

KWV EDUCATION



KWV MATHS 11 & 12

PROBABILITY

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WHERE TO START IN MATHS AND SCIENCE

➤ PROBABILITY

- ✓ Probability is the study of how likely it is that an event will happen.
- ✓ The following questions are typical probability questions:

❖ What is the chance that it will Pirate win over Kaizer Chiefs?

- ✓ Probability scale to decide what chance there is of an event happening.
- ✓ It range from 0 – 1

0 - Impossible $\frac{1}{4}$ Poor chance; $\frac{1}{2}$ -Fair chance; $\frac{3}{4}$ Good chance; 4/4 Certain; 1

0% , 25% ,50% , 75% ,100%

□ The formula: **Probability** = $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{n(E)}{n(S)}$

- ✓ This ratio can be expressed as a common fraction, a decimal fraction or a percentage.

❖ DEFINITIONS

- ✓ Sum : the result of addition
- ✓ Difference : the result of subtraction
- ✓ Product : the result of multiplication
- ✓ Natural numbers : whole numbers greater than or equal to 1
- ✓ Integer : a positive or negative whole number or zero.
- ✓ Even numbers : any integer that can be divided exactly by 2
- ✓ Odd numbers : any integer that cannot be divided exactly by 2
- ✓ Factor : a number that divides exactly into a whole number with no remainder
- ✓ Divisible : a number is divisible if, after dividing, there is no remainder.
- ✓ Prime number : numbers that have only two factors, the number itself and 1.
- ✓ Multiple : the product of two natural numbers
- ✓ An EVENT : is a happening or an activity that has outcomes or results.
- ✓ An OUTCOME : is the possible result of an event.
- ✓ The SAMPLE SPACE: is the set of all possible outcomes.
- ✓ Theoretical outcome : the result, in theory, that is expected for a certain event to happen.
- ✓ Experiment : a trial situation involving chances.(it involves many trials)
- ✓ Union means **or**
- ✓ Intersection means **and**

❖ VENN DIAGRAMS

- ✓ Venn diagrams to help us to represent different events.
- ✓ **Venn diagram consists of:**
 - Circles and a rectangle.
 - The rectangle S represents the sample space (all of the possible outcomes).
 - Each circle inside S represents different events.
 - If the two circles intersect, the intersection shows which outcomes belong to both events.
 - If three event given:
- ✓ start with the number or letter include all three events
- ✓ then followed by include two events ○ Take note of non- included number or letter.

❖ MUTUALLY EXCLUSIVE EVENTS

- ✓ Mutually exclusive events are events that cannot happen at the same time.
- ✓ There is no intersection between the events.
- ✓ when two events are mutually exclusive, $P(A \text{ and } B) = 0$
- ✓ $\therefore P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events
- ✓ If $P(A \text{ and } B) = 0$ or if $P(A \text{ or } B) = P(A) + P(B)$, then the events are mutually exclusive.

❖ COMPLEMENTARY EVENTS

- ✓ Events that are mutually exclusive and make up the whole sample space are called complementary events.
- ✓ There is no intersection and no elements from the sample set are outside the two sets

❖ EXHAUSTIVE EVENTS

- ✓ Events which use up the full sample space, that is , all possible outcomes:
- ✓ Rule: $P(A) + P(B) = 1$
- ✓ The complementary rule: $P(A') + P(A) = 1$ or $P(A') = 1 - P(A)$.
- ✓ $P(A')$ means probability of „not A“.

❖ EVENTS WHICH ARE NOT MUTUALLY EXCLUSIVE

- ✓ Events that have some outcomes that are the same.
- ✓ Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

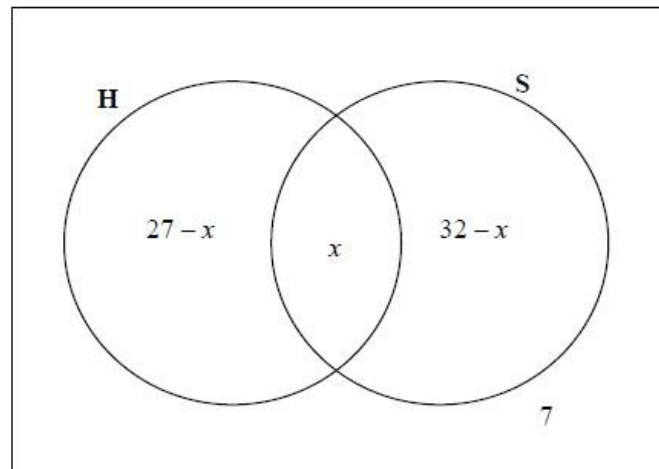
KWV QP 01

QUESTION 8

8.1 In a certain class of 42 boys:

- 27 play hockey (H)
- 32 play soccer (S)
- 7 do not play hockey or soccer
- An unknown number (x) play both hockey and soccer

The information is represented in the Venn diagram below.



8.1.1 Calculate the value of x . (2)

8.1.2 If a boy from the class is chosen at random, calculate the probability that he:

(a) Does not play hockey or soccer (1)

(b) Plays only soccer (2)

8.2 A bag contains 3 blue balls and x yellow balls.

8.2.1 Write down the total number of balls in the bag. (1)

8.2.2 If a ball is drawn from the bag, write down the probability that it is blue. (2)

8.3 8.3.1 Complete the following statement:

If A and B are two mutually exclusive events, then
 $P(A \text{ and } B) = \dots$ (1)

8.3.2 Given that A and B are mutually exclusive events. The probability that event A occurs is 0,55. The probability that event B does not occur is 0,7.

