

Concepts to cover

- 1. Normal probability (Sample Spacing, Tree Diagram etc etc)
- 2. Conditional probability
- 3. Independent and mutually exclusive

Probability: Chance of an occurrence of an event. It can be expressed as

1. Fraction
2. Decimal
3. Percentage

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

① MCQs {A, B, C, D}

$$P(\text{Right}) = \frac{1}{4} \text{ or } 25\%$$

$$P(\text{Wrong}) = \frac{3}{4} \text{ or } 75\% \quad P(\text{Wrong}) = 1 - P(\text{Right})$$

② Die Roll {1, 2, 3, 4, 5, 6}

Fair/Unbiased: Equal Probabilities
Biased: Unequal Probabilities

$$P(6) = \frac{1}{6}$$

Key Points

1. Sum of probability or total probability is always 1 or 100%
2. $P(\text{Win}) = 1 - P(\text{Lose})$ or $P(\text{same}) = 1 - P(\text{different})$ or $P(A') = 1 - P(A)$

$\xrightarrow{\text{event not occurring}}$
 $\xrightarrow{\text{event occurring}}$

3. AND/BOTH/ALL: MULTIPLY the probabilities
4. OR: ADD the probabilities

Whenever there are multiple ways of obtaining a certain outcome.

Usually happens when order/quantity is not specified then we find probabilities of all possible orders and add them up

Q. One is red & other is blue: A. $BR + RB$

Q. At least one is Blue: A. $P(1 \text{ Blue}) + P(2 \text{ Blue}) \dots$

Probability distributions, possible diagrams and sample spacing

If we need to find the sum, product or difference of 2 things then in order to calculate their probabilities we have to draw a probability distribution diagram also known as sample spacing.

Example

1. ^{fair}Two dice are rolled and their sum is recorded, find the probability that the score is
- a. A perfect square —
 - b. A prime number .
 - c. The difference between the two scores is 5.

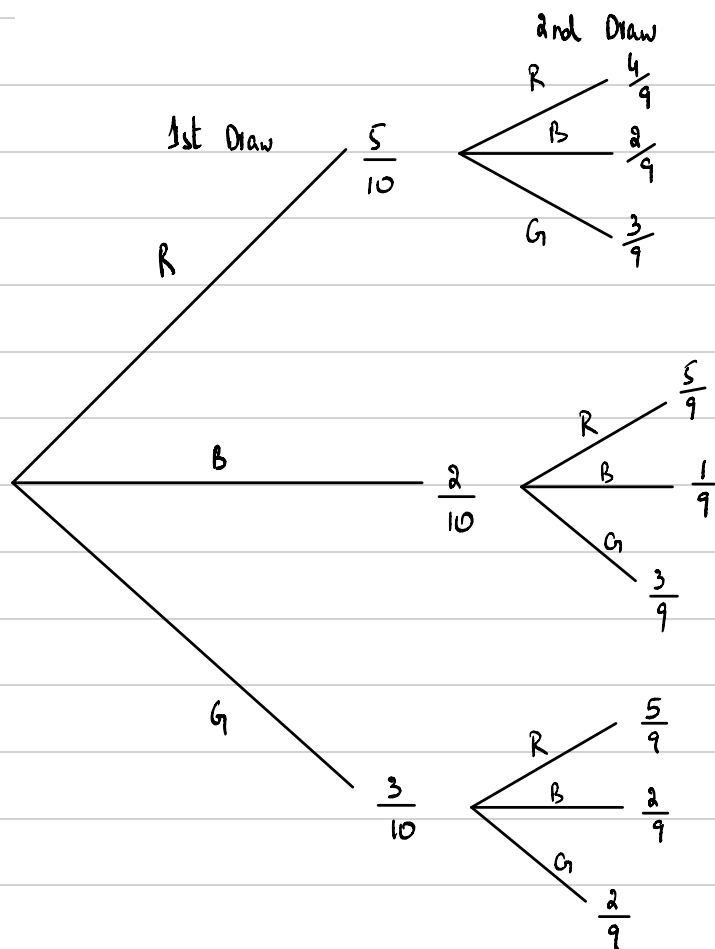
+	1	2	3	4	5	6	
1	2	3	4	5	6	7	a. $\frac{7}{36}$
2	3	4	5	6	7	8	
3	4	5	6	7	8	9	
4	5	6	7	8	9	10	b. $\frac{15}{36}$
5	6	7	8	9	10	11	
6	7	8	9	10	11	12	

c

-	1	2	3	4	5	6	
1						✓	c. $\frac{2}{36}$
2							
3							
4							
5							
6	✓						

