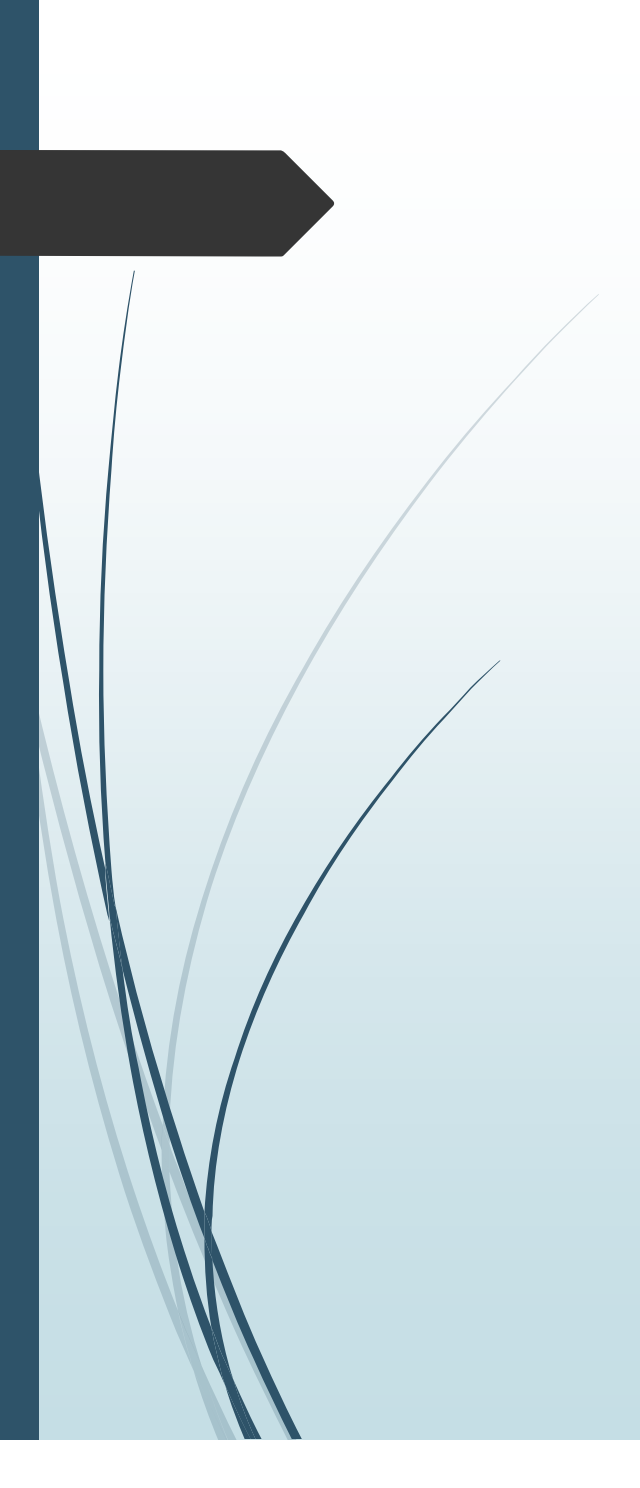




EXPONENTS, EQUATIONS AND INEQUALITIES

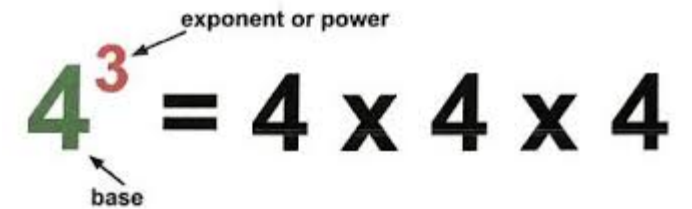
EXPONENTS

Exponents

- ❑ An exponent is a mathematical expression, written as a^n involving two numbers or terms.
- ❑ The base a and the exponent or power n .
- ❑ The exponent is usually shown as a superscript to the right of the base.



A diagram showing the expression 7^3 . A green arrow points from the word "Base" to the number 7. A purple arrow points from the word "Exponent" to the number 3.



A diagram showing the expression $4^3 = 4 \times 4 \times 4$. A black arrow points from the word "base" to the number 4 on the left. Another black arrow points from the words "exponent or power" to the number 3.



