

Grade 12

Mathematics

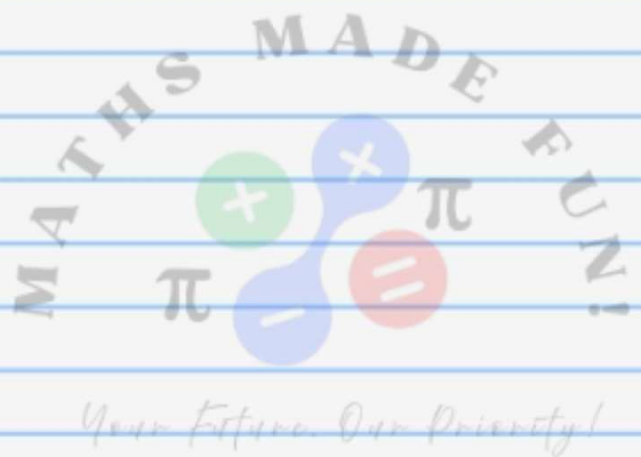


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Financial Maths



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Grade 10/11 Revision

Simple Interest	$A = P(1 + in)$	A is the final (accumulated) amount. P is the original (principal) amount. i is the interest rate per period. n is the number of periods.
Compound Interest or Inflation	$A = P(1 + i)^n$	A is the final (accumulated) amount. P is the original (principal) amount. i is the interest rate per compounding period. n is the number of periods.
Straight Line Depreciation	$A = P(1 - in)$	A is the final (depreciated) value. P is the original value. i is the depreciation rate per period. n is the number of periods.
Reducing Balance Depreciation	$A = P(1 - i)^n$	A is the final (depreciated) value. P is the original value. i is the depreciation rate per period. n is the number of periods.
Effective and Nominal Interest Conversion	$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$	i_{eff} is the effective annual interest rate. i_{nom} is the nominal interest rate (rate p.a. compounded more than once a year). m is the number of times interest is compounded in one year.

Example 1:

① Lwandle deposits R100 000 into a savings account which offers an interest rate of 12% p.a. compounded quarterly. How long will she have to wait to have R180 000 in the account?

quarterly - 4 times
Semi-annually - 2 times
Monthly - 12 times

$$P = R100\ 000$$

$$i = 12\% = 0.12$$

$$n = ?$$

$$A = R180\ 000$$

$$A = P(1+i)^n$$

$$\frac{180\ 000}{100\ 000} = \frac{100\ 000}{100\ 000} \left(1 + \frac{0.12}{4}\right)^n$$

$$\left(1 + \frac{0.12}{4}\right)^n = 1.8$$

$$n = \log_{1.03} 1.8$$

$$n = \frac{19.88533}{4}$$

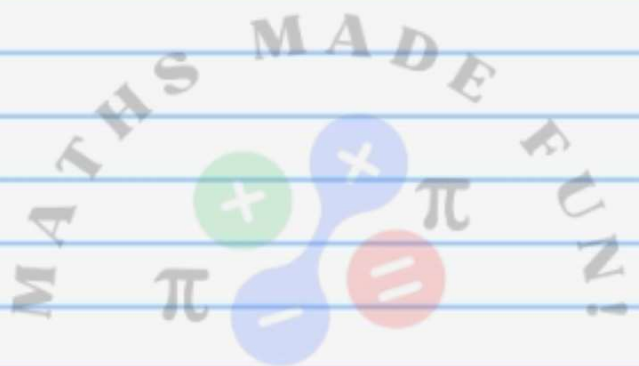
$$n = 4.97 \text{ years}$$



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Example 2:

A manufacturing machine is purchased for R1 000 000. Its value depreciates at 35% p.a. on the reducing balance. The machine will be sold once its value reaches R600 000. How long will the machine be used before it is sold?

