

Exponents, equations and inequalities Memo

Examples 1 and 2 to do in class

Example 3

Solutions

$$\begin{aligned} \text{(a)} \quad & \frac{16^2 \cdot (3^2)^4}{2^7 \cdot 2^5 \cdot 81} \\ &= \frac{(2^4)^2 \cdot 3^8}{2^{12} \cdot 3^4} \\ &= \frac{2^8 \cdot 3^8}{2^{12} \cdot 3^4} \\ &= \frac{3^4}{2^4} = \frac{81}{16} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{(2^x)^4 \cdot 9^x}{12^{2x}} \\ &= \frac{2^{4x} \cdot (3^2)^x}{(2^2 \cdot 3)^{2x}} \\ &= \frac{2^{4x} \cdot 3^{2x}}{2^{4x} \cdot 3^{2x}} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \frac{16^{x-1}}{2^{4x} \cdot 32} \\ &= \frac{(2^4)^{x-1}}{2^{4x} \cdot 2^5} \\ &= \frac{2^{4x-4}}{2^{4x+5}} \\ &= 2^{4x-4-(4x+5)} \\ &= 2^{4x-4-4x-5} \\ &= 2^{-9} \end{aligned}$$

Example 4

Solution

$$\frac{(4x^3)^{\frac{1}{2}} \cdot x^{-\frac{1}{2}}}{x^{-3}}$$

$$= \frac{(2^2 x^3)^{\frac{1}{2}} \cdot x^{-\frac{1}{2}}}{x^{-3}}$$

[write 4 to the base 2]

$$= \frac{2x^{\frac{3}{2}} \cdot x^{-\frac{1}{2}}}{x^{-3}}$$

[multiply exponents]

$$= \frac{2x^{\frac{3}{2}-\frac{1}{2}}}{x^{-3}} = \frac{2x^1}{x^{-3}} = 2x^1 \cdot x^3 = 2x^4$$

[apply exponential laws and simplify]

Example 5

$$(a) \quad \frac{3^{x+1} + 2 \cdot 3^{x+2}}{7 \cdot 3^x}$$
$$= \frac{3^x \cdot 3^1 + 2 \cdot 3^x \cdot 3^2}{7 \cdot 3^x}$$

[apply the rule $a^{m+n} = a^m \cdot a^n$]

$$= \frac{3^x (3^1 + 2 \cdot 3^2)}{7 \cdot 3^x}$$

[factorise numerator]

$$= \frac{3^x (3 + 2 \cdot 9)}{7 \cdot 3^x}$$

$$= \frac{3^x (21)}{7 \cdot 3^x}$$

$$= 3$$

[simplify]

$$(b) \quad \frac{9^x + 3^x - 2}{9^x - 4}$$

$$= \frac{(3^2)^x + 3^x - 2}{(3^2)^x - 4}$$

[write 9^x to base 3]

$$= \frac{(3^x)^2 + 3^x - 2}{(3^x)^2 - 4}$$

[apply the rule $(a^m)^n = (a^n)^m$]

$$= \frac{(3^x + 2)(3^x - 1)}{(3^x + 2)(3^x - 2)}$$

[factorise the numerator and denominator]

$$= \frac{(3^x - 1)}{(3^x - 2)}$$

[simplify]

Example 6

- (a) $4 \cdot 25^{x+3} = 4$
 $\therefore 25^{x+3} = 1$ [divide both sides by 4]
 $\therefore (5^2)^{x+3} = 5^0$ [write 25 to base 5 and 1 as 5^0]
 $\therefore 5^{2x+6} = 5^0$ [multiply exponents]
 $\therefore 2x+6 = 0$ [equate exponents]
 $\therefore 2x = -6$
 $\therefore x = -3$
- (b) $(0,5)^{x-1} = \left(\frac{1}{4}\right)^x$
 $\therefore \left(\frac{1}{2}\right)^{x-1} = \left(\frac{1}{2^2}\right)^x$ [write 0,5 as a fraction and 4 as 2^2]
 $\therefore (2^{-1})^{x-1} = (2^{-2})^x$ [apply the definition $\frac{1}{a^n} = a^{-n}$]
 $\therefore 2^{-x+1} = 2^{-2x}$ [multiply exponents]
 $\therefore -x+1 = -2x$ [equate exponents]
 $\therefore x = -1$ [solve]
- (c) $9^x + 3^{2x+1} = 36$
 $\therefore (3^2)^x + 3^{2x} \cdot 3 = 36$ [write 9 to base 3]
 $\therefore 3^{2x} + 3^{2x} \cdot 3 = 36$ [apply the rule $a^{m+n} = a^m \cdot a^n$]
 $\therefore 3^{2x}(1+3) = 36$ [factorise]
 $\therefore 3^{2x}(4) = 36$ [simplify]
 $\therefore 3^{2x} = 9$ [divide both sides by 4]
 $\therefore 3^{2x} = 3^2$ [write 9 to base 3]
 $\therefore 2x = 2$ [equate exponents]
 $\therefore x = 1$ [solve]

Example 7

Solutions

(a) $x^{\frac{1}{3}} = 4$

$$\therefore (x^{\frac{1}{3}})^3 = 4^3$$

[raise both sides to the reciprocal of the exponent]

$$\therefore x = 81$$

(b) $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$

$$\therefore (x^{\frac{1}{4}})^2 - 5x^{\frac{1}{4}} + 6 = 0$$

[change the expression $x^{\frac{1}{2}}$ to $(x^{\frac{1}{4}})^2$]

$$\therefore (x^{\frac{1}{4}} - 3)(x^{\frac{1}{4}} - 2) = 0$$

[factorise the trinomial]

$$\therefore x^{\frac{1}{4}} = 3 \quad \text{or} \quad x^{\frac{1}{4}} = 2$$

$$\therefore (x^{\frac{1}{4}})^4 = 3^4 \quad \text{or} \quad (x^{\frac{1}{4}})^4 = 2^4$$