A diagram showing the general expression b^n . A green arrow points from the word "Base" to the letter b . A purple arrow points from the word "Exponent" to the letter n .

b^n means "b raised to the power of n".

Exponent rules,

- They are also known as the 'laws of exponents' or the 'properties of exponents' make the process of simplifying expressions involving exponents easier.

Product Rule	$a^m \times a^n = a^{m+n}$
Quotient Rule	$a^m \div a^n = a^{m-n}$
Power of a Power Rule	$(a^m)^n = a^{mn}$
Power of a Product Rule	$(ab)^m = a^m b^m$
Power of a Quotient Rule	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero Exponent Rule	$a^0 = 1$
Negative Exponent Rule	$a^{-m} = \frac{1}{a^m}$
Fractional Exponent Rule	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

LAWS OF EXPONENTS

Law

Example

1. $a^m a^n = a^{m+n}$

$3^2 \cdot 3^5 = 3^{2+5} = 3^7$
2. $\frac{a^m}{a^n} = a^{m-n}$

$\frac{3^5}{3^2} = 3^{5-2} = 3^3$
3. $(a^m)^n = a^{mn}$

$(3^2)^5 = 3^{2 \cdot 5} = 3^{10}$
4. $(ab)^n = a^n b^n$

$(3 \cdot 4)^2 = 3^2 \cdot 4^2$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$
6. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$
7. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

$\frac{3^{-2}}{4^{-5}} = \frac{4^5}{3^2}$

Example 1

Simplify the following

a) $2p^3 \times 8p^2$

b) $2^5 \cdot 2^3 \cdot 2$

c) $4 \cdot 2^3$

d) $(3x^3y^5)^2$

e) $(2^5 \cdot 6^7)^2$

f) $\frac{x^5}{x^9}$

g) $\frac{-12a^{14}}{18a^6}$

h) $\frac{(3x^2y^3)^2}{(3xy)(3xy^7)}$

Definitions

$$a^0 = 1$$

$$1^a = 1 \text{ with } a \in \mathbb{R}$$

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{1}{x^{-n}} = x^n$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\frac{1}{ax^{-n}} = \frac{x^n}{a}$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

$$3^0 = 1$$

$$1^{1018} = 1$$

$$x^{-5} = \frac{1}{x^5}$$

$$\frac{1}{x^{-3}} = x^3$$

$$\sqrt[3]{a^4} = a^{\frac{4}{3}}$$

$$\frac{1}{(3x)^{-4}} = \frac{x^4}{3}$$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$$

$$\left(\frac{a^{-m}}{b^{-n}}\right) = \frac{b^n}{a^m}$$

$$\frac{a}{x^{-n}} = ax^n$$

$$\frac{1}{(ax)^{-n}} = (ax)^n$$

$$ax^{-n} = \frac{a}{x^n}$$

Definitions

$$\left(\frac{x^{-3}}{y^{-4}}\right) = \frac{y^4}{x^{-3}}$$

$$\frac{3}{x^{-4}} = 3x^4$$

$$\frac{1}{(3x)^{-4}} = (3x)^4$$

$$3x^{-4} = \frac{3}{x^4}$$

Example 2

a) $2p^0$

b) $2^3 \cdot 2^{-5}$

c) $(3a)^{-3}$

d) $\left(\frac{5}{3}\right)^{-3}$

e) $\frac{3x^{-4} \cdot y^{-3}}{9x^{-2}y^{-1}}$

f) $\left(\frac{x^{-4} \cdot y^8}{x^2 \cdot y^{-3}}\right)^2$

g) $3x^3 \cdot x - 4x \cdot x^3$

h) $(5a)^5(4a^5) - 15a \cdot a^9 + 10b^{10}$

i) $-3(-2a^3)^2 + 2(-2a^2)^3$

Example 3

Simplify the following

a)
$$\frac{16^2 \cdot (3^2)^4}{2^7 \cdot 2^5 \cdot 81}$$

b)
$$\frac{(2^x)^4 \cdot 9^x}{12^{2x}}$$

c)
$$\frac{16^{x-1}}{2^{4x} \cdot 32}$$

EXPRESSIONS WITH RATIONAL EXPONENTS

- Exponents need not always be restricted to integers.
- It is possible to work with powers that include other rational exponents such as fractions. The next example works with fractional exponents.