$$A = P(1 - i)^n$$

$$i = \frac{35}{100} = 0.35$$

$$\frac{600\ 000}{1\ 000\ 000} = \frac{1\ 000\ 000}{1\ 000\ 000} (1 - 0.35)^n$$

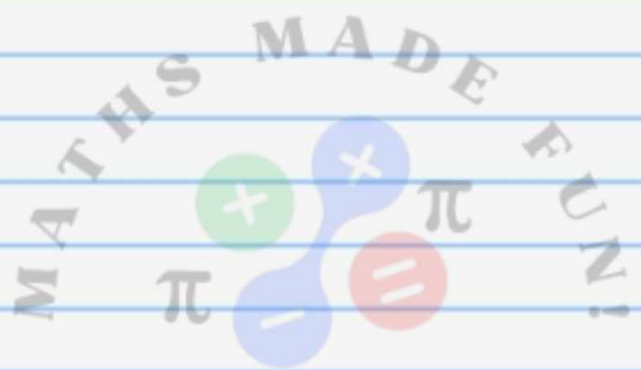
$$0.65^n = 0.6$$

$$n = \log_{0.65} 0.6$$

$$n = 1.1858$$

$$(0.1858 \times 12 = 2.2)$$

$\therefore$ Approx 1 year 2 months



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Annuities

► An Annuity is a SERIES of Equal payments made at regular time intervals.

Examples :

- ① Loan Repayments.
- ② Retirement Annuities
- ③ Regular Savings
- ④ Bond payments.

The Future Value Formula

$$F = \frac{x[(1+i)^n - 1]}{i}$$

F - future Value

x - Regular Payments

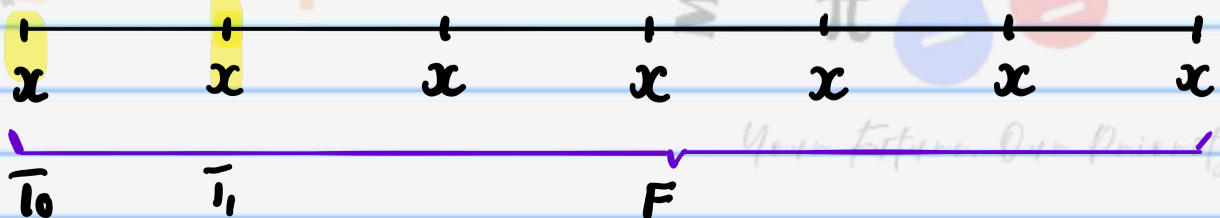
i - Interest rate per interval

n - number of payments.

► Future value formula is used when (x-payments) are made upfront and the total (future value) amount is collected later.

i.e. - Savings for University Tuitions.
- Retirement fund

ON A TIMELINE:



Example 3:

Griffen wants to move out of his parents' house when he is done with school. He decides to save R2 500 per quarter for the next two years, starting immediately and ending at the end of the two years. How much will he have saved up by the end of the 2 years, if interest is calculated at 9% p.a. compounded quarterly?

$$x = R2\,500$$

$$n = 2 \text{ years}$$

$$F = ?$$

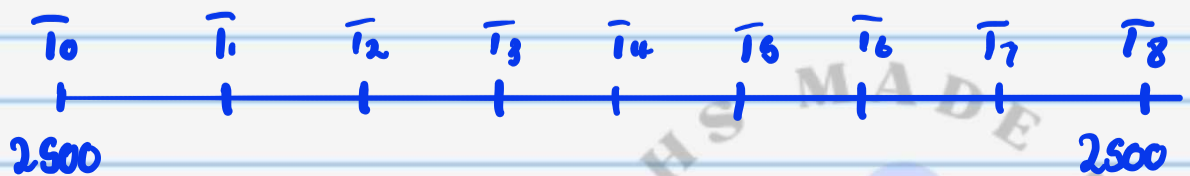
$$i = 9\% = 0.09$$

$$F = \frac{x [(1+i)^n - 1]}{i}$$

$$= \frac{2500 \left[\left(1 + \frac{0.09}{4}\right)^9 - 1 \right]}{\left(\frac{0.09}{4}\right)}$$

$$= R24\,634.98 \rightarrow$$

$$\frac{2500 \left[\left(\frac{409}{400}\right)^9 - 1 \right]}{0.0225}$$



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Example 4:

Katlego opens a savings account to save for an engagement ring for his girlfriend. One month after opening the account, he deposits R400 and continues to deposit R400 at the end of each month thereafter. Interest is calculated at 11,5% p.a. compounded monthly. How much will he have in the account 3 years after it was opened?