$$\therefore x = 81 \quad \text{or} \quad x = 16$$

PAST EXAM PAPERS

2022 Gauteng November Paper 1 Q 1.2.2 and Q 1.2.3

1.2.2	$x^{-3} \div x^{-5} + (x^{-2})^{-1}$ $= x^{-3-(-5)} + x^2$ $= x^2 + x^2$ $= 2x^2$	✓ division x^{-3+5} and brackets/ <i>deel x^{-3+5} en hakies</i> ✓ simplify/vereenvoudig	(2)
1.2.3	$\frac{5^{x+1} \cdot (4^x)^3}{2^{5x-2} \cdot 10^{x-1}}$ $= \frac{5^{x+1} \cdot (2^{2x})^3}{2^{5x-2} \cdot (5 \cdot 2)^{x-1}}$ $= \frac{5^{x+1} \cdot 2^{6x}}{2^{5x-2} \cdot 5^{x-1} \cdot 2^{x-1}}$ $= 5^{x+1-x+1} \cdot 2^{6x-5x+2-x+1}$ $= 5^2 \cdot 2^3$ $= 200$	✓ bases primefactors/ <i>basisse priemfaktore</i> ✓ simplify numerator/ <i>vereenvoudig teller</i> ✓ simplify denominator/ <i>vereenvoudig noemer</i> ✓ final answer/ <i>finale antwoord</i>	(4)

2025 KwaZulu Natal March Paper Q 2.1.3-2.1.4

2.1.3	$3^{5x-4} - 729 = 0$ $3^{5x-4} = 729$ $3^{5x-4} = 3^6$ $5x - 4 = 6$ $5x = 10$ $x = 2$	✓ A prime base ✓ CA simplification ✓ CA answer	(3)
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2.1.2	$5^x = \frac{1}{125}$ $5^x = \frac{1}{5^3}$ $5^x = 5^{-3}$ $x = -3$	✓ 5^{-3} ✓ answer/antwoord	(2)
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2019 November Paper 1 Q 1.2.2

1.2.3	$\frac{3^{x+1} + 3^x}{27 \cdot 3^{-1+x}}$ $= \frac{3^x \cdot 3 + 3^x}{27 \cdot 3^{-1} \cdot 3^x}$ $= \frac{3^x(3+1)}{3^x \cdot \frac{27}{3}}$ $= \frac{4}{9}$ OR/OF $\frac{3^{x+1} + 3^x}{27 \cdot 3^{-1+x}}$ $= \frac{3^x \cdot 3 + 3^x}{27 \cdot 3^{-1} \cdot 3^x}$ $= \frac{4 \cdot 3^x}{3^x \cdot \frac{27}{3}}$ $= \frac{4}{9}$	✓ separating bases/skei basisse ✓ common factor/ gemeenskaplike faktor ✓ answer/antwoord (3)
	$\frac{3^{x+1} + 3^x}{27 \cdot 3^{-1+x}}$ $= \frac{3^x \cdot 3 + 3^x}{27 \cdot 3^{-1} \cdot 3^x}$ $= \frac{4 \cdot 3^x}{3^x \cdot \frac{27}{3}}$ $= \frac{4}{9}$	✓ separating bases/skei basis ✓ addition/optelling ✓ answer/antwoord (3)

2017 November Paper 1 Q 1.3.3

1.3.3	$\frac{3^t - 3^{t-2}}{2 \cdot 3^t - 3^t}$ $= \frac{3^t(1 - 3^{-2})}{3^t(2 - 1)}$ $= \frac{1 - \frac{1}{9}}{1}$ $= \frac{8}{9}$	✓ factors/faktore ✓ simpl./vereenv ✓ answ/antw	(3)
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2025 Mpumalanga March Paper Q 2.2

2.2	$\frac{4^x - 3 \cdot 2^x - 18}{2^x + 3} = \frac{2^{2x} - 3 \cdot 2^x - 18}{2^x + 3}$ $= \frac{(2^x + 3)(2^x - 6)}{2^x + 3}$ $= 2^x - 6$	✓ 2^{2x} ✓✓ factors ✓ answer	(4)
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2024 Free State June Paper 1 Q 2.3 and Q 3.1.4

2.2.3	$\frac{3 \cdot 2^m - 4 \cdot 2^m \cdot 2^{-2}}{2^m - 2^m \cdot 2^{-1}}$ $= \frac{2^m \left(3 - 4 \cdot \frac{1}{4} \right)}{2^m \left(1 - \frac{1}{2} \right)}$ $= \frac{2}{1}$ $= 2$	✓ common factor ✓ simplification ✓ answer	(3)
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3.1.4	$2^x \cdot 2^{x+1} = 32$ $2^{x+x+1} = 2^5$ $2x+1 = 5$ $x = 2$	✓ base 2 ✓ equating exponents ✓ answer	(3)
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2012 Exemplar Paper 1 Q 2.1.2

2.1.2	$96 = 3x^{\frac{5}{4}}$ $32 = x^{\frac{5}{4}}$ $x = (32)^{\frac{4}{5}}$ $= (2^5)^{\frac{4}{5}}$ $= 2^4$ $= 16$	✓ $32 = x^{\frac{5}{4}}$ ✓ $x = (32)^{\frac{4}{5}}$ ✓ answer	(3)
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2018 Gauteng June Paper 1 Q 2.1

2.1	2.1.1	$\frac{(3x)^2(-2xy)^3}{2x^5y^4}$ $= \frac{9x^2 \cdot -8x^3y^3}{2x^5y^4}$ $= \frac{-72x^5y^3}{2x^5y^4}$ $= -36x^{5-5}y^{3-4}$ $= -36y^{-1}$ $= -\frac{36}{y}$	$\checkmark 9x^2 \cdot -8x^3y^3$ $\checkmark -36y^{-1}$ $\checkmark -\frac{36}{y}$ (3)
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2025 KwaZulu Natal November Paper 1 Q 1.1.3 and Q 2.1.1

1.1.3	$= \frac{2^{3x} \cdot 3^{3x}}{2^{3x} \cdot 3^{2x} \cdot 3^x}$ $= 2^0 \times 3^0$ $= 1$	$\checkmark$ A prime bases $\checkmark$ CA simplification $\checkmark$ CA answer (3)
2.1.1	$2^{3x-4} = 2^{-9}$ $3x - 4 = -9$ $x = -\frac{5}{3}$ 	$\checkmark$ A 2^{-9} $\checkmark$ CA equating exp. $\checkmark$ CA answer (3)

Example 8

(a) $3 - 4(k + 2) = 3k + 16$
 $\therefore 3 - 4k - 8 = 3k + 16$ [use the distributive law]
 $\therefore -4k - 5 = 3k + 16$ [collect like terms]
 $\therefore -4k - 3k = 16 + 5$ [add 5 and subtract $3k$ from both sides]
 $\therefore -7k = 21$ [collect like terms]
 $\therefore k = -3$ [divide both sides by -7]
To verify that this value is the solution, substitute into the left and right sides:
Left side $= 3 - 4((-3) + 2) = 7$
Right side $= 3 - 4((-3) + 2) = 7$
 $\therefore$ Left side = Right side
 $\therefore k = -3$ is the solution.

