

Pure 1 summary

→ Indices and Surds

$$\textcircled{1} \sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\textcircled{2} a^m \times a^n = a^{m+n}$$

$$\textcircled{3} (a^m)^n = a^{mn}$$

$$\textcircled{4} \frac{a^m}{a^n} = a^{m-n}$$

$$\textcircled{5} (ab)^n = a^n b^n$$

$$\textcircled{6} \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\textcircled{7} \sqrt{a} = (a)^{\frac{1}{2}}$$

$$\textcircled{8} \sqrt[3]{a} = a^{\frac{1}{3}}$$

$$\textcircled{9} \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\textcircled{10} a^0 = 1$$

$$\textcircled{11} \frac{c}{\sqrt{a}+b} \times \frac{\sqrt{a}-b}{\sqrt{a}-b} \quad (\text{rationalize})$$

$$\textcircled{12} (\sqrt{a}+b)(\sqrt{a}-b) = (\sqrt{a})^2 - (b)^2$$

→ Coordinate Geometry

$$\textcircled{1} \text{Midpoint of 2 points} \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$\textcircled{2} \text{Distance between 2 points} \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$

$$\textcircled{3} \text{gradient}(m) = \frac{y_2-y_1}{x_2-x_1} \quad \begin{matrix} \text{(rise)} \\ \text{(run)} \end{matrix}$$

$$\textcircled{4} \text{Line equation:}$$

$$y = mx + c \rightarrow c \text{ is the y-intercept}$$

or $y - y_1 = m(x - x_1)$

- ⑤ - gradients of parallel lines are equal
 - gradients of perpendicular lines satisfy the rule $m_1 \times m_2 = -1$

→ Discriminant ~~is~~ wordings:

[discriminant = $b^2 - 4ac$]

- ① intersect at 2 distinct points → $(b^2 - 4ac > 0)$
- ② don't touch or intersect → $(b^2 - 4ac < 0)$
- ③ Line is a tangent to the curve → $(b^2 - 4ac = 0)$
- ④ intersect / meet at 1 point → $(b^2 - 4ac = 0)$
- ⑤ intersect at least once → $(b^2 - 4ac \geq 0)$
- ⑥ intersect at most once → $(b^2 - 4ac \leq 0)$
 → never came before

→ Quadratic graphs

general quadratic form:
 $y = ax^2 + bx + c$

① $y = a(x-b)(x-c)$

- when given 2 x-intercepts and a point

② $y = a(x-b)^2 + c$ note (parabola is symmetrical from turning point)

- when given a turning point and a point

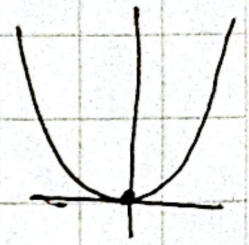
③ $y = ax^2 + bx + c$ → only used when the

- givens of the other 2 forms isn't available ^{and} so you substitute the given points and solve simultaneously

Complete the square formula:

$$\textcircled{1} \quad a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

$$\textcircled{2} \quad a\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$



→ when solving inequalities having quadratic equations:

$$\textcircled{1} \quad \text{quadratic equation} > 0$$

answers: $x > a$ or $x < b$

$$\textcircled{2} \quad \text{quadratic equation} < 0$$

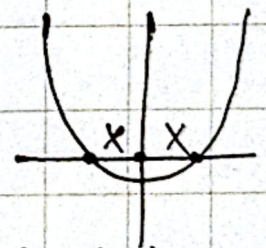
answers: $b < x < a$

$$\textcircled{3} \quad \text{quadratic equation} \geq 0$$

answers: $x \geq a$ or $x \leq b$

$$\textcircled{4} \quad \text{quadratic equation} \leq 0$$

answers: $b \leq x \leq a$



- parabola is symmetrical from T.P

- equation of line of symmetry is the x-coordinate of the T.P

$$x = \square$$

where a and b are the roots/answers/solutions of your quadratic equation and $a > b$

Cubic Graphs

[general form:
 $y = ax^3 + bx^2 + cx + d$]

① $y = a(x-b)(x-c)(x-d)$

- when 3 x-intercepts are given and a point

② $y = a(x-b)^2(x-c)$

- when 2 x-intercepts are given but one of them is a turning point (touches) which takes the squared bracket, and the other x-intercept (passes through) which takes the other bracket.

