

Geometry – Past Papers - Questions & Solutions**November 2008****QUESTION 7**

7.1 Complete the statements below by filling in the missing word(s) so that the statements are CORRECT:

7.1.1 The angle subtended by a chord at the centre of a circle is (1)

7.1.2 The angle between the tangent and a chord is (1)

7.1.3 The opposite angles of a cyclic quadrilateral are (1)

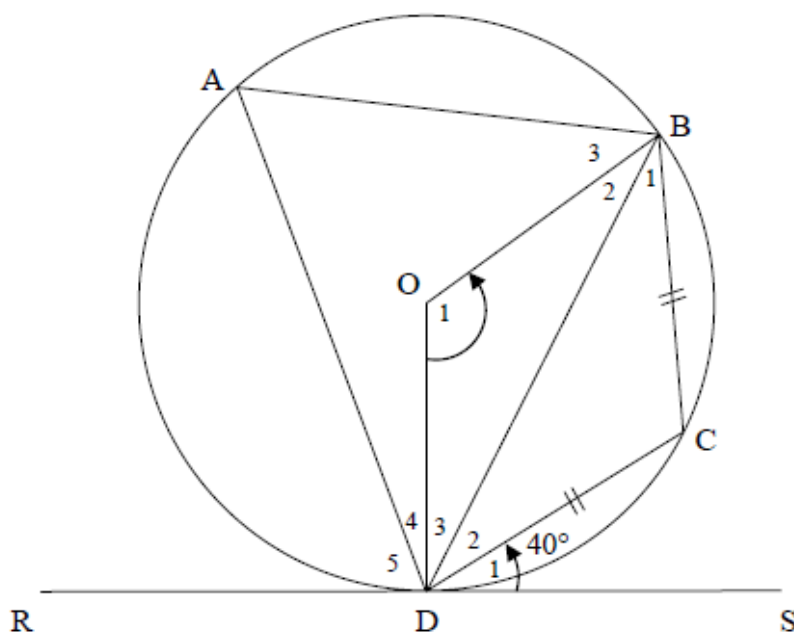
7.2 In the figure below, RDS is a tangent to circle O at D. If $BC = DC$ and $\hat{CDS} = 40^\circ$, calculate, with reasons, the measures of:

7.2.1 $\hat{BDC}$ (2)

7.2.2 $\hat{C}$ (1)

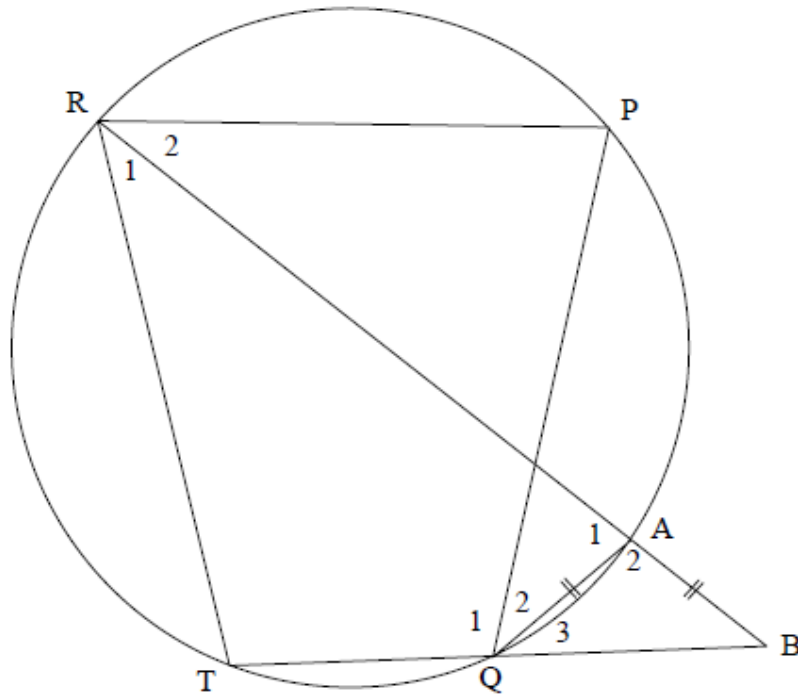
7.2.3 $\hat{A}$ (1)

7.2.4 $\hat{O_1}$ (2)

**[9]**

QUESTION 8

In the diagram below, points R, P, A, Q and T lie on a circle. RA bisects $\hat{R}$ and $AB = AQ$. RA and TQ produced meet at B. RA and TQ produced meet at B.



Prove that:

- 8.1 AQ bisects $\hat{PQB}$ (3)
 - 8.2 $TR = TB$ (2)
 - 8.3 $\hat{P} = \hat{TRP}$ (3)
- [8]

QUESTION 7

7.1.1 equal to twice the angle subtended by the same chord at the circle.

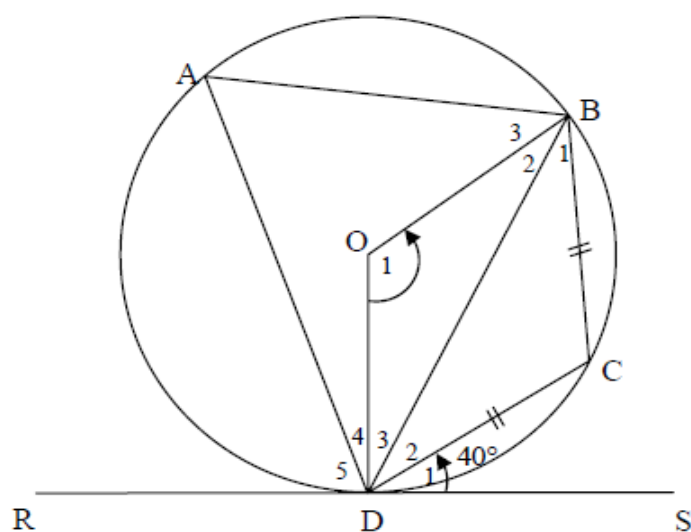
✓ answer (1)

7.1.2 equal to the angle subtended by the same chord in the alternate segment.

✓ answer (1)

7.1.3 supplementary.

✓ answer (1)

7.2.1 $\hat{D}_1 = \hat{B}_1 = 40^\circ$... (angle between tangent and chord) $\therefore \hat{D}_2 = \hat{B}_1 = 40^\circ$... (CD = CB)✓ statement & reason
✓ statement

(2)

7.2.2 $\therefore \hat{C} = 180^\circ - (40^\circ + 40^\circ)$
 $= 100^\circ$... (angle sum of triangle)✓ statement $\hat{C} = 100^\circ$

(1)

7.2.3 $\hat{A} = 180^\circ - 100^\circ$
 $= 80^\circ$ (Opposite angles of a cyclic quad are supp.)✓ statement $\hat{A} = 80^\circ$

(1)

7.2.4 $\hat{O}_1 = 2\hat{A} = 160^\circ$ (angle at the centre is twice...)✓ statement $\hat{O}_1 = 160^\circ$
✓ reason

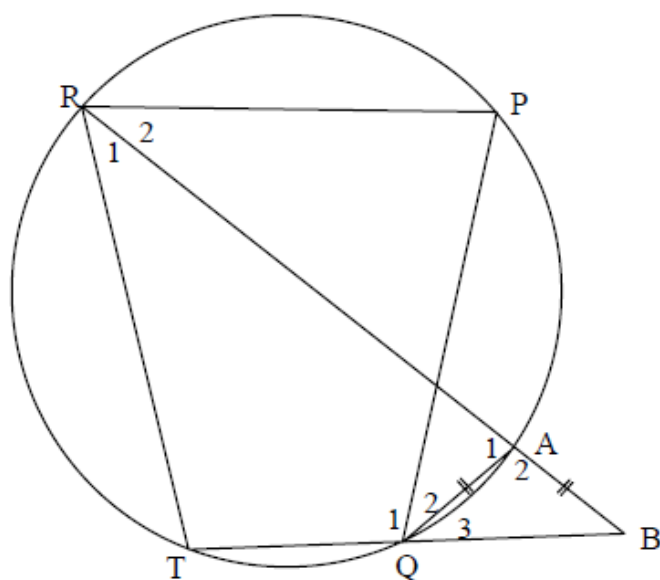
(2)

ALTERNATIVEFrom 7.2.1 $\hat{D}_2 = \hat{B}_1 = 40^\circ$ Now $\hat{D}_3 = 90^\circ - (40^\circ + 40^\circ) = 10^\circ$... (tan $\perp$ radius) $\therefore \hat{O}_1 = 180^\circ - (10^\circ + 10^\circ) = 160^\circ$... (sum of angles in triangles)✓ $\hat{D}_3 = 10^\circ$ ✓ $\hat{O}_1 = 160^\circ$

(2)

[9]

QUESTION 8



8.1 $\hat{Q}_3 = \hat{R}_1 = \hat{R}_2 = x$... (ext angle of cyclic quad...) and
(RA bisects $\hat{R}$)

$\hat{R}_2 = \hat{Q}_2 = x$... (angles in the same segment)

Now $\hat{Q}_2 = \hat{Q}_3$

OR

$\hat{Q}_2 + \hat{Q}_3 = \hat{R}_1 + \hat{R}_2$ (ext angle of cyclic quad.)

but $\hat{Q}_2 = \hat{R}_2 = \hat{R}_1$ (angles in same segment, RA bisect...)

$\therefore \hat{Q}_3 = \hat{Q}_2$

OR

$\hat{Q}_2 + \hat{Q}_2 = \hat{R}_1 + \hat{R}_2$ (ext angle cyclic quad.)

but $\hat{Q}_2 = \hat{R}_2$ (angles in same segment)

$\Rightarrow \hat{Q}_3 = \hat{R}_1$

but $\hat{R}_1 = \hat{R}_2 = \hat{Q}_1$ (given)

$\Rightarrow \hat{Q}_3 = \hat{Q}_2$

$\therefore$ AQ bisects $\hat{PQB}$

8.2 $\hat{Q}_3 = \hat{B} = x$... (angles opp equal sides, $AQ = AB$)

$\hat{R}_1 = \hat{B} = x$... (from 8.1)

$\therefore TR = TB$ (sides opp equal angles)

✓ $\hat{R}_1 = \hat{R}_2$

✓ reason

✓ $\hat{R}_2 = \hat{Q}_2 = x$

If no valid conclusion
2/3

(3)

Follow
candidates'
argument.

To get full marks
candidate must
reach a valid
conclusion

✓ $\hat{Q}_3 = \hat{B} = x$

✓ $\hat{R}_1 = \hat{B} = x$

(2)

8.3 $\hat{P} = \hat{A}_1$ ($\angle$ in same segment)
 $\hat{A}_1 = \hat{Q}_3 + \hat{B}$ (ext $\angle$ of $\triangle ABC$ = sum into opp $\angle$'s)
 $\hat{Q}_3 + \hat{B} = 2\hat{Q}_3$ ($\hat{Q}_3 = \hat{B}$ $\angle$'s opp equal sides)
 $2\hat{Q}_3 = 2\hat{R}_1$ (from 8.1)
 $2\hat{R}_1 = \hat{PRT}$ (given)

OR

$\hat{TRP} = 2x$ (from above)
 $\hat{A}_1 = \hat{Q}_3 + \hat{B} = 2x$ (exterior angle of triangle)
 And $\hat{P} = \hat{A}_1 = 2x$ (angles in the same segment)
 $= \hat{TRP}$

✓ $\hat{P} = \hat{A}_1 = 2x$

✓ $\hat{A}_1 = \hat{Q}_3 + \hat{B} = 2x$

✓ $2\hat{Q}_3 = 2\hat{R}_1$

✓ $\hat{R}_1 + \hat{R}_2 = 2x$

✓ $\hat{A}_1 = \hat{Q}_3 + \hat{B} = 2x$

✓ $\hat{P} = \hat{A}_1 = 2x$

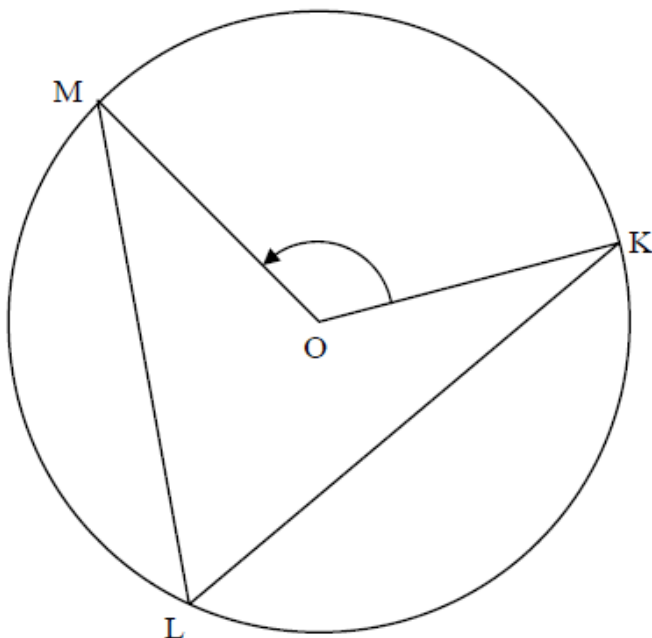
(3)

[8]

November 2009

QUESTION 8

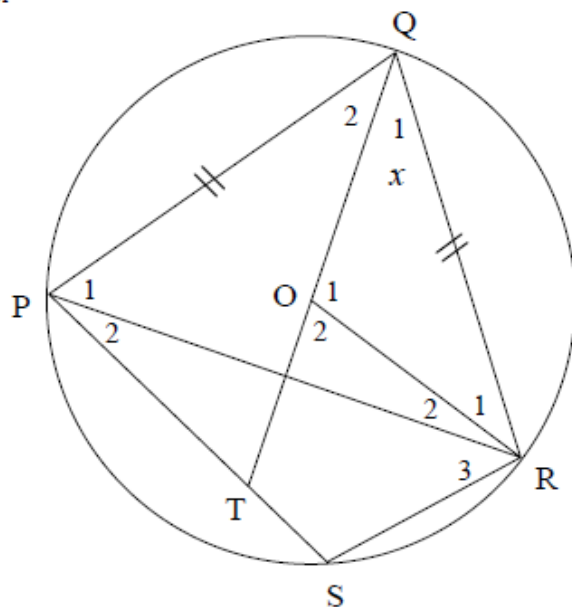
- 8.1 In the diagram below, O is the centre of the circle. K, L and M are points on the circumference of the circle.



Prove that the obtuse angle at O, $\hat{KOM} = 2\hat{KLM}$.

(6)

- 8.2 In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS. $PQ = QR$ and $\hat{Q}_1 = x$.

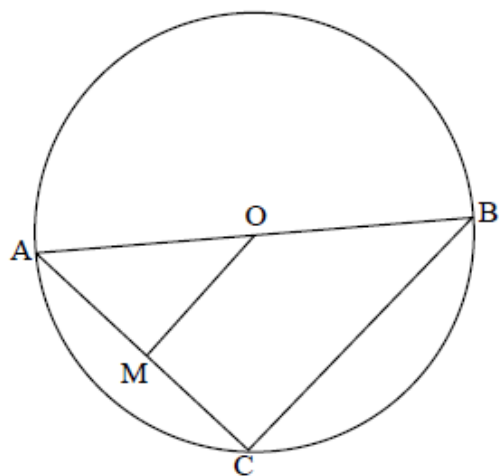


- 8.2.1 Calculate, with reasons, $\hat{P}_1$ in terms of x . (3)
- 8.2.2 Show that TQ bisects $\hat{PQR}$. (3)
- 8.2.3 Show that STOR is a cyclic quadrilateral. (3)

[15]

QUESTION 9

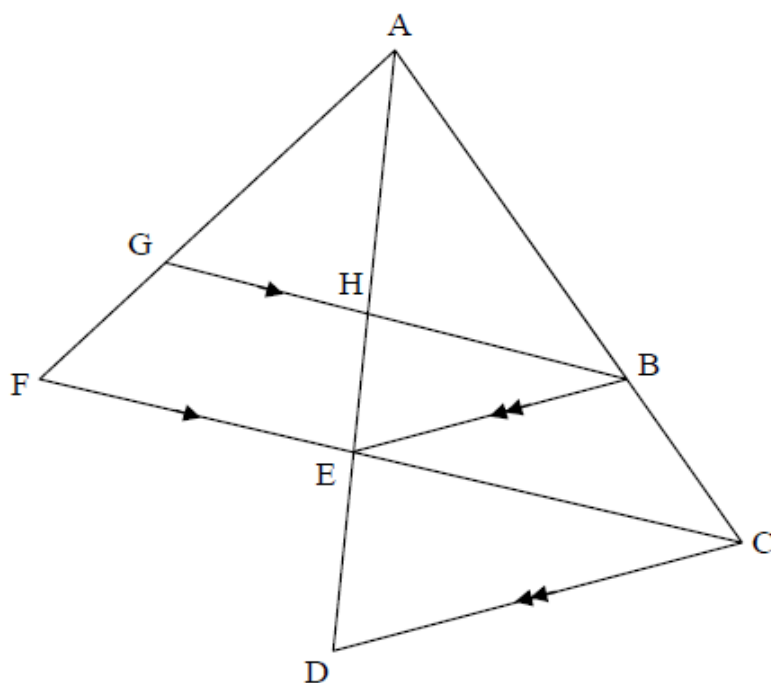
O is the centre of the circle below. $OM \perp AC$. The radius of the circle is equal to 5 cm and $BC = 8$ cm.



- 9.1 Write down the size of $\hat{BCA}$. (1)
- 9.2 Calculate: (3)
- 9.2.1 The length of AM, with reasons (3)
- 9.2.2 Area $\triangle AOM$: Area $\triangle ABC$ (3)

QUESTION 10

In the figure below, $GB \parallel FC$ and $BE \parallel CD$. $AC = 6$ cm and $\frac{AB}{BC} = 2$.



10.1 Calculate with reasons:

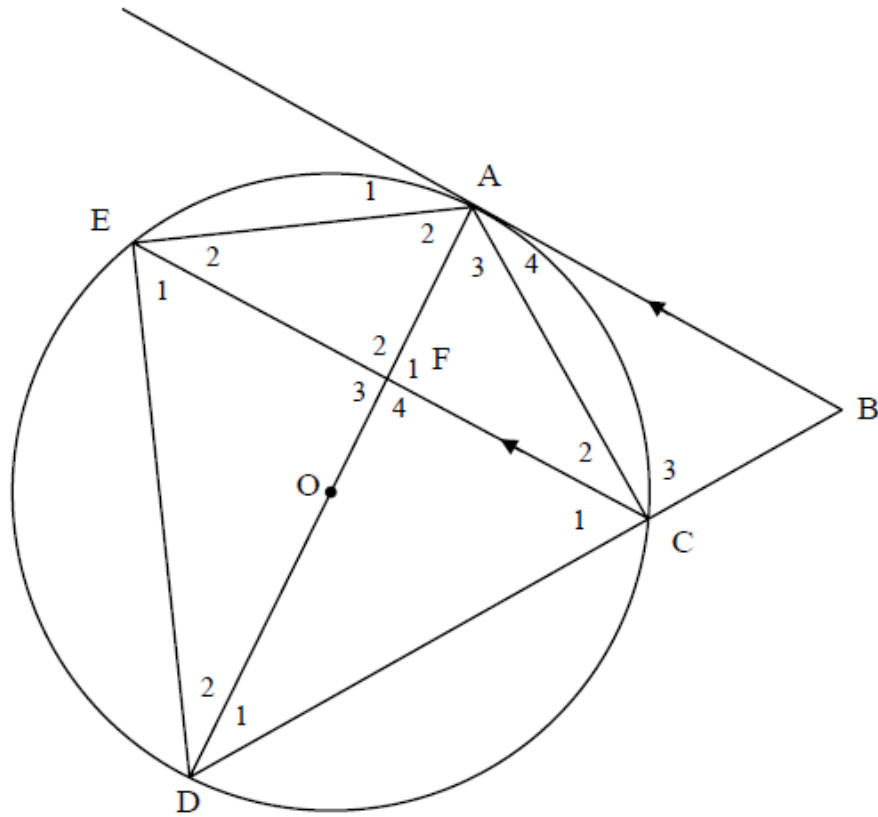
10.1.1 $AH : ED$ (4)

10.1.2 $\frac{BE}{CD}$ (2)

10.2 If $HE = 2$ cm, calculate the value of $AD \times HE$. (2)
[8]

QUESTION 11

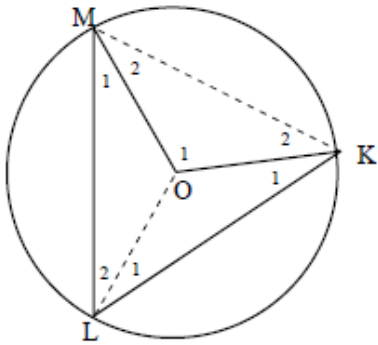
In the figure below, AB is a tangent to the circle with centre O. $AC = AO$ and $BA \parallel CE$. DC produced, cuts tangent BA at B.



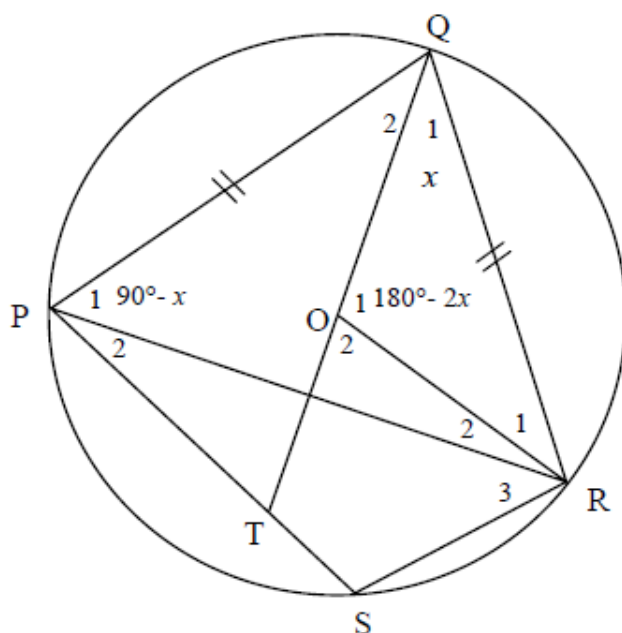
- 11.1 Show $\hat{C}_2 = \hat{D}_1$. (3)
 - 11.2 Prove that $\triangle ACF \parallel \triangle ADC$. (3)
 - 11.3 Prove that $AD = 4AF$. (4)
- [10]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 8

8.1	Construct OL and extend to J $\hat{L}_2 = \hat{LMO} \quad (MO = OL)$ $\hat{O}_2 = \hat{LMO} + \hat{L}_2 \quad (\text{ext } \angle \text{ of } \triangle OML)$ $\hat{O}_2 = 2\hat{L}_2$ Similarly $\hat{O}_1 = 2\hat{L}_1$ $\hat{O}_1 + \hat{O}_2 = 2\hat{L}_1 + 2\hat{L}_2$ $\hat{KOM} = 2(\hat{L}_1 + \hat{L}_2)$ $\hat{KOM} = 2\hat{KLM}$	✓ construction ✓ S/R ✓ S/R ✓ $\hat{O}_1 = 2\hat{L}_1$ ✓ ✓ $\hat{O}_1 + \hat{O}_2 = 2\hat{L}_1 + 2\hat{L}_2$ ✓ $\hat{KOM} = 2(\hat{L}_1 + \hat{L}_2)$ (6)
	If candidate writes: $\hat{KOM} = 2\hat{KLM}$ ($\angle$ circ centre = $2\angle$ at circumference): 0 / 6 Note: Construction can be stated or drawn.	
	OR Join M to K and O to L $\hat{L}_2 = \hat{M}_1 = x \quad (MO = OL)$ $\hat{K}_2 = \hat{M}_2 = y \quad (MO = OK)$ $\hat{K}_1 = \hat{L}_1 = z \quad (OL = OK)$ $\hat{O}_1 = 180^\circ - 2y \quad (\angle \text{ sum } \triangle)$ $2y + 2z + 2x = 180^\circ \quad (\angle \text{ sum } \triangle)$ $2z + 2x = 180^\circ - 2y$ $2(z + x) = 180^\circ - 2y$ $2(\hat{L}_1 + \hat{L}_2) = \hat{KOM}$ $\hat{KOM} = 2\hat{KLM}$	

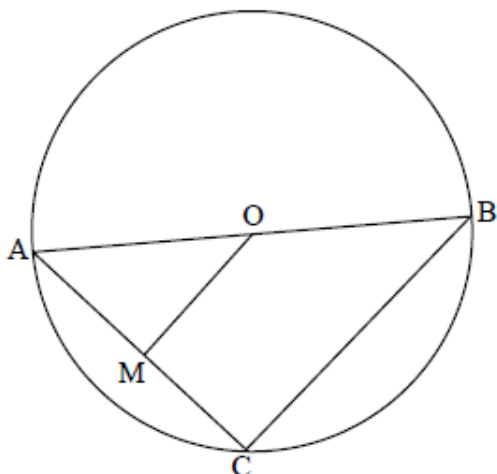
- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.



8.2.1	$\hat{R}_1 = x$ ($\angle$'s opp = radii) $\hat{O}_1 = 180^\circ - 2x$ ($\angle$ sum in ΔQRT) $\hat{P}_1 = 90^\circ - x$ ($\angle$ circle centre = twice $\angle$ at circumference)	✓ S/R ✓ $\hat{O}_1 = 180^\circ - 2x$ ✓ S/R $\hat{P}_1 = 90^\circ - x$ (3)
8.2.2	$PQ = QR$ (given) $\hat{QRP} = 90^\circ - x$ ($\angle$ opp = sides in Δ) $\hat{PQR} = 2x$ ($\angle$ sum in ΔPQR) $x + \hat{Q}_2 = 2x$ $\hat{Q}_2 = x$ TQ bisects $\hat{PQR}$	✓ S/R ✓ Statement ✓ $\hat{Q}_2 = x$ (3)
8.2.3	$\hat{PQR} = 2x$ $\hat{S} = 180^\circ - 2x$ (opp $\angle$'s of cyclic quad are supplementary) $\hat{O}_1 = 180^\circ - 2x$ $\hat{O}_1 = \hat{S}$ $STOR$ is a cyclic quadrilateral ... (converse – ext $\angle$ of cyclic quad = int opp. $\angle$) (ext $\angle$ quad = int opp $\angle$)	✓ S/R ✓ Statement ✓ Reason (3) [15]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

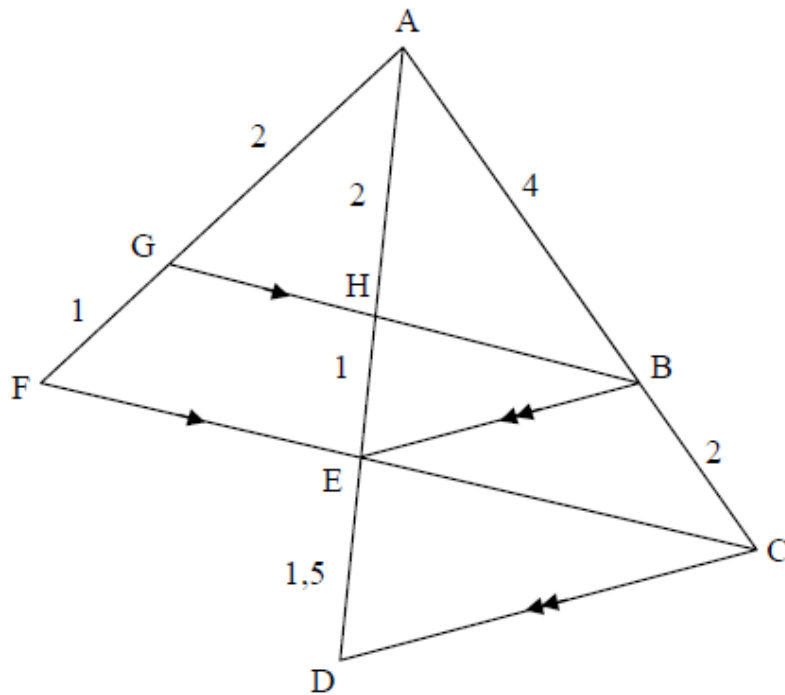
QUESTION 9



9.1	$\hat{BCA} = 90^\circ$ ($\angle$'s in a semi-circle)	✓ answer (1)
9.2.1	$AC = \sqrt{10^2 - 8^2}$ (Pythagoras) $= \sqrt{36}$ $= 6$ $AM = 3$ (line from circle centre $\perp$ chord bisects chord OR midpoint theorem)	✓ diameter = 10 ✓ AC ✓ AM (3)
9.2.2	$OM = \sqrt{5^2 - 3^2}$ (Pythagoras) $= 4$ (OR midpoint theorem) Area $\triangle AOM$: Area $\triangle ABC$ $= \frac{1}{2} \cdot 4 \cdot 3 : \frac{1}{2} \cdot 8 \cdot 6$ $= 6 : 24$ $= 1 : 4$ OR Area $\triangle AOM$: Area $\triangle ABC$ $= \frac{1}{2} \cdot AM \cdot OM \cdot \sin \hat{OAM} : \frac{1}{2} \cdot AC \cdot AB \cdot \sin \hat{BAC}$ $= \frac{1}{2} \cdot 4 \cdot 3 : \frac{1}{2} \cdot 8 \cdot 6$ $= 6 : 24$	✓ OM ✓ substitution ✓ answer (3) [7]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

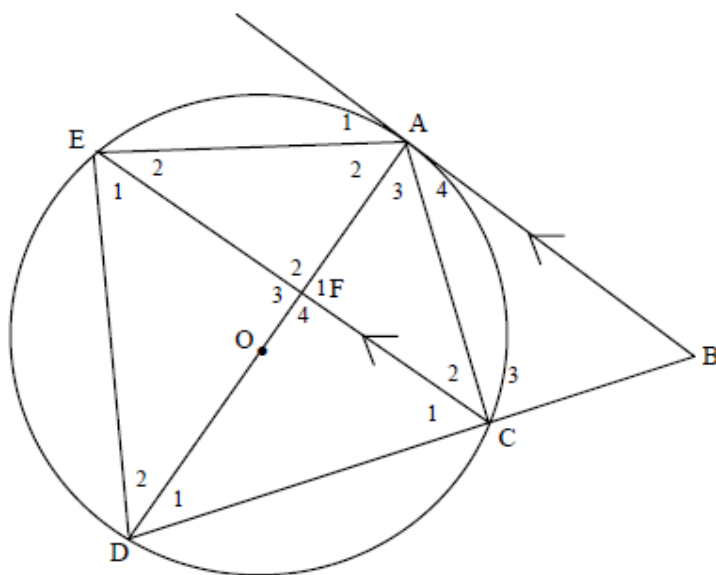
QUESTION 10



10.1.1	$\frac{AH}{HE} = \frac{2}{1}$ $AH = 2y$ $HE = y$ $\frac{AE}{ED} = \frac{2}{1}$ $ED = 1,5y$ $\frac{AH}{ED} = \frac{2}{1,5}$ $\frac{AH}{ED} = \frac{4}{3}$	$(GHB \parallel FEC)$ If learner stops at 2 : 1,5 : no penalty	✓ statement ✓ reason ✓ ED = 1,5y ✓ answer (4)
10.1.2	$\frac{BE}{CD} = \frac{4}{6}$ $= \frac{2}{3}$	$(\triangle AEB \parallel \triangle ADC)$	✓ answer ✓ reason (2)
10.2	$HE = 2 \text{ cm}$ $AH = 4 \text{ cm}$ $ED = 3 \text{ cm}$ $AD \cdot HE = (AH + HE + ED) \cdot HE$ $= (4 + 2 + 3) \cdot (2)$ $= 18$	(given)	✓ AH and ED ✓ AD = AH + HE + ED (2) [8]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 11

[illegible]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

11.2	In $\triangle ACF$ and $\triangle ADC$ 1. $\hat{A}_3$ is common 2. $\hat{C}_2 = \hat{D}_1$ (proved) $\triangle ACF \parallel \triangle ADC$ ($\angle\angle\angle$) OR In $\triangle ACF$ and $\triangle ADC$ 1. $\hat{A}_3$ is common 2. $\hat{C}_2 = \hat{D}_1$ (proved) 3. $\hat{F}_1 = \hat{C}_D$ (remaining $\angle$s in triangles) $\triangle ACF \parallel \triangle ADC$	✓ $\hat{A}_3$ is common ✓ $\hat{C}_2 = \hat{D}_1$ ✓ Reason (3)
11.3	$\frac{AF}{AC} = \frac{AC}{AD}$ (sim Δ's $\therefore$ sides in proportion) $AF = \frac{AC \cdot AC}{AD}$ $AC = AO = \frac{1}{2}AD$ (2radius = diameter) $AF = \frac{\frac{1}{2}AD \cdot \frac{1}{2}AD}{AD}$ $AF = \frac{AD}{4}$ $4AF = AD$ OR $\triangle AOC$ is equilateral $\therefore \hat{AOC} = \hat{A}_3 = 60^\circ$ $\cos 60^\circ = \frac{AF}{AC} = \frac{1}{2}$ $AF = \frac{1}{2}AC = \frac{1}{2}AO$ $AF = \frac{1}{2}(\frac{1}{2}AD)$ (2radius = diameter) $AF = \frac{1}{4}AD$ $AD = 4AF$	✓ statement ✓ Statement ✓ Simplification ✓ Substitution (4) ✓ S/R ✓ Statement ✓ Simplification ✓ Substitution (4) [10]

February 2010

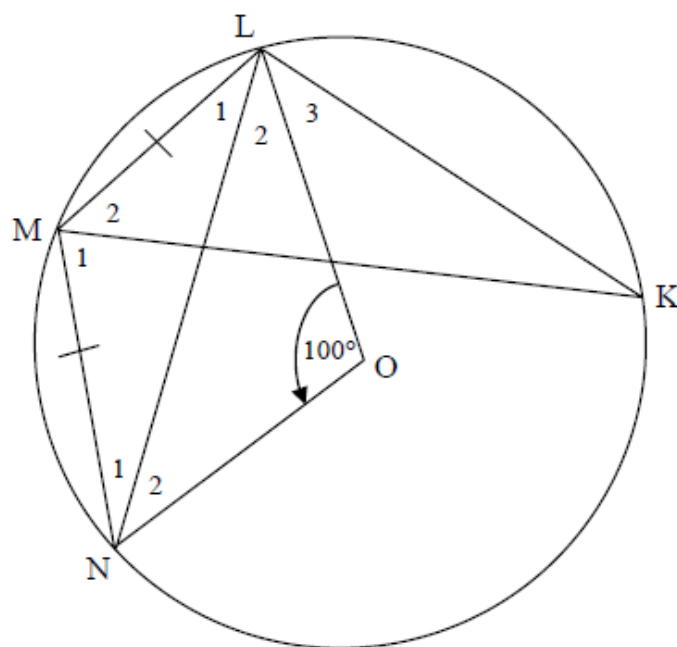
QUESTION 8

8.1 Complete the statement:

The sum of the angles around a point is ...

(1)

8.2 In the figure below, O is the centre of the circle. K, L, M and N are points on the circumference of the circle such that $LM = MN$. $\angle LON = 100^\circ$.



Determine, with reasons, the values of the following:

8.2.1 $\angle LMN$

(3)

8.2.2 $\angle LKM$

(3)

[7]

QUESTION 9

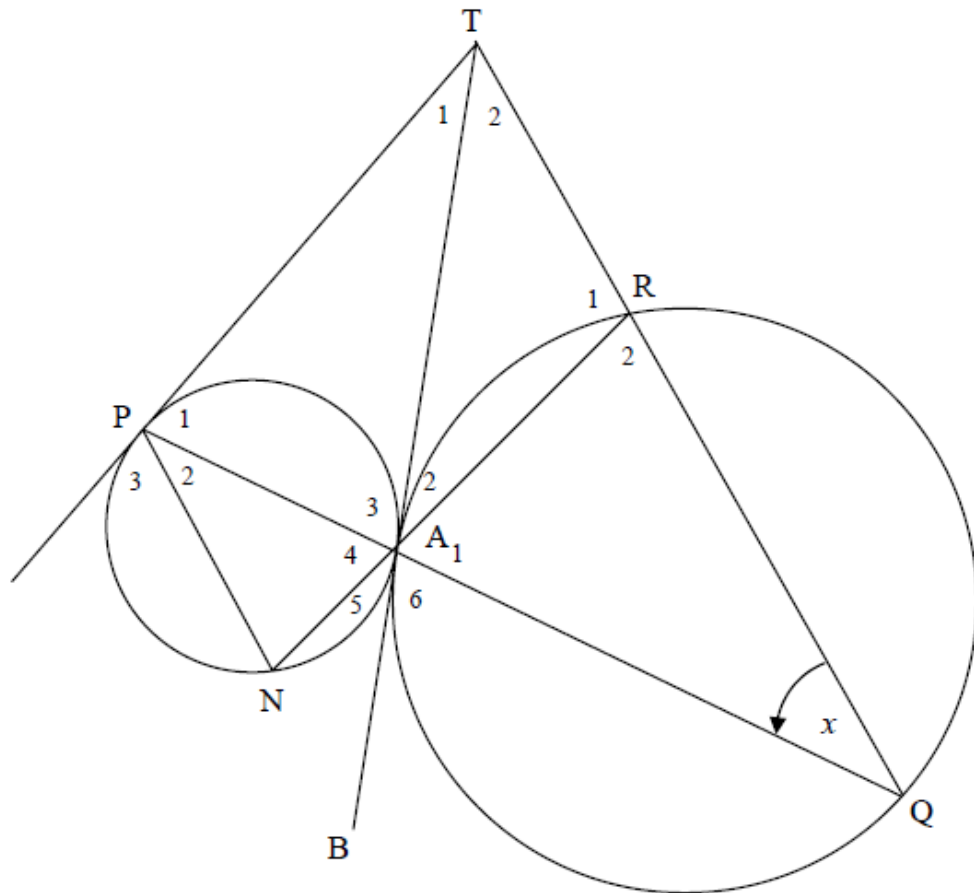
9.1 Complete the following statement:

The angle between the tangent and the chord ...

(1)

9.2 In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines. PAQ, QRT and NAR are straight lines.

Let $\hat{Q} = x$.



9.2.1 Name, with reasons, THREE other angles equal to x .

(5)

9.2.2 Prove that APTR is a cyclic quadrilateral.

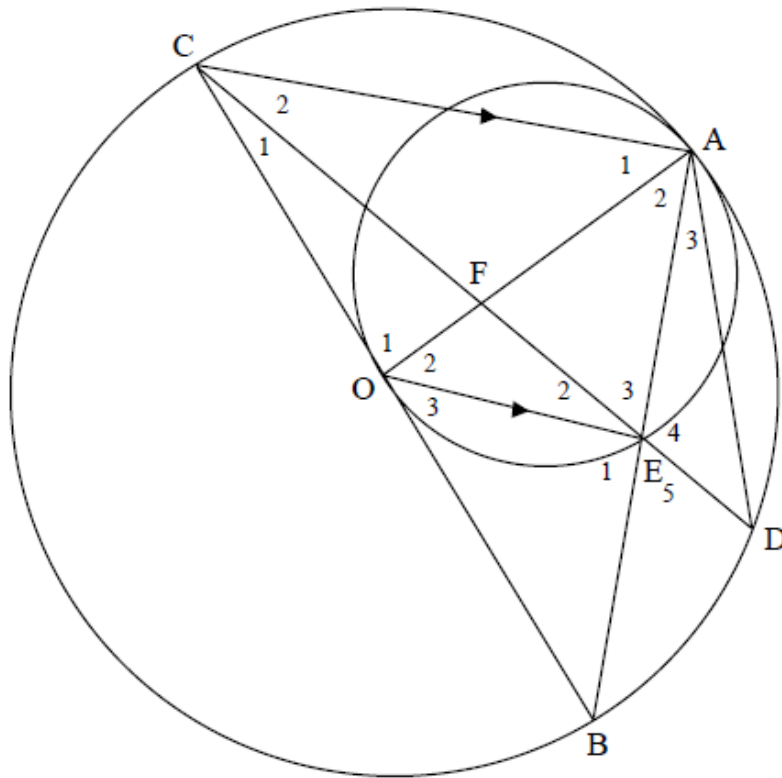
(5)

[11]

QUESTION 10

Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle.

$OE \parallel CA$.



- 10.1 Prove, with reasons, that $AE = BE$. (2)
- 10.2 Prove that $\triangle AED \parallel \triangle CEB$. (3)
- 10.3 Hence, or otherwise, show that $AE^2 = DE \cdot CE$. (2)
- 10.4 If $AE \cdot EB = EF \cdot EC$, show that E is the midpoint of DF. (3)

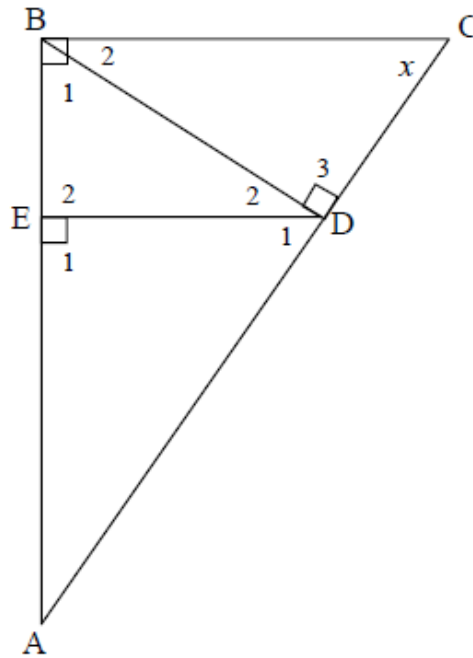
[10]

QUESTION 11

$\triangle ABC$ is a right-angled triangle with $\hat{B} = 90^\circ$. D is a point on AC such that $BD \perp AC$ and E is a point on AB such that $DE \perp AB$. E and D are joined.

$$AD : DC = 3 : 2.$$

$$AD = 15 \text{ cm.}$$



- 11.1 Prove that $\triangle BDA \parallel \triangle CDB$. (3)
- 11.2 Calculate BD (Leave your answer in surd form). (3)
- 11.3 Calculate AE (Leave your answer in surd form). (6)
- [12]**

QUESTION 8

8.1	Equal to 360°	✓ answer (1)
8.2.1	$\text{reflex } \hat{O} = 360^\circ - 100^\circ = 260^\circ$ ($\angle$'s round a point) $2\hat{LMN} = \text{reflex } \hat{O}$ ($\angle$ circ centre = 2 $\angle$ circumference) $\therefore \hat{LMN} = \frac{260^\circ}{2} = 130^\circ$	✓ reflex $\hat{O} = 260^\circ$ ✓ reason ✓ $\hat{LMN} = 130^\circ$ (3)
8.2.2	$\hat{N}_1 = \frac{180^\circ - 130^\circ}{2} = 25^\circ$ (base angles LM = MN) $\therefore \hat{K} = 25^\circ$ (angles in same segment)	✓ = 25° ✓ answer ✓ reason (3) [7]

QUESTION 9

9.1	Is equal to the angle subtended by the chord in the alternate segment	✓ answer (1)
9.2.1	$\hat{A}_2 = x$ (tangent chord theorem) $\hat{A}_5 = x$ (vertically opp. angles) $\hat{P}_2 = x$ (tangent chord theorem)	✓ answer ✓ reason ✓ answer ✓ answer ✓ reason (5)
9.2.2	PT = TA (tangents drawn from same point) $\hat{P}_1 = \hat{A}_3$ (angles opp equal sides); PT = TA $\hat{A}_3 = \hat{A}_6$ (vertical opp angles) $\hat{A}_6 = \hat{R}_2$ (tangent chord theorem) $\therefore \hat{P}_1 = \hat{R}_2$ $\therefore$ APTR is a cyclic quadrilateral (converse : ext angle of cycl.quad.)	✓ statement ✓ statement ✓ statement ✓ equal angles ✓ reason (5) [11]

QUESTION 10

10.1	$OC = OB$ (radii) Hence $AE = BE$ (midpoint theorem) OR $\hat{CAB} = 90^\circ$ (diameter subtends right angle) $\hat{OEB} = \hat{CAB} = 90^\circ$ (corresponding angles $AC \parallel OE$) $\therefore AE = BE$ (line drawn from centre, perpend. to chord or midpoint theorem)	✓ $OC = OB$ ✓ conclusion and reason (2) ✓ $\hat{OEB} = \hat{CAB} = 90^\circ$ ✓ conclusion and reason (2)
10.2	In $\triangle AED$ and $\triangle CEB$ $\hat{AED} = \hat{CEB}$ (vertically opp angles) $\hat{D} = \hat{B}$ (angles in same segment) $\hat{A}_3 = \hat{C}_1$ (angles in same segment) $\therefore \triangle AED \sim \triangle CEB$ (equi - angular)	✓ statement ✓ statement ✓ statement (3)
10.3	$\frac{AE}{DE} = \frac{CE}{BE}$ (deduction) $AE \cdot BE = DE \cdot CE$ but $AE = BE$ (proven) $\therefore AE^2 = DE \cdot CE$	✓ $\frac{AE}{DE} = \frac{CE}{BE}$ ✓ $AE = BE$ (2)
10.4	$AE \cdot BE = DE \cdot CE$ But $AE \cdot BE = EF \cdot CE$ $\therefore DE \cdot CE = EF \cdot CE$ $DE = EF$ $\therefore E$ is the midpoint of DF OR $AE^2 = DE \cdot CE$ $AE \cdot BE = EF \cdot CE$ $\Rightarrow AE^2 = EF \cdot CE$ $\therefore EF \cdot CE = DE \cdot CE$ $EF = DE$ $\therefore E$ is the midpoint of DF	✓ $AE \cdot BE = DE \cdot CE$ ✓ $AE \cdot BE = EF \cdot CE$ ✓ $DE \cdot CE = EF \cdot CE$ (3) ✓ $AE \cdot BE = EF \cdot CE$ ✓ $\Rightarrow AE^2 = EF \cdot CE$ ✓ $\therefore EF \cdot CE = DE \cdot CE$ (3) [10]

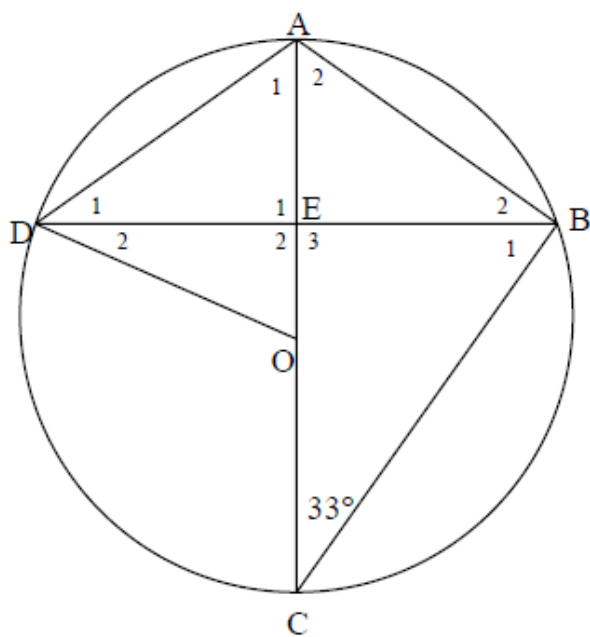
QUESTION 11

11.1	In $\triangle BDA$ and $\triangle CDB$ $\hat{BDA} = \hat{CDB} = 90^\circ$ $\hat{B}_1 = \hat{C}$ (both = x) $\hat{A} = \hat{B}_2$ (remaining angles) $\triangle BDA \text{ } \parallel \triangle CDB$ (equiangular)	$\checkmark \hat{BDA} = \hat{CDB}$ $\checkmark \hat{B}_1 = \hat{C}$ $\checkmark \hat{A} = \hat{B}_2$ (3)
11.2	$AD : DC = 3 : 2$ $\therefore CD = \frac{2}{3} \times 15 = 10$ But $\frac{BD}{AD} = \frac{CD}{BD}$ $\therefore BD^2 = AD \cdot CD$ $BD^2 = 15 \cdot 10$ $= 150$ $BD = \sqrt{150}$	$\checkmark CD$ $\checkmark \frac{BD}{AD} = \frac{CD}{BD}$ $\checkmark BD$ (3)
11.3	$AB^2 = (\sqrt{150})^2 + (15)^2$ (Theorem of Pythagoras) $= 150 + 225$ $= 375$ $AB = \sqrt{375}$ $\hat{E}_1 = \hat{ABC} = 90^\circ$ $\therefore BC \parallel DE$ $\frac{AE}{AB} = \frac{AD}{AC}$ (proportion theorem) $\frac{AE}{\sqrt{375}} = \frac{15}{25}$ $AE = \frac{15 \times \sqrt{375}}{25} = \sqrt{135} = 3\sqrt{15}$	$\checkmark$ using Pythagoras $\checkmark$ answer $\checkmark = 90^\circ$ $\checkmark \therefore BC \parallel DE$ $\checkmark \frac{AE}{AB} = \frac{AD}{AC}$ $\checkmark$ answer (6) [11]

November 2010

QUESTION 7

In the diagram below AC is a diameter of the circle with centre O. AC and chord BD intersect at E. AB, BC and AD are also chords of the circle. OD is joined. $AE \perp BD$.