Example 4

Simplify:

$$\frac{(4x^3)^{\frac{1}{2}} \cdot x^{-\frac{1}{2}}}{x^{-3}}$$

Example 5

Simplify the following

a)
$$\frac{3^{x+1} + 2 \cdot 3^{x+2}}{7 \cdot 3^x}.$$

b)
$$\frac{9^x + 3^x - 2}{9^x - 4}$$

EXPONENTIAL EQUATIONS

In Grade 9 you learnt that in an exponential equation, the exponent is the unknown.

Example 6

Solve the following equations

a) $4.25^{x+3} = 4$

b) $(0,5)^{x-1} = \left(\frac{1}{4}\right)^x$

a) $9^x + 3^{2x+1} = 36$

EQUATIONS WITH RATIONAL EXPONENTS (FRACTIONS)

Example 7

a) $x^{\frac{1}{3}} = 4$

b) $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$

PAST EXAM PAPERS

2022 Gauteng November Paper 1 Q 1.2.2 and Q 1.2.3

Simplify the following:

1.2.2 $x^{-3} \div x^5 + (x^{-2})^{-1}$ (2)

1.2.3 $\frac{5^{x+1} \cdot (4^x)^3}{2^{5x-2} \cdot 10^{x-1}}$ (4)

2025 KwaZulu Natal March Paper Q 2.1.3-2.1.5

Simplify the following:

$$2.1.3 \quad 3^{5x-4} - 729 = 0 \quad (3)$$

$$2.1.4 \quad 2x^{-\frac{3}{2}} = 128 \quad (4)$$

$$2.1.5 \quad 2^{x+1} + 2^{x-1} = 20 \quad (5)$$

2015 November Paper 1 Q 1.2.2 and 2.1.2

1.2.2 Simplify the following expressions fully:

$$\frac{10^{2x+3} \cdot 4^{1-x}}{25^{2+x}}$$

(4)

2.1.2

$$5^x = \frac{1}{125}$$

(2)

2019 November Paper 1 Q 1.2.2

Simplify the following expressions fully:

$$\frac{3^{x+1} + 3^x}{27 \cdot 3^{-1+x}}$$

(3)

2017 November Paper 1 Q 1.3.3

Simplify the following expressions fully:

$$\frac{3^t - 3^{t-2}}{2 \cdot 3^t - 3^t}$$

(3)

2025 Mpumalanga March Paper Q 2.2

Simplify :

$$\frac{4^x - 3 \cdot 2^{x+1} - 27}{2^x + 3}$$

(4)

2022 Free State June Paper 1 Q 2.3 and Q 3.1.4

2.3 Simplify :

$$\frac{3 \cdot 2^m - 4 \cdot 2^{m-2}}{2^m - 2^{m-1}}$$

(3)

3.1.4 Solve for

$$2^x \cdot 2^{x+1} = 32$$

(3)

2012 Exemplar Paper 1 Q 2.1.2

Determine, **without the use of a calculator**, the value of x in each of the following:

$$96 = 3x^{\frac{5}{4}}$$

(3)

2018 Gauteng June Paper 1 Q 2.1

Simplify:

$$\frac{(3x)^2(-2xy)^3}{2x^5y^4}$$

(3)

