

⇒ In case of a nucleus, there is a large mass occupying a very small volume ∴ high density

⇒ In case of an atom, almost the same mass occupies a significantly larger volume ∴ low density

Diameter of gold nucleus = 10^{-14} m
Diameter of gold atom = 10^{-10} m
] Learn!

Calculate $\frac{\rho \text{ of gold nucleus } (\rho_N)}{\rho \text{ of gold atom } (\rho_A)}$

$$\rho_N/\rho_A = \frac{M_N/V_N}{M_A/V_A} = V_A/V_N = \frac{4/3\pi\left[10^{-10}/2\right]^3}{4/3\pi\left[10^{-14}/2\right]^3} = 10^{12}/1$$

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Why was gold foil preferred in the experiment?

- Gold is chemically inactive
- It is a heavy nucleus i.e. it contains large number of protons, therefore, it is capable of applying strong repulsive forces.

Why are α-particles preferred over β-particles?

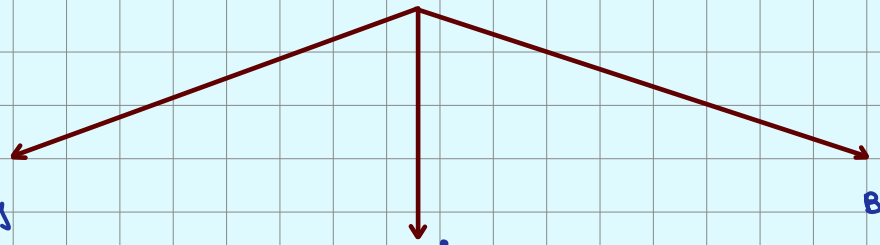
- β-particles are attracted to the nucleus
- Force on β-particles varies because they have a range of K.E.
- β-particles can be easily influenced by orbital e^-.

Suggest changes, if any, if an isotope of gold is used

No change is observed because a difference in number of neutrons does not impact the experiment

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Fundamental Particles



Hadron family:-

1) Proton [uud] } Baryons
2) Neutron [udd] }
3) Meson [$s\bar{s}$]

Hadrons are not fundamental particles i.e. they are believed to be composed of quarks and anti-quarks.

Quarks:-

- Top quark [t] = $+\frac{2}{3}e$
- Bottom quark [b] = $-\frac{1}{3}e$
- Up quark [u] = $+\frac{2}{3}e$
- Down quark [d] = $-\frac{1}{3}e$
- Charm quark [c] = $+\frac{2}{3}e$
- Strange quark [s] = $-\frac{1}{3}e$

Anti-quarks:-

- Top anti quark [$\bar{t}$] = $-\frac{2}{3}e$
- Bottom anti quark [$\bar{b}$] = $+\frac{1}{3}e$
- Up anti quark [$\bar{u}$] = $-\frac{2}{3}e$
- Down anti quark [$\bar{d}$] = $+\frac{1}{3}e$
- Charm anti quark [$\bar{c}$] = $-\frac{2}{3}e$
- Strange anti quark [$\bar{s}$] = $+\frac{1}{3}e$

where e: charge of an $\bar{e}$

Quarks are fundamental particles. They are believed to be particles that make the core of the universe

Baryons are hadrons which are known to be composed of three quarks.

Mesons are hadrons which are known to be composed of one quark and one anti-quark.

Proton [uud] = $+\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = 1e$

Neutron [udd] = $+\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

Meson [$s\bar{s}$] = $-\frac{1}{3}e + \frac{1}{3}e = 0$

Composition of an α-particle

2p + 2n

2(uud) + 2(udd)

6u, 6d

Example 1: Hadron → 3 identical quarks → +2e

$3x = +2e$

$x = +\frac{2}{3}e$

∴ 3u OR 3t OR 3c

Example 2: Hadron → 2 quarks → +1e. One of these is a down anti quark

$+\frac{1}{3}e + x = +1e$

$x = +\frac{2}{3}e$

∴ u OR t OR c

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Example 3:- Hadron → 3 quarks (non-identical) → +1e. One of these is a strange quark.

$-1/3e + 2x = 1$

$x = +2/3e$

∴ (u, t) OR (u, c) OR (t, c)

Example 4:- Hadron → 2 quarks → -1e. One of these is a down quark

$-1/3e + x = -1$

$x = -2/3e$

∴ $\bar{u}$ OR $\bar{t}$ OR $\bar{c}$

Lepton family:-

1) Electron $[{}^{0}_{-1}e, {}^{0}_{-1}\beta, \beta^{-}]$
2) Positron $[{}^{0}_{+1}e, {}^{0}_{+1}\beta, \beta^{+}]$
3) Neutrino $[{}^{0}_{0}\nu, \nu]$
4) Antineutrino $[{}^{0}_{0}\bar{\nu}, \bar{\nu}]$

Electron and positron have the same mass i.e. 9.11×10^{-31} kg but carry opposite charges.

Neutrinos and antineutrinos have zero charge, negligible mass and they are believed to travel with a range of speeds / KE.

How are these particles produced ?

a) neutron → proton + electron + antineutrino

${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$

udd → uud [Quark Model]

Since antineutrino travels with a range of speeds, therefore e^- are also known to be emitted with a range of speeds / KE [90% - 99%]

Note:- The weak interaction force or the weak nuclear force is responsible for converting a neutron into a proton or in terms of a quark model this force is responsible to convert a down quark into an up quark.

b) proton → neutron + positron + neutrino

${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + {}^{0}_{0}\nu$

uud → udd

Since neutrino travels with a range of speeds, therefore e^+ are also known to be emitted with a range of speeds / KE

Note:- The weak interaction force or the weak nuclear force is responsible for converting a proton into a neutron or in terms of a quark model this force is responsible to convert an up quark into a down quark.

Boson family:-

1) γ - rays [γ - photons] → high frequency EMR

How are γ - rays produced ?

electron + positron → 2 gamma rays

${}^{0}_{-1}e + {}^{0}_{+1}e \rightarrow 2\gamma$

${}^{0}_{-1}\beta + {}^{0}_{+1}\beta \rightarrow 2\gamma$

$\beta^{-} + \beta^{+} \rightarrow 2\gamma$

when an ${}^{0}_{-1}e$ combines with a ${}^{0}_{+1}e$, their mass gets converted into energy and this energy is released in the form of high - frequency EMR known as γ - radiations or γ - rays or γ - photons.

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