Calculate $P(A \text{ or } B)$. (3) [12]

KWV M 01

QUESTION 8/VR44G 8		
8.1.1	$27 - x + x + 32 - x + 7 = 42$ $-x = 42 - 66$ $x = 24$	✓ equation/vergelyking ✓ answer/antwoord (2)
8.1.2 (a)	P(does not play hockey or soccer/ <i>speel nie hokkie of sokker</i>) $= \frac{7}{42}$ $= \frac{1}{6}$	✓ answer/antwoord (1)
8.1.2 (b)	P(soccer only/ <i>slegs sokker</i>) $= \frac{8}{42}$ $= \frac{4}{21}$ OR/OF P(soccer only/ <i>slegs sokker</i>) $= 1 - \left(\frac{3 + 24 + 7}{42} \right)$ $= \frac{8}{42}$ $= \frac{4}{21}$	✓✓ answer/antwoord (2) ✓✓ answer/antwoord (2)
8.2.1	$x + 3$	✓ answer/antwoord (1)
8.2.2	$P(\text{blue/blou}) = \frac{3}{x + 3}$	✓✓ ans/antwoord (2)
8.3.1	$P(A \text{ and/en } B) = 0$	✓ answer/answer (1)
8.3.2	$P(B) = 1 - P(B')$ $= 1 - 0,7$ $= 0,3$ $P(A \text{ or/of } B) = P(A) + P(B)$ $= 0,55 + 0,3$ $= 0,85$	✓ $P(B) = 0.3$ ✓ subst./vervang ✓ answer/antwoord (3) [12]

KWV QP 02

QUESTION 7

- 7.1 Two events, A and B, are complementary and make up the entire sample space. Also, $P(A') = 0,35$.

7.1.1 Complete the statement: $P(A) + P(B) = \dots$ (1)

7.1.2 Write down the value of $P(A \text{ and } B)$. (1)

7.1.3 Write down the value of $P(B)$. (1)

- 7.2 A survey was conducted among 150 learners in Grade 10 at a certain school to establish how many of them owned the following devices: smartphone (S) or tablet (T).

The results were as follows:

- 8 learners did not own either a smartphone or a tablet.
- 20 learners owned both a smartphone and a tablet.
- 48 learners owned a tablet.
- x learners owned a smartphone.

7.2.1 Represent the information above in a Venn diagram. (4)

7.2.2 How many learners owned only a smartphone? (3)

7.2.3 Calculate the probability that a learner selected at random from this group:

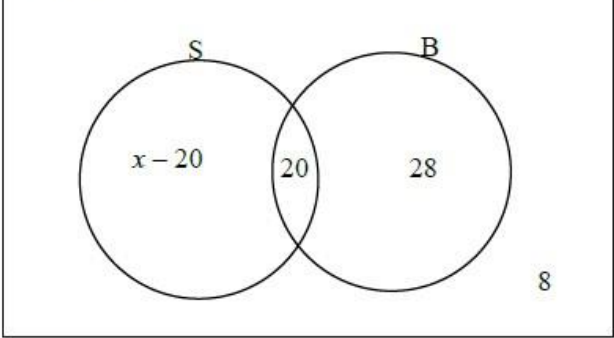
(a) Owned only a smartphone (1)

(b) Owned at most one type of device (2)

[13]

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QUESTION/VRAAG 7

7.1.1	$P(A) + P(B) = 1$	✓ answ/antw (1)
7.1.2	$P(A \text{ and } B) = 0$	✓ answ/antw (1)
7.1.3	$P(B) = P(A')$ $= 0,35$	✓ answ/antw (1)
7.2.1	$S = 150$ 	✓ 20 ✓ 28 ✓ $x - 20$ ✓ 8 (4)
7.2.2	$x - 20 + 20 + 28 + 8 = 150$ $x = 114$ Smartphone only/<i>Slegs slimfoon</i> = $114 - 20$ $= 94$	✓ equation/verg. ✓ value/waarde of/van x ✓ answ/antw (3)
7.2.3 (a)	$P(\text{S only/slegs}) = \frac{94}{150} = 0,63$	✓ answ/antw (1)
7.2.3 (b)	$P(\text{S or/of T or neither/of geeneen}) = \frac{94}{150} + \frac{28}{150} + \frac{8}{150}$ $= \frac{130}{150}$ $= \frac{13}{15}$ $= 0,87$	✓ addition/optel ✓ answ/antw (2)
		[13]

KWV QP 03

QUESTION 7

Read the advertisement below.

**Buy a Samsung J5 for only
R229 per month.**

**You have 24 months to pay.
No deposit is required.**

- 7.1 Calculate the total amount to be paid over a period of 24 months. (1)
- 7.2 The monthly instalment, quoted in the advertisement, is calculated on a hire purchase agreement which charges interest of 7,5% p.a. on the cash price of the cellphone. Show that the price of the cellphone is R4 779,13. (2)
- 7.3 Calculate the total interest paid over a period of 24 months if the cellphone is bought with this hire purchase agreement. (1)
- 7.4 The cellphone is insured at 11,5% p.a. of the cash price. The total insurance is calculated and then split up over 24 months. It is then added to the monthly instalment. Calculate the new monthly instalment if the customer wants to insure the cellphone. (3)
- 7.5 The cost of the cellphone is subject to inflation and increases to a cash price of R5 100,00 after 2 years. Calculate the annual inflation rate. (4)
- [11]**