2025 KwaZulu Natal November Paper 1 Q 1.1.3 and Q 2.1.1

1.1.3 Simplify the following completely

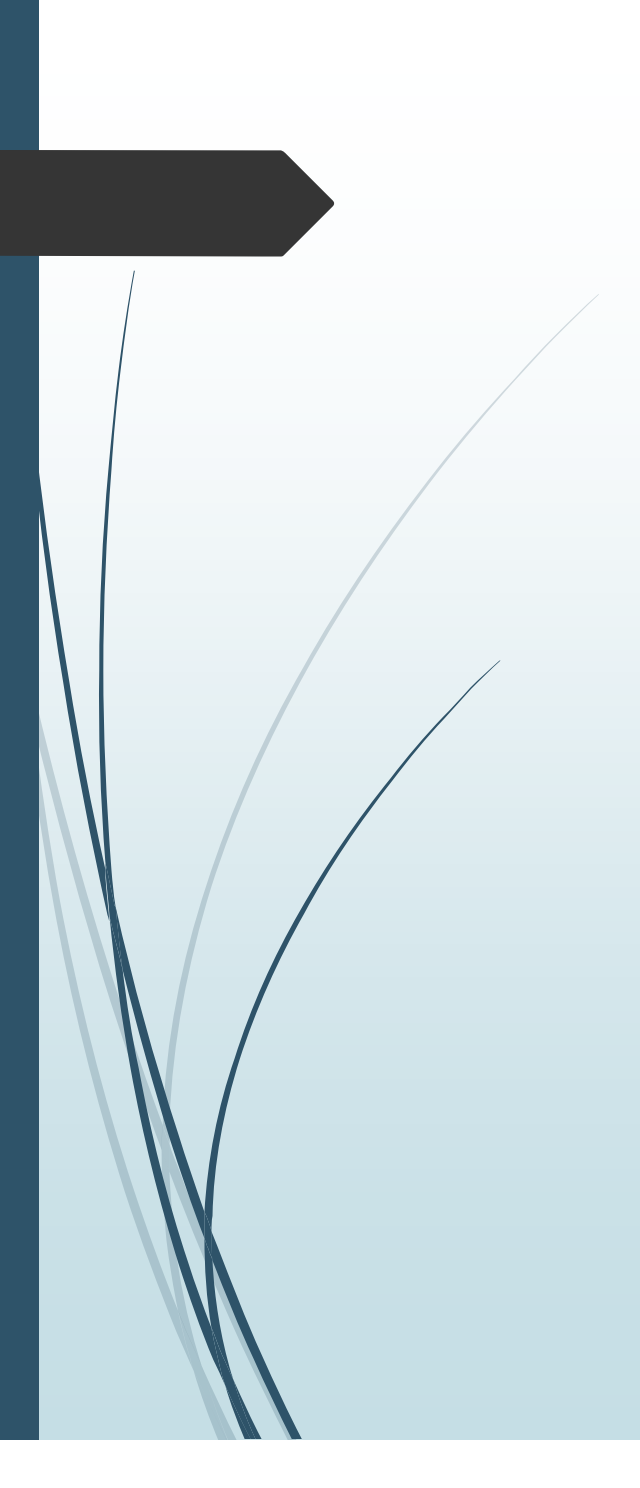
$$\frac{(2^x)^3 \cdot 27^x}{8^x \cdot (3^2)^x \cdot 3^x}$$

(3)

2.1.1 Solve for x , **without using a calculator**:

$$2^{3x-4} = \frac{1}{512}$$

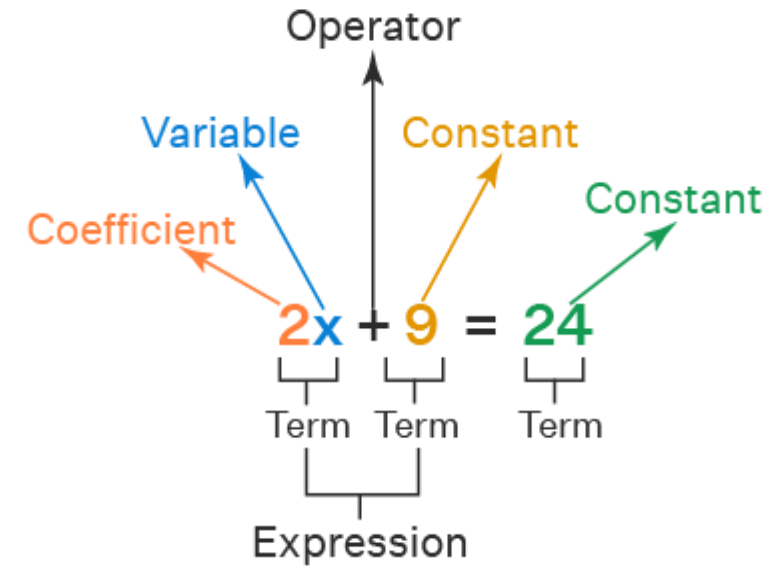
(3)



EQUATIONS

Equation

- An equation is a mathematical statement asserting that two expressions are equal, joined by an equal sign (=)
- It acts as a balanced scale, where the left side has the same value as the right side, often containing variables (unknowns), numbers, and operators
- A linear equation is an equation of degree one with at most **one solution**.



