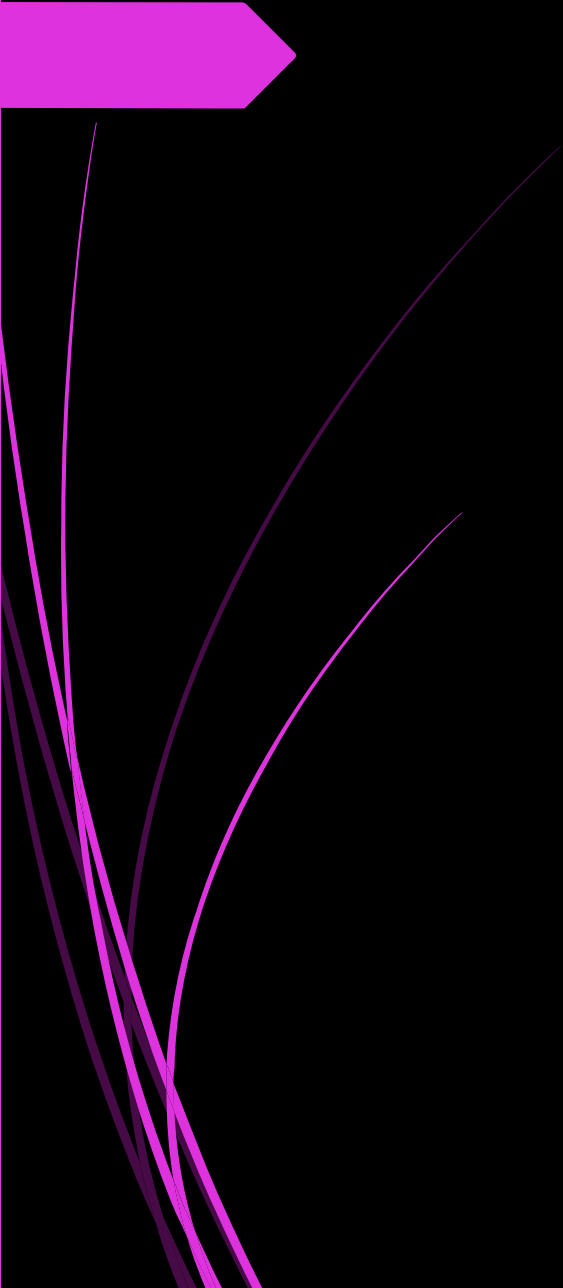
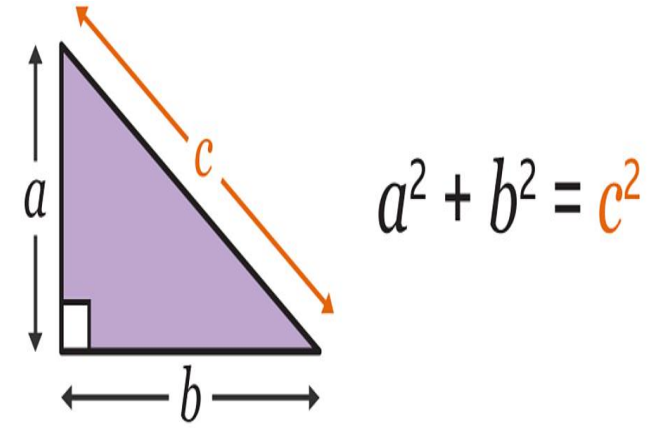


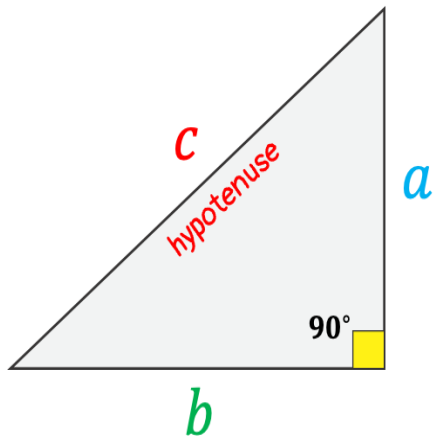


INTRODUCTION TO TRIGONOMETRY PART ONE

- ❖ Trigonometry is **the study of the relationship between the sides of a triangle and its angles.**
- ❖ Right angled triangles are fundamental to the study of trigonometry
- ❖ Theorem of Pythagoras which describes the relationship between the three sides of a right-angled triangle.
- ❖ The Theorem of Pythagoras **applies exclusively to right-angled triangles (triangles with a 90° angle).**



PYTHAGOREAN THEOREM

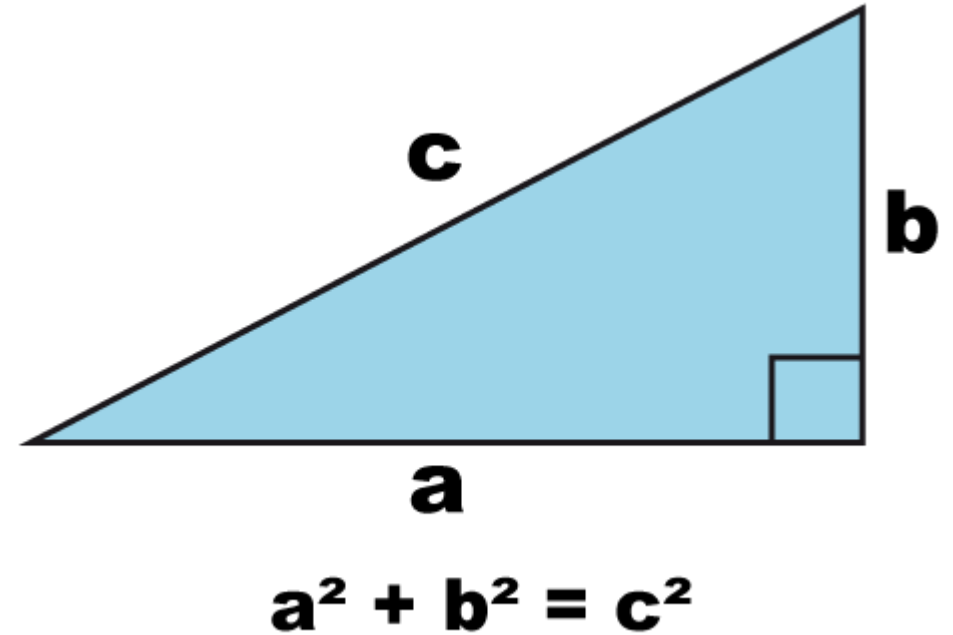


$$c^2 = a^2 + b^2$$

$$\star c = \sqrt{a^2 + b^2}$$

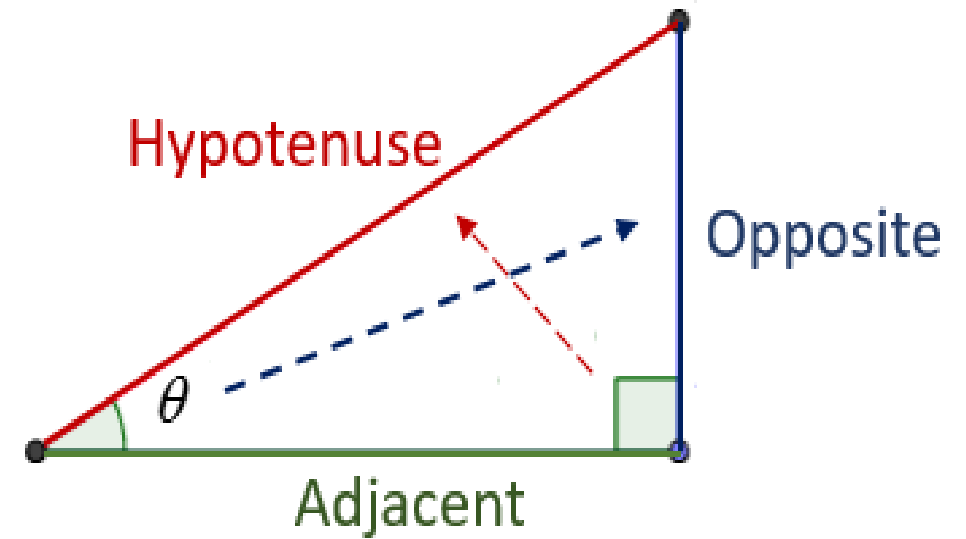
$$\star a = \sqrt{c^2 - b^2}$$

$$\star b = \sqrt{c^2 - a^2}$$



Sides of a Triangle

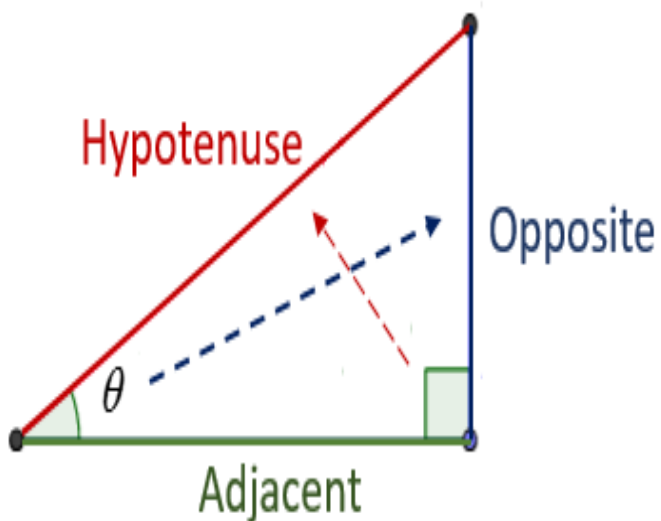
- ❑ In a **right-angled triangle**, we refer to the lengths of the three sides according to how they are placed in relation to the angle θ .
- ❑ The side **opposite to the right** angle is labeled “**hypotenuse**”
- ❑ the side **opposite θ** is labelled “**opposite**”
- ❑ the **side next to θ** is labeled “**adjacent**”.