(b) $\frac{4x-1}{3} - \frac{7x+2}{6} = \frac{x}{2}$

The equation contains fractions. Insert brackets around the numerators if they are expressions of more than a single term. Determine the lowest common multiple of the denominators (LCD).

$$\frac{(4x-1)}{3} - \frac{(7x+2)}{6} = \frac{x}{2} \quad \text{LCD} = 6$$

In Grade 9 two approaches were discussed. Method 1 is the preferred approach.

Method 1

Multiply both sides by the LCD

$$\begin{aligned} \frac{(4x-1)}{3} \times 6 - \frac{(7x+2)}{6} \times 6 &= \frac{x}{2} \times 6 \\ \therefore \frac{6(4x-1)}{3} - \frac{6(7x+2)}{6} &= \frac{6x}{2} \\ \therefore 2(4x-1) - (7x+2) &= 3x \\ \therefore 8x - 2 - 7x - 2 &= 3x \\ \therefore x - 4 &= 3x \\ \therefore x - 3x &= 4 \\ \therefore -2x &= 4 \\ \therefore x &= -2 \end{aligned}$$

Method 2

Write every term as a fraction with the common denominator

$$\begin{aligned} \frac{(4x-1)}{3} \times \frac{2}{2} - \frac{(7x+2)}{6} \times \frac{1}{1} &= \frac{x}{2} \times \frac{3}{3} \\ \therefore \frac{2(4x-1)}{6} - \frac{(7x+2)}{6} &= \frac{3x}{6} \\ \therefore 2(4x-1) - (7x+2) &= 3x \\ \therefore 8x - 2 - 7x - 2 &= 3x \\ \therefore x - 4 &= 3x \\ \therefore x - 3x &= 4 \\ \therefore -2x &= 4 \\ \therefore x &= -2 \end{aligned}$$

Example 9

Solutions

(a) $x^2 - 5x - 14 = 0$
 $\therefore (x-7)(x+2) = 0$ [factorise]
 $\therefore x-7 = 0$ or $x+2 = 0$ [apply the zero-factor law]
 $\therefore x = 7$ or $x = -2$

(b) $5x + 2x^2 = 3$
 $\therefore 2x^2 + 5x - 3 = 0$ [standard form]
 $\therefore (2x-1)(x+3) = 0$ [factorise]
 $\therefore 2x-1 = 0$ or $x+3 = 0$ [apply the zero-factor law]
 $\therefore 2x = 1$
 $\therefore x = \frac{1}{2}$ or $x = -3$

(c) $2x^2 = 4x$ Alternately:
 $\therefore 2x^2 - 4x = 0$ [standard form] First divide by 2:
 $\therefore 2x(x-2) = 0$ [factorise] $\therefore x^2 = 2x$
 $\therefore 2x = 0$ or $x-2 = 0$ $\therefore x^2 - 2x = 0$
 $\therefore x = 0$ or $x = 2$ [apply the zero-factor law] $\therefore x(x-2) = 0$
 $\therefore x = 0$ or $x = 2$

However, please take note that you may **never divide both sides by the variable you are solving for**. The reason for this is that you will lose one of the solutions.

(d) $8x - 16 - x^2 = 0$ (e) $(3x-1)(x+2) = 0$
 $\therefore 0 = x^2 - 8x + 16$ [standard form] $\therefore 3x-1 = 0$ or $x+2 = 0$
 $\therefore (x-4)(x-4) = 0$ [factorise] $\therefore 3x = 1$
 $\therefore x-4 = 0$ [no need to write both] $\therefore x = \frac{1}{3}$ or $x = -2$
 $\therefore x = 4$

(f) $(3x-1)(x+2) = 10$ The right side is not zero so proceed as follows:
 $\therefore 3x^2 + 6x - x - 2 = 10$ [expand the product]
 $\therefore 3x^2 + 5x - 12 = 0$ [standard form]
 $\therefore (3x-4)(x+3) = 0$ [factorise]
 $\therefore 3x = 4$ [apply the zero-factor law]
 $\therefore x = \frac{4}{3}$ or $x = -3$

Example 10

(a) $x^2 = 36$
 $\therefore x^2 - 36 = 0$ [standard form]
 $\therefore (x-6)(x+6) = 0$ [factorise]
 $\therefore x-6=0$ or $x+6=0$
 $\therefore x=6$ or $x=-6$

Alternatively:
 $x^2 = 36$
 $\sqrt{x^2} = \pm\sqrt{36}$
 $\therefore x = \pm 6$
 $\therefore x=6$ or $x=-6$

(b) $3x^2 = 48$
 $\therefore x^2 = 16$ [divide by 3]
 $\therefore x^2 - 16 = 0$ [standard form]
 $\therefore (x-4)(x+4) = 0$ [factorise]
 $\therefore x=4$ or $x=-4$

Alternatively:
 $\therefore x^2 = 16$
 $\therefore \sqrt{x^2} = \pm\sqrt{16}$
 $\therefore x = \pm 4$
 $\therefore x=4$ or $x=-4$

(c) $x^2 + 25 = 0$
The expression on the left cannot be factorised into real factors.
Proceed as follows:
 $x^2 = -25$
 $\therefore x = \pm\sqrt{-25}$
Since $\sqrt{-25}$ is non-real, the equation has no real solutions.

(d) $(x+5)^2 = 9$ [the right side is not zero]
 $\therefore x^2 + 10x + 25 = 9$ [expand the left side]
 $\therefore x^2 + 10x + 16 = 0$ [standard form]
 $\therefore (x+2)(x+8) = 0$ [factorise]
 $\therefore x = -2$ or $x = -8$ [apply zero-factor law]

Alternatively:
 $(x+5)^2 = 9$
 $\therefore \sqrt{(x+5)^2} = \pm\sqrt{9}$
 $\therefore x+5 = \pm 3$
 $\therefore x+5 = -3$ or $x+5 = 3$
 $\therefore x = -8$ or $x = -2$

(e) $(x+5)^2 = -9$
 $\therefore x+5 = \pm\sqrt{-9}$
There are no real solutions

Example 11

- (a) In this example, there are variables in the denominator. Since the denominators can never equal zero, it is important to note that $x \neq 0$.