Left hand side = Right hand side

Example 8

Solve the following linear equations:

a) $3 - 4(k + 2) = 3k + 16$

b) $\frac{4x - 1}{3} - \frac{7x + 2}{6} = \frac{x}{2}$

Quadratic Equation

- ❑ A quadratic equation is a second degree equation with a standard form $ax^2 + bx + c = 0$
- ❑ It has at most **two real solutions**.
- ❑ From the standard form it is clear that one side of the equation has to equal zero.
- ❑ When the right hand side is zero then the quadratic expression on the left hand side has to be factorised.

A **quadratic equation** in x is an equation that can be written in the **standard form**;

$$ax^2 + bx + c = 0$$

Where a, b , and c are real numbers and $a \neq 0$.

quadratic expression MUST be isolated on one side of the equation


$$ax^2 + bx + c = 0$$



one side must ONLY contain zero

Example 9

Solve each of the following equations

a) $x^2 - 5x - 14 = 0$

b) $5x + 2x^2 = 3$

c) $2x^2 = 4x$

d) $8x - 16 - x^2 = 0$

e) $(3x - 1)(x + 2) = 0$

d) $(3x - 1)(x + 2) = 10$

Quadratic equations of the form $x^2 = \text{constant}$

Example 10

Solve each of the following equations:

a) $x^2 = 36$

b) $3x^2 = 48$

c) $x^2 + 25 = 0$

d) $(x + 5)^2 = 9$

e) $(x + 5)^2 = -9$

Equations with fractions (variables in the denominator)

- When solving equations with fractions (variables in the denominator) it is important to remember that **division by 0 is not permissible**.
- You must always check that your solutions do not make any one of the denominators 0.
- That is the reason why we must state restrictions when solving equations with variables in the denominators.

Example 11

Solve the following equations. Consider the solution(s) carefully as the solution(s) may have been stated as a restriction.

a) $10 + \frac{16}{x} = \frac{4(x + 4)}{x}$

b) $\frac{4}{x - 5} - \frac{10}{x} = \frac{2}{x^2 - 5x}$

c) $\frac{2x}{x - 3} + \frac{5x - 3}{9 - x^2} = \frac{x}{x + 3}$

Tips:

Apply the sign-change rule if necessary and then factorise the denominators.

2. State the restrictions.

3. Multiply every term by the lowest common denominator (LCD).

4. Identify the equation as linear or quadratic and solve the equation.

Simultaneous linear equations

- Simultaneous linear equations are a set of two or more equations the same variables (e.g., x and y) that are true at the same time.
- They model scenarios requiring multiple conditions to be met simultaneously.
- The goal is to find specific values for the variables that satisfy all equations, typically solved using **substitution**, **elimination**, or **graphical methods**

Simultaneous equations

$$2x + 3y = 8$$

$$3x - y = 23$$

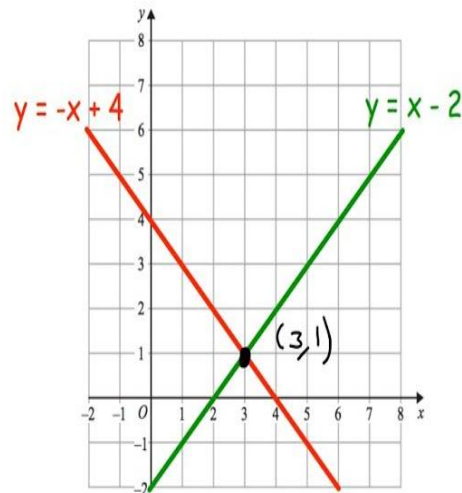
Solving Simultaneous Equations Graphically

Solve the following pair of simultaneous equations using the graph to help:

$$y = -x + 4$$

$$y = x - 2$$

$$x = 3 \quad y = 1$$



Eliminate y

$$\begin{array}{r} 4x + y = 5 \\ + \quad x - y = 10 \\ \hline 5x = 15 \\ x = 3 \end{array}$$

$$\begin{array}{r} x - y = 10 \\ 3 - y = 10 \\ -y = 7 \\ y = -7 \end{array}$$

Solution:

$$\begin{array}{l} x = 3 \\ y = -7 \end{array}$$

SYSTEMS OF
LINEAR
EQUATIONS

Substitute $(10+y)$ for x

$$\begin{array}{r} 4x + y = 5 \\ x - y = 10 \\ +y \quad +y \\ \hline x = 10 + y \end{array}$$

$$\begin{array}{r} 4(10+y) + y = 5 \\ 40 + 4y + y = 5 \\ 40 + 5y = 5 \\ 5y = -35 \\ y = -7 \end{array}$$

$$\begin{array}{r} x - y = 10 \\ x - (-7) = 10 \\ x + 7 = 10 \\ x = 3 \end{array}$$

The method of elimination

This method is the preferred method when solving two linear equations simultaneously.