If $\hat{C} = 33^\circ$, calculate, with reasons, the size of:

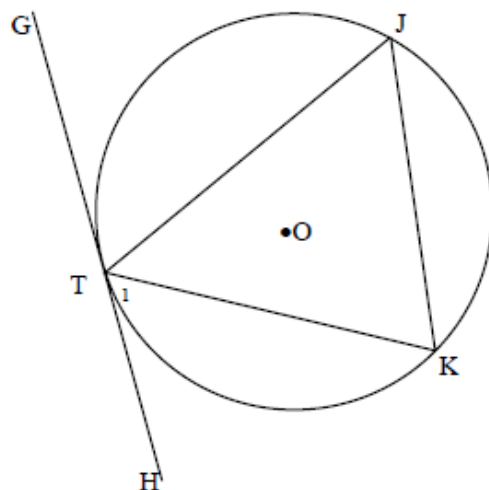
7.1 $\hat{A}_1$ (3)

7.2 $\hat{D}_2$ (2)

7.3 Show that AE bisects $\hat{DAB}$ (3)
[8]

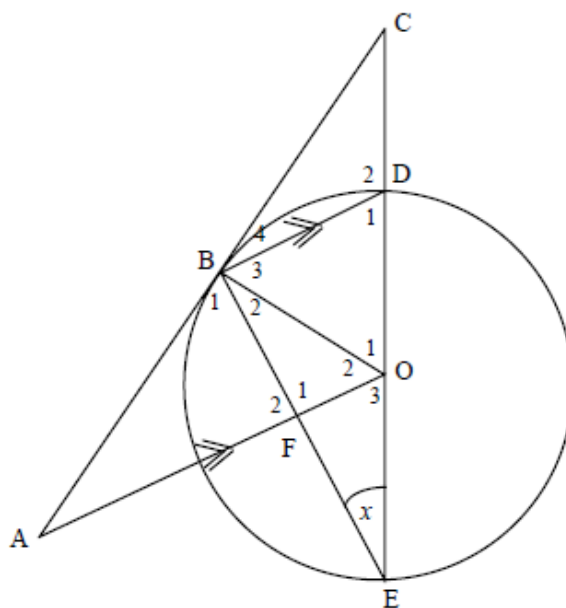
QUESTION 8

- 8.1 In the diagram below O is the centre of the circle. GH is a tangent to the circle at T. J and K are points on the circumference of the circle. TJ, TK and JK are joined.



Prove the theorem that states $\hat{T}_1 = \hat{TJK}$. (5)

- 8.2 ED is a diameter of the circle, with centre O. ED is extended to C. CA is a tangent to the circle at B. AO intersects BE at F. $BD \parallel AO$. $\hat{E} = x$.



- 8.2.1 Write down, with reasons, THREE other angles equal to x . (4)
- 8.2.2 Determine, with reasons, $\hat{CBE}$ in terms of x . (3)
- 8.2.3 Prove that F is the midpoint of BE. (4)
- 8.2.4 Prove that $\triangle CBD \parallel \triangle CEB$. (2)
- 8.2.5 Prove that $2EF \cdot CB = CE \cdot BD$. (3)

[21]

QUESTION 9

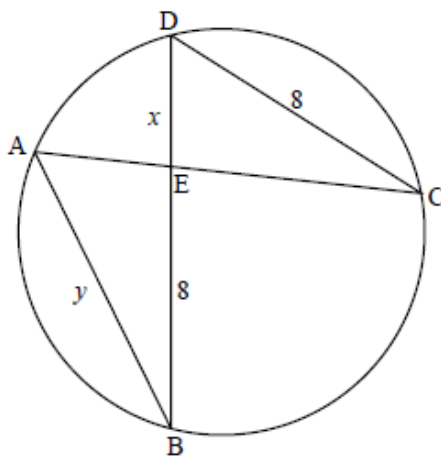
In the diagram below A, B, C and D are points on the circumference of the circle.

BD and AC intersect at E. Also,

$EB = 8 \text{ cm}$,

$DC = 8 \text{ cm}$ and

$AE : EC = 4 : 7$.

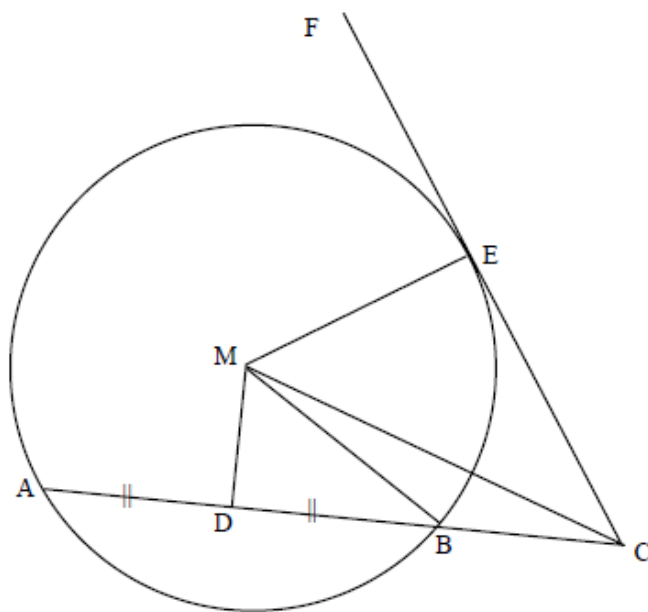


If $DE = x$ units and $AB = y$ units, calculate x and y .

[6]

QUESTION 10

In the diagram below M is the centre of the circle. FEC is a tangent to the circle at E. D is the midpoint of AB.



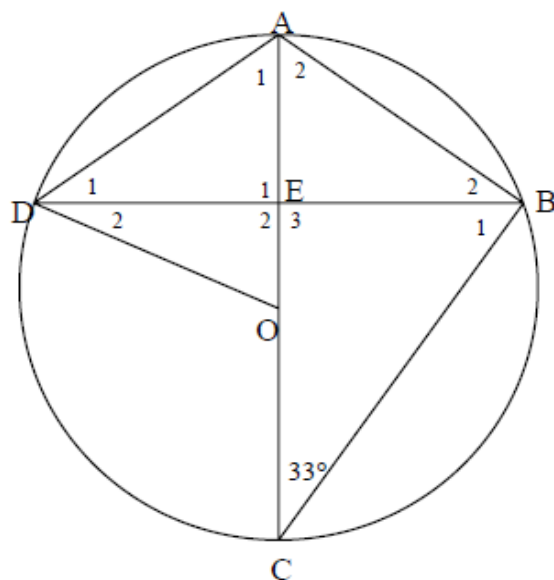
10.1 Prove MDCE is a cyclic quadrilateral. (3)

10.2 Prove that $MC^2 = MB^2 + DC^2 - DB^2$. (3)

10.3 Calculate CE if $AB = 60 \text{ mm}$, $ME = 40 \text{ mm}$ and $BC = 20 \text{ mm}$. (4)

[10]

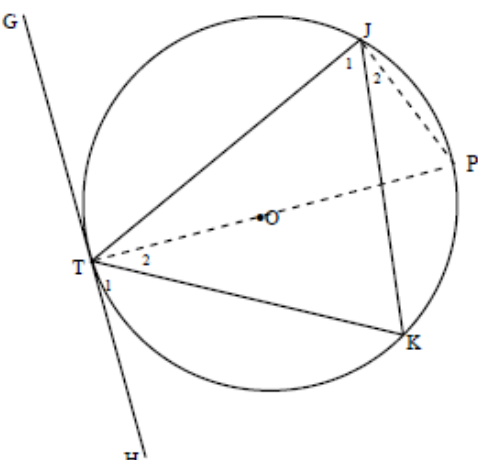
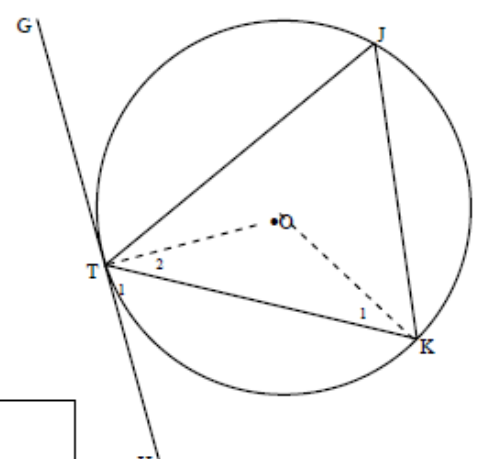
QUESTION 7



7.1	$\hat{D}_1 = 33^\circ$ ($\angle$ in same segment) $\hat{A}ED = 90^\circ$ (given) $\hat{A}_1 = 57^\circ$ ($\angle$ sum Δ) OR $\hat{B}EC = 90^\circ$ (given) $\hat{B}_1 = 57^\circ$ ($\angle$ sum Δ) $\hat{A}_1 = 57^\circ$ ($\angle$ in same segment) OR $DE = EB$ (line from circ cent $\perp$ ch bis ch) AE is common $\hat{A}ED = \hat{E}_1 = 90^\circ$ (given) $\Delta AED \equiv \Delta AEB$ (SAS) $\hat{A}BC = 90^\circ$ ($\angle$s in semi-circle) $\hat{A}_1 = \hat{A}_2 = 57^\circ$ ($\angle$ sum Δ)	$\checkmark \hat{D}_1 = 33^\circ$ $\checkmark \angle$ in same segment $\checkmark \hat{A}_1 = 57^\circ$ (3) $\checkmark \hat{B}_1 = 57^\circ$ $\checkmark \hat{A}_1 = 57^\circ$ $\checkmark \angle$ in same segment (3) $\checkmark DE = EB$ (S/R) $\checkmark \Delta AED \equiv \Delta AEB$ (SAS) $\checkmark$ answer (3)
7.2	$\hat{D}_2 + \hat{D}_1 = 57^\circ$ ($OD = OA = \text{radii}$) $\hat{D}_2 = 24^\circ$ OR $\hat{D}OC = 114^\circ$ ($OD = OA = \text{radii}$) OR $\angle$ at the centre theorem $\hat{E}_2 = 90^\circ$ $\hat{D}_2 = 114^\circ - 90^\circ$ $= 24^\circ$	$\checkmark \hat{D}_2 + \hat{D}_1 = 57^\circ$ $\checkmark$ answer (2) $\checkmark \hat{D}OC = 114^\circ$ $\checkmark$ answer (2)

7.3	$\hat{A}BC = 90^\circ$ ($\angle$ in semi-circle) $\hat{A}_2 = 57^\circ$ ($\angle$ sum Δ) $= \hat{A}_1$ AE bisects $\hat{D}AB$ OR $DE = EB$ (line from circ centre bis ch) AE is common $\hat{E}_1 = \hat{A}EB = 90^\circ$ (given) $\triangle ADE \equiv \triangle ABE$ (SAS) $\hat{A}_2 = \hat{A}_1$	$\checkmark \hat{A}BC = 90^\circ$ $\checkmark \angle$ in semi-circle $\checkmark \hat{A}_2 = \hat{A}_1$ or AE bisects $\hat{D}AB$ (3) $\checkmark DE = EB$ (S/R) $\checkmark \triangle AED \equiv \triangle AEB$ (SAS) $\checkmark \hat{A}_2 = \hat{A}_1$ or AE bisects $\hat{D}AB$ (3) [8]
-----	--	---

QUESTION 8

8.1	Draw diameter TP. Join P to J. $\hat{T}_1 + \hat{T}_2 = 90^\circ \quad (\text{tan} \perp \text{diameter})$ $\hat{J}_1 + \hat{J}_2 = 90^\circ \quad (\angle \text{ in semi-circle})$ $\hat{J}_2 = \hat{T}_2 \quad (\angle \text{ in same seg})$ $\hat{TJK} = \hat{T}_1$ OR Draw radii OT and OK Let $\hat{T}_2 = x$ $\hat{K}_1 = x \quad (\angle \text{ opp} = \text{radii})$ $\hat{T}_1 = 90^\circ - x \quad (\text{rad} \perp \text{tan})$ $\hat{TOK} = 180^\circ - 2x \quad (\angle \text{ sum } \Delta)$ $\hat{TJK} = 90^\circ - x \quad (\angle \text{ circ cent})$ $\hat{TJK} = \hat{T}_1 \quad (= 90^\circ - x)$ NOTE: If there is no construction: 0 / 5 marks If candidate changes lettering and states "Similarly": max full marks  	✓ construction ✓ $\hat{T}_1 + \hat{T}_2 = 90^\circ$ ✓ tan $\perp$ diameter ✓ S/R ✓ S/R (5) ✓ construction ✓ $\hat{T}_1 = 90^\circ - x$ ✓ rad $\perp$ tan ✓ S/R ✓ S/R (5) ✓ construction ✓ S/R ✓ S/R ✓ $\hat{T}_1 + \hat{T}_2 = 90^\circ$ ✓ tan $\perp$ radius (5)
-----	--	---

OR Construct OT, OJ and OK $\hat{T}_1 = \hat{J}_1 = x$ (radii) $\hat{T}_2 = \hat{K}_1 = z$ (radii) $\hat{K}_2 = \hat{J}_2 = y$ (radii) $2x + 2y + 2z = 180^\circ$ ($\angle$ sum Δ) $x + y + z = 90^\circ$ $x + y = 90^\circ - z$ $\hat{O}\hat{T}H = 90^\circ$ (rad $\perp$ tan)) $\hat{T}_3 = 90^\circ - z$ $= 90^\circ - (90^\circ - (x + y))$ $= 90^\circ - z$ $= \hat{T}\hat{J}K$		✓ construction ✓ S/R ✓ S ✓ $\hat{T}_3 + \hat{T}_2 = 90^\circ$ ✓ rad $\perp$ tan (5)
--	-------------	--

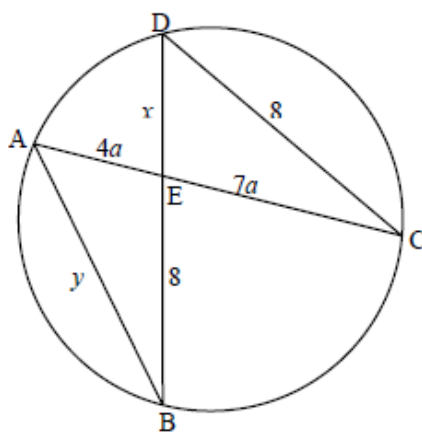
8.2			
8.2.1	$\hat{B}_4 = x$ (tan chord theorem) $\hat{A} = \hat{B}_4 = x$ (corres $\angle$; $BD \parallel AO$) $\hat{B}_2 = x$ ($BO = EO = \text{radii}$)	Note: If start with $\hat{A} = x$ and do not use tan ch th: max 2 marks	✓ $\hat{B}_4 = x$ ✓ tan chord theorem ✓ $\hat{A} = \hat{B}_4 = x$ with reason ✓ $\hat{B}_2 = x$ (4)
8.2.2	$\hat{DBE} = 90^\circ$ ($\angle$ in semi-circle) $\hat{CBE} = 90^\circ + x$ OR $\hat{CBO} = 90^\circ$ (rad $\perp$ tan) $\hat{CBE} = 90^\circ + x$ OR $\hat{O}_1 = 2x$ ($\angle$ circ cent) $\hat{B}_3 = \hat{D}_1 = 90^\circ - x$ (radii) $\hat{CBE} = x + (90^\circ - x) + x = 90^\circ + x$		✓ $\hat{DBE} = 90^\circ$ ✓ $\angle$ in semi-circle ✓ $\hat{CBE} = 90^\circ + x$ (3) ✓ $\hat{CBO} = 90^\circ$ ✓ rad $\perp$ tan ✓ $\hat{CBE} = 90^\circ + x$ (3) ✓ $\hat{O}_1 = 2x$ ✓ $\angle$ circ cent ✓ $\hat{CBE} = 90^\circ + x$ (3)
8.2.3	$\hat{DBE} = 90^\circ$ (proved in 8.2.2) $\hat{BFO} = 90^\circ$ (co-int angles supp; $BD \parallel AO$) $BF = FE$ (line from circ cent $\perp$ ch bisect ch) F is the midpoint of EB		✓ $\hat{DBE} = 90^\circ$ ✓ $\hat{BFO} = 90^\circ$ and reason ✓ $BF = FE$ ✓ line from circ cent $\perp$ ch bisect ch) (4)

	OR $OD = OE$ (radii) $BF = FE$ ($BD \parallel AO$) F is the midpoint of EB OR $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) OF is common $BO = OE$ (radii) $\triangle BOF \equiv \triangle EOF$ (90°HS) $BF = FE$ ($\equiv \Delta s$) OR $\hat{B}_2 = \hat{A} = x$ (proven) $\hat{O}_2$ is common $\triangle AOB \parallel \triangle BOF$ (AAA) $\hat{ABO} = \hat{BFO}$ $\hat{ABO} = 90^\circ$ (proven) $\hat{ABO} = \hat{BFO} = 90^\circ$ $BF = FE$ (line from circ cent $\perp$ ch bisects ch) OR $\hat{DBE} = 90^\circ$ ($\angle$ in semi-circle) $\hat{B}_3 = 90^\circ - x$ $\hat{O}_2 = 90^\circ - x$ (alt $\angle s$; $BD \parallel FO$) $\hat{F}_1 = 90^\circ$ ($\angle$ sum Δ) $BF = FE$ (line from circ cent $\perp$ ch bisects ch) OR In $\triangle OBF$ and $\triangle OEF$ 1. $OB = OE$ (radii) 2. $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) 3. $\hat{B}_2 = \hat{E}$ (radii) $\triangle OBF \equiv \triangle OEF$ (AAS) $BF = FE$	✓ $OD = OE$ ✓ radii ✓ $BF = FE$ ✓ $BD \parallel AO$ (4) ✓ $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) ✓ $BO = OE$ ✓ $\triangle BOF \equiv \triangle EOF$ ✓ $BF = FE$ (4) ✓ $\triangle AOB \parallel \triangle BOF$ ✓ $\hat{ABO} = \hat{BFO}$ ✓ $BF = FE$ ✓ line from circ cent $\perp$ ch bisects ch (4) ✓ $\hat{DBE} = 90^\circ$ ✓ $\hat{F}_1 = 90^\circ$ ✓ $BF = FE$ ✓ line from circ cent $\perp$ ch bisects ch (4) ✓ $OB = OE$ ✓ $\hat{BFO} = \hat{EFO} = 90^\circ$ ($BD \parallel AO$) ✓ $\triangle OBF \equiv \triangle OEF$ ✓ $BF = FE$ (4)
8.2.4	In $\triangle CBD$ and $\triangle CEB$ 1. $\hat{E} = \hat{B}_4 = x$ (proven in 8.2.1) 2. $\hat{C}$ is common 3. $\hat{D}_4 = \hat{CBE} = 90^\circ + x$ $\triangle CBD \parallel \triangle CEB$ (AAA)	✓ $\hat{E} = \hat{B}_4 = x$ ✓ $\hat{C}$ is common Or ✓ $\hat{D}_4 = \hat{CBE} = 90^\circ + x$ Any two of the above (2)

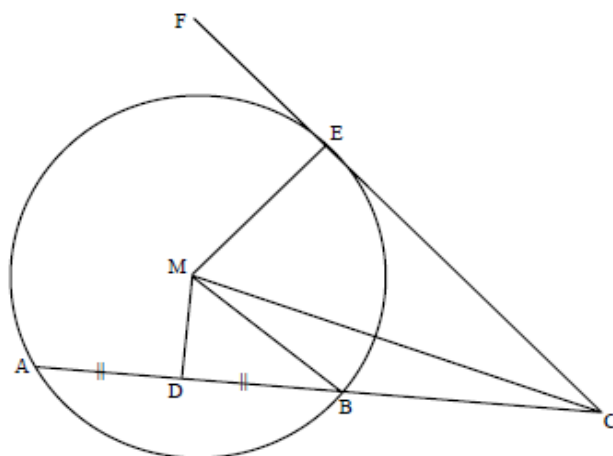
8.2.5	$\frac{EB}{BD} = \frac{CE}{CB}$ (sim Δ s $\therefore$ sides in proportion) $EB \cdot CB = CE \cdot BD$ but $EB = 2EF$ (F is the midpoint of BE) $2EF \cdot CB = CE \cdot BD$	$\checkmark \frac{EB}{BD} = \frac{CE}{CB}$ $\checkmark EB \cdot CB = CE \cdot BD$ $\checkmark EB = 2EF$ (3) [21]
-------	--	---

QUESTION 9

9.	$\hat{A} = \hat{D}$ ($\angle$ in same seg) $\hat{B} = \hat{C}$ ($\angle$ in same seg) $\hat{AEB} = \hat{DEC}$ (vert opp $\angle$ s) $\Delta DEC \parallel \Delta AEB$ ($\angle \angle \angle$) $\frac{DE}{AE} = \frac{EC}{EB} = \frac{DC}{AB}$ (sides in prop) Let $AC = 11a$ $\frac{x}{4a} = \frac{7a}{8}$ $x = \frac{28a^2}{8}$ $x = 3,5a^2$ $\frac{y}{8} = \frac{8}{7a}$ (sides in prop) $y = \frac{64}{7a}$ If candidate proves similarity of two triangles: full marks. If candidate does not prove similarity max 3 marks. The triangles have to be in the correct order in order to be given 3 marks.	$\checkmark \hat{A} = \hat{D}$ S/R $\checkmark \hat{B} = \hat{C}$ S/R $\checkmark \hat{AEB} = \hat{DEC}$ $\checkmark \checkmark \checkmark \Delta DEC \parallel \Delta AEB$ $(\angle \angle \angle)$ [6]
----	--	--



QUESTION 10

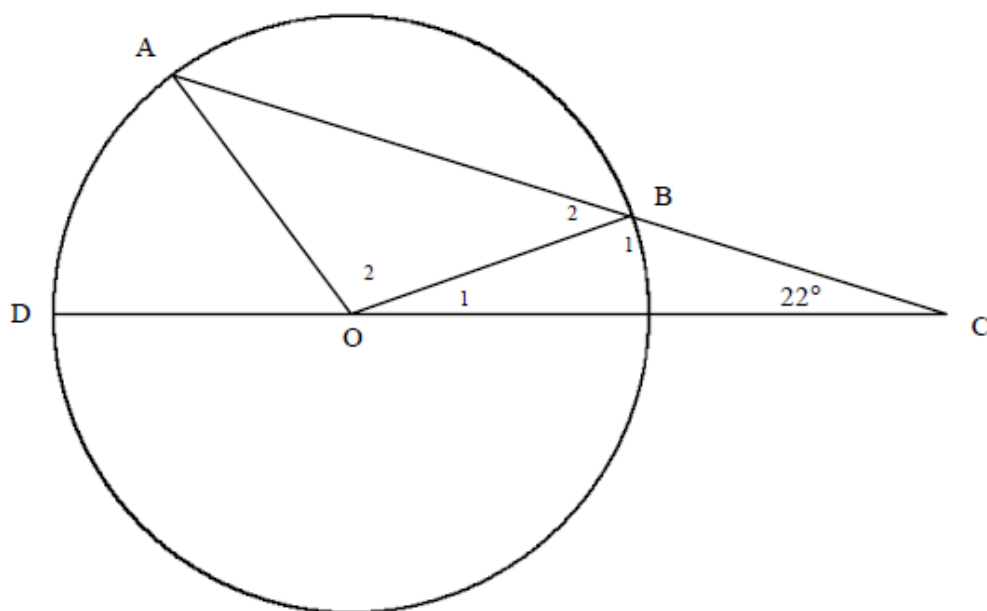


10.1	$\hat{M}\hat{E}C = 90^\circ$ (tan $\perp$ rad) $\hat{M}\hat{D}C = 90^\circ$ (line from cent bisects ch) $\hat{M}\hat{E}C + \hat{M}\hat{D}C = 180^\circ$ $\therefore MDCE$ a cyclic quad (opp $\angle$ s of quad supplementary)	$\checkmark \hat{M}\hat{E}C = 90^\circ$ (tan $\perp$ rad) $\checkmark \hat{M}\hat{D}C = 90^\circ$ $\checkmark$ opp $\angle$ s of quad supplementary (3)
	OR $\hat{M}\hat{E}C = 90^\circ$ (tan $\perp$ rad) $\hat{M}\hat{D}A = 90^\circ$ (line from cent bisects ch) $\hat{M}\hat{E}C = \hat{M}\hat{D}A$ $\therefore MDCE$ a cyclic quad (ext $\angle$ quad = int opp)	$\checkmark \hat{M}\hat{E}C = 90^\circ$ (tan $\perp$ rad) $\checkmark \hat{M}\hat{D}A = 90^\circ$ $\checkmark$ ext $\angle$ quad = int opp (3)
10.2	$MD^2 = MB^2 - DB^2$ (Pythagoras; $\triangle MBD$) $MC^2 = MD^2 + DC^2$ (Pythagoras; $\triangle MDC$) $= MB^2 - DB^2 + DC^2$	$\checkmark MD^2 = MB^2 - DB^2$ $\checkmark$ Pythagoras $\checkmark MC^2 = MD^2 + DC^2$ (3)
10.3	$DB = 30$ (given) $MB = 40$ (radii) $MC^2 = (40)^2 + (50)^2 - (30)^2$ $= 3\,200$ $MC = 40\sqrt{2} = 56,57$ $MC^2 = ME^2 + CE^2$ (Pythagoras) $CE^2 = 3\,200 - 1\,600$ $CE^2 = 1\,600$ $CE = 40$ mm	$\checkmark MB = ME$ $\checkmark DB = 30$ $\checkmark MC^2 = 3200$ or $MC = 40\sqrt{2}$ or $MC = 56,57$
	OR $MC^2 = CE^2 + ME^2 - 2CE \cdot ME \cdot \cos \hat{M}\hat{E}C$ $3200 = CE^2 + (40)^2 - 2CE \cdot (40) \cdot \cos 90^\circ$ $= CE^2 + 1600$ $CE^2 = 1600$ $CE = 40$	$\checkmark$ answer (4)
	OR $MC^2 = CE^2 + ME^2 - 2CE \cdot ME \cdot \cos \hat{M}\hat{E}C$ $3200 = CE^2 + (40)^2 - 2CE \cdot (40) \cdot \cos 90^\circ$ $= CE^2 + 1600$ $CE^2 = 1600$ $CE = 40$	$\checkmark$ cosine rule $\checkmark ME = 40$ $\checkmark MC^2 = 3200$ $\checkmark$ answer (4)

[10]

QUESTION 8

O is the centre of the circle. AB produced and DO produced meet at C.
BC = OA and $\angle ACO = 22^\circ$.



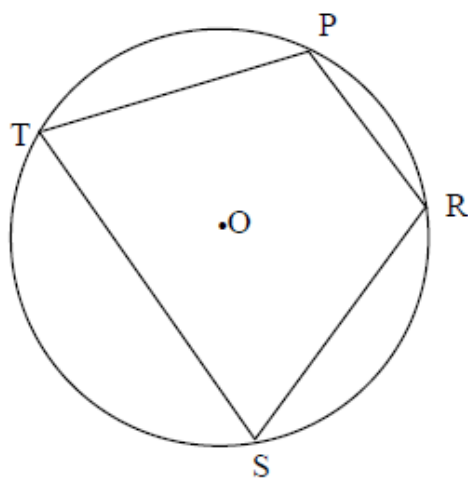
Calculate, with reasons, $\angle AOD$.

[5]

QUESTION 9

9.1 In the figure below O is the centre of the circle and PRST is a cyclic quadrilateral.

Prove the theorem that states $\angle PRS + \angle PTS = 180^\circ$.

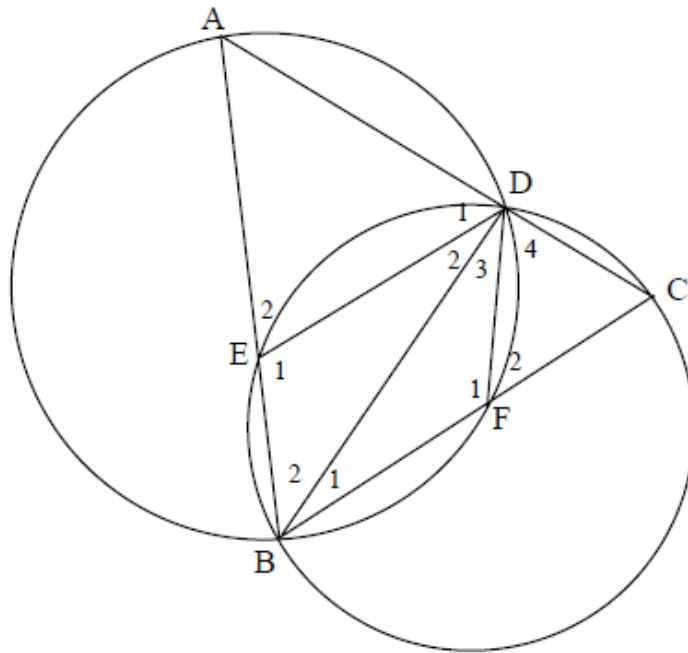


(5)

- 9.2 In the diagram below two circles intersect one another at D and B. AB is a straight line such that it intersects the circle BCD at point E. BC is a straight line such that it intersects the circle ABD at F. DE, DB and DF are joined.

$$\hat{F}_2 = 180^\circ - 2x$$

$$FC = FD$$



- 9.2.1 Calculate, with reasons, in terms of x :

(a) $\hat{DEB}$ (3)

(b) $\hat{A}$ (2)

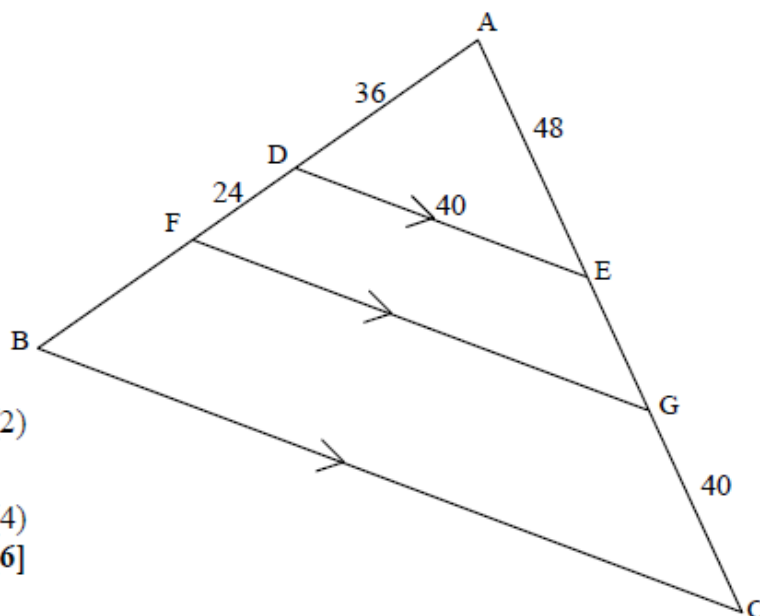
- 9.2.2 Hence, or otherwise, prove $ED \parallel BC$. (3)

[13]

QUESTION 10

In the figure below $DE \parallel FG \parallel BC$.

$AD = 36$ cm, $DF = 24$ cm, $AE = 48$ cm and $DE = GC = 40$ cm.



Determine, with reasons, the lengths of:

10.1 EG

(2)

10.2 BC

(4)

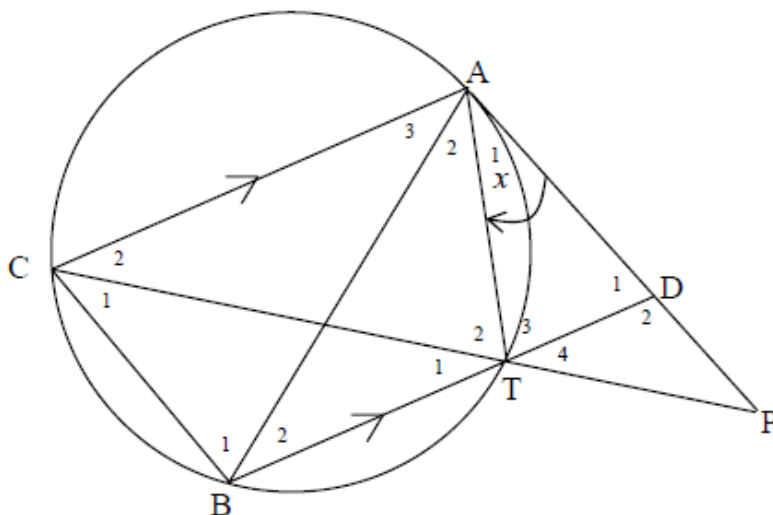
[6]

QUESTION 11

In the diagram below DA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are joined.

$AC \parallel BD$

Let $\hat{A}_1 = x$



11.1 Prove that $\triangle ABC \parallel \triangle ADT$.

(6)

11.2 Prove that PT is a tangent to the circle ADT at T.

(3)

11.3 Prove that $\triangle APT \parallel \triangle TPD$.

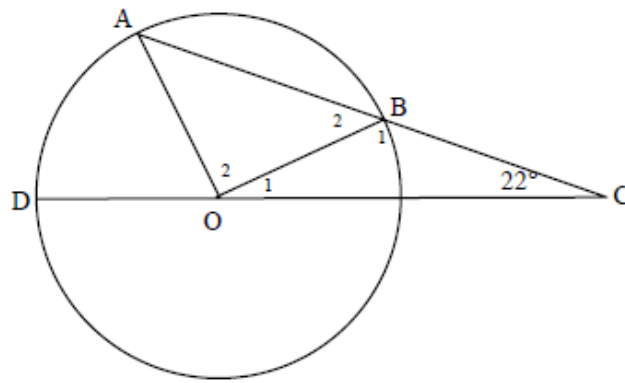
(3)

11.4 If $AD = \frac{2}{3} AP$, show that $AP^2 = 3PT^2$.

(4)

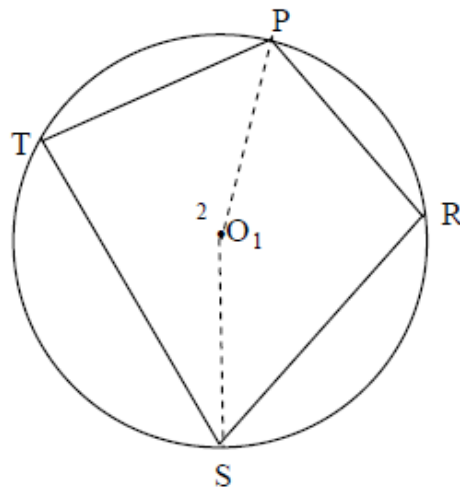
[16]

QUESTION 8

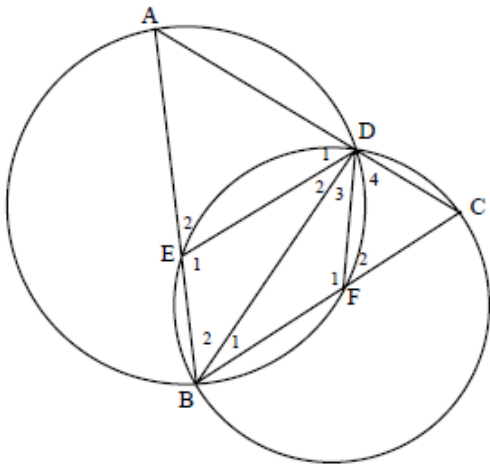


8.	$AO = OB$ (radii) $AO = BC$ (given) $OB = BC$ $\hat{O}_1 = 22^\circ$ ($\angle$ s opp = radii) $\hat{B}_2 = 44^\circ$ (ext $\angle \Delta$ = sum int opp) $\hat{A} = 44^\circ$ ($\angle$ s opp = radii) $\hat{AOD} = 66^\circ$ (ext $\angle \Delta$ = sum int opp)	✓ S ✓ S ✓ S/R ✓ S ✓ S ✓ answer
		[5]

QUESTION 9

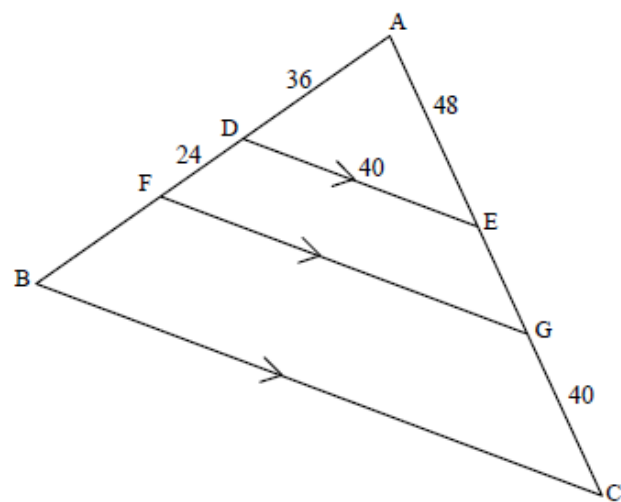


9.1	Join PO and OS Let $\hat{O}_1 = 2x$ $\hat{T} = x$ ($\angle$ at circ centre = 2 $\angle$ at circumference) $\hat{O}_2 = 360^\circ - 2x$ ($\angle$ s round a point) $\hat{R} = 180^\circ - x$ ($\angle$ at circ centre = 2 $\angle$ at circumference) $\hat{T} + \hat{R} = x + 180^\circ - x$ $= 180^\circ$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S
		(5)



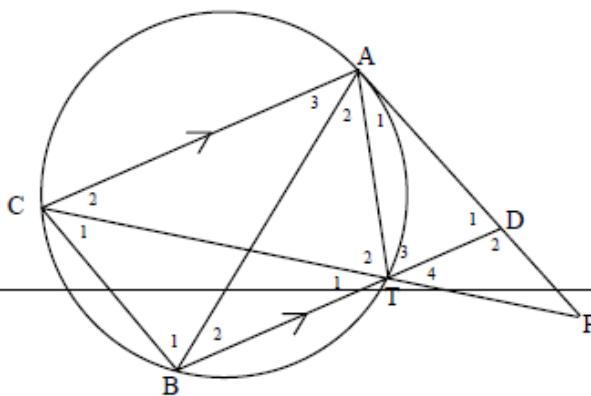
9.2.1(a)	$\hat{D}_4 = \hat{C}$ ($\angle$ s opp = sides) $\hat{C} = x$ ($\angle$ sum Δ) $\hat{DEB} = 180^\circ - x$ (opp $\angle$ cyclic quad supp)	✓ S/R ✓ S ✓ S/R (3)
9.2.1(b)	$\hat{A} = 180^\circ - 2x$ (ext $\angle$ cyclic quad = int opp $\angle$)	✓ S ✓ R (2)
9.2.2	$\hat{D}_1 + \hat{A} = \hat{E}_1$ (ext $\angle \Delta =$ sum int opp) $\hat{D}_1 = x$ $\hat{C} = x$ ($\angle$ sum Δ) OR proved above $\hat{D}_1 = \hat{C} = x$ $DE \parallel CB$ (corres $\angle$ s =)	✓ S/R ✓ statement ✓ Reason (3) [13]

QUESTION 10



10.1	$\frac{EG}{48} = \frac{24}{36} \quad (DE \parallel FG)$ $EG = \frac{48 \times 24}{36}$ $EG = 32 \text{ cm}$	✓ S/R ✓ answer (2)
10.2	$\frac{BC}{DE} = \frac{AC}{AE}$ $BC = \frac{120 \times 40}{48}$ $= 100 \text{ cm}$ OR $\frac{AB}{AD} = \frac{AC}{AE}$ $AB = \frac{120 \times 36}{48}$ $AB = 90$ $\triangle ABC \parallel \triangle ADE \quad (\angle \angle \angle)$ $\frac{BC}{DE} = \frac{AB}{AD} \quad (\text{sides in proportion})$ $BC = \frac{90 \times 40}{36}$ $BC = 100 \text{ cm}$ OR $\triangle ABC \parallel \triangle ADE \quad (\angle \angle \angle)$ $\frac{BC}{DE} = \frac{AC}{AE} \quad (\text{sides in proportion})$ $BC = \frac{120 \times 40}{36}$ $BC = 100 \text{ cm}$	✓ statement ✓✓ substitution ✓ answer (4) ✓ S ✓ S ✓ 90 ✓ answer (4) ✓ S ✓ S ✓ substitution ✓ answer (4) [6]

QUESTION 11



11.1	Let $\hat{A}_1 = x$ In $\triangle ABC$ and $\triangle ADT$ $\hat{A}_1 = \hat{B}_2 = x$ (tan ch th) $\hat{B}_2 = \hat{A}_3 = x$ ($AC \parallel BD$ alt $\angle$s) $\hat{A}_1 = \hat{A}_3$ $\hat{T}_3 = \hat{BCA}$ (ext $\angle$ cyclic quad) $\hat{B}_1 = \hat{D}_1$ (3^{rd} $\angle$ on triangle) $\triangle ABC \parallel \triangle ADT$ ($\angle\angle\angle$)	✓ statement ✓ reason ✓ statement ✓ statement ✓ reason ✓ statement (6)
11.2	$\hat{A}_1 = \hat{C}_2 = x$ (tan ch th) $\hat{T}_1 = \hat{C}_2 = x$ ($AC \parallel BD$; alt $\angle$s) $\therefore \hat{T}_1 = \hat{A}_1 = x$ $\hat{T}_4 = x$ (vert opp angles) $\hat{T}_4 = \hat{A}_1$ ($= x$) PT is a tangent (conv tan ch th) OR $\hat{A}_1 = \hat{B}_2 = \hat{A}_3 = x$ ($AC \parallel BT$) $\hat{A}_3 = \hat{T}_1 = \hat{T}_4 = x$ ($\angle$s in same segment) $\hat{A}_1 = \hat{T}_4 = x$ PT is a tangent (conv tan ch th) OR $\hat{B}_1 = \hat{T}_2$ ($\angle$s in same seg) $\hat{B}_1 = \hat{D}_1$ ($\parallel \Delta$s) $\hat{D}_1 = \hat{T}_2$ PT is a tangent (conv tan ch th)	✓ S/R ✓ S/R ✓ Reason (3) ✓ S/R ✓ S/R ✓ Reason (3) ✓ S/R ✓ S/R ✓ Reason (3)
11.3	In $\triangle APT$ and $\triangle TPD$ $\hat{P}$ is common. $\hat{T}_4 = \hat{A}_1$ (proven) $\hat{ATP} = \hat{D}_2$ (3^{rd} $\angle$ on triangle) $\triangle APT \parallel \triangle TPD$ ($\angle\angle\angle$)	✓ S/R ✓ S/R ✓ S (3)

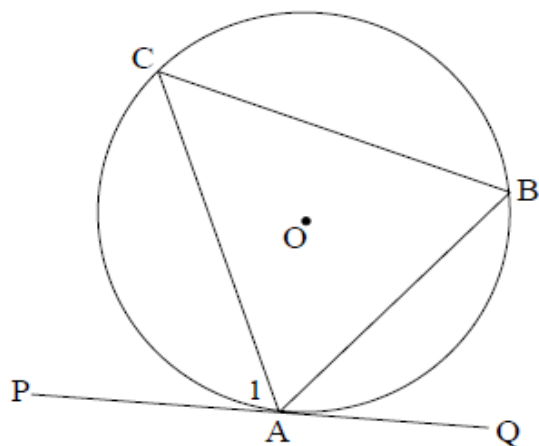
11.4	$\frac{AP}{PT} = \frac{PT}{PD} \quad (\triangle APT \parallel \triangle TPD)$ $AP \cdot PD = PT \cdot PT$ $AP \cdot \frac{1}{3} AP = PT^2$ $AP^2 = 3PT^2$	✓ statement ✓ reason ✓ $DP = \frac{1}{3} AP$ ✓ substitution (4) [16]
------	---	--

November 2011

QUESTION 8

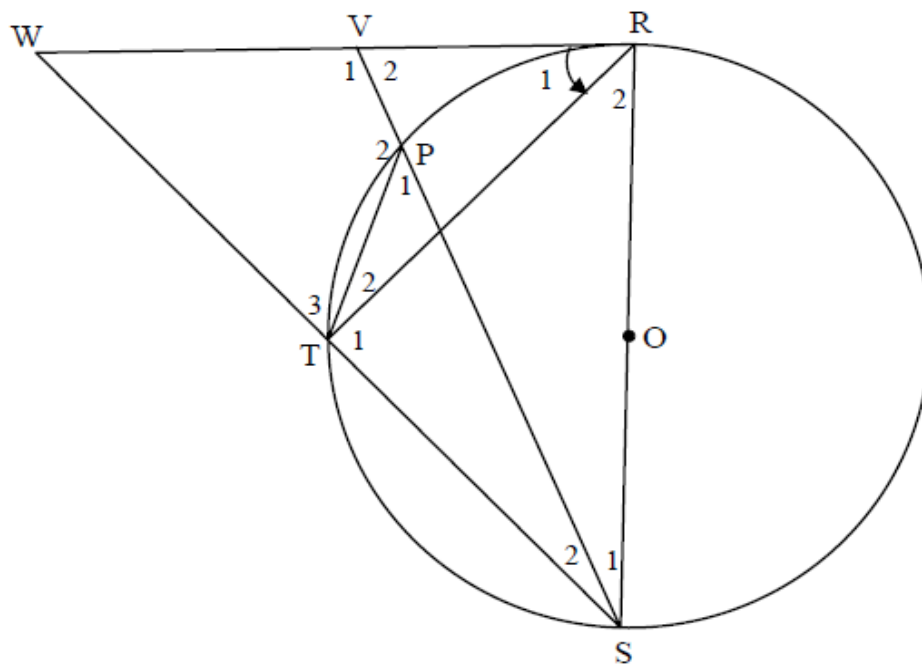
- 8.1 In the diagram below, O is the centre of the circle. PQ is a tangent to the circle at A . B and C are points on the circumference of the circle. AB , AC and BC are joined.

Prove the theorem that states $\hat{CAP} = \hat{ABC}$.



(5)

- 8.2 RS is a diameter of the circle with centre O . Chord ST is produced to W . Chord SP produced meets the tangent RW at V . $\hat{R}_1 = 50^\circ$.



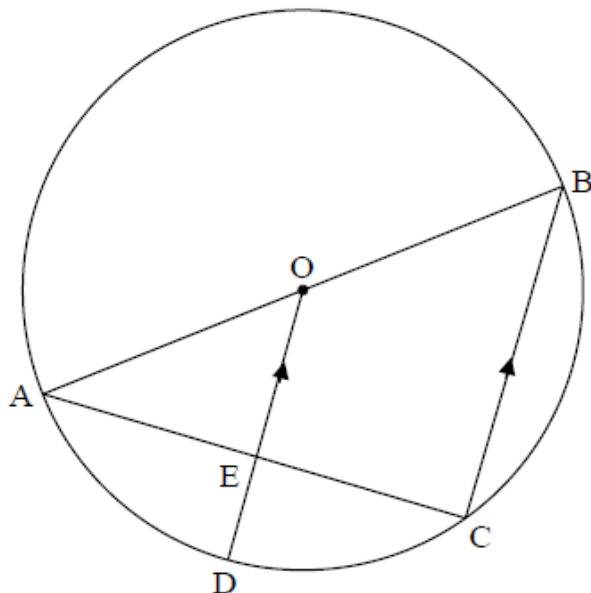
Calculate the size of:

- 8.2.1 $\hat{WRS}$ (1)
- 8.2.2 $\hat{W}$ (2)
- 8.2.3 $\hat{P}_1$ (3)
- 8.2.4 Prove that $\hat{V}_1 = \hat{PTS}$. (4)

[15]

QUESTION 9

AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC in E.

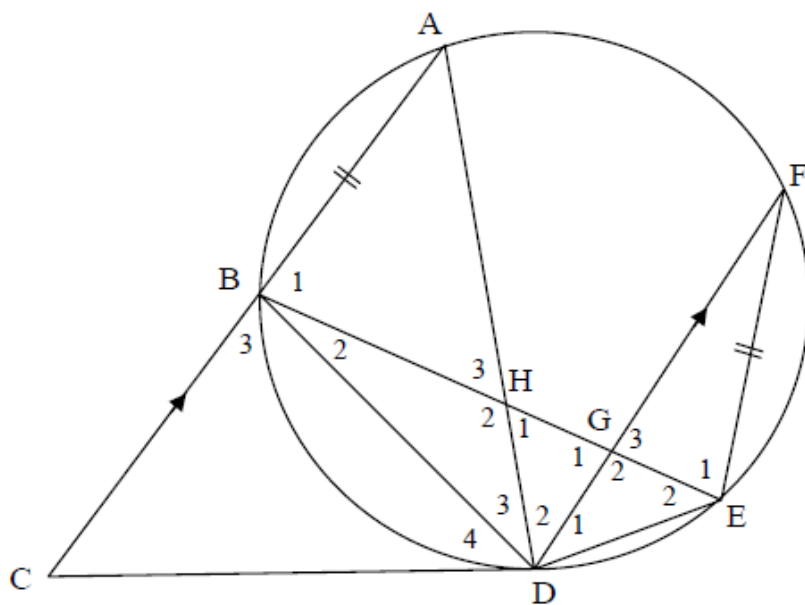


If the radius is 10 cm and $AC = 16$ cm, calculate the length of ED.

[5]

QUESTION 10

CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. $AC \parallel FD$ and $FE = AB$. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.