$$x = R400$$

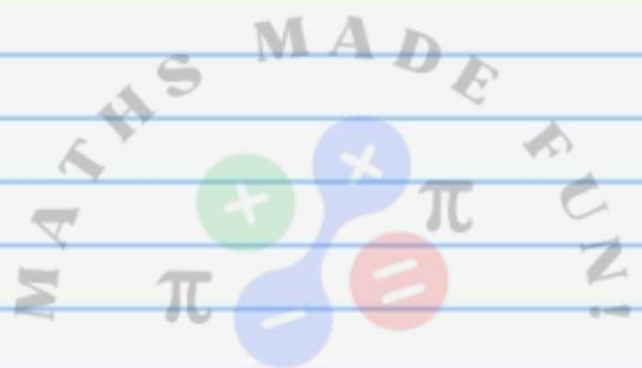
$$i = 11.5\% = 0.115$$

$$n = 3 \text{ years}$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$= \frac{R400 \left[\left(1 + \frac{0.115}{12} \right)^{36} - 1 \right]}{\left(\frac{0.115}{12} \right)}$$

$$= \underline{R17099.37} \rightarrow$$



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The PRESENT VALUE Formula:

$$P = \frac{x [1 - (1+i)^{-n}]}{i}$$

P - Present value

x - Regular Payments

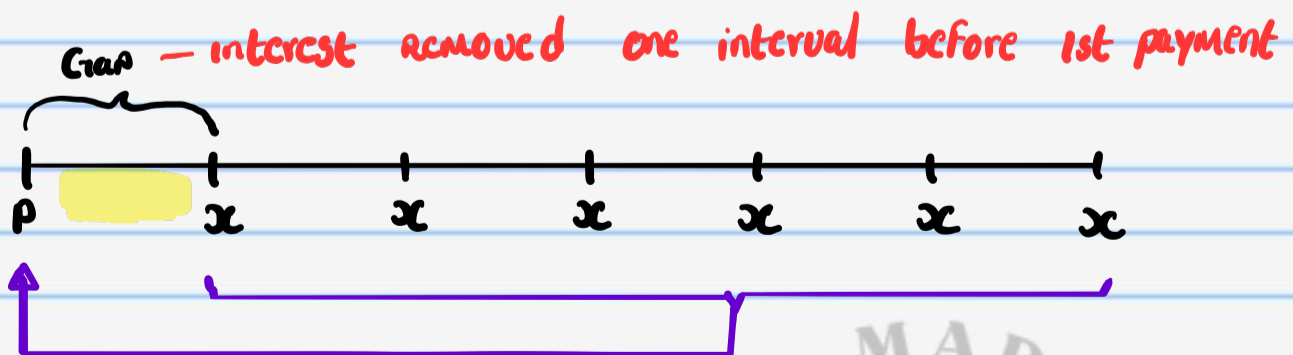
i - interest rate per interval

n - number of payments.

▷ The present value formula is used whenever the total amount (Present value) is received and payments are made later.

i.e. - Loans (Home / car loans)

- Money set aside to make regular withdrawals in future. (Lottery Payout)



Example 5:

Donald wants to buy a house worth R1,2 million. His bank offers him a home loan at an interest rate of 11,5% p.a. compounded monthly. He will pay off the loan with monthly payments over a period of 20 years. He will make his first payment a month after taking out the loan. Calculate

(1) Donald's monthly payment.

(2) the total interest on the loan.

$$P = R1.2 \text{ M}$$

$$i = 11.5\% = 0.115$$

$$n = 20 \text{ years} \times 12 = 240$$

$$P = \frac{x [1 - (1+i)^{-n}]}{i}$$

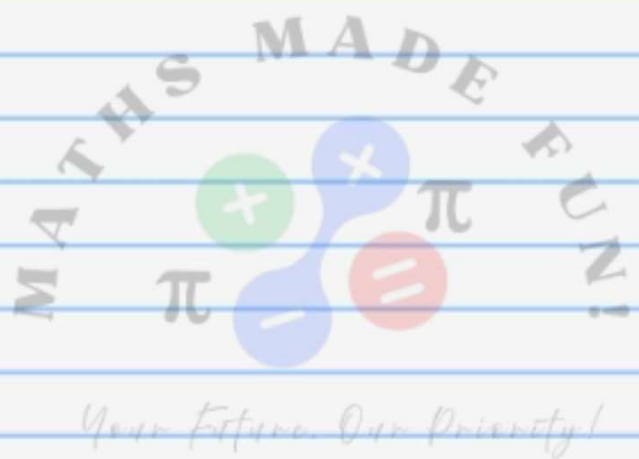
$$R 1200\ 000 = \frac{x [1 - (1 + \frac{0.115}{12})^{-240}]}{(\frac{0.115}{12})}$$