Trigonometric ratios

- There are three special ratios in the study of Trigonometry, namely the **sine**, **cosine** and **tangent** ratios.
- When these ratios are applied to a right-angled triangle, they define the relationships between its sides and angles.
- The trigonometric ratios of sine, cosine and tangent relate to **right angled triangles only**.

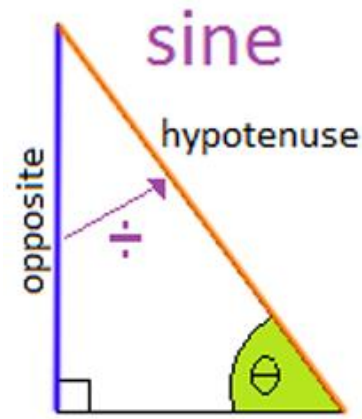
SOHCAHTOA



SOH $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

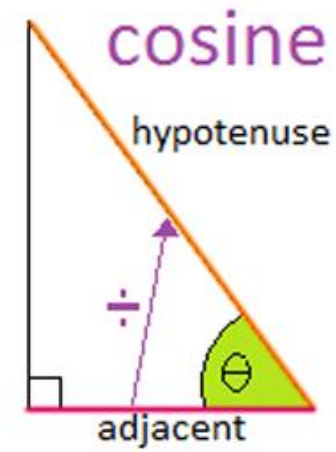
CAH $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$



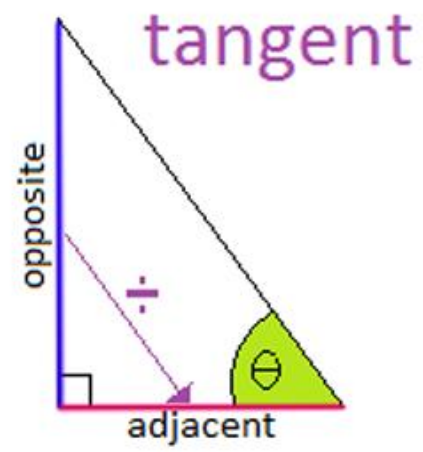
$$\frac{\text{Opposite}}{\text{hypotenuse}}$$

SOH



$$\frac{\text{adjacent}}{\text{hypotenuse}}$$

CAH



$$\frac{\text{Opposite}}{\text{adjacent}}$$

TOA

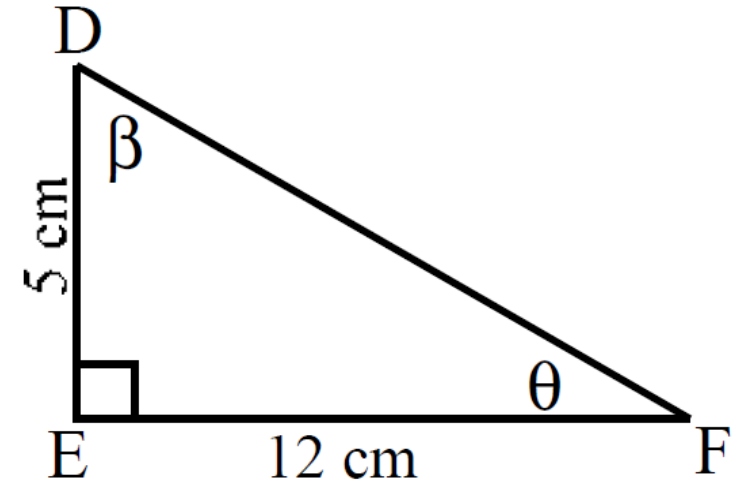
Example 1

In $\triangle DEF$, $DE = 5$, $EF = 12$, $\angle E = 90^\circ$, $\angle D = \beta$ and $\angle F = \theta$.

(a) Determine the length of the hypotenuse DF .

(b) Write the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

(c) Write the value of $\sin \beta$, $\cos \beta$ and $\tan \beta$.

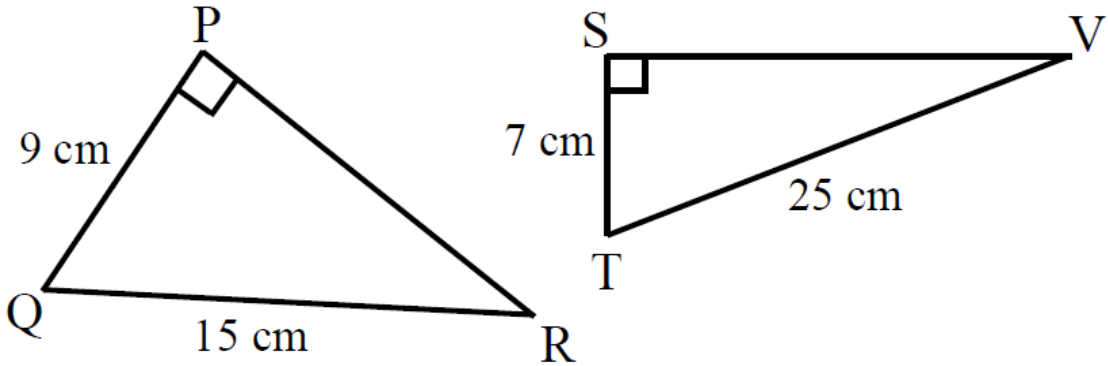


Example 2

Determine:

(a) $\tan Q$

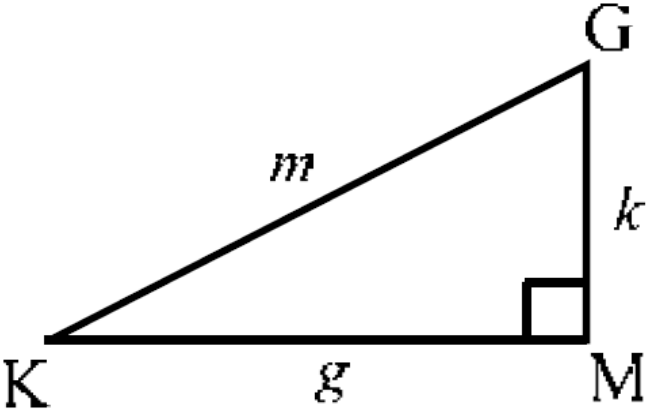
(b) $\cos V$



Example 3

State the following in terms of g , k and m :

- a) $\sin K$
- b) $\cos K$
- c) $\tan K$
- d) $\sin G$
- e) $\cos G$
- f) $\tan G$



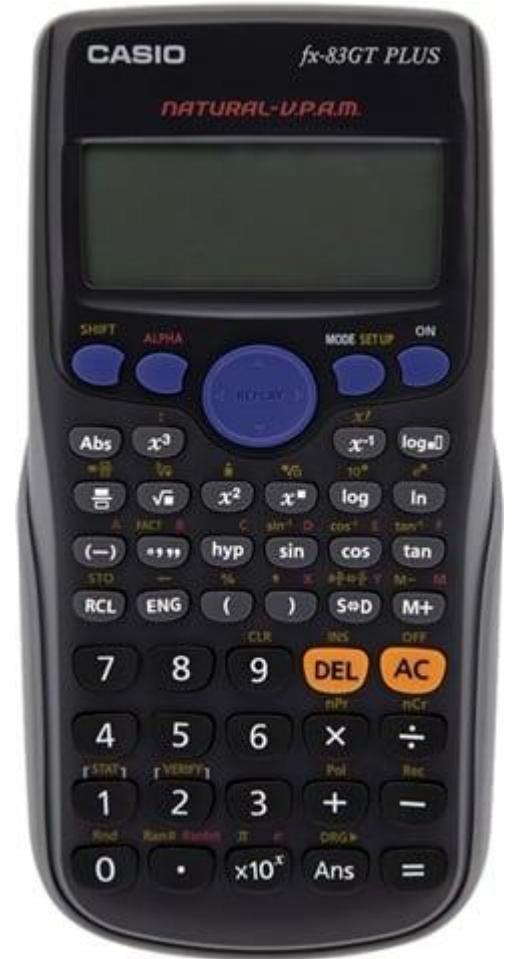
CALCULATING THE TRIGONOMETRIC RATIOS OF A GIVEN ANGLE

The **sine, cosine and tangent ratio** can be calculated with the use of a calculator which must be on the DEG (degree) mode.