10.1 Determine THREE other angles that are each equal to x . (6)

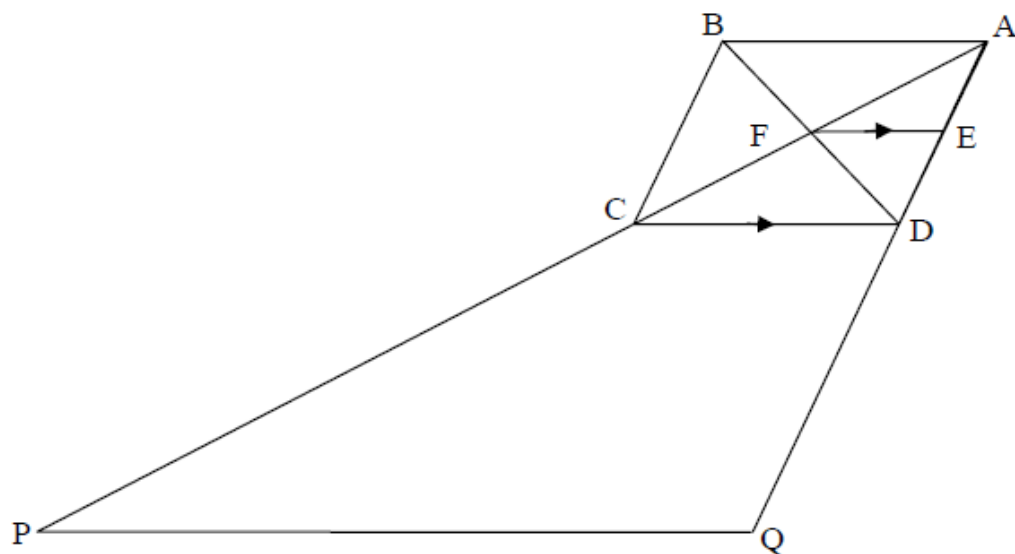
10.2 Prove that $\triangle BHD \parallel \triangle FED$. (5)

10.3 Hence, or otherwise, prove that $AB \cdot BD = FD \cdot BH$. (2)

[13]

QUESTION 11

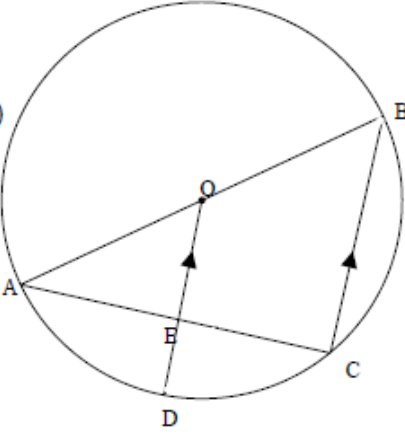
ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that $PC = 2AC$ and AD is produced to Q such that $DQ = 2AD$.



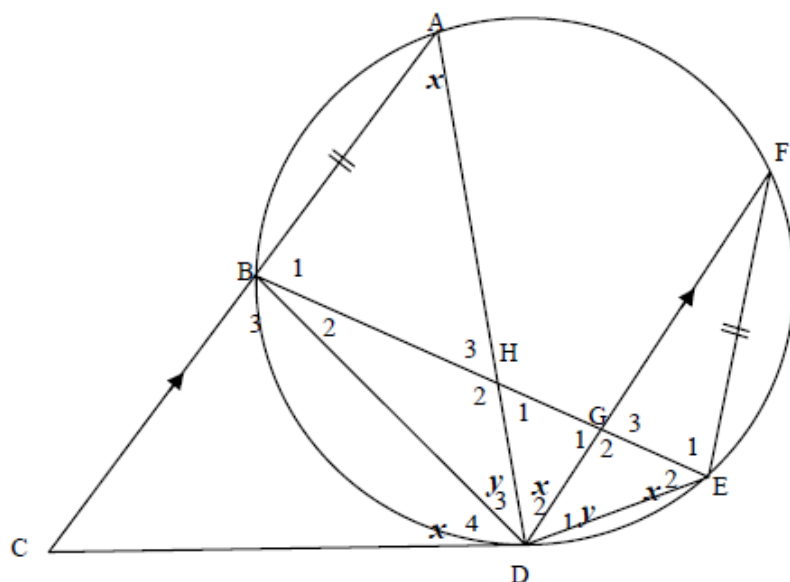
- 11.1 Show that E is the midpoint of AD. (2)
- 11.2 Prove $PQ \parallel FE$. (3)
- 11.3 If PQ is 60 cm, calculate the length of FE. (5)
- [10]**

8.2.4	$\hat{P}_1 = \hat{W}$ ($= 40^\circ$) WVPT is a cyclic quadrilateral (ext $\angle$ = int opp) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ (ext $\angle$ cyclic quad) OR $\hat{T}_1 = 90^\circ$ ($\angle$s in semi circle) $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ $\hat{T}_2 = \hat{S}_1$ ($\angle$s in same seg) $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{S}_1$ $\hat{V}_1 = 90^\circ + \hat{S}_1$ (ext $\angle$ Δ) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ OR $\hat{P}_2 = 140^\circ$ ($\angle$s on str line) $\hat{W} + \hat{P}_2 = 180^\circ$ WVPT is cyclic quad (opp $\angle$s suppl) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ (ext $\angle$ cyclic quad) OR $\hat{V}_1 = \hat{R}_1 + \hat{R}_2 + \hat{S}_1$ (ext $\angle$ Δ) $\hat{V}_1 = 90^\circ + \hat{S}_1$ $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ But $\hat{T}_2 = \hat{S}_1$ ($\angle$s in same seg) $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ OR In $\Delta P\hat{T}\hat{S}$ and $\Delta W\hat{V}\hat{S}$ $\hat{P}_1 = \hat{W}$ ($= 40^\circ$) $\hat{S}_2$ is common $\hat{V}_1 = \hat{P}\hat{T}\hat{S}$ ($\angle$ sum Δ)	✓ $\hat{P}_1 = \hat{W}$ ✓ WVPT is a cyclic quadrilateral ✓ ext $\angle$ = int opp ✓ ext $\angle$ cyclic quad (4) ✓ $\angle$s in semi circle ✓ $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ ✓ $\hat{T}_2 = \hat{S}_1$ ✓ $\angle$s in same seg (4) ✓ $\hat{W} + \hat{P}_2 = 180^\circ$ ✓ WVPT is a cyclic quadrilateral ✓ opp $\angle$ suppl ✓ ext $\angle$ cyclic quad (4) ✓ $\hat{V}_1 = 90^\circ + \hat{S}_1$ ✓ $\hat{P}\hat{T}\hat{S} = 90^\circ + \hat{T}_2$ ✓ $\hat{T}_2 = \hat{S}_1$ ✓ $\angle$s in same seg (4) ✓ identification of triangles ✓ $\hat{P}_1 = \hat{W}$ ✓ $\hat{S}_2$ is common ✓ $\angle$ sum Δ (4) [15]
-------	--	---

QUESTION 9

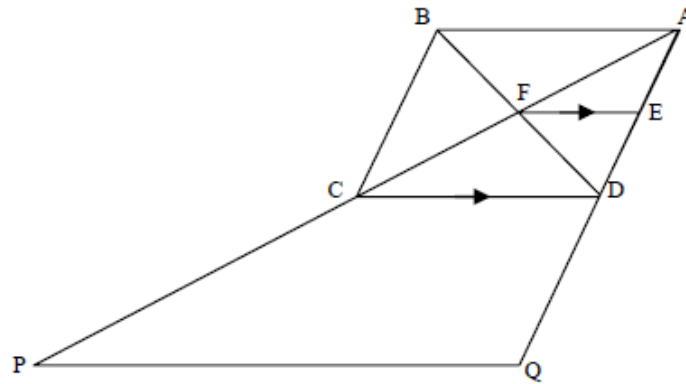
9.	$\hat{C} = 90^\circ$ ($\angle$s in semi circle) $\hat{OEA} = 90^\circ$ (corres $\angle$s; $OD \parallel BC$) $AE = 8 \text{ cm}$ (line from circ cent $\perp$ ch bis ch) $OE = 6 \text{ cm}$ (Pythagoras) $ED = 10 - 6$ $= 4 \text{ cm}$ OR $\hat{C} = 90^\circ$ ($\angle$s in semi circle) $\hat{OEA} = 90^\circ$ (corres $\angle$s; $OD \parallel BC$) $OE \parallel BC$ (given) $OA = OB$ (radii) $AE = EC = 8 \text{ cm}$ (midpoint theorem) $OE = 6 \text{ cm}$ (Pythagoras) $ED = 10 - 6$ $= 4 \text{ cm}$ OR $\hat{C} = 90^\circ$ ($\angle$s in semi circle) $BC^2 = (20)^2 - (16)^2$ $BC^2 = 144$ $BC = 12$ $OE = \frac{1}{2} BC$ (midpoint theorem) $OE = 6 \text{ cm}$ $OD = 10 \text{ cm}$ $ED = 10 - 6$ $= 4 \text{ cm}$ OR $\hat{C} = 90^\circ$ ($\angle$s in semi circle) $BC^2 = (20)^2 - (16)^2$ $BC^2 = 144$ $BC = 12$ $OE = \frac{1}{2} BC$ (midpoint theorem) $OE = 6 \text{ cm}$ $ED = 4 \text{ cm}$	 $\checkmark \hat{C} = 90^\circ$ $\checkmark \hat{OEA} = 90^\circ$ $\checkmark$ line from circ cent $\perp$ ch bis ch $\checkmark OE = 6 \text{ cm}$ $\checkmark ED = 4 \text{ cm}$ $\checkmark \hat{C} = 90^\circ$ $\checkmark \hat{OEA} = 90^\circ$ $\checkmark$ midpoint theorem $\checkmark OE = 6 \text{ cm}$ $\checkmark ED = 4 \text{ cm}$ $\checkmark \hat{C} = 90^\circ$ $\checkmark BC = 12$ $\checkmark$ reason $\checkmark OE = 6 \text{ cm}$ $\checkmark ED = 4 \text{ cm}$ [5] $\checkmark \hat{C} = 90^\circ$ $\checkmark BC = 12$ $\checkmark$ reason $\checkmark OE = 6 \text{ cm}$ $\checkmark ED = 4 \text{ cm}$ [5]
-----------	---	--

QUESTION 10



10.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) OR ($\angle$ s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$ s; $CA \parallel DF$)	✓ $\hat{A} = x$ ✓ tan ch th ✓ $\hat{E}_2 = x$ ✓ reason ✓ $\hat{D}_2 = x$ ✓ alt $\angle$ s; $CA \parallel DF$ (6)
10.2	In $\triangle BHD$ and $\triangle FED$ 1. $\hat{B}_2 = \hat{F}$ ($\angle$ s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$ s) $\triangle BHD \parallel \triangle FED$ ($\angle\angle\angle$)	✓ $\hat{B}_2 = \hat{F}$ ✓ $\angle$ s in same seg ✓ $\hat{D}_3 = \hat{D}_1$ ✓ = chs subt = $\angle$ s ✓ $\angle\angle\angle$ (5)
10.3	$\frac{FE}{BH} = \frac{FD}{BD}$ ($\parallel \Delta$ s) But $FE = AB$ (given) $\frac{AB}{BH} = \frac{FD}{BD}$ $AB \cdot BD = FD \cdot BH$	✓ $\frac{FE}{BH} = \frac{FD}{BD}$ ✓ $FE = AB$ (2) [13]

QUESTION 11



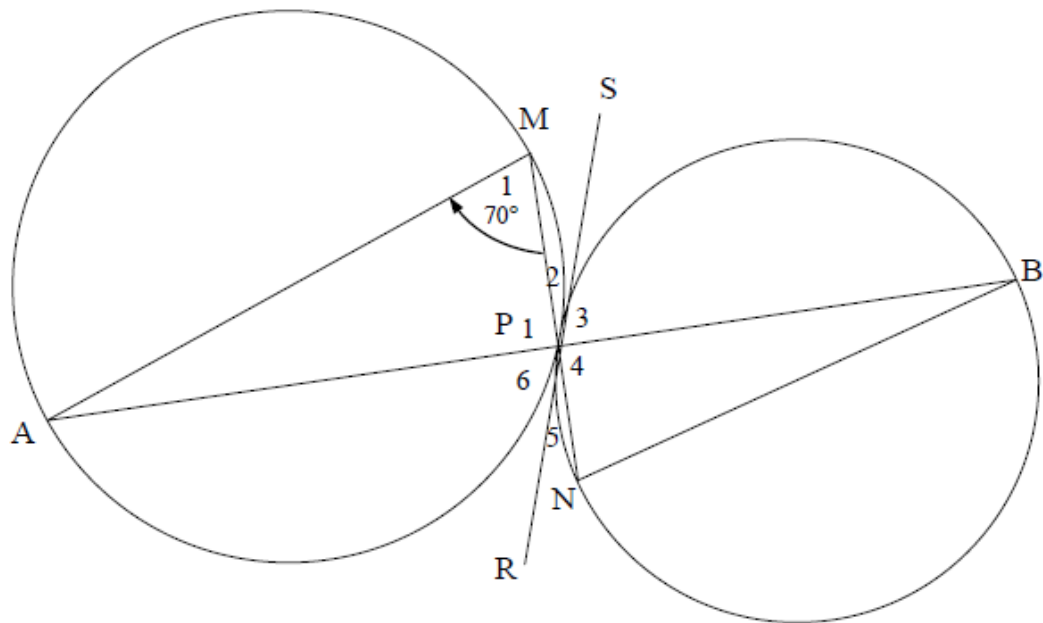
11.1	$AF = FC$ $FE \parallel CD$ $AE = ED$ (diags of parallelogram bisect) (Prop Th; $FE \parallel CD$) OR (Midpoint Theorem)	$\checkmark AF = FC$ $\checkmark$ reason (2)
11.2	$\frac{AC}{CP} = \frac{1}{2}$ (given) $\frac{AD}{DQ} = \frac{1}{2}$ (given) $\frac{AC}{CP} = \frac{AD}{DQ}$ $CD \parallel PQ$ (converse proportionality theorem) $CD \parallel FE$ (given) $\therefore PQ \parallel FE$ OR $\frac{AC}{AP} = \frac{1}{3}$ $\frac{AD}{AQ} = \frac{1}{3}$ $\frac{AC}{AP} = \frac{AD}{AQ}$ $CD \parallel PQ$ (converse proportionality theorem) $CD \parallel FE$ (given) $\therefore PQ \parallel FE$ OR $\frac{AF}{AP} = \frac{1}{6}$ $\frac{AE}{AQ} = \frac{1}{6}$ $\frac{AF}{AP} = \frac{AE}{AQ}$ $\therefore PQ \parallel FE$ (converse proportionality theorem)	$\checkmark$ ratios equal $\checkmark CD \parallel PQ$ $\checkmark$ reason: converse prop th and conclusion (3) $\checkmark$ ratios equal $\checkmark CD \parallel PQ$ $\checkmark$ reason: converse prop th and conclusion (3) $\checkmark \frac{AF}{AP} = \frac{1}{6}$ $\checkmark \frac{AF}{AP} = \frac{AE}{AQ}$ $\checkmark$ conv prop theorem

11.3	In $\triangle AEF$ and $\triangle APQ$ 1. $\hat{A}$ is common 2. $\hat{AEF} = \hat{AQP}$ (corres $\angle$s; $FE \parallel PQ$) 3. $\hat{AFE} = \hat{APQ}$ (corres $\angle$s; $FE \parallel PQ$) $\therefore \triangle AEF \parallel \triangle AQP$ ($\angle\angle\angle$) $\frac{FE}{PQ} = \frac{AF}{AP} \quad (\parallel \Delta s)$ $\frac{FE}{60} = \frac{1}{6}$ $FE = 10 \text{ cm}$ OR In $\triangle ADC$ and $\triangle APQ$ 1. $\hat{A}$ is common 2. $\hat{ADC} = \hat{AQP}$ (corres $\angle$s; $CD \parallel PQ$) 3. $\hat{ACD} = \hat{APQ}$ (corres $\angle$s; $CD \parallel PQ$) $\therefore \triangle ADC \parallel \triangle AQP$ ($\angle\angle\angle$) $\frac{AC}{AP} = \frac{AD}{AQ} = \frac{1}{3} \quad (\parallel \Delta s)$ $CD = \frac{1}{3} PQ$ $CD = 20 \text{ cm}$ But $AF = FC$ $AE = ED$ (Midpoint Theorem) $FE = \frac{1}{2} CD$ $FE = 10 \text{ cm}$ NOTE: If the similarity has not been proven, then max 3/5 marks	✓ first pair of angles equal with reason ✓ second pair of angles equal with reason ✓ $\frac{AF}{AP} = \frac{1}{6}$ ✓ $\frac{FE}{PQ} = \frac{AF}{AP}$ ✓ answer (5) ✓ first pair of angles equal with reason ✓ second pair of angles equal with reason ✓ $CD = \frac{1}{3} PQ$ ✓ $FE = \frac{1}{2} CD$ ✓ answer (5) [10]
------	---	---

February 2012

QUESTION 8

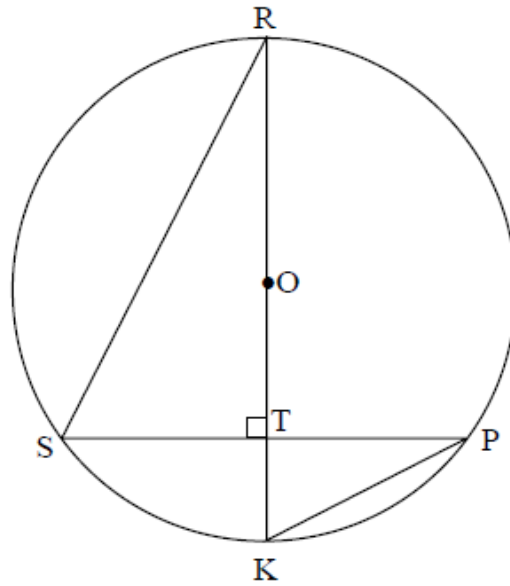
In the diagram below, AM is the diameter of the bigger circle AMP . RPS is a common tangent to both circles at P . APB and MPN are straight lines.



- 8.1 State the size of $\hat{P}_1$. (1)
- 8.2 Hence, show that BN is the diameter of the smaller circle. (2)
- 8.3 If $\hat{M}_1 = 70^\circ$, calculate the size of each of the following angles:
- 8.3.1 $\hat{A}$ (1)
- 8.3.2 $\hat{P}_6$ (1)
- 8.3.3 $\hat{B}$ (2)
- [7]

QUESTION 9

In the diagram below, O is the centre of the circle with diameter RK .
 $PS \perp RK$.
 RK intersects PS at T .



- 9.1 If $PS = 4x$, write down the length of ST in terms of x . (1)
- 9.2 Prove that $\triangle RST \cong \triangle PKT$. (3)
- 9.3 If it is further given that $TK = x$ and $RT = 320$ mm, calculate the value of x . (3)
- [7]

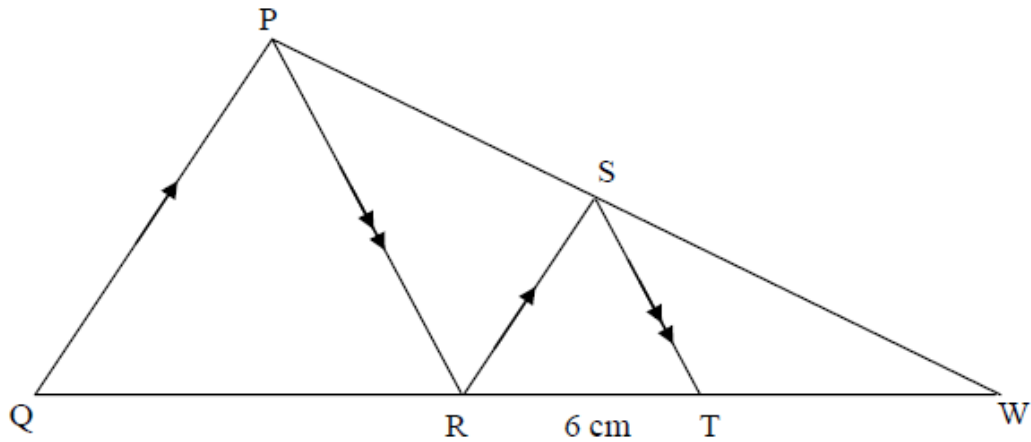
QUESTION 10

In $\triangle PQW$, S is a point on PW and R is a point on QW such that $SR \parallel PQ$.

T is a point on QW such that $ST \parallel PR$.

$RT = 6 \text{ cm}$

$WS : SP = 3 : 2$



Calculate:

10.1 WT (3)

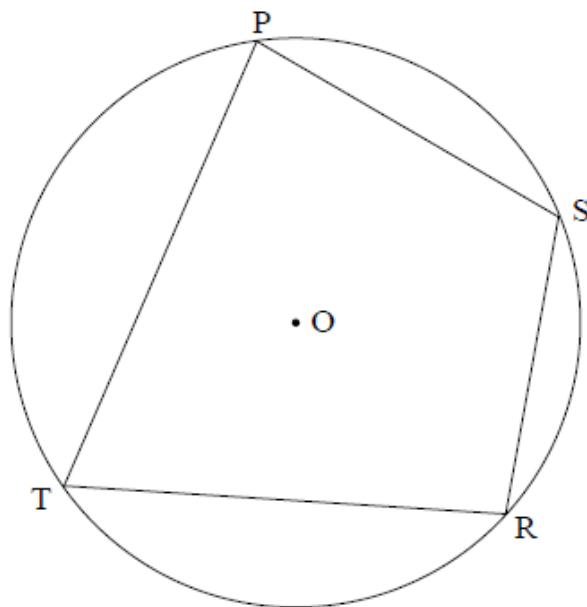
10.2 WQ (4)

[7]

QUESTION 11

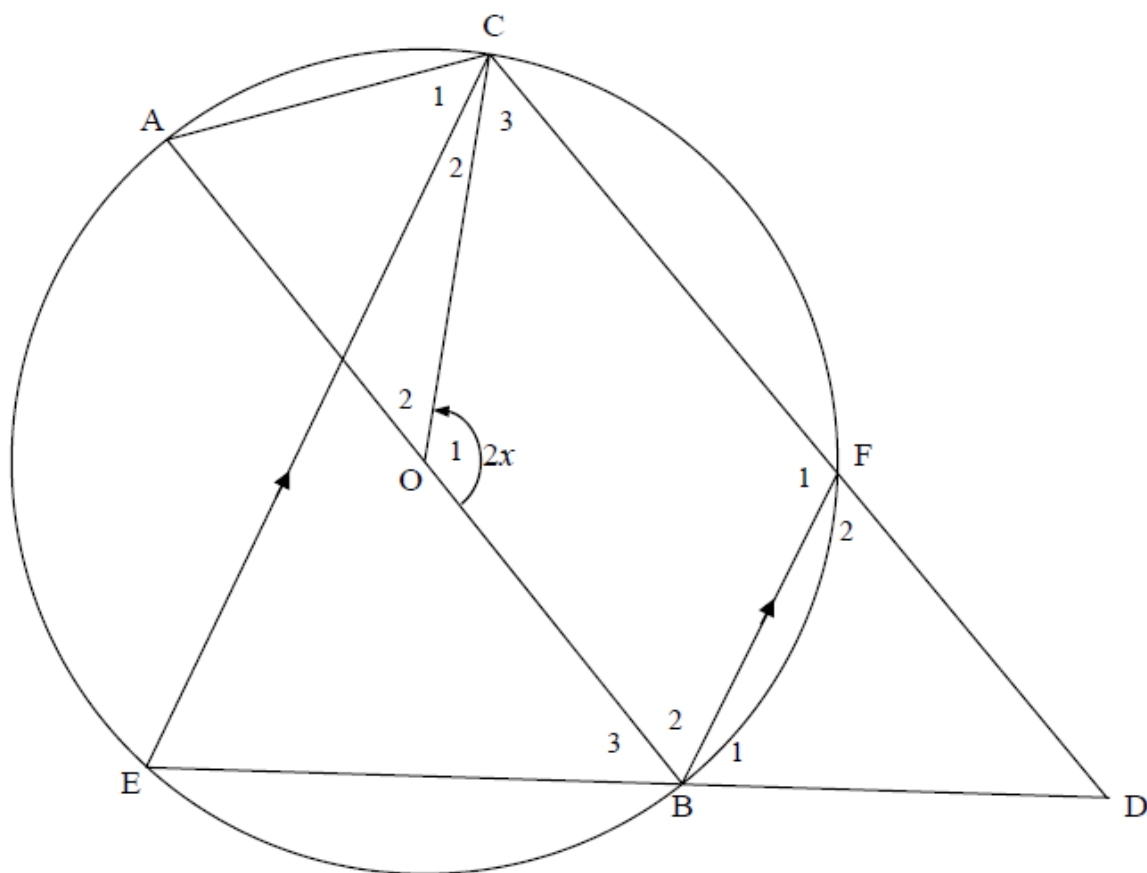
11.1 In the diagram below, O is the centre of the circle. PSRT is a cyclic quadrilateral.

Prove the theorem that states $\hat{PTR} + \hat{PSR} = 180^\circ$.



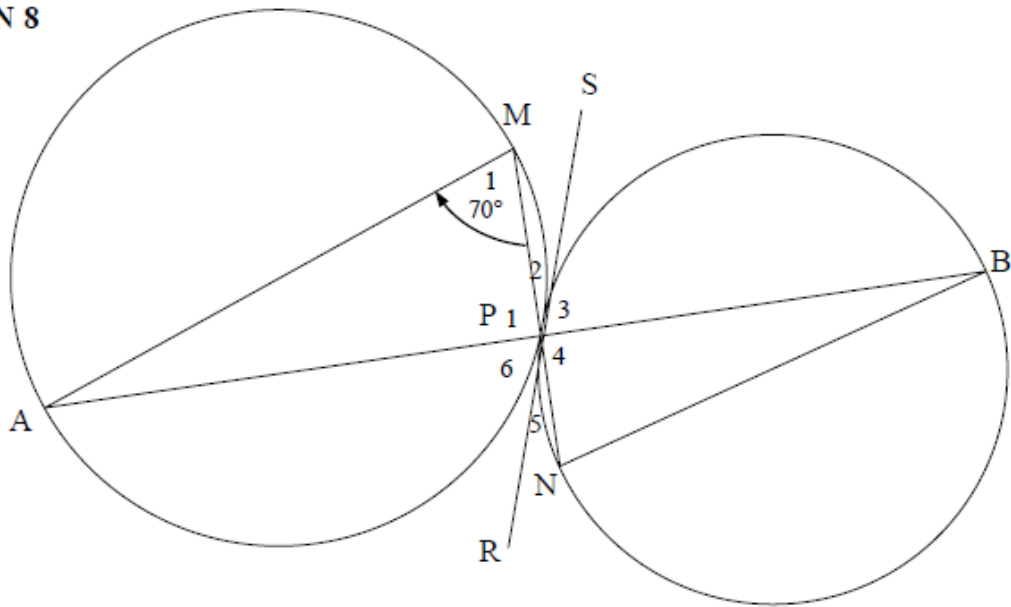
(6)

- 11.2 In the diagram below, O is the centre of the circle. AB is a diameter of the circle. Chord CF produced meets chord EB produced at D . Chord EC is parallel to chord BF . CO and AC are joined. Let $\hat{O}_1 = 2x$



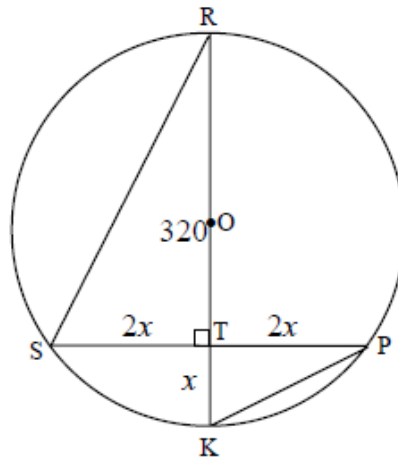
- 11.2.1 Determine, in terms of x , the size of $\hat{F}_1$. (4)
- 11.2.2 Prove that $DF = BD$. (4)
- 11.2.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)
- 11.2.4 If $DF = 5$ cm and $OA = 6$ cm, calculate area $\triangle BFD$: area $\triangle AOC$. (4)
- [22]

QUESTION 8



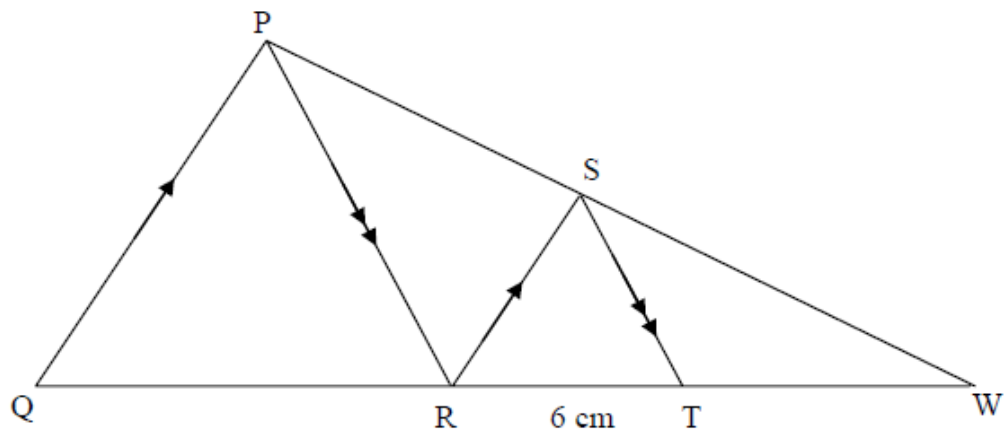
8.1	$\hat{P}_1 = 90^\circ$ ($\angle$ in semicircle)	$\checkmark \hat{P}_1 = 90^\circ$ (1)
8.2	$\hat{P}_4 = 90^\circ$ (vert opp $\angle$ s) BN is a diameter (chord subtends 90°)	$\checkmark \hat{P}_4 = 90^\circ$ $\checkmark$ chord subtends 90° (2)
8.3.1	$\hat{A} = 20^\circ$ ($\angle$ sum Δ)	$\checkmark$ answer (1)
8.3.2	$\hat{P}_6 = 70^\circ$ (tan ch th)	$\checkmark$ answer (1)
8.3.3	$\hat{P}_3 = 70^\circ$ (vert opp $\angle$ s) $\hat{N} = 70^\circ$ (tan ch th) $\hat{P}_4 = 90^\circ$ (proven) $\hat{B} = 20^\circ$ ($\angle$ sum Δ) OR $\hat{P}_5 = 20^\circ$ ($\angle$ s on str line) $\hat{B} = 20^\circ$ (tan ch th)	$\checkmark \hat{N} = 70^\circ$ $\checkmark \hat{B} = 20^\circ$ (2) $\checkmark \hat{P}_5 = 20^\circ$ $\checkmark \hat{B} = 20^\circ$ (2) [7]

QUESTION 9



9.1	$ST = PT = 2x$ (line from circ centre $\perp$ ch bis ch)	✓ S/R (1)
9.2	In $\triangle RST$ and $\triangle PKT$ 1. $\hat{R} = \hat{P}$ ($\angle$ in same seg) 2. $\hat{S} = \hat{K}$ ($\angle$ in same seg) 3. $\hat{RTK} = \hat{PTK} = 90^\circ$ (given) $\triangle RST \equiv \triangle PKT$ ($\angle\angle\angle$)	✓ S/R ✓ S/R ✓ R (3)
9.3	$\frac{ST}{KT} = \frac{RT}{PT}$ ($\equiv \Delta s$) $\frac{2x}{x} = \frac{320}{2x}$ $4x = 320$ $x = 80 \text{ mm}$ OR $\frac{ST}{RT} = \frac{KT}{PT}$ $\frac{2x}{320} = \frac{x}{2x}$ $2x = 160$ $x = 80 \text{ mm}$	✓ $\frac{ST}{KT} = \frac{RT}{PT}$ ✓ substitution ✓ answer (3) [7]

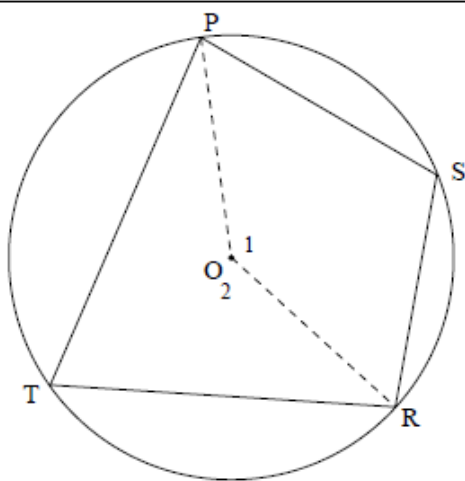
QUESTION 10



10.1	$\frac{WS}{SP} = \frac{3}{2}$ $\frac{WS}{SP} = \frac{WT}{RT} = \frac{3}{2} \quad (ST \parallel PR; \text{Prop th})$ $WT = \frac{3 \times 6}{2}$ $WT = 9 \text{ cm}$	$\checkmark \frac{WS}{SP} = \frac{WT}{RT}$ $\checkmark ST \parallel PR; \text{Prop th}$ $\checkmark \text{ answer}$ (3)
10.2	$\frac{WS}{SP} = \frac{WR}{RQ} = \frac{3}{2} \quad (SR \parallel PQ; \text{Prop th})$ $\frac{9+6}{RQ} = \frac{3}{2}$ $RQ = 10 \text{ cm}$ $WQ = 10 + 9 + 6$ $= 25 \text{ cm}$	$\checkmark \frac{WS}{SP} = \frac{WR}{RQ} = \frac{3}{2}$ $\checkmark \frac{9+6}{RQ} = \frac{3}{2}$ $\checkmark RQ = 10$ $\checkmark WQ = 25$ (4) [7]

QUESTION 11

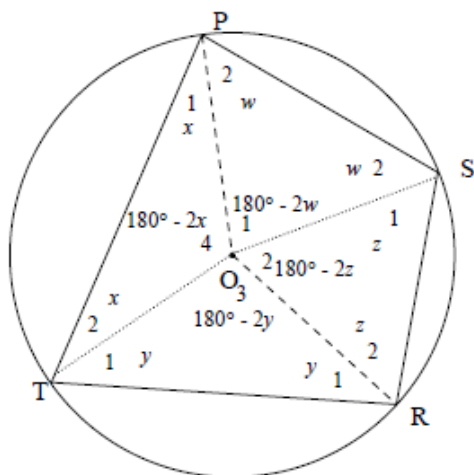
11.1



Join RO and OP

Let $\hat{O}_1 = 2x$ $\hat{O}_2 = 360^\circ - 2x$ ($\angle$ s in a rev) $\hat{T} = x$ ($\angle$ circ centre = 2 $\angle$ circumference) $\hat{S} = 180^\circ - x$ ($\angle$ circ centre = 2 $\angle$ circumference) $\hat{S} + \hat{T} = x + 180^\circ - x$ $= 180^\circ$

OR



Draw radii OP, OS, OR and OT

Let $\hat{P}_1 = x$, $\hat{S}_2 = w$, $\hat{S}_1 = z$ and $\hat{R}_1 = y$ $\therefore \hat{T}_2 = x$, $\hat{P}_2 = w$, $\hat{R}_2 = z$ and $\hat{T}_1 = y$ ($\angle$ s opp = radii) $\hat{O}_1 = 180^\circ - 2w$ ($\angle$ sum Δ)

Similarly

 $\hat{O}_2 = 180^\circ - 2z$, $\hat{O}_3 = 180^\circ - 2y$, $\hat{O}_4 = 180^\circ - 2x$

By angles in a revolution

 $\hat{O}_1 + \hat{O}_2 + \hat{O}_3 + \hat{O}_4 = 180^\circ - 2w + 180^\circ - 2z + 180^\circ - 2y + 180^\circ - 2x$

$$360^\circ = 720^\circ - 2(w + z + y + x)$$

$$2(w + z + y + x) = 360^\circ$$

$$w + z + y + x = 180^\circ$$

OR

✓ construction

✓ $\hat{O}_1 = 2x$ ✓ $\hat{O}_2 = 360^\circ - 2x$ ✓ $\hat{T} = x$ ✓ $\hat{S} = 180^\circ - x$

✓ reason

(6)

✓ construction

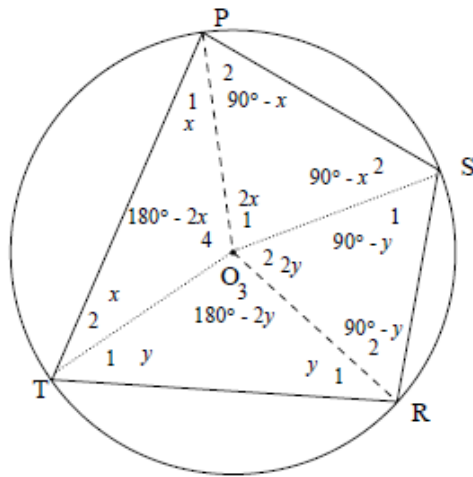
✓ Let $\hat{P}_1 = x$, $\hat{S}_2 = w$, $\hat{S}_1 = z$ and $\hat{R}_1 = y$ ✓ $\hat{O}_1 = 180^\circ - 2w$ ($\angle$ sum Δ)

✓ setting up equation

✓ simplification

✓ conclusion

(6)



Draw radii OP, OS, OR and OT

Let $\hat{P}_1 = x$ and $\hat{R}_1 = y$

$\therefore \hat{T}_2 = x$ and $\hat{T}_1 = y$ ($\angle$ s opp = radii)

$\hat{O}_3 = 180^\circ - 2y$ ($\angle$ sum Δ)

Similarly

$\hat{O}_4 = 180^\circ - 2x$

$\hat{S}_2 = \hat{P}_2$ and $\hat{S}_1 = \hat{R}_1$ ($\angle$ s opp = radii)

$\hat{T}_1 + \hat{T}_2 + \hat{S}_1 + \hat{S}_2 = x + y + 90^\circ - x + 90^\circ - y$
 $= 180^\circ$

✓ construction

✓ Let $\hat{P}_1 = x$ and

$\hat{R}_1 = y$

✓ $\hat{O}_3 = 180^\circ - 2y$

($\angle$ sum Δ)

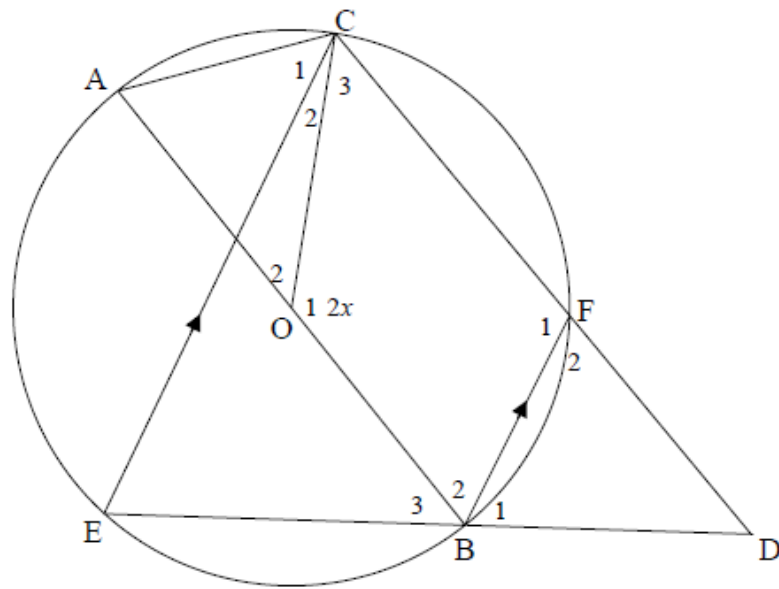
✓ $\hat{S}_2 = \hat{P}_2$ and

$\hat{S}_1 = \hat{R}_1$

✓ $\hat{T}_1 + \hat{T}_2 + \hat{S}_1 + \hat{S}_2$

✓ conclusion

(6)



11.2.1	$\hat{A} = x$ ($\angle$ circ centre = $2 \angle$ circumference) $\hat{F}_1 = 180^\circ - x$ (opp $\angle$ s of cyclic quad = 180°) OR $\text{reflex } \hat{BOC} = 360^\circ - 2x$ ($\angle$ in revolution) $\hat{F}_1 = 180^\circ - x$ ($\angle$ circ centre = $2 \angle$ circumference)	$\checkmark \hat{A} = x$ $\checkmark \angle$ circ centre = $2 \angle$ circumference $\checkmark \hat{F}_1 = 180^\circ - x$ $\checkmark$ opp $\angle$ s of cyclic quad = 180° (4)
11.2.2	$\hat{F}_2 = x$ (ext $\angle$ = int opp $\angle$) $\hat{B}_1 = x = \hat{E}$ (corres $\angle$ s; $EC \parallel BF$) $\hat{F}_2 = \hat{B}_1$ $DF = BD$ (sides opp = $\angle$ s) OR $\hat{E} = x$ ($\angle$ circ centre = $2 \angle$ circumference) $\hat{B}_1 = \hat{E} = x$ (corres $\angle$ s; $EC \parallel BF$) $\hat{F}_2 = x$ (ext $\angle$ cyclic quad) $\hat{F}_2 = \hat{B}_1$ $DF = BD$ (sides opp = $\angle$ s)	$\checkmark \hat{F}_2 = x$ $\checkmark$ ext $\angle$ = int opp $\angle$ $\checkmark \hat{B}_1 = x$ $\checkmark$ sides opp = $\angle$ s (4)

11.2.3	$\hat{C}_1 + \hat{C}_2 = x$ ($\angle$ s opp equal sides in Δ) $\hat{C}_3 + \hat{C}_2 = x$ (corresp $\angle$ s CE $\parallel$ BF) $\hat{C}_1 = \hat{C}_3$ OR $\hat{D} = 180^\circ - 2x$ ($\angle$ sum Δ) $\hat{O}_1 + \hat{D} = 180^\circ$ OCDB is a cyclic quad (opp $\angle$ s suppl) $\hat{B}_3 = \hat{C}_3$ (ext $\angle$ cyclic quad) $\hat{B}_3 = \hat{C}_1$ ($\angle$ same seg) $\hat{C}_1 = \hat{C}_3$	$\checkmark \hat{C}_1 + \hat{C}_2 = x$ $\checkmark \angle$ s opp equal sides in Δ $\checkmark \hat{C}_2 + \hat{C}_3 = x$ $\checkmark$ corresp $\angle$ s CE $\parallel$ BF $\checkmark$ opp $\angle$ s suppl $\checkmark \hat{B}_3 = \hat{C}_3$ $\checkmark$ ext $\angle$ cyclic quad $\checkmark \hat{B}_3 = \hat{C}_1$ (4)
11.2.4	$\hat{D} = 180^\circ - 2x$ ($\angle$ sum Δ) $\hat{O}_2 = 180^\circ - 2x$ ($\angle$ on straight line) $\therefore \hat{D} = \hat{O}_2$ $\therefore \sin \hat{D} = \sin \hat{O}_2$ $\frac{\text{area } \Delta BFD}{\text{area } \Delta AOC}$ $= \frac{\frac{1}{2} DF \cdot BD \cdot \sin \hat{D}}{\frac{1}{2} AO \cdot OC \cdot \sin \hat{O}_2}$ $= \frac{\frac{1}{2} 5.5 \cdot \sin \hat{D}}{\frac{1}{2} 6.6 \cdot \sin \hat{O}_2}$ $= \frac{5.5}{6.6}$ $= \frac{25}{36}$	$\checkmark \hat{D} = 180^\circ - 2x$ $\checkmark \hat{O}_2 = 180^\circ - 2x$ $\checkmark \frac{\frac{1}{2} DF \cdot BD \cdot \sin \hat{D}}{\frac{1}{2} AO \cdot OC \cdot \sin \hat{O}_2}$ $\checkmark \frac{25}{36}$ (4)

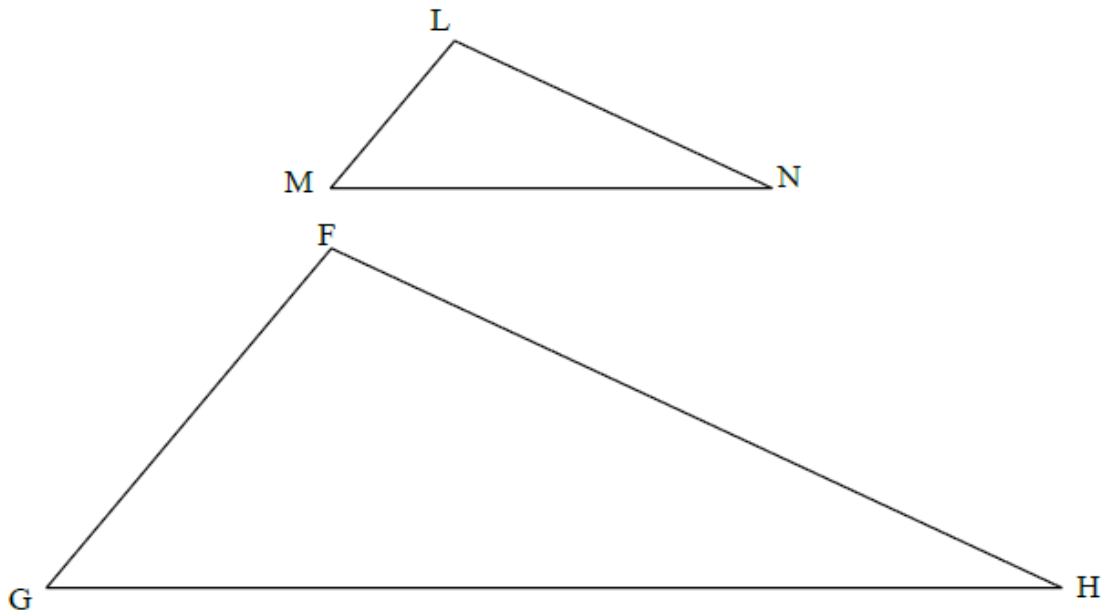
[22]

November 2012

NOTE: Give reasons for all statements made in QUESTION 7, QUESTION 8, QUESTION 9 and QUESTION 10.

QUESTION 7

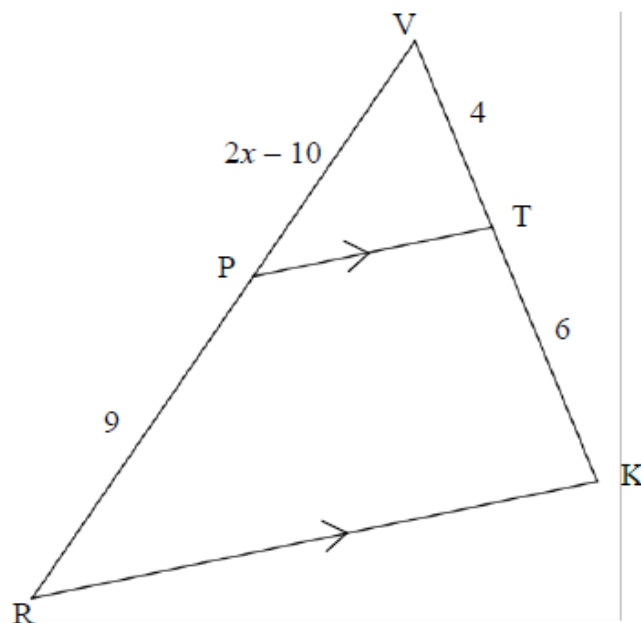
- 7.1 If in $\triangle LMN$ and $\triangle FGH$ it is given that $\hat{L} = \hat{F}$ and $\hat{M} = \hat{G}$, prove the theorem that states $\frac{LM}{FG} = \frac{LN}{FH}$.



(7)

- 7.2 In the diagram below, $\triangle VRK$ has P on VR and T on VK such that $PT \parallel RK$. $VT = 4$ units, $PR = 9$ units, $TK = 6$ units and $VP = 2x - 10$ units.

Calculate the value of x .



(4)
[11]

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord is equal ...

(1)

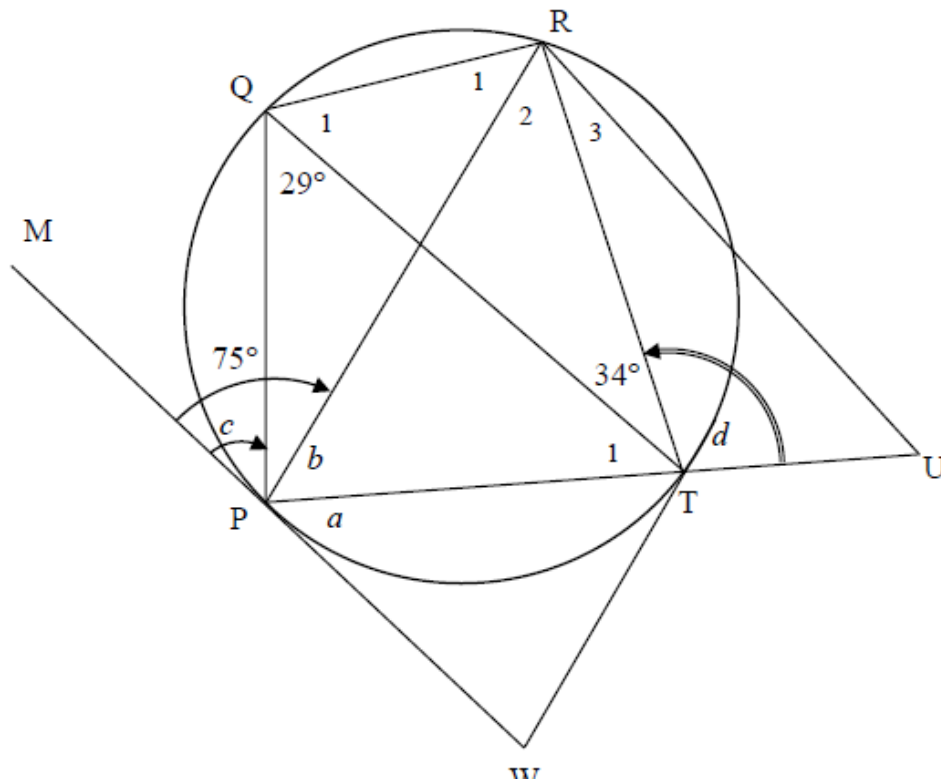
8.2 In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U.

$$\hat{MPR} = 75^\circ$$

$$\hat{PQT} = 29^\circ$$

$$\hat{QTR} = 34^\circ$$

Let $\hat{TPW} = a$, $\hat{RPT} = b$, $\hat{MPQ} = c$ and $\hat{RTU} = d$, calculate the values of a , b , c and d .