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QUESTION/VRAAG 8		
8.1.1 (a)	$P(B) = 1 - P(B')$ $= 1 - \frac{3}{8}$ $= \frac{5}{8}$	✓ formula ✓ answer/antwoord (2)
8.1.1(b)	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $\frac{5}{7} = \frac{2}{5} + \frac{5}{8} - P(A \text{ and } B)$ $P(A \text{ and } B) = \frac{87}{280}$ $= 0,31$	✓ identity ✓ subst./verv. ✓ answer/antwoord (3)
8.1.2	Not mutually exclusive events. $P(A \text{ and } B) \neq 0$	✓ NOT/NIE ✓ reason/rede (2)
8.2.1	$P(A \cap B)$ OR $P(A \text{ and } B)$	✓ answer/antwoord (1)
8.2.2	$P(A \cup B)'$ OR $P(A \text{ or } B)'$	✓ answer/antwoord (1)
8.2.3	$P(A \cap B)'$ OR $P(A \text{ and } B)'$	✓ answer/antwoord (1)
8.3	8.2.3	✓ answer/antwoord (1)
		[11]

➤ **TREE DIAGRAMS**

❖ **INDEPENDENT EVENTS**

- ✓ Two successive events are independent if the outcomes of the one event do not influence the outcomes of the other event.
- ✓ **Rule:** $P(A \text{ and } B) = P(A) \times P(B)$ if the events are independent

❖ **DEPENDENT EVENTS**

- ✓ Events are dependent if the probability of one event happening is influenced by how another event happens.
- ✓ **Rule:** $P(A \text{ and } B) \neq P(A) \times P(B)$ If the events are dependant events

KEY!

- ✓ For mutually inclusive which is independent :

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= P(A) + P(B) - (P(A) \times P(B))$$

- ✓ If certain percentage is given for one event then for other one subtract from 100%

➤ **CONTINGENCY TABLES**

- ✓ It used to represent all possible outcomes of events.
- ✓ Numbers in each row add up to totals on the right.
- ✓ Numbers in each column add up to totals below the table.

❖ **proof dependent and independent events:**

✚ For independent event : $P(A \text{ and } B) = P(A) \times P(B)$

✚ For dependent event: $P(A \text{ and } B) \neq P(A) \times P(B)$.

✚ Use right –angle triangle rule: @ an angle of 90 degrees that is intersection

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QUESTION 9

- 9.1 Given: $P(A) = 0,2$
 $P(B) = 0,5$
 $P(A \text{ or } B) = 0,6$ where A and B are two different events

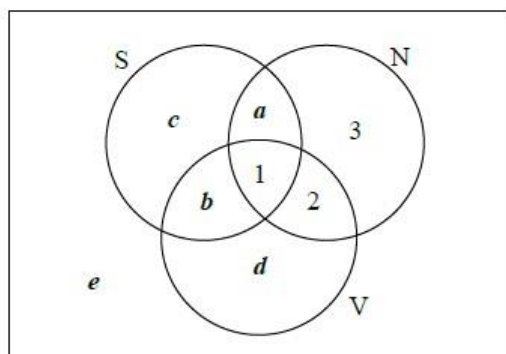
9.1.1 Calculate $P(A \text{ and } B)$. (2)

9.1.2 Are the events A and B independent? Show your calculations. (3)

- 9.2 A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball and volleyball. The results are shown below.

- 55 learners play soccer (S)
- 21 learners play netball (N)
- 7 learners play volleyball (V)
- 3 learners play netball only
- 2 learners play soccer and volleyball
- 1 learner plays all 3 sports

The Venn diagram below shows the information above.



9.2.1 Determine the values of a , b , c , d and e . (5)

9.2.2 What is the probability that one of the learners chosen at random from this group plays netball or volleyball? (2)

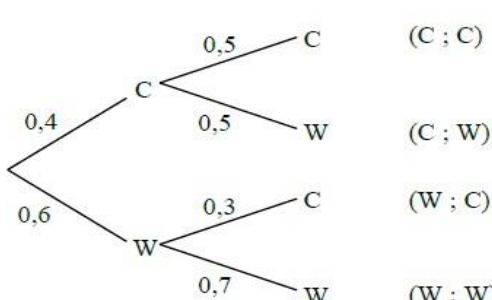
- 9.3 The probability that the first answer in a maths quiz competition will be correct is 0,4. If the first answer is correct, the probability of getting the next answer correct rises to 0,5. However, if the first answer is wrong, the probability of getting the next answer correct is only 0,3.

9.3.1 Represent the information on a tree diagram. Show the probabilities associated with each branch as well as the possible outcomes. (3)

9.3.2 Calculate the probability of getting the second answer correct. (3)
[18]

KWV M 01

QUESTION/VRAAG 9

9.1	Given/Gegee: $P(A) = 0,2$ $P(B) = 0,5$ $P(A \text{ or } B) = 0,6$	
9.1.1	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,6 = 0,2 + 0,5 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,1$	$\checkmark 0,6 = 0,2 + 0,5 - P(A \text{ and } B)$ $\checkmark P(A \text{ and } B) = 0,1$ (2)
9.1.2	$P(A \text{ and } B) = 0,1$ $P(A) \times P(B) = 0,2 \times 0,5$ $= 0,1$ $\therefore P(A \text{ and } B) = P(A) \times P(B)$ $\therefore A \text{ and } B \text{ are independent}$	$\checkmark P(A) \times P(B) = 0,1$ $\checkmark P(A \text{ and } B) = P(A) \times P(B)$ $\checkmark \text{conclusion}$ (3)
9.2.1	$a = 15$ $b = 1$ $c = 38$ $d = 3$ $e = 37$	$\checkmark a = 15$ $\checkmark b = 1$ $\checkmark c = 38$ $\checkmark d = 3$ $\checkmark e = 37$ (5)
9.2.2	$P(\text{one learner plays netball or volleyball}) = \frac{25}{100} = \frac{1}{4}$	$\checkmark 25$ $\checkmark \text{answer/antwoord}$ (2)
9.3.1		$\checkmark \text{branch at first level}$ $\checkmark \text{branches at second level}$ $\checkmark \text{probabilities and outcomes}$ (3)

