

A LEVEL PHYSICS

DERIVATIONS

**40 of the most important
derivations for A Level Physics**



Derivations | Introduction

Thank you for downloading this guide, I really hope you find it useful!

I have brief derivations for forty of the most important equations that you may come across during your A Level Physics course. Some of these are listed in the specification, many have come up in previous exam questions and the others just help you understand the underlying physics.

Derivation | Period of a Pendulum

HOW TO DERIVE:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Worksheet 1

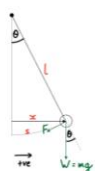
Worksheet 2

Solution

Don't forget that I also have short videos where I explain each of these derivations in a little more detail at [ALevelPhysicsOnline.com](https://www.ALevelPhysicsOnline.com). There are also worksheets (with varying levels of support) you can use when you're revising these.

Derivation | Period of a Pendulum

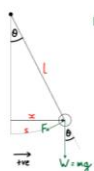
Derive the expression for time period for a pendulum undergoing SHM: $T = 2\pi \sqrt{\frac{l}{g}}$



1

Derivation | Period of a Pendulum

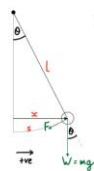
Derive the expression for time period for a pendulum undergoing SHM: $T = 2\pi \sqrt{\frac{l}{g}}$



2

Derivation | Period of a Pendulum

Derive the expression for time period for a pendulum undergoing SHM: $T = 2\pi \sqrt{\frac{l}{g}}$



$F = -mg \sin \theta$ $F = ma$
 $-mg \sin \theta = ma$
 $-g \sin \theta = a$
 $\sin \theta = \theta$ or θ is small
 $-g \theta = a$
 $\theta = \frac{s}{l}$ $s = x$ $\theta = \frac{x}{l}$
 $a = -\left(\frac{g}{l}\right)x$ $\frac{a}{x} = -\frac{g}{l}$
 $\frac{a}{x} = -\frac{4\pi^2}{T^2}x$ $\frac{g}{l} = \frac{4\pi^2}{T^2}$
 $T^2 = \frac{4\pi^2 l}{g}$ $T = 2\pi \sqrt{\frac{l}{g}}$

While you're on the website, don't forget that you can sign up for a **Premium Plan** (or your teachers can buy a **School Subscription**) to access hundreds more videos and resources.

There is also a workbook called the **Daily Workout** with short questions you can complete every single day – available to buy on Amazon.



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Derivation | Electrical Power

Derive the series of expressions linking power, current, potential difference and resistance:

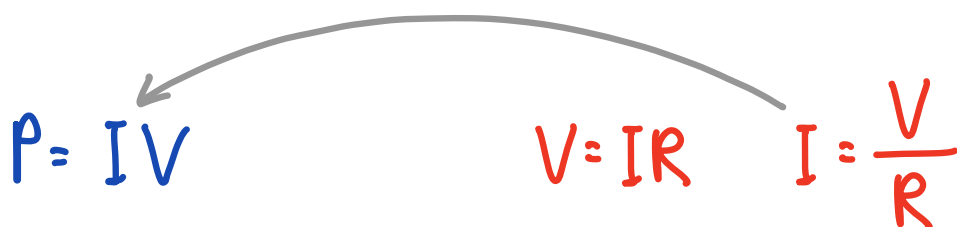
$$P = I^2 R$$

$$P = V^2 / R$$


$$P = IV \quad V = IR$$

$$P = I IR$$

$$P = I^2 R$$


$$P = IV \quad V = IR \quad I = \frac{V}{R}$$

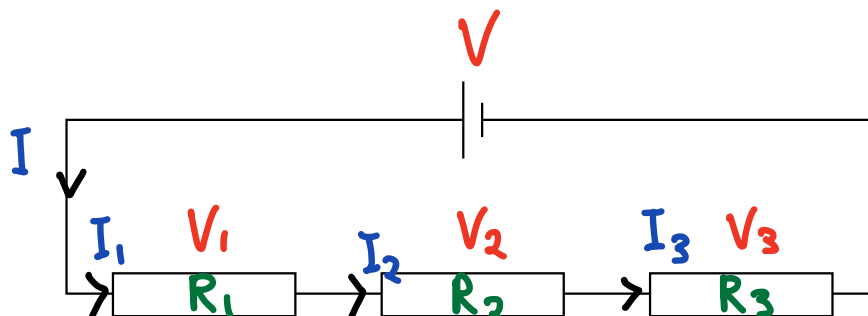
$$P = \frac{V V}{R}$$

$$P = \frac{V^2}{R}$$

Derivation | Resistors in Series

Derive the expression for calculating the total resistance of resistors in series:

$$R_T = R_1 + R_2 + \dots + R_n$$



$$V = V_1 + V_2 + V_3$$

$$V = I R_T \quad V_1 = I_1 R_1 \quad V_2 = I_2 R_2 \quad V_3 = I_3 R_3$$

$$I R_T = I_1 R_1 + I_2 R_2 + I_3 R_3$$

$$I = I_1 = I_2 = I_3$$

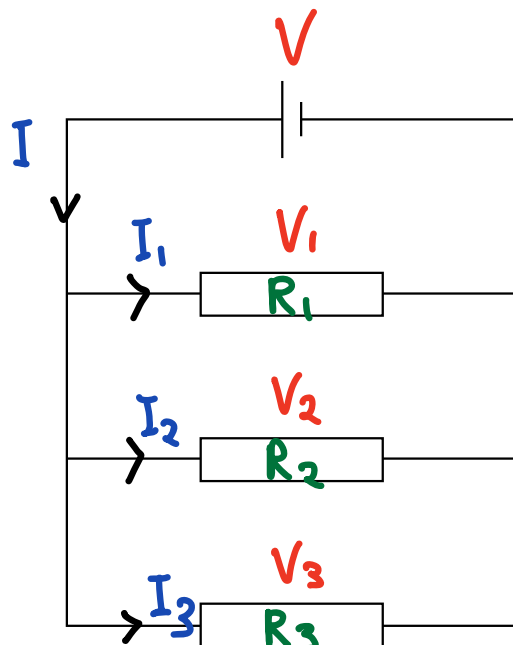
$$R_T = R_1 + R_2 + R_3$$

$$\therefore R_T = R_1 + R_2 + \dots + R_n$$

Derivation | Resistors in Parallel

Derive the expression for calculating the total resistance of resistors in parallel:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$



$$I = I_1 + I_2 + I_3$$
$$I = \frac{V}{R_T} \quad I_1 = \frac{V_1}{R_1} \quad I_2 = \frac{V_2}{R_2} \quad I_3 = \frac{V_3}{R_3}$$

$$\frac{V}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$V = V_1 = V_2 = V_3$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\therefore \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

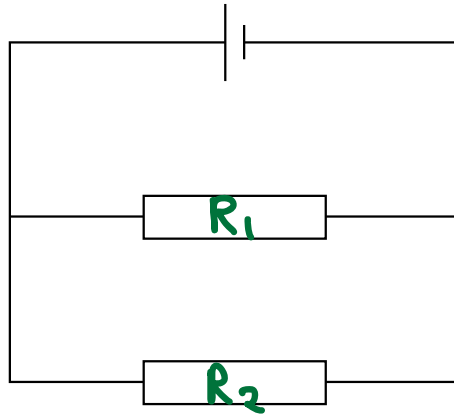
ALEvelPhysicsOnline.com/derivations



Derivation | Two Resistors in Parallel

Derive the expression for calculating the total resistance of resistors in parallel:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$\times R_T$

$$1 = \frac{R_T}{R_1} + \frac{R_T}{R_2}$$

$\times R_1$

$$R_1 = R_T + \frac{R_T R_1}{R_2}$$

$\times R_2$

$$R_1 R_2 = R_T R_2 + R_T R_1$$

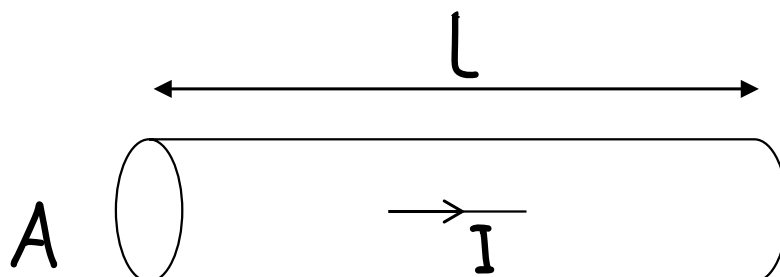
$$R_1 R_2 = R_T (R_1 + R_2)$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Derivation | Drift Velocity