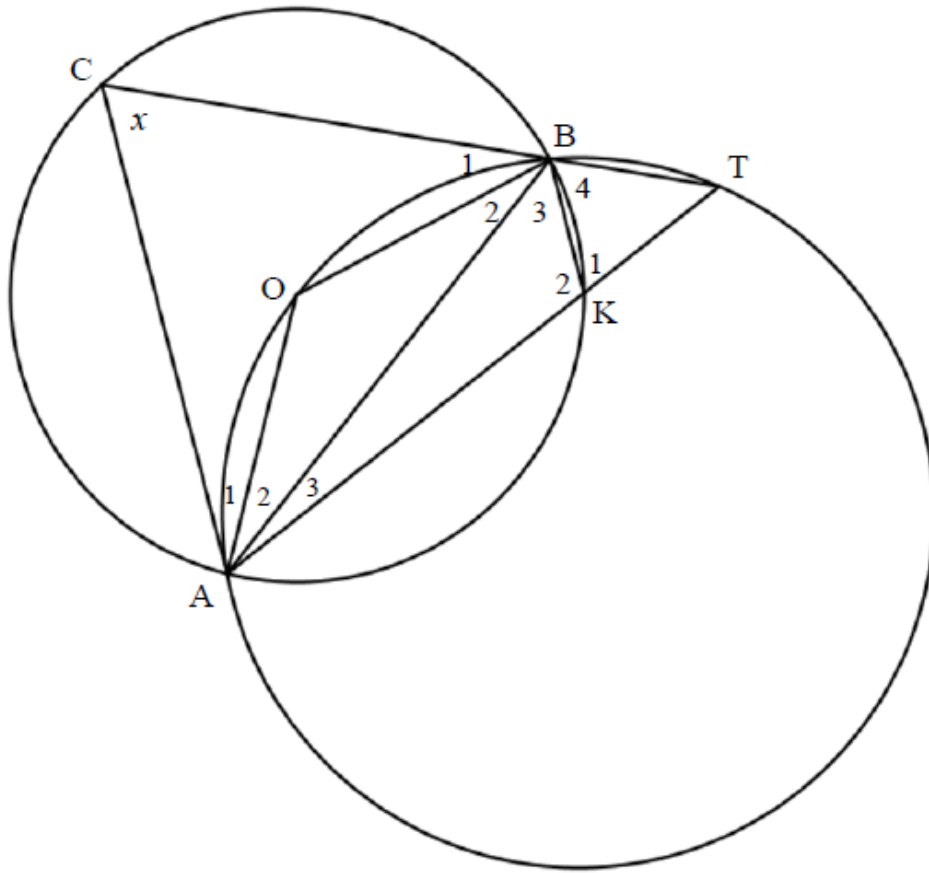
(9)
[10]

QUESTION 9

O is the centre of the circle CAKB.

AK produced intersects circle AOBT at T.

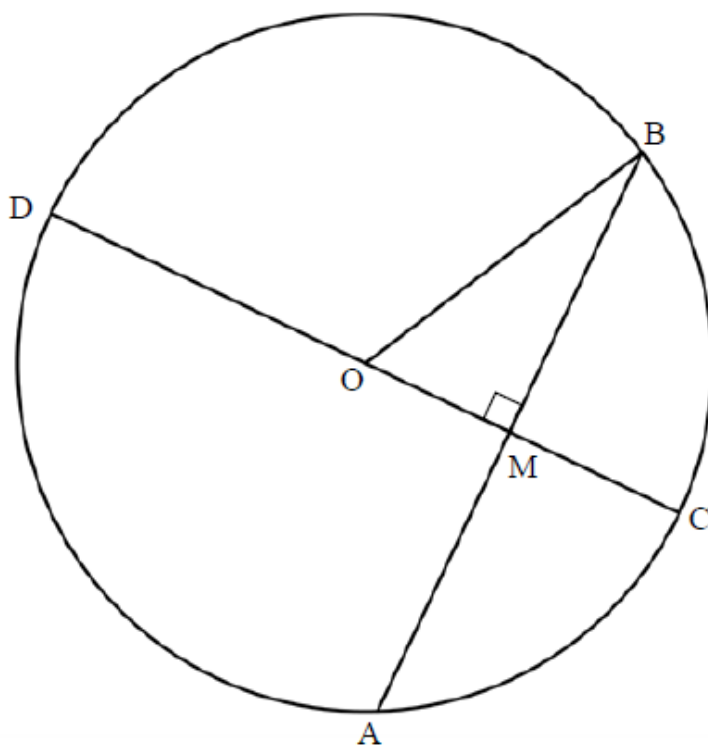
$\hat{ACB} = x$



- 9.1 Prove that $\hat{T} = 180^\circ - 2x$. (3)
- 9.2 Prove $AC \parallel KB$. (5)
- 9.3 Prove $\triangle BKT \parallel \triangle CAT$ (3)
- 9.4 If $AK : KT = 5 : 2$, determine the value of $\frac{AC}{KB}$ (3)
- [14]**

QUESTION 10

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC. $CM : MD = 4 : 9$ and $AB = 24$ units.



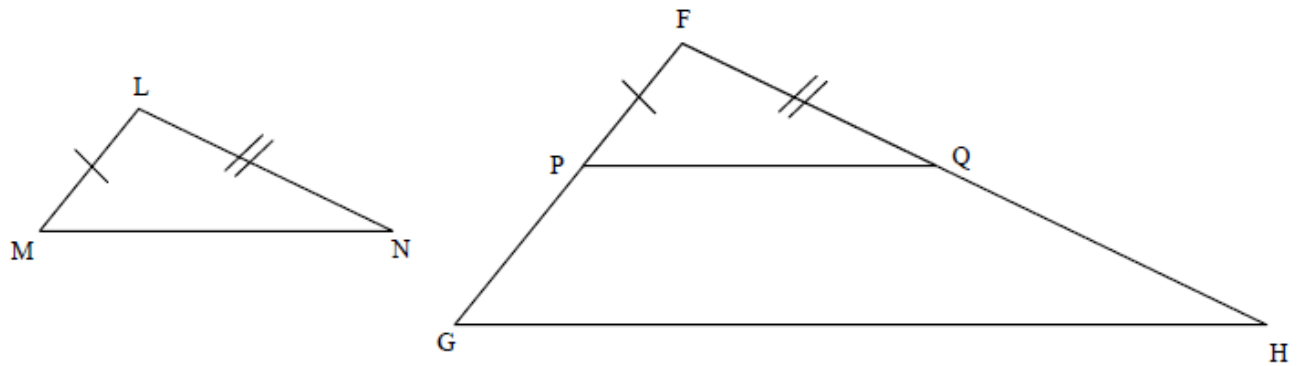
10.1 Determine an expression for DC in terms of x if $CM = 4x$ units. (1)

10.2 Determine an expression for OM in terms of x . (2)

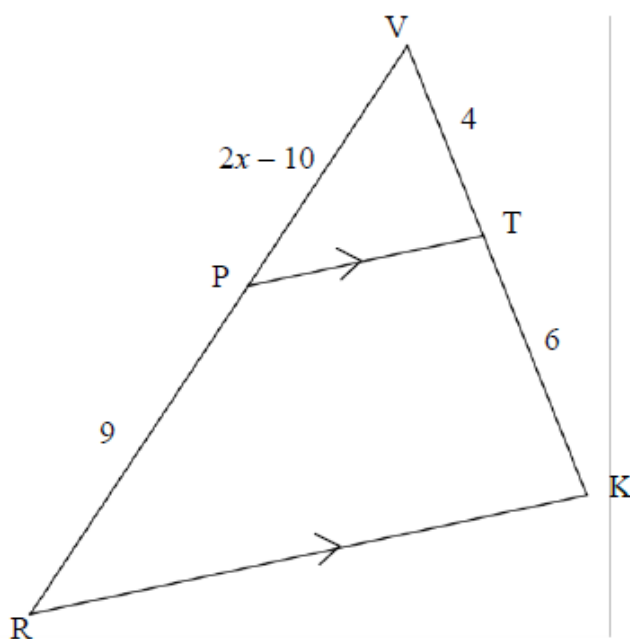
10.3 Hence, or otherwise, calculate the length of the radius. (4)

[7]

QUESTION 7



7.1	Draw a point P on FG such that $FP = LM$ and a point Q on FH such that $FQ = LN$. In $\triangle FPQ$ and $\triangle LMN$ 1. $\hat{F} = \hat{L}$ (given) 2. $FP = LM$ (construction) 3. $FQ = LN$ (construction) $\therefore \triangle FPQ \equiv \triangle LMN$ (SAS) $\hat{FPQ} = \hat{LMN}$ ($\equiv \Delta s$) But $\hat{FGH} = \hat{LMN}$ (given) $\hat{FPQ} = \hat{FGH}$ $PQ \parallel GH$ (corresponding angles =) $\frac{FP}{FG} = \frac{FQ}{FH}$ ($PQ \parallel GH$; Prop Th) $\frac{LM}{FG} = \frac{LN}{FH}$	✓ construction ✓ All three statements must be given ✓ $\triangle FPQ \equiv \triangle LMN$ (SAS) ✓ $\hat{FPQ} = \hat{LMN}$ ✓ $\hat{FPQ} = \hat{FGH}$ ✓ $PQ \parallel GH$ ✓ $\frac{FP}{FG} = \frac{FQ}{FH}$ (7)
-----	--	---

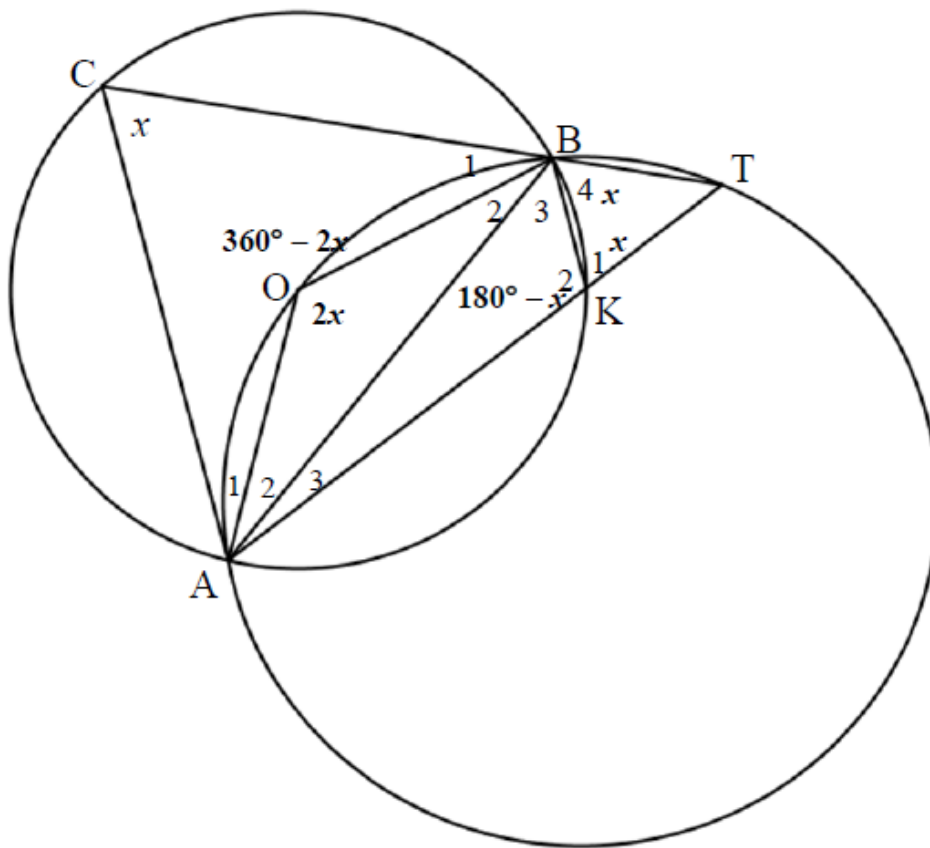


7.2	$\frac{VP}{PR} = \frac{VT}{TK} \text{ (PT} \parallel \text{RK; Prop Th)}$ $\frac{2x-10}{9} = \frac{4}{6}$ $2x-10 = 6$ $2x = 16$ $x = 8$ OR $\frac{VP}{VR} = \frac{VT}{VK} \text{ (PT} \parallel \text{RK; Prop Th)}$ $\frac{2x-10}{2x-1} = \frac{4}{10}$ $20x-100 = 8x-4$ $12x = 96$ $x = 8$	$\checkmark \frac{VP}{PR} = \frac{VT}{TK}$ $\checkmark \text{ (PT} \parallel \text{RK; Prop Th)}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (4) $\checkmark \frac{VP}{VR} = \frac{VT}{VK}$ $\checkmark \text{ (PT} \parallel \text{RK; Prop Th)}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (4) [11]
-----	--	--

QUESTION 8

8.1	... equal to the angle subtended by the chord in the alternate segment.	✓ answer (1)
8.2	$a = 29^\circ$ (tan ch.thm) $\hat{QPR} = 34^\circ$ ($\angle$s in same seg) $c = 41^\circ$ $b = 76^\circ$ (adj $\angle$s on str. line) $\hat{Q}_1 = 76^\circ$ ($\angle$s in same seg) $d = 105^\circ$ (ext $\angle$ cyclic quad) OR $a = 29^\circ$ (tan ch. thm) $\hat{T}_1 = c$ (tan ch. thm) $c + 34^\circ = 75^\circ$ (tan ch. thm) $c = 41^\circ$ $b = 76^\circ$ (adj $\angle$s on str. line) $d = 105^\circ$ (adj $\angle$s on str. line) OR An alternative solution for calculating d: $\hat{Q}_1 = \hat{RPT} = 76^\circ$ ($\angle$s in same seg) $d + \hat{RTQ} = \hat{PQT} + \hat{QPR} + \hat{RPT}$ (ext $\angle \Delta$) $d + 34^\circ = 29^\circ + 34^\circ + 76^\circ$ $d = 105^\circ$	✓ $a = 29^\circ$ ✓ tan ch. thm ✓ $\hat{QPR} = 34^\circ$ ✓ $\angle$s in same seg ✓ $c = 41^\circ$ ✓ $b = 76^\circ$ ✓ $\hat{Q}_1 = 76^\circ$ ✓ $d = 105^\circ$ ✓ ext $\angle$ cyclic quad (9) ✓ $a = 29^\circ$ ✓ tan ch. thm ✓ $\hat{T}_1 = c$ ✓ tan ch. thm ✓ $c + 34^\circ = 75^\circ$ ✓ tan ch. thm ✓ $c = 41^\circ$ ✓ $b = 76^\circ$ ✓ $d = 105^\circ$ (9) [10]

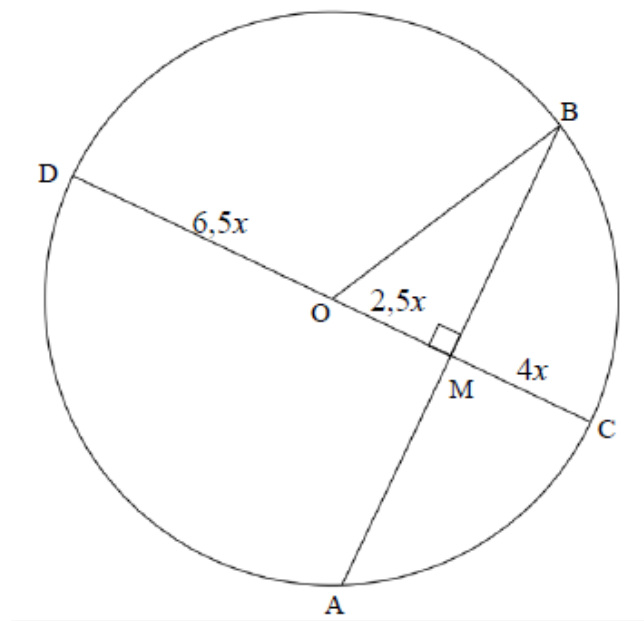
QUESTION 9



9.1	$\hat{AOB} = 2x$ ($\angle$ circ centre = 2 $\angle$ circumference) $\hat{T} = 180^\circ - 2x$ (opp $\angle$ cyclic quad suppl)	$\checkmark \hat{AOB} = 2x$ $\checkmark \angle$ circ centre = 2 $\angle$ circumference $\checkmark$ opp $\angle$ cyclic quad suppl (3)
9.2	$\hat{CAT} = x$ ($\angle$ sum Δ) $\hat{K}_1 = x$ (ext $\angle$ cyclic quad) $\hat{CAT} = \hat{K}_1$ $BK \parallel AC$ (corresponding $\angle$ s =) OR $\hat{K}_1 = \hat{C} = x$ (ext $\angle$ cyclic quad) $\hat{B}_4 = x$ ($\angle$ sum Δ) $\hat{B}_4 = \hat{C} = x$ $BK \parallel CA$ (corresponding $\angle$ s =) OR $\hat{CAT} = x$ ($\angle$ sum Δ) $\hat{BKA} = 180^\circ - x$ (opp $\angle$ cyclic quad) $\hat{CAT} + \hat{BKA} = 180^\circ$ $BK \parallel AC$ (co-int $\angle$ s suppl)	$\checkmark \hat{CAT} = x$ $\checkmark \angle$ sum Δ $\checkmark \hat{K}_1 = x$ $\checkmark$ ext $\angle$ cyclic quad $\checkmark$ corresponding $\angle$ s = (5) $\checkmark \hat{K}_1 = \hat{C} = x$ $\checkmark$ ext $\angle$ cyclic quad $\checkmark \hat{B}_4 = x$ $\checkmark \angle$ sum Δ $\checkmark$ corresponding $\angle$ s = (5) $\checkmark \hat{CAT} = x$ $\checkmark \angle$ sum Δ $\checkmark \hat{BKA} = 180^\circ - x$ $\checkmark$ opp $\angle$ cyclic quad $\checkmark$ co-int $\angle$ s suppl (5)

9.3	In $\triangle BKT$ and $\triangle CAT$ 1. $\hat{CAT} = \hat{K}_1$ ($= x$) 2. $\hat{T}$ is common 3. $\hat{ACT} = \hat{B}_4$ ($\angle$ sum Δ) $\triangle BKT \parallel \triangle CAT$ ($\angle\angle\angle$)	✓ $\hat{CAT} = \hat{K}_1$ ✓ $\hat{T}$ is common ✓ $\angle\angle\angle$ (3)
9.4	$\frac{AC}{KB} = \frac{AT}{KT}$ ($\parallel \Delta s$) $\frac{AC}{KB} = \frac{7}{2}$	✓ $\frac{AC}{KB} = \frac{AT}{KT}$ ✓ $\parallel \Delta s$ ✓ answer (3) [14]

QUESTION 10

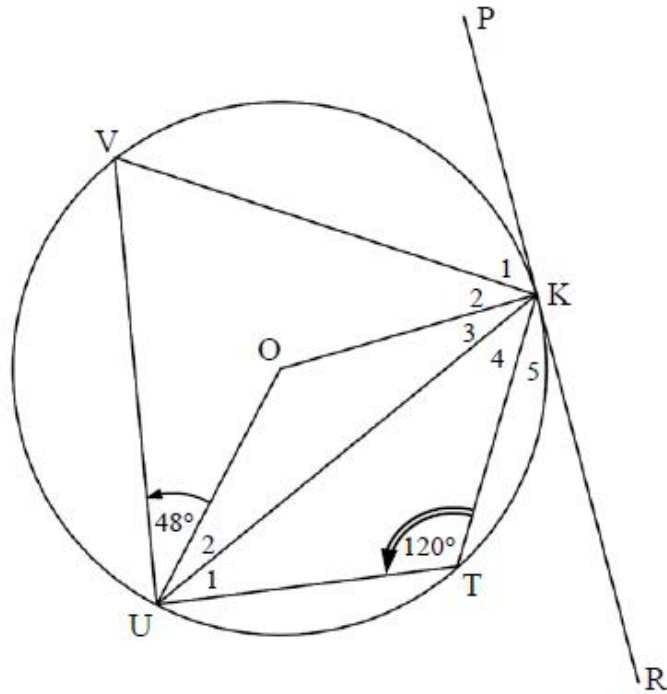


10.1	$DC = 13x$	✓ $CD = 13x$ (1)
10.2	$OD = \frac{13}{2}x$ $OM = \frac{5}{2}x$	✓ $OD = \frac{13}{2}x$ ✓ answer (2)
10.3	$BO = OD$ (radii) $AM = MB = 12$ units (line from circ cent $\perp$ ch) $12^2 + \left(\frac{5}{2}x\right)^2 = \left(\frac{13}{2}x\right)^2$ (Pythagoras) $144 + \frac{25x^2}{4} = \frac{169x^2}{4}$ $144 = \frac{144x^2}{4}$ $x^2 = 4$ $x = \pm 2$ $x = 2$ ($x > 0$) The radius $= \frac{13}{2}(2)$ $= 13$ units.	✓ $MB = 12$ ✓ $12^2 + \left(\frac{5}{2}x\right)^2 = \left(\frac{13}{2}x\right)^2$ or $12^2 + 6,25x^2 = 42,25x^2$ or $12^2 + \frac{25}{4}x^2 = \frac{169}{4}x^2$ ✓ answer ✓ answer (4) [7]

February 2013

QUESTION 8

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K. $\angle UV = 48^\circ$ and $\angle KTU = 120^\circ$.

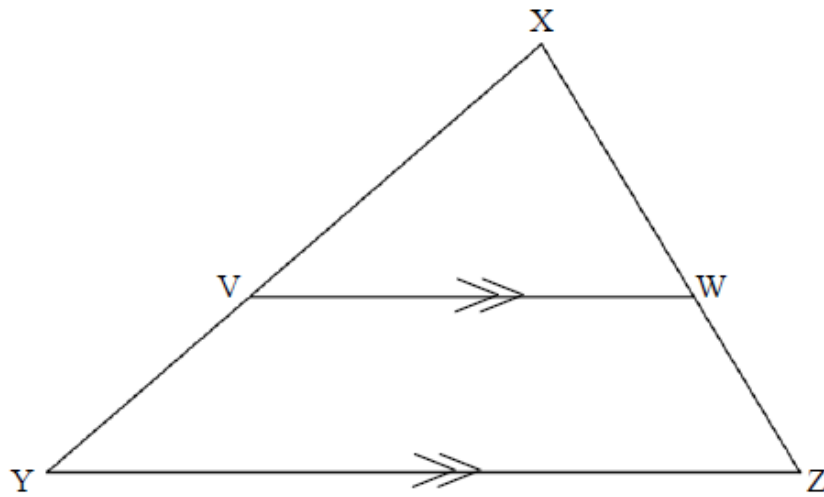


Calculate, with reasons, the sizes of the following angles:

- | | | |
|-----|--------------|-------------|
| 8.1 | $\hat{V}$ | (2) |
| 8.2 | $\angle KOU$ | (2) |
| 8.3 | $\hat{U}_2$ | (2) |
| 8.4 | $\hat{K}_1$ | (2) |
| 8.5 | $\hat{K}_2$ | (2) |
| | | [10] |

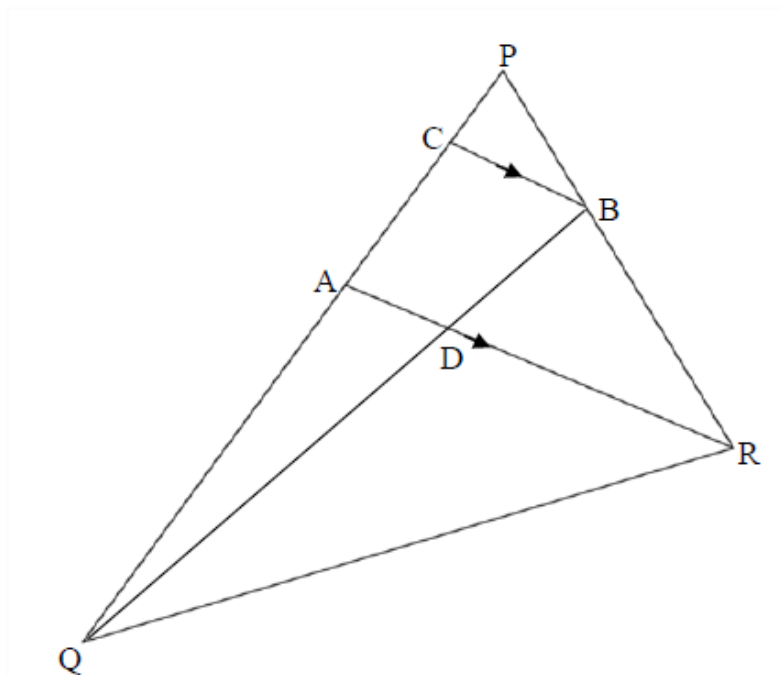
QUESTION 9

- 9.1 Use the diagram below to prove the theorem which states that if $VW \parallel YZ$ then $\frac{XV}{VY} = \frac{XW}{WZ}$.



(6)

- 9.2 In ΔPQR below, B lies on PR such that $2PB = BR$. A lies on PQ such that $PA : PQ = 3 : 8$. BC is drawn parallel to AR.



- 9.2.1 Write down the value of $\frac{\text{area of } \Delta PRA}{\text{area of } \Delta QRA}$ (2)

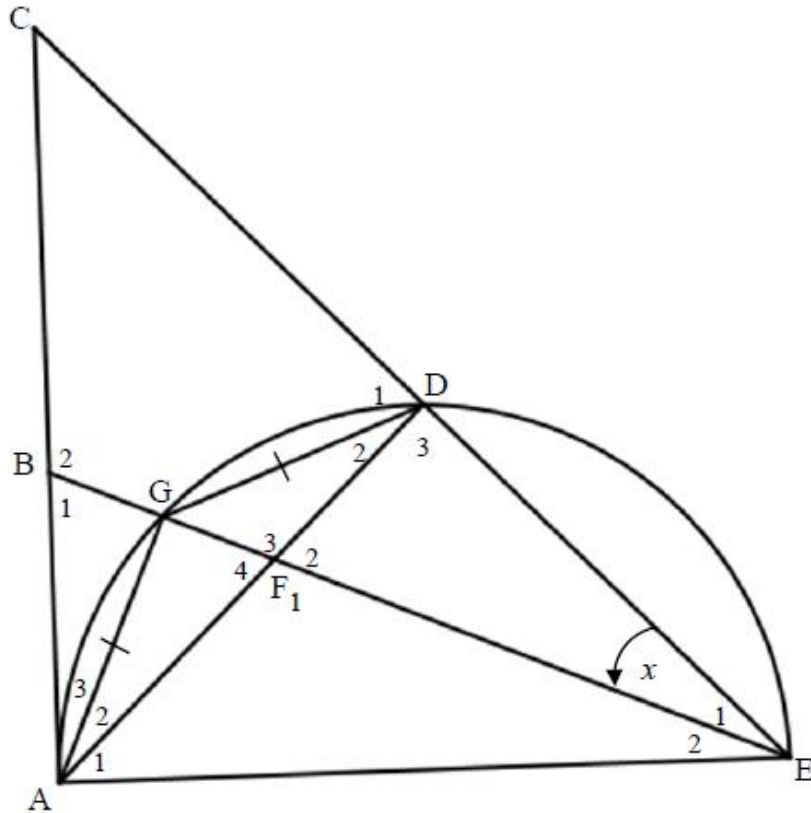
- 9.2.2 Calculate the value of the ratio $\frac{BD}{BQ}$. Show all working to support your answer. (5)

[13]

QUESTION 10

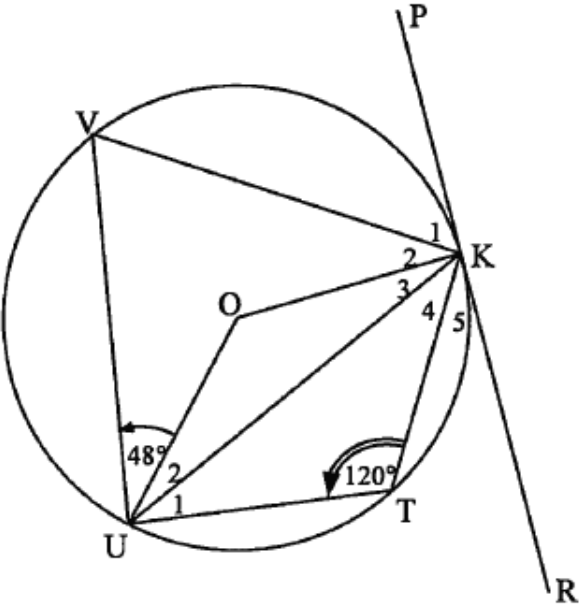
In the figure AGDE is a semicircle. AC is the tangent to the semicircle at A and EG produced intersects AC at B. AD intersects BE in F.

$AG = GD$. $\hat{E}_1 = x$.



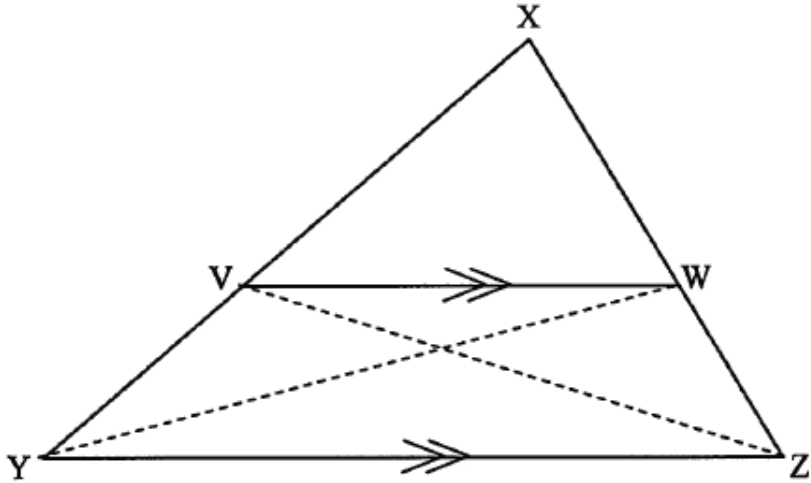
- | | | |
|------|---|-------------|
| 10.1 | Write down, with reasons, FOUR other angles each equal to x . | (8) |
| 10.2 | Prove that $BE \cdot DE = AE \cdot FE$ | (7) |
| 10.3 | Prove that $\hat{B}_1 = \hat{D}_1$ | (4) |
| | | [19] |

QUESTION 8

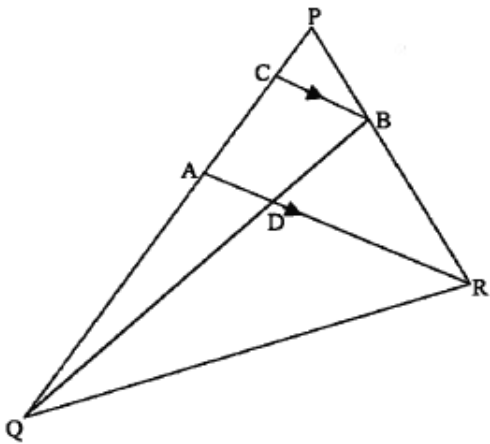


8.1	$\hat{V} = 180^\circ - 120^\circ = 60^\circ$ [Opp angles of cyclic quad are supp]	✓ 60° ✓ reason (2)
8.2	$\angle K\hat{O}U = 2(60^\circ) = 120^\circ$ [Angle at centre = twice angle at circum.]	✓ 120° ✓ reason (2)
8.3	$\hat{U}_2 = \frac{180^\circ - 120^\circ}{2} = 30^\circ$ [Base angles of isosceles $\triangle UOS$; $OU = OK$ = radii]	✓ 30° ✓ reason (2)
8.4	$\hat{K}_1 = 48^\circ + 30^\circ = 78^\circ$ [tan-chord theorem]	✓ 78° ✓ reason (2)
8.5	$\hat{K}_2 = 90^\circ - 78^\circ = 12^\circ$ [tan $\perp$ radius]	✓ 12° ✓ reason (2) [10]

QUESTION 9

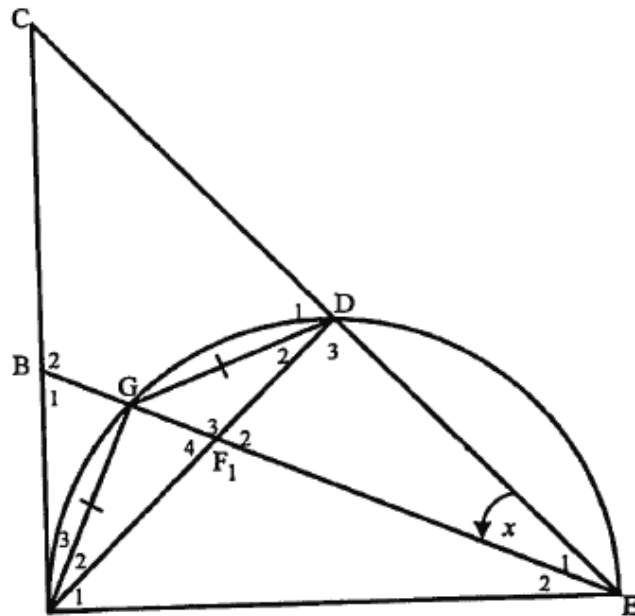


9.1	Construct VZ and WY $\frac{\text{area } \triangle XVW}{\text{area } \triangle VWY} = \frac{XV}{VY} \text{ (equal altitudes)}$ $\frac{\text{area } \triangle XVW}{\text{area } \triangle WVZ} = \frac{XW}{WZ} \text{ (equal altitudes)}$ $\text{area } \triangle YVW = \text{area } \triangle VWZ \quad (VW \parallel YZ)$ $\text{area } \triangle XVW \text{ is common}$ $\frac{XW}{WZ} = \frac{XV}{VY}$	✓ construction $\checkmark \frac{\text{area } \triangle XVW}{\text{area } \triangle VWY} = \frac{XV}{VY}$ $\checkmark \frac{\text{area } \triangle XVW}{\text{area } \triangle WVZ} = \frac{XW}{WZ}$ ✓ $\text{area } \triangle YVW = \text{area } \triangle VWZ$ ✓ $VW \parallel YZ$ ✓ conclusion (6)
-----	---	--



9.2.1	$\frac{\text{area } \triangle PRA}{\text{area } \triangle QRA} = \frac{PA}{QA}$ (equal altitudes) $\frac{\text{area } \triangle PRA}{\text{area } \triangle QRA} = \frac{3}{5}$	$\checkmark \frac{\text{area } \triangle PRA}{\text{area } \triangle QRA} = \frac{PA}{QA}$ $\checkmark$ answer (2)
9.2.2	$\frac{BD}{DQ} = \frac{CA}{AQ}$ (AR CB) $\frac{PC}{CA} = \frac{1}{2}$ (AR CB) PC = y units CA = 2y units CQ = 5y units $\frac{BD}{BQ} = \frac{2}{7}$	$\checkmark \frac{BD}{DQ} = \frac{CA}{AQ}$ $\checkmark$ reason $\checkmark \frac{PC}{CA} = \frac{1}{2}$ $\checkmark$ CQ = 5y units $\checkmark \frac{BD}{BQ} = \frac{2}{5}$ (5) [13]

QUESTION 10



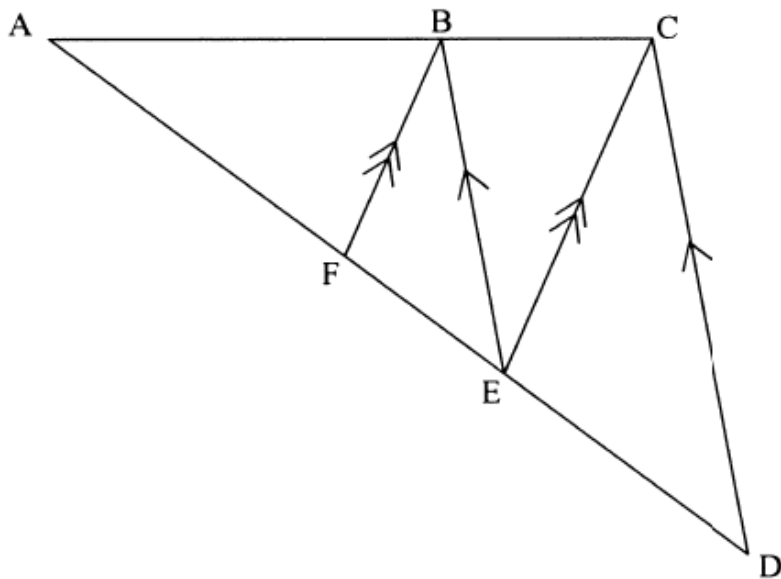
10.1	$\hat{A}_2 = x$ ($\angle$ s in same seg) $\hat{D}_2 = x$ ($\angle$ s opp = sides) $\hat{E}_2 = x$ (= chs = $\angle$ s) or ($\angle$ s in same seg) $\hat{A}_3 = x$ (tan-chord theorem)	$\checkmark \hat{A}_2 = x$ $\checkmark$ reason $\checkmark \hat{D}_2 = x$ $\checkmark$ reason $\checkmark \hat{E}_2 = x$ $\checkmark$ reason $\checkmark \hat{A}_3 = x$ $\checkmark$ reason	(8)
10.2	In ΔABE and ΔDFE 1. $\hat{E}_2 = \hat{E}_1$ ($= x$) 2. $\hat{D}_3 = 90^\circ$ ($\angle$ s in semicircle) $\hat{BAE} = 90^\circ$ (tan $\perp$ rad) $\hat{BAE} = \hat{D}_3$ $\Delta ABE \parallel \Delta DFE$ ($\angle\angle\angle$) $\frac{BE}{FE} = \frac{AE}{DE}$ ($\parallel \Delta$ s) $BE.DE = AE.FE$	$\checkmark \hat{E}_2 = \hat{E}_1$ $\checkmark \hat{D}_3 = 90^\circ$ $\checkmark$ reason $\checkmark \hat{BAE} = 90^\circ$ $\checkmark$ reason $\checkmark \frac{BE}{FE} = \frac{AE}{DE}$ $\checkmark \parallel \Delta$ s	(7)
10.3	$\hat{D}_1 = 90^\circ - x$ ($\angle$ s on str line) $\hat{B}_1 = 90^\circ - x$ ($\angle$ sum Δ) $\hat{B}_1 = \hat{D}_1$	$\checkmark \hat{D}_1 = 90^\circ - x$ $\checkmark$ reason $\checkmark \hat{B}_1 = 90^\circ - x$ $\checkmark$ reason	(4)

[19]

November 2013

QUESTION 9

In $\triangle ADC$, E is a point on AD and B is a point on AC such that $EB \parallel DC$.
F is a point on AD such that $FB \parallel EC$. It is also given that $AB = 2BC$.

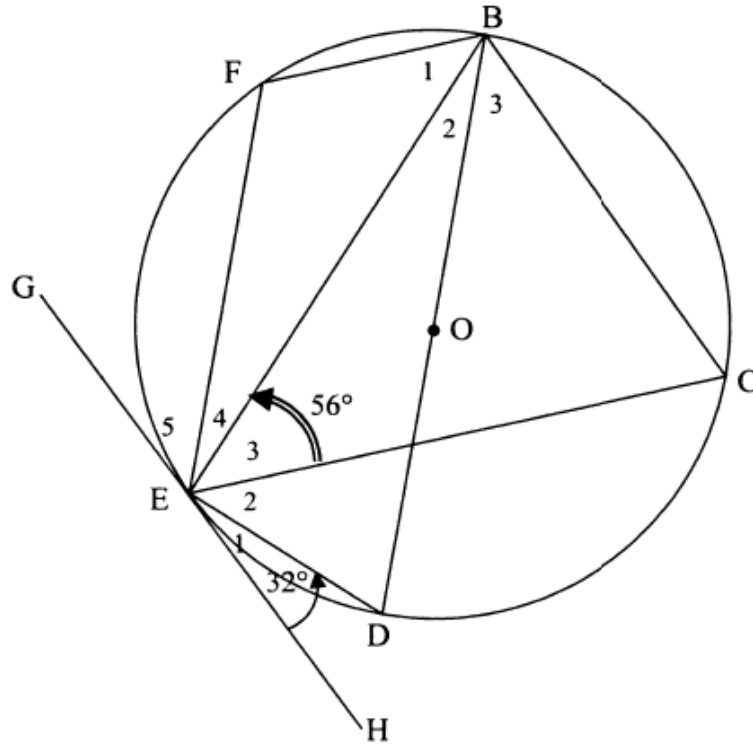


- 9.1 Determine the value of $AF : FE$ (2)
- 9.2 Calculate the length of ED if $AF = 8$ cm. (4)
- [6]

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$$\hat{E}_1 = 32^\circ \text{ and } \hat{E}_3 = 56^\circ.$$



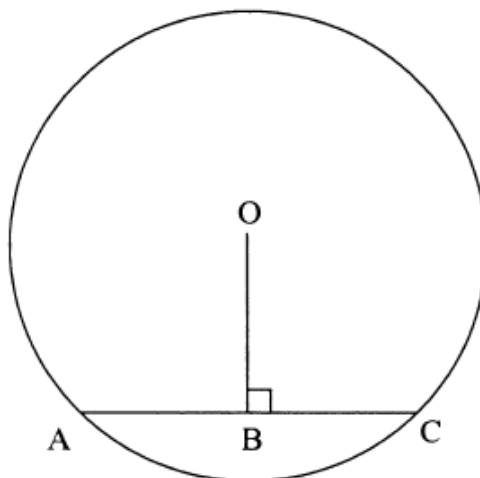
Calculate, with reasons, the values of:

- | | | |
|------|--------------|-----|
| 10.1 | $\hat{E}_2$ | (2) |
| 10.2 | $\angle EBC$ | (3) |
| 10.3 | $\hat{F}$ | (4) |
| | | [9] |

QUESTION 11

In the diagram below, O is the centre of the circle and OB is perpendicular to the chord AC.

Prove, using Euclidean geometry methods, the theorem that states $AB = BC$.

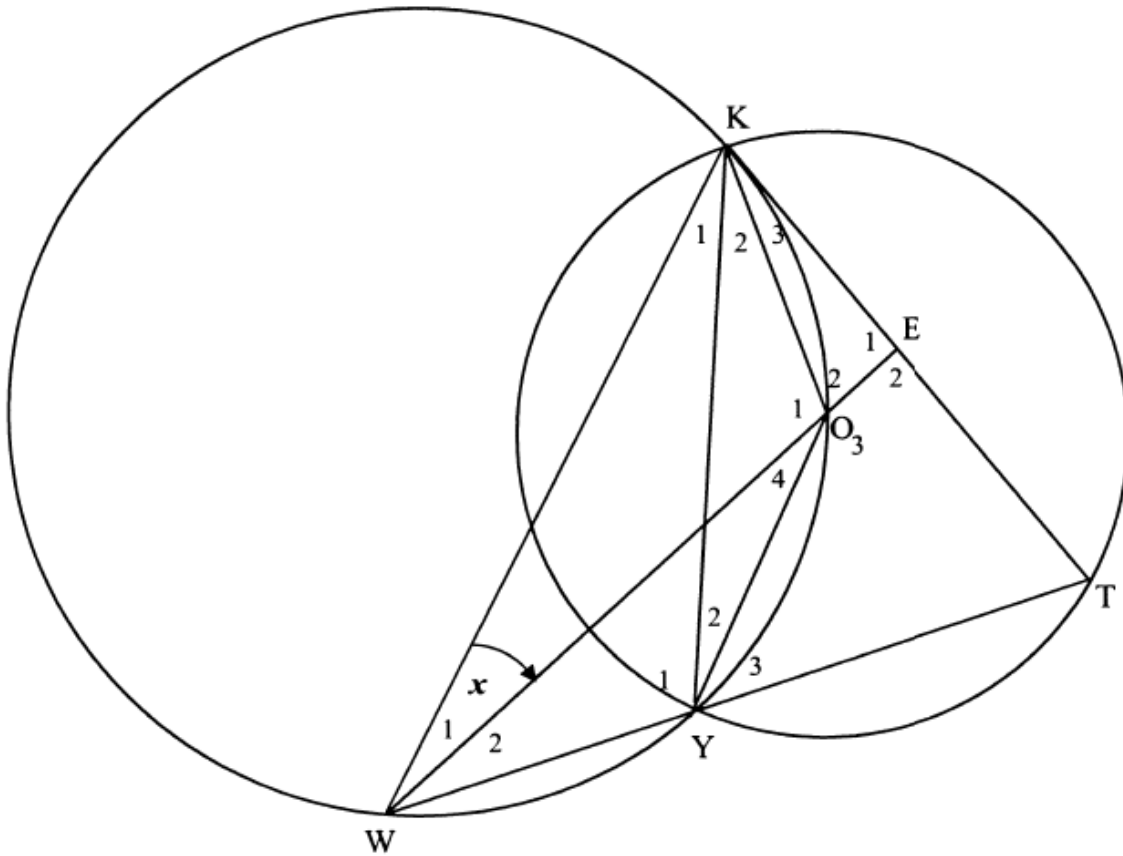


[5]

QUESTION 12

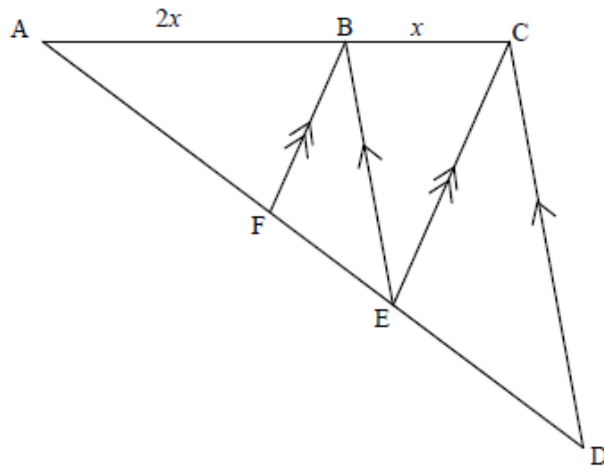
In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W. WO produced meets KT at E.