Example 12

Solve for x and y simultaneously

$$3x - 2y = 8$$

$$4x + 2y = 6$$

The method of substitution

- The elimination method works mainly for two linear equations only, but the method of substitution is a method you can use for other systems of equations as well.
- It may seem a little longer than the elimination method but its process is much easier to understand.

Example 13

Solve for x and y simultaneously

$$3x - 2y = 8$$

$$4x + 2y = 6$$

Example 14

Solve for x and y simultaneously

a) $y = 3x + 4$ and $y = -3x - 4$

b) $y = \frac{1}{4}x + 2$ and $y = 2x - 7$

c) $x - 3y - 5 = 0$ and $2x + 3y + 8 = 0$

d) $5y - 3 = \frac{1}{2}x$ and $9y = 4x - 7$

Literal Equations

- A literal equation is one in which letters of the alphabet are used as coefficients and constants.
- These equations, usually referred to as formulae, are used a great deal in Mathematics, Science and Technology.
- The aim is to solve the equation or formula for a specific letter (or to make that letter the subject of the formula).
- This is a very useful skill in Grade 12.

LITERAL EQUATIONS

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A **literal equation** is an equation involving **two** or **more variables**. Unlike traditional equations where you often solve for one variable in terms of numbers, in a literal equation, you solve for **one variable** in terms of the **other variables**.



EXAMPLES:

$$A = \frac{1}{2}bh \quad F = \frac{9}{5}C + 32 \quad V = \pi r^2 h \quad s = \frac{at^2}{2}$$

Example 15

a) Make t the subject of the formula $v = u + at$

b) The area of a circle is given by $A = \pi r^2$. Make r the subject of the formula

c) The surface area of a cylinder is given by
 $A = 2\pi r(h + r)$

1) Show that:

$$h = \frac{A - 2\pi r^2}{2\pi r}$$

2) Hence determine the value of h if the radius is 3cm and the surface area is 86 cm^2 . Round off your answer to one decimal place

Example 16

Make the variable given after each equation the subject of the formula

a) $V = \pi r^2 h$ for h

b) $A = P(1 + in)$ for i

c) $A = P(1 + i)^n$ for i

d) $3 = \frac{-b\sqrt{b^2 - 4ac}}{2a}$ for c

e) $s = ut + \frac{1}{2}at^2$ for a

h) $SA = 2(lb + lh + bh)$ for h

PAST EXAM PAPERS

2018 Gauteng June Paper 1 Q 3.1

Solve for x :

$$\frac{x+2}{x^2-3x-4} = \frac{3}{x-4} - \frac{1}{2+2x} \quad (5)$$

2025 KwaZulu Natal March Q 2.1.1, 2.1.2 and 2.4

Solve for x

$$2.1.1 \quad 3x(x - 8) = 0 \quad (2)$$

$$2.1.2 \quad 7x^2 - 5x - 2 = 0 \quad (2)$$

2.4 Solve for x and y simultaneously

$$2x - y = 17$$

$$4y + 3x = 9$$

(4)

2019 November Paper 1 Q 2.1.1 and Q 2.1.2

Solve for x

$$2.1.1 \quad 2x^2 - 10x = 0 \quad (2)$$

$$2.1.2 \quad px - kx = k - p \quad (3)$$

2012 Exemplar Paper 1 Q 2.1.1 and Q 2.2.

Determine, **without the use of a calculator**, the value of x in each of the following:

2.1.1 $x^2 - 4x = 21$ (3)

2.1.3 Make x the subject of the formula in the following equation

$$R = \frac{2\sqrt{x}}{3S}$$

(2)

2.2 Solve for p and q simultaneously if:

$$6q + 7p = 3$$

$$2q + p = 5$$

(5)

2024 North West November Paper 1 Q 2.1.3; 2.2 and Q 2.3

Solve for x

$$2.1.3 \quad 2x^2 - x - 10 = 0 \quad (4)$$

2.2 How much must be added to $(x + 5)(x - 5)$ so that it will be equal to $(x + 1)(x - 25)$ (4)

2.3 Solve for x and y simultaneously:

$$y - 3x = -2$$

$$7x - 2y = 8$$

(4)

