

DISTRIBUTIONS

DISCRETE

Fixed values of outcomes

1) BINOMIAL DISTRIBUTION

$X \sim B(n, p)$ a) Repeated
b) Success/failure
(p) (q = 1 - p)

(c) n, p, q are constants.

2) GEOMETRIC DISTRIBUTION

a) Repeated
b) Repeat until you
get success for the
first time.
c) Success (p), failure (q = 1 - p)

3) POISSON DISTRIBUTION

CONTINUOUS

Data can take any value

1) NORMAL DISTRIBUTION

$X \sim N(\mu, \sigma^2)$ a) Continuous
b) Symmetrical data on
both sides of mean.

POISSON DISTRIBUTION

CONDITIONS:

- 1) Events occur Singularly
- 2) Random
- 3) Independent
- 4) at a constant rate.

$$X \sim P_o(\lambda)$$

λ = Parameter of Poisson

λ = Mean

λ = Average rate

- Example:**
- a) People arrive at airport counter at an average rate of 3 persons/min
 - (b) Internet gets disconnected on average every 48 hours.
 - (c) Computer breakdowns occur at average every 8 days at an office.

FORMULA

$$P(X=n) = \frac{e^{-\lambda} \lambda^n}{n!}$$

$$X = 0, 1, 2, 3, \dots, \infty$$

IN POISSON DISTRIBUTION

1) $MEAN = E(X) = \lambda$

2) $VAR(X) = \lambda \longrightarrow SD(X) = \sqrt{\lambda}$

3) $MODE(X)$

(a) λ IS INTEGER There are two modes.

$$MODE 1 = \lambda$$

$$MODE 2 = \lambda - 1$$

eg: $\lambda = 5$ Mode 1 = 5
 Mode 2 = 4

(b) λ IS NOT INTEGER There is one mode.

MODE IS INTEGER BELOW λ

eg $\lambda = 8.9$ mode = 8
 $\lambda = 3.1$ mode = 3

LETS GET USED TO FORMULA

$$P(X=n) = \frac{e^{-\lambda} \lambda^n}{n!}$$

1) $P(X=0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-\lambda}$

2) $P(X=1) = \frac{e^{-\lambda} \lambda^1}{1!} = e^{-\lambda} \lambda$

3) $P(X \leq 3) = P(0, 1, 2, 3)$
$$\frac{e^{-\lambda} \lambda^0}{0!} + \frac{e^{-\lambda} \lambda^1}{1!} + \frac{e^{-\lambda} \lambda^2}{2!} + \frac{e^{-\lambda} \lambda^3}{3!}$$

$$= e^{-\lambda} \left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} \right]$$

$$\begin{aligned} 4) P(X > 3) &= P(4, 5, 6, \dots, \infty) \\ &= 1 - P(0, 1, 2, 3) \\ &= 1 - e^{-\lambda} \left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} \right] \end{aligned}$$

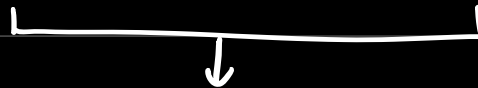
SOMETIMES TWO DIFFERENT VARIABLES
WITH POISSON DISTRIBUTION ARE ADDED

Internet Breakdown

$$X \sim P_0(\lambda_1)$$

Computer breakdown

$$Y \sim P_0(\lambda_2)$$



$$\begin{array}{l} \text{Total} = X + Y \sim P_0(\lambda_1 + \lambda_2) \\ \text{(Sum)} \end{array}$$

SUPER IMPORTANT: BE CAREFUL WITH UNITS OF λ .

Computer breakdowns occur randomly on average once every 48 hours of use.

- (i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use. [3]

$\lambda =$ once every 48 hours $\rightarrow$ This can only tell probabilities of events in 48 hours.

To find probability for 60 hours we have to modify λ by using ratio method.

mean (λ)	Hours
1	48
λ	60

$$48\lambda = 60 \times 1$$

$$\lambda = 1.25 \text{ (for 60 hrs)}$$

$$\begin{aligned} P(X < 4) &= P(0, 1, 2, 3) \\ &= e^{-1.25} \left[\frac{1.25^0}{0!} + \frac{1.25^1}{1!} + \frac{1.25^2}{2!} + \frac{1.25^3}{3!} \right] \\ &= 0.962 \end{aligned}$$

Computer breakdowns occur randomly on average once every 48 hours of use.