Example 4

Evaluate the following trigonometric ratios rounded off to two decimal places where necessary.

- a) $\cos 35^\circ$
- b) $\tan 36^\circ$
- c) $\sin 83^\circ$
- d) $\sin 43^\circ$
- e) $\cos 14^\circ$
- f) $\tan 45^\circ$
- g) $5\cos 60^\circ$
- h) $\frac{\sin 12^\circ}{12}$
- i) $\frac{1}{3}\tan 36^\circ$
- j) $\frac{20,35}{\sin 38^\circ}$
- k) $\sin(35^\circ + 75^\circ)$
- l) $\sin 35^\circ + \sin 75^\circ$



Example 5

Determine the decimal value of the following if
 $A = 23,8^\circ$ and $B = 18,1^\circ$.

(Round off your answers to one decimal place)

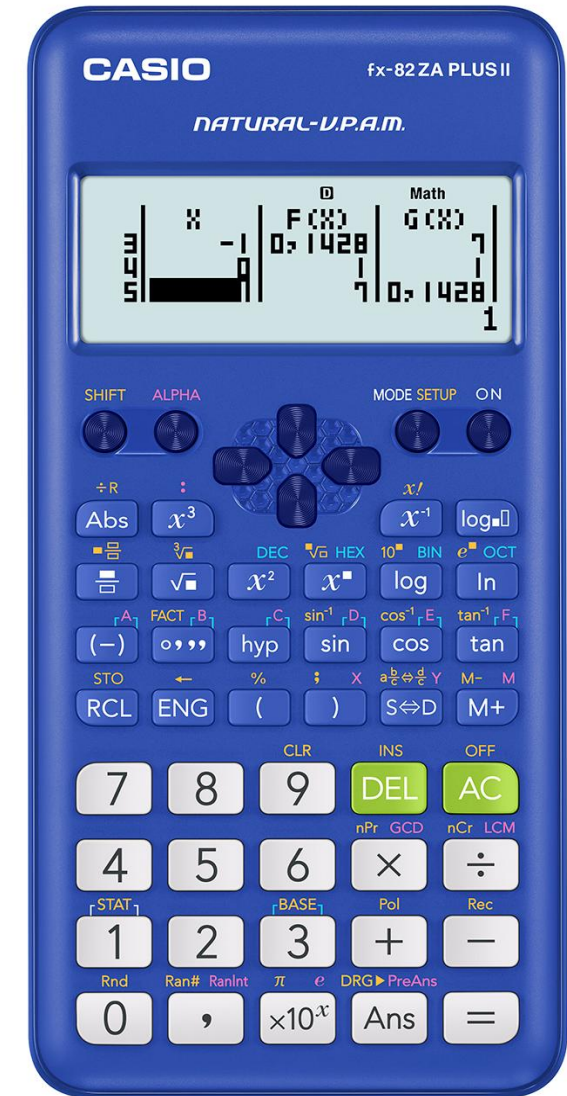
a) $\sin(A + B)$

b) $\tan 2B$

c) $\cos^2(2A - 10^\circ)$

SOLVING SIMPLE TRIGONOMETRIC EQUATIONS

- This process is **finding the size of an angle** when given its sine, cosine or tangent ratio.
- Consider the trigonometric equation $\cos \theta = 0,5$.
- We want to find the size of the angle that will result in the ratio 0,5.
- To do this, we will make use of the $\cos^{-1}$ function on the calculator.
- This function can be found above the “cos” button.
- To use this function, you must press the “SHIFT” button on your calculator.
- Therefore, the sequence of buttons to press on most of the calculators will be as follows:
 - Press the SHIFT button followed by the “cos” button. The $\cos^{-1}$ will appear on your calculator screen.
 - Enter the ratio value (in this case 0.5)
 - Press the “=” button (some calculators expect you to close the brackets first)
- Some calculators may have the button INV or 2nd F instead of SHIFT. Always ensure that your calculator is on the degree mode [DEG]



Example 6

a) Determine the size of the acute angle θ in each of the following trigonometric equations. Round your answers off to one decimal place where necessary.

1) $\cos\theta = 0,5$

2) $\tan\theta = 4,123$

3) $\sin\theta = 0,707$

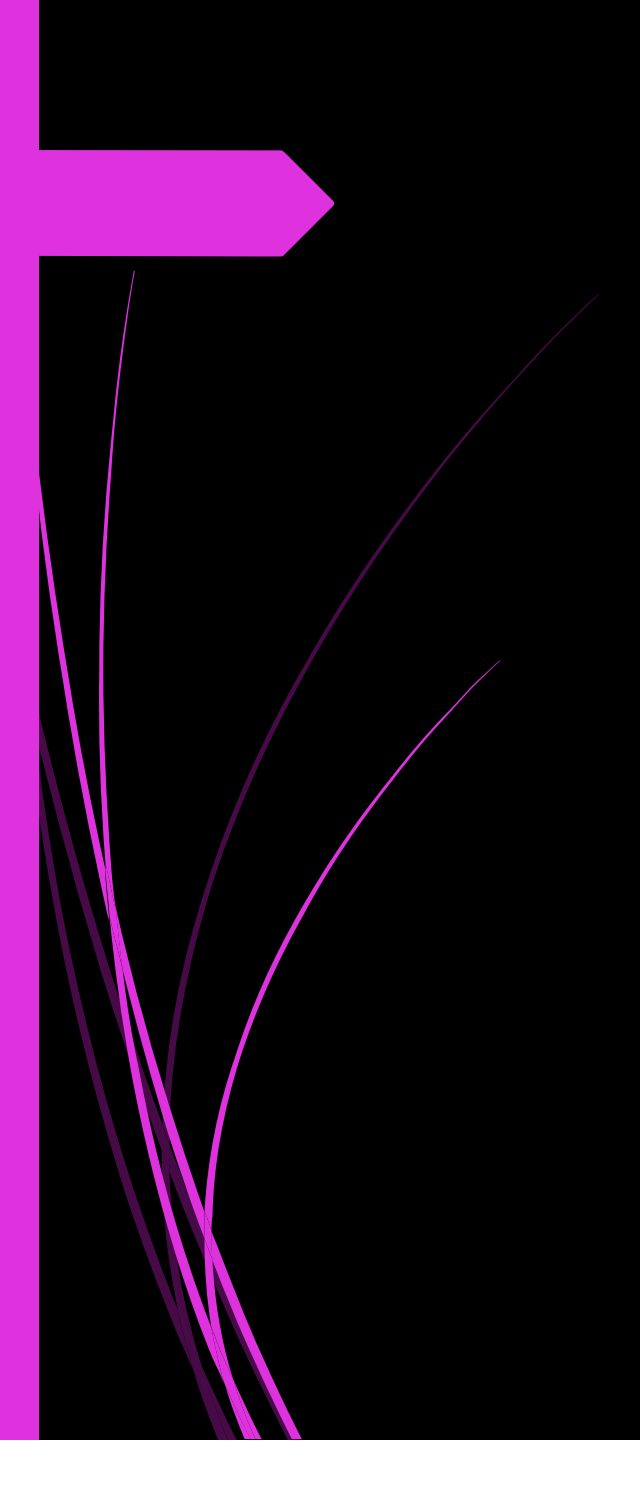
b) Solve the following equations. Round your answers off to one decimal place where necessary. All angles are acute.