Probability applied to without replacement events using Tree Diagram

1. A bag contains 5 red, 2 blue and 3 green balls. Two balls are chosen at random without replacement. Find the probability that
 - a. The 1st ball is blue
 - b. Both balls are red
 - c. 1st is red and 2nd is green
 - d. One is red the other is green
 - e. Both are of the same colour
 - f. Both are of different colours



a. $\frac{2}{10} = \frac{1}{5}$

b. $P(R \& R)$

$$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$$

c. $P(\text{Red \& Green})$

$$\frac{5}{10} \times \frac{3}{9} = \frac{15}{90} = \frac{1}{6}$$

d. $P(RG) \text{ or } P(GR)$

$$\left(\frac{5}{10} \times \frac{3}{9} \right) + \left(\frac{3}{10} \times \frac{5}{9} \right)$$

$$\frac{15}{90} + \frac{15}{90} = \frac{30}{90} = \frac{1}{3}$$

e. $P(RR) + P(BB) + P(GG)$

$$\left(\frac{5}{10} \times \frac{4}{9} \right) + \left(\frac{2}{10} \times \frac{1}{9} \right) + \left(\frac{3}{10} \times \frac{2}{9} \right)$$

$$\frac{20}{90} + \frac{2}{90} + \frac{6}{90}$$

$$\frac{28}{90} = \frac{14}{45}$$

f. RB, BR, GB, BG, RG, GR

or

$$P(\text{Different}) = 1 - P(\text{same})$$

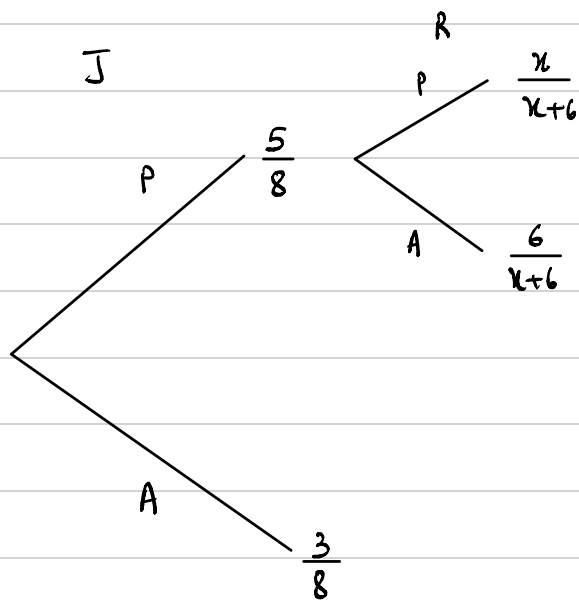
$$1 - \frac{14}{45} = \frac{31}{45}$$

$\frac{31}{45}$

Past Paper Questions

①

- 2 Jameel has 5 plums and 3 apricots in a box. Rosa has x plums and 6 apricots in a box. One fruit is chosen at random from Jameel's box and one fruit is chosen at random from Rosa's box. The probability that both fruits chosen are plums is $\frac{1}{4}$. Write down an equation in x and hence find x . [3]



$$P(J_P \text{ \& } R_P) = \frac{1}{4}$$

$$\frac{5}{8} \times \frac{x}{x+6} = \frac{1}{4}$$

$$\frac{5x}{2x+12} = 1$$

$$5x = 2x + 12$$

$$3x = 12$$

$$x = 4$$

2

- 1 A biased die has faces numbered 1 to 6. The probabilities of the die landing on 1, 3 or 5 are each equal to 0.1. The probabilities of the die landing on 2 or 4 are each equal to 0.2. The die is thrown twice. Find the probability that the sum of the numbers it lands on is 9. [4]

$$\begin{aligned} P(1) &= 0.1 \\ P(2) &= 0.2 \\ P(3) &= 0.1 \\ P(4) &= 0.2 \\ P(5) &= 0.1 \\ P(6) &= 0.3 \end{aligned}$$