9.3.2	$P(\text{second answer correct}) = P(C \text{ and } C) + P(W \text{ and } C)$ $= (0,4 \times 0,5) + (0,6 \times 0,3)$ $= 0,38$	$\checkmark \text{addition of probabilities}$ $\checkmark \text{substitution}$ $\checkmark \text{answer/antw.}$ (3) [18]
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KWV QP 02

QUESTION 8

- 8.1 A bag contains 3 blue marbles and 2 red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded and then put aside.
- 8.1.1 Draw a tree diagram to represent the information above. Show the probabilities associated with EACH branch, as well as the possible outcomes. (3)
- 8.1.2 Determine the probability of first taking a red marble and then taking a blue marble, in that order. (2)
- 8.2 A and B are two events. The probability that event A will occur is 0,4 and the probability that event B will occur is 0,3. The probability that either event A or event B will occur is 0,58.
- 8.2.1 Are events A and B mutually exclusive?
Justify your answer with appropriate calculations. (3)
- 8.2.2 Are events A and B independent?
Justify your answer with appropriate calculations. (3)
- [11]

QUESTION 9

A survey was done among 80 learners on their favourite sport.
The results are shown below.

- 52 learners like rugby (R)
 - 42 learners like volleyball (V)
 - 5 learners like chess (C) only
 - 14 learners like rugby and volleyball but not chess
 - 12 learners like rugby and chess but not volleyball
 - 15 learners like volleyball and chess but not rugby
 - x learners like all 3 types of sport
 - 3 learners did not like any sport
- 9.1 Draw a Venn diagram to represent the information above. (5)
- 9.2 Show that $x = 8$. (2)
- 9.3 How many learners like only rugby? (1)
- 9.4 Calculate the probability that a learner, chosen randomly, likes at least TWO different types of sport. (3)
- [11]

KWV M 02

QUESTION/VRAAG 8

8.1.1	(3)	✓ branches/takke ✓ probabilities/waarskynlikhede ✓ outcomes/uitkomst (3)
8.1.2	$P(R, B) = \frac{2}{5} \times \frac{3}{4}$ $= \frac{3}{10}$	✓ $\frac{2}{5} \times \frac{3}{4}$ ✓ answer/antwoord (2)
8.2.1	$P(A) = 0,4$ $P(B) = 0,3$ $P(A \text{ or } B) = 0,58$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,58 = 0,4 + 0,3 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,12 \neq 0$ Events A and B are not mutually exclusive/<i>Gebeurtenis A en B is nie onderlinguitsluitend nie</i>	✓ $0,58 = 0,4 + 0,3 - P(A \text{ and } B)$ ✓ $P(A \text{ and } B) = 0,12 \neq 0$ ✓ Not mutually exclusive/<i>nie onderling uitsluitend nie</i> (3)
8.2.2	$P(A \text{ and } B) = 0,12$ $P(A) \times P(B) = 0,4 \times 0,3$ $= 0,12$ $\therefore P(A \text{ and } B) = P(A) \times P(B)$ A and B are independent events/<i>is onafhanklik</i>	✓ $P(A) \times P(B) = 0,4 \times 0,3$ ✓ $P(A \text{ and } B) = P(A) \times P(B)$ ✓ A and B are independent/<i>is onafhanklik</i> (3)
		[11]

QUESTION/VRAG 9

9.1		✓ 14 or/of 12 or/of 15 ✓ $26 - x$ ✓ $13 - x$ ✓ 5 ✓ 3 (5)
9.2	$26 - x + 14 + x + 12 + 5 + 15 + 13 - x + 3 = 80$ $88 - 80 = x$ $x = 8$	✓ $26 - x + 14 + x + 12 + 5 + 15 + 13 - x + 3$ ✓ equating to/gelyk aan 80 (2)
9.3	Number who chose Rugby only/aantal wat net rugby kies $= 26 - 8$ $= 18$	✓ answer/antw. (1)
9.4	P(At least 2 types of sports /ten minste 2 sportsoorte) $= \frac{12 + 14 + 15 + 8}{80}$ $= \frac{49}{80}$ OR/OF P(at least 2 types of sport/ten minste 2 sportsoorte) $= 1 - \frac{18 + 5 + 5 + 3}{80}$ $= 1 - \frac{31}{80}$ $= \frac{49}{80}$	✓ numerator/Noemer ✓ denominator/Teller ✓ answer/antw. (3) ✓ $\frac{18 + 5 + 5 + 3}{80}$ ✓ method/metode ✓ answer/antw. (3) [11]

KWV QP 03

QUESTION 8

A bag contains 6 red balls, 8 green balls and an unknown number of yellow balls. The probability of randomly choosing a green ball from the bag is 25%.

8.1 Show that there are 32 balls in the bag. (1)

8.2 A ball is drawn from the bag, the colour is recorded and it is not returned to the bag. Thereafter another ball is drawn from the bag, the colour is recorded and it is also not returned to the bag.

Draw a tree diagram to represent ALL the possible ways in which the two balls could have been drawn from the bag. Show the probabilities associated with EACH branch, as well as the outcomes. (4)

8.3 Calculate the probability that the two balls drawn from the bag will have the same colour. (4)
[9]

QUESTION 9

9.1 On a flight, passengers could choose between a vegetarian snack and a chicken snack. The snacks selected by the passengers were recorded. The results are shown in the table below.

SNACK	MALE	FEMALE	TOTAL
Vegetarian	12	20	32
Chicken	55	63	118
TOTAL	67	83	150

Was the choice of snack on this flight independent of gender? Motivate your answer with the necessary calculations. (5)

9.2 For any two events, A and B, it is given that $P(A \text{ and } B) = 0,12$, $P(A \text{ or } B) = 0,83$ and $P(B) = 4 P(A)$.