LCD = x and the restriction is $x \neq 0$

$$\therefore 10 \times x + \frac{16}{x} \times x = \frac{4(x+4)}{x} \times x \quad [\text{multiply every term by the LCD}]$$

$$\therefore 10x + 16 = 4(x+4)$$

$$\therefore 10x + 16 = 4x + 16 \quad [\text{this is a linear equation}]$$

$$\therefore 6x = 0$$

$$\therefore x = 0$$

However, since $x \neq 0$, the equation does not have a solution.

- (b) With the addition and subtraction of algebraic fractions, you needed to **factorise the denominators first** before finding the LCD. With equations you will do the same.

$$\frac{4}{x-5} - \frac{10}{x} = \frac{2}{x^2-5x}$$

$$\therefore \frac{4}{x-5} - \frac{10}{x} = \frac{2}{x(x-5)}$$

$\therefore$ LCD = $x(x-5)$ and the restrictions are:

$$x \neq 0 \quad \text{and} \quad x-5 \neq 0$$

$$x \neq 5$$

$$\therefore \frac{4}{x-5} \times x(x-5) - \frac{10}{x} \times x(x-5) = \frac{2}{x(x-5)} \times x(x-5) \quad [\text{multiply by the LCD}]$$

$$\therefore 4x - 10(x-5) = 2$$

$$\therefore 4x - 10x + 50 = 2$$

$$\therefore -6x = 2 - 50$$

$$\therefore -6x = -48$$

$$\therefore x = 8$$

This is a valid solution since it is not stated as a restriction.

$$\begin{aligned}
\text{(c)} \quad & \frac{2x}{x-3} + \frac{5x-3}{9-x^2} = \frac{x}{x+3} \\
& \therefore \frac{2x}{x-3} + \frac{5x-3}{-(x^2-9)} = \frac{x}{x+3} \quad [\text{change in sign: } 9-x^2 = -(x^2-9)] \\
& \therefore \frac{2x}{x-3} + \frac{5x-3}{-(x-3)(x+3)} = \frac{x}{x+3} \quad [\text{factorise}] \\
& \therefore \frac{2x}{x-3} - \frac{5x-3}{(x-3)(x+3)} = \frac{x}{x+3} \\
& \therefore \text{LCD} = (x-3)(x+3) \text{ and the restrictions are: } x \neq 3 \text{ and } x \neq -3 \\
& \therefore \frac{2x}{x-3} \times (x-3)(x+3) - \frac{5x-3}{(x-3)(x+3)} \times (x-3)(x+3) = \frac{x}{x+3} \times (x-3)(x+3) \\
& \therefore 2x(x+3) - (5x-3) = x(x-3) \\
& \therefore 2x^2 + 6x - 5x + 3 = x^2 - 3x \quad [\text{this is a quadratic equation}] \\
& \therefore x^2 + 4x + 3 = 0 \quad [\text{standard form}] \\
& \therefore (x+1)(x+3) = 0 \\
& \therefore x = -1 \text{ or } x = -3 \\
& \text{Since } x \neq -3, \text{ the solution is } x = -1
\end{aligned}$$

Example 12

Method 1 (Eliminating y)

Label each equation as follows:

$$3x - 2y = 8 \dots\dots\dots \text{A}$$

$$4x + 2y = 6 \dots\dots\dots \text{B}$$

By vertically adding these two equations you will eliminate one of the variables (y in this case) and be able to solve for the other variable (x in this case):

$$3x - 2y = 8 \dots\dots\dots \text{A}$$

$$4x + 2y = 6 \dots\dots\dots \text{B}$$

$$\begin{aligned}
\therefore (A+B) \text{ is } & \begin{array}{r} 3x - 2y = 8 \\ 4x + 2y = 6 \\ \hline 7x + 0y = 14 \end{array} & \text{Add the like terms of equation A and B} \\
& \therefore 7x = 14 \\
& \therefore x = 2
\end{aligned}$$

Now solve for y if $x = 2$. Therefore, substitute $x = 2$ into either A or B to get y :

If using equation A:

$$\text{If } x = 2 \text{ then } 3(2) - 2y = 8$$

$$\therefore 6 - 2y = 8$$

$$\therefore -2y = 2$$

$$\therefore y = -1$$

If using equation B:

$$\text{If } x = 2 \text{ then } 4(2) + 2y = 6$$

$$\therefore 8 + 2y = 6$$

$$\therefore 2y = -2$$

$$\therefore y = -1$$

Method 2 (Eliminating x)

$$3x - 2y = 8 \dots\dots\dots A$$

$$4x + 2y = 6 \dots\dots\dots B$$

If we want the terms in x to be eliminated when added, the coefficients have to be additive inverses (1 and -1 or 2 and -2 or 3 and -3 etc.) . Let us therefore consider the lowest common multiple (LCM) of 3 and 4 (the coefficients of x) in order to attain that. The LCM is 12. We want the coefficient of x of equation A to be 12 and that of equation B to be -12 .

Multiply each term of equation A by 4
and call the new equation C:

$$3x \times 4 - 2y \times 4 = 8 \times 4 \dots\dots\dots (A \times 4)$$

$$\therefore 12x - 8y = 32 \dots\dots\dots C$$

Multiply each term of equation B by -3
and call the new equation D:

$$4x \times -3 + 2y \times -3 = 6 \times -3 \dots\dots\dots (B \times -3)$$

$$-12x - 6y = -18 \dots\dots\dots D$$

Now by vertically adding the equations C and D you will eliminate the x :

$$12x - 8y = 32 \dots\dots\dots C$$

$$\underline{-12x - 6y = -18 \dots\dots\dots D}$$

$$\therefore (C + D) \text{ is } 0x - 14y = 14 \quad \text{Add the like terms of equation C and D}$$

$$\therefore -14y = 14$$

$$\therefore y = -1$$

If using equation A:

$$\text{If } y = -1 \text{ then } 3x - 2(-1) = 8$$

$$\therefore 3x + 2 = 8$$

$$\therefore 3x = 6$$

$$\therefore x = 2$$

If using equation B:

$$\text{If } y = -1 \text{ then } 4x + 2(-1) = 6$$

$$\therefore 4x - 2 = 6$$

$$\therefore 4x = 8$$

$$\therefore x = 2$$

Example 13

Method 1 (Making y the subject of one of the equations)