0.7

+	1	2	3	4	5	6
1						
2						
3						9
4					9	
5				9		
6			9			

$$P(3 \text{ \& } 6) + P(4 \text{ \& } 5) + P(5 \text{ \& } 4) + P(6 \text{ \& } 3)$$

$$0.1 \times 0.3 + 0.2 \times 0.1 + 0.1 \times 0.2 + 0.3 \times 0.1$$

$$0.03 + 0.02 + 0.02 + 0.03 = 0.10$$

3

A bag contains 5 green balls and 3 yellow balls. Ronnie and Julie play a game in which they take turns to draw a ball from the bag at random without replacement. The winner of the game is the first person to draw a yellow ball. Julie draws the first ball. Find the probability that Ronnie wins the game. [4]

1	2	
J	R	OR
G	Y	

1	2	3	4	
J	R	J	R	OR
G	G	G	Y	

1	2	3	4	5	6
J	R	J	R	J	R
G	G	G	G	G	Y

$$\left(\frac{5}{8} \times \frac{3}{7}\right) + \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}\right) + \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}\right)$$

$$\frac{15}{56} + \frac{3}{28} + \frac{1}{56}$$

$\frac{11}{28}$

Conditional Probability: It means to find the number of favourable outcomes under a specific condition

$$\frac{\text{No of favourable outcomes with the given condition}}{\text{Probability of the given condition.}}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \longrightarrow \frac{\text{Probability of A \& B occurring together}}{\text{Probability of B}}$$

Find that probability that the game is delayed given that it rains

A

B

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{Delayed \& it rains})}{P(\text{Rains})}$$

Find the probability that the student is a male give that the student is left handed.

A

B

$$P(A|B) = \frac{P(\text{Student is male \& left handed})}{P(\text{Left handed})}$$

Find the probability that Ali travels by car given that he is not late

A

B

$$P(A|B) = \frac{P(\text{Ali travels by car \& not late})}{P(\text{Not late})}$$

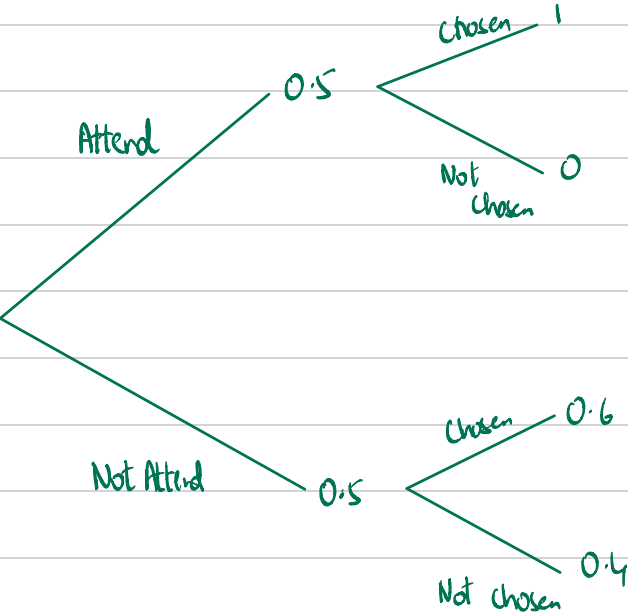
Examples

1

2 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]