Let $\hat{W}_1 = x$



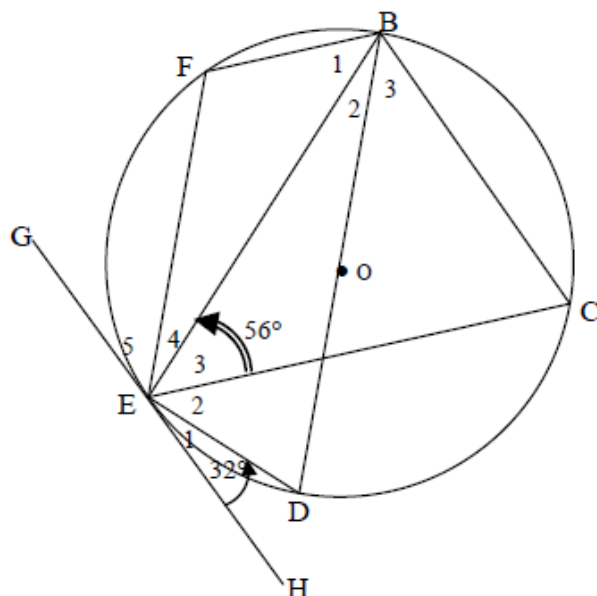
- | | | |
|------|--|-------------|
| 12.1 | Determine FOUR other angles, each equal to x . | (8) |
| 12.2 | Prove that $\hat{T} = 90^\circ - x$. | (3) |
| 12.3 | Prove that $KE = ET$. | (3) |
| 12.4 | Prove that $KE^2 = OE \cdot WE$ | (6) |
| | | [20] |

QUESTION/VR4AG 9



9.1	$AF : FE = 2 : 1$ (Prop Th; $FB \parallel EC$) (Ewaredigheid St; $FB \parallel EC$)	✓ answer ✓ reason (2)
9.2	$\frac{AF}{FE} = \frac{2}{1}$ $FE = \frac{AF}{2} = \frac{8}{2} = 4 \text{ cm}$ $AE = 12 \text{ cm}$ $\frac{ED}{AE} = \frac{1}{2}$ (BE $\parallel$ DC; Prop Th) / (BE $\parallel$ DC; Ewaredigheid St) $\frac{ED}{12} = \frac{1}{2}$ $ED = 6 \text{ cm}$	✓ $FE = 4 \text{ cm}$ ✓ $AE = 12 \text{ cm}$ ✓ $\frac{ED}{AE} = \frac{1}{2}$ ✓ answer (4) [6]

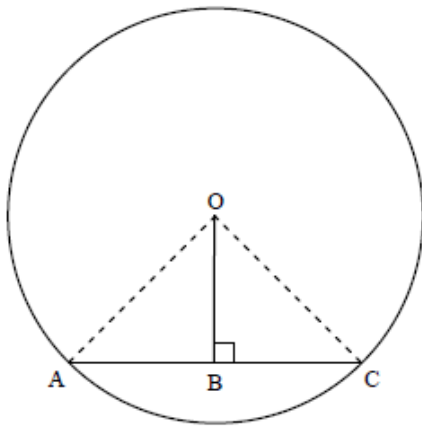
QUESTION/VRAAG 10



10.1	$\hat{E}_2 = 90^\circ - 56^\circ$ ($\angle$ s in a semi circle) $\hat{E}_2 = 34^\circ$ ($\angle$ e in 'n halwe sirkel)	$\checkmark \hat{E}_2 = 34^\circ$ $\checkmark \angle$ s in a semi circle (2)
10.2	$C\hat{E}H = \hat{E}_1 + \hat{E}_2 = 66^\circ$ $\therefore E\hat{B}C = \hat{B}_2 + \hat{B}_3 = 66^\circ$ (tan ch th) (raaklyn koord) OR $\hat{B}_2 = 32^\circ$ (tan ch th) (raaklyn koord) $\hat{B}_3 = 34^\circ$ ($\angle$ s in the same segment) ($\angle$ e in selfde seg) $E\hat{B}C = 66^\circ$	$\checkmark C\hat{E}H = 66^\circ$ $\checkmark E\hat{B}C = 66^\circ$ $\checkmark$ tan ch th (3) $\checkmark$ tan ch th $\checkmark \hat{B}_3 = 34^\circ$ $\checkmark$ answer (3)
10.3	$\hat{E}_1 + \hat{E}_2 + \hat{E}_3 = 122^\circ$ $\hat{F} = 122^\circ$ (tan ch th) / (raaklyn koord) OR $\hat{C} = 58^\circ$ (sum of int $\angle$ s of Δ) / (som van binne $\angle$ e Δ) $\hat{F} = 122^\circ$ (opp $\angle$ s cyclic quad) / (oorst $\angle$ koordevierhoek) OR $\hat{D} = 58^\circ$ (sum of int $\angle$ s of Δ) / (som van binne $\angle$ e Δ) $\hat{F} = 122^\circ$ (opp $\angle$ s cyclic quad) / (oorst $\angle$ koordevierhoek)	$\checkmark\checkmark \hat{F} = 122^\circ$ $\checkmark\checkmark$ reason $\checkmark \hat{C} = 58^\circ$ $\checkmark \angle$ sum Δ $\checkmark \hat{F} = 122^\circ$ $\checkmark$ opp $\angle$ s cyclic quad $\checkmark \hat{D} = 58^\circ$ $\checkmark \angle$ sum Δ $\checkmark \hat{F} = 122^\circ$ $\checkmark$ opp $\angle$ s cyclic quad (4)

[9]

QUESTION/VR4AG 11



Construct radii OA and OC.

In $\triangle OAB$ and $\triangle OCB$

- i. OB is common
- ii. $OA = OC$ (radii)
- iii. $\angle OBA = \angle OBC = 90^\circ$ (given)

$\triangle OAB \equiv \triangle OCB$ (90°HS)

$AB = BC$ ($\equiv \Delta s$)

OR

Construct radii OA and OC.

In $\triangle OAB$ and $\triangle OCB$

- i. OB is common **OR** $OA = OC$ (radii)
- ii. $\hat{A} = \hat{C}$ ($\angle s$ opp = radii)
- iii. $\angle OBA = \angle OBC = 90^\circ$ (given)

$\triangle OAB \equiv \triangle OCB$ (SAA)

$AB = BC$ ($\equiv \Delta s$)

OR

Construct radii OA and OC.

$\angle OBA = \angle OBC = 90^\circ$ (given)

$OA^2 = OB^2 + AB^2$ (Pythagoras)

$OC^2 = OB^2 + BC^2$ (Pythagoras)

$OC = OA$ (radii)

OB is common

$AB^2 = BC^2$

$AB = BC$

✓ construction

✓ OB common

✓ radii

✓ $\angle OBA = \angle OBC = 90^\circ$

✓ 90°HS

✓ construction

✓ OB common /
OA = OC

✓ $\angle s$ opp = radii

✓ $\angle OBA = \angle OBC = 90^\circ$

✓ SAA

✓ construction

✓ $OA^2 = OB^2 + AB^2$

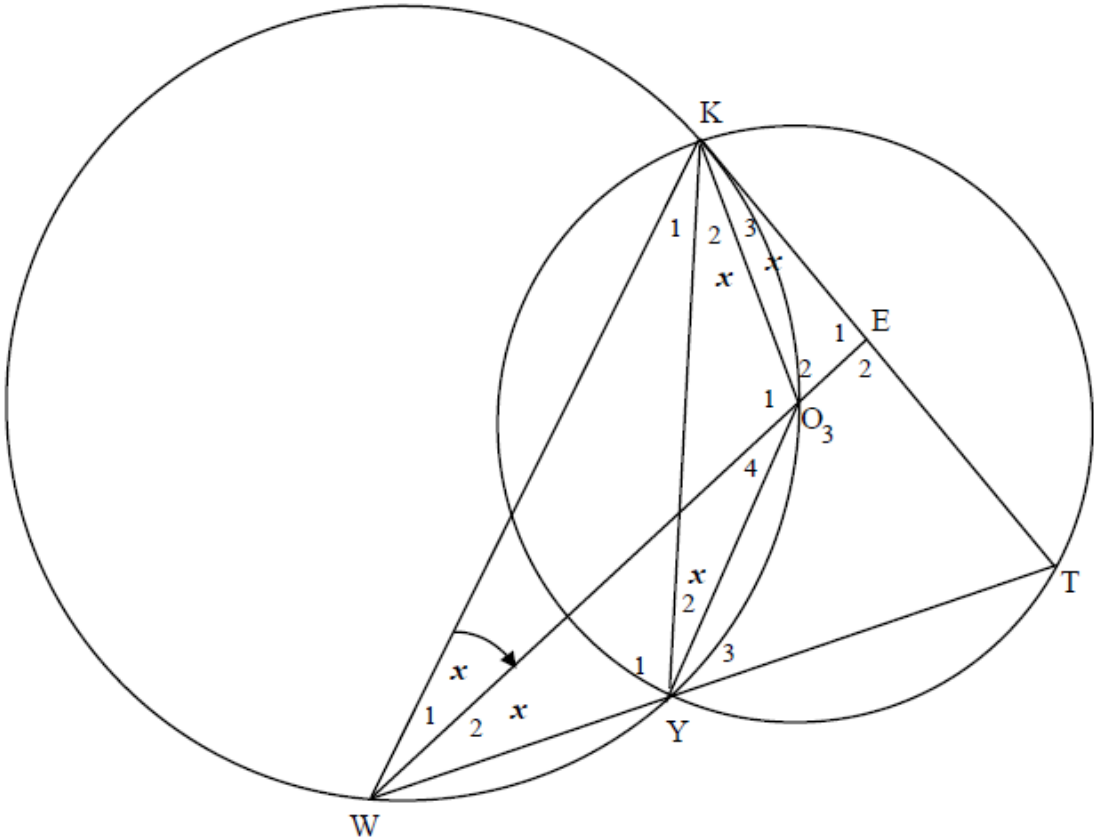
✓ $OC^2 = OB^2 + BC^2$

✓ $OA = OC$ / radii

✓ $AB^2 = BC^2$

[5]

QUESTION/VR4AG 12



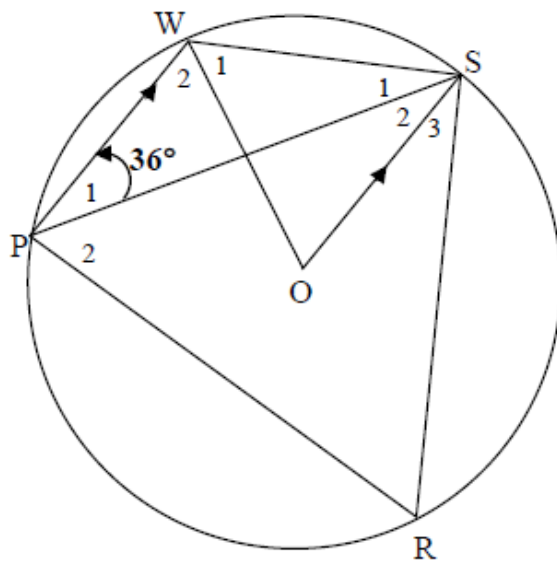
12.1	$\hat{K}_3 = x$ (tan ch th) (<i>raaklyn koord</i>) $\hat{Y}_2 = x$ ($\angle$ s in same seg) (<i>$\angle$e in selfde seg</i>) $\hat{K}_2 = x$ ($\angle$ s opp = radii) / (= chs subt = $\angle$ s) (<i>$\angle$e oork = radiusse</i>) / (= <i>koorde</i> = $\angle$ e) $\hat{W}_2 = x$ ($\angle$ s in same seg) / (= chs subt = $\angle$ s) (<i>$\angle$e in selfde seg</i>) / (= <i>koorde</i> = $\angle$ e)	$\checkmark \hat{K}_3 = x$ $\checkmark$ tan ch th $\checkmark \hat{Y}_2 = x$ $\checkmark \angle$ s in same seg $\checkmark \hat{K}_2 = x$ $\checkmark$ reason $\checkmark \hat{W}_2 = x$ $\checkmark$ reason (8)
12.2	$\hat{O}_1 + \hat{O}_4 = 180^\circ - 2x$ (sum of int $\angle$'s of Δ) / (opp $\angle$ cyclic quad) (<i>som van binne $\angle$e Δ</i>) / (<i>oorst $\angle$ koordevierhoek</i>) $\hat{T} = 90^\circ - x$ ($\angle$ at circ cent = $2\angle$ at circumference)	$\checkmark \hat{O}_1 + \hat{O}_4 = 180^\circ - 2x$ $\checkmark$ reason $\checkmark$ reason (3)

12.3	$\hat{E}_2 = 90^\circ$ (sum of int $\angle$'s of Δ) / (som van binne $\angle$ e Δ) $KE = ET$ ($\perp$ from centre to chord bisects chord) ($\perp$ van middelpunt tot koord halveer koord) OR $\hat{K}_1 + \hat{K}_2 + \hat{K}_3 = 90^\circ - x$ (sum of int $\angle$'s of Δ) In ΔKWE and ΔTEW 1. $\hat{K}_1 + \hat{K}_2 + \hat{K}_3 = \hat{T} = 90^\circ - x$ (proven above) 2. $\hat{W}_1 = \hat{W}_2 = x$ (Proven in 12.1) 3. WE is common $\therefore \Delta KWE \equiv \Delta TEW$ ($\angle\angle S$) $KE = ET$	$\checkmark \hat{E}_2 = 90^\circ$ $\checkmark$ sum of int $\angle$'s of Δ $\checkmark \perp$ from centre to chord bisects chord (3) $\checkmark \hat{K}_1 + \hat{K}_2 + \hat{K}_3 = 90^\circ - x$ $\checkmark$ sum of int $\angle$'s of Δ $\checkmark \Delta KWE \equiv \Delta TEW$ (3)
12.4	In ΔKOE and ΔWTE i. $\hat{K}_3 = \hat{W}_2 = x$ (proven) ii. $\hat{E}_2 = \hat{E}_1 = 90^\circ$ ($\angle$ s on str line / sum of int $\angle$'s of Δ) ($\angle$ e op reguit lyn / som van binne $\angle$ e Δ) iii. $\hat{O}_2 = \hat{T} = 90^\circ - x$ (3^{rd} $\angle$ of Δ) $\Delta KOE \parallel \Delta WTE$ ($\angle\angle\angle$) $\frac{KE}{WE} = \frac{OE}{TE}$ ($\parallel \Delta$ s) $KE = TE$ (proven) $KE \cdot TE = OE \cdot WE$ $KE^2 = OE \cdot WE$ OR In ΔKOE and ΔKWE i. $\hat{K}_3 = \hat{W}_1 = x$ (proven) ii. $\hat{E}_1$ is common iii. $\hat{O}_2 = \hat{K}_1 + \hat{K}_2 + \hat{K}_3$ (3^{rd} $\angle$ of Δ) $\Delta KOE \parallel \Delta WKE$ ($\angle\angle\angle$) $\frac{KE}{WE} = \frac{OE}{KE}$ ($\parallel \Delta$ s) $KE^2 = OE \cdot WE$	$\checkmark \Delta KOE$ and ΔWTE $\checkmark \hat{K}_3 = \hat{W}_2 = x$ $\checkmark \hat{E}_2 = \hat{E}_1 = 90^\circ$ $\checkmark \hat{O}_2 = \hat{T} = 90^\circ - x$ $\checkmark \frac{KE}{WE} = \frac{OE}{TE}$ $\checkmark KE = TE$ (6) $\checkmark \Delta KOE$ and ΔWTE $\checkmark \hat{K}_3 = \hat{W}_1 = x$ $\checkmark \hat{E}_1$ is common $\checkmark \angle\angle\angle$ $\checkmark \checkmark \frac{KE}{WE} = \frac{OE}{KE}$ (6) [20]

February 2014

QUESTION 7

In the diagram, O is the centre of the circle. $PWSR$ is a cyclic quadrilateral. PS , WO and OS are drawn. $PW \parallel OS$ and $\hat{P}_1 = 36^\circ$.



Calculate the sizes of the following angles:

7.1 $\hat{SOW}$ (2)

7.2 $\hat{W}_2$ (2)

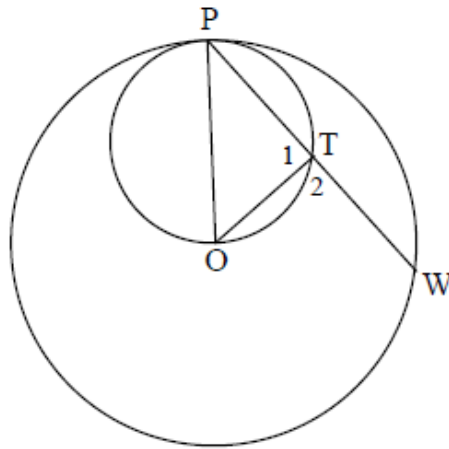
7.3 $\hat{OSW}$ (3)

7.4 $\hat{R}$ (3)

[10]

QUESTION 8

In the diagram, OP is the diameter of the smaller circle. O is the centre of the larger circle. Also, PTW is a chord of the larger circle and T lies on the smaller circle. OT is joined.

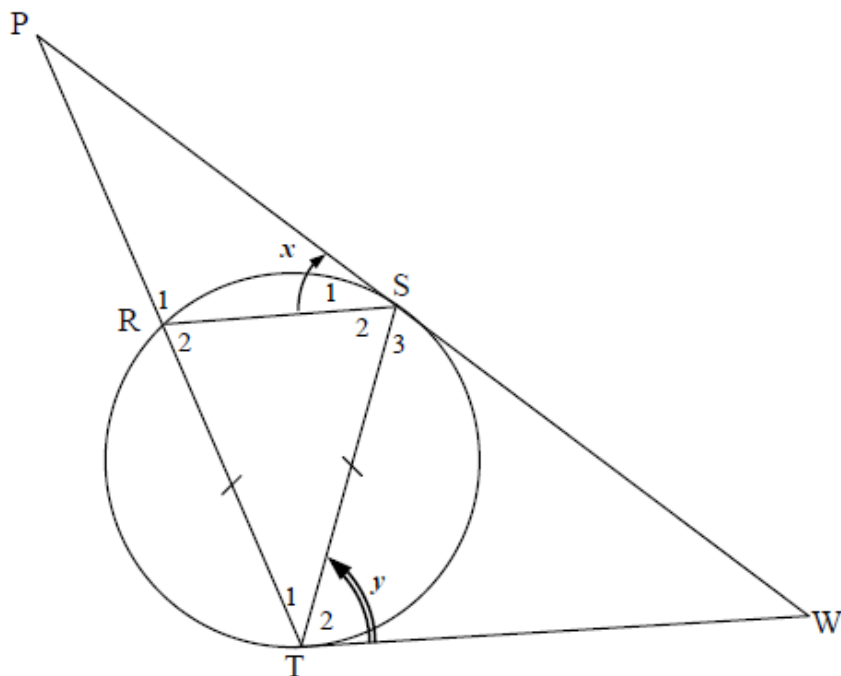


If $OT = 10$ cm and $PW = 48$ cm, calculate the length of the radius of the smaller circle.

[5]**QUESTION 9**

In the diagram, PSW and WT are tangents to circle RST at S and T respectively. PT is drawn and intersects the circle at R . RS and ST are joined. $RT = TS$.

Let $\hat{S}_1 = x$ and $\hat{T}_2 = y$.



9.1 Name, with reasons, THREE angles each equal to y . (6)

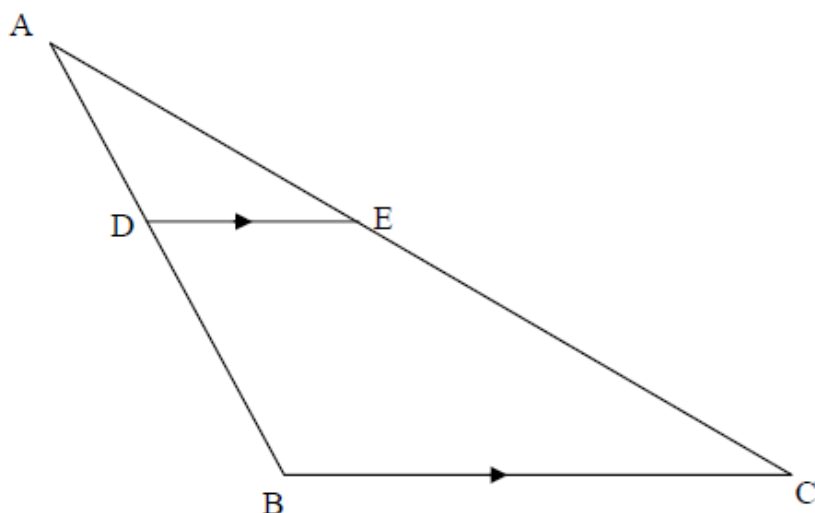
9.2 Prove that $\triangle PRS \parallel \triangle PST$. (3)

9.3 Prove that $PS \times RT = RS \times PT$. (3)

[12]

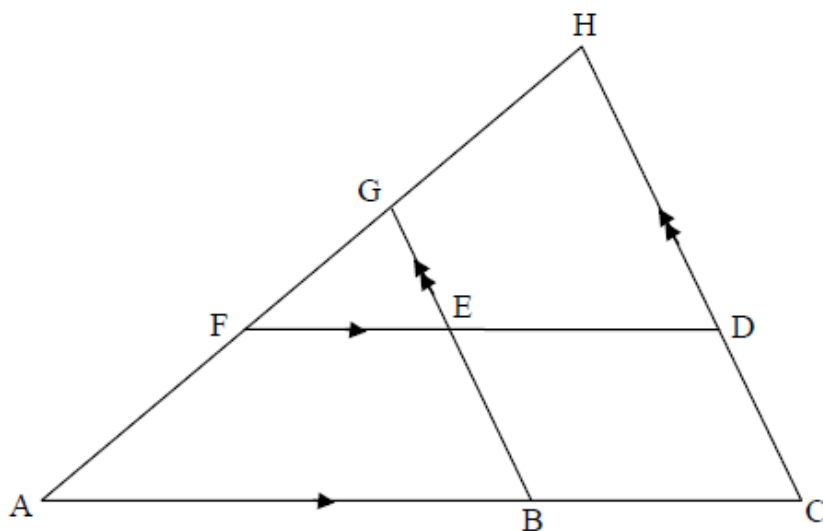
QUESTION 10

- 10.1 In $\triangle ABC$ below, D is a point on AB and E is a point on AC such that $DE \parallel BC$.
Prove, using Euclidean geometry methods, the theorem that states $\frac{AD}{DB} = \frac{AE}{EC}$.



(7)

- 10.2 In the diagram below, ACH is a triangle with point B on AC and point G on AH such that $BG \parallel CH$. F is a point on AH and D is a point on HC such that $FD \parallel AC$. GB intersects FD at E.
It is also given that $HD : DC = 5 : 3$ and $AB = 2BC$.

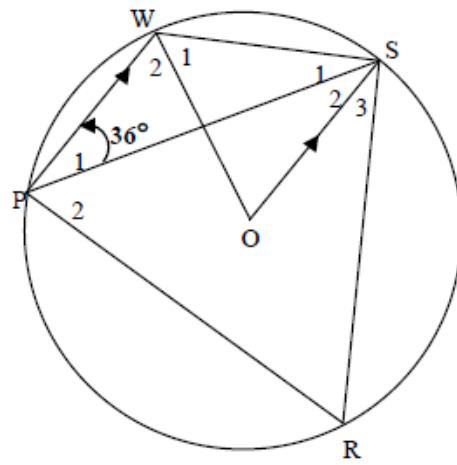


If $AH = 48$ cm, calculate the following with reasons:

- 10.2.1 HF (3)
 10.2.2 FG (3)
 10.2.3 $EF : ED$ (2)

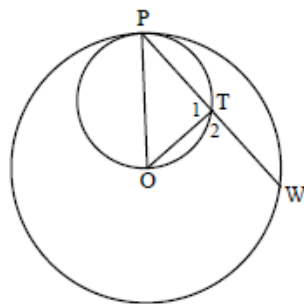
[15]

QUESTION/VRAAG 7

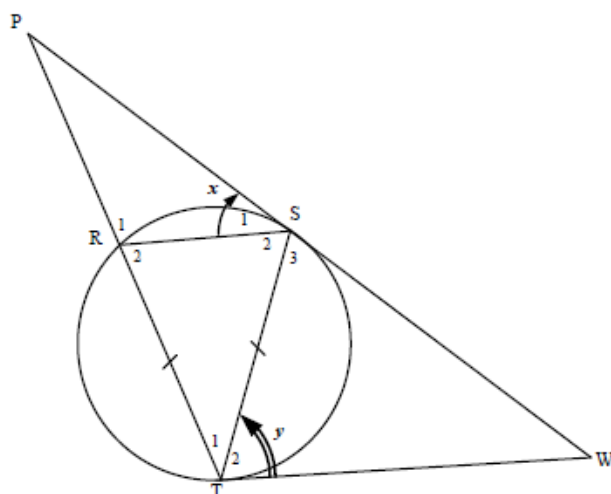


7.1	$\hat{S}OW = 72^\circ$ ($\angle$ circ cent = 2 $\angle$ circumference) (middelpunts $\angle$ = 2 omtreks $\angle$)	✓ $\hat{S}OW = 72^\circ$ ✓ $\angle$ circ cent = 2 $\angle$ circumference (2)
7.2	$\hat{W}_2 = 72^\circ$ (alt $\angle$ s; $PW \parallel SA$) / (verw $\angle$ e; $PW \parallel SA$)	✓ $\hat{W}_2 = 72^\circ$ ✓ $PW \parallel SO$ (2)
7.3	$\hat{O}SW = \hat{W}_1$ ($\angle$ s opp = radii) / ($\angle$ e teenoor = radiusse) $2\hat{O}SW + 72^\circ = 180^\circ$ ($\angle$ sum Δ) / (som van binne $\angle$ e Δ) $2\hat{O}SW = 108^\circ$ $\hat{O}SW = 54^\circ$	✓ $\hat{O}SW = \hat{W}_1$ ✓ $\angle$ s opp = radii ✓ answer (3)
7.4	$\hat{R} + \hat{W}_1 + \hat{W}_2 = 180^\circ$ (opp $\angle$ s cyclic quad) / (oorst $\angle$ e koordevierhoek) $\hat{R} + 54^\circ + 72^\circ = 180^\circ$ $\hat{R} = 54^\circ$	✓ $\hat{R} + \hat{W}_1 + \hat{W}_2 = 180^\circ$ ✓ opp $\angle$ s cyclic quad ✓ answer (3) [10]

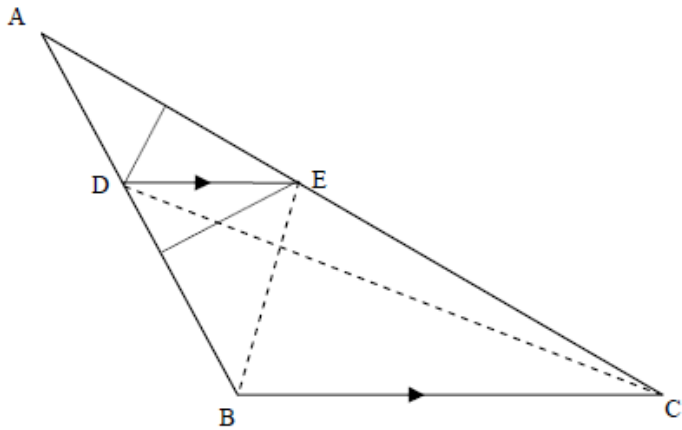
QUESTION/VRAAG 8



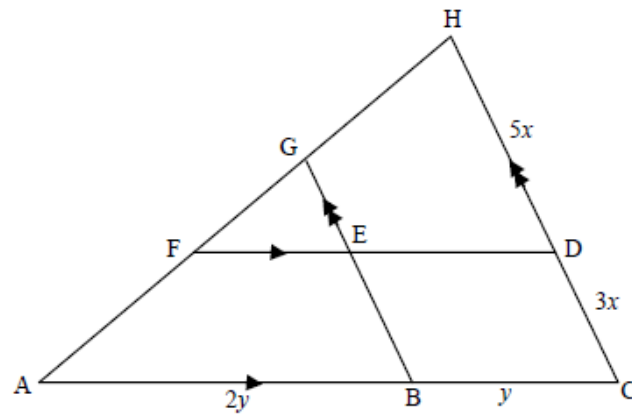
8.	$\hat{T}_1 = 90^\circ$ ($\angle$ s in a semi-circle) / ($\angle$ e in 'n halwe sirkel) $PT = TW = 24$ cm (line from circ cent $\perp$ ch) (lyn van middelpunt $\perp$ koord) $OP^2 = OT^2 + PT^2$ (Pythagoras) $OP^2 = (10)^2 + (24)^2$ $OP^2 = 676$ $OP = 26$ cm Radius of smaller circle = 13 cm	✓ $\hat{T}_1 = 90^\circ$ ✓ $\angle$ s in a semi-circle ✓ line from circ cent $\perp$ ch ✓ $OP = 26$ cm ✓ answer (5)
----	---	--

Page 93 of 166

QUESTION 10



10.1	Join E to B and D to C. $\frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{AD}{DB} \quad \begin{array}{l} \text{(common altitudes)/} \\ \text{(gemeenskaplike hoogtelyne)} \end{array}$ $\frac{\text{area } \triangle AED}{\text{area } \triangle DEC} = \frac{AD}{EC} \quad \text{(common altitudes)}$ area $\triangle AED$ is common area $\triangle DEB = \text{area } \triangle DEC$ ($DE \parallel BC$; same base BC) $\frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{\text{area } \triangle AED}{\text{area } \triangle DEC}$ $\frac{AD}{DB} = \frac{AD}{EC}$	✓ construction $\checkmark \frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{AD}{DB}$ ✓ = alts $\therefore$ ratio areas = ratio bases $\checkmark \frac{\text{area } \triangle AED}{\text{area } \triangle DEC} = \frac{AD}{EC}$ ✓ area $\triangle DEB = \text{area } \triangle DEC$ and ✓ $DE \parallel BC$; same base BC $\checkmark \frac{\text{area } \triangle AED}{\text{area } \triangle DEB} = \frac{\text{area } \triangle AED}{\text{area } \triangle DEC}$ (7)
------	--	--



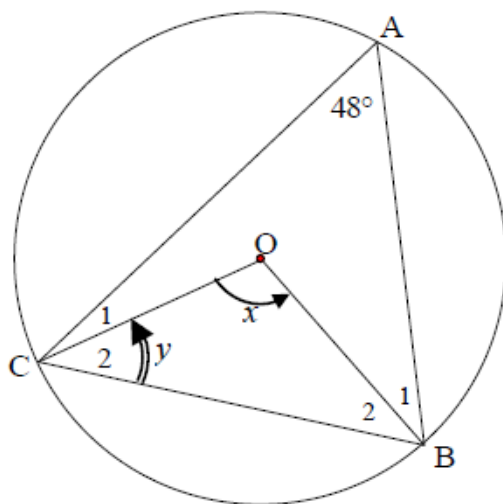
10.2.1	$\frac{HF}{AH} = \frac{5}{8}$ (FD $\parallel$ AC; Prop Th/ <i>Verhouding St</i>) $HF = \frac{5}{8} AH$ $HF = \frac{5}{8} (48)$ $HF = 30 \text{ cm}$	$\checkmark \frac{HF}{AH} = \frac{5}{8}$ $\checkmark FD \parallel AC$ $\checkmark \text{ answer}$
10.2.2	$AF = 18 \text{ cm}$ $\frac{AF + FG}{HF - FG} = \frac{2}{1}$ (BG $\parallel$ CH; Prop Th/ <i>Verhouding St</i>) $\frac{18 + FG}{30 - FG} = 2$ $18 + FG = 2(30 - FG)$ $18 + FG = 60 - 2FG$ $3FG = 42$ $FG = 14 \text{ cm}$ OR $\frac{GH}{AH} = \frac{BC}{AC}$ (BG $\parallel$ CH; Prop Th/ <i>Verhouding St</i>) $\frac{GH}{48} = \frac{1}{3}$ $GH = 16 \text{ cm}$ $FG = HF - GH$ $= 30 - 16$ $= 14 \text{ cm}$	$\checkmark AF = 18$ $\checkmark \frac{AF + FG}{HF - FG} = \frac{2}{1}$ $\checkmark \text{ answer}$ $\checkmark \frac{GH}{AH} = \frac{BC}{AC}$ $\checkmark GH = 16 \text{ cm}$ $\checkmark \text{ answer}$
10.2.3	$EF : ED = GF : GH$ (BG $\parallel$ CH; Prop Th/ <i>Verhouding St</i>) $EF : ED = 14 : 16$ $= 7 : 8$	$\checkmark EF : ED = GF : GH$ $\checkmark \text{ answer}$

(2)
[15]

November 2014

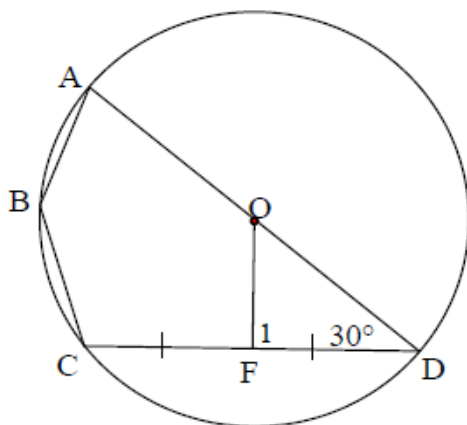
QUESTION 8

- 8.1 In the diagram, O is the centre of the circle passing through A, B and C.
 $\hat{CAB} = 48^\circ$, $\hat{COB} = x$ and $\hat{C}_2 = y$.



Determine, with reasons, the size of:

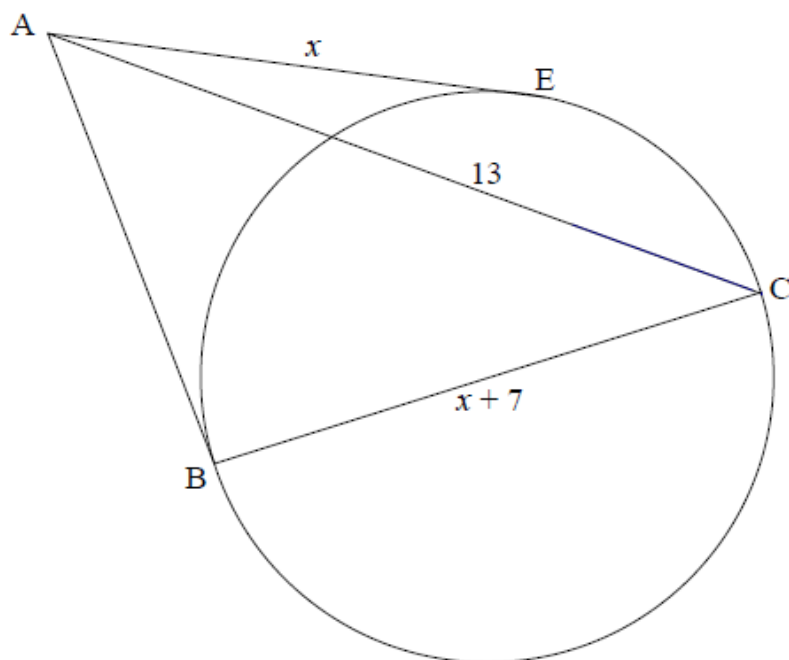
- 8.1.1 x (2)
- 8.1.2 y (2)
- 8.2 In the diagram, O is the centre of the circle passing through A, B, C and D.
 AOD is a straight line and F is the midpoint of chord CD. $\hat{ODF} = 30^\circ$ and OF are joined.



Determine, with reasons, the size of:

- 8.2.1 $\hat{F}_1$ (2)
- 8.2.2 $\hat{ABC}$ (2)

- 8.3 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. $AC = 13$, $AE = x$ and $BC = x + 7$.



- 8.3.1 Give reasons for the statements below.
Complete the table on **DIAGRAM SHEET 3**.

	Statement	Reason
(a)	$\hat{A}BC = 90^\circ$	
(b)	$AB = x$	

(2)

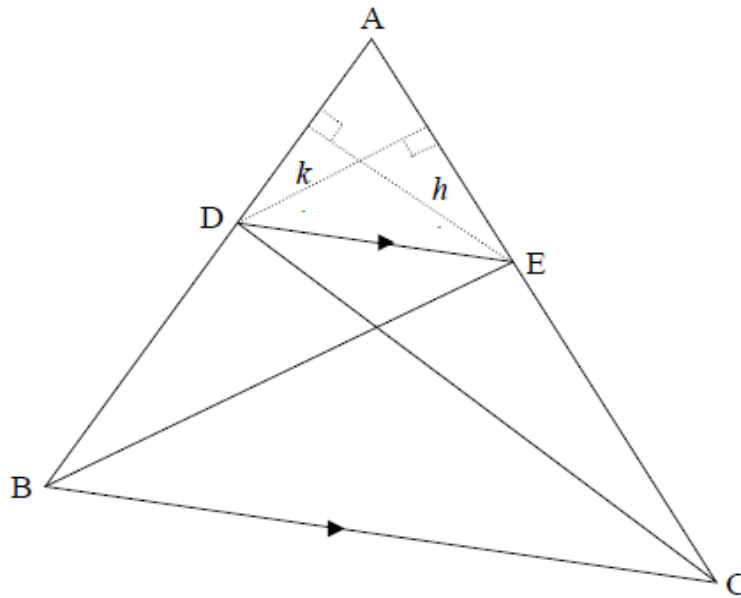
- 8.3.2 Calculate the length of AB.

(4)

[14]

QUESTION 9

- 9.1 In the diagram, points D and E lie on sides AB and AC of $\triangle ABC$ respectively such that $DE \parallel BC$. DC and BE are joined.



- 9.1.1 Explain why the areas of $\triangle DEB$ and $\triangle DEC$ are equal. (1)

- 9.1.2 Given below is the partially completed proof of the theorem that states that if in any $\triangle ABC$ the line $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$.

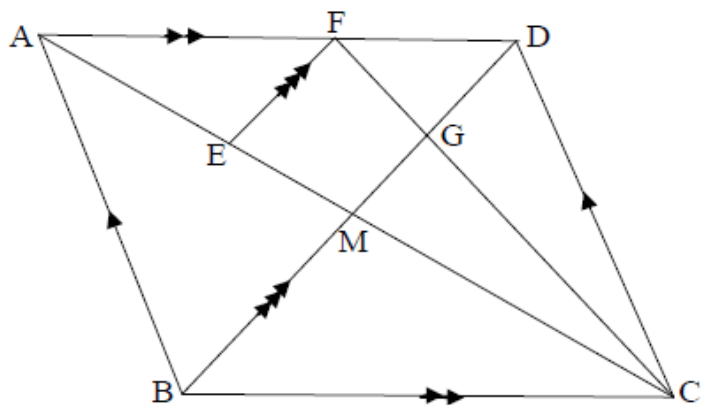
Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 4.

Construction: Construct the altitudes (heights) h and k in $\triangle ADE$.

$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \dots\dots\dots$
$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \dots\dots\dots = \frac{AE}{EC}$
But area $\triangle DEB = \dots\dots\dots$ (reason: $\dots\dots\dots$)
$\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \dots\dots\dots$
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

(5)

- 9.2 In the diagram, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that $AF : FD = 4 : 3$. E is a point on AM such that $EF \parallel BD$. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

9.2.1 $\frac{EM}{AM}$ (3)

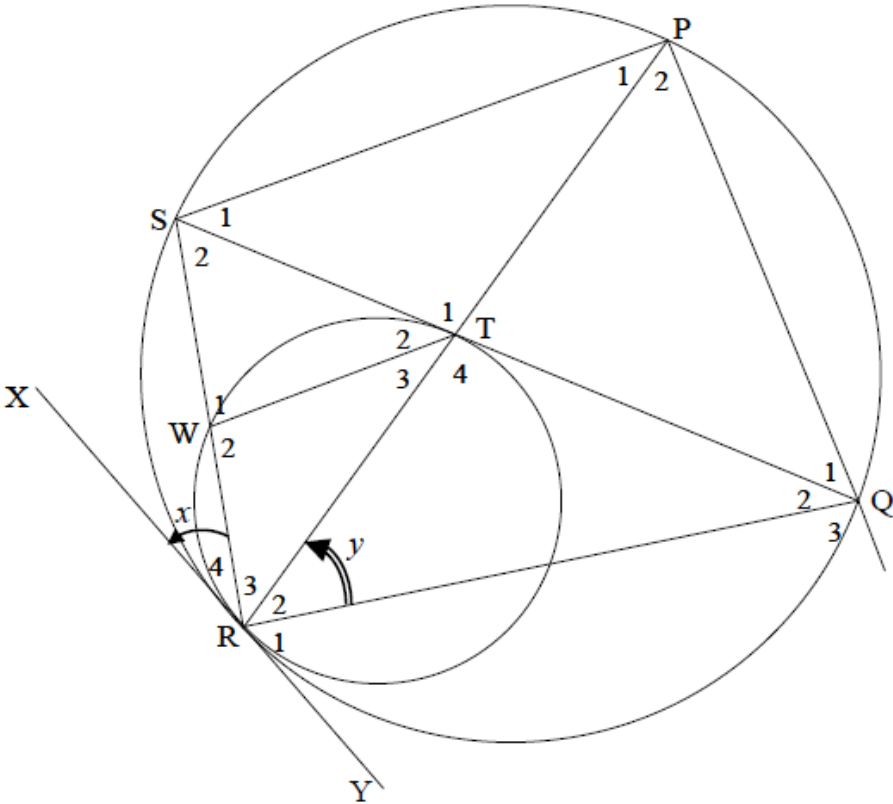
9.2.2 $\frac{CM}{ME}$ (3)

9.2.3 $\frac{\text{area } \triangle FDC}{\text{area } \triangle BDC}$ (4)
[16]

QUESTION 10

The two circles in the diagram have a common tangent XRY at R . W is any point on the small circle. The straight line RWS meets the large circle at S . The chord STQ is a tangent to the small circle, where T is the point of contact. Chord RTP is drawn.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$



10.1 Give reasons for the statements below.
Complete the table on **DIAGRAM SHEET 6**.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	Reason
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	$WT \parallel SP$	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

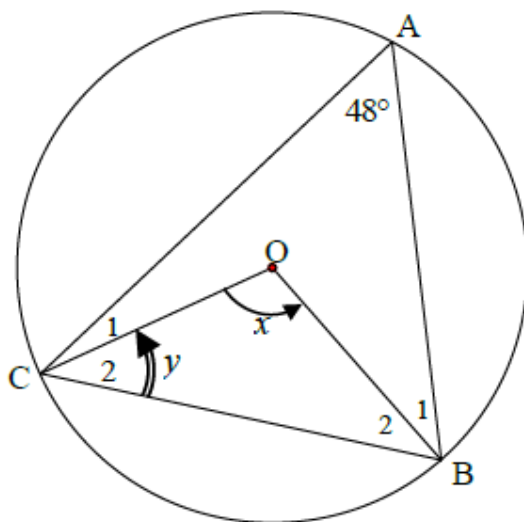
(5)

- 10.2 Prove that $RT = \frac{WR \cdot RP}{RS}$ (2)
- 10.3 Identify, with reasons, another TWO angles equal to y . (4)
- 10.4 Prove that $\hat{Q}_3 = \hat{W}_2$. (3)
- 10.5 Prove that $\Delta RTS \parallel \Delta RQP$. (3)
- 10.6 Hence, prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)

[20]

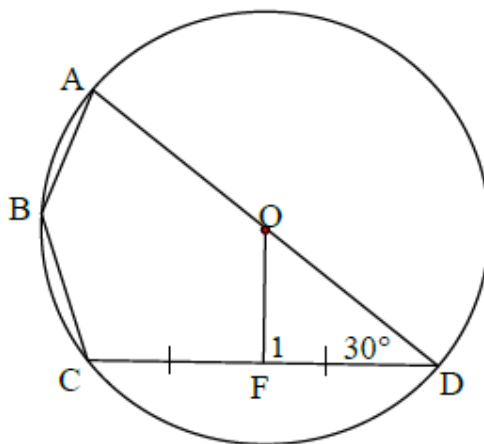
QUESTION/VRAAG 8

8.1



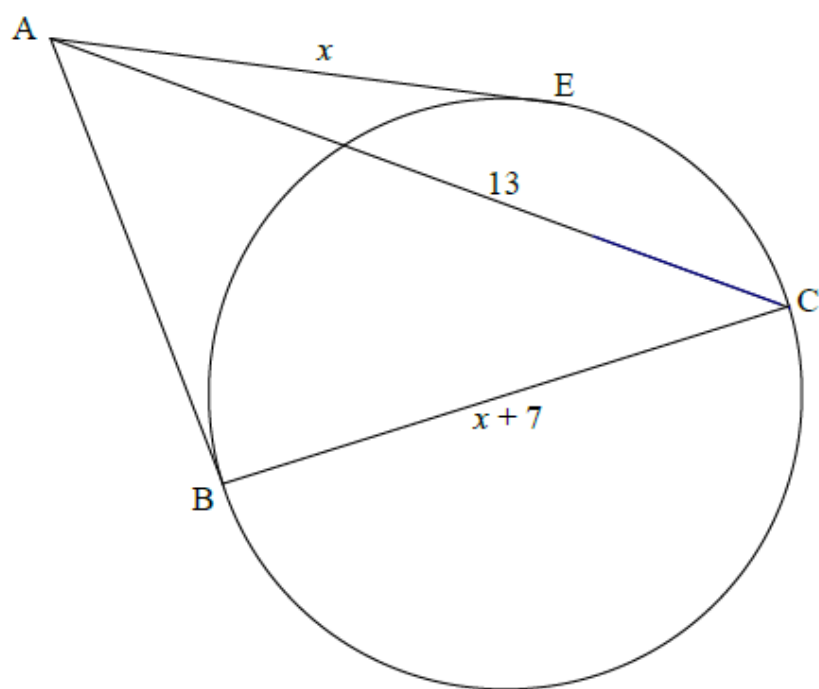
8.1.1	$x = 96^\circ$ ($\angle$ at centre = $2\angle$ at circumference/ $\angle$ by midpt = $2\angle$ by omtrek)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180^\circ - 96^\circ = 84^\circ$ (sum of $\angle$ s in Δ / som v $\angle$ e in Δ) $y = \hat{B}_2 = 42^\circ$ ($\angle$ s opp = sides/ $\angle$ e teenoor = sye)	✓ S ✓ S (2)

8.2



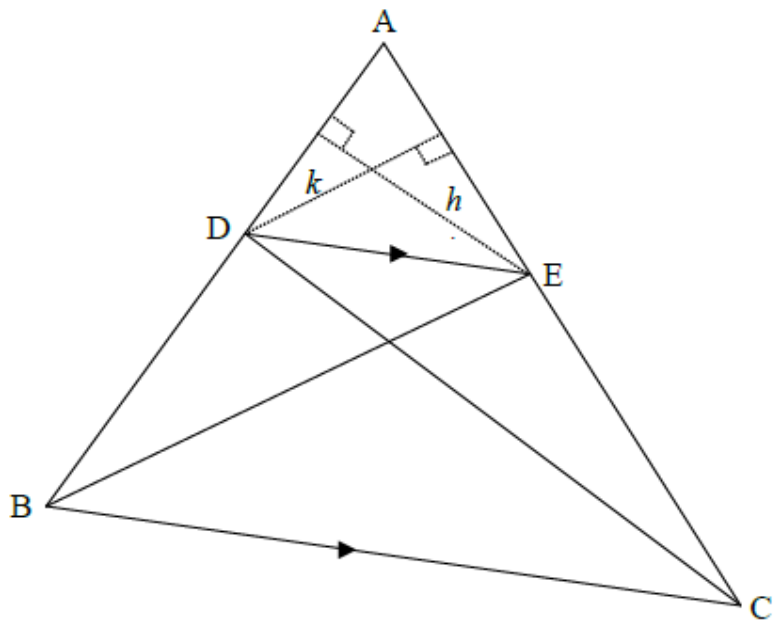
8.2.1	$\hat{F}_1 = 90^\circ$ (line from centre to midpt chord/ <i>lyn vanaf midpt na midpt kd</i>)	✓ S ✓ R (2)
8.2.2	$\hat{A}BC = 150^\circ$ (opposite $\angle$ s of cyclic quad/ <i>tos $\angle$e v koordevh</i>)	✓ S ✓ R (2)

8.3



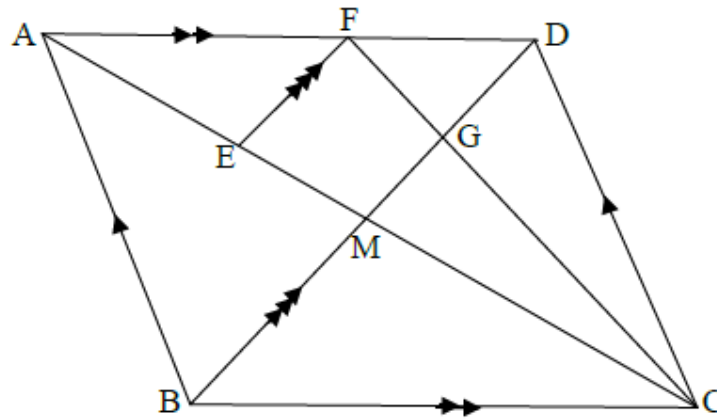
8.3.1 (a)	tangent $\perp$ radius/diameter / <i>raaklyn $\perp$ radius/middellyn</i>	$\checkmark$ R (1)
8.3.1 (b)	tangents from common pt OR tangents from same pt / <i>raaklyne v gemeensk pt OF raaklyne vanaf dies pt</i>	$\checkmark$ R (1)
8.3.2	$AB^2 + BC^2 = AC^2$ $x^2 + (x + 7)^2 = 13^2 \quad (\text{Theorem of/Stelling van Pythagoras})$ $x^2 + x^2 + 14x + 49 = 169$ $2x^2 + 14x - 120 = 0$ $x^2 + 7x - 60 = 0$ $(x - 5)(x + 12) = 0$ $x = 5 \quad (x \neq -12)$	$\checkmark$ $AB^2 + BC^2 = AC^2$ $\checkmark$ $x^2 + (x + 7)^2 = 13^2$ $\checkmark$ standard form $\checkmark$ answer (4) [14]

9.1

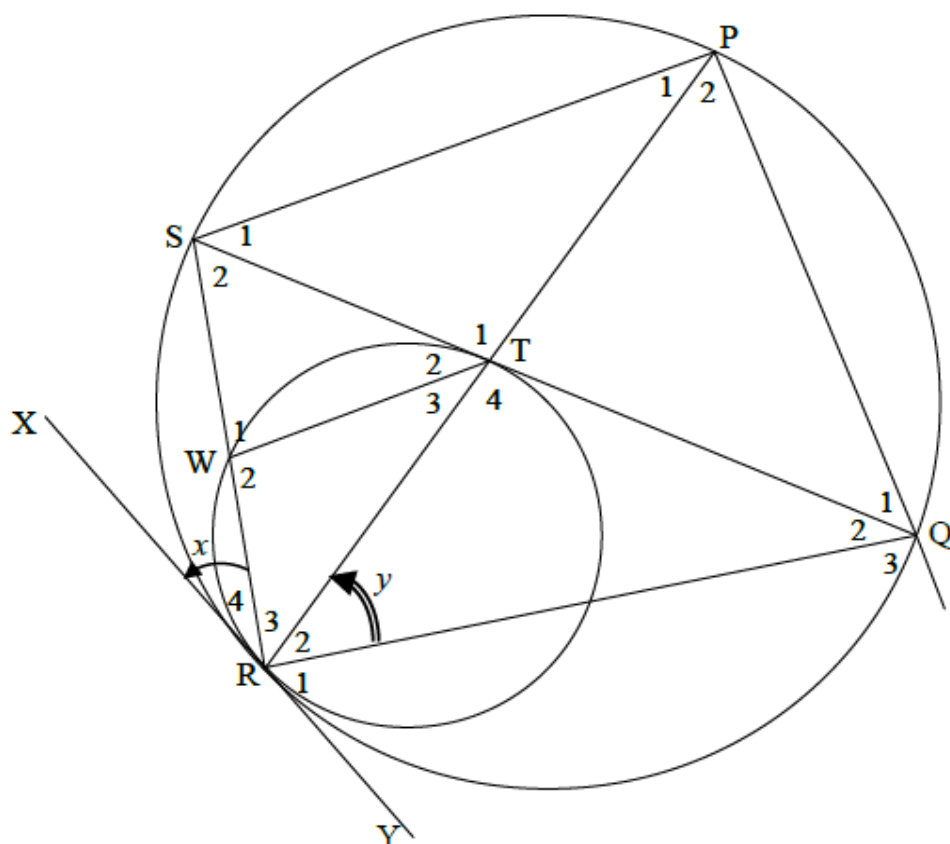


9.1.1	Same base (DE) and same height (between parallel lines) <i>Dieselfde basis (DE) en dieselfde hoogte (tussen ewewydige lyne)</i>	✓ same base/ <i>dies basis</i> between lines/ <i>tussen lyne</i> (1)
9.1.2	$\frac{\frac{AD}{DB} \times \frac{1}{2} AE \times k}{\frac{1}{2} EC \times k}$ But/<i>Maar</i> area $\triangle DEB$ = area $\triangle DEC$ (Same base and same height/<i>dieselfde basis en dieselfde hoogte</i>) $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ S ✓ S ✓ S ✓ R ✓ S (5)