1) $2 \sin \theta = 1,124$

2) $\sin 2\theta = 0,435$

3) $\frac{1}{2}\tan 2x = 3$

4) $1 + 2 \cos(x + 10^\circ) = 2,356$

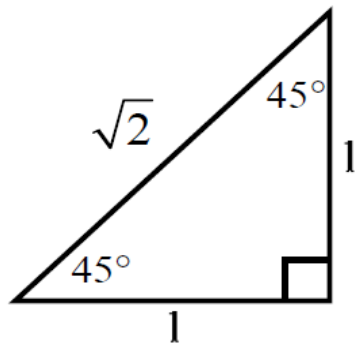


INTRODUCTION TO TRIGONOMETRY PART TWO

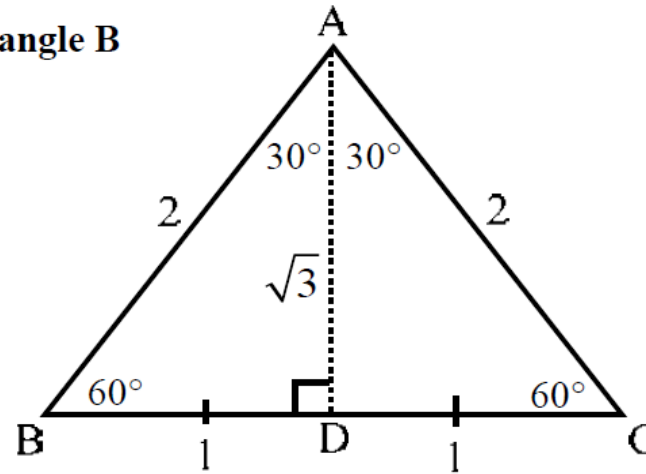
TRIGONOMETRIC RATIOS OF THE SPECIAL ANGLES 30° , 45° AND 60°

- ❖ The angles 30° , 45° and 60° are referred to as special angles.
- ❖ The reason for them being called special angles is because the trigonometric ratios of these angles can be evaluated without using a calculator.
- ❖ The following two triangles can be used to determine trigonometric ratios of special angles.

Triangle A



Triangle B

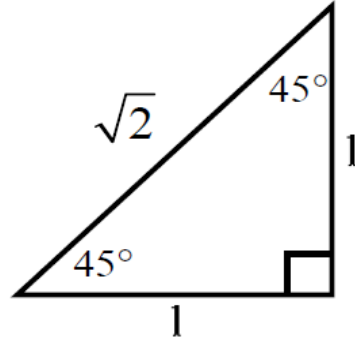


Example 7

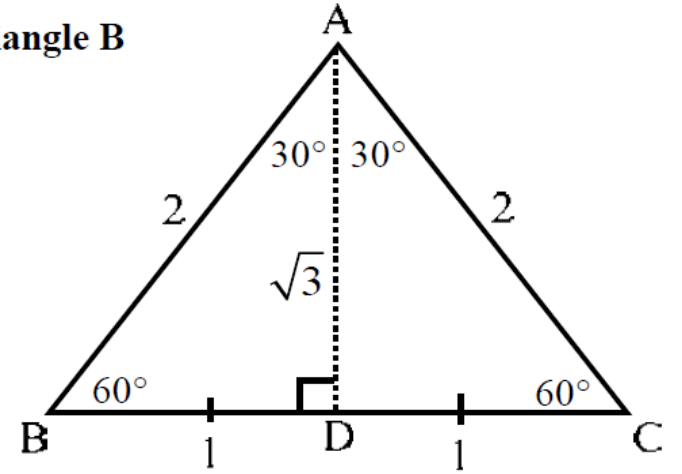
Evaluate the following without using a calculator. Use the triangles alongside to assist you.

- a) $\cos 45^\circ$
- b) $\cos 60^\circ$
- c) $\tan 45^\circ$
- d) $\sin 30^\circ$

Triangle A



Triangle B



Example 8

Calculate the following without using a calculator:

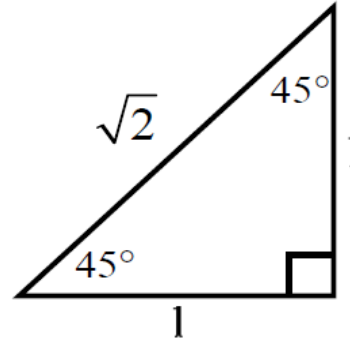
a) $\sin 30^\circ + \cos 60^\circ$

b) $\tan 60^\circ - \cos 30^\circ$

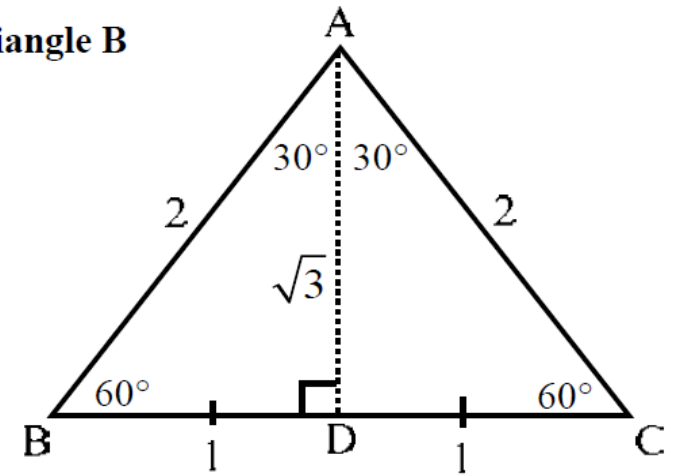
c) $\frac{\sin 60^\circ}{\tan 30^\circ}$

d) $\cos^2 45^\circ$

Triangle A



Triangle B



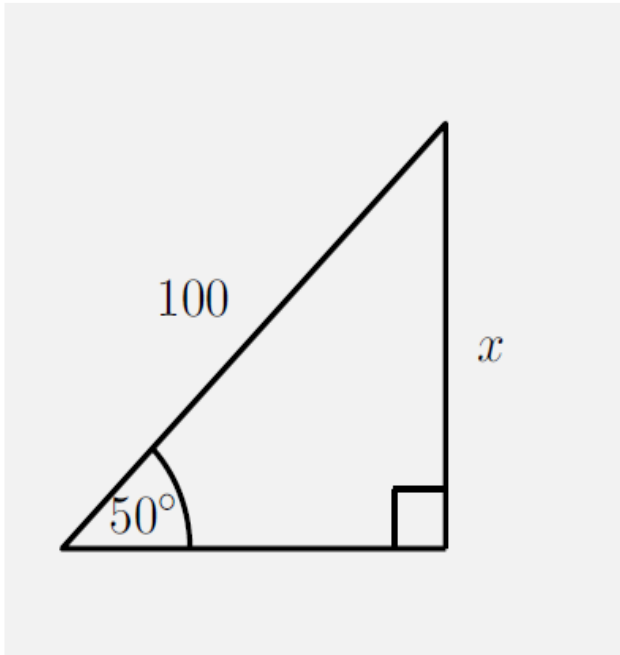
Example 9

Solve the equation $2\cos x - \sqrt{3} = 0$ without using a calculator where x is an acute angle.

Finding sides and angles using trigonometric ratios

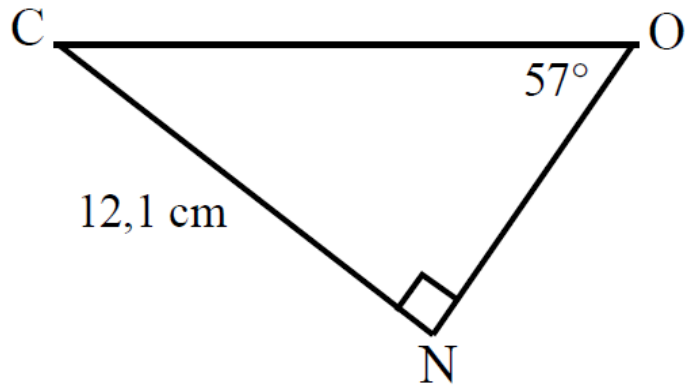
Example 10 (Finding the length of a side)

Find the length of x in the following right-angled triangle:



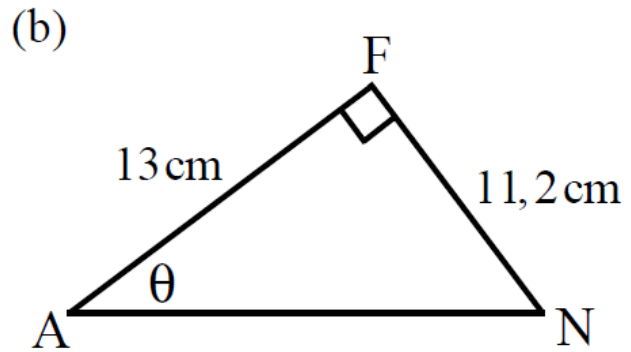
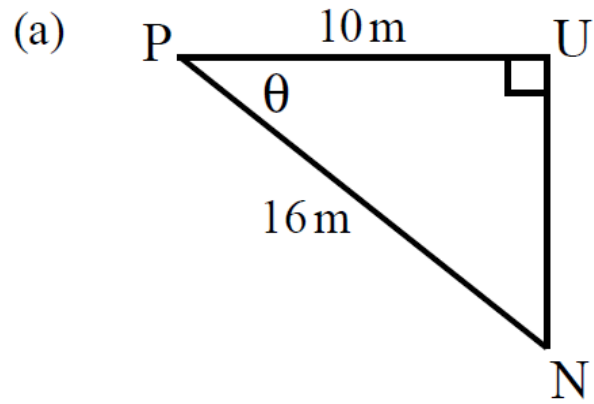
Example 11 (Finding the length of a side)

Consider the triangle sketched below.
Calculate the length of ON correct to one decimal place.