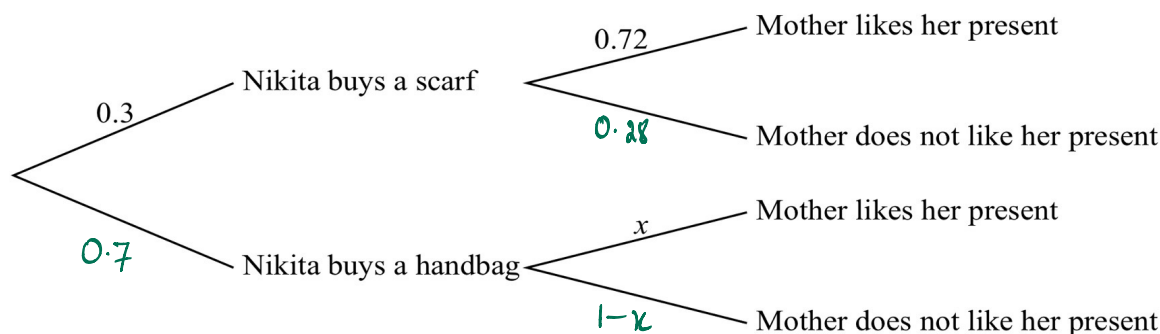
i $P(A \cup C) + P(N.A \cup C)$

$(0.5 \times 1) + (0.5 \times 0.6)$

0.8

ii $P(A|B) = \frac{P(\text{Attended} \cap \text{Chosen})}{P(\text{Chosen})}$

$\frac{0.5 \times 1}{0.8} = \frac{5}{8}$



Nikita goes shopping to buy a birthday present for her mother. She buys either a scarf, with probability 0.3, or a handbag. The probability that her mother will like the choice of scarf is 0.72. The probability that her mother will like the choice of handbag is x . This information is shown on the tree diagram. The probability that Nikita's mother likes the present that Nikita buys is 0.783.

(i) Find x .

[3]

(ii) Given that Nikita's mother does not like her present, find the probability that the present is a scarf.

[4]

$$(0.3 \times 0.72) + (0.7 \times x) = 0.783$$

$$x = 0.81$$

$$P(A|B) = \frac{P(\text{Scarf \& Not like})}{P(\text{Not like})}$$

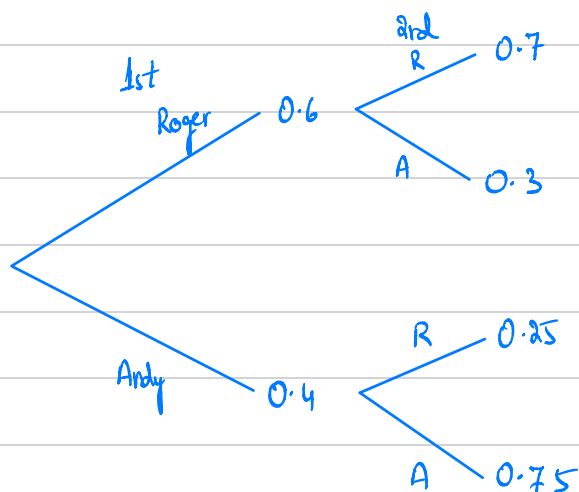
$$\frac{0.3 \times 0.28}{1 - 0.783} = \frac{12}{31}$$

$$0.217$$

- 3 Roger and Andy play a tennis match in which the first person to win two sets wins the match. The probability that Roger wins the first set is 0.6. For sets after the first, the probability that Roger wins the set is 0.7 if he won the previous set, and is 0.25 if he lost the previous set. No set is drawn.

(i) Find the probability that there is a winner of the match after exactly two sets. [3]

(ii) Find the probability that Andy wins the match given that there is a winner of the match after exactly two sets. [2]



$$i \quad \begin{array}{ccccc} \text{1st} & \text{2nd} & \text{or} & \text{1st} & \text{2nd} \\ R_w & R_w & & A_w & A_w \\ (0.6 \times 0.7) & & + & (0.4 \times 0.75) & \end{array}$$

$$\boxed{0.72}$$

$$ii \quad P(A|B) = \frac{P(\text{Andy wins after two sets})}{P(\text{winner after two sets})}$$