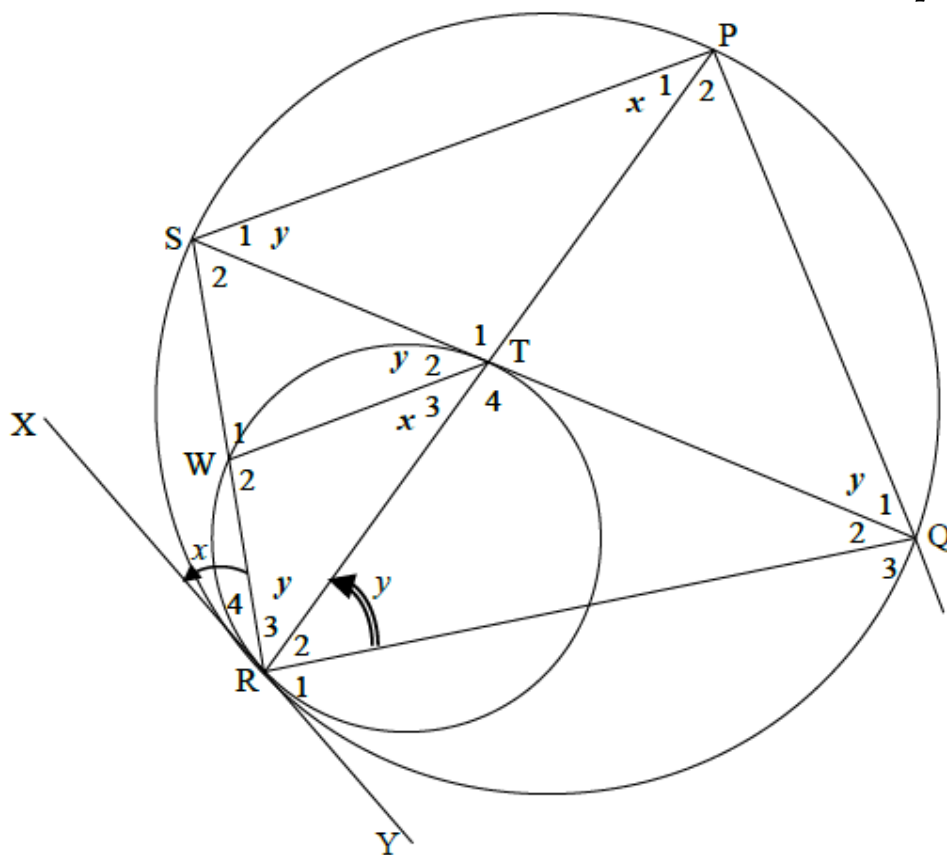
9.2



9.2.1	$\frac{EM}{AM} = \frac{FD}{AD}$ (Line parallel one side of Δ OR prop th; $EF \parallel BD$) (<i>Lyn ewewydig aan sy v Δ</i> OF eweredigst; $EF \parallel BD$) $\frac{EM}{AM} = \frac{3}{7}$	✓ S ✓ R ✓ answer/antw (3)
9.2.2	$CM = AM$ $\frac{CM}{ME} = \frac{AM}{ME} = \frac{7}{3}$ (diags of parm bisect/<i>hoekl parm halv</i>) (from 9.2.1/<i>vanaf 9.2.1</i>)	✓ S ✓ R ✓ answer/antw (3)
9.2.3	$h \text{ of } \Delta FDC = h \text{ of } \Delta BDC$ $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{\frac{1}{2}FD \cdot h}{\frac{1}{2}BC \cdot h}$ $= \frac{FD}{AD}$ $= \frac{3}{7}$ (opp sides of parm =) (<i>tos sye v parm =</i>) OR/OF $\frac{\text{area } \Delta FDC}{\text{area } \Delta ADC} = \frac{FD}{AD} = \frac{3}{7}$ (same heights) (<i>dieselfde hoogtes</i>) But Area $\Delta ADC =$ Area ΔBDC (diags of parm bisect area) (<i>hoekl v parm halv opp</i>) $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC} = \frac{3}{7}$	✓ $AD \parallel BC$ ✓ subst into area form/ <i>subst in opp formule</i> ✓ S ✓ answer/antw (4) ✓ S ✓ R ✓ S ✓ answer/antw (4) [16]



10.1.1	Tangent chord theorem/ <i>Raaklyn-koordstelling</i>	✓ R (1)
10.1.2	Tangent chord theorem/ <i>Raaklyn-koordstelling</i>	✓ R (1)
10.1.3	Corresponding angles equal/ <i>Ooreenkomstige ∠e gelyk</i>	✓ R (1)
10.1.4	∠s subtended by chord PQ OR ∠s in same segment <i>∠e onderspan deur dieselfde koord OF ∠e in dieselfde segment</i>	✓ R (1)
10.1.5	alternate ∠s/ <i>verwisselende ∠e</i> ; WT SP	✓ R (1)
10.2	$\frac{RW}{RS} = \frac{RT}{RP}$ (Line parallel one side of Δ OR prop th; WT SP) $\therefore RT = \frac{WR.RP}{RS}$ <i>(Lyn ewewydig aan sy v Δ OF eweredighst: WT SP)</i> OR/OF $\Delta RTW \Delta RPS$ ($\angle$; $\angle$; $\angle$) $\therefore \frac{RW}{RS} = \frac{RT}{RP}$ ($\Delta RTW \Delta RPS$) $\therefore RT = \frac{RW.RP}{RS}$	✓ S ✓ R (2) ✓ S ✓ S (2)
10.3	$y = \hat{T}_2 = \hat{R}_3$ <i>(tan chord theorem/Rkl-koordst)</i> $y = \hat{R}_3 = \hat{Q}_1$ <i>(∠s in same segment/∠e in dieselfde segment)</i>	✓ S ✓ R ✓ S ✓ R (4)



10.4	$\hat{Q}_3 = \hat{P}\hat{S}\hat{R}$ (ext $\angle$ of cyc quad/ <i>buite</i> $\angle$ v <i>kdvh</i>) $\hat{P}\hat{S}\hat{R} = \hat{W}_2$ (corresp $\angle$ s/ <i>ooreenk</i> $\angle$ e ; WT SP) $\therefore \hat{Q}_3 = \hat{W}_2$ OR/OF $\hat{Q}_2 = x$ ($\angle$ s in same segment/ $\angle$ e in <i>dies</i> segment) $\hat{Q}_3 = 180^\circ - (x + y)$ ($\angle$ s on straight line/ $\angle$ e <i>op reguitlyn</i>) $\hat{W}_2 = 180^\circ - (x + y)$ ($\angle$ s of ΔWRT / $\angle$ e v ΔWRT) $\therefore \hat{Q}_3 = \hat{W}_2$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S	
10.5	In ΔRTS and ΔRQP : $\hat{R}_3 = \hat{R}_2 = y$ (proven above/ <i>hierbo bewys</i>) $\hat{S}_2 = \hat{P}_2$ ($\angle$ s in same segment/ $\angle$ e in <i>dies</i> segment) $\hat{R}\hat{T}\hat{S} = \hat{R}\hat{Q}\hat{P}$ (3^{rd} angle of Δ) $\therefore \Delta RTS \parallel \Delta RQP$ ($\angle$; $\angle$; $\angle$)	$\checkmark$ S $\checkmark$ S/R $\checkmark$ S OR/OF ($\angle$; $\angle$; $\angle$)	

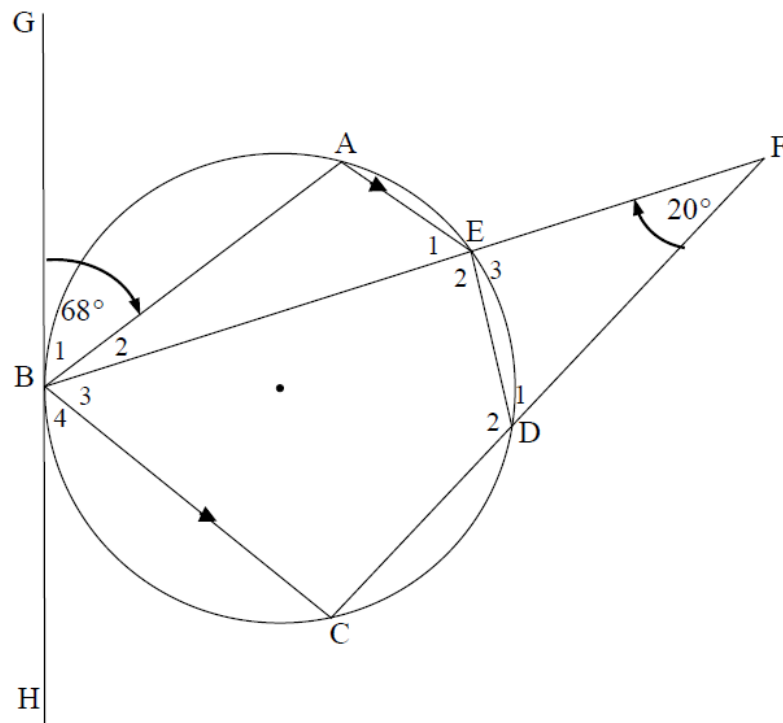
10.6	$\frac{RT}{RQ} = \frac{RS}{RP} \quad (\Delta RTS \Delta RQP)$ $\frac{RS}{RP} \times \frac{RS}{RP} = \frac{RT}{RQ} \times \frac{RS}{RP}$ $\left(\frac{RS}{RP}\right)^2 = \left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ $= \left(\frac{RW}{RS}\right)\left(\frac{RS}{RQ}\right) \quad (\text{proven in 10.2/bewys in 10.2})$ $= \frac{RW}{RQ}$ OR/OF $\frac{RT}{RQ} = \frac{RS}{RP} \quad (\Delta RTS \Delta RQP)$ But $RT = \frac{WR.RP}{RS}$ (proven in 10.2/bewys in 10.2) $\therefore \frac{RT}{RQ} = \frac{WR.RP}{RQ.RS} = \frac{RS}{RP}$ $WR.RP^2 = RQ.RS^2$ $\therefore \frac{WR}{RQ} = \frac{RS^2}{RP^2}$ OR/OF $\frac{RT}{RS} = \frac{RQ}{RP} \quad (\Delta RTS \Delta RQP)$ $RQ = \frac{RT.RP}{RS}$ and $WR = \frac{RT.RS}{RP}$ (proven in 10.2/bewys in 10.2) $\frac{WR}{RQ} = \frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$ $= \frac{RT.RS}{RP} \times \frac{RS}{RT.RP}$ $= \frac{RS^2}{RP^2}$	✓ S ✓ $\times \frac{RS}{RP}$ on both sides ✓ $\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ (3) ✓ S ✓ $RT = \frac{WR.RP}{RS}$ ✓ multiplication/ vermenigvuldig (3) ✓ S ✓ $WR = \frac{RT.RS}{RP}$ ✓ simplification/ vereenvoudiging (3) [20]
------	---	---

Exemplar Paper 2 – 2014**QUESTION 8**

8.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to ... (1)

8.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that $AE \parallel BC$. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68^\circ$ and $\hat{F} = 20^\circ$.



Determine the size of each of the following:

8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

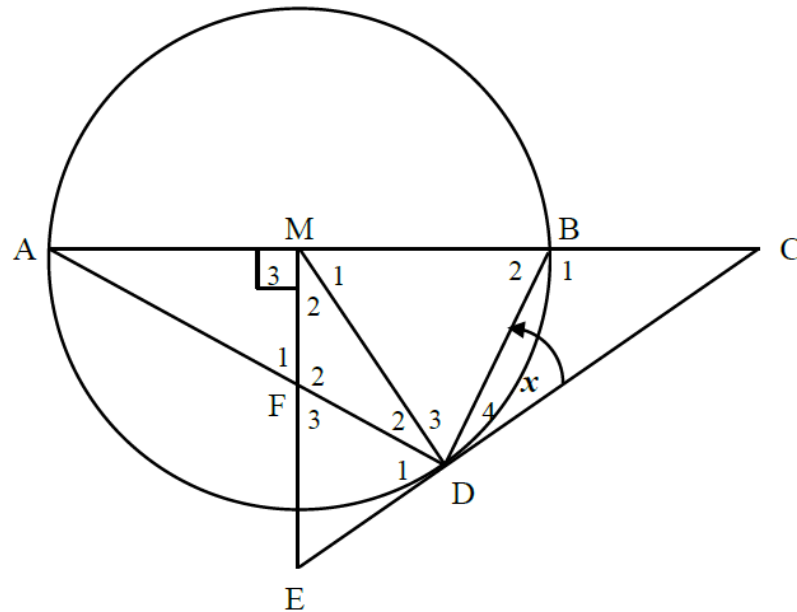
8.2.3 $\hat{D}_1$ (2)

8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2)
[9]

QUESTION 9

In the diagram, M is the centre of the circle and diameter AB is produced to C . ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D . ME and chord AD intersect at F . $MB = 2BC$.

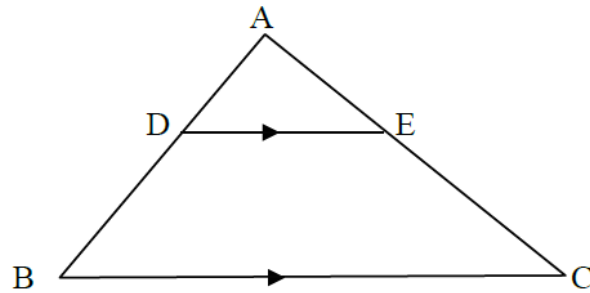


- 9.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x . (3)
- 9.2 Prove that CM is a tangent at M to the circle passing through M , E and D . (4)
- 9.3 Prove that $FMBD$ is a cyclic quadrilateral. (3)
- 9.4 Prove that $DC^2 = 5BC^2$. (3)
- 9.5 Prove that $\triangle DBC \parallel \triangle DFM$. (4)
- 9.6 Hence, determine the value of $\frac{DM}{FM}$. (2)

[19]

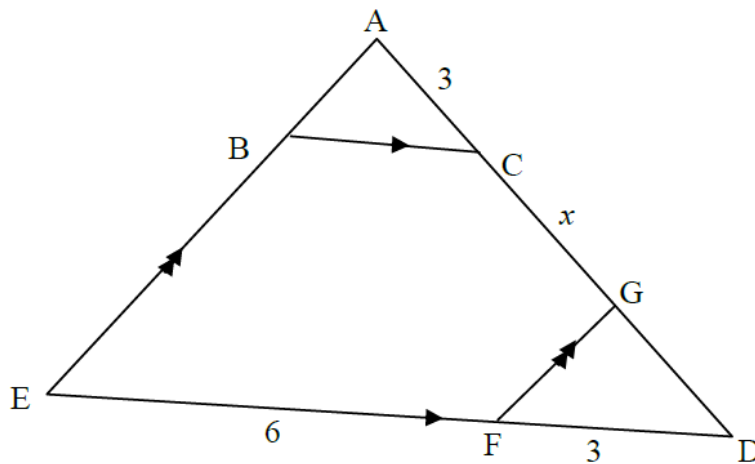
QUESTION 10

- 10.1 In the diagram, points D and E lie on sides AB and AC respectively of $\triangle ABC$ such that $DE \parallel BC$. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

- 10.2 In the diagram, ADE is a triangle having $BC \parallel ED$ and $AE \parallel GF$. It is also given that $AB : BE = 1 : 3$, $AC = 3$ units, $EF = 6$ units, $FD = 3$ units and $CG = x$ units.



Calculate, giving reasons:

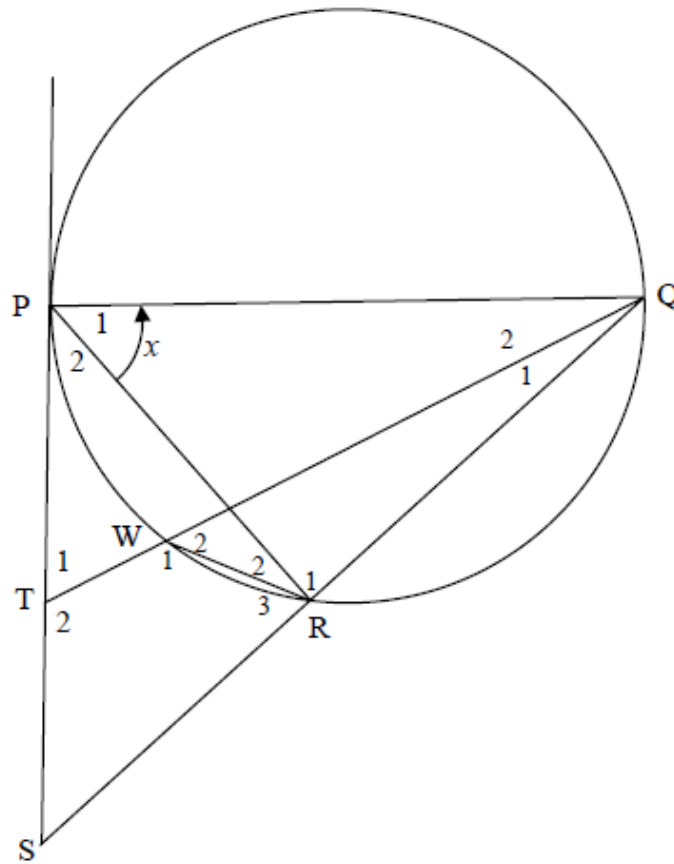
- 10.2.1 The length of CD (3)
- 10.2.2 The value of x (4)
- 10.2.3 The length of BC (5)
- 10.2.4 The value of $\frac{\text{area } \triangle ABC}{\text{area } \triangle GFD}$ (5)

[23]

QUESTION 9

In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.

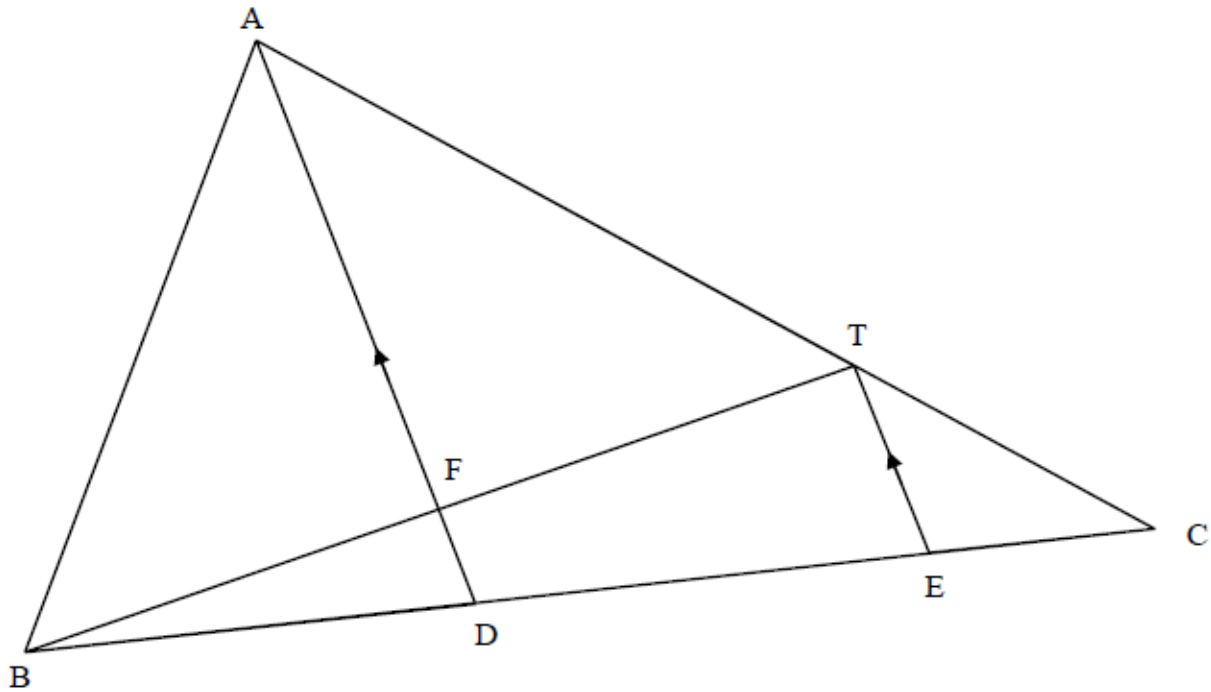
Let $\hat{P}_1 = x$



- 9.1 Why is $\hat{PRQ} = 90^\circ$? (1)
- 9.2 Prove that $\hat{P}_1 = \hat{S}$. (3)
- 9.3 Prove that SRWT is a cyclic quadrilateral. (3)
- 9.4 Prove that $\triangle QWR \parallel \triangle QST$. (3)
- 9.5 If $QW = 5$ cm, $TW = 3$ cm, $QR = 4$ cm and $WR = 2$ cm, calculate the length of:
- 9.5.1 TS (3)
- 9.5.2 SR (3)
- [16]

QUESTION 10

In the figure below, $\triangle ABC$ has D and E on BC. $BD = 6$ cm and $DC = 9$ cm.
 $AT : TC = 2 : 1$ and $AD \parallel TE$.



- 10.1 Write down the numerical value of $\frac{CE}{ED}$ (1)
- 10.2 Show that D is the midpoint of BE. (2)
- 10.3 If $FD = 2$ cm, calculate the length of TE. (2)
- 10.4 Calculate the numerical value of:
- 10.4.1 $\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$ (1)
- 10.4.2 $\frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC}$ (3)
- [9]

QUESTION 8

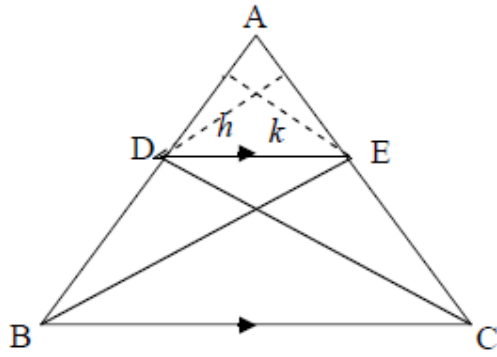
8.1	the angle subtended by the chord in the alternate segment	✓ correct theorem (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68^\circ$ (tan chord theorem)	✓ $\hat{E}_1 = 68^\circ$ ✓ reason (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68^\circ$ (alt $\angle$ s; $AE \parallel BC$)	✓ $\hat{B}_3 = 68^\circ$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68^\circ$ (ext $\angle$ of cyclic quad)	✓ $\hat{D}_1 = 68^\circ$ ✓ reason (2)
8.2.4	$\hat{E}_2 = 20^\circ + 68^\circ$ $= 88^\circ$ (ext $\angle$ of Δ)	✓ $\hat{E}_2 = 88^\circ$ (S/R) (1)
8.2.5	$\hat{C} = 180^\circ - 88^\circ$ $= 92^\circ$ (opp $\angle$ s of cyclic quad)	✓ $\hat{C} = 92^\circ$ ✓ reason (2) [9]

QUESTION 9

9.1	$\hat{D}_4 = \hat{A} = x$ (tan chord theorem) $\hat{A} = \hat{D}_2 = x$ ($\angle$ s opp equal sides)	$\checkmark \hat{A} = x$ $\checkmark$ reason $\checkmark \hat{A} = \hat{D}_2 = x$ (S/R) (3)
9.2	$\hat{M}_1 = 2x$ (ext $\angle$ of Δ) or ($\angle$ at centre = $2\angle$ at circum) $\hat{MDE} = 90^\circ$ (radius $\perp$ tan) $\hat{M}_2 = 90^\circ - 2x$ $\therefore \hat{E} = 180^\circ - (90^\circ + 90^\circ - 2x)$ (sum of $\angle$ s in ΔMDE) $= 2x$ $\therefore$ CM is a tangent (converse tan chord theorem)	$\checkmark \hat{M}_1 = 2x$ (S/R) $\checkmark \hat{MDE} = 90^\circ$ (S/R) $\checkmark \hat{E} = 2x$ $\checkmark$ reason (4)
9.3	$\hat{M}_3 = 90^\circ$ (EM $\perp$ AC) $\hat{ADB} = 90^\circ$ ($\angle$ in semi-circle) $\therefore$ FMBD a cyclic quad (ext $\angle$ of quad = int opp $\angle$) OR $\hat{EMC} = 90^\circ$ (EM $\perp$ AC) $\hat{ADB} = 90^\circ$ ($\angle$ in semi-circle) $\therefore$ FMBD a cyclic quad (opp $\angle$ s of quad supp)	$\checkmark \hat{M}_3 = 90^\circ$ $\checkmark \hat{ADB} = 90^\circ$ (S/R) $\checkmark$ reason (3) $\checkmark \hat{EMC} = 90^\circ$ $\checkmark \hat{ADB} = 90^\circ$ (S/R) $\checkmark$ reason (3)
9.4	$DC^2 = MC^2 - MD^2$ (Theorem of Pythagoras) $= (3BC)^2 - (2BC)^2$ (MB = MD = radii) $= 9BC^2 - 4BC^2$ $= 5BC^2$	$\checkmark$ Th of Pythagoras $\checkmark$ substitution $\checkmark 9BC^2 - 4BC^2$ (3)
9.5	In ΔDBC and ΔDFM : $\hat{D}_4 = \hat{D}_2 = x$ (proven in 9.1) $\hat{B}_1 = \hat{F}_2$ (ext $\angle$ of cyclic quad) $\hat{C} = \hat{M}_2$ $\therefore \Delta DBC \parallel \Delta DFM$ ($\angle$; $\angle$; $\angle$)	$\checkmark \hat{D}_4 = \hat{D}_2$ $\checkmark \hat{B}_1 = \hat{F}_2$ $\checkmark$ reason $\checkmark \hat{C} = \hat{M}_2$ or $(\angle; \angle; \angle)$ (4)
9.6	$\frac{DM}{FM} = \frac{DC}{BC}$ ($\Delta DBC \parallel \Delta DFM$) $= \frac{\sqrt{5}BC}{BC}$ $= \sqrt{5}$	$\checkmark$ S $\checkmark$ answer (2) [19]

QUESTION 10

10.1



Construction: Join DC and BE and heights k and h

$$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\frac{1}{2} \cdot AD \cdot k}{\frac{1}{2} \cdot DB \cdot k} = \frac{AD}{DB} \quad (\text{equal heights})$$

$$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2} \cdot AE \cdot h}{\frac{1}{2} \cdot EC \cdot h} = \frac{AE}{EC} \quad (\text{equal heights})$$

But Area $\triangle DEB$ = Area $\triangle DEC$ (same base, same height)

$$\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

✓ construction

$$\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{AD}{DB}$$

✓ reason

$$\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{AE}{EC}$$

✓ Area $\triangle DEB$ = Area $\triangle DEC$ (S/R)

$$\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$$

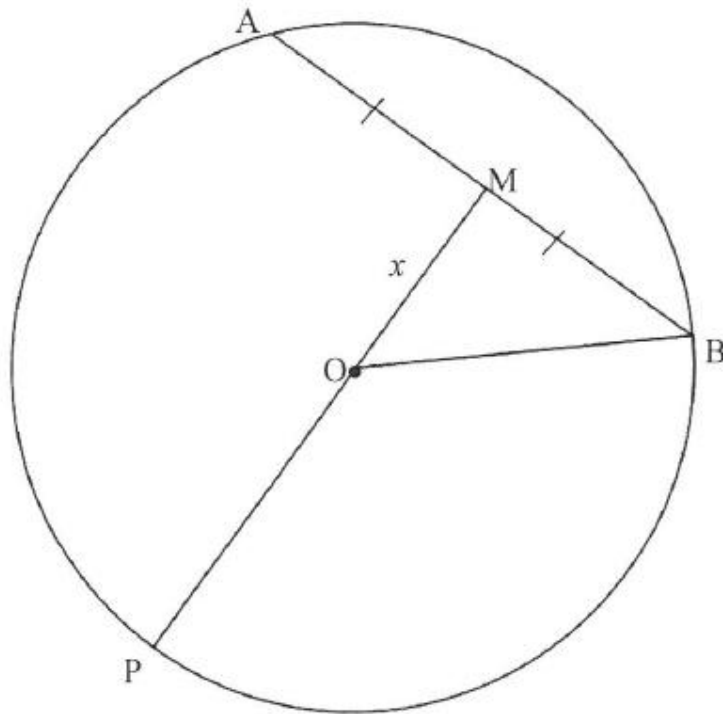
(6)

10.2.1	$\frac{AB}{BE} = \frac{AC}{CD} \quad (\text{Prop Th; } BC \parallel ED)$ $\frac{1}{3} = \frac{3}{CD}$ $\therefore CD = 9 \text{ units}$	$\checkmark \frac{AB}{BE} = \frac{AC}{CD} \text{ (S/R)}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (3)
10.2.2	$\frac{DG}{GA} = \frac{FD}{FE} \quad (\text{Prop Th; } FG \parallel EA)$ $\frac{9-x}{3+x} = \frac{3}{6}$ $54 - 6x = 9 + 3x$ $-9x = -45$ $x = 5$	$\checkmark \frac{DG}{GA} = \frac{FD}{FE} \text{ (S/R)}$ $\checkmark \text{ substitution}$ $\checkmark \text{ simplification}$ $\checkmark \text{ answer}$ (4)
10.2.3	In $\triangle ABC$ and $\triangle AED$: $\hat{A}$ is common $\hat{ABC} = \hat{E} \quad (\text{corres } \angle\text{s; } BC \parallel ED)$ $\hat{ACB} = \hat{D} \quad (\text{corres } \angle\text{s; } BC \parallel ED)$ $\triangle ABC \sim \triangle AED \text{ (}\angle, \angle, \angle\text{)}$ $\therefore \frac{BC}{ED} = \frac{AC}{AD}$ $\frac{BC}{9} = \frac{3}{12}$ $BC = 2\frac{1}{4} \text{ units}$	$\checkmark \hat{A} \text{ is common}$ $\checkmark \hat{ABC} = \hat{E} \text{ (S/R)}$ $\checkmark \hat{ACB} = \hat{D} \text{ (S/R)}$ or ($\angle$; $\angle$; $\angle$) $\checkmark \frac{BC}{ED} = \frac{AC}{AD}$ $\checkmark \text{ answer}$ (5)
10.2.4	$\frac{\text{area } \triangle ABC}{\text{area } \triangle GFD} = \frac{\frac{1}{2} AC \cdot BC \cdot \sin \hat{ACB}}{\frac{1}{2} GD \cdot FD \cdot \sin \hat{D}}$ $= \frac{\frac{1}{2} (3) (2\frac{1}{4}) \sin \hat{D}}{\frac{1}{2} (4) (3) \sin \hat{D}} \quad (\text{corres } \angle\text{s; } BC \parallel ED)$ $= \frac{9}{16}$	$\checkmark \text{ use of area rule}$ $\checkmark \text{ correct sides and angles}$ $\checkmark \text{ substitution of values}$ $\checkmark \sin \hat{ACB} = \sin \hat{D} \text{ (S/R)}$ $\checkmark \text{ answer}$ (5) [23]

February 2015

QUESTION 7

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. $OM = x$ units, $AB = 20$ units and $\frac{PM}{OM} = \frac{5}{2}$.



- 7.1 Write down the length of MB. (1)
- 7.2 Give a reason why $OM \perp AB$. (1)
- 7.3 Show that $OP = \frac{3x}{2}$ units. (2)
- 7.4 Calculate the value of x . (3)
- [7]

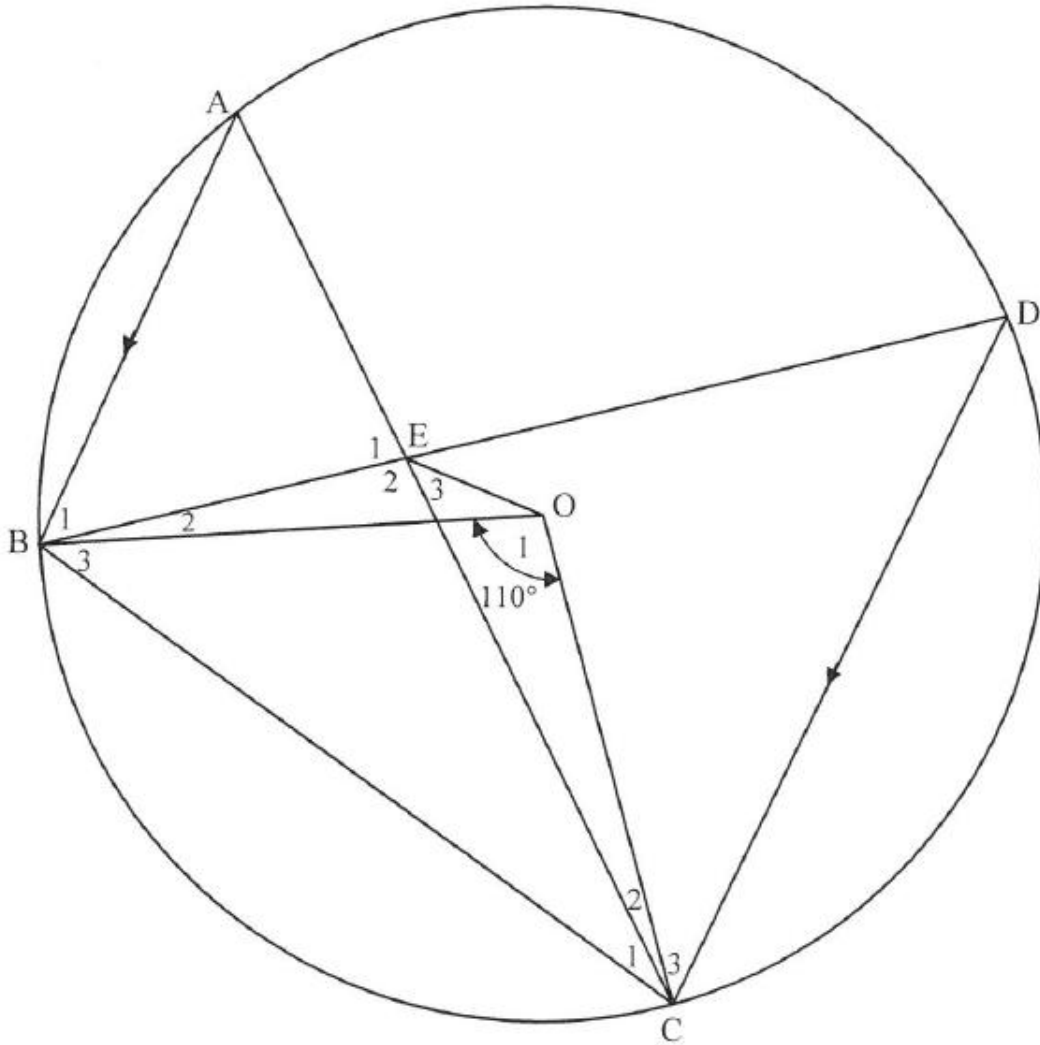
QUESTION 8

In the diagram below, the circle with centre O passes through A , B , C and D .

$AB \parallel DC$ and $\hat{BOC} = 110^\circ$.

The chords AC and BD intersect at E .

EO , BO , CO and BC are joined.



8.1 Calculate the size of the following angles, giving reasons for your answers:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

8.2 Prove that $BEOC$ is a cyclic quadrilateral. (2)

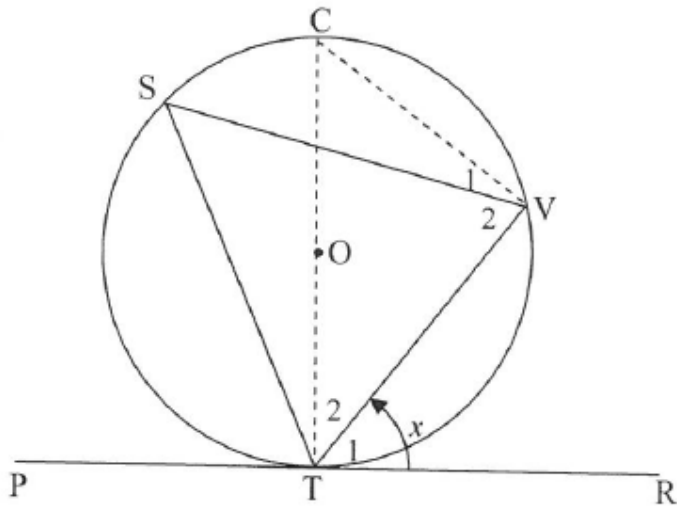
[10]

QUESTION 9

9.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



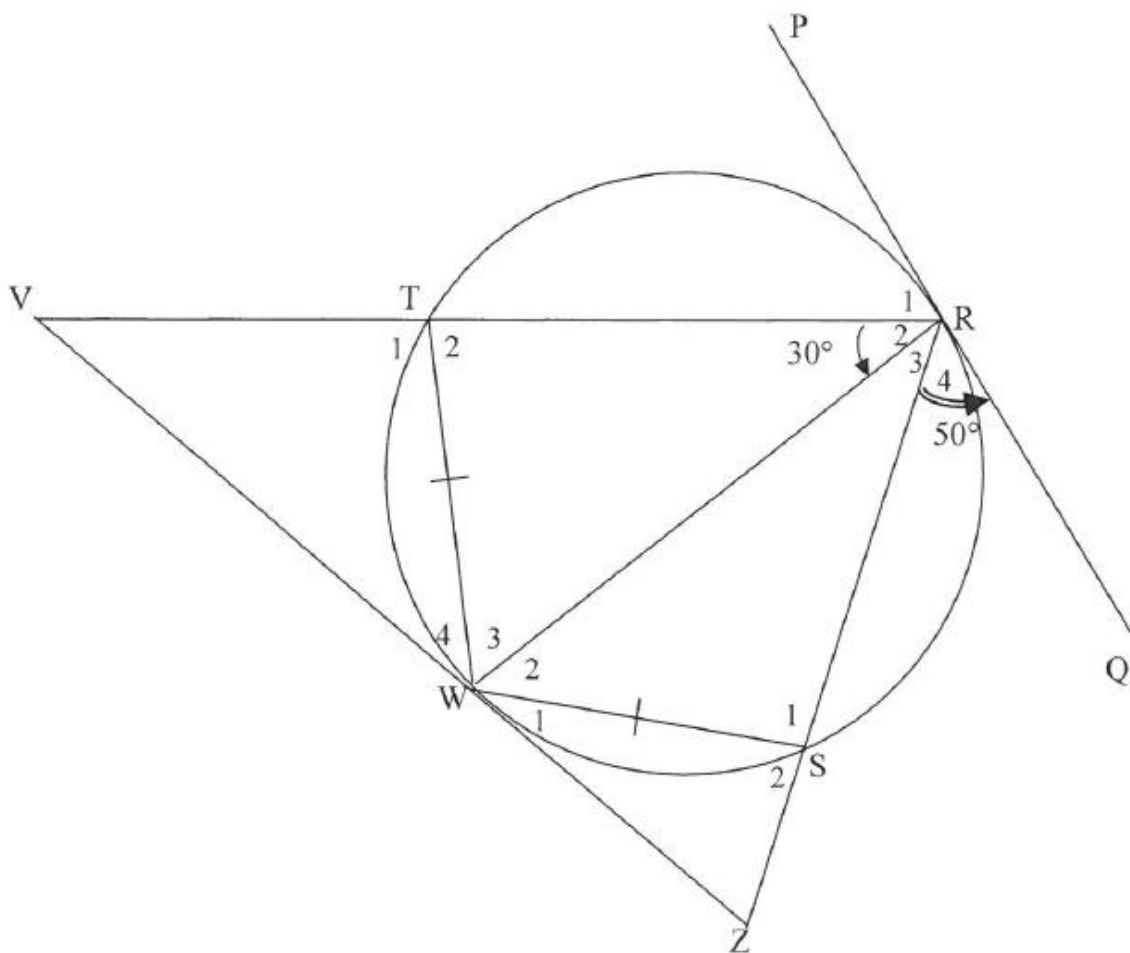
Given below is the partially completed proof of the theorem that states that $\hat{VTR} = \hat{S}$. Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.

Construction: Draw diameter TC and join CV.

Statement	Reason
Let: $\hat{VTR} = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 = \dots\dots\dots$	$\dots\dots\dots$
$\hat{T}_2 = 90^\circ - x$	$\dots\dots\dots$
$\therefore \hat{C} = \dots\dots\dots$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	$\dots\dots\dots$
$\therefore \hat{VTR} = \hat{S}$	

(5)

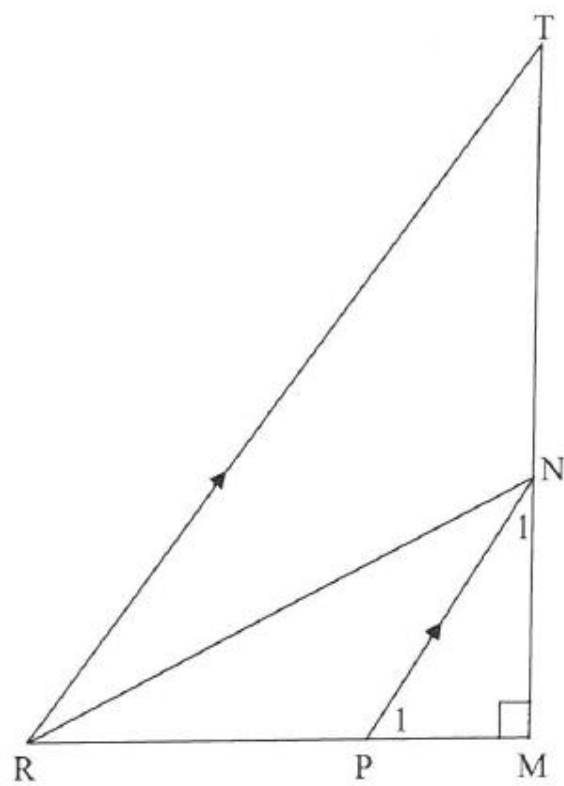
- 9.3 In the figure, TRSW is a cyclic quadrilateral with $TW = WS$. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30^\circ$ and $\hat{R}_4 = 50^\circ$.



- 9.3.1 Give a reason why $\hat{R}_3 = 30^\circ$. (1)
- 9.3.2 State, with reasons, TWO other angles equal to 30° . (3)
- 9.3.3 Determine, with reasons, the size of:
- (a) $\hat{S}_2$ (3)
- (b) $\hat{V}$ (4)
- 9.3.4 Prove that $WR^2 = RV \times RS$. (5)
- [22]

QUESTION 10

In $\triangle TRM$, $\hat{M} = 90^\circ$. NP is drawn parallel to TR with N on TM and P on RM . It is further given that $RT = 3PN$.



