

Applications of Integration – Formula Sheet:

Area under the Curve: $A = \sum_{i=1}^n f(x_i) \Delta x \quad \Delta x = \frac{b-a}{n}$ $A = \int_a^b f(x) dx \quad \text{Interval} \rightarrow [a, b]$	Antiderivatives: $F'(x) = f(x)$ $\int f(x) dx = F(x) + C$
Summation Formulas: $\sum_{i=1}^n c = cn \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}$ $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$	Rectilinear Motion: $v(t) = s'(t)$ $a(t) = v'(t)$ $\int v(t) dt = s(t) + C$ $\int a(t) dt = v(t) + C$
Summation Formulas: $\sum_{i=1}^n i^4 = \frac{6n^5 + 15n^4 + 10n^3 - n}{30}$	Definition of the Definite Integral: $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$
Summation Formulas: $\sum_{i=1}^n i^5 = \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}$	Evaluating Definite Integrals: $\int_a^b f(x) dx = F(b) - F(a)$
Area – Riemann Sums: (Left, Right, & Midpoint) $A_L = \Delta x [f(x_0) + f(x_1) + f(x_2) + \dots + f(x_{n-1})]$ $A_M = \Delta x [f(x_{0.5}) + f(x_{1.5}) + f(x_{2.5}) + \dots + f(x_{n-0.5})]$ $A_R = \Delta x [f(x_1) + f(x_2) + f(x_3) + \dots + f(x_n)]$ $\Delta x = \frac{b-a}{n}$	Properties of Definite Integrals: $\int_a^b f(x) dx = - \int_b^a f(x) dx$ $\int_a^a f(x) dx = 0 \quad \int_a^b c dx = c(b-a)$ $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

Area – Trapezoidal Rule: (Approximate Integration)

$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$Area = \int_a^b f(x) dx \approx T_n \quad \Delta x = \frac{b-a}{n} \quad x_i = a + i\Delta x$$

Area – Simpson's Rule: (Approximate Integration)

$$S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$n \rightarrow \text{even} \quad \Delta x = \frac{b-a}{n}$$

Error Bounds – Trapezoidal & Midpoint: $|f''(x)| \leq K$

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Error Bounds – Simpson's Rule:

$$|E_S| \leq \frac{K(b-a)^5}{180n^4} \quad |f^{(4)}(x)| \leq K \text{ on } [a, b]$$

Integral of Even Functions:

$$f(-x) = f(x)$$

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

Integral of Odd Functions:

$$f(-x) = -f(x) \quad \int_{-a}^a f(x) dx = 0$$

Fundamental Theorem of Calculus – Part 1:

$$g(x) = \int_a^x f(t) dt \quad g'(x) = f(x)$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

If $f(x)$ is continuous on the interval $[a, b]$, then $g(x)$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) .

Natural Log defined as an integral:

$$\ln(x) = \int_1^x \frac{1}{t} dt \quad x > 0$$

Fundamental Theorem of Calculus – Part 2:

$$\int_a^b f(x) dx = F(b) - F(a)$$

U-Substitution:

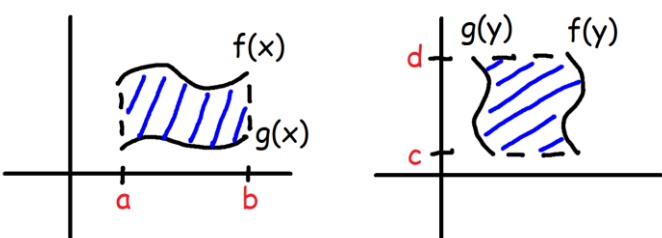
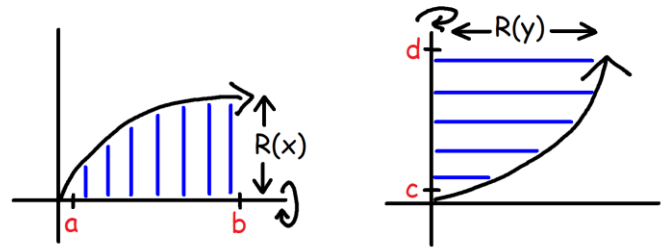
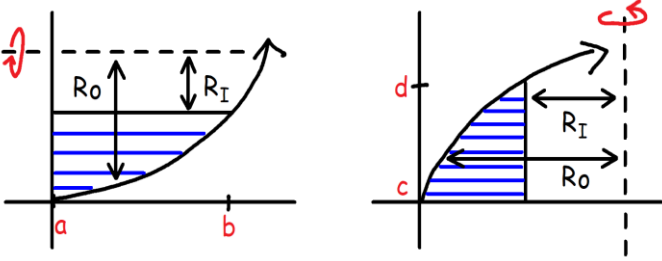
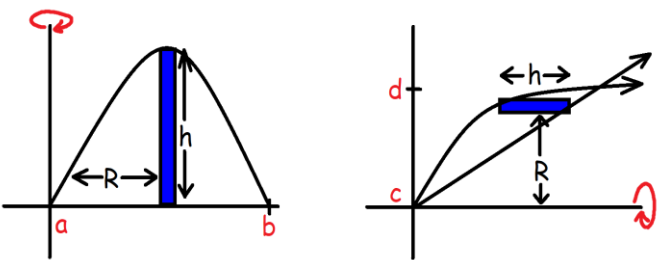
$$\int f[g(x)] \cdot g'(x) dx = \int f(u) du \quad u = g(x)$$

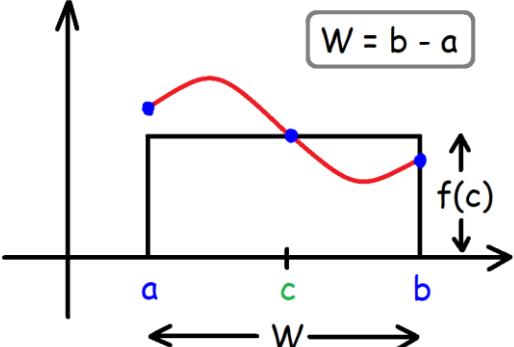
$$\int_a^b f[g(x)] g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

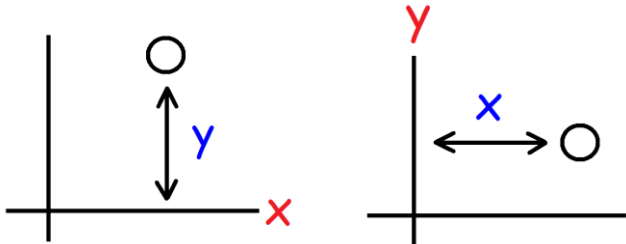
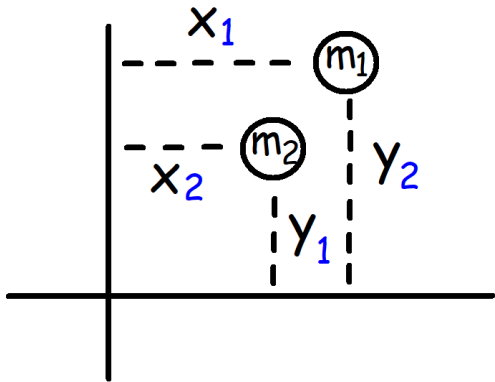
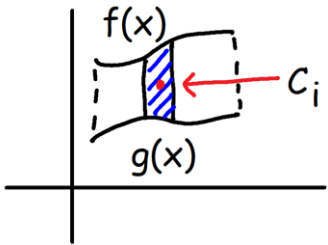
Net Change Theorem:

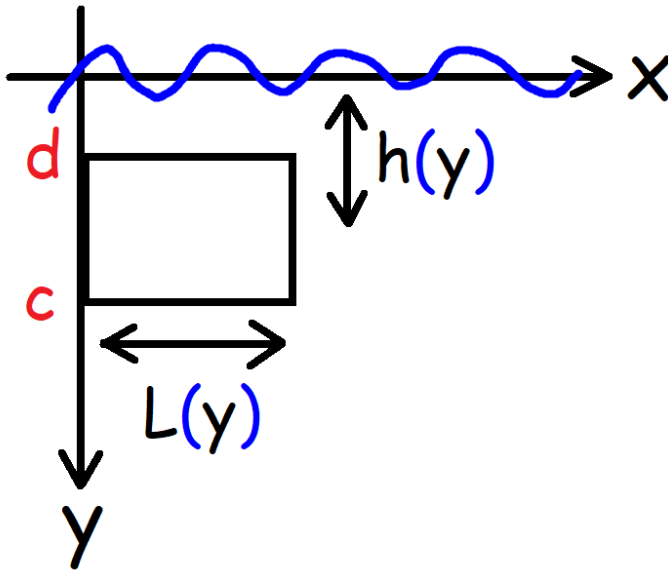
$$\int_a^b F'(x) dx = F(b) - F(a)$$

$$\int_a^b V'(t) dt = V(b) - V(a)$$

Area Between Curves: 	Area Between Curves: $A = \int_a^b [f(x) - g(x)] dx \quad (\text{top} - \text{bottom})$ $A = \int_c^d [f(y) - g(y)] dy \quad (\text{right} - \text{left})$
Disk Method: 	Disk Method: $V = \pi \int_a^b R^2(x) dx$ $V = \pi \int_c^d R^2(y) dy$
Washer Method: 	Washer Method: $V = \pi \int_a^b [R_o^2(x) - R_i^2(x)] dx$ $V = \pi \int_c^d [R_o^2(y) - R_i^2(y)] dy$
Shell Method: 	Shell Method: $V = 2\pi \int_a^b R(x) h(x) dx$ $V = 2\pi \int_c^d R(y) h(y) dy$
Improper Integrals: $\int_a^\infty f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$ $\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$	Volume by Cross Sections: $V = \int_a^b A(x) dx \quad cs \perp x - axis$ $V = \int_c^d A(y) dy \quad cs \perp y - axis$

Work done by a Force: $W = Fd \quad W = \int_a^b F(x) dx$ Note: F(x) is a function of force with respect to position.	Arc Length: $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx \quad L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ $L = \int_c^d \sqrt{1 + [g'(y)]^2} dy \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
Gravitational Force: $F = mg \quad F = \frac{GM_1M_2}{R^2}$ Restoring Force of Springs – Hooke's Law: $F(x) = -kx$	Area of a Surface of Revolution: $S = 2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} dx$ $S = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
Work required to pump water out of a tank: $W = pg \int_a^b V(x) D(x) dx$ Density of Water: $\rho_{H_2O} = 62.5 \text{ lbs/ft}^3 = 1000 \text{ kg/m}^3$	Area of a Surface of Revolution: $S = 2\pi \int_c^d g(y) \sqrt{1 + [g'(y)]^2} dy$ $S = 2\pi \int_c^d x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
Work done by an expanding gas: $W = \int_{V_1}^{V_2} P dV$	Average Value of a function: $f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$
Mean Value Theorem for Integrals: 	Mean Value Theorem for Integrals: $\int_a^b f(x) dx = f(c)(b-a)$ $A_{curve} = A_{rectangle}$ $Width = b - a \quad Height = f(c)$ $f(c) = f_{ave}$

	Moment around the x-axis: $M_x = my$ Moment around the y-axis: $M_y = mx$
Center of Mass: 	Coordinates of the Center of Mass: $(\bar{x}, \bar{y})$ $\bar{x} = \frac{M_y}{m_T} \quad \bar{y} = \frac{M_x}{m_T}$ $M_y = \sum_{i=1}^n m_i x_i \quad M_x = \sum_{i=1}^n m_i y_i$ $\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \bar{y} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$
Centroid: $C_i(\bar{x}_i, \frac{1}{2}[f(\bar{x}_i) + g(\bar{x}_i)])$ 	Coordinates of the Center of Mass: $(\bar{x}, \bar{y})$ $\bar{x} = \frac{1}{A} \int_a^b x[f(x) - g(x)] dx$ $\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2}[f^2(x) - g^2(x)] dx$
Mass of the Plate: ($p \rightarrow$ surface density) $m = pA = p \int_a^b [f(x) - g(x)] dx$ Area of the Plate / Laminar: $A = \int_a^b [f(x) - g(x)] dx$ Surface Density: $p = \frac{\text{mass}}{\text{Area}} \quad (\text{kg/m}^2)$	Moment of the system around the x-axis: $M_x = p \int_a^b \frac{1}{2}[f^2(x) - g^2(x)] dx$ Moment of the system around the y-axis: $M_y = p \int_a^b x[f(x) - g(x)] dx$ Linear Density: (Not used in the formulas above) $p = \frac{\text{mass}}{\text{length}} \quad (\text{kg/m})$