Sketching graphs tips:

To sketch any cubic / quadratic curves:

① Get the x-intercepts $\rightarrow y=0$
and solve

② Get the y-intercept $\rightarrow x=0$

③ Check your x^2 / x^3 coefficient to know the general shape

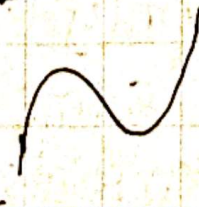
$y = ax^2$ +ve



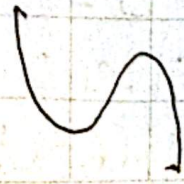
$y = -ax^2$ -ve



$y = ax^3$ +ve



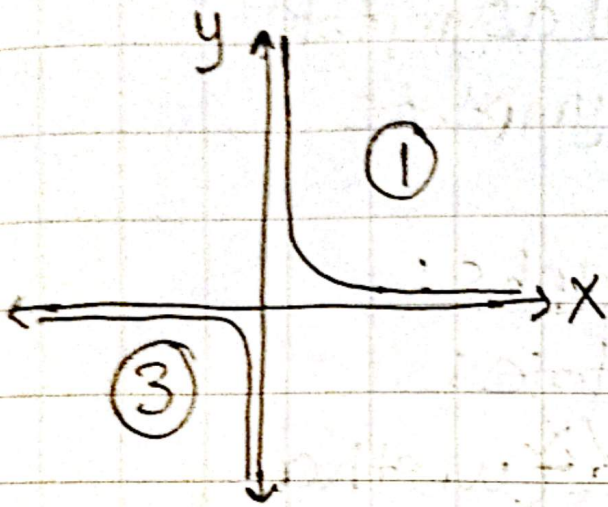
$y = -ax^3$ -ve



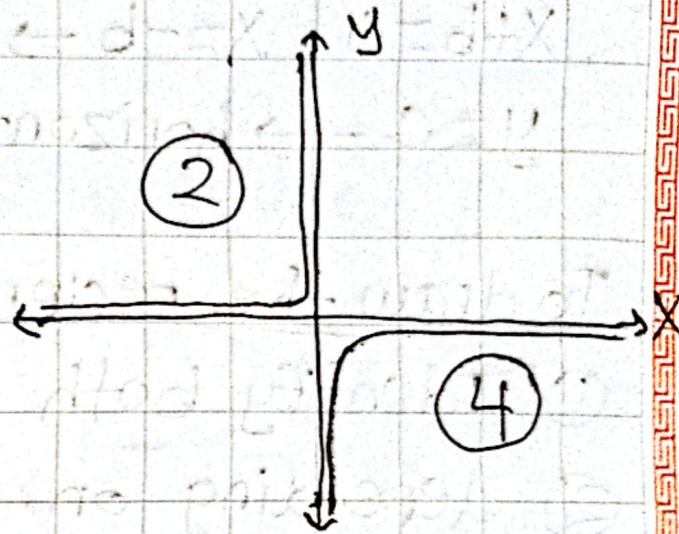
Reciprocal graphs

(a is a number)
constant

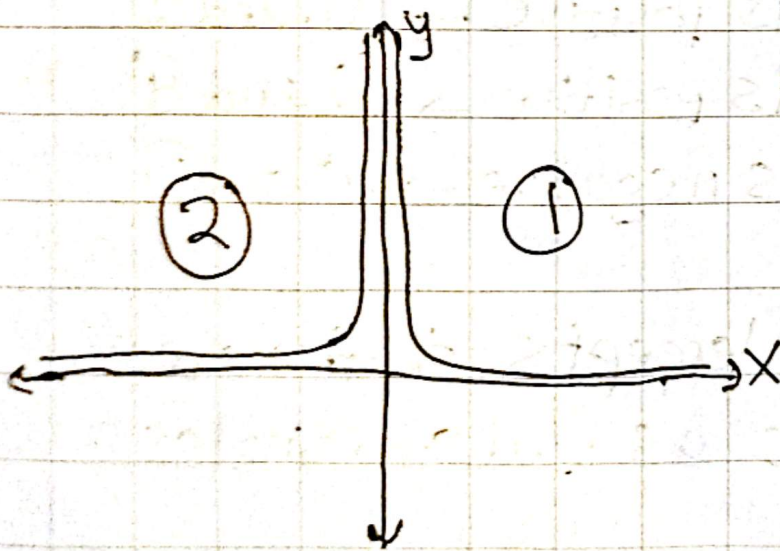
① $y = \frac{a}{x}$



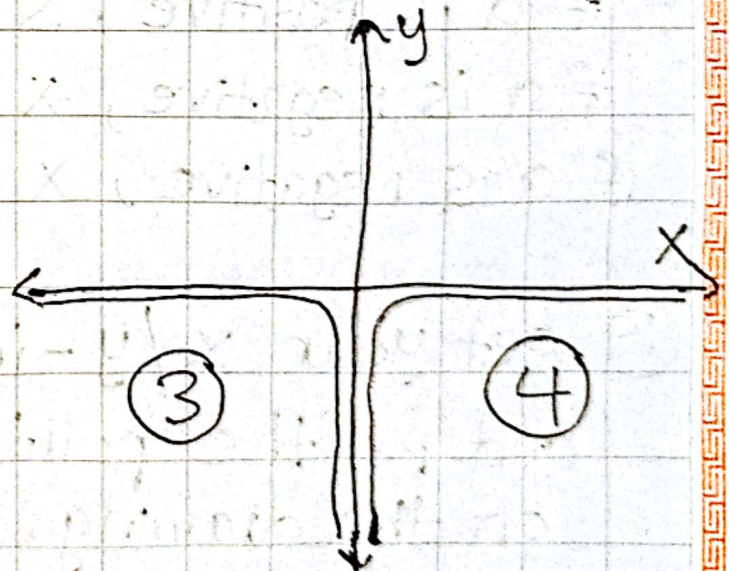
② $y = -\frac{a}{x}$



③ $y = \frac{a}{x^2}$



④ $y = -\frac{a}{x^2}$



all those have:

- horizontal asymptote $\rightarrow y=0$ (x-axis)
- Vertical asymptote $\rightarrow x=0$ (y-axis)

general reciprocal form:

$$\textcircled{1} \quad y = \frac{a}{x+b} + c$$

$x+b=0$, $x=-b \rightarrow$ vertical asymptote

$y=c \rightarrow$ horizontal asymptote

To draw the reciprocal graphs:

① Identify both asymptotes

② depending on your $\frac{a}{x}$ (whether positive or negative) you decide which quadrant the drawing is in

if a is positive, x is positive $\rightarrow$ ① and ③

if a is negative, x is positive $\rightarrow$ ② and ④

if a is negative, x is negative $\rightarrow$ ① and ③

③ get your x/y -intercepts of the curve and put the points as full coordinates on the drawing.

x -intercept $\rightarrow$ intersection with x -axis

y -intercept $\rightarrow$ intersection with y -axis

(quadratic formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

examples for transformations

- $f(x-2)$ $\rightarrow$ +2 for each x-coordinate

translation $\rightarrow$ 2 units horizontally to the right

- $f(x+2)$ $\rightarrow$ -2 for each x-coordinate