$$\frac{11500}{0.898637} = \frac{x (0.898637)}{0.898637}$$

$$x = \underline{R12\ 797.16} \rightarrow$$

$$\textcircled{6} \quad \begin{array}{r} R12\ 797.16 \times 240 \\ = R3\ 071\ 317.339 \end{array} - 1\ 200\ 000$$

$$\therefore \text{Total interest} = \underline{R1\ 871\ 317.34}$$



EXAMPLE 6:

Otto repays a loan over a period of 5 years, by means of equal semi-annual payments of R15 000, starting 6 months after the loan was granted. What is the value of the loan, if the interest rate is 14,8% p.a. compounded semi-annually?

$$n = 5 \text{ years} \times 2$$

$$x = R15\,000$$

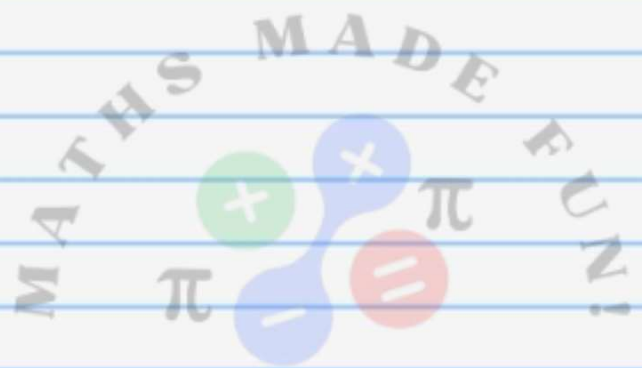
$$p = ?$$

$$i = 14.8\% = 0.148$$

$$p = \frac{x [1 - (1 + i)^{-n}]}{i}$$

$$= \frac{15000 [1 - (1 + \frac{0.148}{2})^{-10}]}{(\frac{0.148}{2})}$$

$$= \underline{R103433} \rightarrow$$



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Outstanding Balance On A Loan

Outstanding Balance = Loan with interest to date — Repayments with interest to date
(Use compound Interest) (use FV formula)

$$OB = P(1+i)^n - \frac{x[(1+i)^n - 1]}{i}$$

↪ For normal loans where first x payment is made after the first interval

Example 7:

Thabo pays off a loan of R80 000 over a period of 5 years. He makes half-yearly payments, starting 6 months after the loan was granted and ending at the end of the 5 year period. The interest rate is 18% p.a. compounded semi-annually.

- (1) What is his half-yearly payment?
- (2) How much does he owe immediately after the 10th payment?

① $P = 80\,000$
 $n = 5 \text{ years}$
 $i = 18\% = 0.18$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$R80\,000 = \frac{x[1 - (1 + \frac{0.18}{2})^{-10}]}{(\frac{0.18}{2})}$$

$$\frac{7200}{0.577589} = \frac{x(0.577589)}{0.577589}$$

$$\therefore x = \underline{R\ 12\ 465.61} \rightarrow$$

② No payment will be made

$$0 = P(1+i)^n - \frac{x[(1+i)^n - 1]}{i}$$

$$= 80\ 000 \left(1 + \frac{0.12}{2}\right)^{10} - \frac{12\ 465.61 [(1 + \frac{0.12}{2})^{10} - 1]}{(\frac{0.12}{2})}$$

$$= R189389 - R189389$$

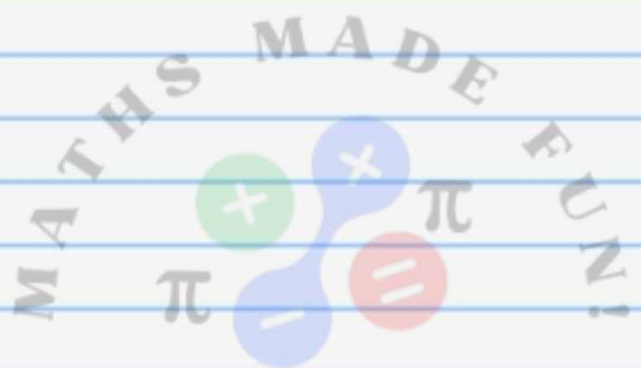
$$= \underline{0} \rightarrow$$



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The Last Payment

$$\text{Last payment} = \text{Outstanding Balance} \times (1+i)$$

After the last full payment.