10.1 Give reasons for the statements below.
Use **DIAGRAM SHEET 5**.

	Statement	Reason
	In $\triangle PNM$ and $\triangle RTM$:	
10.1.1	$\hat{N}_1 = \hat{T}$	
	$\hat{M}$ is common	
10.1.2	$\therefore \triangle PNM \parallel \triangle RTM$	

(2)

10.2 Prove that $\frac{PM}{RM} = \frac{1}{3}$.

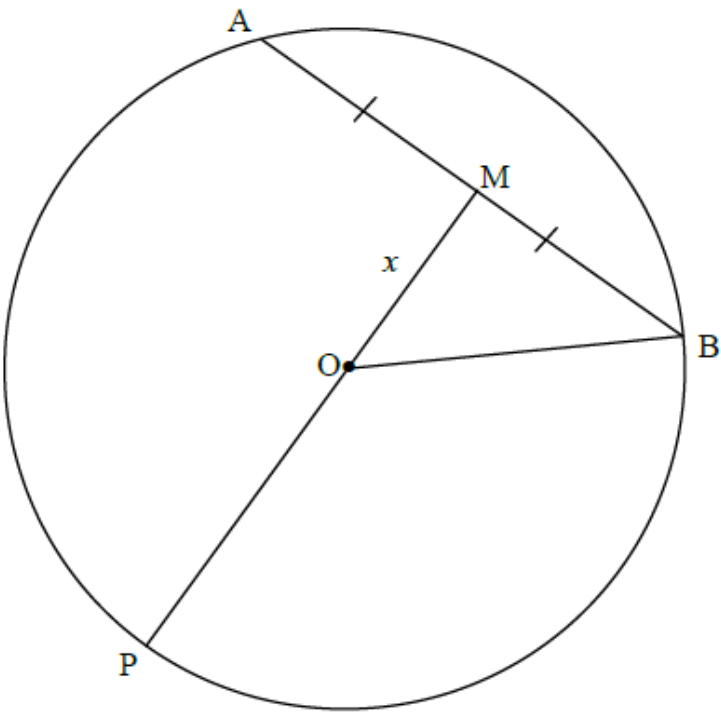
(2)

10.3 Show that $RN^2 - PN^2 = 2RP^2$.

(4)

[8]

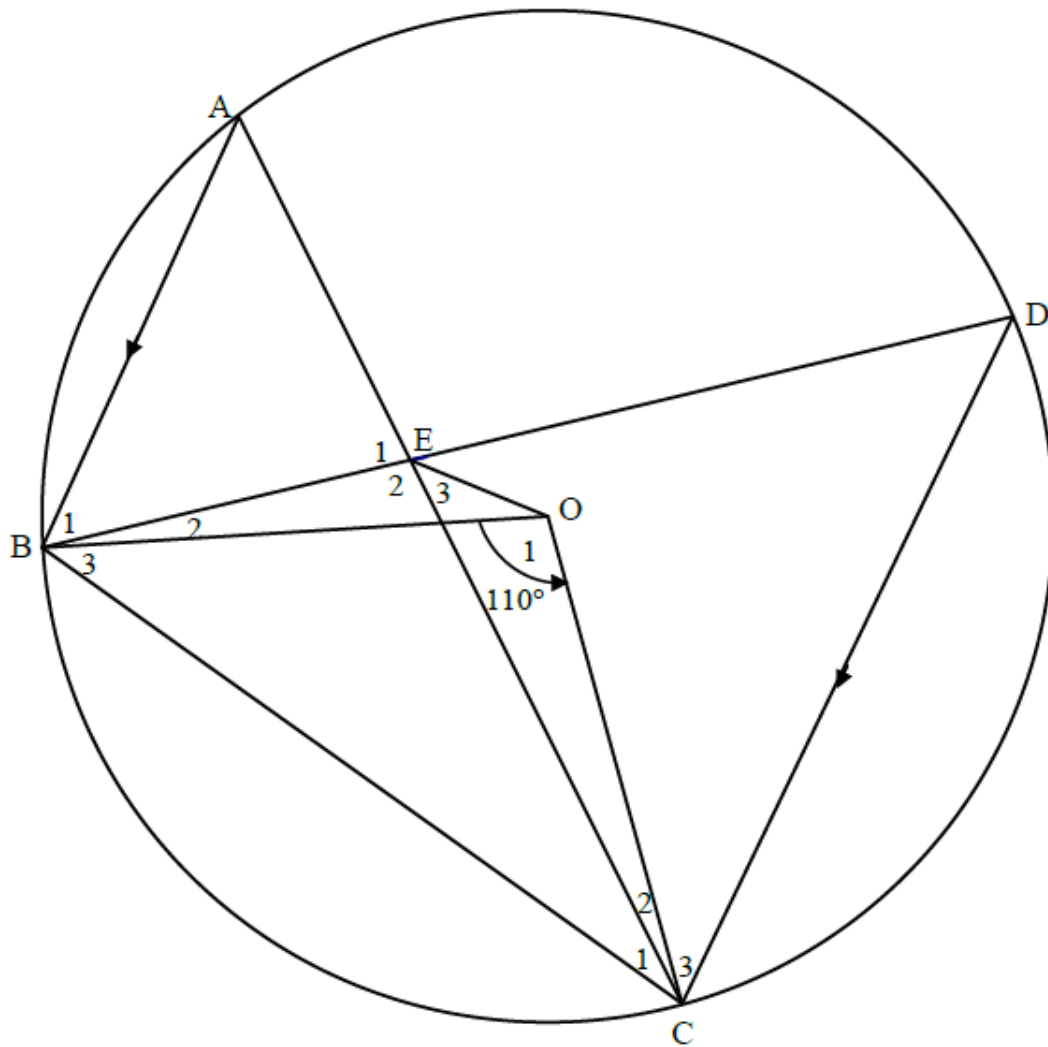
QUESTION/VRAAG 7



7.1	MB = 10 cm	✓ answer/antw (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/lyn vanaf midpt na midpt van koord is loodreg op koord OR/OF line from centre bisects chord/lyn vanaf midpt halveer koord	✓ answer/antw (1) ✓ answer/antw (1)
7.3	$\frac{MP}{OM} = \frac{5}{2}$ $\frac{x + OP}{x} = \frac{5}{2}$ $2x + 2OP = 5x$ $OP = \frac{3x}{2}$ OR/OF $\frac{OP}{OM} = \frac{3}{2}$ $OP = \frac{3x}{2}$	✓ $\frac{x + OP}{x} = \frac{5}{2}$ ✓ $OP = \frac{3x}{2}$ (2) ✓ $\frac{OP}{OM} = \frac{3}{2}$ ✓ $OP = \frac{3x}{2}$ (2)

7.4	$OM^2 + MB^2 = OB^2$ $x^2 + 10^2 = \left(\frac{3x}{2}\right)^2$ $4x^2 + 400 = 9x^2$ $5x^2 = 400$ $x^2 = 80$ $x = 8,94 \text{ or } 4\sqrt{5} \text{ or } \sqrt{80}$	✓ subst into/<i>subst</i> Pythagoras ✓ $4x^2 + 400 = 9x^2$ ✓ answer/<i>antw</i> (3) [7]
-----	--	---

QUESTION/VRAAG 8

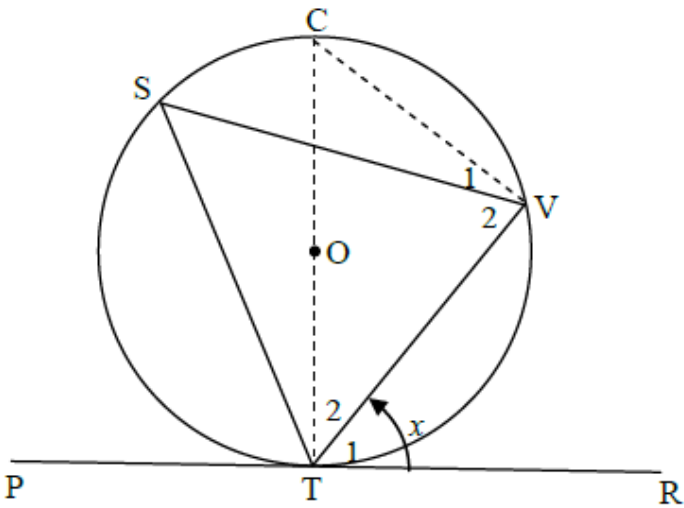


8.1.1	$\hat{D} = \frac{1}{2} \hat{O}_1 = 55^\circ$ ($\angle$ at centre = $2 \times \angle$ at circ / $\angle$ by midpt = $2 \times \angle$ by omt)	✓S ✓R (2)
8.1.2	$\hat{A} = \frac{1}{2} \hat{O}_1 = 55^\circ$ ($\angle$ at centre = $2 \times \angle$ at circ / $\angle$ by midpt = $2 \times \angle$ by omt)	✓S ✓R (2)
	OR/OF	
	$\hat{A} = \hat{D} = 55^\circ$ ($\angle$ s in same segment / $\angle$ e in dieselfde segment)	✓S ✓R (2)
8.1.3	$\hat{B}_1 = \hat{D} = 55^\circ$ (alternate $\angle$ s / <i>verwiss</i> $\angle$ e; $AB \parallel DC$)	✓S ✓R
	$\hat{E}_2 = \hat{B}_1 + \hat{A}$ (ext $\angle$ of Δ = sum of opp $\angle$ s / <i>buite</i> $\angle$ v Δ = <i>som v tos</i> $\angle$ e)	✓R
	$= 55^\circ + 55^\circ$	
	$\hat{E}_2 = 110^\circ$	✓ answer / <i>antw</i> (4)
8.2	$\hat{E}_2 = \hat{O}_1 = 110^\circ$ (proven in / <i>bewys in</i> 8.1.3)	✓S
	BEOC is a cyclic quadrilateral (equal $\angle$ s subtended by line / <i>gelyke</i> $\angle$ e <i>onderspan</i> <i>deur</i> <i>lyn</i>)	✓R (2)
		[10]

QUESTION/VRAAG 9

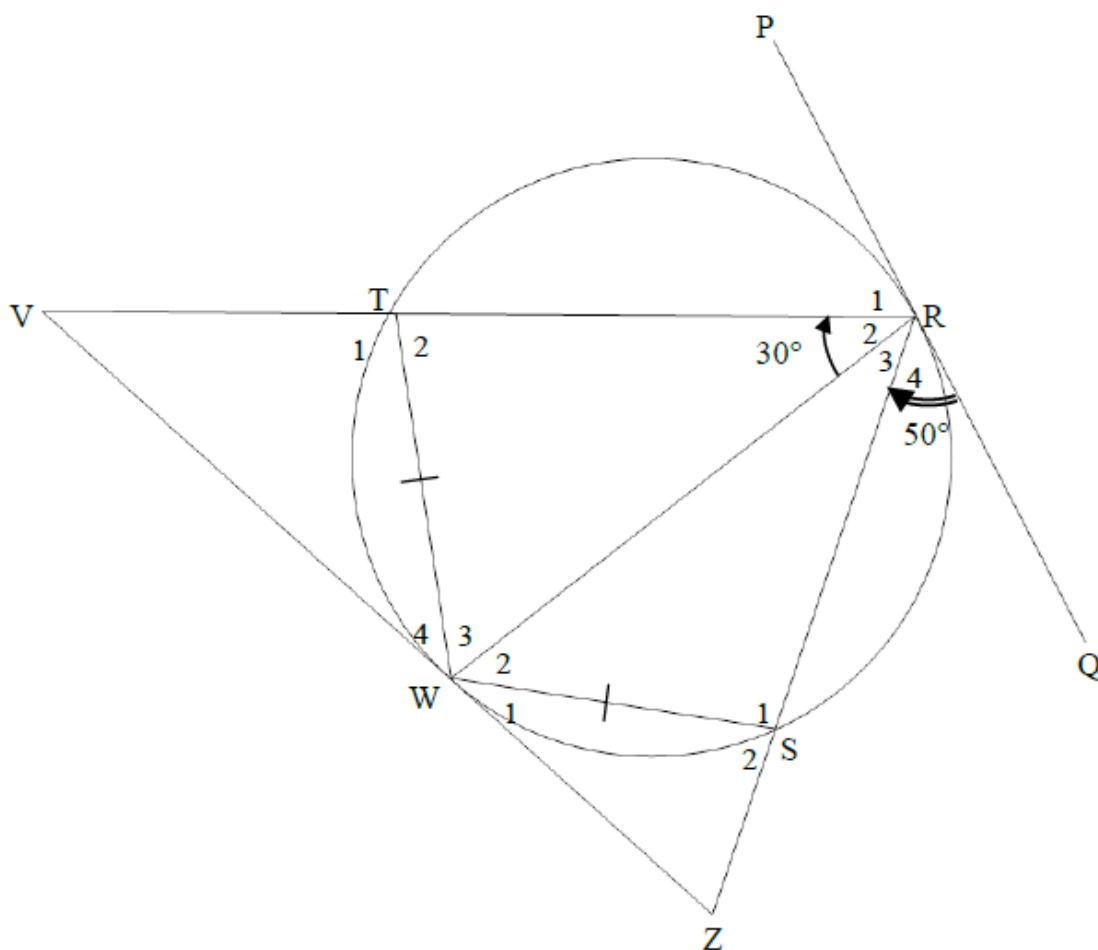
9.1	the interior opposite angle/ <i>die teenoorstaande binnehoek</i> .	✓ answer/antw (1)
-----	--	----------------------

9.2



Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

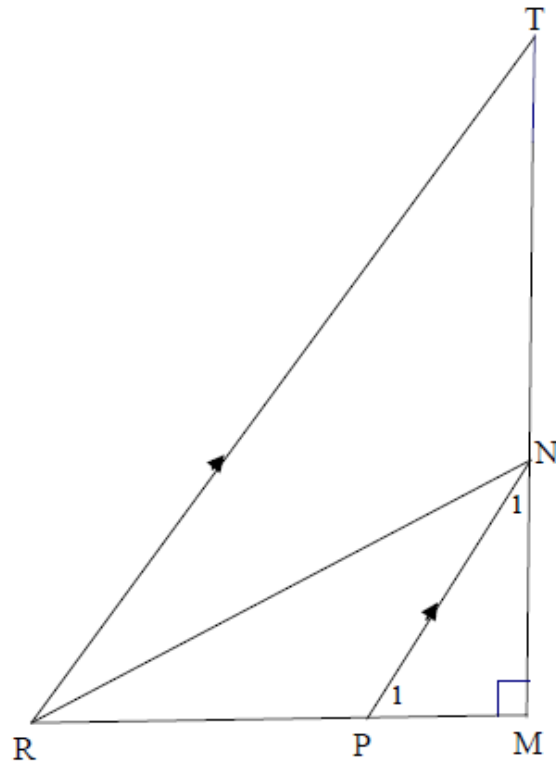
$\hat{V}_1 + \hat{V}_2 = 90^\circ$	$\angle$ in semi-circle/ $\angle$ in halfsirkel	✓ S ✓ R
$\hat{T}_2 = 90^\circ - x$	Tangent $\perp$ diameter/radius/ <i>raaklyn $\perp$ middellyn/radius</i>	✓ R
$\therefore \hat{C} = x$	Sum of the angles of triangle/ <i>Som van die hoeke van 'n driehoek</i>	✓ S
$\therefore \hat{S} = x$	$\angle$'s same segment/ <i>$\angle$e in dieselfde segment</i>	✓ R
$\therefore \hat{VTR} = \hat{S}$		(5)



9.3.1	Equal chords subtend equal $\angle$ s/ <i>Gelyke koorde onderspan gelyke $\angle$e</i>	✓ R (1)
9.3.2	$\hat{W}_4 = 30^\circ$ (tan chord theorem/ <i>rkl-koordst</i>) $\hat{W}_1 = 30^\circ$	✓ answer/ <i>antw</i> ✓ reason/ <i>rede</i> ✓ answer/ <i>antw</i> (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50^\circ$ (tan chord theorem/ <i>rkl-koordst</i>) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext $\angle$ of Δ / <i>buite $\angle$ v Δ</i>) $\therefore \hat{S}_2 = 80^\circ$ OR/OF $\hat{R}_2 = \hat{R}_3 = 30^\circ$ (= chords subtend = $\angle$ s / = <i>kde onderspan = $\angle$e</i>) $\hat{R}_4 = \hat{W}_2 = 50^\circ$ (tan chord theorem/ <i>rkl-koordst</i>) $\therefore \hat{S}_2 = 80^\circ$	✓ S ✓ R ✓ S (3) ✓ S ✓ R ✓ S (3)

9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80^\circ$ (ext $\angle$ of cyclic quad/ <i>buite</i> $\angle$ van koordevh) $V + \hat{W}_4 = \hat{T}_2$ (ext $\angle$ of Δ / <i>buite</i> $\angle$ van Δ) $\therefore \hat{V} = 50^\circ$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S (4)
9.3.4	In ΔRVW and/<i>en</i> ΔRWS: $\hat{R}_2 = \hat{R}_3 = 30^\circ$ (proven/<i>bewys</i> in 9.3.1) $\hat{V} = \hat{W}_2 = 50^\circ$ (proven/<i>bewys</i> in 9.3.3) $V\hat{W}R = \hat{S}_1$ (3rd $\angle$ in Δ) $\therefore \Delta RVW \parallel \Delta RWS$ ($\angle\angle\angle$) $\therefore \frac{WR}{RV} = \frac{RS}{WR}$ ($\Delta RVW \parallel \Delta RWS$) $\therefore WR^2 = RV \cdot RS$	$\checkmark$ using the correct Δ s/ <i>gebruik korrekte Δe</i> $\checkmark$ S $\checkmark$ S $\checkmark$ R (3rd $\angle$ in Δ) or ($\angle\angle\angle$) $\checkmark$ S (5) [22]

QUESTION/VRAAG 10



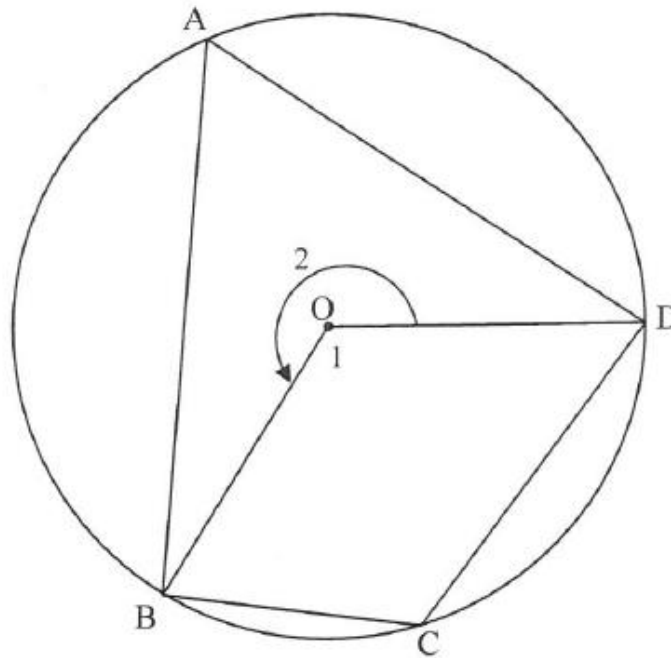
10.1.1	corresponding $\angle$ s/ooreenkomstige $\angle$ e; $PN \parallel RT$	✓ answer/antw (1)
10.1.2	$\angle$; $\angle$; $\angle$ OR/OF $\angle$; $\angle$	✓ answer/antw (1)
10.2	$\frac{PM}{RM} = \frac{PN}{RT} \quad (\triangle PNM \parallel \triangle RTM)$ $= \frac{PN}{3PN}$ $= \frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM} = \frac{1}{3} \quad \therefore \frac{RP}{RM} = \frac{2}{3}$ $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2) \quad (\text{Pyth})$ $= RM^2 - PM^2$ $= \left(\frac{3}{2}RP\right)^2 - \left(\frac{1}{2}RP\right)^2$ $= \frac{9}{4}RP^2 - \frac{1}{4}RP^2$ $= 2RP^2$ OR/OF	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = \frac{3}{2}RP$ ✓ $PM = \frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

	$ \begin{aligned} RN^2 - PN^2 &= (RM^2 + NM^2) - (PM^2 + NM^2) \quad (\text{Pyth}) \\ &= RM^2 - PM^2 \\ &= (3PM)^2 - PM^2 \\ &= 8PM^2 \\ &= 2(2PM)^2 \\ &= 2RP^2 \end{aligned} $ OR/OF $ \begin{aligned} RN^2 - PN^2 &= (RM^2 + NM^2) - (PM^2 + NM^2) \quad (\text{Pyth}) \\ &= RM^2 - PM^2 \\ &= (RP + PM)^2 - PM^2 \\ &= RP^2 + 2RP \cdot PM + PM^2 - PM^2 \\ &= RP^2 + 2RP \cdot \frac{1}{2}RP \\ &= 2RP^2 \end{aligned} $	$ \begin{aligned} &\checkmark \text{ Use of Pyth. for } RN^2 \text{ and } PN^2 \\ &\checkmark RM = RP + PM \\ &\checkmark (3PM)^2 - PM^2 \\ &\checkmark RP = 2PM \end{aligned} $ (4) $ \begin{aligned} &\checkmark \text{ Use of Pyth. for } RN^2 \text{ and } PN^2 \\ &\checkmark RM = RP + PM \\ &\checkmark \text{ expansion/} \\ &\quad \text{uitbreiding} \\ &\checkmark PM = \frac{1}{2} RP \end{aligned} $ (4) [8]
--	---	--

November 2015

QUESTION 8

8.1 In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.



8.1.1 Complete the following statement:

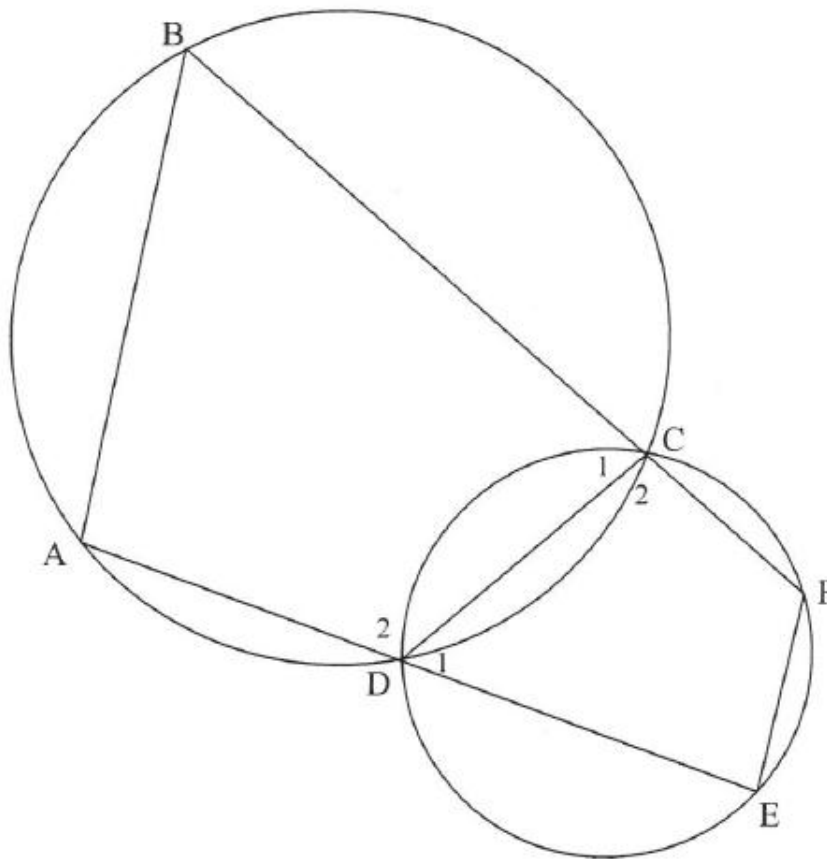
The angle subtended by a chord at the centre of a circle is ... the angle subtended by the same chord at the circumference of the circle.

(1)

8.1.2 Use QUESTION 8.1.1 to prove that $\hat{A} + \hat{C} = 180^\circ$.

(3)

- 8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.

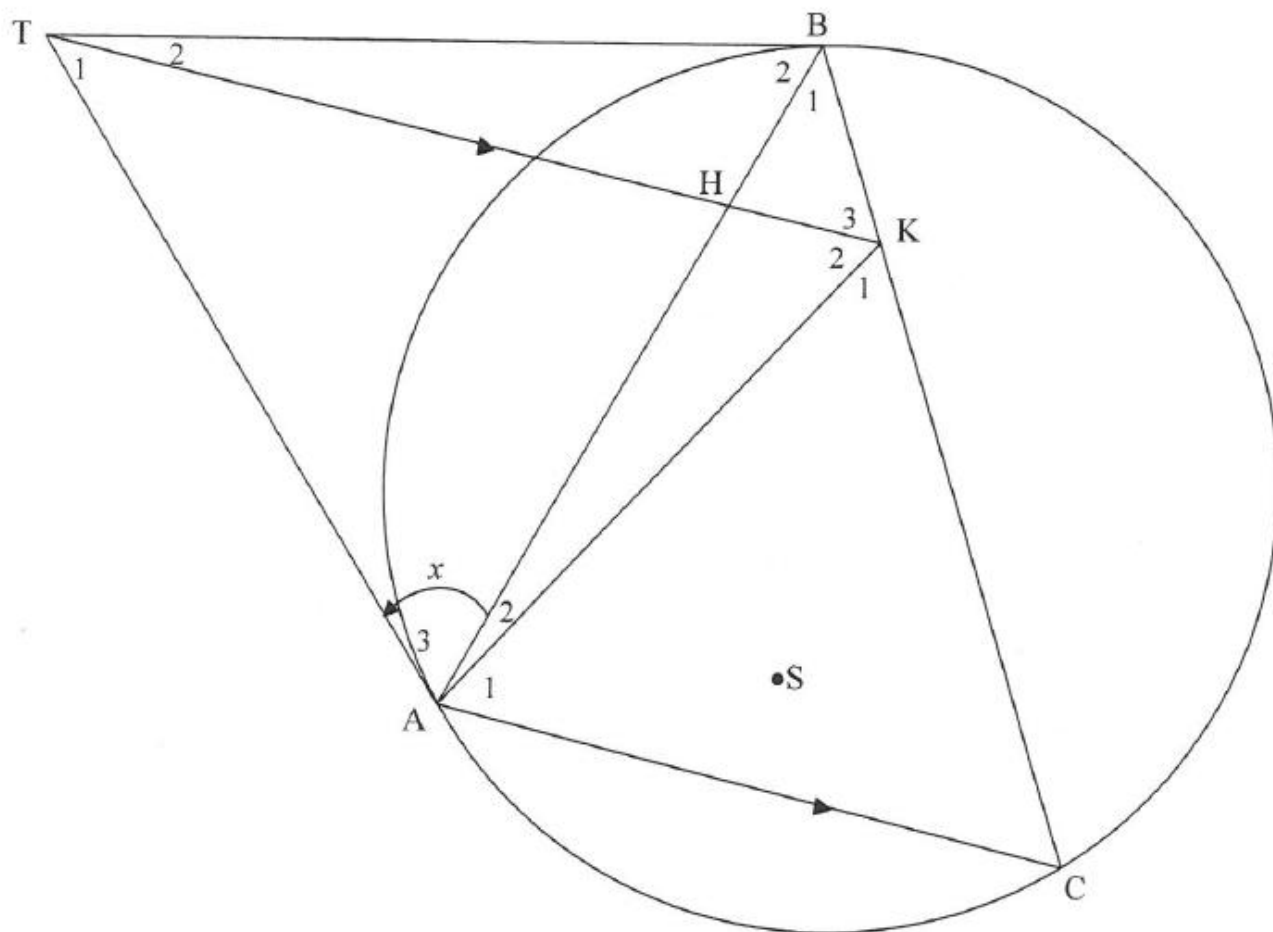


Prove that $EF \parallel AB$.

(5)
[9]

QUESTION 9

In the diagram below, $\triangle ABC$ is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC . AK is drawn. Let $\hat{A}_3 = x$.

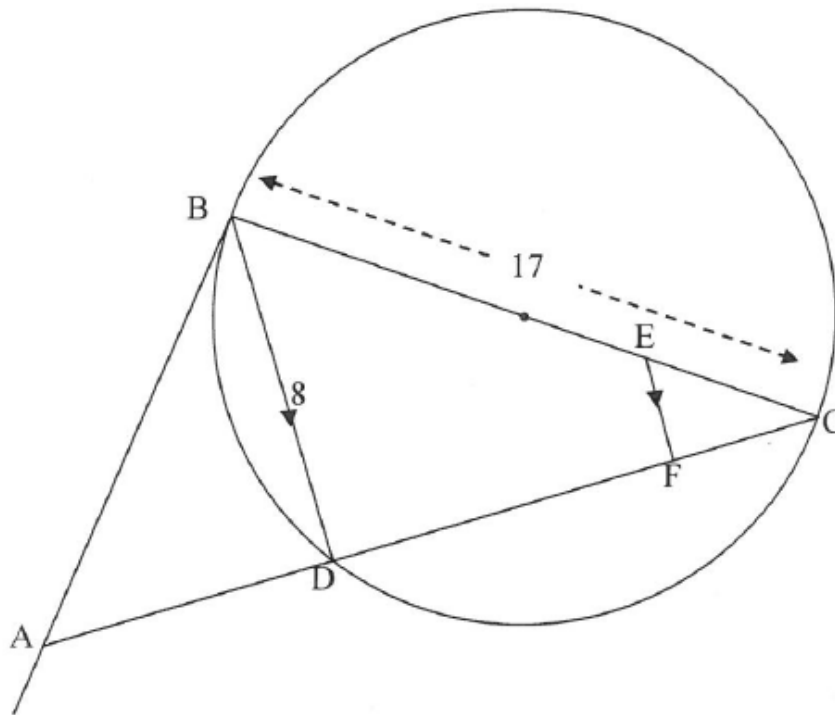


- 9.1 Prove that $\hat{K}_3 = x$. (4)
- 9.2 Prove that $AKBT$ is a cyclic quadrilateral. (2)
- 9.3 Prove that TK bisects $\hat{AKB}$. (4)
- 9.4 Prove that TA is a tangent to the circle passing through the points A , K and H . (2)
- 9.5 S is a point in the circle such that the points A , S , K and B are concyclic. Explain why A , S , B and T are also concyclic. (2)

[14]

QUESTION 10

In the diagram below, $BC = 17$ units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A .



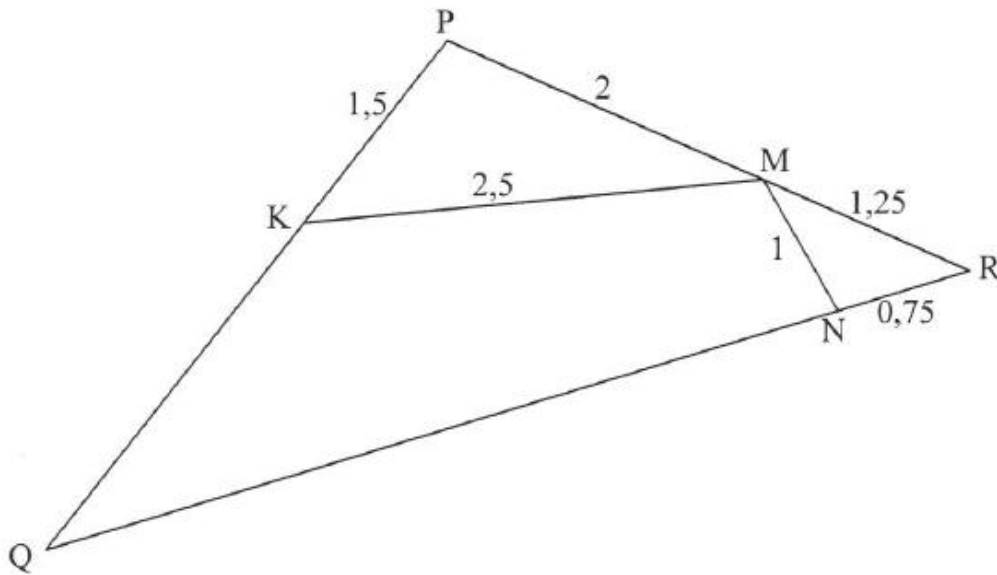
- 10.1 Calculate, with reasons, the length of DC . (3)
- 10.2 E is a point on BC such that $BE : EC = 3 : 1$. EF is parallel to BD with F on DC .
- 10.2.1 Calculate, with reasons, the length of CF . (3)
- 10.2.2 Prove that $\triangle BAC \parallel \triangle FEC$. (5)
- 10.2.3 Calculate the length of AC . (4)
- 10.2.4 Write down, giving reasons, the radius of the circle passing through points A , B and C . (2)
- [17]**

QUESTION 11

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are ... (1)

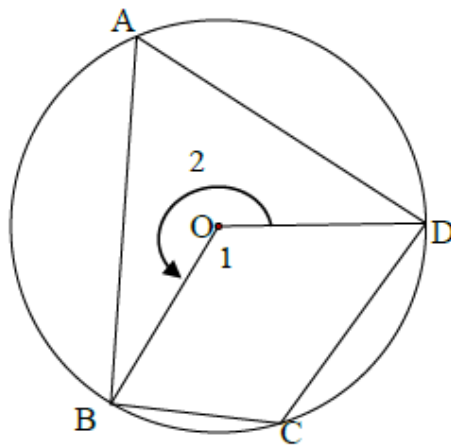
11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of $\triangle PQR$. $KP = 1,5$; $PM = 2$; $KM = 2,5$; $MN = 1$; $MR = 1,25$ and $NR = 0,75$.



11.2.1 Prove that $\triangle KPM \parallel \triangle RNM$. (3)

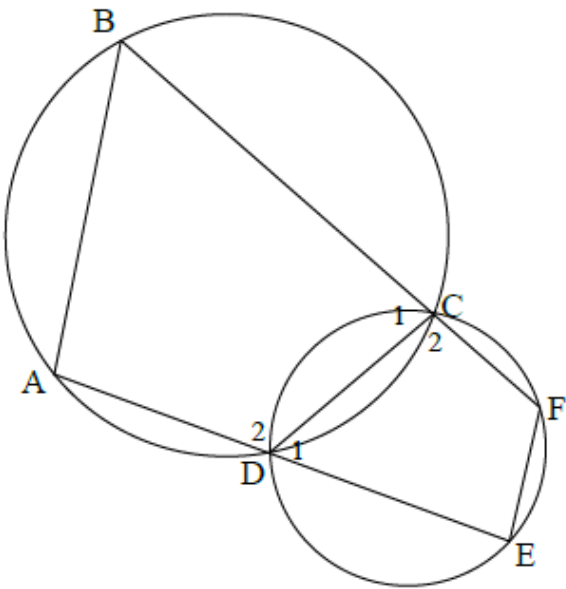
11.2.2 Determine the length of NQ. (6)
[10]

QUESTION/VRAAG 8



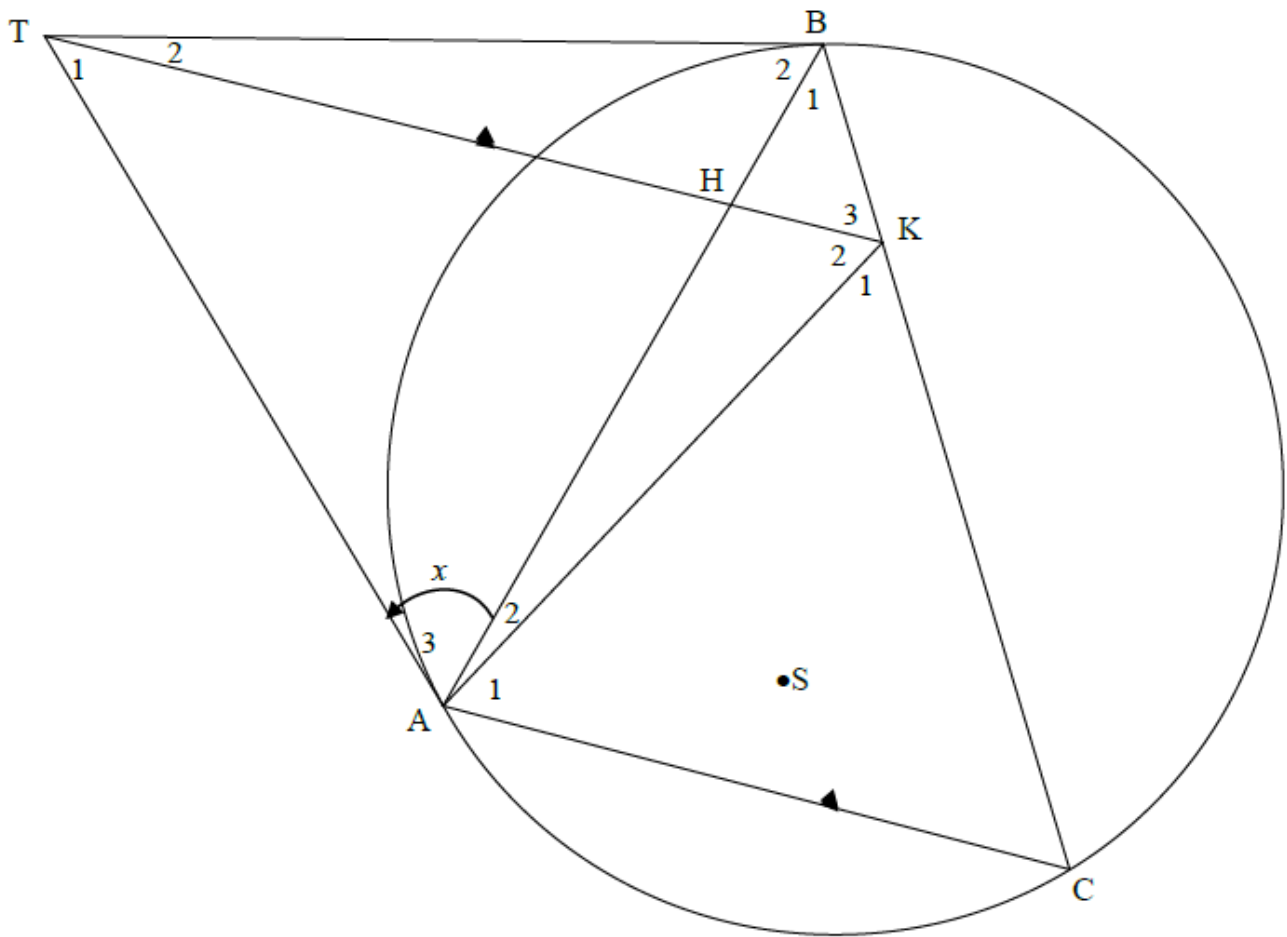
8.1.1	twice or double / <i>twee keer of dubbel</i>	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [$\angle$ at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times \text{omtreks } \angle$] $\hat{O}_2 = 2\hat{C}$ [$\angle$ at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times \text{omtreks } \angle$] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [$\angle$ s in a rev/ $\angle$ e in omw of om 'n pt] $2\hat{A} + 2\hat{C} = 360^\circ$ $\therefore \hat{A} + \hat{C} = 180^\circ$ OR/OF Let/ <i>Gestel</i> $\hat{O}_1 = 2x$ $\hat{A} = x$ [$\angle$ at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times \text{omtreks } \angle$] $\hat{O}_2 = 360^\circ - 2x$ [$\angle$ s in a rev/ $\angle$ e in omw of om 'n pt] $\hat{C} = 180^\circ - x$ [$\angle$ at centre = $2 \times \angle$ at circ/midpts $\angle = 2 \times \text{omtreks } \angle$] $\therefore \hat{A} + \hat{C} = 180^\circ$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

8.2



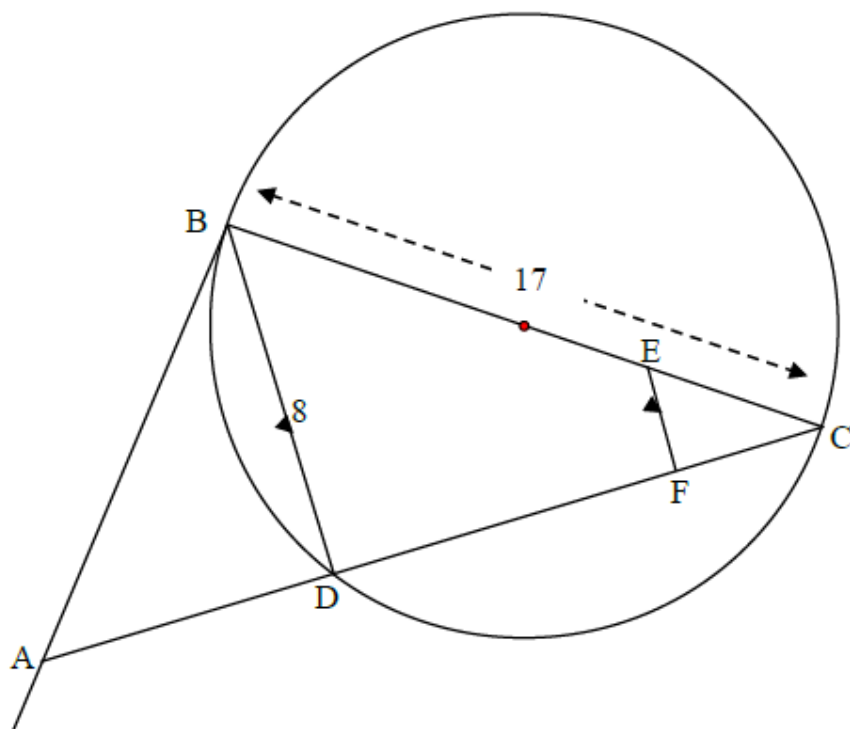
8.2	$\hat{A} = \hat{C}_2$	[ext $\angle$ of cyclic quad/ <i>buite</i> $\angle$ v <i>kdvh</i>]	$\checkmark$ S $\checkmark$ R	(5)
	$\hat{E} = 180^\circ - \hat{C}_2$	[opp $\angle$ s of cyclic quad/ <i>tos</i> $\angle$ e v <i>kdvh</i>]	$\checkmark$ S $\checkmark$ R	
	$\therefore \hat{E} = 180^\circ - \hat{A}$			
	$\therefore EF \parallel AB$	[co-interior $\angle$ s 180° / <i>ko-binne</i> $\angle$ e 180°]	$\checkmark$ R	
	OR/OF			
	$\hat{B} = \hat{D}_1$	[ext $\angle$ of cyclic quad/ <i>buite</i> $\angle$ v <i>kdvh</i>]	$\checkmark$ S $\checkmark$ R	
	$\hat{F} = 180^\circ - \hat{D}_1$	[opp $\angle$ s of cyclic quad/ <i>tos</i> $\angle$ e v <i>kdvh</i>]	$\checkmark$ S $\checkmark$ R	
	$\therefore \hat{F} = 180^\circ - \hat{B}$			
	$\therefore EF \parallel AB$	[co-interior $\angle$ s 180° / <i>ko-binne</i> $\angle$ e 180°]	$\checkmark$ R	

QUESTION/VRAAG 9



9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp $\angle$ s/ooreenk $\angle e$; CA KT] [tan-chord th/raakl-koordst]	✓ S ✓ R ✓ S ✓ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ $\therefore$ AKBT is cyc quad	[proved/bewys in 9.1] [line (BT) subtends equal $\angle$ s/ lyn (BT) onderspan gelyke $\angle e$] OR/OF [converse $\angle$ s in same segment/ omgek $\angle e$ in dies segment]	✓ S ✓ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ $\therefore$ TK bisects/halveer $\hat{A}KB$ OR/OF $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/raakl-koordst] [$\angle$ s in the same segm/ $\angle e$ in dies segm] [$\angle$ s in the same seg/ $\angle e$ in dies segm] [tans from same pt; $\angle$ s opp equal sides/ rkle v dies pt; $\angle e$ to gelyke sye]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R

	$\therefore \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/halveer $\hat{AKB}$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/bewys] $\therefore$ TA tangent [converse tan chord theorem OR $\angle$ between line and chord/ <i>omgekeerde raakl-kdst OF $\angle$ tussen lyn en koord]</i>	$\checkmark$ S $\checkmark$ R (2)
9.5	$\hat{BSA} = \hat{BKA} = 2x$ [A,S,K & B concyclic/konsiklies] $\hat{ATB} = 180^\circ - 2x$ [A,T,B & K concyclic/konsiklies] $\therefore$ points A, S, B and T are also concyclic/ <i>punte A, S, B en T is ook konsiklies</i> [opp $\angle$ s of quad = 180° /tos $\angle$ e van vierhoek= 180°] OR/OF A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic. OR/OF The circle passing through points A, K and B contains the point S on the circumference (A, S, K and B concyclic)./ <i>Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).</i> The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./ <i>Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).</i> $\therefore$ points A, S, B and T are also concyclic/ <i>punte A, S, B en T is konsiklies</i>	$\checkmark$ S (both/beide statements/bewerings) $\checkmark$ R (2) $\checkmark$ S $\checkmark$ S (2) $\checkmark$ S $\checkmark$ S (2) [14]



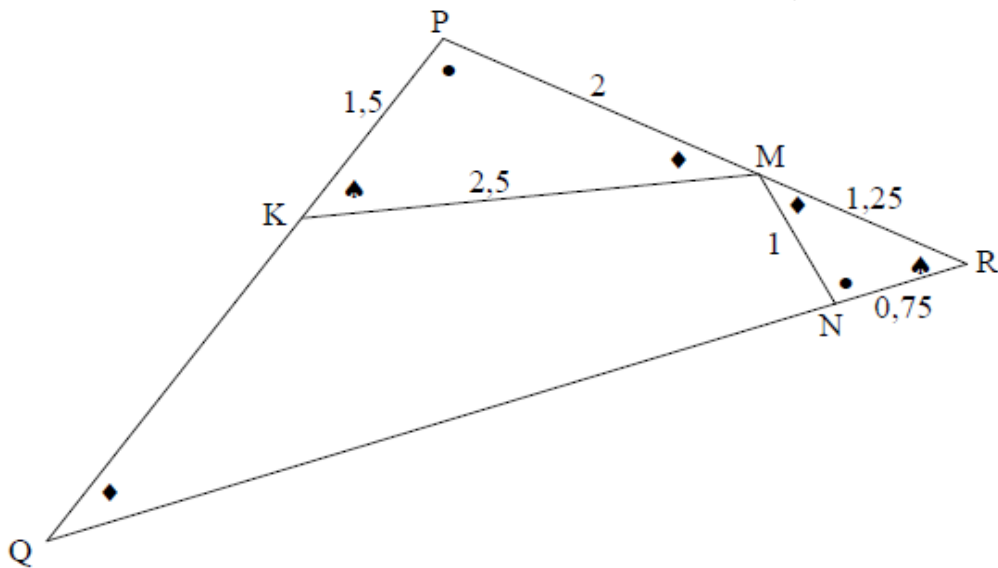
10.1	$\hat{BDC} = 90^\circ$ $DC^2 = 17^2 - 8^2$ $= 225$ $\therefore DC = 15$	$[\angle \text{ in semi circle } / \angle \text{ in halfsirkel}]$ $[\text{Th of/stelling v Pythagoras}]$	$\checkmark$ S $\checkmark$ using/gebruik Pyth <i>korrek/</i> correctly $\checkmark$ answ/antw (3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3,75$	$[\text{line } \text{ one side of } \Delta / \text{lyn } \text{ een sy van } \Delta]$ OR/OF $\triangle CEF \triangle CBD$	$\checkmark$ S/R $\checkmark$ subst correctly/ <i>korrek</i> $\checkmark$ answ/antw (3)
10.2.2	$\hat{BDC} = 90^\circ$ $\hat{EFC} = \hat{BDC}$ $\hat{ABC} = 90^\circ$ In $\triangle BAC$ and/en $\triangle FEC$: $\hat{ABC} = \hat{EFC}$ $\hat{C} = \hat{C}$ $\therefore \triangle BAC \triangle FEC$	$[\angle \text{ in semi circle } / \angle \text{ in halfsirkel}]$ $[\text{corresp } \angle \text{s/ooreenk } \angle \text{e; EF} \parallel \text{BD}]$ $[\text{tan } \perp \text{ diameter/raakl } \perp \text{ middellyn}]$ $[\text{proven/bewys}]$ $[\text{common/gemeen}]$ $[\angle \angle \angle]$	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (5)
	OR/OF $\hat{BDC} = 90^\circ$ $\hat{EFC} = \hat{BDC}$ $\hat{ABC} = 90^\circ$ In $\triangle BAC$ and/en $\triangle FEC$: $\hat{ABC} = \hat{EFC}$ $\hat{C} = \hat{C}$	$[\angle \text{ in semi circle } / \angle \text{ in halfsirkel}]$ $[\text{corresp } \angle \text{s/ooreenk } \angle \text{e; EF} \parallel \text{BD}]$ $[\text{tan } \perp \text{ diameter/raakl } \perp \text{ middellyn}]$ $[\text{proven/bewys}]$ $[\text{common/gemeen}]$	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S

	$\hat{BAC} = \hat{FEC}$ [$\angle$ sum in $\Delta / \angle$ som van Δ] $\therefore \Delta BAC \parallel \Delta FEC$	$\checkmark$ S (5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC}$ [$\Delta BAC \parallel \Delta FEC$] $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27$ or/of $19\frac{4}{15}$ OR/OF $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27$ or/of $19\frac{4}{15}$ OR/OF $\Delta BCA \parallel \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27$ or/of $19\frac{4}{15}$ OR/OF $\hat{C} = \hat{ABD}$ [tan-chord theorem/ <i>rkl-kdstelling</i>] $\frac{AD}{8} = \tan \hat{ABD}$ $= \tan \hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27$ or/of $19\frac{4}{15}$	$\checkmark$ length of/ <i>lengte v</i> EC $\checkmark$ S $\checkmark$ subst correctly/ <i>korrek</i> $\checkmark$ answ/ <i>antw</i> (4) $\checkmark \checkmark$ correct ratios/ <i>korrekte verh's</i> $\checkmark$ subst correctly/ <i>korrek</i> $\checkmark$ answ/ <i>antw</i> (4) $\checkmark$ S OR Pyth th $\checkmark$ correct ratio $\checkmark$ subst $\checkmark$ answ/ <i>antw</i> (4) $\checkmark$ S $\checkmark$ correct ratio $\checkmark$ subst $\checkmark$ answ/ <i>antw</i> (4)

10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° OR converse $\angle$ in semi circle] <i>AC is middellyn van die sirkel wat deur die punte A, B en C gaan</i> [koord onderspan 90° OF omgek $\angle$ in halfsirkel] $\therefore \text{radius} = \frac{1}{2} \times 19,27 = 9,63$ or/of $9\frac{19}{30}$ or/of $\frac{1}{2}AC$	✓ S/R ✓ answ/antw (2) [17]
--------	--	---

QUESTION/VRAAG 11

11.1	equiangular or similar/ <i>gelykhoekig of gelykvormig</i>	✓ answ/antw (1)
------	---	--------------------



11.2.1	$\frac{KP}{RN} = \frac{1,5}{0,75} = 2 ; \frac{PM}{NM} = \frac{2}{1} = 2 ; \frac{KM}{RM} = \frac{2,5}{1,25} = 2$ $\therefore \frac{KP}{RN} = \frac{PM}{NM} = \frac{KM}{RM}$ $\therefore \Delta KPM \parallel \parallel \Delta RNM \quad [\text{Sides of } \Delta \text{ in prop/sye v } \Delta \text{ eweredig}]$ OR/OF $\frac{RN}{KP} = \frac{0,75}{1,5} = \frac{1}{2} ; \frac{NM}{PM} = \frac{1}{2} ; \frac{RM}{KM} = \frac{1,25}{2,5} = \frac{1}{2}$ $\therefore \frac{RN}{KP} = \frac{NM}{PM} = \frac{RM}{KM}$ $\therefore \Delta KPM \parallel \parallel \Delta RNM \quad [\text{Sides of } \Delta \text{ in prop/sye v } \Delta \text{ eweredig}]$ OR/OF In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore \hat{MNR} = 90^\circ \quad [\text{converse Pyth theorem}]$ In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90^\circ \quad [\text{converse Pyth theorem}]$ $\cos \hat{PKM} = \frac{1,5}{2,5} = \frac{3}{5} \text{ and } \cos \hat{R} = \frac{0,75}{1,25} = \frac{3}{5}$ $\therefore \hat{PKM} = \hat{R}$ In ΔKPM and ΔRNM $\hat{PKM} = \hat{R} \quad [\text{proved}]$ $\hat{P} = \hat{MNR} \quad [\text{proved}]$ $\therefore \Delta KPM \parallel \parallel \Delta RNM \quad [\angle; \angle; \angle \text{ OR } 3^{\text{rd}} \angle]$	✓✓✓ all 3 statements/ al 3 bewerings (3) ✓✓✓ all 3 statements/ al 3 bewerings (3) ✓ $\hat{P} = \hat{MNR}$ ✓ $\hat{PKM} = \hat{R}$ ✓ $[\angle; \angle; \angle \text{ OR } 3^{\text{rd}} \angle]$ (3)
--------	--	---

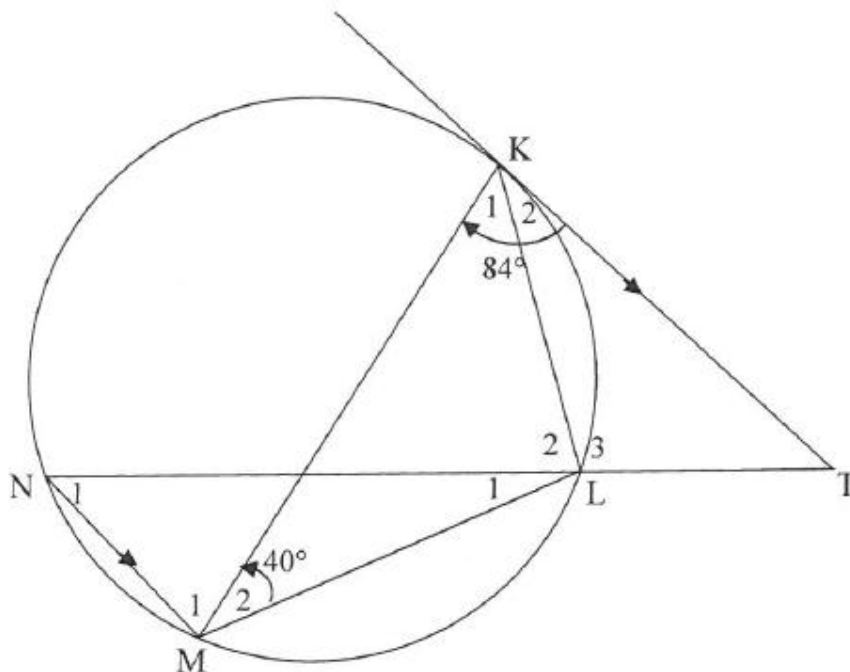
11.2.2	$\hat{P}\hat{K}\hat{M} = \hat{R}$ [ΔKPM ΔRNM] $\therefore \hat{P}$ is common/ <i>gemeen</i> $\therefore \Delta RPQ \Delta KPM$ [∠∠∠] $\frac{RP}{KP} = \frac{RQ}{KM}$ [ΔRPQ ΔKPM] $\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}$ $\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	✓ S ✓ ΔRPQ ΔKPM ✓ S ✓ subst correctly/ <i>korrek</i> ✓ $RQ = 5\frac{5}{12}$ ✓ $NQ = \text{answ/antw}$ (6)
	OR/OF $\hat{R}\hat{N}\hat{M} = \hat{P}$ [ΔKPM ΔRNM] $\therefore \hat{R}$ is common/ <i>gemeen</i> $\therefore \Delta RNM \Delta RPQ$ [∠∠∠] $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ΔRPQ] $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	✓ S ✓ ΔRNM ΔRPQ ✓ S ✓ subst correctly/ <i>korrek</i> ✓ $RQ = 5\frac{5}{12}$ ✓ $NQ = \text{answ/antw}$ (6)
	OR/OF In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore \hat{M}\hat{N}\hat{R} = 90^\circ$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90^\circ$ [converse Pyth theorem] In ΔMNR and ΔQPR $\angle R$ is common $\hat{M}\hat{N}\hat{R} = \hat{P} = 90^\circ$ $\therefore \Delta MNR \Delta QPR$ [∠∠∠] $\frac{RP}{RN} = \frac{RQ}{RM}$ [ΔRNM ΔRPQ] $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	✓ S ✓ ΔMNR ΔQPR ✓ S ✓ subst correctly/ <i>korrek</i> ✓ $RQ = 5\frac{5}{12}$ ✓ $NQ = \text{answ/antw}$ (6)

[10]

Give reasons for ALL statements in QUESTIONS 8, 9 and 10.

