



# JENN

**Training and Consultancy**

**The path to enlightened education**

**SUBJECT: MATHEMATICS**

**CONTENT: ALGEBRA, EQUATIONS, AND INEQUALITIES**

**ACTIVITY MANUAL**

**LEARNER/TEACHER MANUAL**

**Algebra, equations, and  
inequalities**



**JENN TRAINING: CONTENT MANUAL TEACHER/LEARNERS:**

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# Algebra, Equations and Inequalities

## Outline :

1. Simplify expressions and solve equations using the laws of exponents for rational exponents where  
$$x^{\frac{p}{q}} = \sqrt[q]{x^p}; x > 0; q > 0$$
2. Add, subtract, multiply and divide simple surds.
3. Quadratic equations (by factorisation and by using the quadratic formula)
4. Quadratic inequalities in one unknown (Interpret solutions graphically.)  
NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs
5. Equations in two unknowns, one of which is linear and the other quadratic
6. Nature of roots

## Parts

- Part 1: Quadratic equations
- Part 2: Quadratic Inequalities
- Part 3: Exponential equations
- Part 4: Surd equations
- Part 5: Simultaneous equations
- Part 6: Nature of roots
- Part 7: Fusion
- Part 8: More on Algebra, equations and inequalities

Formulae to be used in this section are not limited to the following:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac \quad (\text{not provided in the information sheet})$$

# Part 1

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $x^2 = 5x - 4$
2.  $x(3 - x) = -3$
3.  $x(x - 4) = 5$
4.  $4x^2 - 20x + 1 = 0$
5.  $x(x - 1) = 30$
6.  $3x^2 - 5x + 1 = 0$
7.  $3x + \frac{1}{x} = 4$
8.  $x = \frac{5}{3x - 2}$

# Part 2

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $3 - x < 2x^2$
2.  $15x - 4 < 9x^2$
3.  $4 + 5x > 6x^2$
4.  $4x^2 + 1 \geq 5x$
5.  $x^2 + 7x - 8 < 0$
6.  $(x + 1)(x - 3) > 12$
7.  $(x + 1)(4 - x) > 0$
8. Given:  $(x + 2)(x - 3) < -3x + 2$ 
  - 8.1 Solve for  $x$  if:  $(x + 2)(x - 3) < -3x + 2$
  - 8.2 Hence or otherwise, determine the sum of all the integers satisfying the expression  $x^2 + 2x - 8 < 0$ .

## Part 3

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $2.3^x = 81 - 3^x$
2.  $9.2^{x-1} = 2.3^x$
3.  $2^{x\sqrt{x}} = 2^{27}$
4.  $27^{x^2+x} = 3^{3x^2} \times 9$
5.  $5^x = \frac{1}{125}$

## Part 4

Solve for  $x$ , rounded off to TWO decimal places where necessary:

1.  $\sqrt{2x+1} = x-1$
2.  $2x-3 = \sqrt{2x-3}$
3.  $2\sqrt{x-3} = x-3$
4.  $x - \sqrt{x} = 6$

## Part 5

1. Determine the values of  $x$  and  $y$  if they satisfy both the following equations simultaneously:

$$\begin{aligned} 2x + y &= 3 \\ x^2 + y + x &= y^2 \end{aligned}$$

2. Solve simultaneously for  $x$  and  $y$ :

$$\begin{aligned} y - x + 3 &= 0 \\ x^2 - x &= 6 + y \end{aligned}$$

3. Consider the equation:  $x^2 + 5xy + 6y^2 = 0$

3.1 Calculate the values of the ratio  $\frac{x}{y}$ .

3.2 Hence, calculate the values of  $x$  and  $y$  if  $x + y = 8$ .

4. Solve for  $x$  and  $y$  if:  $3^{x-10} = 3^{3x}$  and  $y^2 + x = 20$ .

5. Given:  $2^x + 2^{x+2} = -5y + 20$

5.1 Express  $2^x$  in terms of  $y$ .

5.2 How many solutions for  $x$  will the equation have if  $y = -4$ ?

5.3 Solve for  $x$  if  $y$  is the largest possible integer value for which  $2^x + 2^{x+2} = -5y + 20$  will have solutions.

## Part 6

1. Determine the nature of the roots of  $x^2 + 9 = 0$ .
2. Determine the values of  $k$  for which the equation  $x^2 - 3x + (k + 1) = 0$  has:
  - 2.1 Real roots
  - 2.2 One root equal to 1
3. Show that the equation  $x^2 - px - p^2 = 2$  has two real and unequal roots for all real values of  $p$ .
4. The coefficients of a quadratic equation are rational numbers, and its discriminant is  $\Delta = 2k(2k - 9)$ .