9.2.1 Are events A and B mutually exclusive? Justify your answer. (2)

9.2.2 Calculate $P(B)$. (4)

9.2.3 Calculate $P(\text{not } A)$. (2)
[13]

KWV M 03

QUESTION/VRAAG 8

8.1	Given/Gegee: $P(G) = 0,25$ Let x be the total number of balls $P(G) = \frac{8}{x} = \frac{1}{4}$ $x = 32$ $n(S) = 32$ OR/OF Let x be the number of yellow balls $\therefore x+14$ be the total number of balls $P(G) = \frac{8}{x+14} = \frac{1}{4}$ $x+14=32$ $n(S)=32$	✓ $\frac{8}{x} = \frac{1}{4}$ (1) ✓ $\frac{8}{x+14} = \frac{1}{4}$ (1)
8.2		✓ 18 (number of yellow balls/ aantal geel balle) ✓ branches/takke ✓ probabilities/waarskynlikhede ✓ outcomes/uitkomst (4)
8.3	$P(G, G) + P(R, R) + P(Y, Y)$ $= \left(\frac{8}{32} \times \frac{7}{31}\right) + \left(\frac{6}{32} \times \frac{5}{31}\right) + \left(\frac{18}{32} \times \frac{17}{31}\right)$ $= \frac{49}{124}$	✓ $\left(\frac{8}{32} \times \frac{7}{31}\right)$ ✓ $\left(\frac{6}{32} \times \frac{5}{31}\right)$ ✓ $\left(\frac{18}{32} \times \frac{17}{31}\right)$ ✓ answer/ antw (4) [9]

QUESTION/VRAAG 9

9.1	$P(V) \times P(M)$ $\frac{32}{150} \times \frac{67}{150} = 0,095$ $P(V \text{ and/en } M) = \frac{12}{150} = 0,08$ $P(V \text{ and/en } M) \neq P(V) \times P(M)$ The events are not independent/ <i>Die gebeurtenisse is nie onafhanklik</i> OR/OF $P(V) \times P(F)$ $\frac{32}{150} \times \frac{83}{150} = 0,118$ $P(V \text{ and } F) = \frac{20}{150} = 0,133$ $P(V \text{ and } F) \neq P(V) \times P(F)$ The events are not independent/ <i>Die gebeurtenisse is nie onafhanklik</i>	$\checkmark \frac{32}{150}$ $\checkmark \frac{67}{150}$ $\checkmark P(V) \times P(M) = 0,095$ $\checkmark P(V \text{ and/en } M) = 0,08$ $\checkmark \text{conclusion/gevolgtr.}$ (5) $\checkmark \frac{32}{150}$ $\checkmark \frac{83}{150}$ $\checkmark P(V) \times P(F) = 0,118$ $\checkmark P(V \text{ and } F) = 0,133$ $\checkmark \text{conclusion/gevolgtr.}$ (5)
9.2.1	$P(A \text{ and/en } B) = 0,12 \neq 0$ Events are not mutually exclusive/ <i>Gebeurtenisse nie onderling uitsluitend nie</i>	$\checkmark P(A \text{ and } B) \neq 0$ $\checkmark \text{conclusion/gevolgtr.}$ (2)
9.2.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,83 = P(A) + 4P(A) - 0,12$ $0,95 = 5P(A)$ $P(A) = 0,19$ $P(B) = 4(0,19) = 0,76$	$\checkmark \text{formula/ formule}$ $\checkmark \text{substitution/verv.}$ $\checkmark P(A)$ $\checkmark P(B)$ (4)
9.2.3	$P(\text{not } A) = 1 - P(A)$ $= 1 - 0,19$ $= 0,81$	$\checkmark P(\text{not } A) = 1 - P(A)$ $\checkmark \text{answer/antw.}$ (2)

[13]

➤ COUNTING PRINCIPLES

- ✓ It is used when there are not too many outcomes or possibilities.
- ✓ A way of grouping elements of a group in a specific order.

➤ USE OF COUNTING PRINCIPLES IN PROBABILITY

- ✓ Firstly calculate the total possible outcomes
- ✓ And then the selected event

➤ ARRANGEMENTS

- ✓ The number of permutations of n different items.
- ✓ Also uses n factorial ($n!$): The exclamation mark $!$ is called the factorial symbol
- ✓ Permutations of outcomes that are not all distinct (some are the same)
- ✓ the number of permutations of m distinct objects taken n at a time
- ✓ **TREAT AS FIXED IF:**
 - To be next to each other.
 - Start and end with
- ✓ Repeated implies exponential form
- ✓ Not repeated implies factorial ($n! = n \times (n-1) \times (n-2) \times (n-3) \times \dots$)

➤ EXACTLY ONE

- ✓ The number must be fixed but changing direction in the different positions given.

➤ **AT LEAST**

- ✓ It means one or more.
- ✓ Complimentary rule should be applied: meaning we calculate without that number.

➤ **SITTING ARRANGEMENT**

- ✓ Sitting next to each other:
 - Single unit is formed but in different ways (not fixed).
 - It reduces the total number outcomes.

➤ **DIGITS**

- ✓ Are from 0 to 9, means **10** possible outcomes.

➤ **DIVISIBLE ONLY**

- ✓ check the first and last digit.

➤ **GREATER AND DIVISIBLE**

- ✓ last digit must be divisible by that number.
- ✓ 0 can be divisible by any number.

➤ **GREATER THAN OR EQUAL TO**

- ✓ From that number and above (number included)

➤ **GREATER THAN**

- ✓ The number not included (excluded).
- ✓ And that means 1 must be subtracted from the final answer.