Label each equation as follows:

$$3x - 2y = 8 \dots\dots\dots A$$

$$4x + 2y = 6 \dots\dots\dots B$$

Now pick either one of the equations and solve for one of the variables. Let's solve for the variable y in equation B as it will be the easier one because the resultant equation will have no fractions.

$$4x + 2y = 6$$

$$\therefore 2y = -4x + 6$$

$$\therefore y = -2x + 3 \dots\dots\dots C \quad \text{[divide both sides by 2]}$$

Now replace the variable y in equation A with $-2x + 3$ and solve for x :

$$3x - 2y = 8$$

$$\therefore 3x - 2(-2x + 3) = 8$$

$$\therefore 3x + 4x - 6 = 8$$

$$\therefore 7x = 14$$

$$\therefore x = 2$$

Now substitute $x = 2$ into either equation A, B or C. Equation C will always be the easiest choice.

$$\text{If } x = 2 \text{ then } y = -2(2) + 3 = -4 + 3 = -1$$

Method 2 (Making y the subject of both equations)

$$3x - 2y = 8 \dots\dots\dots A$$

$$4x + 2y = 6 \dots\dots\dots B$$

Solve for the variable y in both equation A and B:

$$3x - 2y = 8$$

$$\therefore -2y = -3x + 8$$

$$\therefore y = \frac{3x}{2} - 4 \dots\dots\dots C$$

$$4x + 2y = 6$$

$$\therefore 2y = -4x + 6$$

$$\therefore y = -2x + 3 \dots\dots\dots D$$

Now you equate equations C and D because both equations are equal to y .

$$\frac{3x}{2} - 4 = -2x + 3$$

$$\therefore 3x - 8 = -4x + 6 \quad [\text{multiply by the LCD of 2}]$$

$$\therefore 3x + 4x = 8 + 6$$

$$\therefore 7x = 14$$

$$\therefore x = 2$$

Now substitute $x = 2$ into either A, B, C or D

Equation C or D will be the easiest. We will choose equation D.

$$\therefore y = -2(2) + 3 = -4 + 3 = -1$$

Method 3 (Making x the subject of one of the equations)

$$3x - 2y = 8 \dots\dots\dots \text{A}$$

$$4x + 2y = 6 \dots\dots\dots \text{B}$$

Choose either one of the equations and solve for x . Let's use equation A:

$$3x - 2y = 8$$

$$\therefore 3x = 2y + 8$$

$$\therefore x = \frac{2y}{3} + \frac{8}{3} \dots\dots\dots \text{C} \quad [\text{divide both sides by 3}]$$

Now replace the variable x in equation B with $\frac{2y}{3} + \frac{8}{3}$ and solve for y :

$$4x + 2y = 6$$

$$\therefore 4\left(\frac{2y}{3} + \frac{8}{3}\right) + 2y = 6$$

$$\therefore \frac{8y}{3} + \frac{32}{3} + 2y = 6$$

$$\therefore 8y + 32 + 6y = 18 \quad [\text{multiply by 3}]$$

$$\therefore 14y = -14$$

$$\therefore y = -1$$

$$\therefore \text{If } y = -1 \text{ then } x = \frac{2(-1)}{3} + \frac{8}{3} = \frac{-2}{3} + \frac{8}{3} = 2$$

Example 14

a) $y = 3x + 4 \dots 1$ and $y = -3x - 4 \dots 2$

Subs 1 into 2

$$\therefore 3x + 4 = -3x - 4$$

$$\therefore 3x + 3x = -4 - 4$$

$$\therefore 6x = -8$$

$$\therefore x = -\frac{8}{6} = -\frac{4}{3}$$

Subs back into 1:

$$\therefore y = 3\left(-\frac{4}{3}\right) + 4$$

$$\therefore y = -4 + 4$$

$$\therefore y = 0 \qquad \therefore \left(-\frac{4}{3}; 0\right)$$

b) $y = \frac{1}{4}x + 2 \dots 1$ and $y = 2x - 7 \dots 2$

Subs 1 into 2:

$$\therefore \frac{1}{4}x + 2 = 2x - 7$$

$$\therefore \frac{1}{4}x - 2x = -7 - 2$$

$$\therefore -\frac{7}{4}x = -9$$

$$\therefore -7x = -36$$

$$\therefore x = \frac{36}{7} = 5\frac{1}{7}$$

Subs back into 2:

$$\therefore y = 2\left(5\frac{1}{7}\right) - 7$$

$$\therefore y = 3\frac{2}{7} \qquad \therefore \left(5\frac{1}{7}; 3\frac{2}{7}\right)$$

c)

$x - 3y - 5 = 0$ and $2x + 3y + 8 = 0 \dots 2$

$x = 3y + 5 \dots 1$

Subs 1 into 2

$$\therefore 2(3y + 5) + 3y + 8 = 0$$

$$\therefore 6y + 10 + 3y + 8 = 0$$

$$\therefore 9y = -18$$

$$\therefore y = -2$$

Subs back into 1:

$$\therefore x = 3(-2) + 5$$

$$\therefore x = -1 \qquad \therefore (-1; -2)$$

d)

$$5y - 3 = \frac{1}{2}x \quad \text{and} \quad 9y = 4x - 7 \quad \dots 2$$

$$10y - 6 = x \quad \dots 1$$

Subs 1 into 2

$$\therefore 9y = 4(10y - 6) - 7$$

$$\therefore 9y = 40y - 24 - 7$$

$$\therefore 9y - 40y = -31$$

$$\therefore -31y = -31$$

$$\therefore y = 1$$

Subs back into 1:

$$\therefore x = 10(1) - 6$$

$$\therefore x = 4$$

$$\therefore (4; 1)$$

Example 15

(a) $v = u + at$

Subtract u from both sides:

$$\therefore v - u = at$$

Divide both sides by a where $a \neq 0$:

$$\therefore \frac{v - u}{a} = t$$

(b) $A = \pi r^2$

Divide both sides by π :

$$\therefore \frac{A}{\pi} = r^2$$

Square root both sides:

$$\therefore \sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

$$\therefore r = \sqrt{\frac{A}{\pi}} \text{ (not } \pm \text{ as } r \text{ is positive)}$$