translation $\rightarrow$ 2 units horizontally to the left

- $f(x) + 2$ $\rightarrow$ +2 for each y-coordinate

translation $\rightarrow$ 2 units vertically upwards

- $f(x) - 2$ $\rightarrow$ -2 for each y-coordinate

translation $\rightarrow$ 2 units vertically downwards

$\rightarrow$ x-transformations $\rightarrow$ left or right

$\rightarrow$ y-transformations $\rightarrow$ up or down

examples (continued)

→ $f(2X) \rightarrow \frac{1}{2}X$ (each X -coordinate)

- stretch in the X -direction by scale factor of $\frac{1}{2}$ → (reciprocal of 2)

→ $f(\frac{1}{2}X) \rightarrow 2X$ (each X -coordinate)

- stretch in the X -direction by scale factor of 2

→ $2f(X) \rightarrow 2Y$ (each y -coordinate)

- stretch in the y -direction by scale factor of 2

$\frac{1}{2}f(X) \rightarrow \frac{1}{2}Y$ (each y -coordinate)

- stretch in the y -direction by scale factor of $\frac{1}{2}$

2 → can be replaced with any **number**
(1, 3, -4, 5, 7)

the concept is the most imp.

Regions:

- If your Region is above the line so
 $y \geq (\text{line equation})$ / $y > (\text{line equation})$
- If your Region is below the line so
 $y \leq (\text{line equation})$ / $y < (\text{line equation})$
- If your Region is above a curve so
 $y \geq (\text{curve equation})$ / $y > (\text{curve equation})$
- If your Region is below a curve so
 $y \leq (\text{curve equation})$ / $y < (\text{curve equation})$

If the drawing is solid $\rightarrow$ put or equal

If the drawing is dashed $\rightarrow$ don't put or equal

- Don't forget the inequalities related to the y-values / x-values in the region

General notes:

- the largest side of a triangle is always opposite to the greatest angle
- the smallest side of a triangle is opposite to the smallest angle.