➤ **GREATER AND EVEN**

- ✓ The number involved must be the first digit in the positions given.
- ✓ Even number must be the last digit in the positions given.
- ✓ Subtract one in the final answer if the first digit must be greater than

➤ **BETWEEN TWO NUMBERS**

- ✓ Means the codes lie between.
- ✓ The first digit must start with that number.
- ✓ And the arrange the remaining digits.

➤ **DIGITS NUMBER**

- ✓ Zero cannot start the digit number
- ✓ Repeated give us exponential form
- ✓ Not repeated give us factorial

➤ **DIGITS CODE**

- ✓ Zero can start the code
- ✓ Repeated give us exponential form
- ✓ Not repeated give us factorial

KWV QP 01

QUESTION 10

- 10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

- 10.1.1 What is the probability that a member selected at random is:
- (a) Female (2)
 - (b) Female and plays tennis (1)
- 10.1.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)

- 10.2 The probability of events A and B occurring are denoted by $P(A)$ and $P(B)$ respectively.

For any two events A and B it is given that:

- $P(B') = 0,28$
- $P(B) = 3P(A)$
- $P(A \text{ or } B) = 0,96$

Are events A and B mutually exclusive? Justify your answer. (4)
[10]

QUESTION 11

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

- 11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)
- 11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)
[6]

KWV M 01

QUESTION/VRAAG 10

10.1.1 (a)	$P(\text{Female/Vroulik}) = \frac{70}{150} = \frac{7}{15} = 0,47$	✓ 70 ✓ answer (2)
10.1.1 (b)	$P(\text{Female playing tennis/Vroulik speel tennis}) = \frac{20}{150} = \frac{2}{15} = 0,13$	✓ answer (1)

10.1.2	$P(\text{Female/Vroulik}) = \frac{70}{150}$ $P(\text{Playing/Speel tennis}) = \frac{70}{150}$ $P(\text{Female playing tennis/Vrouliks speel tennis}) = \frac{20}{150} = 0,13$ $P(\text{Female/Vroulik}) \times P(\text{Playing/Speel tennis}) = \left(\frac{70}{150}\right)\left(\frac{70}{150}\right) = \frac{4900}{22500} = 0,22$ $P(\text{Female playing tennis/Vroulik speel tennis}) \neq P(\text{Female/Vroulik}) \times P(\text{Playing/Speel tennis})$ Therefore the event of playing tennis is not independent of gender./ Dus is die gebeurtenis om tennis te speel nie onafhanklik van geslag nie OR/OF $P(\text{Male/Manlik}) = \frac{80}{150}$ $P(\text{Playing/Speel tennis}) = \frac{70}{150}$ $P(\text{Male playing tennis/Manlik speel tennis}) = \frac{50}{150} = 0,33333$ $P(\text{Male/Manlik}) \times P(\text{Playing/Speel tennis}) = \left(\frac{80}{150}\right)\left(\frac{70}{150}\right) = \frac{5600}{22500} = 0,25$ $P(\text{Male playing tennis/Manlik speel tennis}) \neq P(\text{Male/Manlik}) \times P(\text{Playing/Speel tennis})$ Therefore the event of playing tennis is not independent of gender./ Dus is die gebeurtenis om tennis te speel nie onafhanklik van geslag nie. OR/OF	$\checkmark P(\text{Play ten}) = \frac{70}{150}$ $\checkmark$ $\left(\frac{70}{150}\right)\left(\frac{70}{150}\right) = \frac{4900}{22500} = 0,22$ $\checkmark P(\text{F play t}) \neq P(\text{F}) \times P(\text{Play t})$ Not independent (3) $\checkmark P(\text{Play ten}) = \frac{70}{150}$ $\checkmark$ $\left(\frac{80}{150}\right)\left(\frac{70}{150}\right) = \frac{5600}{22500} = 0,25$ $\checkmark P(\text{M play t}) \neq P(\text{M}) \times P(\text{Play t})$ Not independent (3)
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	$P(\text{Male}) = \frac{80}{150}$ $P(\text{Not playing tennis}) = \frac{80}{150}$ $P(\text{Male not playing tennis}) = \frac{80}{150} = 0,53333$ $P(\text{Male}) \times P(\text{Not playing tennis}) = \left(\frac{80}{150}\right)\left(\frac{80}{150}\right) = \frac{6400}{22500} = 0,28$ $P(\text{Male not playing tennis}) \neq P(\text{Male}) \times P(\text{Not playing tennis})$ Therefore the event of playing tennis is not independent of gender. OR/OF $P(\text{Female}) = \frac{70}{150}$ $P(\text{Not playing tennis}) = \frac{80}{150}$ $P(\text{Female not playing tennis}) = \frac{50}{150} = 0,3333$ $P(\text{Female}) \times P(\text{Not playing tennis}) = \left(\frac{70}{150}\right)\left(\frac{80}{150}\right) = \frac{5600}{22500} = 0,25$ $P(\text{Female not playing tennis}) \neq P(\text{Female}) \times P(\text{Not playing tennis})$ Therefore the events of playing tennis and gender are not independent.	$\checkmark P(\text{not play ten}) = \frac{80}{150}$ $\checkmark$ $\left(\frac{80}{150}\right)\left(\frac{80}{150}\right) = \frac{6400}{22500} = 0,28$ $\checkmark P(M \text{ not play } t) \neq P(M) \times P(\text{Not play } t)$ Not independent (3) $\checkmark P(\text{not play ten}) = \frac{80}{150}$ $\checkmark$ $\left(\frac{70}{150}\right)\left(\frac{80}{150}\right) = \frac{5600}{22500} = 0,25$ $\checkmark P(F \text{ not play } t) \neq P(F) \times P(\text{Not play } t)$ Not independent (3)
10.2	$P(B) = 1 - P(B')$ $= 1 - 0,28$ $= 0,72$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,96 = 0,24 + 0,72 - P(A \text{ and } B)$ $0,96 = 0,96 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0$ Events A and B are mutually exclusive <i>Gebeurtenis A en B is onderling uitsluitend</i>	$\checkmark P(B) = 0,72$ $\checkmark P(A) = 0,24$ $\checkmark \text{substitution into correct formula}$ $\checkmark P(A \text{ and } B) = 0$ (4)