Describe the nature of the roots of the equation if  $k = 6$ .
5. If 5 is one root of the equation  $x^2 + kx - 15 = 0$ , determine the value of  $k$ , and the other root.
6. Given:  $f(x) = 3x^2 - 6x + m$ 

Determine the values of  $m$  if  $f(x) = 0$  has non-real roots.
7. Given:  $f(x) = x^2 - 5x + c$ 

Determine the value of  $c$  if it is given that the solutions of  $f(x) = 0$  are  $\frac{5 \pm \sqrt{41}}{2}$ .
8. Given:  $f(x) = 5x^2 + 6x - 7$ 
  - 8.1 Solve for  $x$  if  $f(x) = 0$  (correct to TWO decimal places).
  - 8.2 Hence, or otherwise, calculate the value of  $d$  for which  $5x^2 + 6x - d = 0$  has equal roots.
9. The solutions of a quadratic equation are given by  $x = \frac{-2 \pm \sqrt{2p + 5}}{7}$ 

For which value(s) of  $p$  will this equation have:

  - 9.1 Two equal solutions
  - 9.2 No real solutions

## Part 7

1. Given  $x = 999\,999\,999\,999$ , determine the exact value of  $\frac{x^2 - 4}{x - 2}$ .  
Show ALL your calculations.
2. Explain why the equation  $\frac{x^4 + 1}{x^4} = \frac{1}{2}$  has no real roots.
3. If  $m$  and  $n$  are rational numbers such that  $\sqrt{m} + \sqrt{n} = \sqrt{7 + \sqrt{48}}$ , calculate a possible value of  $m^2 + n^2$ .
4. Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}} \quad (\text{Show ALL calculations.})$$

5. Calculate the integer that is the closest approximation to:

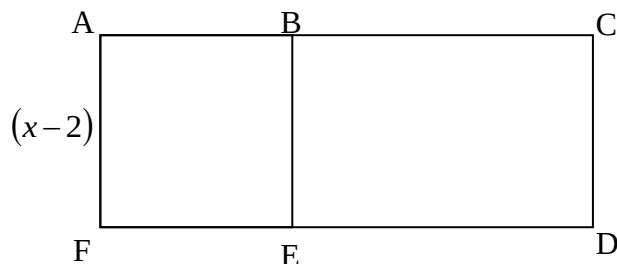
$$\frac{5^{2007} + 5^{2010}}{5^{2008} + 5^{2009}} \quad (\text{Show ALL workings.})$$

6. The volume of a box with a rectangular base is  $3\,072\text{ cm}^3$ . The lengths of the sides are in the ratio  $1 : 2 : 3$ . Calculate the length of the shortest side.
7. Mary gave one third of her money to Nazeem and one fifth of her money to Elwethu. Elwethu received R28 less than Nazeem. How much money did Mary have originally?
8. Given:  $m + \frac{1}{m} = 3$

$$8.1 \quad \text{Determine the value of: } m^2 - 1 + \frac{1}{m^2}.$$

$$8.2 \quad \text{Hence determine the value of: } m^3 + \frac{1}{m^3}.$$

9. ACDF is a rectangle with an area of  $x^2 + 2x - 8\text{ cm}^2$ . B is a point on AC and E is a point on FD such that ABEF is a square with sides of length  $(x - 2)$  cm each.



Calculate the length of ED.

## QUESTION 1

- 1.1 Solve for  $x$ :
- 1.1.1  $(x-2)(4+x) = 0$  (2)
- 1.1.2  $3x^2 - 2x = 14$  (correct to TWO decimal places) (4)
- 1.1.3  $2^{x+2} + 2^x = 20$  (3)
- 1.2 Solve the following equations simultaneously:
- $$x = 2y + 3$$
- $$3x^2 - 5xy = 24 + 16y$$
- (6)
- 1.3 Solve for  $x$ :  $(x-1)(x-2) < 6$  (4)
- 1.4 The roots of a quadratic equation are:  $x = \frac{3 \pm \sqrt{-k-4}}{2}$   
For which values of  $k$  are the roots real? (2)  
[21]

November 2015

## QUESTION 1

- 1.1 Solve for  $x$ :
- 1.1.1  $x^2 - 9x + 20 = 0$  (3)
- 1.1.2  $3x^2 + 5x = 4$  (correct to TWO decimal places) (4)
- 1.1.3  $2x^{\frac{-5}{3}} = 64$  (4)
- 1.1.4  $\sqrt{2-x} = x-2$  (4)
- 1.1.5  $x^2 + 7x < 0$  (3)
- 1.2 Given:  $(3x-y)^2 + (x-5)^2 = 0$   
Solve for  $x$  and  $y$ . (4)
- 1.3 For which value of  $k$  will the equation  $x^2 + x = k$  have no real roots? (4)  
[26]