(i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use. [3]

(iii) Independently of the computer breaking down, the computer operator receives phone calls randomly on average twice in every 24-hour period. Find the probability that the total number of phone calls and computer breakdowns in a 60-hour period is exactly 4. [3]

COMPUTER BREAKDOWNS (X)

mean (λ) Hours

1 48
 λ_1 60

$$48\lambda = 60 \times 1$$

$$\lambda_1 = 1.25 \text{ (for 60 hrs)}$$

PHONE CALLS (Y)

mean (λ) Hours

2 24
 λ_2 60

$$24\lambda_2 = 2 \times 60$$

$$\lambda_2 = 5$$

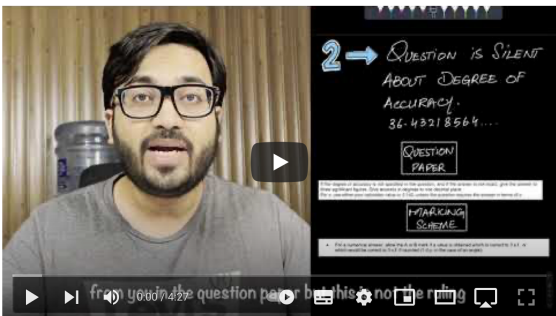
TOTAL (X+Y) $\longrightarrow P_0(\lambda_1 + \lambda_2)$

$$P_0(1.25 + 5)$$

$$\lambda_T = 6.25$$

$$P(T=4) = \frac{e^{-\lambda} \lambda^r}{r!} = \frac{e^{-6.25} (6.25)^4}{4!}$$

$$= 0.122735$$



2 $\rightarrow$ QUESTION IS SILENT ABOUT DEGREE OF ACCURACY. 36.43218564....

QUESTION PAPER

MARKING SCHEME

PAPER ATTEMPTING TECHNIQUES

HOW MANY SIGNIFICANT FIGURES SHOULD YOU LEAVE YOUR FINAL ANSWER IN O & A LEVELS EXAMS

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APPROXIMATIONS

In S1 we did Binomial $\longrightarrow$ Normal

Conditions: $np > 5$ $nq > 5$

1. BINOMIAL $\longrightarrow$ POISSON

Conditions: $n > 50$
 $p < 0.1$

$$X \sim B(n, p)$$

$$X \sim P_0(\lambda)$$

$$E(X) = \lambda = np$$

Discrete

Discrete

NO CORRECTION OF
CONTINUITY NEEDED

2. POISSON $\longrightarrow$ NORMAL

Condition: $\lambda > 15$

$$X \sim P_0(\lambda)$$



$$\text{Mean} = \lambda$$

$$\text{Var} = \lambda$$

$$X \sim N(\mu, \sigma^2)$$

Mean

Var



$$X \sim N(\lambda, \lambda)$$

[Imp]

$$\text{Mean} = \mu = \lambda$$

$$\text{Var} = \sigma^2 = \lambda$$

$$\hookrightarrow \text{SD} = \sigma = \sqrt{\lambda}$$

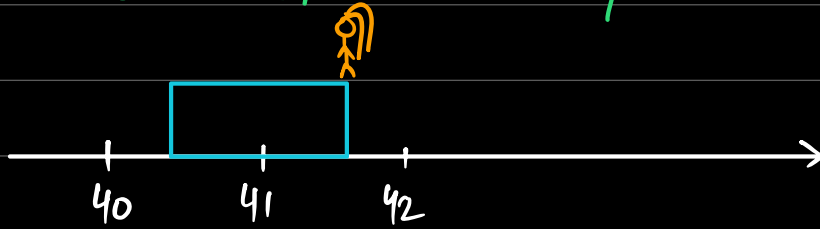
Because you need SD for finding
z in normal distribution.

$$Z = \frac{X - \mu}{\sigma}$$

POISSON
(DISCRETE)

NORMAL
(CONTINUOUS)

CORRECTION OF CONTINUITY NEEDED



POISSON
 $P(X > 41)$
right

NORMAL
 $P(X > 41.5)$

$P(X \geq 41)$
right

$P(X \geq 40.5)$

$P(X < 41)$
left

$P(X < 40.5)$

$P(X \leq 41)$
left

$P(X < 41.5)$

$P(X = 41)$

$P(40.5 < X < 41.5)$

DISCRETE
DISTRIBUTIONS

(Binomial, Geometric, Poisson)

$>$ and $\geq$ are different
 $<$ and $\leq$ are different.

$P(X > 3) = P(4, 5, 6, \dots)$
 $P(X \geq 3) = P(3, 4, 5, 6, \dots)$

CONTINUOUS
DISTRIBUTIONS

(Normal)

$>$ and $\geq$ means same
 $<$ and $\leq$ means same

The number of goals scored per match by Everly Rovers is represented by the random variable X which has mean 1.8.