[10]

QUESTION/VRAAG 11

11.1	$2 \times 2! \times 7! = 20\,160$	✓ $2 \times 2!$ ✓ $7!$ ✓ $20\,160$ (3)
11.2	All seated in $9! = 362\,880$ ways Girls seated together in $4!$ ways. With the girls as one unit they can all be seated in $4! \cdot 6! = 17\,280$ <i>Almal sit op $9! = 362\,880$ maniere</i> <i>Meisies sit saam op $4!$ maniere.</i> <i>Met die meisies as 'n eenheid kan almal op $4! \cdot 6!$ maniere = $17\,280$ sit</i> $ \begin{aligned} P(\text{all girls seated together/al die meisies sit saam}) &= \frac{4! \cdot 6!}{9!} \\ &= \frac{17\,280}{362\,880} \\ &= \frac{1}{21} \\ &= 0,047619\dots \\ &= 4,76\% \end{aligned} $	✓ $9!$ or $362\,880$ ✓ $4! \cdot 6!$ or $17\,280$ ✓ $\frac{17\,280}{362\,880}$ or $\frac{1}{21}$ or $0,047619$ or $4,76\%$ (3) [6]

KWV QP 02

QUESTION 11

- 11.1 The letters of the word EQUATION are randomly used to form a new word consisting of five letters. How many of these words are possible if letters may not be repeated? (2)
- 11.2 It is given that two events, A and B, are independent. $P(A) = \frac{2}{5}$ and $P(B) = 0,35$. Calculate $P(A \text{ or } B)$. (4)
- 11.3 Grade 12 learners in a certain town may choose to attend any one of three high schools. The table below shows the number of Grade 12 learners (as a percentage) attending the different schools in 2016 and the matric pass rate in that school (as a percentage) in 2016.

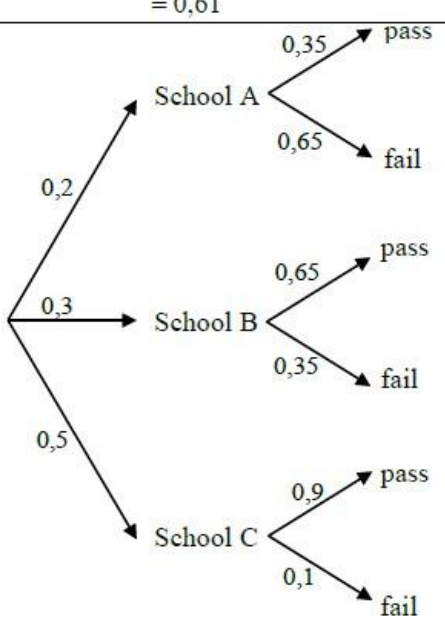
SCHOOLS	NUMBER OF LEARNERS ATTENDING (%)	MATRIC PASS RATE (%)
A	20	35
B	30	65
C	50	90

If a learner from this town, who was in Grade 12 in 2016, is selected at random, determine the probability that the learner:

- 11.3.1 Did not attend School A (2)
- 11.3.2 Attended School B and failed Grade 12 in 2016 (3)
- 11.3.3 Passed Grade 12 in 2016 (4)
- [15]

KWV M 02

QUESTION/VRAG 11

11.1	$8 \times 7 \times 6 \times 5 \times 4$ or $\frac{8!}{3!}$ $= 6720$	$\checkmark 8 \times 7 \times 6 \times 5 \times 4 / \frac{8!}{3!}$ $\checkmark 6720$ (2)
11.2	$P(A \text{ and } B) = P(A) \times P(B)$ $= 0,4 \times 0,35$ $= 0,14$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= 0,4 + 0,35 - 0,14$ $= 0,61$	$\checkmark 0,4 \times 0,35$ $\checkmark 0,14$ $\checkmark$ substitution $\checkmark$ answer (4)
		
11.3.1	$100\% - 20\%$ or/of $1 - 0,2$ $= 80\%$ $= 0,8$ OR/OF $30\% + 50\% = 80\%$ or/of $0,3 + 0,5 = 0,8$	$\checkmark 100\% - 20\%$ or $1 - 0,2$ $\checkmark 80\%$ or $0,8$ $\checkmark 30\% + 50\%$ or $0,3 + 0,5$ $\checkmark 80\%$ or $0,8$ (2)
11.3.2	$0,3 \times 0,35 = 0,105$ $= 10,5\%$	$\checkmark 0,3$ $\checkmark 0,35$ $\checkmark 0,105 = 10,5\%$ (3)
11.3.3	$(0,2 \times 0,35) + (0,3 \times 0,65) + (0,5 \times 0,9)$ $= 0,715$ $= 71,5\%$	$\checkmark 0,2 \times 0,35$ $\checkmark 0,3 \times 0,65$ $\checkmark 0,5 \times 0,9$ $\checkmark$ answer (4)

[15]

KWV QP 03

QUESTION 10

Ben, Nhlanhla, Owen, Derick and 6 other athletes take part in a 100 m race. Each athlete will be allocated a lane in which to run. The athletic track has 10 lanes.