2022 Gauteng November Paper 1 Q 2.1.1 and Q 2.3

Solve for x

$$2.1.1 \quad (3x - 1)(x + 2) = 0 \quad (2)$$

2.3 Solve for x and y simultaneously:

$$x - y = 4$$

$$\frac{x}{5} + \frac{y}{2} = 5$$

(5)

2016 November Paper 1 Q 2.1.1; 2.1.2 and Q 2.3

2.1 Solve for x

2.1.1 $x(x - 1) = 20$ (4)

2.1.2 $\frac{3x - 2}{2} = x + 1$ (3)

2.3 Given $4x^2 - y^2 = 171$ and $2x - y = 9$

2.3.1 Calculate the value of $2x + y$ (2)

2.3.2 Solve for x and y simultaneously (3)

2023 KwaZulu Natal November Paper 1 Q 1.4 and Q 2.3

1.4 Determine the numerical value of $(3x + y)^2$ if it is given that $9x^2 + y^2 = 12$ and $xy = -3$ (3)

2.3 Solve for x and y simultaneously if:

$$2x - y = 17$$

$$4y + 3x = 9$$

(4)

INEQUALITIES

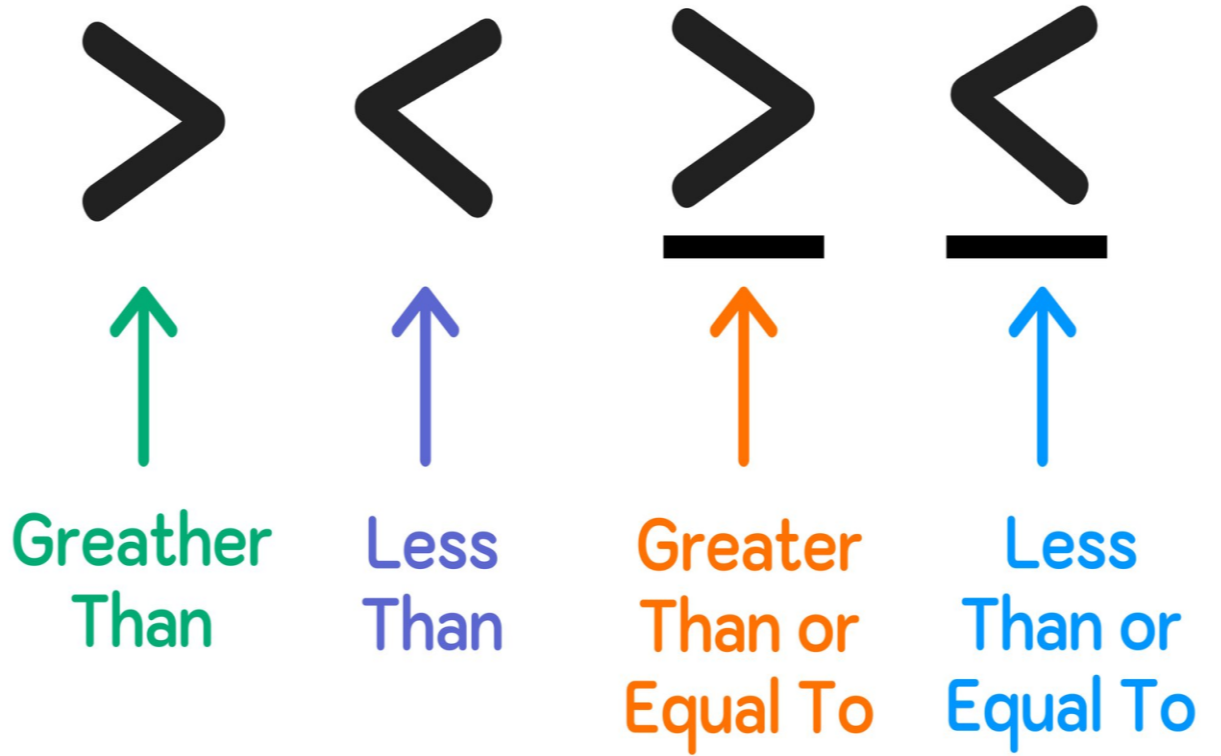
Inequalities

- ❑ The relationship between two values that are not equal is defined by **inequalities**.
- ❑ Inequality means not equal.
- ❑ The relationship between variables can be shown using the symbols:
 - $<$: "less than "
 - $\leq$: "less than or equal to"
 - $\neq$: "not equal to"
 - $>$: "greater than"
 - $\geq$: "greater than or equal to"