(i) State two conditions for X to be modelled by a Poisson distribution. [2]

Assume now that $X \sim \text{Po}(1.8)$.

$$\lambda = 1.8$$

(ii) Find $P(2 < X < 6)$. [2]

$$P(X \geq 1)$$

(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. [3]

(i) Random - Independent - Singular - Constant Rate.

$$\begin{aligned} \text{(ii)} \quad P(2 < X < 6) &= P(3, 4, 5) \\ &= e^{-1.8} \left(\frac{1.8^3}{3!} + \frac{1.8^4}{4!} + \frac{1.8^5}{5!} \right) \\ &= 0.2590 \end{aligned}$$

(iii) $P(\text{at least one})$

$$\begin{aligned} &= P(X \geq 1) = P(1, 2, 3, 4, \dots, \infty) \\ &= 1 - P(X = 0) \\ &= 1 - e^{-1.8} \left(\frac{1.8^0}{0!} \right) \end{aligned}$$

$$P(\text{at least one}) = 0.8437 \quad \text{This is for one match.}$$

For 10 Matches Think of it as binomial

$$n = 10$$

$$p = 0.8437$$

$$q = 1 - 0.8437 = 0.1563$$

$$\begin{aligned} P(X = 10) &= {}^{10}C_{10} (0.8437)^{10} (0.1563)^0 \\ &= 0.1642 \end{aligned}$$

A hotel kitchen has two dish-washing machines. The numbers of breakdowns per year by the two machines have independent Poisson distributions with means 0.7 and 1.0. Find the probability that the total number of breakdowns by the two machines during the next two years will be less than 3.

[4]

$X = \text{Machine 1}$

$$\lambda_1 = 0.7$$

Mean Time

$$0.7 \quad 1$$

$$\lambda \quad 2$$

$$\lambda_1 = 1.4$$

$Y = \text{Machine 2}$

$$\lambda_2 = 1.0$$

Mean Time

$$1 \quad 1$$

$$\lambda \quad 2$$

$$\lambda_2 = 2$$

$$P(X+Y < 3)$$

$$\text{New } \lambda = 1.4 + 2 = 3.4$$

$$P(T < 3) = P(0, 1, 2)$$

$$= e^{-3.4} \left[\frac{3.4^0}{0!} + \frac{3.4^1}{1!} + \frac{3.4^2}{2!} \right]$$

$$= 0.3397$$

Customers arrive at an enquiry desk at a constant average rate of 1 every 5 minutes.

- (i) State one condition for the number of customers arriving in a given period to be modelled by a Poisson distribution. [1]

Assume now that a Poisson distribution is a suitable model.

- (ii) Find the probability that exactly 5 customers will arrive during a randomly chosen 30-minute period. [2]
 $x < 3$
- (iii) Find the probability that fewer than 3 customers will arrive during a randomly chosen 12-minute period. [3]
- (iv) Find an estimate of the probability that fewer than 30 customers will arrive during a randomly chosen 2-hour period. [4]
 $x < 30$

(ii)

Mean	Time
1	5
λ	30

$$5\lambda = 30$$
$$\lambda = 6$$

$$P(x=5) = e^{-6} \left(\frac{6^5}{5!} \right) = 0.16062$$

(iii)

Mean	Time
1	5
λ	12

$$5\lambda = 12$$
$$\lambda = 2.4$$

$$P(x < 3) = P(0, 1, 2)$$
$$= e^{-2.4} \left(\frac{2.4^0}{0!} + \frac{2.4^1}{1!} + \frac{2.4^2}{2!} \right)$$
$$= 0.5697$$

(iii)

Mean	Time
1	5
λ	120

$$2 \text{ hours} = 120 \text{ min}$$
$$5\lambda = 120$$
$$\lambda = 24$$

Now we use approximation of
Poisson $\longrightarrow$ NORMAL
Mean = 24 $\mu = 24$

$$\text{Var} = 24$$

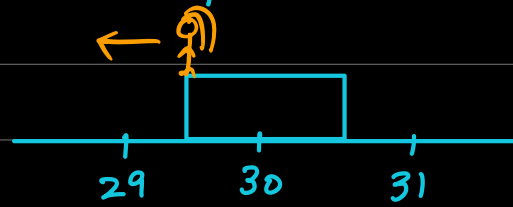
$$\sigma^2 = 24 \rightarrow \sigma = \sqrt{24}$$