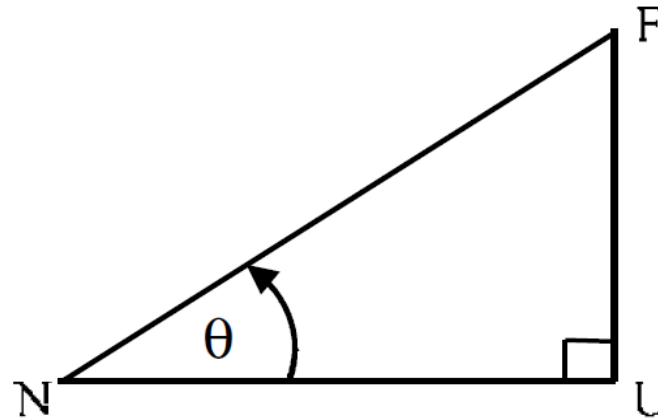
Example 12 (Finding an angle)

Calculate the size of θ correct to one decimal place in each case.

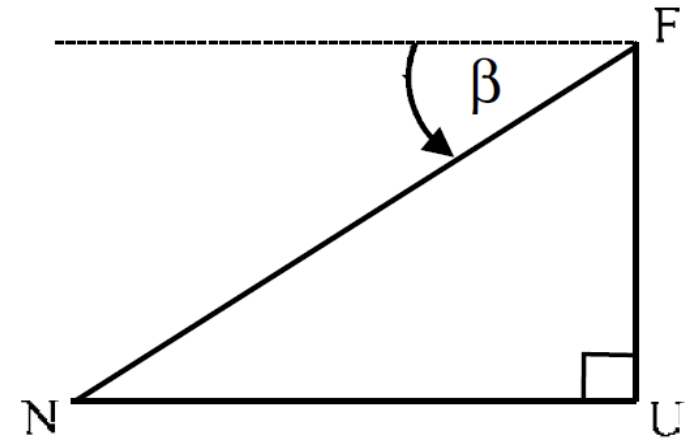


Angles of Elevation and Depression

- Angles of elevation and depression are trigonometric concepts measuring the angle between a horizontal line of sight and an object.
- Elevation looks upward, while depression looks downward.
- Because they are alternate interior angles, the angle of elevation from point N to F is equal to the angle of depression from F to N as shown below



θ is the **angle of elevation** of F from N.

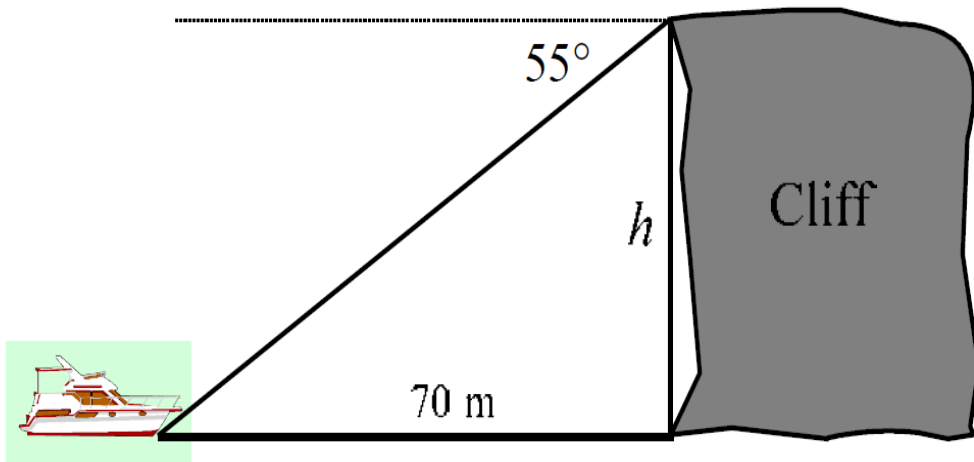


β is the **angle of depression** of N from F.
[Note that $\hat{N} = \beta$ since alt $\angle$ s are equal]

Example 13

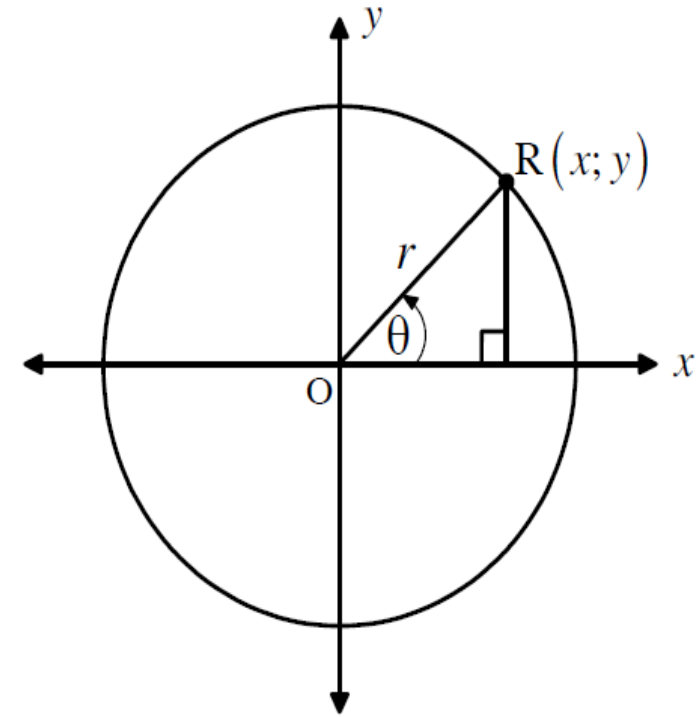
The angle of depression of a boat on the ocean from the top of a cliff is 55° . The boat is 70 metres from the foot of the cliff.

- (a) What is the angle of elevation of the top of the cliff from the boat?
- (b) Calculate the height of the cliff.



Angles in the Cartesian Plane

- Trigonometric definitions can be extended to include angles in the interval $[0^\circ; 360^\circ]$.
- Consider a circle with center $O(0; 0)$ and radius r with $R(x; y)$ any point on the circle.
- θ is the angle measured anti-clockwise from the positive side of the x -axis to the radius OR which is referred to as the **terminal arm** and θ is said to be in **standard position**.
- Note that for every point $R(x; y)$ on the circumference of the circle, $x^2 + y^2 = r^2$



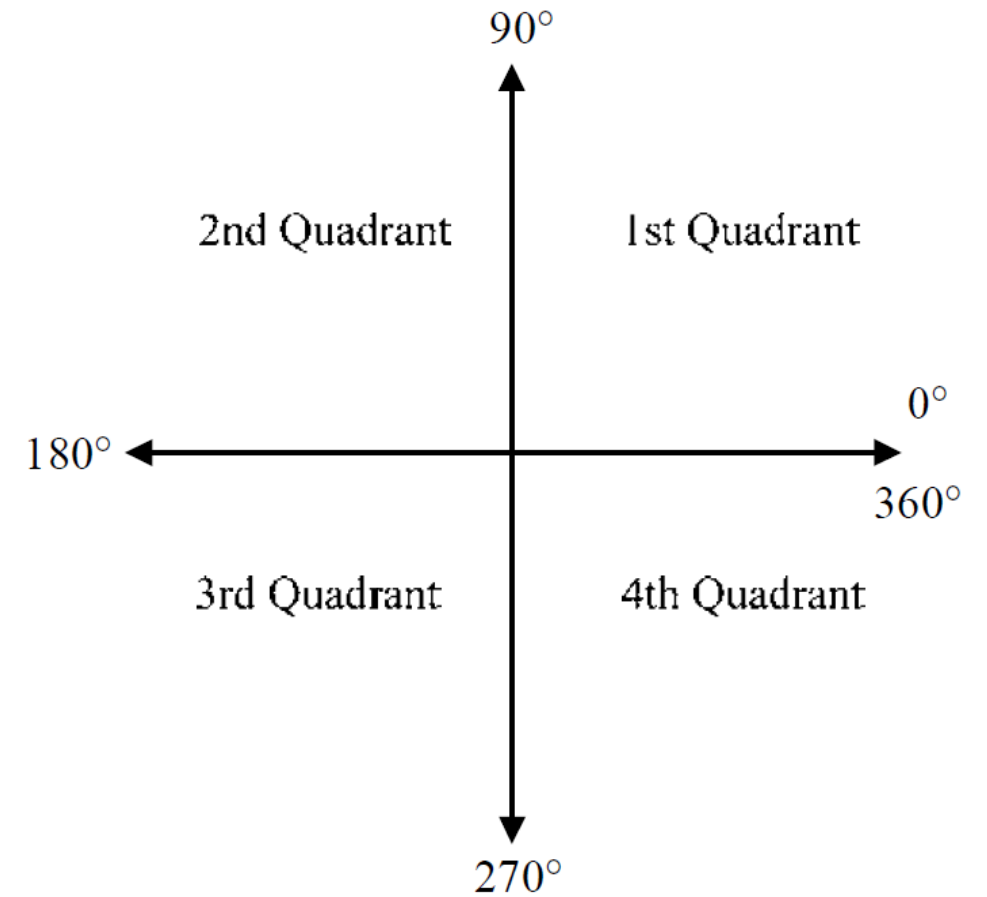
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