Examples of an inequality expressions are:

- $2x < -6$
- $3x - 12 \geq 0$
- $x^2 + 7x - 8 < 0$
- $(x + 1)(4 - x) > 0$
- $2x^2 - 2 \leq 3x$

Inequality Symbols



The following rules are always applicable when working with inequalities:

- Change the direction of the inequality sign whenever you multiply or divide by a negative number.
- Do not change the direction of the inequality sign if you multiply or divide by a positive number.
- Do not change the direction of the inequality sign if you add or subtract by a number or expression
- $<$ and $>$ are represented by open dots
- $\leq$ and $\geq$ are represented by closed dots

Example 17

Solve the following inequalities and then represent their solutions on a number line:

a) $2x > 6$

b) $-2x > -6$

c) $4(2x - 1) > 5x + 2$

d) $\frac{2x - 1}{3} \geq 2x + 1$

Example 18

Solve the following compound inequalities and then represent the solutions on a number line:

a) $-3 < 3x - 1 < 5$

b) $-2 < 3 - \frac{1}{2}x \leq 5$

Interval notation

- Interval notation is a method to represent an interval on a number line.
- In other words, it is a way of writing subsets of the real number line.
- An interval comprises the numbers lying between two specific given numbers.
- For example, the set of numbers x satisfying
- $0 \leq x \leq 50$ is an interval that contains 0, 5, and all numbers between 0 and 5.
- Interval Notation is a represent inequalities.
- We know an interval expressed as $1 < x < 5$ denotes a set of numbers lying between 1 and 5.
- $<$ and $>$ are represented by open brackets (and)
- $\leq$ and $\geq$ are represented by square brackets [and]

Examples

Write the following solutions in interval notation

a) $x > 5$

b) $1 < x < 5$

c) $2 < x \leq 6$

PAST EXAM PAPERS

2025 KwaZulu Natal March Paper Q 2.3

Solve the following inequality

2.3.1 $-15 < 1 - 4x < 5$ and $x \in R$ (3)

2.3.2 Represent your answer above on a number line (2)

2020 Gauteng November Paper 1 Q 4.2

Solve for x and write the answer in interval notation:

$$-4 \leq \frac{1-x}{3} < 3$$

(4)

2015 November Paper 1 Q 2.2

The following inequality is given:

$$3(x + 7) < \frac{x}{2} + 1$$

2.2.1 Solve for x in the inequality. (3)

2.2.2 Represent your answer to QUESTION 2.2.1 on a number line. (1)

2016 Gauteng June Paper 1 Q 3.2

Solve for x and represent your answer on a number line if $x = N_0$

$$2(2x - 3) - 18 \geq 2x \qquad (4)$$

2019 November Paper 1 Q 2.2

2.2 Given:

$$\frac{x + 5}{2} > -2$$

2.2.1 Solve for x . (2)

2.2.2 If $x \in R$,represent the solution to QUESTION 2.2.1 on a number line. (1)

2017 November Paper 1 Q 2.2

2.1 Given: $4 - 2x < 16$ where $x \in R$

2.1.1 Solve the inequality . (2)

2.1.2 Hence represent the solution to QUESTION 2.2.1 on a number line. (1)

2024 Free State June Paper 1 Q 3.1.3

Solve for x

$$-1 < -2x + 3 \leq 5 \quad (3)$$

2022 Gauteng November Paper 1 Q 2.2

The following inequality is given:

$$-1 \leq 1 - \frac{2x}{3} < 5$$

2.2.1 Solve for x in the inequality. (3)

2.2.2 Write down the number of integers that will satisfy the inequality (1)

2016 November Paper 1 Q 2.2

2.1 Given:

$$-4 \leq -\frac{1}{2}m < 5$$

where $m \in R$

2.1.1 Solve for m . (2)

2.1.2 Write the answer to QUESTION 2.2.1 in interval notation.(1)

2018 November Paper 1 Q 2.2

$$-11 \leq 3m - 8 < 4$$

2.2.1 Solve for m (3)

2.2.2 Hence, write down the number of integers that will satisfy the inequality (1)