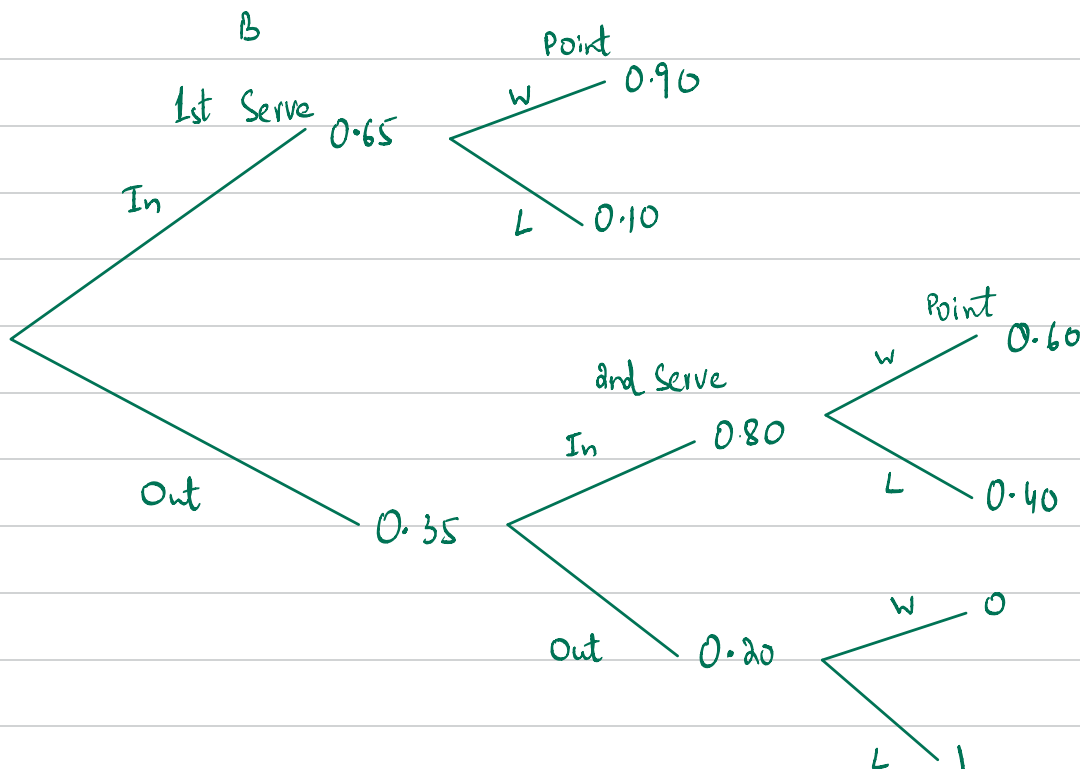
$$\frac{0.4 \times 0.75}{0.72} = \boxed{\frac{5}{12}}$$

- 6 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]



$$\text{ii } (0.65 \times 0.10) + (0.35 \times 0.80 \times 0.40) + (0.35 \times 0.20 \times 1) = \boxed{0.247}$$

$$\text{iii } P(A|B) = \frac{P(\text{First serve is correct \& loses the point})}{P(\text{loses the point})}$$

$$\frac{0.65 \times 0.10}{0.247} = \boxed{0.263}$$

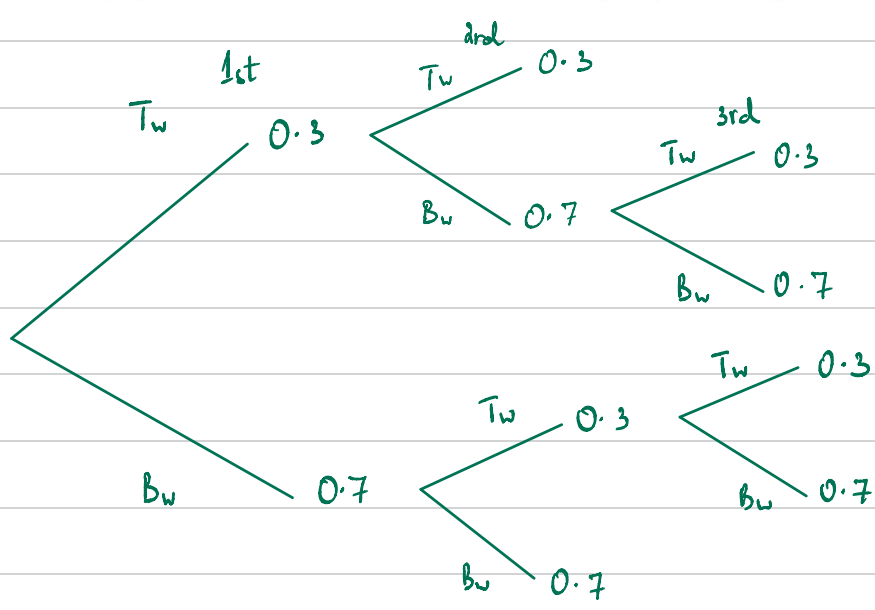
5

6 Tom and Ben play a game repeatedly. The probability that Tom wins any game is 0.3. Each game is won by either Tom or Ben. Tom and Ben stop playing when one of them (to be called the champion) has won two games.