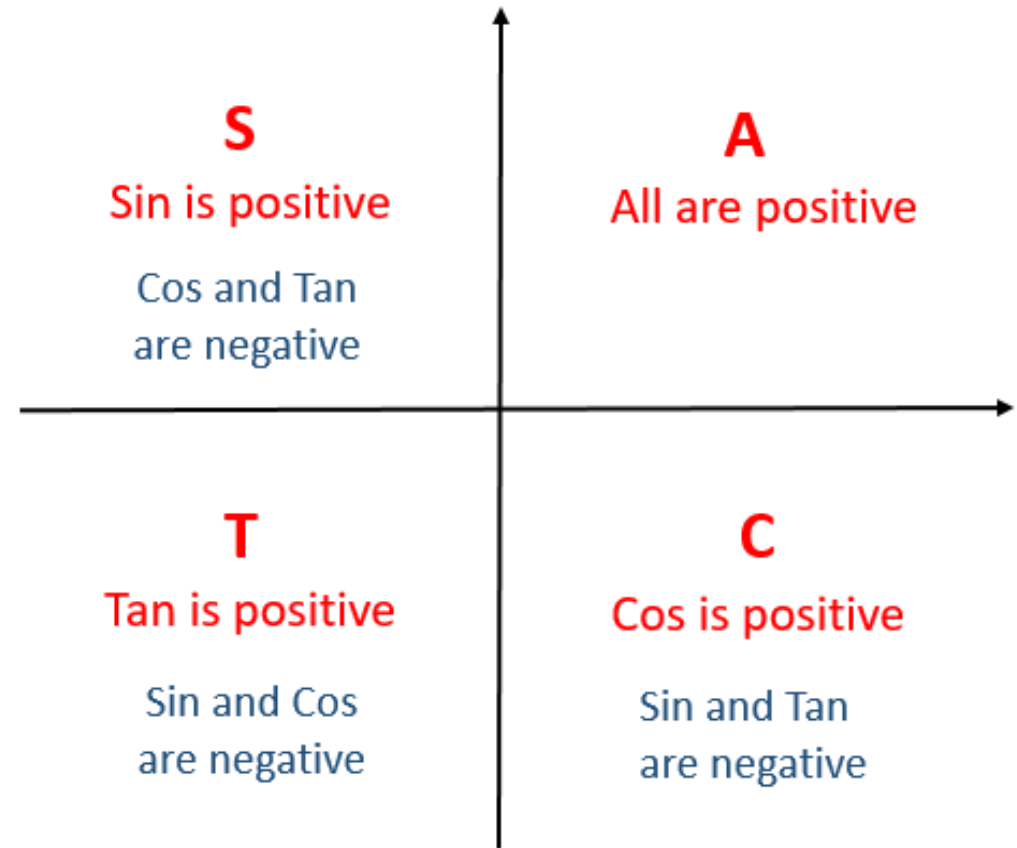
The Cartesian plane is divided into four quadrants allowing for angles in the interval $(0^\circ ; 360^\circ)$

- Angles in the first quadrant will lie in the interval $(0^\circ ; 90^\circ)$
- Angles in the second quadrant will lie in the interval $(90^\circ ; 180^\circ)$
- Angles in the third quadrant will lie in the interval $(180^\circ ; 270^\circ)$
- Angles in the fourth quadrant will lie in the interval $(270^\circ ; 360^\circ)$



CAST DIAGRAM

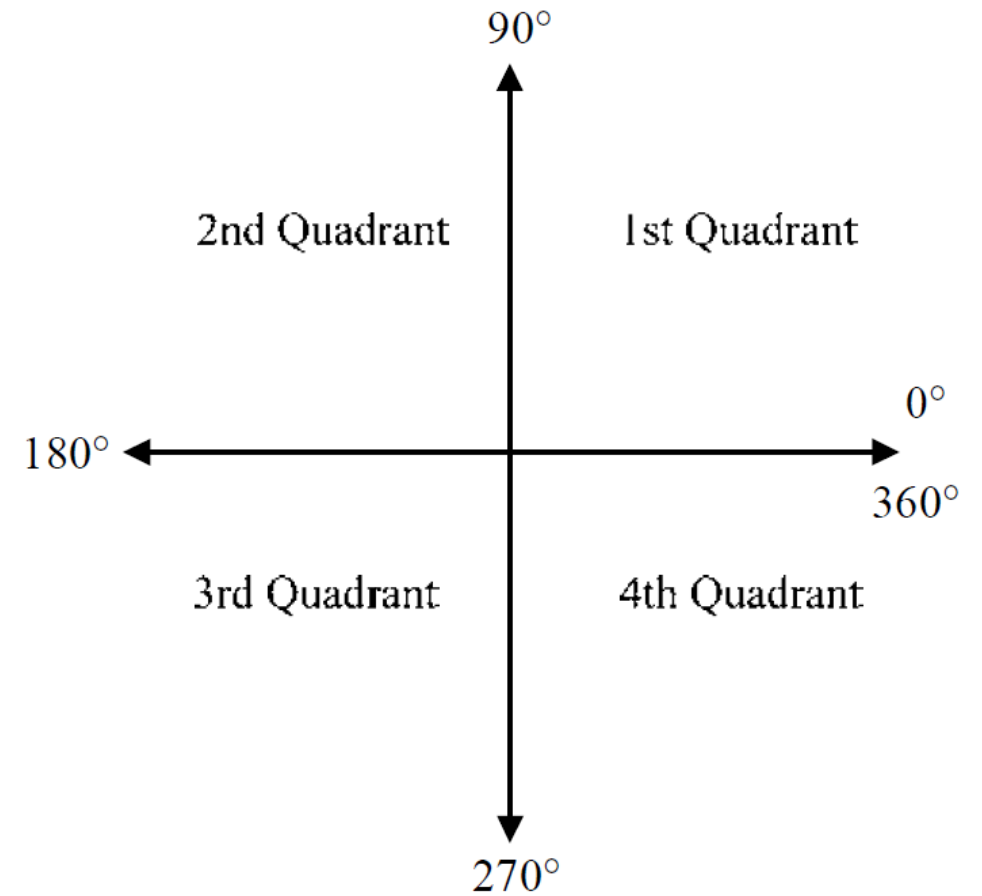
- All trigonometric functions are positive in the first quadrant.
- $\sin\theta$ is positive in the second quadrant and $\tan\theta$ and $\cos\theta$ are negative.
- $\tan\theta$ is positive in the third quadrant and $\sin\theta$ and $\cos\theta$ are negative.
- $\cos\theta$ is positive in the fourth quadrant and $\sin\theta$ and $\tan\theta$ are negative.



Example 14

In which quadrant does the terminal arm of the angle θ lie if:

- a) $\sin \theta > 0$ and $\cos \theta > 0$
- b) $\sin \theta < 0$ and $\cos \theta < 0$
- c) $\tan \theta > 0$ and $\cos \theta < 0$
- d) $\tan \theta < 0$ and $\cos \theta < 0$
- e) $\sin \theta < 0$ and $\theta \in [90^\circ; 270^\circ]$
- f) $\cos \theta < 0$ and $0^\circ < \theta < 180^\circ$



Example 15

If $\cos\theta = \frac{4}{5}$ and $\theta \in [0^\circ; 90^\circ]$ calculate without the use of a calculator and with the aid of a diagram the value of $\tan^2\theta$

Example 16

If $13 \sin \theta = -5$ and $\theta \in [90^\circ; 270^\circ]$ calculate without the use of a calculator and with the aid of a diagram the value of $\cos \theta + \sin \theta$

Reciprocals of the trigonometric ratios

- Reciprocal trig ratios are defined as the multiplicative inverses of the primary trigonometric functions (sine, cosine, and tangent)
- The **cosecant** function (**cosec** θ) is the reciprocal of $\sin \theta$
- The **secant** function **sec** θ is the reciprocal of $\cos \theta$
- The **cotangent** function **cot** θ is the reciprocal of $\tan \theta$

Reciprocal Trig Ratios

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosecant } \theta = \frac{1}{\sin \theta} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{secant } \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cotangent } \theta = \frac{1}{\tan \theta} = \frac{\text{adjacent}}{\text{opposite}}$$