Derive the expression for calculating the drift velocity of charge carriers:

$$I = Anev$$



n = no. of charge carriers per m^3

nV = total no. of charge carriers

nVe = total charge, $Q = nLAe$

$V = LA$

$v = \frac{L}{t}$ $L = vt$

$$Q = It$$

$$I = \frac{Q}{t} = \frac{nLAe}{t} = \frac{nvt \cancel{t} Ae}{\cancel{t}}$$

$$I = n v A e$$

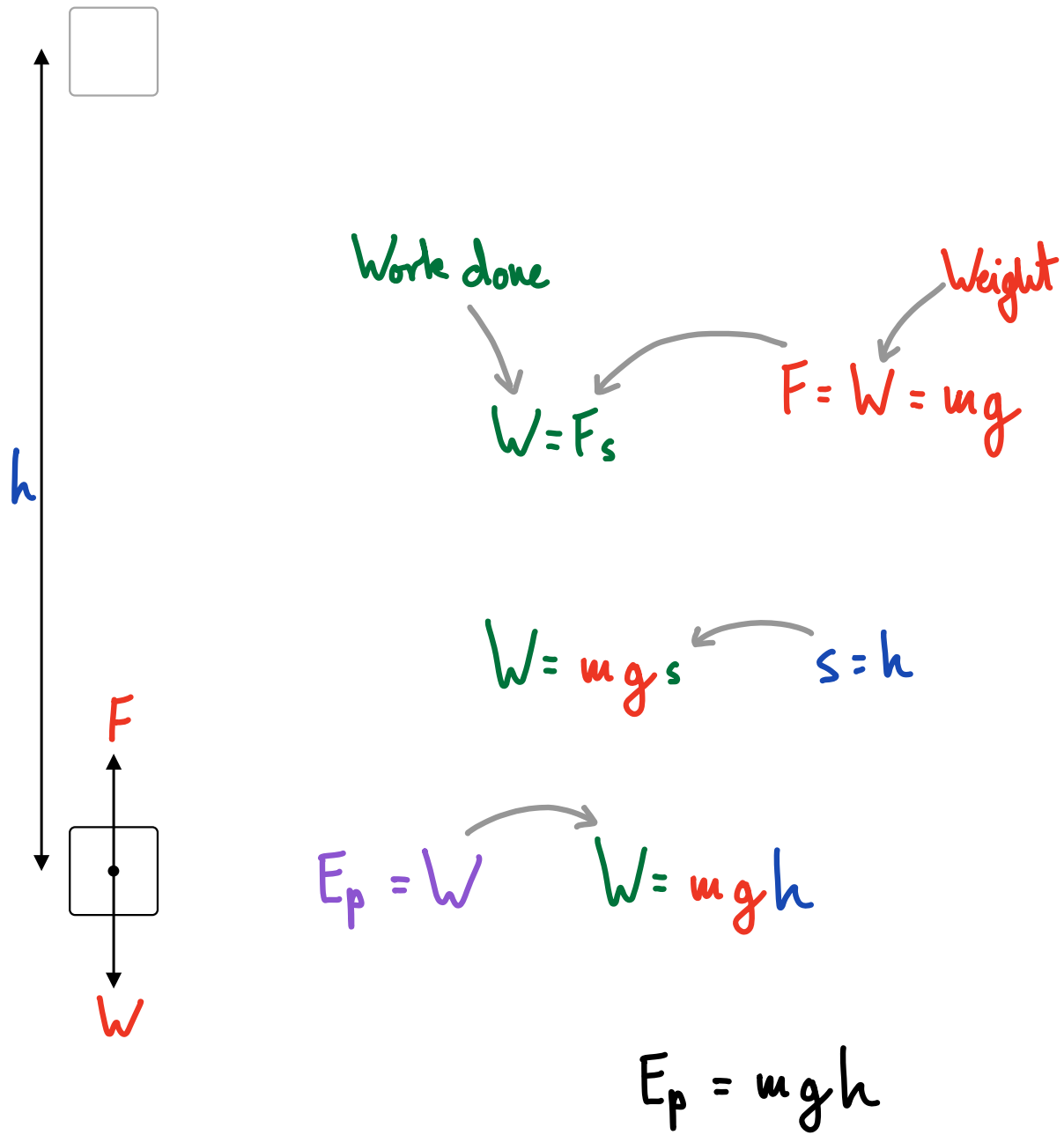
$$I = A n e v$$



Derivation | Potential Energy

Derive the expression for change in gravitational potential energy in a uniform field:

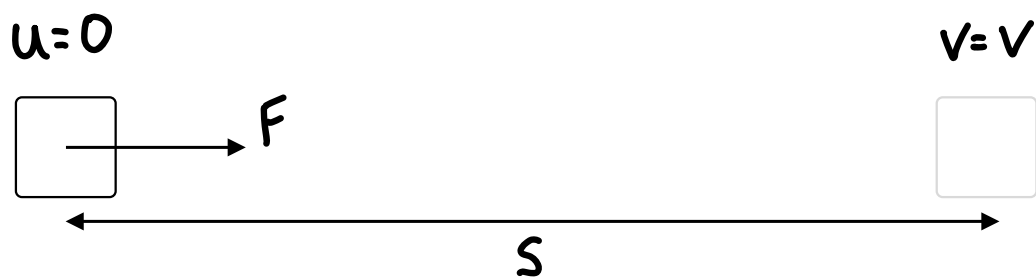
$$E_p = mgh$$



Derivation | Kinetic Energy

Derive the expression for kinetic energy:

$$E_k = \frac{1}{2}mv^2$$



$$W = Fs \quad F = ma \quad v^2 = \cancel{u^2} + 2as$$

$$v^2 = 2as$$

$$W = mas \quad as = \frac{v^2}{2}$$

$$W = E_k \quad W = m \frac{v^2}{2}$$

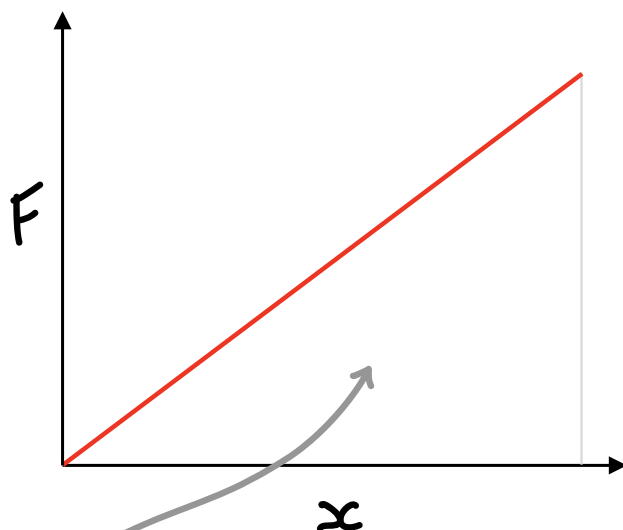
$$E_k = \frac{1}{2}mv^2$$



Derivation | Elastic Potential Energy

Derive the expression for elastic potential energy stored in a spring:

$$E_e = \frac{1}{2} kx^2$$



$$\text{Area} = \text{Energy stored} = \frac{1}{2} Fx$$

$$F = kx$$

$$E = \frac{1}{2} kx x$$

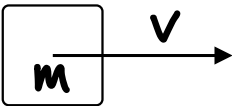
$$E = \frac{1}{2} kx^2$$



Derivation | Energy and Momentum

Derive the expression for kinetic energy in terms of momentum and mass:

$$E_k = \frac{p^2}{2m}$$



$$E_k = \frac{1}{2} m v^2$$

$$p = mv$$

$$v = \frac{p}{m}$$

$$v^2 = \frac{p^2}{m^2}$$

$$E_k = \frac{\cancel{m} p^2}{2 m^2}$$

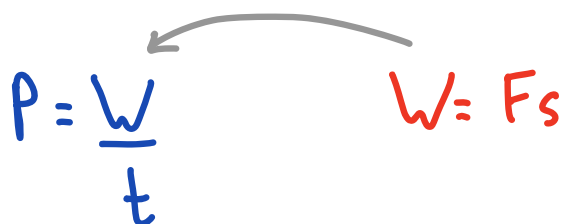
$$E_k = \frac{p^2}{2m}$$

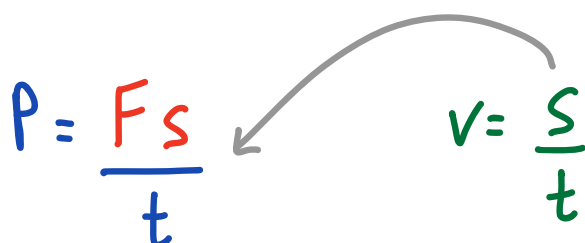


Derivation | Power, Force and Velocity

Derive the expression for power in terms of force and velocity:

$$P = Fv$$


$$P = \frac{W}{t} \qquad W = Fs$$


$$P = \frac{Fs}{t} \qquad v = \frac{s}{t}$$

$$P = Fv$$



Derivation | 'suvat' Equations

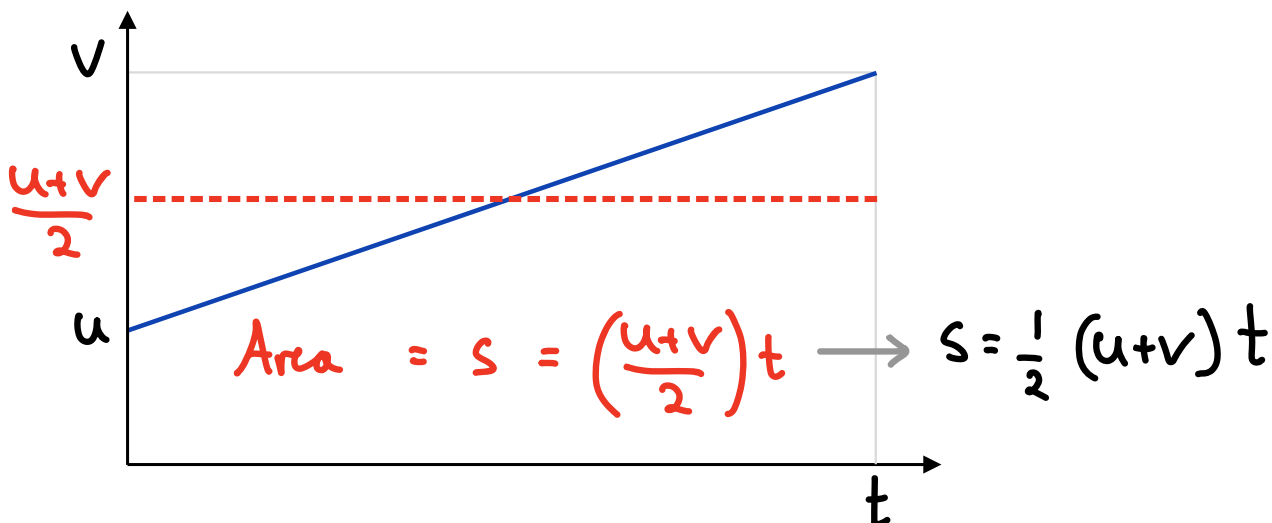
Derive the 'suvat' equations for an object undergoing uniform acceleration:

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$



$$\text{Gradient} = a = \frac{v-u}{t} \rightarrow at = v-u \rightarrow v = u + at$$

$$s = \left(\frac{u+v}{2}\right)t \quad v = u + at$$

$$s = \left(\frac{u+v}{2}\right)t \quad t = \left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{2u}{2}\right)t + \left(\frac{at}{2}\right)t$$

$$s = \frac{(u+v)(v-u)}{2a}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{\cancel{u}v + v^2 - \cancel{u}v - u^2}{2a} = \frac{v^2 - u^2}{2a}$$

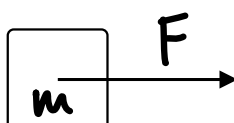
$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

Derivation | Newton's 2nd Law

Derive the expression to show how the resultant force is related to the rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t}$$



$$F = ma \quad a = \frac{v-u}{t}$$

$$F = m \left(\frac{v-u}{t} \right)$$

$$F = \frac{mv - mu}{t} \quad p = mv$$

$$F = \frac{\Delta p}{\Delta t}$$



Derivation | Upthrust

Derive the expression for the upthrust on an object submerged in a fluid:

$$U = \rho V g$$

Diagram illustrating the derivation of upthrust on a submerged object:

Handwritten notes and equations:

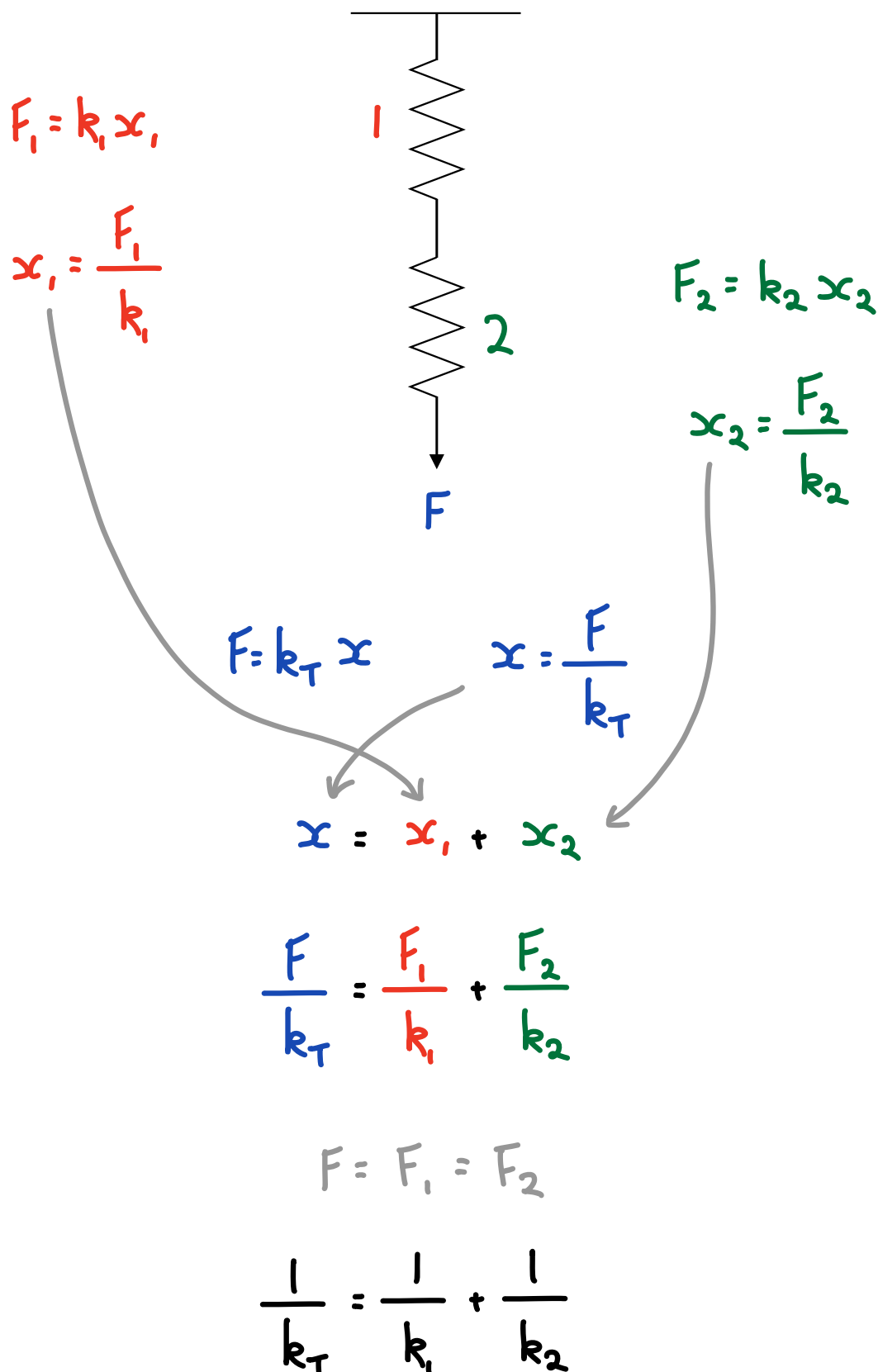
- $\rho = \frac{F}{A}$ (definition of pressure)
- $F = \rho A$ (force on area A)
- $\rho = \rho g h$ (pressure at depth h)
- $F = \rho g h A$ (force on top surface)
- $F_1 = \rho g h A$ (downward force on top surface)
- $F_2 = \rho g (h+x) A$ (upward force on bottom surface)
- $F_2 = \rho g h A + \rho g x A$ (expanded upward force)
- $W = F_2 - F_1$ (net upward force)
- $W = U$ (upthrust U equals weight W)
- $U = \cancel{\rho g h A} + \rho g x A - \cancel{\rho g h A}$ (simplified upthrust)
- $U = \rho g x A$ (upthrust in terms of depth x)
- $x A = V$ (volume of displaced fluid)
- $U = \rho g V$ (upthrust in terms of volume V)
- $U = \rho V g$ (final expression for upthrust)
- Weight = Upthrust



Derivation | Springs in Series

Derive the expression for calculating the combined stiffness of springs in series:

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}$$

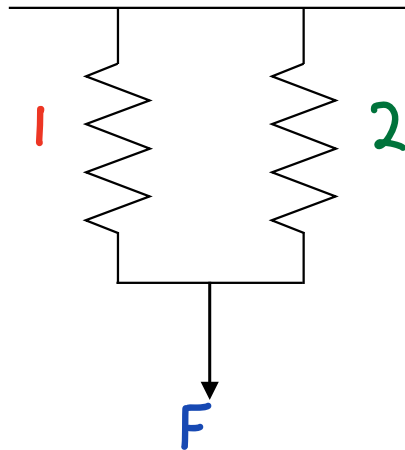




Derivation | Springs in Parallel

Derive the expression for calculating the combined stiffness of springs in parallel:

$$k_T = k_1 + k_2 + \dots + k_n$$



$$F_1 = k_1 x_1$$

$$F = k_T x$$

$$F_2 = k_2 x_2$$

$$F = F_1 + F_2$$

$$k_T x = k_1 x_1 + k_2 x_2$$

$$x = x_1 = x_2$$

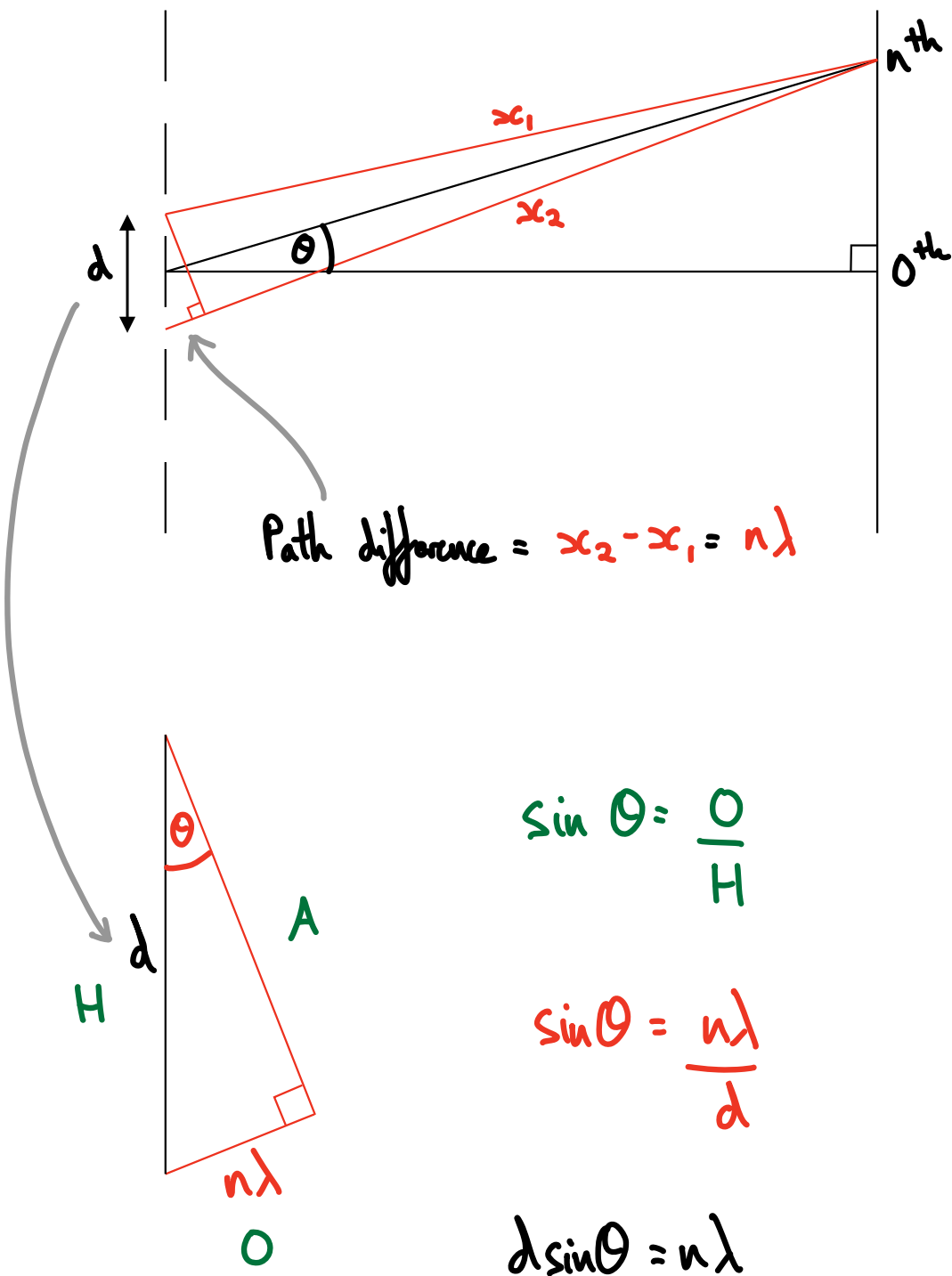
$$k_T = k_1 + k_2$$



Derivation | Diffraction Grating

Derive the expression for a diffraction grating:

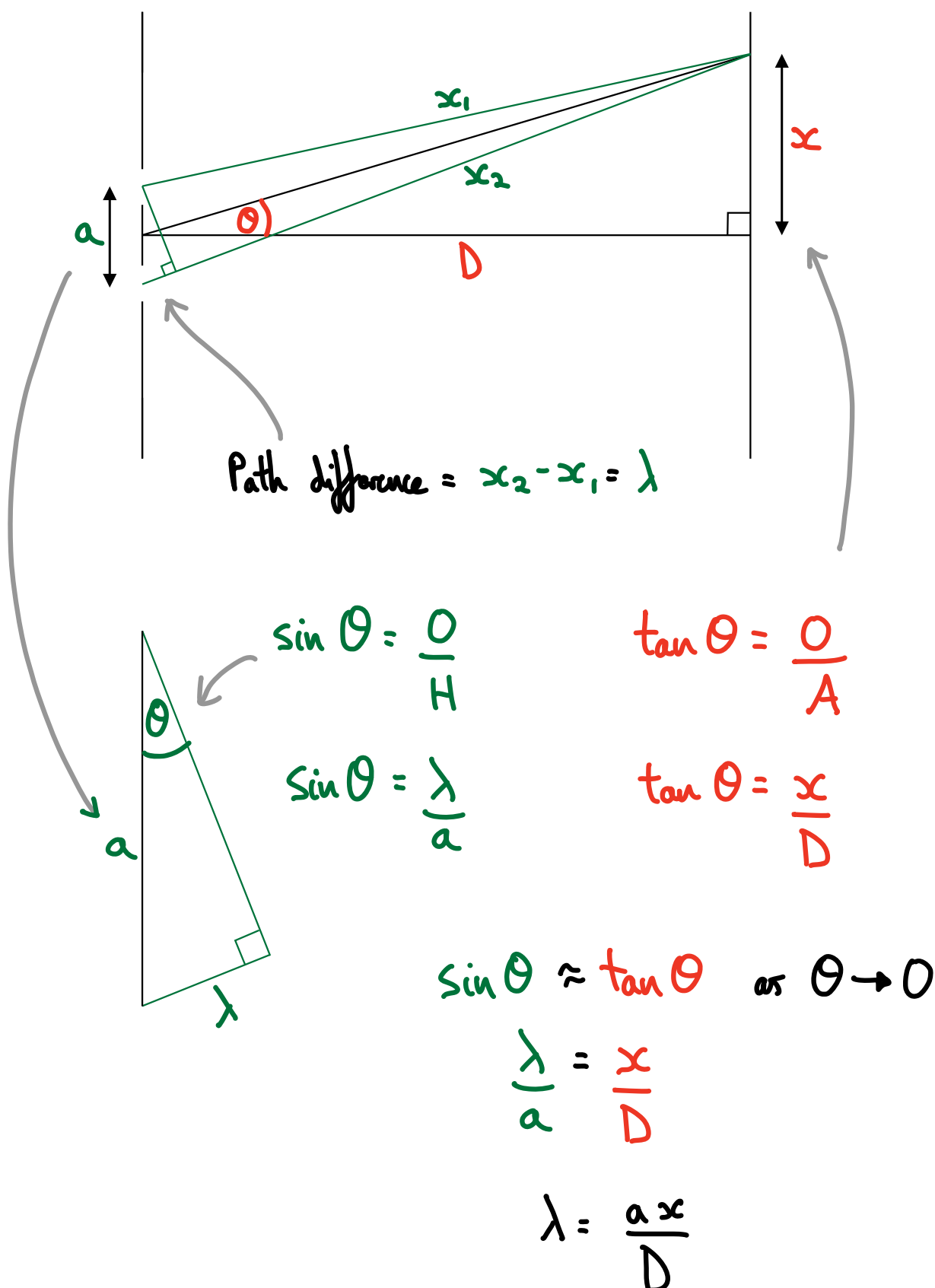
$$d \sin \theta = n \lambda$$



Derivation | Double Slit

Derive the expression for the fringe spacing from a double slit:

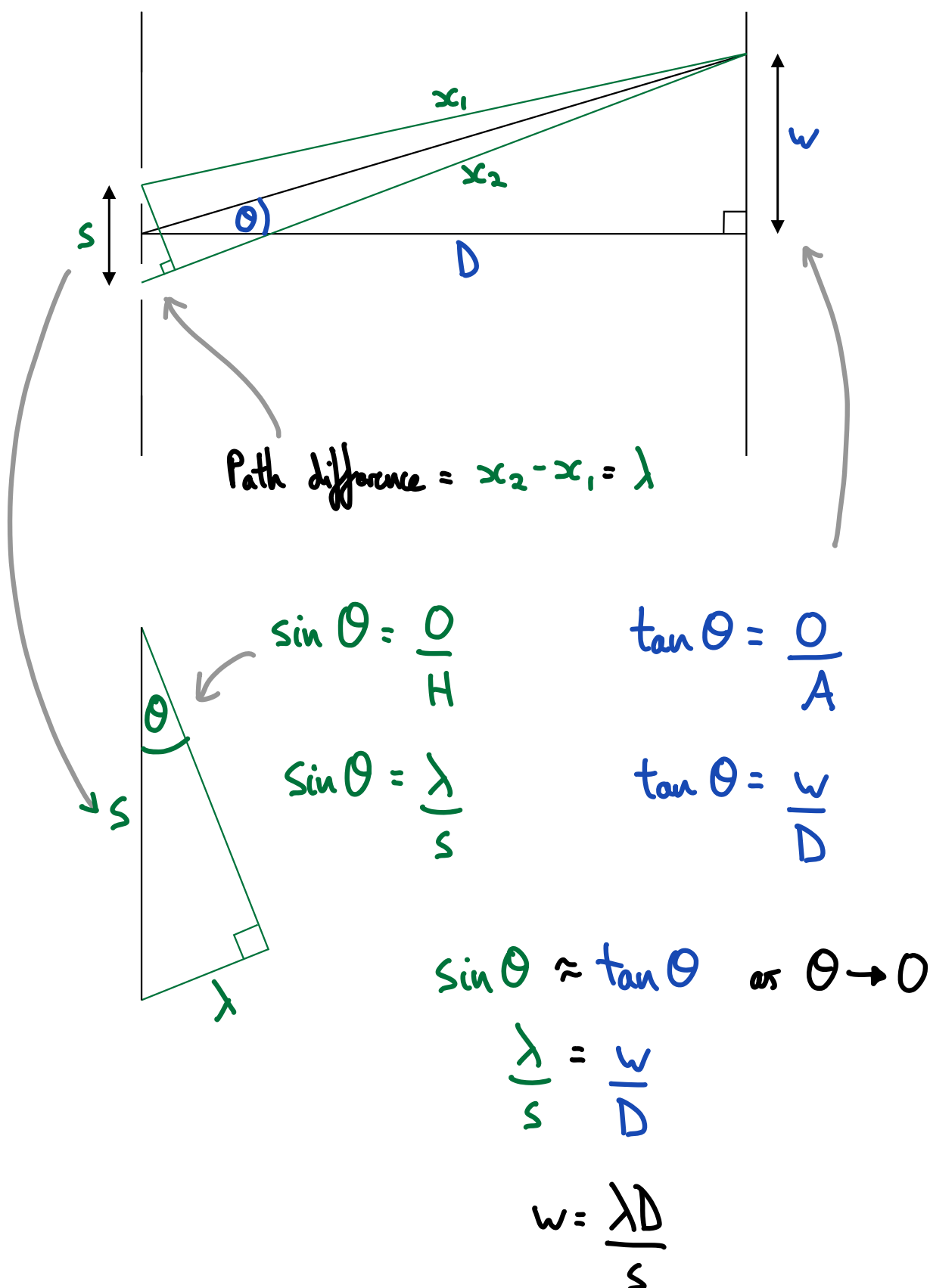
$$\lambda = \frac{ax}{D}$$



Derivation | Double Slit (AQA)

Derive the expression for the fringe spacing from a double slit:

$$w = \frac{\lambda D}{s}$$

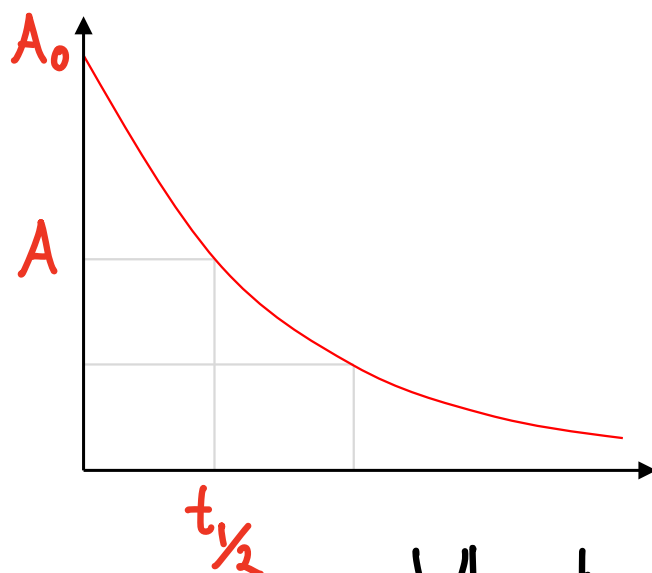




Derivation | Half-life

Derive the expression for the half-life of a radioactive isotope in terms of its decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$



$$A = A_0 e^{-\lambda t}$$

When $t_{1/2}$, $\frac{A}{A_0} = \frac{1}{2}$

$$\frac{A}{A_0} = e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln \frac{1}{2} = -\lambda t_{1/2}$$

$$-\ln \frac{1}{2} = \lambda t_{1/2}$$

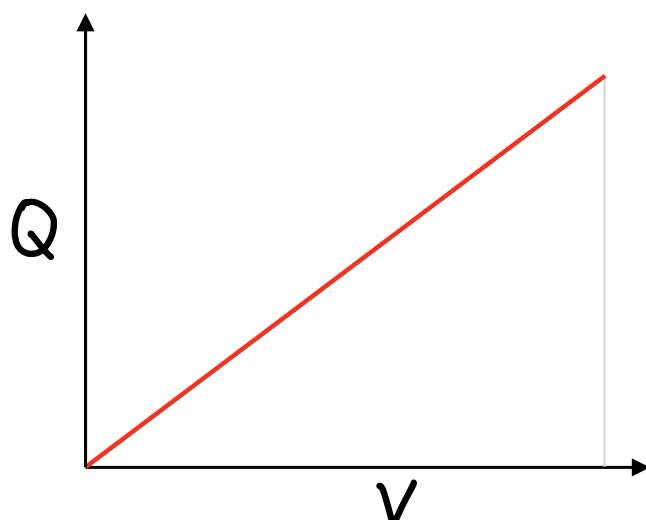
$$-\ln \frac{1}{2} = \ln 2 \quad \therefore \quad \ln 2 = \lambda t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Derivation | Energy in a Capacitor

Derive the series of expressions for the energy stored by a capacitor to its capacitance, charge stored and potential difference:

$$E = \frac{1}{2} CV^2$$
$$E = \frac{1}{2} Q^2 / C$$



Area = $E = \frac{1}{2} QV$ $Q = CV$

$$E = \frac{1}{2} CVV$$

$$E = \frac{1}{2} CV^2$$

$E = \frac{1}{2} QV$ $V = \frac{Q}{C}$

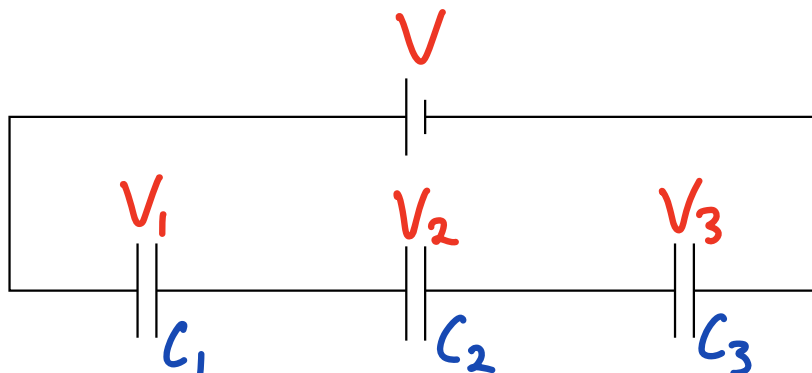
$$E = \frac{1}{2} Q \frac{Q}{C}$$

$$E = \frac{1}{2} Q^2 / C$$

Derivation | Capacitors in Series

Derive the expression for calculating the total capacitance of capacitors in series:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$



$$V = V_1 + V_2 + V_3$$

$$Q = CV \quad V = \frac{Q}{C}$$

$$\frac{Q}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$Q = Q_1 = Q_2 = Q_3$$