QUESTION 8

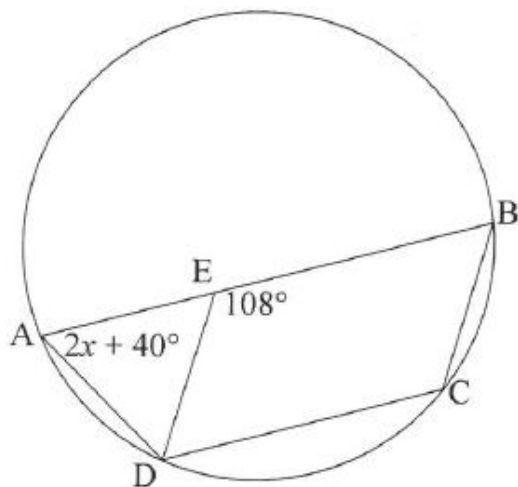
- 8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM . NT cuts the circle at L . $\triangle KML$ is drawn. $\hat{M}_2 = 40^\circ$ and $\hat{MKT} = 84^\circ$.



Determine, giving reasons, the size of:

- | | | |
|-------|-------------|-----|
| 8.1.1 | $\hat{K}_2$ | (2) |
| 8.1.2 | $\hat{N}_1$ | (3) |
| 8.1.3 | $\hat{T}$ | (2) |
| 8.1.4 | $\hat{L}_2$ | (2) |
| 8.1.5 | $\hat{L}_1$ | (1) |

- 8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $\angle DEB = 108^\circ$ and $\angle DAE = 2x + 40^\circ$.

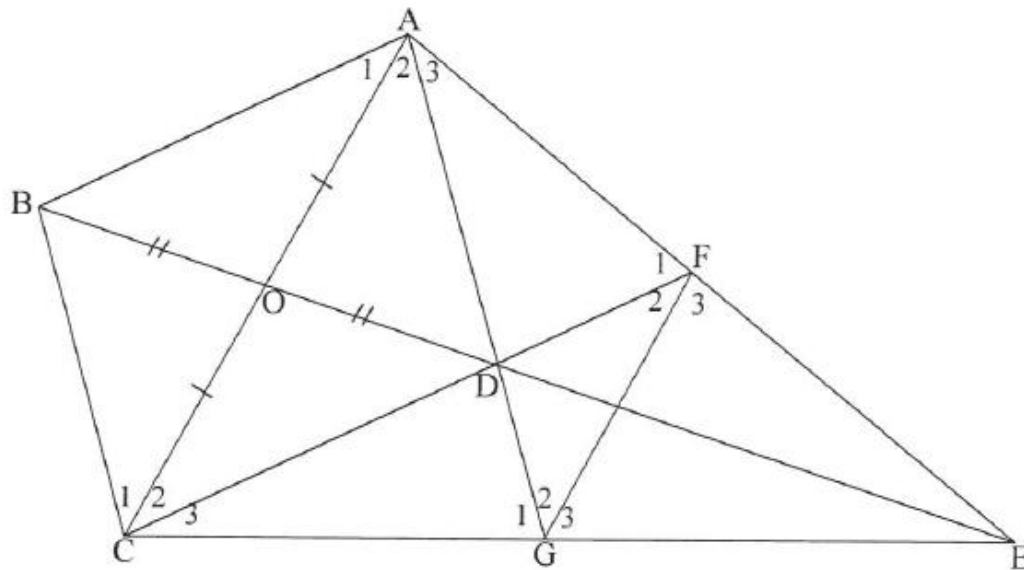


Calculate, giving reasons, the value of x .

(5)
[15]

QUESTION 9

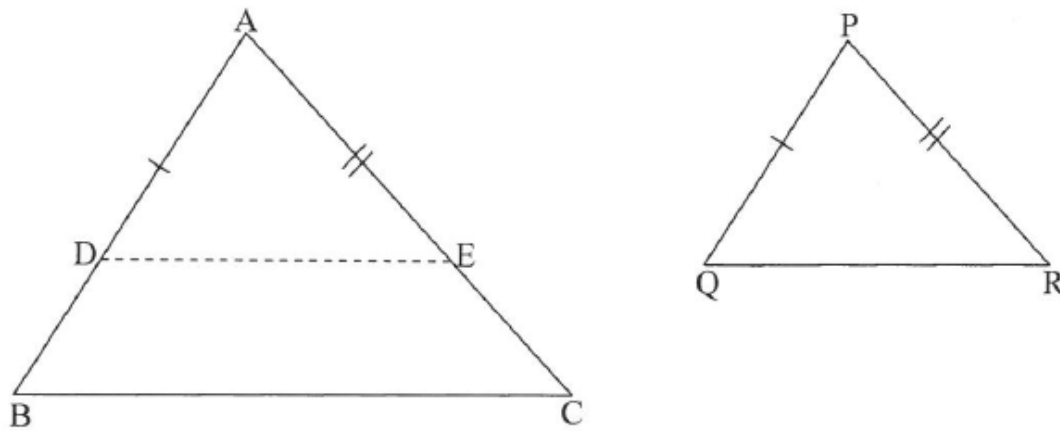
In the diagram below, EO bisects side AC of $\triangle ACE$. EDO is produced to B such that $BO = OD$. AD and CD produced meet EC and EA at G and F respectively.



- 9.1 Give a reason why ABCD is a parallelogram. (1)
- 9.2 Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$. (4)
- 9.3 Prove that $\hat{A}_1 = \hat{F}_2$. (5)
- 9.4 It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral. (3)
- [13]

QUESTION 10

10.1 In the diagram below, $\triangle ABC$ and $\triangle PQR$ are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



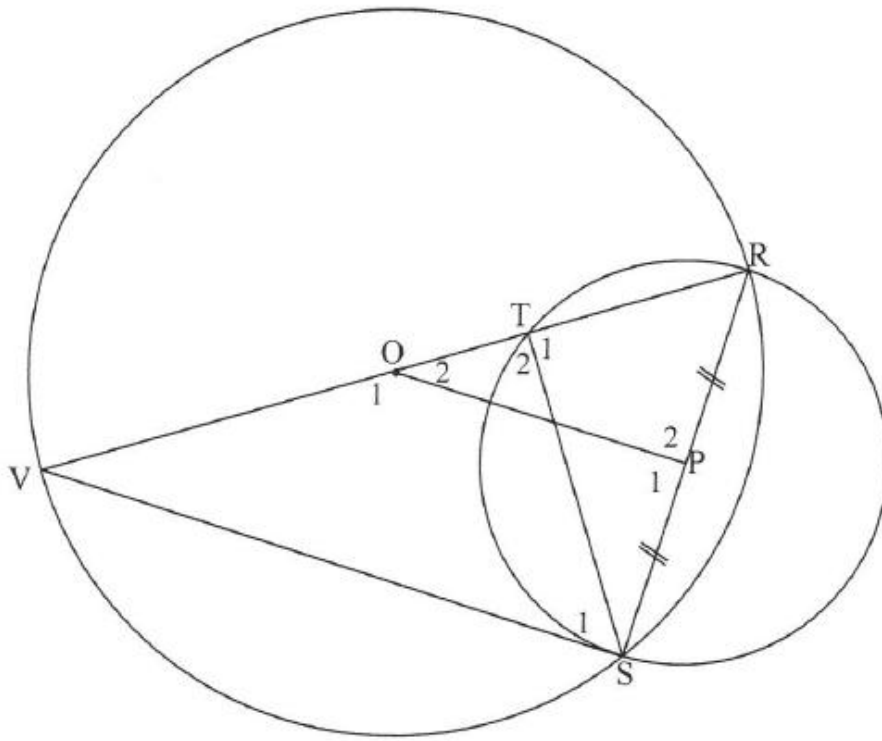
DE is drawn such that $AD = PQ$ and $AE = PR$.

10.1.1 Prove that $\triangle ADE \equiv \triangle PQR$. (2)

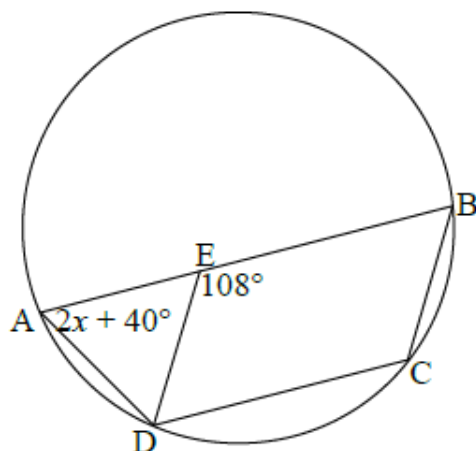
10.1.2 Prove that $DE \parallel BC$. (3)

10.1.3 Hence, prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

- 10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.

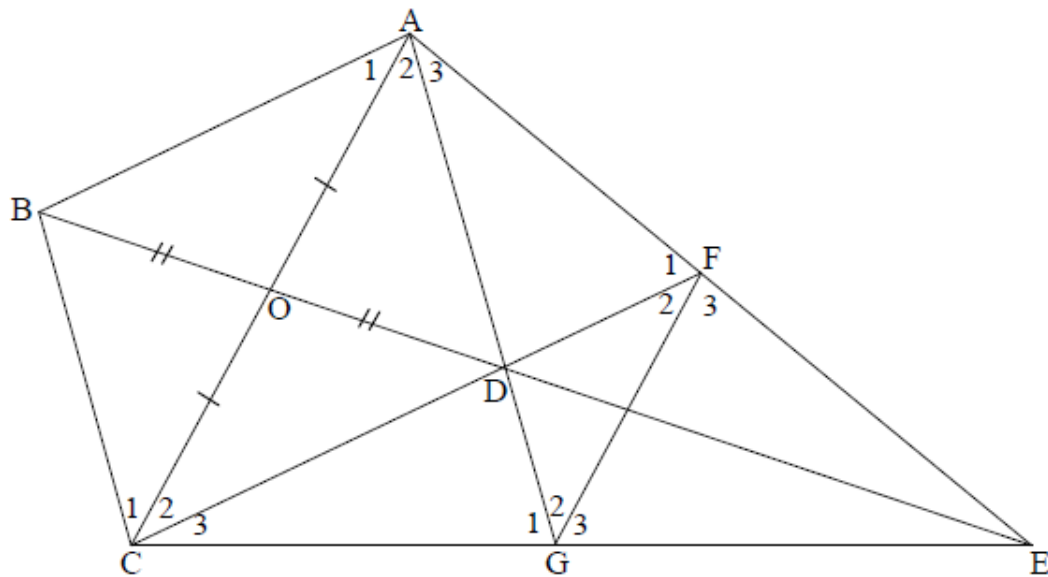


- 10.2.1 Why is $OP \perp PS$? (1)
- 10.2.2 Prove that $\triangle ROP \parallel \triangle RVS$. (4)
- 10.2.3 Prove that $\triangle RVS \parallel \triangle RST$. (3)
- 10.2.4 Prove that $ST^2 = VT \cdot TR$. (6)
- [21]**



8.2	$\hat{C} = 108^\circ$ [opp $\angle$ s of $\parallel m / \text{tos } \angle e \vee \parallel m$] $2x + 40^\circ + 108^\circ = 180^\circ$ [opp $\angle$ s of cyc quad / $\text{tos } \angle e \vee kdv h$] $2x = 32^\circ$ $x = 16^\circ$ OR/OF $\hat{C} = 180^\circ - (2x + 40^\circ)$ [opp $\angle$ s of cyc quad / $\text{tos } \angle e \vee kdv h$] $180^\circ - (2x + 40^\circ) = 108^\circ$ [opp $\angle$ s of $\parallel m / \text{tos } \angle e \vee \parallel m$] $2x = 32^\circ$ $x = 16^\circ$	$\checkmark_S \checkmark_R$ $\checkmark_S \checkmark_R$ $\checkmark_{\text{answ/antw}}$ (5)
		$\checkmark_S \checkmark_R$ $\checkmark_S \checkmark_R$ $\checkmark_{\text{answ/antw}}$ (5)
		[15]

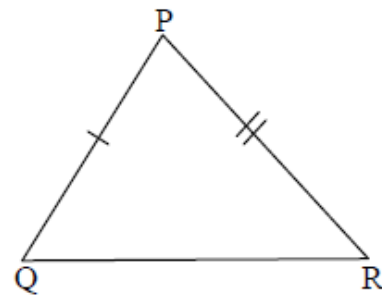
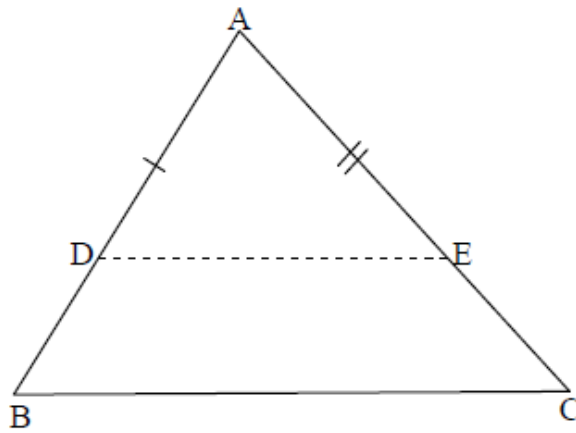
QUESTION/VRAAG 9



9.1	ABCD is a m [diags of quad bisect each other/ hoekl v vh halveer mekaar]	✓ R (1)
9.2	$\frac{ED}{DB} = \frac{FE}{AF}$ [Prop Th/Eweredigh st; DF BA] $\frac{ED}{DB} = \frac{GE}{CG}$ [Prop Th/Eweredigh st; DG BC]	✓ S ✓ R ✓ S ✓ R (4)
9.3	$\frac{FE}{AF} = \frac{GE}{CG}$ [proved/bewys] $\therefore AC \parallel FG$ [line divides two sides of Δ in prop/ lyn verdeel 2 sye van Δ eweredig] $\hat{C}_2 = \hat{F}_2$ [alt/verw $\angle$ s/e; AC FG] $\hat{A}_1 = \hat{C}_2$ [alt/verw $\angle$ s/e; AB CD] $\therefore \hat{A}_1 = \hat{F}_2$	✓ S ✓ S ✓ R ✓ S ✓ S (5)
9.4	$\hat{A}_1 = \hat{A}_2$ [diags of rhombus/hoekl v ruit] $\hat{A}_2 = \hat{F}_2$ [$\hat{A}_1 = \hat{F}_2$] $\therefore ACGF = \text{cyc quad/kdvh}$ [$\angle$ s in the same seg =/ $\angle$ e in dies segm =] OR/OF $\hat{C}_2 = \hat{A}_2$ [$\angle$ s opp equal sides of rhombus/ $\angle$ e to gelyke sye v ruit] $\hat{A}_2 = \hat{G}_2$ [alt/verw- $\angle$ s/e; AC FG] $\therefore \hat{C}_2 = \hat{G}_2$ $\therefore ACGF$ is a cyc quad/kdvh [$\angle$ s in the same seg =/ $\angle$ e in dies segm =]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) [13]

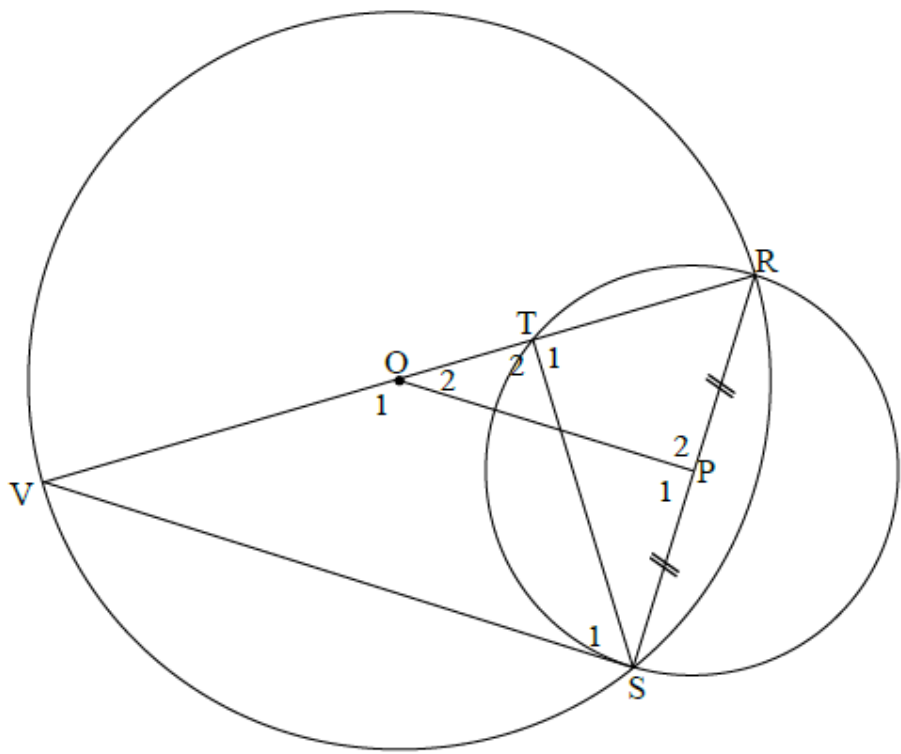
QUESTION/VRAAG 10

10.1



10.1.1	In $\triangle ADE$ and/en $\triangle PQR$: $AD = PQ$ [construction/konstr] $\hat{A} = \hat{P}$ [given/gegee] $AE = PR$ [construction/konstr] $\therefore \triangle ADE \equiv \triangle PQR$ [SAS]	✓ all/al 3 S's/e ✓ reason/rede (2)
10.1.2	$\hat{ADE} = \hat{Q}$ [$\triangle s \equiv \therefore$ corres/ooreenk $\angle s/e =$] But $\hat{B} = \hat{Q}$ [given/gegee] $\therefore \hat{ADE} = \hat{B}$ $\therefore DE \parallel BC$ [corres/ooreenk $\angle s/e =$]	✓ $\hat{ADE} = \hat{Q}$ ✓ $\hat{ADE} = \hat{B}$ ✓ reason/rede (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/Ewaredigh st; $DE \parallel BC$] But/Maar $AD = PQ$ and/en $AE = PR$ [construction/konstr] $\therefore \frac{AB}{PQ} = \frac{AC}{PR}$	✓ S/R ✓ S (2)

10.2



10.2.1	line from centre to midpt of chord/ <i>lyn van midpt na midpt van koord</i>	✓ answ/antw (1)
10.2.2	OP VS [Midpt Theorem/<i>Midpt-stelling</i>] In ΔROP and/en ΔRVS: $\hat{R} = \hat{R}$ [common/<i>gemeen</i>] $\hat{O}_2 = \hat{V}$ [corresp/<i>ooreenk</i> ∠s/<i>e</i>; OP VS] ∴ ΔROP ΔRVS [∠,∠,∠] OR/OF In ΔROP and/en ΔRVS: $\hat{P}_2 = \hat{VSR}$ [corresponding ∠s/<i>ooreenkomstige</i> ∠'e] $\hat{R} = \hat{R}$ [common/<i>gemeen</i>] ∴ ΔROP ΔRVS [∠,∠,∠]	✓ S ✓ R ✓ S ✓ S & ∠;∠;∠ OR/OF 3 angles/<i>hoeke</i> (4) ✓ S ✓ R ✓ S ✓ S & ∠;∠;∠ OR/OF 3 angles/<i>hoeke</i> (4)

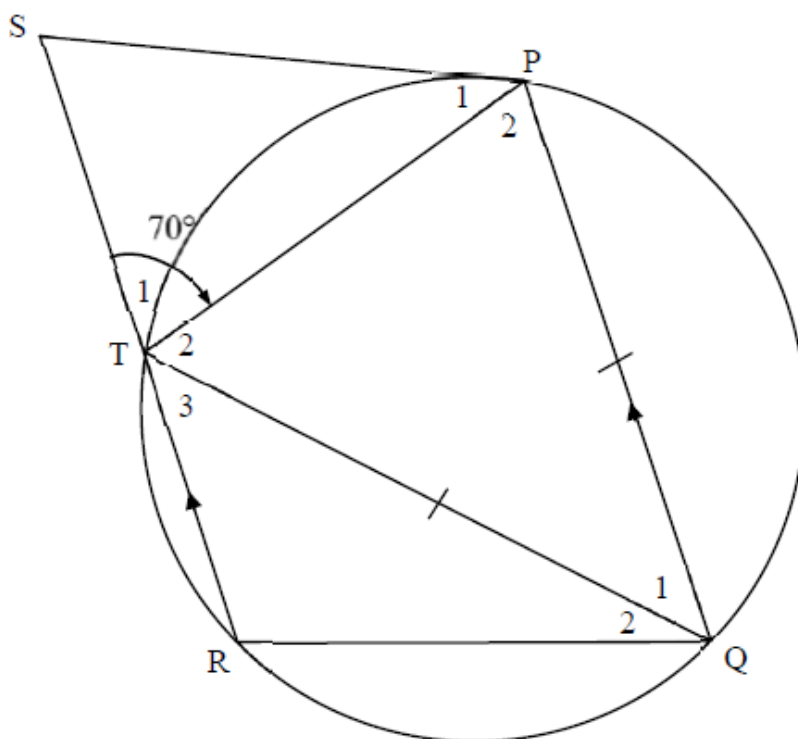
10.2.3	In ΔRVS and/en ΔRST: $\hat{VSR} = \hat{STR} = 90^\circ$ [$\angle$ in semi-circle/$\angle$ in halfsirkel] $\hat{R}$ is common/<i>gemeen</i> $\hat{V} = \hat{TSR}$ $\therefore \Delta RVS \parallel \Delta RST$ [$\angle, \angle, \angle$]	✓S ✓R ✓S & $\angle; \angle; \angle$ OR/OF 3 angles/<i>hoeke</i> (3)
10.2.4	In ΔRTS and/en ΔSTV: $\hat{RTS} = \hat{VTS} = 90^\circ$ [$\angle$ s on straight line/$\angle$ e op <i>rt lyn</i>] $\hat{R} = 90^\circ - \hat{TSR}$ $= \hat{TSV}$ $\hat{TSR} = \hat{V}$ $\therefore \Delta RTS \parallel \Delta STV$ [$\angle, \angle, \angle$] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT \cdot TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/<i>met motivering</i>) ✓$\Delta RTS \parallel \Delta STV$ ✓ratio/<i>verh</i> (6)
		[21]

November 2016

Give reasons for ALL statements and calculations in QUESTIONS 8, 9 and 10.

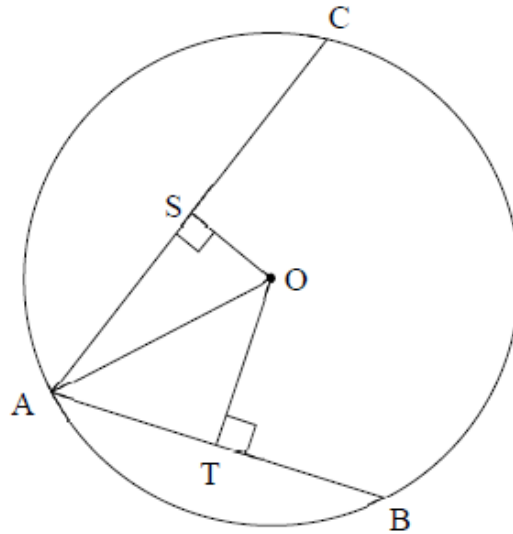
QUESTION 8

- 8.1 In the diagram below PQRT is a cyclic quadrilateral having $RT \parallel QP$. The tangent at P meets RT produced at S. $QP = QT$ and $\hat{PTS} = 70^\circ$.



- 8.1.1 Give a reason why $\hat{P}_2 = 70^\circ$. (1)
- 8.1.2 Calculate, with reasons, the size of:
- (a) $\hat{Q}_1$ (3)
- (b) $\hat{P}_1$ (2)

- 8.2 A, B and C are points on the circle having centre O. S and T are points on AC and AB respectively such that $OS \perp AC$ and $OT \perp AB$. $AB = 40$ and $AC = 48$.



8.2.1 Calculate AT. (1)

8.2.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle. (5)

[12]

The diagram illustrates a geometric setup for a problem involving a circle and several points. The points are labeled A, B, C, D, E, and F. Lines connect these points, forming several triangles. Angles are labeled with numbers 1, 2, 3, 4 and variables x and y . Arrows indicate the direction of light rays.

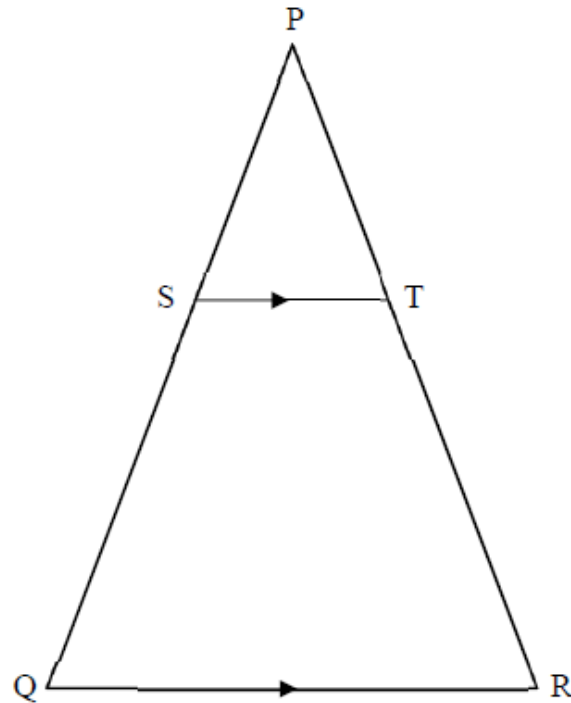
- Point A is on the left, with a line segment AB.
- Point B is on the circle's circumference. At B, there are four angles labeled 1, 2, 3, and 4, formed by lines AB, BE, BF, and BC respectively.
- Point C is at the bottom right. At C, there are two angles labeled 1 and 2, formed by lines CB and CD respectively. A line segment AC is also shown.
- Point D is on the right. At D, there are two angles labeled 1 and 2, formed by lines DE and DC respectively.
- Point E is on the circle's circumference. At E, there are three angles labeled 1, 2, and 3, formed by lines BE, DE, and FE respectively. Two arcs labeled x are also shown at E, one between BE and DE, and another between DE and FE.
- Point F is on the circle's circumference. At F, there is one angle labeled 1, formed by lines BF and FE.

Arrows are placed on segments AB, BE, and CD, indicating a path or direction of light rays.

- Page 157 of 166

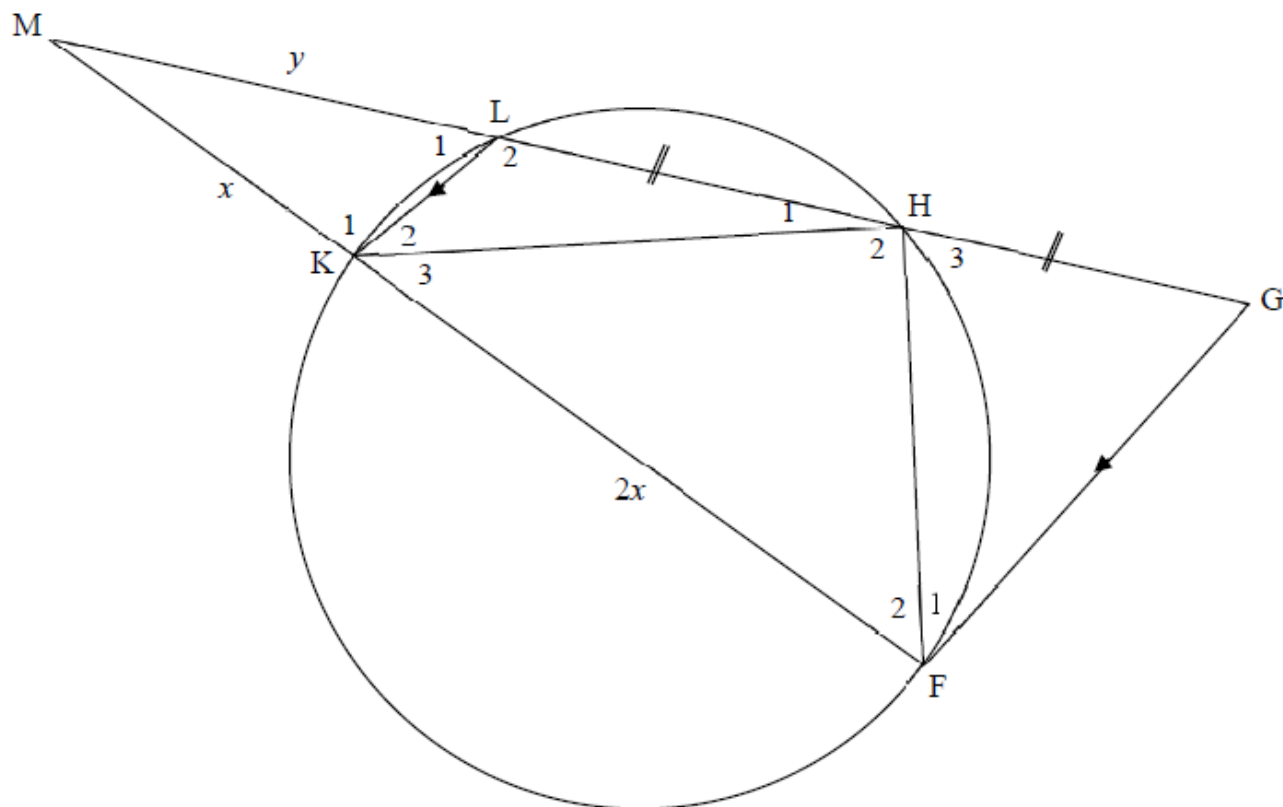
QUESTION 10

- 10.1 In the diagram $\triangle PQR$ is drawn. S and T are points on sides PQ and PR respectively such that $ST \parallel QR$.



Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

- 10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. $MK = x$, $KF = 2x$, $ML = y$ and $LH = HG$.



10.2.1 Give a reason why $\hat{GFM} = \hat{LKM}$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

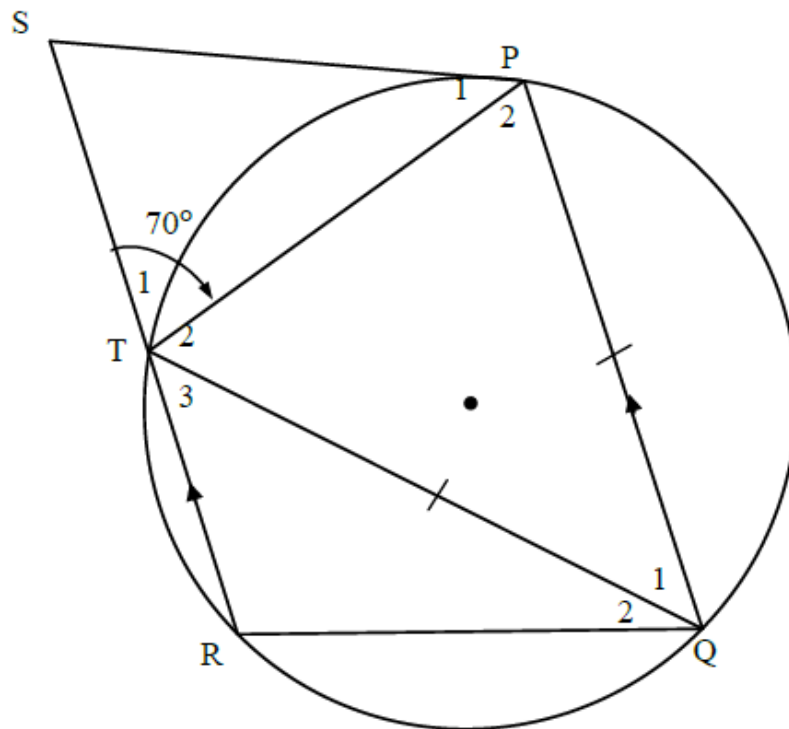
(b) $\triangle MFH \sim \triangle MGF$ (5)

(c) $\frac{GF}{FH} = \frac{3x}{2y}$ (2)

10.2.3 Show that $\frac{y}{x} = \sqrt{\frac{3}{2}}$ (3)
[20]

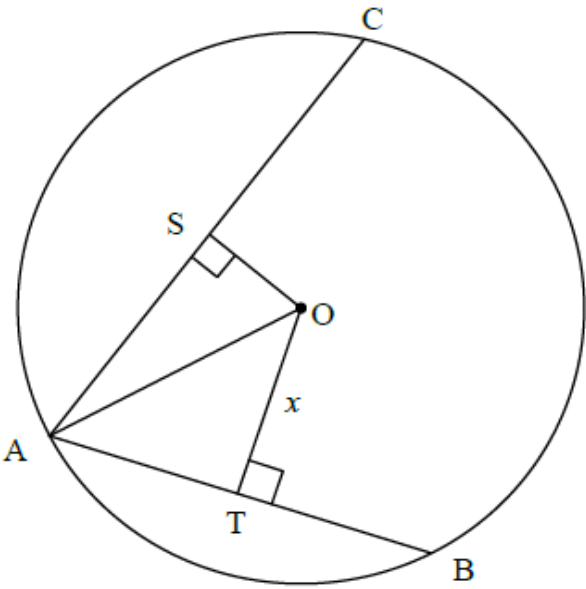
QUESTION/VRAAG 8

8.1



8.1.1	Alternate angles / <i>verwiss hoeke</i> , PQ SR	✓ R	(1)
8.1.2(a)	$\hat{T}_2 = 70^\circ$ [∠s opp = sides/∠e teenoor = sye] $\therefore \hat{Q}_1 = 180^\circ - 2(70^\circ)$ [∠s/e Δ = 180°] $= 40^\circ$	✓ S ✓ R ✓ answer	(3)
8.1.2(b)	$\hat{P}_1 = 40^\circ$ [tangent chord th/raakl-koordst]	✓ S ✓ R	(2)

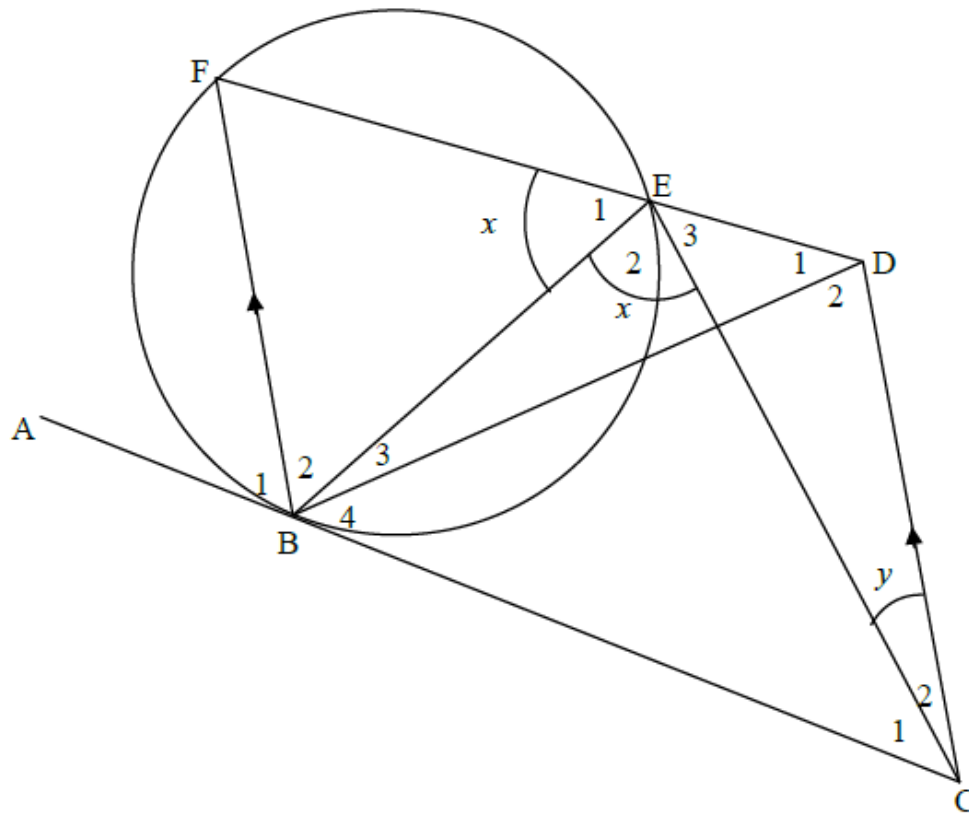
8.2



8.2.1	AT = 20 [line from centre $\perp$ to chord/lyn vanaf midpt $\perp$ koord]	$\checkmark$ S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre $\perp$ to chord/lyn vanaf midpt $\perp$ koord] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ OR/OF Let OS = 7, then OT = 15 In ΔAOT : $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS : $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore OA = 25$ OR/OF	$\checkmark$ equating $\checkmark$ AS = 24 $\checkmark$ substitution $OS = \frac{7}{15}OT$ $\checkmark$ OT $\checkmark$ radius (5) $\checkmark\checkmark$ testing in ΔAOT $\checkmark\checkmark$ testing in ΔAOS $\checkmark$ conclusion (5)

	$AO^2 = OS^2 + AS^2 \quad [\text{Pyth : } \triangle AOS]$ $OT^2 + AT^2 = OS^2 + AS^2 \quad [\text{Pyth : } \triangle AOT]$ Let $OT = 15x$. Then $OS = 7x$ But $AS = 24$ [line from centre $\perp$ to chord/lyn vanaf midpt $\perp$ koord] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ OR/OF $AS = 24$ [line from centre $\perp$ to chord/lyn vanaf midpt $\perp$ koord] $AO^2 = OS^2 + AS^2 \quad [\text{Pyth : } \triangle AOS]$ $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2 \quad [\text{Pyth : } \triangle AOT]$ $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ equating ✓ $AS = 24$ ✓ substitution ✓ $x = 1$ ✓ radius (5) ✓ $AS = 24$ ✓ substitution $OS = \frac{7}{15}OT$ ✓ equating ✓ subst Pyth ✓ radius (5) [12]
--	---	---

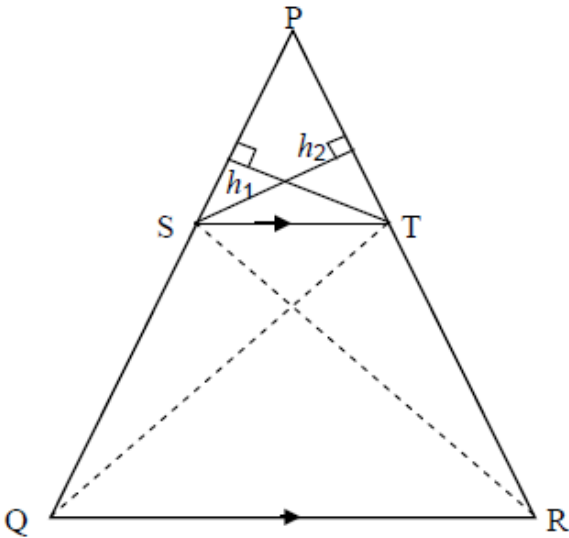
QUESTION/VRAAG 9



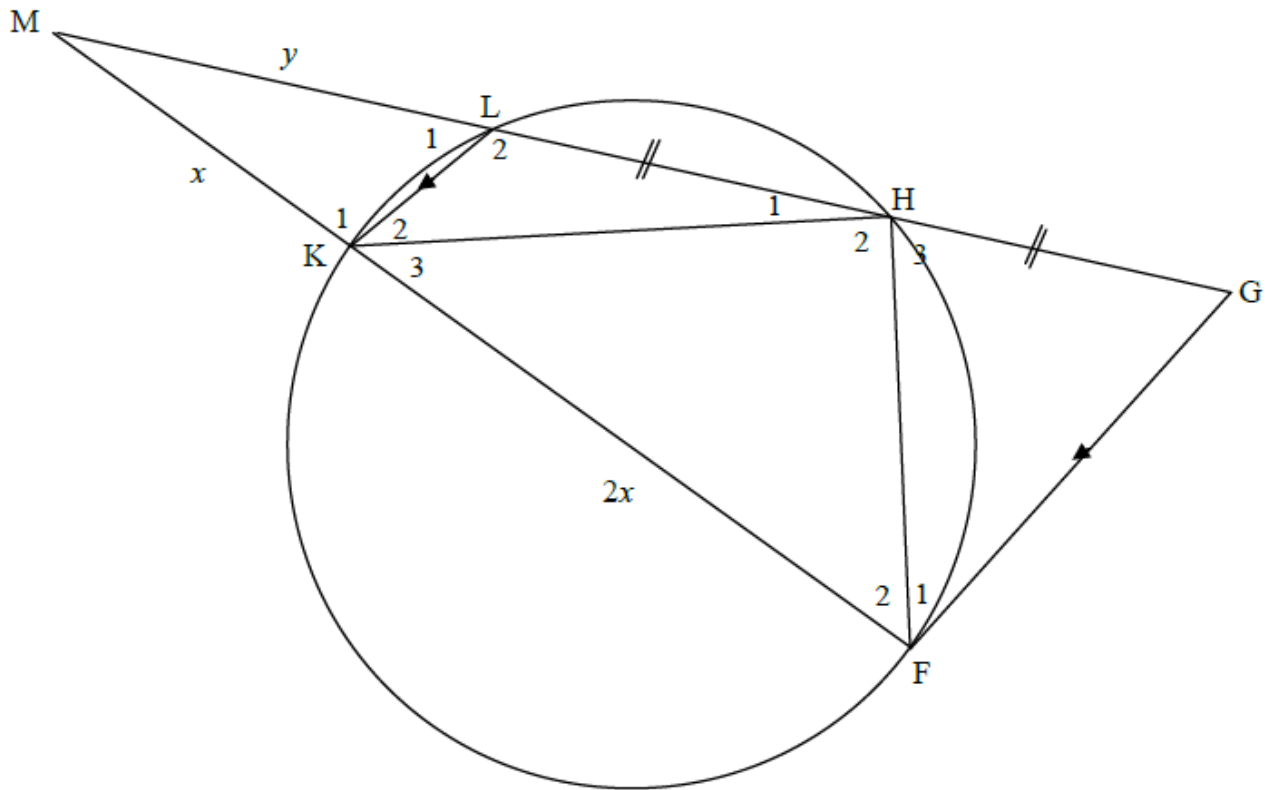
9.1.1	tangent chord theorem/raaklyn-koordstelling	✓ R	(1)
9.1.2	corresponding/ooreenkomstige $\angle$ s/e; $FB \parallel DC$	✓ R	(1)
9.2	$\hat{E}_1 = \hat{C}_D$ $\therefore BCDE = \text{cyclic quad}$ [converse ext $\angle$ cyc quad/omgek: buite $\angle$ kdvh]	✓ S ✓ R	(2)
9.3	$\hat{D}_2 = \hat{E}_2$ [$\angle$ s in the same segment/ $\angle$ e in dies segment] $\hat{D}_2 = \hat{F}_B D$ [alt $\angle$ s, $BF \parallel CD$ /verwiss $\angle$ e, $BF \parallel CD$]	✓ S ✓ S	(2)
9.4	$\hat{B}_3 = y$ OR $\hat{B}_3 = \hat{C}_2$ [$\angle$ s in the same segment/ $\angle$ e in dies segment] $\hat{B}_2 = x - y$ OR $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ OR/OF In $\triangle BFE$ and $\triangle BEC$ $\hat{E}_1 = \hat{E}_2$ [= x] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \triangle BFE \sim \triangle BEC$ [$\angle, \angle, \angle$] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S ✓ identifying Δ 's ✓ S ✓ S	(3) [9]

QUESTION/VRAG 10

10.1



10.1	Constr : Join S to R and T to Q and draw h_1 from S $\perp$ PT and h_2 from T $\perp$ PS/ Verbind SR en TQ en trek h_1 van S $\perp$ PT en h_2 van T $\perp$ PS] Proof : $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2} = \frac{PS}{SQ}$$\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2}PT \times h_1}{\frac{1}{2}TR \times h_1} = \frac{PT}{TR}$ equal altitudes equal altitudes area ΔPST = area ΔPST But area ΔQST = area ΔSTR $\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$ [common] [same base, height; ST QR]	✓ constr/konstruksie $\frac{\text{area } \Delta PST}{\text{area } \Delta QST}$$= \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2}$$\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{PT}{TR}$ ✓ S ✓ R ✓ S (6)
------	--	---



10.2.1	Corresponding/Ooreenkomstige $\angle$ s/e; GF LK	✓ R (1)
10.2.2(a)	$\frac{GL}{LM} = \frac{FK}{KM} \quad \text{OR} \quad \frac{GL}{y} = \frac{2x}{x} \quad [\text{prop theorem/eweredighst; GF LK}]$ $\frac{2GH}{y} = \frac{2x}{x} \quad [LH = HG]$ $\therefore GH = y$	✓ S ✓ R ✓ GL = 2GH (3)

10.2.2(b)	$\hat{K}_1 = \hat{G}\hat{F}\hat{M}$ [corresponding/ooreenkomst $\angle$ s; $\hat{G}\hat{F} \parallel \hat{L}\hat{K}$] $\hat{L}\hat{K}\hat{M}$ or $\hat{K}_1 = \hat{M}\hat{H}\hat{F}$ [ext $\angle$ cyclic quad/buite $\angle$ koordevh] $\hat{M}\hat{H}\hat{F} = \hat{G}\hat{F}\hat{M}$ In $\Delta\hat{M}\hat{F}\hat{H}$ and $\Delta\hat{M}\hat{G}\hat{F}$: $\hat{M} = \hat{M}$ [common/gemeen] $\hat{M}\hat{H}\hat{F} = \hat{G}\hat{F}\hat{M}$ [proven/bewys] $\therefore \Delta\hat{M}\hat{F}\hat{H} \parallel \Delta\hat{M}\hat{G}\hat{F}$ [$\angle\angle\angle$] OR/OR $\hat{K}_1 = \hat{G}\hat{F}\hat{M}$ [corresponding/ooreenkomst $\angle$ s; $\hat{G}\hat{F} \parallel \hat{L}\hat{K}$] $\hat{L}\hat{K}\hat{M}$ or $\hat{K}_1 = \hat{M}\hat{H}\hat{F}$ [ext $\angle$ cyclic quad/buite $\angle$ koordevh] $\hat{M}\hat{H}\hat{F} = \hat{G}\hat{F}\hat{M}$ In $\Delta\hat{M}\hat{F}\hat{H}$ and $\Delta\hat{M}\hat{G}\hat{F}$: $\hat{M} = \hat{M}$ [common/gemeen] $\hat{M}\hat{H}\hat{F} = \hat{G}\hat{F}\hat{M}$ [proven/bewys] $\hat{F}_2 = \hat{G}$ [$\angle$ s of $\Delta = 180^\circ$] $\therefore \Delta\hat{M}\hat{F}\hat{H} \parallel \Delta\hat{M}\hat{G}\hat{F}$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ R (5) $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S (5)
10.2.2(c)	$\therefore \frac{\hat{G}\hat{F}}{\hat{F}\hat{H}} = \frac{\hat{M}\hat{F}}{\hat{M}\hat{H}}$ [$\parallel \Delta$ s] $= \frac{3x}{2y}$	$\checkmark$ S $\checkmark$ R (2)
10.2.3	$\frac{\hat{M}\hat{F}}{\hat{M}\hat{H}} = \frac{\hat{M}\hat{G}}{\hat{M}\hat{F}}$ [$\parallel \Delta$ s] $\frac{3x}{2y} = \frac{3y}{3x}$ [from 10.2.2(c)] $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$	$\checkmark$ S $\checkmark$ substitution $\checkmark$ simplification (3) [20]