(i) Find the probability that Ben becomes the champion after playing exactly 2 games. [1]

(ii) Find the probability that Ben becomes the champion. [3]

(iii) Given that Tom becomes the champion, find the probability that he won the 2nd game. [4]



i $0.7 \times 0.7 = 0.49$

ii

1	2	or	1	2	3	or	1	2	3
Bw	Bw		Bw	Tw	Bw		Tw	Bw	Bw

$0.49 + 0.7 \times 0.3 \times 0.7 + 0.3 \times 0.7 \times 0.7$

0.784

iii $P(A|B) = \frac{P(\text{Tom wins the second game \& is champion})}{P(\text{Tom is champ})}$

$$\frac{(0.3 \times 0.3) + (0.7 \times 0.3 \times 0.3)}{1 - 0.784} = \frac{17}{24}$$

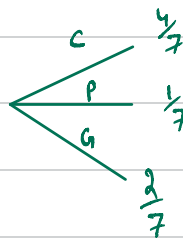
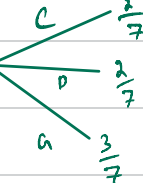
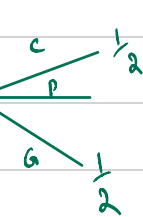
6

- 6 The people living in 3 houses are classified as children (C), parents (P) or grandparents (G). The numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
4C, 1P, 2G	2C, 2P, 3G	1C, 1G

- (i) All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent. [2]
- (ii) A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent. [3]
- (iii) Given that the person chosen by the method in part (ii) is a grandparent, calculate the probability that there is also a parent living in the house. [4]

A

 H_1 $\frac{1}{3}$  H_2 $\frac{1}{3}$  H_3 $\frac{1}{3}$ 

$$\left(\frac{1}{3} \times \frac{2}{7}\right) + \left(\frac{1}{3} \times \frac{3}{7}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right) = \frac{17}{42}$$

iii

$$P(A | B)$$

$$= \frac{P(\text{Parent living if G.P. is chosen})}{P(\text{G.P. is chosen})}$$

$$\left(\frac{1}{3} \times \frac{2}{7}\right) + \left(\frac{1}{3} \times \frac{3}{7}\right)$$

$$\frac{17}{42}$$

$$= \frac{10}{17}$$

Probability with multiple arrangements

Q. A fair coin is tossed five times. Find the probability that

i. Head is obtained five times

$$\frac{H}{\frac{1}{2}} \quad \frac{H}{\frac{1}{2}} \quad \frac{H}{\frac{1}{2}} \quad \frac{H}{\frac{1}{2}} \quad \frac{H}{\frac{1}{2}} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

ii. Head is obtained first three times only

$$\frac{H}{\frac{1}{2}} \quad \frac{H}{\frac{1}{2}} \quad \frac{H}{\frac{1}{2}} \quad \frac{T}{\frac{1}{2}} \quad \frac{T}{\frac{1}{2}} = \frac{1}{32}$$

iii. Head is obtained a total of three times

$$\begin{array}{ccccc} \frac{H}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} \\ \frac{H}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} \\ \frac{T}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} \\ \frac{H}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} & \frac{T}{\frac{1}{2}} & \frac{H}{\frac{1}{2}} \\ & \vdots & & & \end{array} \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times \frac{5!}{3! \times 2!}$$

Note: When a probability condition can be fulfilled multiple ways, we find the probability of one way and multiply it by the number of possible ways to fulfill that condition

Number of arrangements of n unique objects/letters

① ALI

ALI
ILA
IAL
LAI
LIA
AIL

$$3! = 3 \times 2 \times 1 = 6$$

② ALIZA

$$\frac{5!}{2!} = 60$$

③ MASHHOOD

$$= \frac{8!}{2! \cdot 2!} = 10,080$$

$\downarrow \quad \downarrow$
H OO

- 5 A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

(i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$. [1]

(ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time. [3]

i R B G
 $4x + x + 3x = 1$

$$8x = 1$$

$$x = \frac{1}{8}$$

$$P(\text{Blue}) = \frac{1}{8}$$

ii $P(R) = \frac{1}{2}$ $P(G) = \frac{3}{8}$ $P(B) = \frac{1}{8}$

$$\underline{R} \quad \underline{B} \quad \underline{G} : \left(\frac{1}{2} \times \frac{1}{8} \times \frac{3}{8} \right) 3!$$

$$\boxed{\frac{9}{64}}$$

Independent Events

The two events are said to be independent, when the outcome of the first does not change the outcome of the second event

Examples

1. Rolling a die and tossing a coin
2. With replacement

Independence Test

If $P(A) \times P(B) = P(A \cap B)$ then A & B are independent
↳ Both occurring together

If $P(A) \times P(B) \neq P(A \cap B)$ then A & B are NOT independent
↳ Both occurring together

Mutually exclusive

If two events cannot occur together, they are considered mutually exclusive

If $P(A \cap B) = 0$ Mutually exclusive

If $P(A \cap B) \neq 0$ Not Mutually exclusive.

Examples

1

- 3 A fair six-sided die is thrown twice and the scores are noted. Event X is defined as 'The total of the two scores is 4'. Event Y is defined as 'The first score is 2 or 5'. Are events X and Y independent? Justify your answer. [4]

+	1st	2nd	1	2	3	4	5	6
1	2	3	4	5	6	7		
2	3	4	5	6	7	8		
3	4	5	6	7	8	9		
4	5	6	7	8	9	10		
5	6	7	8	9	10	11		
6	7	8	9	10	11	12		

$$X - P(X) = \frac{3}{36}$$

$$Y - P(Y) = \frac{12}{36}$$

$$P(X \cap Y) = \frac{1}{36}$$

$$P(X) \times P(Y) = P(X \cap Y) \quad ?$$

$$\frac{1}{36} \times \frac{12}{36} = \frac{1}{36}$$

$$\frac{1}{36} = \frac{1}{36}$$

$\therefore$ They are independent

- 3 Jason throws two fair dice, each with faces numbered 1 to 6. Event A is 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B is 'the product of the two numbers obtained is even'.

(i) Determine whether events A and B are independent, showing your working. [5]

(ii) Are events A and B mutually exclusive? Justify your answer. [1]

i

	1	2	3	4	5	6
1		x	•	x		x
2	x	x	x	x	x	x
3	•	•		x	•	x
4	x	x	x	x	x	x
5		x	•	x		x
6	x	x	x	x	x	x

$$P(A) \cdot = \frac{16}{36}$$

$$P(B) \times = \frac{27}{36}$$

$$P(A \cap B) = \frac{12}{36}$$

$$\frac{16}{36} \times \frac{27}{36} = \frac{12}{36} \quad ?$$

$$\frac{1}{3} = \frac{1}{3} \quad \therefore A \text{ \& B are independent}$$

ii No because $P(A \cap B) \neq 0$

- 5 Marco has four boxes labelled K , L , M and N . He places them in a straight line in the order K , L , M , N with K on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows.

A : The white marble is in either box L or box M .

B : The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent.

[3]

(A)

$$\begin{array}{cccc} & W & & \\ \hline K & L & M & N \end{array} \quad \text{or} \quad \begin{array}{cccc} & & & W \\ \hline K & L & M & N \end{array}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \quad P(A) = \frac{1}{2}$$

(B)

$$\begin{array}{cccc} & & 3! & \\ \hline R & G & Y & W \\ K & L & M & N \end{array} \quad 6$$

$$\begin{array}{cccc} W & R & G & Y \\ \hline & & 2! & \\ & & 2 & \\ & & 8 & \end{array}$$

$$\text{Total} = 4! = 24$$

$$P(B) = \frac{8}{24}$$

$P(A \cap B)$

$$\begin{array}{cccc} & & 2! & \\ \hline R & W & G & Y \\ K & L & M & N \end{array} \quad 2$$

$$\begin{array}{cccc} & & 2! & \\ \hline R & G & W & Y \\ K & L & M & N \end{array} \quad 2$$

$$\frac{1}{2} \times \frac{8}{24} = \frac{4}{24}$$

$$\frac{1}{6} = \frac{1}{6} \therefore \text{Independent}$$