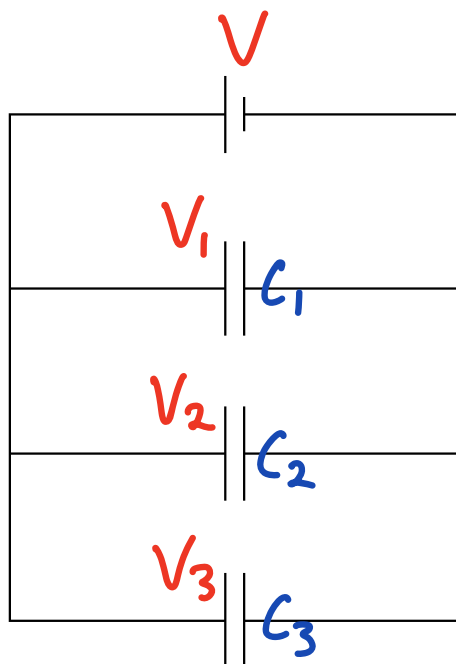
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\therefore \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Derivation | Capacitors in Parallel

Derive the expression for calculating the total capacitance of capacitors in parallel:

$$C_T = C_1 + C_2 + \dots + C_n$$



$$Q = Q_1 + Q_2 + Q_3$$

$$Q = CV$$

$$C_T V = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$V = V_1 = V_2 = V_3$$

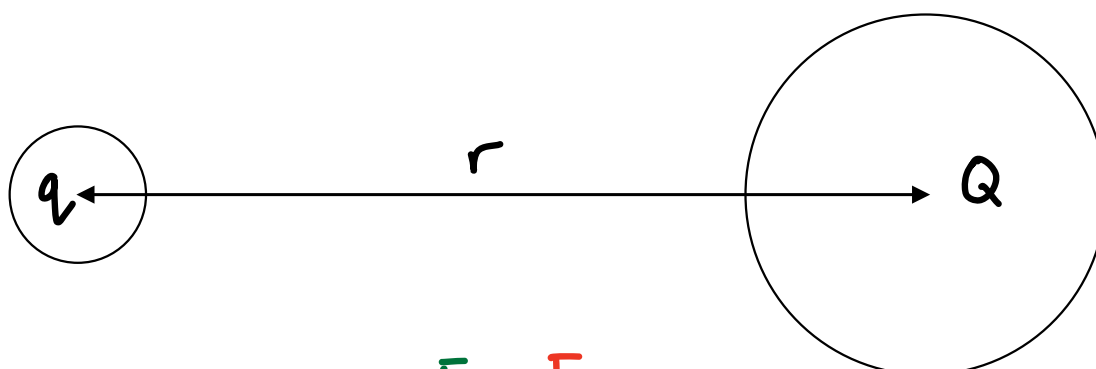
$$C_T = C_1 + C_2 + C_3$$

$$\therefore C_T = C_1 + C_2 + \dots + C_n$$

Derivation | Closest Approach

Derive the expression for the distance of closest approach for an alpha particle to a nucleus:

$$r = \frac{qQ}{2\pi\epsilon_0 mv^2}$$



$$E_K = \frac{1}{2}mv^2 \quad E_K = E_P \quad E_P = \frac{qQ}{4\pi\epsilon_0 r}$$

$$\frac{1}{2}mv^2 = \frac{qQ}{4\pi\epsilon_0 r}$$

$$\frac{1}{2}mv^2 r = \frac{qQ}{4\pi\epsilon_0}$$

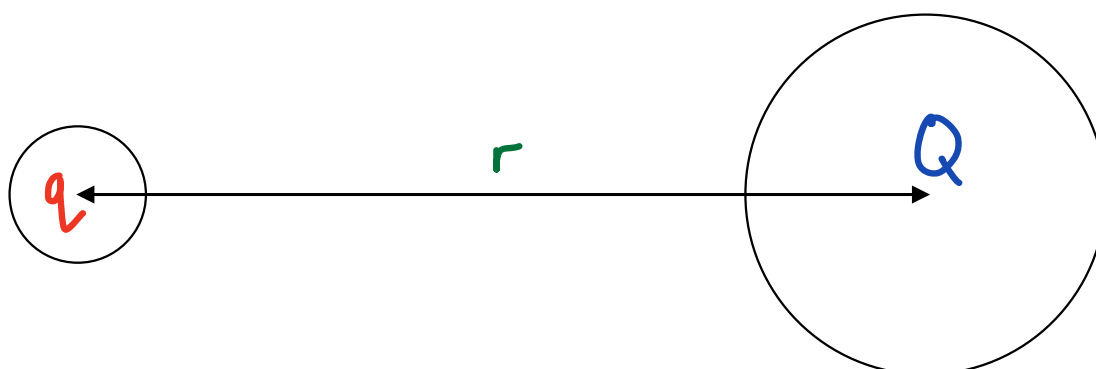
$$r = \frac{2qQ}{4\pi\epsilon_0 mv^2}$$

$$r = \frac{qQ}{2\pi\epsilon_0 mv^2}$$

Derivation | Coulomb's Law

Derive Coulomb's law between two charged objects:

$$F = \frac{qQ}{4\pi\epsilon_0 r^2}$$



$$F \propto q$$

$$F \propto \frac{1}{r^2}$$

$$F \propto Q$$

$$F \propto \frac{qQ}{r^2}$$

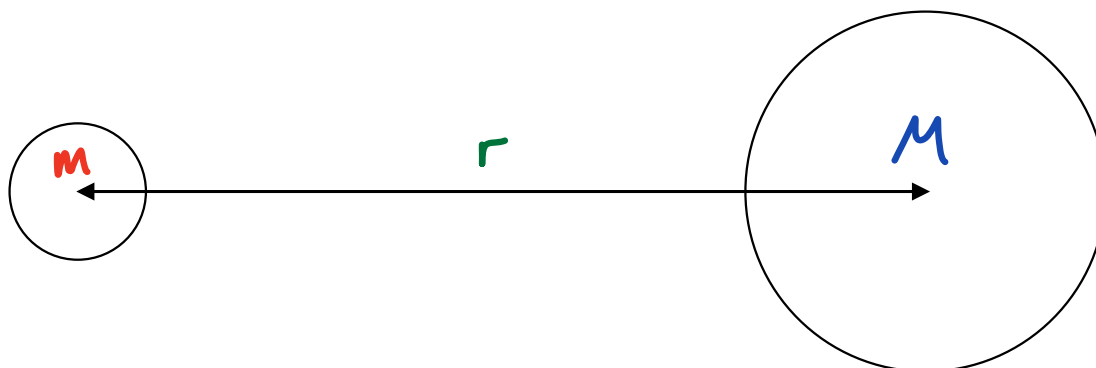
$$F = k \frac{qQ}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$F = \frac{qQ}{4\pi\epsilon_0 r^2}$$

Derivation | Gravitational Attraction

Derive Newton's law of universal gravitation between two objects: $F = \frac{-GMm}{r^2}$