(c) (1) $A = 2\pi r(h + r)$

$$\therefore \frac{A}{2\pi r} = h + r \quad [\text{divide both sides by } 2\pi r]$$

$$\therefore \frac{A}{2\pi r} - r = h$$

$$\therefore \frac{A - 2\pi r^2}{2\pi r} = h \quad [\text{LCD is } 2\pi r]$$

$$(2) \quad h = \frac{86 - 2\pi(3)^2}{2\pi(3)} = 1,6 \text{ cm}$$

Example 16

a) $V = \pi r^2 h$ for h

$$\therefore \frac{V}{\pi r^2} = h$$

b)

$$A = P(1 + in) \quad \text{for } i$$

$$\frac{A}{P} = 1 + in$$

$$\frac{A}{P} - 1 = in$$

$$\therefore \frac{1}{n} \left(\frac{A}{P} - 1 \right) = i$$

c)

$$A = P(1 + i)^n \quad \text{for } i$$

$$\frac{A}{P} = (1 + i)^n$$

$$\sqrt[n]{\frac{A}{P}} = 1 + i$$

$$\therefore \sqrt[n]{\frac{A}{P}} - 1 = i$$

d)

$$3 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{for c} \quad \text{f)}$$

$$6a = -b + \sqrt{b^2 - 4ac}$$

$$6a + b = \sqrt{b^2 - 4ac}$$

$$(6a + b)^2 = b^2 - 4ac$$

$$36a^2 + 12ab + b^2 = b^2 - 4ac$$

$$36a^2 + 12ab = -4ac$$

$$\frac{36a^2}{-4a} + \frac{12ab}{-4a} = c \rightarrow \therefore -9a - 3b = c$$

e)

$$s = ut + \frac{1}{2}at^2 \quad \text{for } u$$

$$s - \frac{1}{2}at^2 = ut$$

$$\frac{1}{t} \left(s - \frac{1}{2}at^2 \right) = u$$

$$\therefore \frac{s}{t} - \frac{at}{2} = u$$

f)

$$SA = 2(lb + lh + bh) \text{ for } h$$

$$\frac{SA}{2} = lb + lh + bh$$

$$\frac{SA}{2} - lb = h(l + b)$$

$$\therefore \frac{\frac{SA}{2} - lb}{l + b} = h$$

PAST EXAM PAPERS

2018 Gauteng June Paper 1 Q 3.1

3.1	$\frac{x+2}{x^2-3x-4} = \frac{3}{x-4} - \frac{1}{2+2x}$ $\frac{x+2}{(x-4)(x+1)} = \frac{3}{x-4} - \frac{1}{2(x+1)}$ CD: $2(x-4)(x+1)$ Restriction: $x \neq 4; x \neq -1$ $2(x+2) = 6(x+1) - 1(x-4)$ $2x+4 = 6x+6-x+4$ $2x-5x = 10-4$ $-3x = 6$ $x = -2$	✓ correct factors ✓ $2(x+2)$ ✓ $6(x+1)$ ✓ $-x+4$ ✓ answer (5)
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2025 KwaZulu Natal March Q 2.1.1, 2.1.2 and 2.4

2.1.1	$3x(x-8) = 0$ $x = 0$ or $x = 8$	✓A ✓A	(2)
2.1.2	$7x^2 - 5x - 2 = 0$ $(7x+2)(x-1) = 0$ $x = -\frac{2}{7}$ or $x = 1$	✓A factors ✓CA ✓CA	(3)

2.4	$2x - y = 17$ equation 1 $4y + 3x = 9$ equation 2 $y = 2x - 17$ equation 3 substitute equation 3 into equation 2 $4(2x - 17) + 3x = 9$ $8x - 68 + 3x = 9$ $11x = 77$ $\therefore x = 7$ substitute x - value into equation 3 $y = 2(7) - 17$ $y = -3$ OR	✓A equation 3 ✓CA substitution ✓CA value of x ✓CA value of y	
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2019 November Paper 1 Q 2.1.1 and Q 2.1.2

2.1.1	$2x^2 - 10x = 0$ $2x(x - 5) = 0$ $x = 0$ or $x = 5$	✓ common factor/ <i>gemeenskaplike faktor</i> ✓ both values of/beide <i>waardes van x</i> (2)
2.1.2	$px - kx = k - p$ $x(p - k) = k - p$ $x = -\frac{k - p}{k - p}$ $x = -1$	✓ common factor/ <i>gemeenskaplike faktor</i> ✓ change of sign/ <i>verander</i> <i>teken</i> ✓ answer/ <i>antwoord</i> (3)

2012 Exemplar Paper 1 Q 2.1.1, 2.1.3 and Q 2.2.

2.1.1	$x^2 - 4x = 21$ $x^2 - 4x - 21 = 0$ $(x + 3)(x - 7) = 0$ $x + 3 = 0$ or $x - 7 = 0$ $x = -3$ or $x = 7$	✓ standard form ✓ factors ✓ answers (3)
2.1.3	$R = \frac{2\sqrt{x}}{3S}$ $\frac{3RS}{2} = \sqrt{x}$ $x = \frac{9R^2S^2}{4}$	✓ Multiply by 3S and divide by 2 ✓ Squaring both sides (2)
2.2	$6q + 7p = 3$ Equation 1 $2q + p = 5$ Equation 2 $6q + 7p = 3$ Equation 1 $14q + 7p = 35$ multiply Equation 2 with 7 Equation 3 Equation 3 – Equation 1: $8q = 32$ $q = 4$ $2(4) + p = 5$ $p = -3$	 ✓ $14q + 7p = 35$ ✓ $8q = 32$ ✓ $q = 4$ ✓ substitution ✓ $p = -3$ (5) [13]