Radians θ in rad, r is radius :

① area of sector = $\frac{1}{2}\theta r^2$

② Length of arc = θr

③ perimeter of arc = $r + r + \theta r$
 $= 2r + \theta r$

- Perimeter $\rightarrow$ sum of outer edges

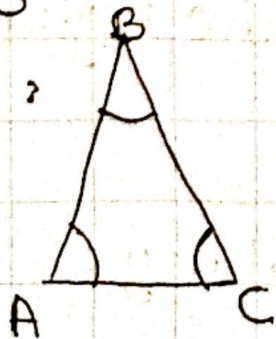
- angles on a straight line add up to $180^\circ (\pi)$

- reflex angle = $2\pi - \theta$
(360)

- obtuse angle $> \frac{\pi}{2} (1.57 \text{ rad})$

to get obtuse angle $\rightarrow \pi - (\text{acute})$
(180)

$$\left[\begin{array}{llll} \frac{\pi}{2} = 90^\circ & \pi = 180^\circ & \frac{3\pi}{2} = 270^\circ & 2\pi = 360^\circ \\ \frac{\pi}{3} = 60^\circ & \frac{\pi}{6} = 30^\circ & \frac{\pi}{4} = 45^\circ & \end{array} \right]$$



(cos rule)

$$\cos(A) = \frac{(AB)^2 + (AC)^2 - (BC)^2}{2(AB)(AC)}$$

$$(BC)^2 = (AB)^2 + (AC)^2 - 2(AB)(AC) \cos(A)$$

(sin rule) $\frac{\sin A}{BC} = \frac{\sin B}{AC} = \frac{\sin C}{AB}$ (tan $x = \frac{\sin x}{\cos x}$)

Differentiation and Integration

$$f(x) \rightarrow f'(x) \text{ (differentiation)}$$

$$f'(x) \rightarrow f(x) \text{ (integration)}$$

- they are both reverse of each other

To differentiate:

① multiply the power by the coefficient

② minus one from the power

~~Complete Square Formula~~

To integrate:

① add one to the old power

② divide by the new power

③ add +c to the answer

ex: $y = 2x^2 + 3x + 5 \rightarrow \frac{dy}{dx} = 2(2)x^{2-1} + 1(3)x^{1-1}$
 $= 4x + 3$

ex: $\int 2x^2 + 3x + 5 \, dx$

$$I = \frac{2x^3}{3} + \frac{3x^2}{2} + 5x + c$$

$$I = \frac{2}{3}x^3 + \frac{3}{2}x^2 + 5x + c$$

note: $\frac{dy}{dx}$ and $f'(x)$ are the same

note: tangent and normal are perpendicular to each other

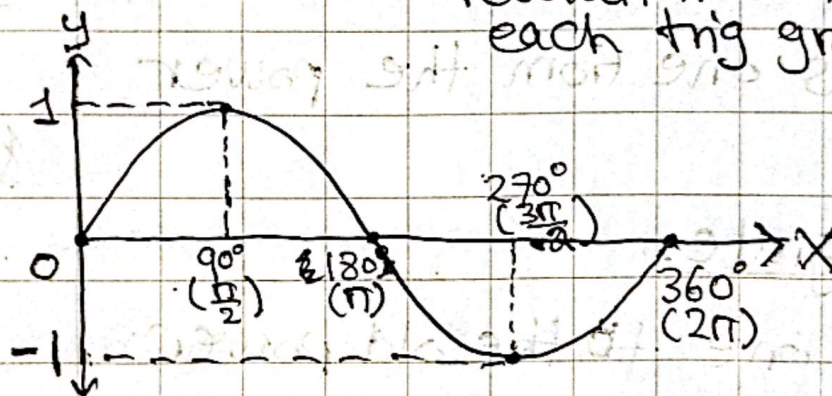
$$m(\text{normal}) = -\frac{1}{m(\text{tangent})}$$