EXAMPLE 8:

Alicia wants to pay off a loan of R450 000. She can afford to pay back R11 000 per month. Interest is calculated at 12,8% p.a. compounded monthly.

- (1) How many payments will Alicia have to make?
- (2) What is the outstanding balance on the loan immediately after the last full payment?
- (3) Determine the value of the last payment, made one month after the last full payment.
- (4) How much interest will Alicia pay on this loan?

$$P = \frac{x [1 - (1+i)^{-n}]}{i}$$

$$450\,000 = \frac{11\,000 [1 - (1 + \frac{0.128}{12})^{-n}]}{(\frac{0.128}{12})}$$

$$\frac{4800}{11000} = 1 - \left(\frac{379}{375}\right)^{-n}$$

$$-\frac{31}{85} = -\left(\frac{379}{375}\right)^{-n}$$

$$\left(\frac{31}{85}\right) = \left(\frac{379}{375}\right)^{-n}$$

$$-n = \log_{\left(\frac{379}{375}\right)} \left(\frac{31}{85}\right)$$

$$-n = -54.03735174$$

$$n = 54.03735174$$

∴ Alicia will make 55 payments in total

- 54 payments of R11000 and 1 lesser payment

$$\textcircled{2} \quad OB = P(1+i)^n - \frac{x[(1+i)^n - 1]}{i}$$

$$= 480\,000 \left(1 + \frac{0.128}{12}\right)^{54} - \frac{11\,000 \left[\left(1 + \frac{0.128}{12}\right)^{54} - 1\right]}{\left(\frac{0.128}{12}\right)}$$

$$= R798070.7517 - R797662.1393$$

$$= R408.61$$

$$\textcircled{3} \quad \text{Last Payment} = OB \times (1+i)$$

$$= 408.61 \times \left(1 + \frac{0.128}{12}\right)$$

$$= R412.97 \rightarrow$$

$$\textcircled{4} \quad R11\,000 \times 54 + R412.97$$

$$R594\,412.97 - R480\,000$$

$$\text{Total interest} = R114\,412.97 \rightarrow$$

Sinking Fund

- ▷ A provision made to replace equipment in the future.
- ▷ Regular payments are made to save up for the replacement of equipment.

Steps to set up a sinking fund:

- ① Calculate what the old machine will be sold for in future. (Use depreciation)
- ② Calculate what new equipment will cost in the future. (Use inflation)
- ③ The difference between these two values (inflated - depreciated) is the future value that has to be saved up for the replacement.

Example 9:

Cleaning equipment is bought for R120 000. The value of the equipment depreciates at 15% p.a. on the reducing balance. The inflation rate is 9% p.a. In 5 years' time, the old cleaning equipment will be sold at scrap value. The proceeds of the sale, together with money saved up in a sinking fund, will be used to purchase replacement cleaning equipment.

- (1) Calculate the scrap value of the old equipment after 5 years.
- (2) What will the new equipment cost in 5 years' time?
- (3) What amount should the company budget for in 5 years' time?
- (4) Calculate the monthly payment to be paid into a sinking fund, paying 12,5% p.a. compounded monthly, in order to have enough money to replace the equipment in 5 years' time? (The first payment is made immediately, and the last payment is made at the end of the 5 years.)