2024 North West November Paper 1 Q 2.1.3; 2.2 and Q 2.3

2.1.3	$2x^2 - x - 10 = 0$ $(2x - 5)(x + 2) = 0$ $2x - 5 = 0$ of $x + 2 = 0$ $x = \frac{5}{2} / 2\frac{1}{2} / 2,5$ or / of $x = -2$	✓ factorising/faktoriseer ✓ equal to 0/stel gelyk aan 0 ✓✓ answers/antwoorde (4)
2.2	$(x + 5)(x - 5) = x^2 - 25$ $(x + 1)(x - 25) = x^2 - 24x - 25$ $-24x$ must be added at $(x + 5)(x - 5)$ $-24x$ moet by $(x + 5)(x - 5)$ bygetel word	✓ $x^2 - 25$ ✓ $x^2 - 25$ ✓ $-24x$ ✓ answer/antwoord (4)
2.3	$y = 3x - 2$ (1) $7x - 2y = 8$ (2) $7x - 2(3x - 2) = 8$ $7x - 6x + 4 = 8$ $x = 4$ $y = 3(4) - 2$ $y = 10$	✓ $3x - 2$ ✓ substitution/vervanging ✓ $x = 4$ ✓ $y = 10$ (4)

2022 Gauteng November Paper 1 Q 2.1.1 and Q 2.3

2.1	2.1.1	$(3x-1)(x+2) = 0$ $x = \frac{1}{3}$ or/of $x = -2$	$\checkmark \frac{1}{3}$ $\checkmark -2$	(2)
2.3	Solve for x and y simultaneously/Los vir x en y gelyktydig op: $x - y = 4$ and/en $\frac{x}{5} + \frac{y}{2} = 5$ $x = y + 4 \dots (1)$ $2x + 5y = 50 \dots (2)$ subst/vervang (1) into/in (2) $2(y+4) + 5y = 50$ $2y + 8 + 5y = 50$ $7y = 42$ $y = 6$ $x = 6$ $x = 10$ OR/OF $y = x - 4 \dots (1)$ subst/vervang (1) into/in (2) $2x + 5(x - 4) = 50$ $2x + 5x - 20 = 50$ $7x = 70$ $x = 10$ $y = 10 - 4$ $y = 6$		$\checkmark$ Make x the subject of the equation (1)/Maak x die onderwerp van die vergelyking (1) $\checkmark$ LCD/KGV $\checkmark$ Substitute/Vervang $\checkmark$ y value/ y -waarde $\checkmark$ x value/ x -waarde $\checkmark$ Make y the subject of the equation (1)/Maak y die onderwerp van die vergelyking (1) $\checkmark$ Substitute/Vervang $\checkmark$ Simplify/Vereenvoudig $\checkmark$ x value/ x -waarde $\checkmark$ y value/ y -waarde	(5)

2016 November Paper 1 Q 2.1.1; 2.1.2 and Q 2.3

QUESTION 2/VRAAG 2		
2.1.1	$x(x-1) = 20$ $x^2 - x - 20 = 0$ $(x-5)(x+4) = 0$ $x = 5$ or/of $x = -4$	✓ removing brackets/verw. hakies ✓ stand.form/stand. vorm ✓ fact/fak ✓ answer/antwoord (4)
2.1.2	$\frac{3x-2}{2} = (x+1)$ $3x-2 = 2(x+1)$ $3x-2 = 2x+2$ $x = 4$ OR/OF $\frac{3x-2}{2} - (x+1) = 0$ $\frac{3x-2-2(x+1)}{2} = 0$ $\frac{3x-2-2x-2}{2} = 0$ $\frac{x-4}{2} = 0$ $x = 4$ OR/OF $\frac{3x}{2} - 1 = x+1$ $\frac{3x}{2} - x = 2$ $\frac{x}{2} = 2$ $x = 4$	✓ multipl./maal ✓ simpl/simp. ✓ answer/antwoord (3) ✓ writing the LHS as a single fraction./ skryf LK as n enkel breuk ✓ simplification/ simpl. ✓ answer/antwoord (3)

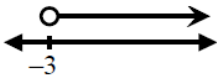
DEPARTMENT OF BASIC
EDUCATION
PRIVATE BAG X595, PRETORIA 0001
2016 -11- 07
APPROVED MARKING GUIDELINE
PUBLIC EXAMINATION

INEQUALITIES

Example 17

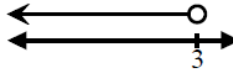
Solutions

(a) $\frac{2x}{2} > \frac{-6}{2}$
 $\therefore x > -3$

A number line with an open circle at -3 and an arrow pointing to the right, representing the inequality x > -3.

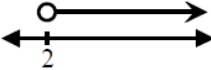
[All real numbers larger than -3]
[The inequality remains the same]

(b) $\frac{-2x}{-2} > \frac{-6}{-2}$
 $\therefore x < 3$

A number line with an open circle at 3 and an arrow pointing to the left, representing the inequality x < 3.

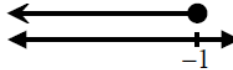
[All real numbers smaller than 3]
[The inequality is reversed]

(c) $4(2x-1) > 5x+2$
 $\therefore 8x-4 > 5x+2$
 $\therefore 8x-5x > 2+4$
 $\therefore 3x > 6$
 $\therefore x > 2$

A number line with an open circle at 2 and an arrow pointing to the right, representing the inequality x > 2.

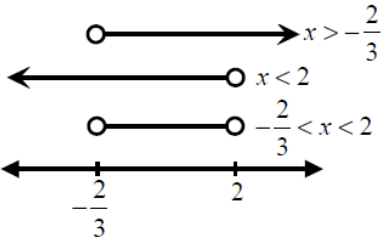
[All real numbers larger than 2]

(d) $\frac{2x-1}{3} \geq 2x+1$
 $\therefore \frac{2x-1}{3} \times 3 \geq (2x+1) \times 3$
 $\therefore 2x-1 \geq 6x+3$
 $\therefore 2x-6x \geq 3+1$
 $\therefore -4x \geq 4$
 $\therefore x \leq -1$

A number line with a closed circle at -1 and an arrow pointing to the left, representing the inequality x ≤ -1.