note: $m(\text{tangent}) = \frac{dy}{dx} = f'(x)$

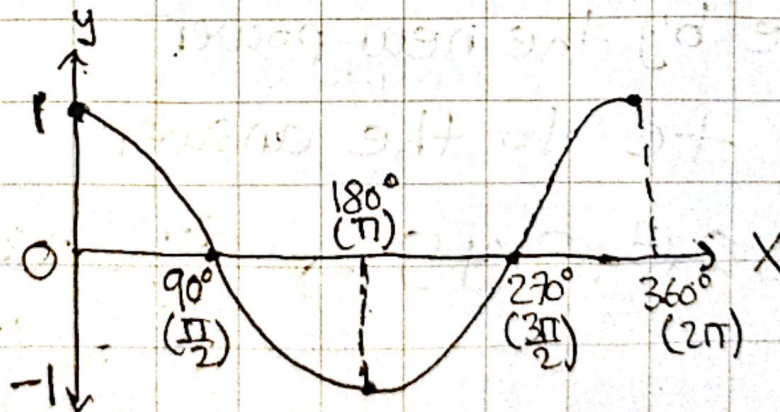
Trigonometry

① how the full revolution looks for each trig graph

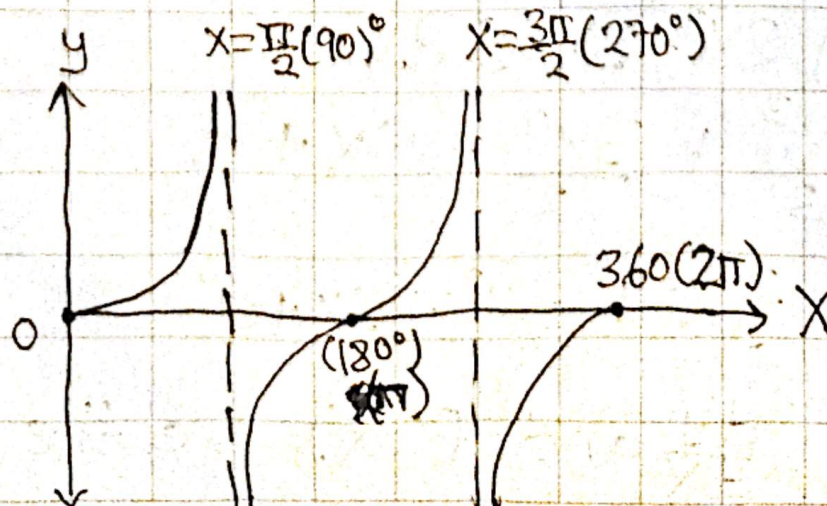
$$y = \sin x$$



$$y = \cos x$$



$$y = \tan x$$



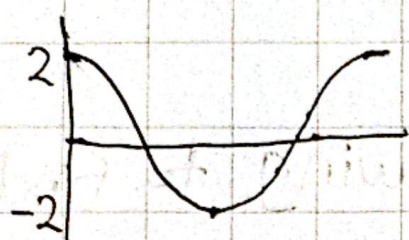
- period $\rightarrow$ is where one full revolution ends
- When solving the (find the number of solutions questions) :

- ① analyze your given drawing to find the number of intersections
- ② check the range if it got changed or not in the question asked
- ③ Check which points won't get repeated (which are mostly the intersections that occur with the y-axis) or with tan graphs there might be only one intersection above the x-axis and all the others are below the x-axis (the one above won't be repeated)
- ④ divide your new range by the given range in the question (which in most cases it's $2\pi (360^\circ)$)
- ⑤ Multiply the answer of your division in ④ by the number of intersections in the original range (and if there is no repeated solution like ③ add it on its own in the end)

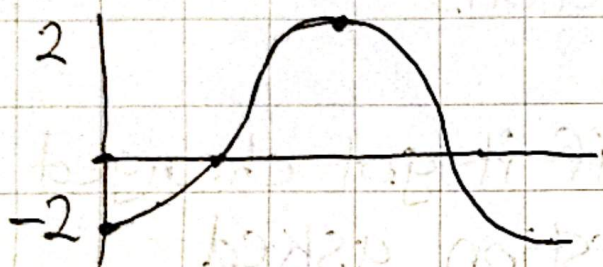
Note:

- A negative multiplied before your function reflects it

ex:



$$y = 2\cos x$$



$$y = -2\cos x$$

Note: You must memorise all the points on the original graphs because when applying transformations we apply them on the original coordinates

$$0 < a < 90\left(\frac{\pi}{2}\right) \quad a \rightarrow \text{acute angle}$$

$$- \sin(a) = \cos(90 - a)$$

$$\sin(a) = \cos\left(\frac{\pi}{2} - a\right)$$

$$- \sin(a) = \sin(180 - a)$$

$$\sin(a) = \sin(\pi - a)$$

$$- \sin(-a) = -\sin(a)$$

$$\sin(-a) = -\sin(a)$$

$$- \sin(a) = \sin(-180 - a)$$

$$\sin(a) = \sin(-\pi - a)$$

degrees

radians

Good luck for everyone !!