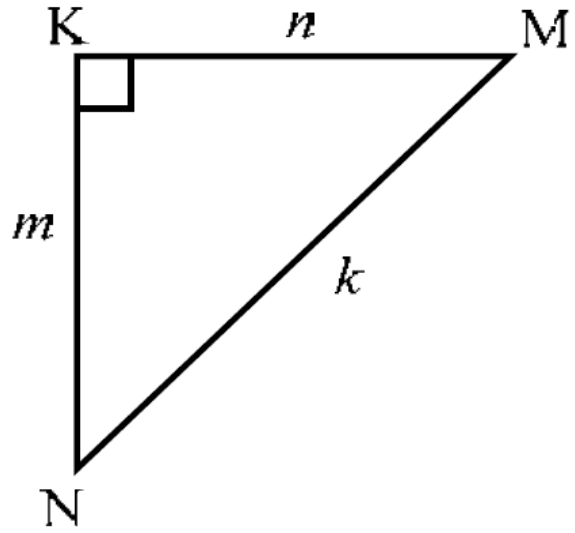
Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Consider the following triangle:



$$\sin N = \frac{n}{k} \text{ and } \operatorname{cosec} N = \frac{k}{n}$$

$$\cos N = \frac{m}{k} \text{ and } \sec N = \frac{k}{m}$$

$$\tan N = \frac{n}{m} \text{ and } \cot N = \frac{m}{n}$$

Example 17

a) Evaluate the following without using a calculator:

1) $\operatorname{cosec}45^\circ$

2) $\sec30^\circ \cdot \tan60^\circ$

b) Evaluate the following rounded off to two decimal places

1) $\cot26^\circ$

2) $\sec36^\circ + \cot63^\circ$

Example 18

Without using a calculator, solve the equation $\operatorname{cosec}\theta = \sqrt{2}$ if θ is an acute angle.

Example 19

Solve the equation $\sec\theta = 1,5$ rounded off to two decimal places if θ is an acute angle.

PAST EXAMS PAPERS

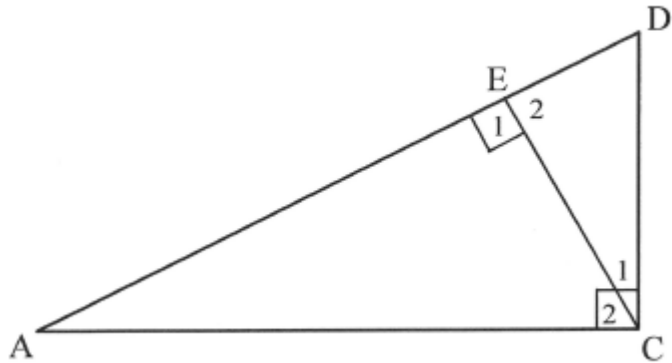
2019 November Paper 2 Q 3

3.1 If $x = 37^\circ$ and $y = 44^\circ$, calculate the value of $\sin^2 x + 2\cos y$ (1)

3.2 Without using a calculator determine the value of

$$\frac{\sin 30^\circ \cdot \cot 45^\circ}{\cos 30^\circ \cdot \tan 60^\circ} \quad (3)$$

3.3 In the diagram below, $\triangle ACD$ is right angled at C. E lies on AD such that CE is perpendicular to AD



3.3.1 Write down the ratio of $\cos D$ in $\triangle ACD$ (1)

3.3.2 Write down the ratio for $\cos D$ in $\triangle CED$ (1)

3.3.3 If $AD = 13$ units and $DC = 5$ units, calculate the length of ED (2)

CONTINUED...

3.4 Given that $\cos \theta = \frac{5}{13}$ and $\sin \theta < 0$

With the aid of a diagram and WITHOUT using a calculator, determine the value of

3.4.1 $\sin \theta$ (3)

3.4.2 $\sec \theta + \tan^2 \theta + 1$ (4)

2023 KwaZulu Natal November Paper 2 Q 4

4.1 Given $\sqrt{3}\operatorname{cosec}\theta + 2 = 0$ and $90^\circ < \theta < 270^\circ$

4.1.1 Use a sketch to determine the value of the following
WITHOUT USING A CALCULATOR

i) $\cos\theta$ (4)

ii) $\frac{\sin\theta \tan\theta}{\cos\theta}$ (4)

4.1.2 If $\theta = 25^\circ$ and $\alpha = 38^\circ$ determine the value
of $\cos(2\alpha - \theta)$ correct to 1 decimal place (3)

4.2 Simplify the following expression WITHOUT using a
calculator : $\operatorname{cosec}^2 60^\circ + \tan^2 45^\circ + \sec 60^\circ$ (6)

4.3 Solve for θ correct to TWO decimal places, if
 $2\tan(\theta - 45^\circ)$ and $0^\circ \leq \theta \leq 90^\circ$ (3)

2025 Limpopo June Paper 2 Q 2

2.1 Determine the value of the following using a calculator

$$\frac{3 \cos 120^\circ}{\sin 150^\circ - \cos 80^\circ}$$

(2)

2.2 Determine the value of each of the following equations, correct to two decimal places:

2.2.1 $4 \sin \theta = 2,1$ (2)

2.2.2 $\cot(\theta + 15^\circ) = 0,845$ (2)

2.3 Without the use of a calculator, determine the value of :

$$\sin 0^\circ + \cos^2 60^\circ + \sqrt{2} \cos 45^\circ$$
 (4)

2.4 Find the value of x without the use of a calculator

$$x \cot 60^\circ = \frac{\sin 50^\circ \cdot \sin 30^\circ \cdot \operatorname{cosec} 50^\circ}{\cot 45^\circ}$$

(5)

2012 Exemplar Paper 2 Q 5

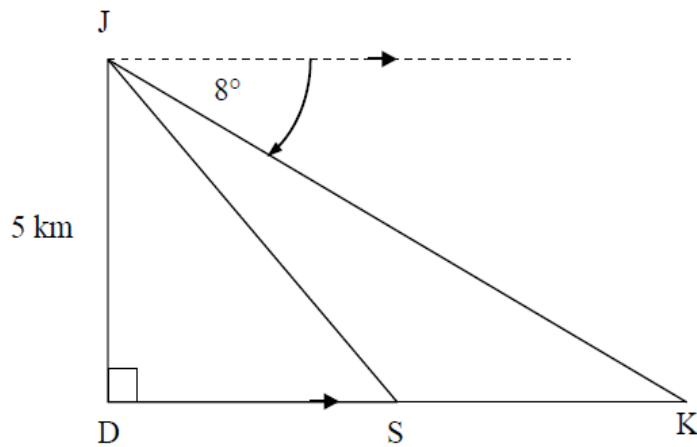
5.1 Solve for x , correct to ONE decimal place, in each of the following equations where $0^\circ \leq x \leq 90^\circ$.

5.1.1 $5\cos x = 3$ (2)

5.1.2 $\tan 2x = 1,19$ (3)

5.1.3 $4\sec x - 3 = 5$ (4)

5.2 An aeroplane at J is flying directly over a point D on the ground at a height of 5 kilometres. It is heading to land at point K. The angle of depression from J to K is 8° . S is a point along the route from D to K.



5.2.4 Calculate the angle of elevation from point S to J, correct to ONE decimal place. (2)

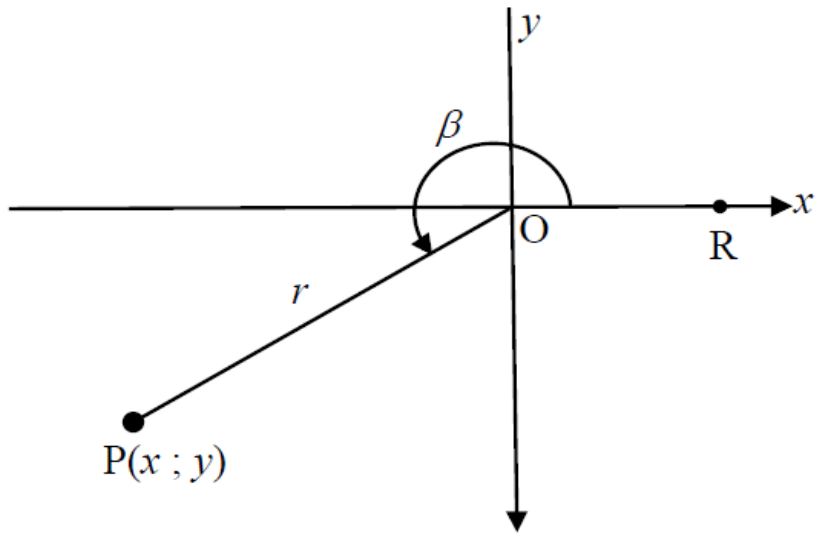
5.2.1 Write down the size of $\hat{JKD}$ (1)

5.2.2 Calculate the distance DK, correct to the nearest metre. (3)

5.2.3 If the distance SK is 8 kilometres, calculate the distance DS. (1)

2015 November Paper 2 Q 5

5.1 In the diagram below, $P(x, y)$ is a point in the third quadrant. $\widehat{R\hat{O}P} = \beta$ and $17 \cos \beta + 15 = 0$.