$$\begin{aligned} P &= 120\,000 \\ i &= 0.15 \\ n &= 5 \end{aligned}$$

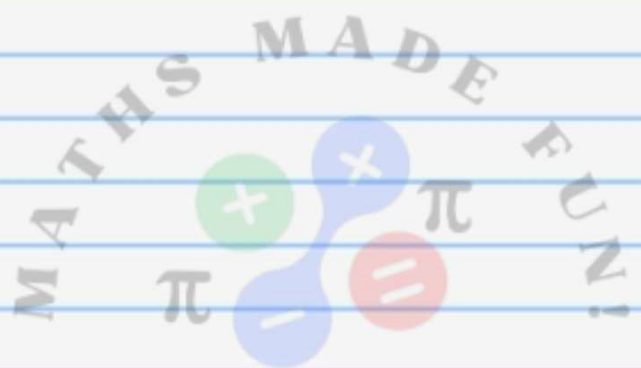
$$\begin{aligned} A &= P(1-i)^n \\ &= 120\,000(1-0.15)^5 \\ &= R53\,244.6375 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad A &= P(1+i)^n \\
 &= 120\,000 (1 + 0.01)^5 \\
 &= \underline{R184\,634.8146} \rightarrow
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad \text{Difference} &= \text{Inflated} - \text{Depreciated} \\
 &= R184\,634.8146 - R63\,244.6375 \\
 &= \underline{R131\,390.2371} \rightarrow
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad F &= \frac{x [(1+i)^n - 1]}{i} \\
 131\,390.2371 &= \frac{x \left[\left(1 + \frac{0.125}{12}\right)^{61} - 1 \right]}{\left(\frac{0.125}{12}\right)}
 \end{aligned}$$

$$x = \underline{R1552.43465} \rightarrow$$



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Homework

QUESTION 6

- 6.1 R12 000 was invested in a fund that paid interest at $m\%$ p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m .

(4)

- 6.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation.

(4)

- 6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

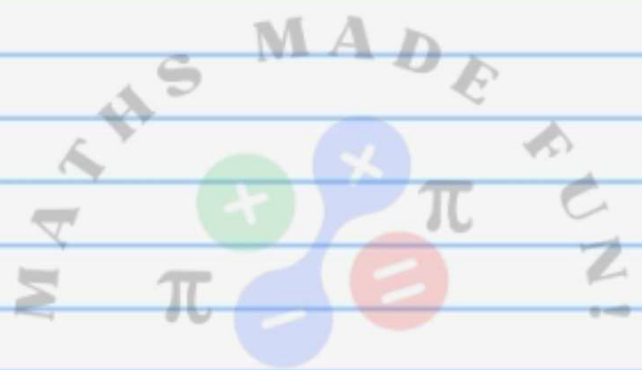
6.3.1 Calculate the value of the loan.

(1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment.

(5)

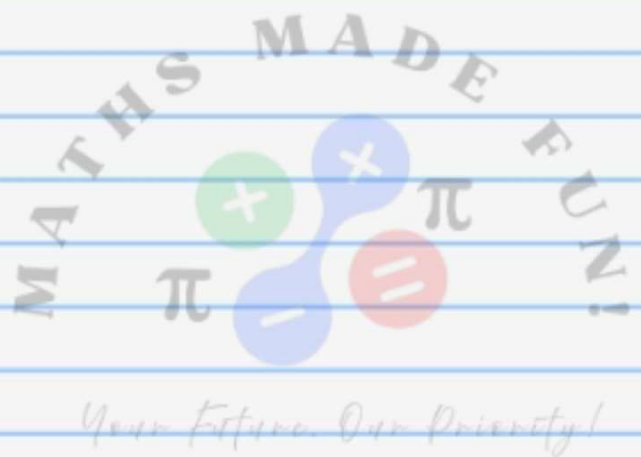
[14]



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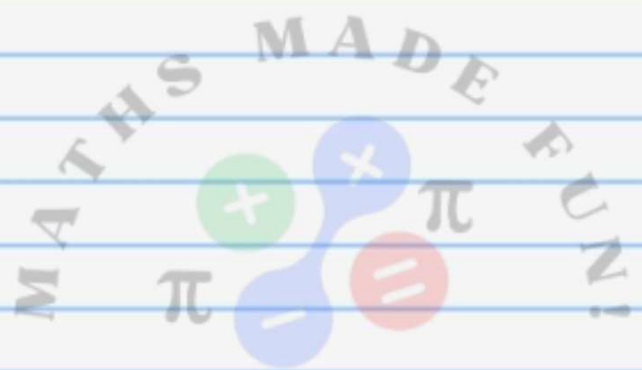
QUESTION 6