$$P(X < 30)$$

Left



Continuity Correction

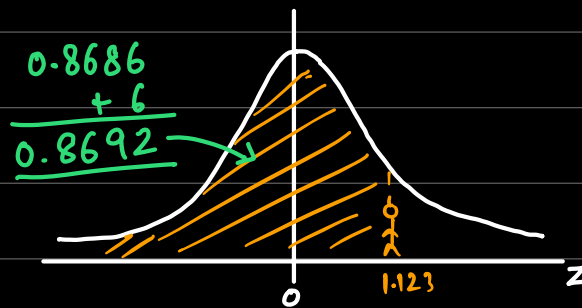


$$P(X < 29.5)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{29.5 - 24}{\sqrt{24}}$$

$$Z = 1.123$$

$$P(Z < 1.123)$$



$$P(X < 30) = 0.8692$$

A random variable X has the distribution $Po(1.6)$.

(i) The random variable R is the sum of three independent values of X . Find $P(R < 4)$. [3]

(ii) The random variable S is the sum of n independent values of X . It is given that

$$P(S = 4) = \frac{16}{3} \times P(S = 2).$$

Find n . [4]

(iii) The random variable T is the sum of 40 independent values of X . Find $P(T > 75)$. [4]

$$R = \boxed{X} + \boxed{X} + \boxed{X}$$

$\lambda = 1.6 \quad \lambda = 1.6 \quad \lambda = 1.6$

$$\downarrow \lambda = 1.6 + 1.6 + 1.6 = 4.8$$

$$\begin{aligned} P(X < 4) &= P(0, 1, 2, 3) \\ &= e^{-4.8} \left(\frac{4.8^0}{0!} + \frac{4.8^1}{1!} + \frac{4.8^2}{2!} + \frac{4.8^3}{3!} \right) \\ &= \boxed{} \end{aligned}$$

$$S = \boxed{X} + \boxed{X} \dots \dots \dots n \text{ times}$$

$$\hookrightarrow \lambda = 1.6 \times n = 1.6n$$

$$P(S=4) = e^{-1.6n} \left(\frac{(1.6n)^4}{4!} \right)$$

$$P(S=2) = e^{-1.6n} \left(\frac{(1.6n)^2}{2!} \right)$$

$$P(S = 4) = \frac{16}{3} \times P(S = 2)$$

$$\cancel{e^{-1.6n}} \left(\frac{(1.6n)^4}{4!} \right) = \frac{16}{3} \times \cancel{e^{-1.6n}} \left(\frac{(1.6n)^2}{2!} \right)$$

$$\frac{(1.6n)^4}{(1.6n)^2} = \frac{16}{3} \times \frac{4!}{2!}$$

$$(1.6n)^2 = 64$$

$$1.6n = 8$$

$$n = 5$$

(iii) $\boxed{T} = \boxed{X} + \boxed{X} \dots \dots 40 \text{ times}$

$$\lambda = 40 \times 1.6 = 64$$

Approximation Poisson $\longrightarrow$ NORMAL

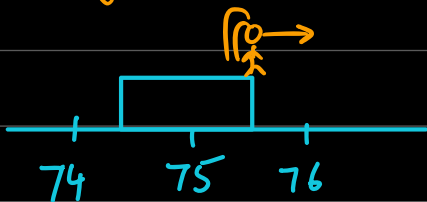
Poisson

$$\text{Mean} = 64$$

$$\text{Variance} = 64$$

$$P(T > 75)$$

right



NORMAL

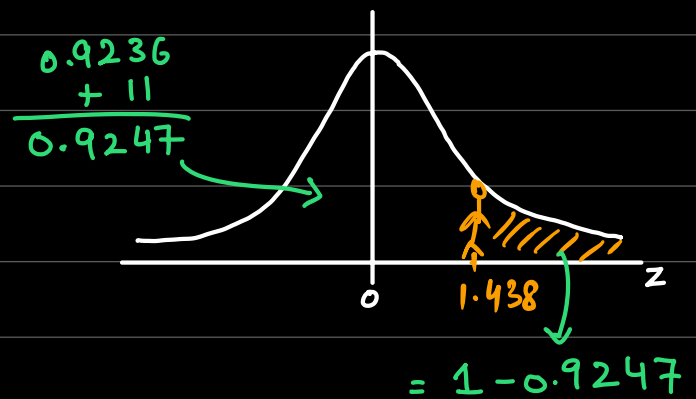
$$\mu = 64$$

$$\sigma^2 = 64, \sigma = 8$$

$$P(X > 75.5)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{75.5 - 64}{8}$$

$$Z = 1.438$$



$$P(X > 75) = 0.0753$$