- 10.1 In how many different ways can all the athletes be allocated a lane? (2)
- 10.2 Four athletes taking part in the event insist on being placed in lanes next to each other. In how many different ways can the lanes be allocated to the athletes now? (3)
- 10.3 If lanes are randomly allocated to athletes, determine the probability that Ben will be placed in lane 1, Nhlanhla in lane 3, Owen in lane 5 and Derick in lane 7. (2)
- [7]

QUESTION 11

A survey on their preference of exercise was conducted among 140 people in two age groups. The information is summarised below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
Older than 35 years	b	21	d	60
	21	49	70	140

- 11.1 If it is given that preferring to play tennis and age are independent of each other, determine the value of a . (3)
- 11.2 If it is given that $a = 12$, determine the probability that a randomly selected person prefers going to the gym or is in the age group 35 years and younger. (5)
- [8]

KWV M 03

QUESTION/VRAG 10

10.1	$10!$ $= 3\,628\,800$	✓ 10! ✓ answer (2)
10.2	$4! \times 7!$ $= 120\,960$ OR/OF $4! \times 6! \times 7$ $= 120\,960$	✓ 4! ✓ 7! ✓ $4! \times 7!$ or 120 960 (3) OR/OF ✓ 4! ✓ $6! \times 7$ ✓ $4! \times 6! \times 7$ or 120 960 (3)
10.3	$\frac{6!}{10!}$ $= \frac{1}{5040}$ or 0,000198	✓ 6! ✓ $\frac{6!}{10!}$ or $\frac{1}{5040}$ or 0,000198 (2) [7]

QUESTION/VRAG 11

11.1	$P(\text{tennis}) \times P(\leq 35 \text{ years}) = P(\text{tennis and } \leq 35 \text{ years})$ $\frac{21}{140} \times \frac{80}{140} = \frac{a}{140}$ $a = 12$	✓ statement ✓ substitution ✓ answer (3)
11.2	$P(\text{gym or } \leq 35 \text{ years})$ $= P(\text{gym}) + P(\leq 35 \text{ years}) - P(\text{gym and } \leq 35 \text{ years})$ $= \frac{70}{140} + \frac{80}{140} - \frac{40}{140}$ $= \frac{110}{140}$ $= \frac{11}{14}$ or 0,79	✓ statement ✓ $\frac{70}{140}$ ✓ $\frac{80}{140}$ ✓ $\frac{40}{140}$ ✓ $\frac{110}{140}$ or $\frac{11}{14}$ or 0,79 (5) [8]

KWV QP 04

QUESTION 10

- 10.1 A bag contains 7 yellow balls, 3 red balls and 2 blue balls. A ball is chosen at random from the bag and not replaced. A second ball is then chosen. Determine the probability that of the two balls chosen, one is red and the other is blue. (4)
- 10.2 Learners at a hostel may choose a meal and a drink for lunch. Their selections on a certain day were recorded and shown in the partially completed table below.

		MEAL		TOTAL
		SANDWICH (S)	PASTA (P)	
DRINK	Fruit Juice (F)	a	30	b
	Bottled Water (W)			
TOTAL		200		250

The probability of a learner choosing fruit juice and a sandwich on that day was 0,48.

- 10.2.1 Calculate the number of learners who chose fruit juice and a sandwich for lunch on that day. (1)
- 10.2.2 Is the choice of fruit juice independent of the choice of a sandwich for lunch on that day? Show ALL calculations to motivate your answer. (4)
- [9]**

QUESTION 11

Two learners from each grade at a high school (Grades 8, 9, 10, 11 and 12) are elected to form a sports committee.

- 11.1 In how many different ways can the chairperson and the deputy chairperson of the sports committee be elected if there is no restriction on who may be elected? (2)
- 11.2 A photographer wants to take a photograph of the sports committee. In how many different ways can the members be arranged in a straight line if:
- 11.2.1 Any member may stand in any position? (1)
- 11.2.2 Members from the same grade must stand next to each other and the Grade 12 members must be in the centre? (3)
- [6]**

KWV M 04

QUESTION/VRAG 10

10.1	$P(\text{One Red and One Blue})$ $= P(\text{Red, Blue}) + P(\text{Blue, Red})$ $= \left(\frac{3}{12}\right) \times \left(\frac{2}{11}\right) + \left(\frac{2}{12}\right) \times \left(\frac{3}{11}\right)$ $= \frac{1}{11}$	$\checkmark \left(\frac{3}{12}\right) \times \left(\frac{2}{11}\right)$ $\checkmark \left(\frac{2}{12}\right) \times \left(\frac{3}{11}\right)$ $\checkmark$ addition of products $\checkmark$ answer (4)
10.2.1	$a = 0,48 \times 250$ $a = 120$	$\checkmark$ answer (1)
10.2.2	$b = 150$ $P(S) \times P(F)$ $= \frac{200}{250} \times \frac{150}{250}$ $= 0,48$ $= P(S \text{ and } F)$ These events are independent / <i>Hierdie gebeurtenisse is onafhanklik</i>	$\checkmark b$ $\checkmark P(S) \times P(F)$ $\checkmark \frac{200}{250}$ and $\frac{150}{250}$ $\checkmark$ conclusion (with realistic probabilities) (4)
		[9]

QUESTION/VRAG 11

11.1	10×9 $= 90$	$\checkmark \checkmark 10 \times 9$ (2)
11.2.1	$10!$ $= 3\,628\,800$	$\checkmark 10!$ (1)
11.2.2	$2! \times 2! \times 2! \times 2! \times 2! \times 4!$ $= 768$	$\checkmark 2! \times 2! \times 2! \times 2! \times 2!$ $\checkmark 4!$ $\checkmark 2! \times 2! \times 2! \times 2! \times 2! \times 4!$ or 768 (3)
		[6]