5.1.1 Write down the values of x , y and r . (4)

5.1.2 WITHOUT using a calculator, determine the value of:

(a) $\sin \beta$ (1)

(b) $\cos^2 30^\circ \tan \beta$ (3)

5.1.3 Calculate the size of $\widehat{R\hat{O}P}$ correct to TWO decimal places. (2)

5.2 In each of the following equations, solve for x where $0^\circ \leq x \leq 90^\circ$.

Give your answers correct to TWO decimal places .

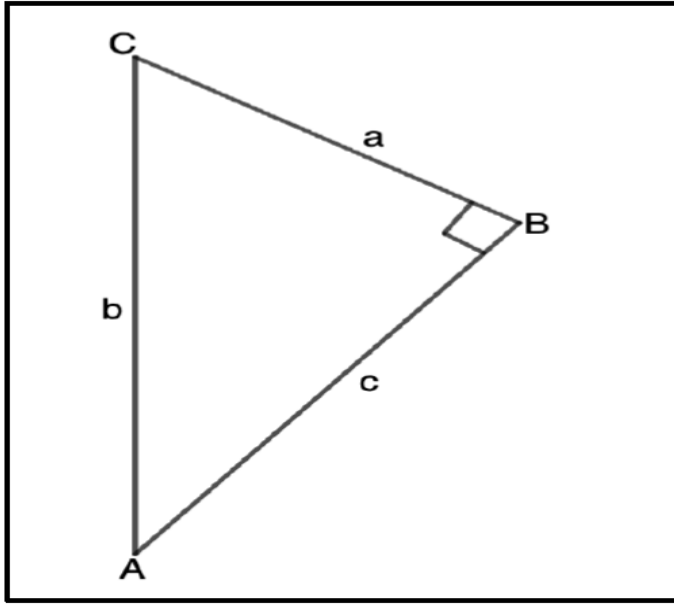
5.2.1 $\tan x = 2,22$ (2)

5.1.2 $\sec(x + 10^\circ) = 5.759$ (3)

5.1.3 $\frac{\sin x}{0,2} - 2 = 1,24$ (3)

2024 North West November Paper 2 Q 4

In the diagram is a right-angled triangle has sides a , b and c and the $\hat{B} = 90^\circ$



4.1 Write down the following in terms of a , b and c .

4.1.1 $\sin A$ (1)

4.1.2 $\cot C$ (1)

4.1.3 If it is given that $a = 5$ and $A = 50^\circ$,
calculate the numerical value of c . (3)

4.2 Given: $13 \cos \theta = 12$, where $0^\circ \leq \theta \leq 90^\circ$. ,**without using a calculator**, use a sketch to determine the following.

4.2.1 $\sin \theta$ (4)

4.2.2 $\tan \theta - \operatorname{cosec}^2 \theta$ (3)

CONTINUED....

4.3 Simplify, **without the use of a calculator.**

$$\operatorname{cosec} 60^{\circ} \cdot \cot 30^{\circ} + \cos 45^{\circ} \cdot \operatorname{cosec} 45^{\circ} \quad (5)$$

4.4 Solve for x , correct to two decimal places:

$$3 + \sec x = 5 \text{ and } 0^{\circ} \leq x \leq 90^{\circ}. \quad (3)$$

2017 November Paper 2 Q 4

4.1 Given $4 \cot \theta + 3 = 0$ and $0^\circ < \theta < 180^\circ$

4.1.1 Use a sketch to determine the value of the following. DO NOT use a calculator.

a) $\cos \theta$ (4)

b) $\frac{3 \sin \theta \sec \theta}{\tan \theta}$ (4)

4.1.2 Hence show that $\sin^2 \theta = 1 - \cos^2 \theta$ (3)

4.2 Simplify the following expression WITHOUT using a calculator.

$\cos 30^\circ \cdot \tan 60^\circ + \operatorname{cosec}^2 45^\circ \sin^2 60^\circ$ (3)

4.2 Solve for θ , correct to TWO decimal places, if:

$\frac{4}{3} \sin \theta = \cos 37^\circ$ and $0^\circ \leq \theta \leq 90^\circ$ (2)

2022 Gauteng November Paper 2 Q 4

4.1 If $x = 66,4^\circ$ and $y = 114,7^\circ$, evaluate the following correct to TWO decimal places.

4.1.1 $\cos(x + y)$ (2)

4.1.2 $2\sin x$ (2)

4.1.3 $\operatorname{cosec} x$ (2)

4.2 Determine the value of θ correct to ONE decimal place.

4.2.1 $\sin \theta + 0,38 = 1$ (2)

4.2.2 $2\cot 2\theta = 3$ (3)

4.2.3 $2\cos(3\theta - 60^\circ) = 1,71$ (4)

2022 Gauteng November Paper 2 Q 5

5.1 Given that $\sin \theta = \frac{4}{5}$ and $\tan \theta < 0$.

Determine with the aid of a diagram and WITHOUT the use of a calculator:

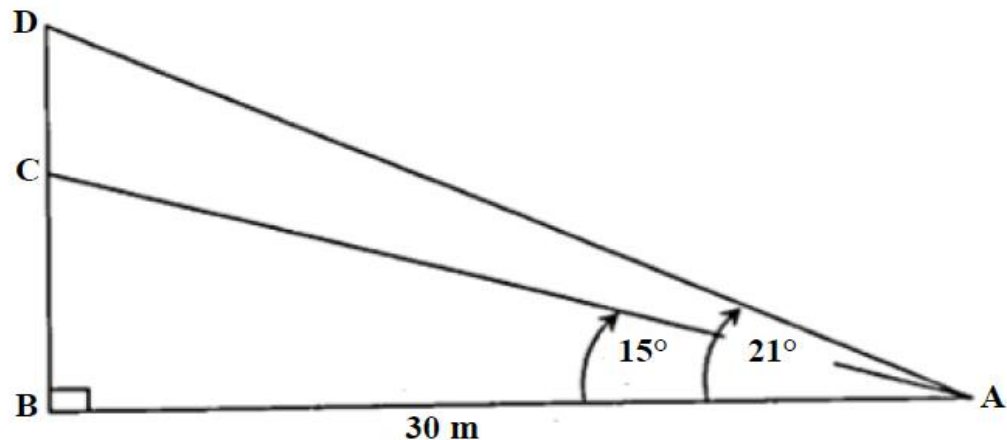
5.1.1 $\tan \theta$ (4)

5.1.2 $2\cos^2 \theta - 1$ (2)

5.2 Simplify the following expression WITHOUT using a calculator.

$\cos 0^\circ + \sin^2 60^\circ + \sqrt{2} \cdot \sec 45^\circ$ (4)

5.3 In the diagram below, a person is standing at point A which is 30 m away from point B. Point A and point B are on the same horizontal plane. The angle of elevation of a first storey window at point C from point A is 15° . The angle of elevation of the second storey window at point D from point A is 21° .

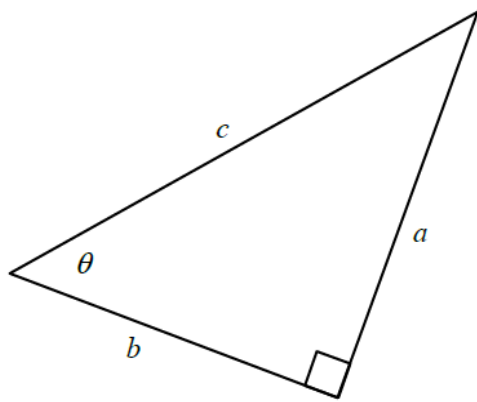


Determine the distance between the two windows.

(6)

2016 November Paper 2 Q 4

4.1 A right-angled triangle has sides a , b and c and the angle θ , as shown below



4.1.1 Write the following in terms of a , b and c :

- a) $\cos\theta$ (1)
- b) $\tan\theta$ (1)
- c) $\sin(90^\circ - \theta)$ (2)

4.1.2 If it is given that $a = 5$ and $A = 50^\circ$, calculate the numerical value of b . (3)

4.2 Given that $\hat{A} = 38,2^\circ$ and $\hat{B} = 146,4^\circ$
Calculate the value of $2\operatorname{cosec}A + \cos 3B$ (3)

CONTINUED....

4.3 Simplify fully, WITHOUT the use of a calculator

$$\frac{\sin 45^\circ \cdot \tan^2 60^\circ}{\cos 45^\circ} \quad (4)$$

4.4 Given that: $5\cos\beta - 3 = 0$ and $0^\circ < \beta < 90^\circ$

If $\alpha + \beta = 90^\circ$ and $0^\circ < \alpha < 90^\circ$, calculate the value of $\cot\alpha$ (4)