- 6.1 Patrick deposited an amount of R18 500 into an account earning $r\%$ interest p.a., compounded monthly. After 6 months, his balance was R19 319,48.
- 6.1.1 Calculate the value of r . (3)
- 6.1.2 Calculate the effective interest rate. (2)
- 6.2 Kuda bought a laptop for R10 000 on 31 January 2019. He will replace it with a new one in 5 years' time on 31 January 2024.
- 6.2.1 The value of the old laptop depreciates annually at a rate of 20% p.a. according to the straight-line method. After how many years will the laptop have a value of R0? (2)
- 6.2.2 Kuda will buy a laptop that costs R20 000. In order to cover the cost price, he made his first monthly deposit into a savings account on 28 February 2019. He will make his 60th monthly deposit on 31 January 2024. The savings account pays interest at 8,7% p.a., compounded monthly. Calculate Kuda's monthly deposit into this account. (4)
- 6.3 Tino wins a jackpot of R1 600 000. He invests all of his winnings in a fund that earns interest of 11,2% p.a., compounded monthly. He withdraws R20 000 from the fund at the end of each month. His first withdrawal is exactly 1 month after his initial investment. How many withdrawals of R20 000 will Tino be able to make from this fund? (5)
- [16]



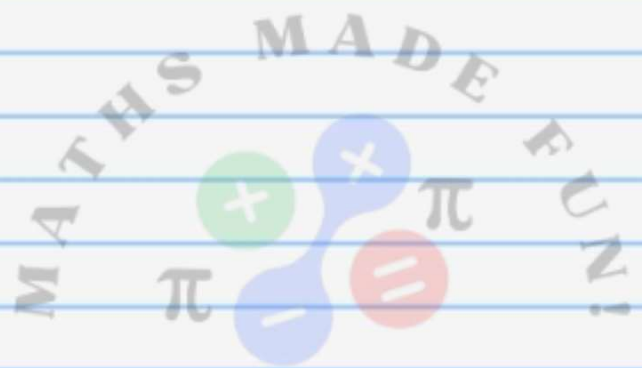
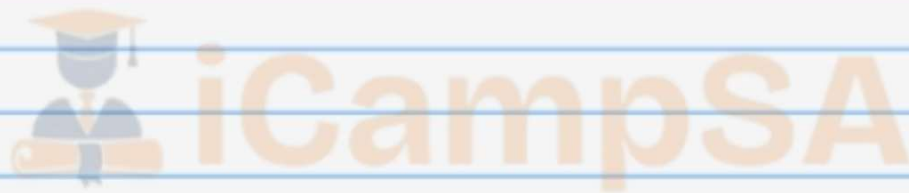
QUESTION 7

- 7.1 Mary's grandparents deposited R5 000 into a savings account on the day that she was born. The account pays interest at a rate of 6,8% p.a., compounded quarterly. Calculate the accumulated amount in this account on Mary's 16th birthday. (3)
- 7.2 After 4 years, the value of a printer was half of its original value. Determine the rate at which the value of the printer depreciated over this period, if depreciation was calculated according to a straight-line method. (2)
- 7.3 Tshepo was granted a loan of R100 000 on 1 March 2022 at an interest rate of 13,5% p.a., compounded monthly. Tshepo agreed to repay the loan over 5 years in monthly instalments of R2 300,98, starting on 1 April 2022.
- 7.3.1 Calculate the total interest that he will pay over the 5 years. (2)
- 7.3.2 Tshepo paid R22 300,98 (his monthly instalment and an additional R20 000) on 1 March 2024 into the loan account. He continues to pay the original monthly instalment thereafter. How many months earlier will Tshepo repay the loan? (7)
- [14]



QUESTION 7

- 7.1 A travel company informs Robert that a holiday to a certain destination costs R40 000 now. The company predicts that the cost of this holiday will increase by 7,8% per annum. What will this holiday cost in 5 years' time? (2)
- 7.2 Sarah opened a savings account that paid interest at a rate of 5,8% p.a., compounded quarterly. She deposited R2 300 into the account on 1 January 2020 and continued to make deposits of R2 300 at the beginning of each quarter thereafter. She made her last deposit on 1 October 2025. Calculate the accumulated amount in the account on 1 January 2026. (4)
- 7.3 The bank granted Rajesh a loan of R900 000 on 28 February 2024 at an interest rate of 6,8% p.a., compounded monthly.
- 7.3.1 Rajesh was unable to make the first three payments. He made his first repayment of R10 000 on 30 June 2024. He continued to make monthly repayments of R10 000 at the end of each month thereafter. How long, in completed months, will it take Rajesh to repay the loan from the time the loan was granted? (5)
- 7.3.2 Calculate the value of the final payment. (4)
- [15]



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