Hydrostatic Force:**Hydrostatic Force:**

$$F = W \int_c^d h(y) L(y) dy$$

Weight Density:

$$W = \frac{\text{weight force}}{\text{volume}} = \frac{mg}{V} = \left(\frac{m}{V}\right) g = \rho g$$

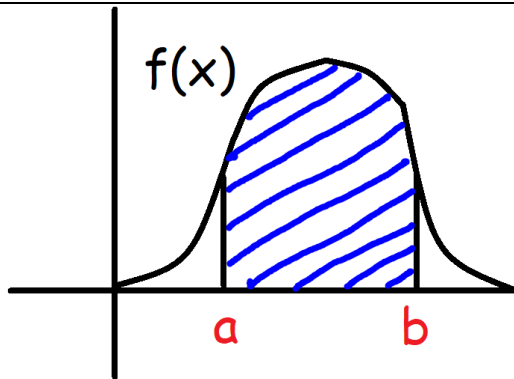
$$W_{H_2O} = 9800 \text{ N/m}^3 = 62.4 \text{ lbs/ft}^3$$

Normal Density:

$$\rho = \frac{\text{mass}}{\text{volume}} \quad (\text{kg/m}^3)$$

$$\rho_{H_2O} = 1000 \text{ kg/m}^3$$

Note: $L(y) \rightarrow \text{Length}$ $h(y) \rightarrow \text{depth}$

**Probability Density Functions:**

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$P(a \leq x \leq b) = \int_a^b f(x) dx$$

Probability Density Functions:**The Mean:**

$$\mu = \int_{-\infty}^{\infty} xf(x) dx$$

The Median:

$$\int_m^{\infty} f(x) dx = \frac{1}{2}$$

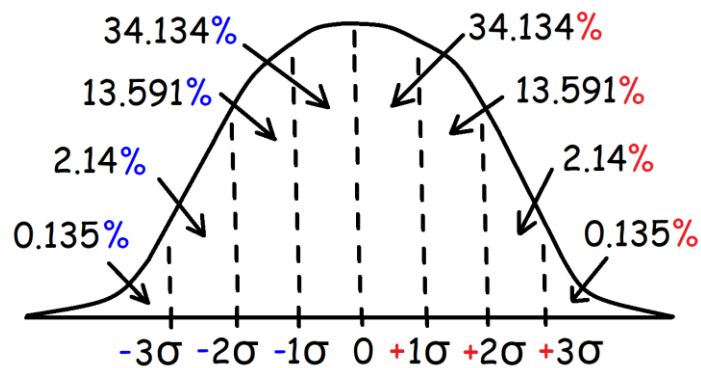
Probability of Exponential Distributions:

$$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{u} e^{-t/u} & t \geq 0 \end{cases}$$

$$P(t > a) = \int_a^{\infty} \frac{1}{u} e^{-t/u} dt$$

$$P(a \leq t \leq b) = \int_a^b \frac{1}{u} e^{-t/u} dt$$

Probability of a Normal Distribution:



Probability of a Normal Distribution:

$$f(x) = \frac{e^{-(x-u)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

$$\int_{-\infty}^{\infty} \frac{e^{-(x-u)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} dx = 1$$

$$P(a \leq x \leq b) = \int_a^b \frac{e^{-(x-u)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} dx$$