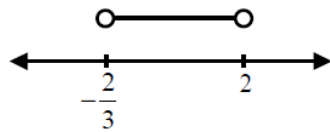
[All real numbers smaller than and including -1]

Example 18

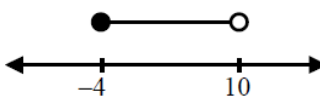
- (a) The inequality $-3 < 3x - 1 < 5$ is read as follows:
 $3x - 1$ is smaller than 5 AND larger than -3 which implies that both the inequality statements must be true.
 $-3 < 3x - 1$ and $3x - 1 < 5$
 $\therefore -3x < 2$ and $3x < 6$
 $\therefore x > -\frac{2}{3}$ and $x < 2$
 $\therefore -\frac{2}{3} < x < 2$
- 

An alternative and relatively easier approach is the following:

$$\begin{aligned} \therefore -3 < 3x - 1 < 5 \\ \therefore -3 + 1 < 3x - 1 + 1 < 5 + 1 & \quad [\text{add 1 to all terms in order to get } x \text{ by itself}] \\ \therefore -2 < 3x < 6 & \quad [\text{divide all three terms by 3}] \\ \therefore -\frac{2}{3} < x < 2 & \quad [\text{inequality signs remain the same}] \end{aligned}$$



[All real numbers greater than $-\frac{2}{3}$ and smaller than 2]

- (b) $-2 < 3 - \frac{1}{2}x \leq 5$
- 
- $\therefore -5 < -\frac{1}{2}x \leq 2$ [subtract 3 to all terms]
 $\therefore 10 > x \geq -4$ [multiply all terms by -2]
 [Note that the inequality signs had to be reversed]
 Rewrite the inequality as follows:
 $\therefore -4 \leq x < 10$

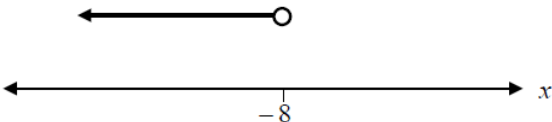
2025 KwaZulu Natal March Paper Q 2.3

2.3.1	$-15 < 1 - 4x < 5$	✓ $-16 < -4x < 4$	
	$-16 < -4x < 4$ $4 > x > -1$ $-1 < x < 4$	✓ end points ✓ notation	(3)
2.3.2		✓✓ answer	(2)

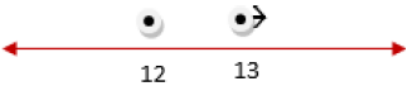
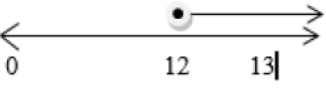
2020 Gauteng November Paper 1 Q 4.2

4.2	$-4 \leq \frac{1-x}{3} < 3$ $-4 \times 3 \leq 1 - x < 3 \times 3$ $-12 \leq 1 - x < 9$ $-12 - 1 \leq -x < 9 - 1$ $-13 \leq x < 8$ $\therefore -8 < x \leq 13$ $x \in (-8; 13]$	✓ $-12 \leq 1 - x < 9$ ✓ $-13 \leq x < 8$ ✓ $-8 < x \leq 13$ ✓ Interval notation/ Intervalnotasie	(4)
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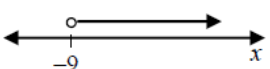
2015 November Paper 1 Q 2.2

2.2.1	$3(x+7) < \frac{x}{2} + 1$ $3x + 21 < \frac{x}{2} + 1$ $6x + 42 < x + 2$ $5x < -40$ $x < -8$	✓ $3x + 21$ ✓ $6x + 42 < x + 2$ ✓ answer/antwoord	(3)
2.2.2		✓ indicating numbers to the left of -8 and -8 not included/ dui getalle links van -8 aan met -8 nie ingesluit	(1)

2016 Gauteng June Paper 1 Q 3.2

3.2	$2(2x-3)-18 \geq 2x$ $4x-6-18 \geq 2x$ $4x-2x \geq 6+18$ $2x \geq 24$ $x \geq 12$	Simplification $4x-6 \checkmark$ $x \geq 12 \checkmark$	(4)	
		Representing on the number line and 12 included $\checkmark$ and arrow $\checkmark$		
		NOTE: If 12 is included with a line - 1mark		

2019 November Paper 1 Q 2.2

2.2.1	$\frac{x+5}{2} > -2$ $x+5 > -4$ $x > -9$	$\checkmark$ simplification/ $\checkmark$ vereenvoudiging $\checkmark$ answer/antwoord (2)
2.2.2		$\checkmark$ number line/getalleylyn (1)

2017 November Paper 1 Q 2.2

2.1.1	$4-2x < 16$ $-2x < 12$ $x > -6$	OR / OF	$4-2x < 16$ $-12 < 2x$ $-6 < x$	$\checkmark$ simpl./vereenv $\checkmark$ answ/antw (2)
2.1.2				$\checkmark$ answ/antw (1)

2024 Free State June Paper 1 Q 3.1.3

3.1.3	$-1-3 < -2x \leq 5-3$ $\frac{-4}{-2} > x \geq \frac{2}{-2}$ $-1 \leq x < 2$	$\checkmark$ simplification $\checkmark \checkmark$ answer (3)
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2022 Gauteng November Paper 1 Q 2.2

2.2	2.2.1	$-1 \leq 1 - \frac{2x}{3} < 5$ $-2 \leq -\frac{2x}{3} < 4$ $-6 \leq -2x < 12$ $3 \geq x > -6$ $-6 < x \leq 3$	✓ Subtract 1 on both sides/ <i>Trek 1 aan albei kante af</i> ✓ $\times 3$ ✓ $\div (-2)$ and change of inequalities/en <i>verandering van</i> <i>ongelykhede</i>	(3)
	2.2.2	9 integers satisfy the equation/9 heelgetalle voldoen aan die vergelyking	✓ Answer/Antwoord	(1)

2016 November Paper 1 Q 2.2

2.2.1	$-4 \leq -\frac{1}{2}m < 5$ $-8 \leq -m < 10$ $8 \geq m > -10$ $-10 < m \leq 8$ OR/OF $-4 \leq -\frac{1}{2}m$ $-\frac{1}{2}m < 5$ $-8 \leq -m$ and /en $-m < 10$	
2.2.2	$(-10 ; 8]$	✓ ans/ant
2.3.1	Given/Geev	(1)

2018 November Paper 1 Q 2.2

2.2.1	$-11 \leq 3m - 8 < 4$ $-3 \leq 3m < 12$ $-1 \leq m < 4$	✓ simplification/ <i>vereenvoudiging</i> ✓ answer/antwoord (2)
2.2.2	5 integers/heelgetalle	✓ answer/antwoord (1)