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x(x-7) = 0$  (2)

1.1.2  $x^2 - 6x + 2 = 0$  (correct to TWO decimal places) (3)

1.1.3  $\sqrt{x-1} + 1 = x$  (5)

1.1.4  $3^{x+3} - 3^{x+2} = 486$  (4)

1.2 Given:  $f(x) = x^2 + 3x - 4$ 

1.2.1 Solve for  $x$  if  $f(x) = 0$  (2)

1.2.2 Solve for  $x$  if  $f(x) < 0$  (2)

1.2.3 Determine the values of  $x$  for which  $f'(x) \geq 0$  (2)

1.3 Solve for  $x$  and  $y$ :  $x = 2y$  and  $x^2 - 5xy = -24$  (4)  
[24]

Feb.–Mar. 2015

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - x - 20 = 0$  (2)

1.1.2  $2x^2 - 11x + 7 = 0$  (correct to TWO decimal places) (3)

1.1.3  $5x^2 + 4 > 21x$  (5)

1.1.4  $2^{2x} - 6.2^x = 16$  (4)

1.2 Solve for  $x$  and  $y$  simultaneously:

$$\begin{aligned} y + 1 &= 2x \\ x^2 - xy + y^2 &= 7 \end{aligned} \quad (6)$$

1.3 The roots of a quadratic equation are given by  $x = \frac{-5 \pm \sqrt{20+8k}}{6}$ ,  
where  $k \in \{-3; -2; -1; 0; 1; 2; 3\}$ .1.3.1 Write down TWO values of  $k$  for which the roots will be rational. (2)1.3.2 Write down ONE value of  $k$  for which the roots will be non-real. (1)1.4 Calculate  $a$  and  $b$  if  $\sqrt{\frac{7^{2014} - 7^{2012}}{12}} = a(7^b)$  and  $a$  is not a multiple of 7. (4)  
[27]

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 + 5x - 6 = 0$  (3)

1.1.2  $4x^2 + 3x - 5 = 0$  (correct to TWO decimal places) (3)

1.1.3  $4x^2 - 1 < 0$  (3)

1.1.4  $\left(\sqrt{\sqrt{32} + x}\right)\left(\sqrt{\sqrt{32} - x}\right) = x$  (4)

1.2 Solve simultaneously for  $x$  and  $y$ :

$y + x = 12$  and  $xy = 14 - 3x$  (5)

1.3 Consider the product  $1 \times 2 \times 3 \times 4 \times \dots \times 30$ .Determine the largest value of  $k$  such that  $3^k$  is a factor of this product. (4)**[22]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - 6x = 0$  (2)

1.1.2  $x^2 + 10x + 8 = 0$  (correct to TWO decimal places) (3)

1.1.3  $(1-x)(x+2) < 0$  (3)

1.1.4  $\sqrt{x+18} = x-2$  (5)

1.2 Solve simultaneously for  $x$  and  $y$ :

$x + y = 3$  and  $2x^2 + 4xy - y = 15$  (6)

1.3 If  $n$  is the largest integer for which  $n^{200} < 5^{300}$ , determine the value of  $n$ . (3)**[22]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - 6x - 16 = 0$  (3)

1.1.2  $2x^2 + 7x - 1 = 0$  (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to  $x^2 - 25 < 0$ . (4)1.3 Solve for  $x$  and  $y$ :

$-2y + x = -1$  and  $x^2 - 7 - y^2 = -y$  (6)

1.4 Evaluate:  $\frac{3^{2018} + 3^{2016}}{3^{2017}}$  (2)

1.5 Given:  $t(x) = \frac{\sqrt{3x-5}}{x-3}$

1.5.1 For which values of  $x$  will  $\frac{\sqrt{3x-5}}{x-3}$  be real? (3)

1.5.2 Solve for  $x$  if  $t(x) = 1$ . (4)

**[26]**

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 + 2x - 15 = 0$  (3)

1.1.2  $5x^2 - x - 9 = 0$  (Leave your answer correct to TWO decimal places.) (3)

1.1.3  $x^2 \leq 3x$  (4)

1.2 Given:  $a + \frac{64}{a} = 16$ 1.2.1 Solve for  $a$ . (3)1.2.2 Hence, solve for  $x$ :  $2^x + 2^{6-x} = 16$  (3)1.3 **Without using a calculator**, calculate the value of  $\sqrt{\frac{2^{1002} + 2^{1006}}{17(2)^{998}}}$  (4)1.4 Solve for  $x$  and  $y$  simultaneously:

$$2x - y = 2 \quad \text{and} \quad \frac{1}{x} - 3y = 1$$

(6)  
**[26]**

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