$$F \propto m$$

$$F \propto \frac{1}{r^2}$$

$$F \propto M$$

$$F \propto \frac{m M}{r^2}$$

$$F = k \frac{m M}{r^2}$$

$$k = G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

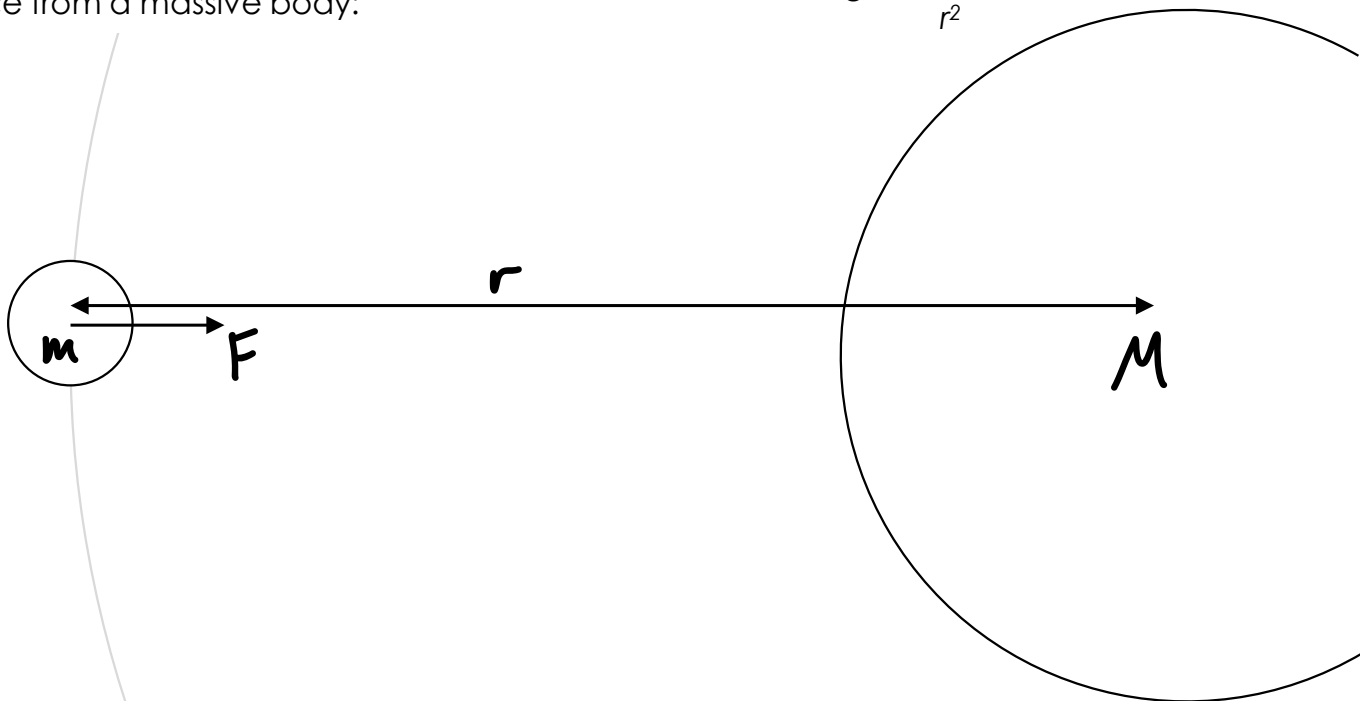
Attractive force $\nearrow$ $F = \frac{-G m M}{r^2}$



Derivation | Gravitational Field Strength

Derive the expression for gravitational field strength at a distance from a massive body:

$$g = \frac{-GM}{r^2}$$



$$F = mg$$

$$F = \frac{-GmM}{r^2}$$

$$\cancel{mg} = \frac{-G\cancel{m}M}{r^2}$$

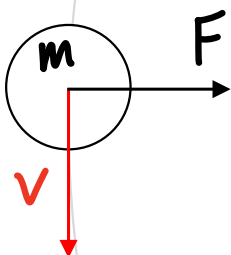
$$g = \frac{-GM}{r^2}$$



Derivation | Radial Path

Derive the expression for the radius of a path of a charged particle in a magnetic field:

$$r = \frac{p}{BQ}$$



$$F = \frac{mv^2}{r}$$

$$F = BQv$$

$$\frac{mv^2}{r} = BQv$$

$$\frac{mv}{r} = BQ$$

$$r = \frac{mv}{BQ}$$

$$p = mv$$

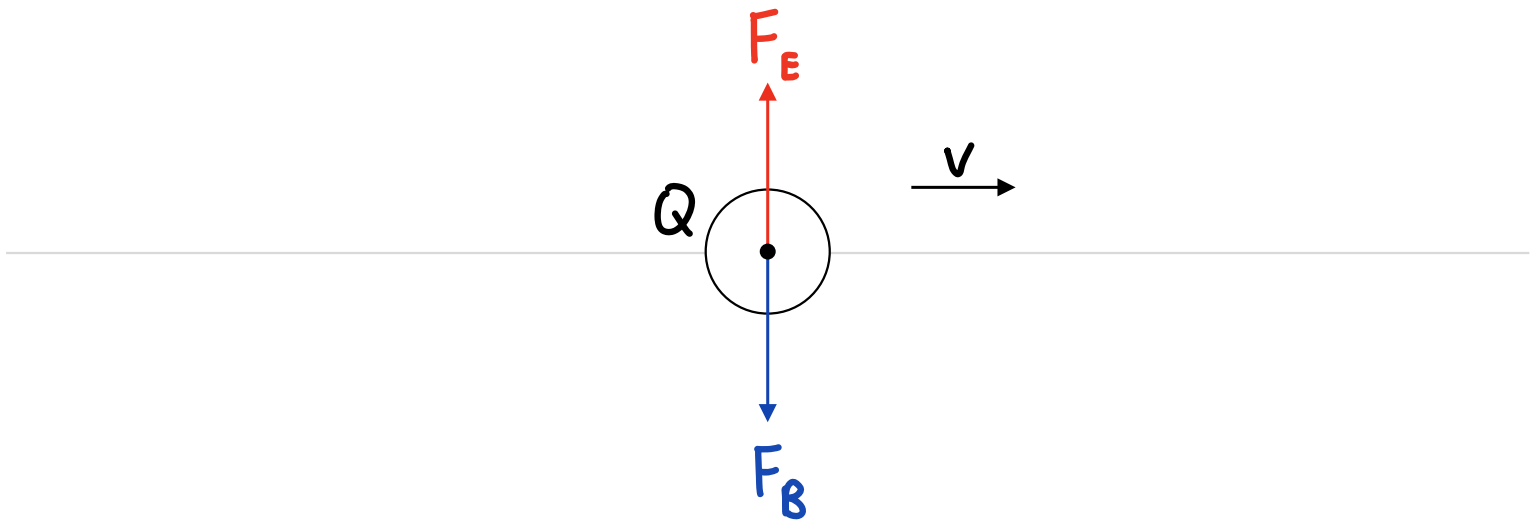
$$r = \frac{p}{BQ}$$



Derivation | Velocity Selector

Derive the expression for the velocity of a particle travelling in a straight line in a magnetic and electric field:

$$v = \frac{E}{B}$$



$$F_B = BQv$$

$$F_E = EQ$$

$$F_B = F_E$$

$$\cancel{BQv} = \cancel{EQ}$$

$$Bv = E$$

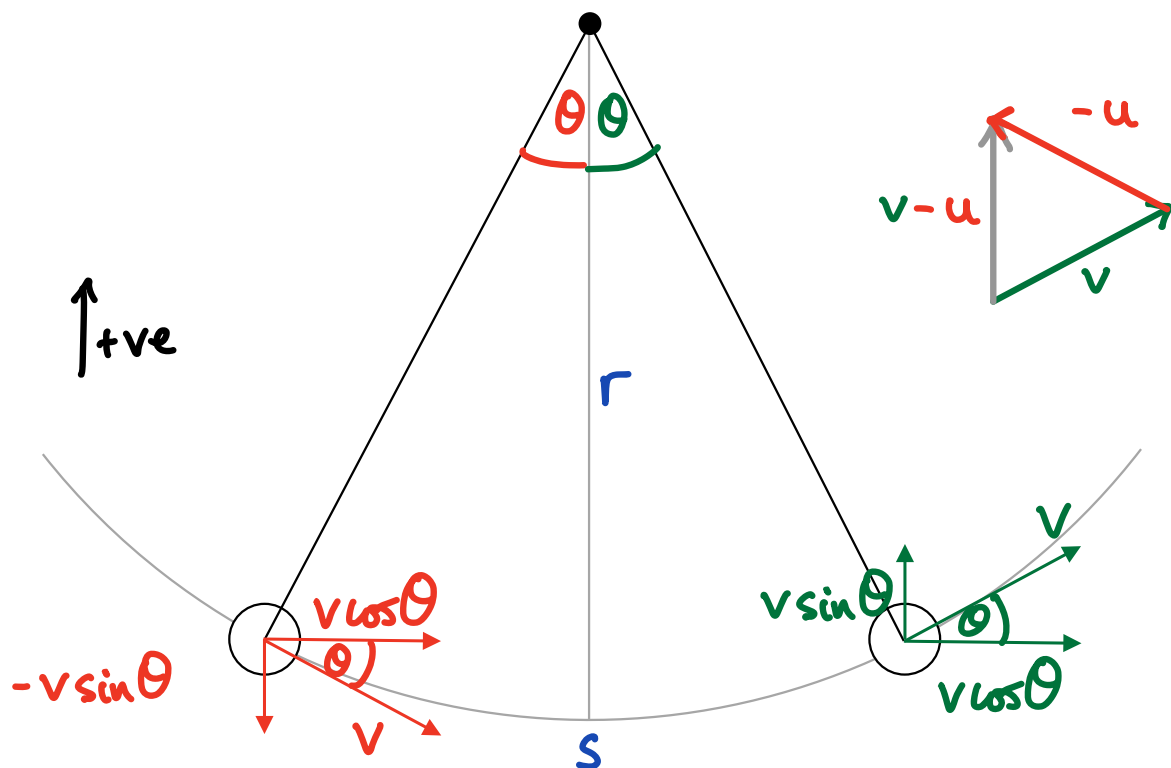
$$v = \frac{E}{B}$$



Derivation | Centripetal Acceleration

Derive the expression for centripetal acceleration:

$$a = \frac{v^2}{r}$$



$$a = \frac{v - u}{t} \quad a = \frac{v \sin \theta - (-) v \sin \theta}{t} = \frac{2v \sin \theta}{t}$$

$$t = \frac{s}{v} = \frac{2r\theta}{v}$$

$$a = \frac{2v \sin \theta}{2r\theta/v} = \frac{2v^2 \sin \theta}{2r\theta} = \frac{v^2 \sin \theta}{r\theta}$$

$$\frac{\sin \theta}{\theta} \approx 1 \quad \text{as } \theta \rightarrow 0$$

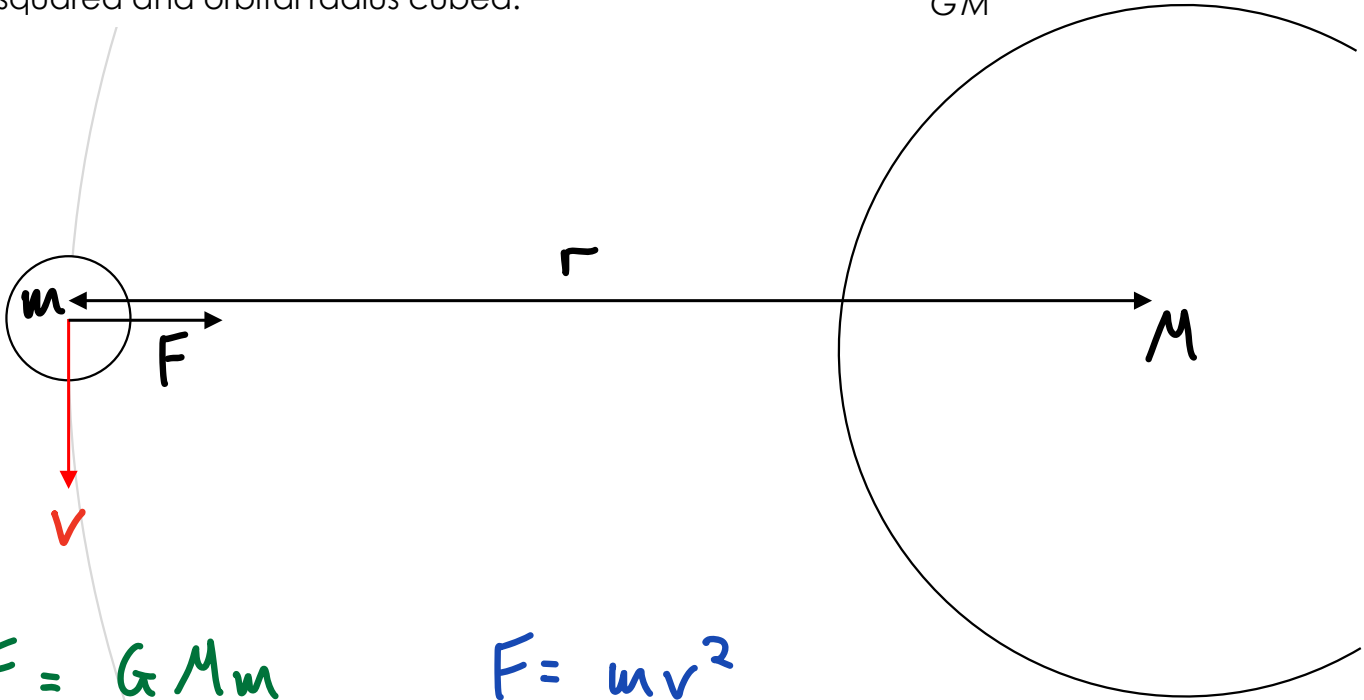
$$a = \frac{v^2}{r}$$



Derivation | Orbital Period and Radius

Derive the expression for the relationship between orbital period squared and orbital radius cubed:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$



$$F = \frac{GMm}{r^2}$$

$$F = \frac{mv^2}{r}$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = v^2$$

$$v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

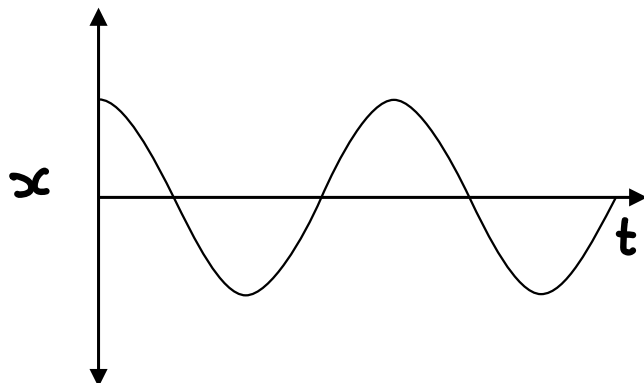
$$T^2 = \frac{4\pi^2 r^3}{GM}$$



Derivation | Simple Harmonic Motion

Show that for a system undergoing simple harmonic motion:

$$a \propto -x$$



$$x = A \cos(\omega t)$$

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t)$$

$$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t)$$

$$a = -A\omega^2 \cos(\omega t)$$

$$a = -\omega^2 x \quad (\omega = 2\pi f)$$

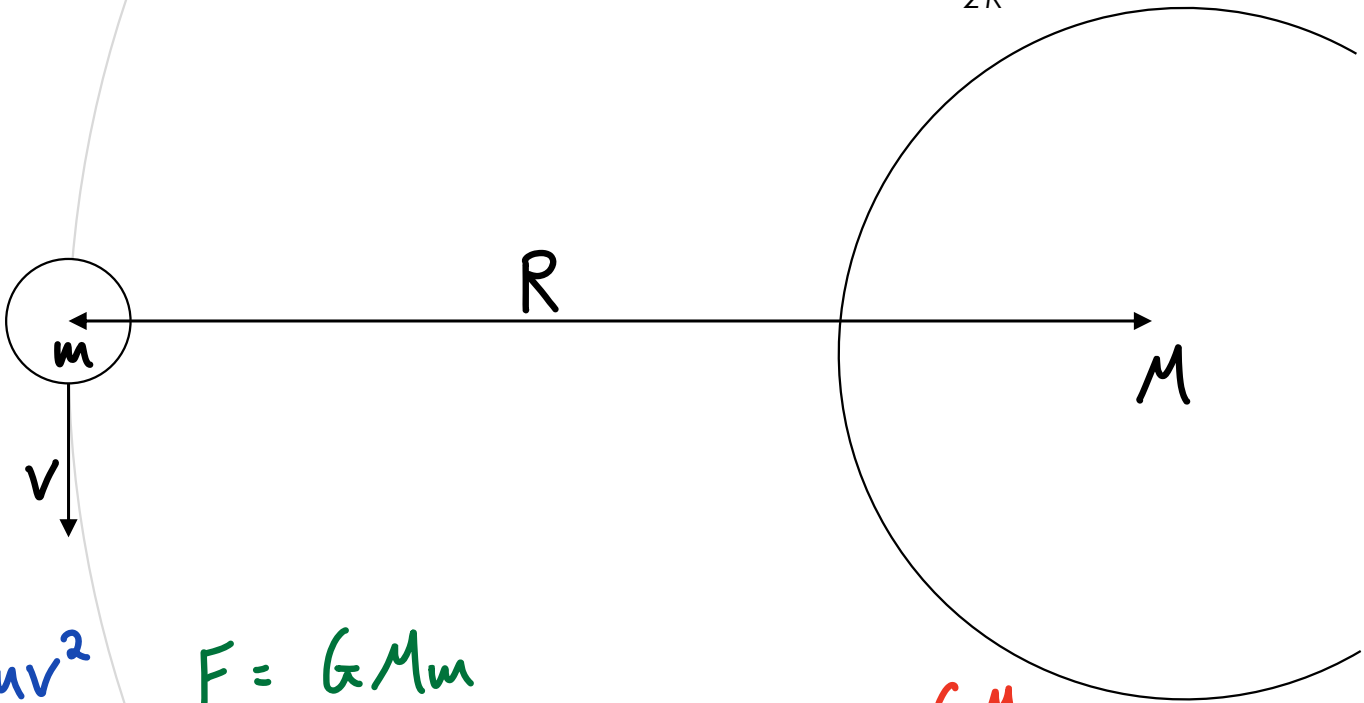
$$\therefore a \propto -x$$



Derivation | Total Energy of a Satellite

Derive the expression for the total energy of a satellite:

$$E_T = \frac{-GmM}{2R}$$



$$F = \frac{mv^2}{R}$$

$$F = \frac{GMm}{R^2}$$

$$\frac{\cancel{mv^2}}{\cancel{R}} = \frac{GM\cancel{m}}{R^2}$$

$$v^2 = \frac{GM}{R}$$

$$E_K = \frac{1}{2}mv^2$$

$$E_K = \frac{1}{2}m \frac{GM}{R}$$

$$E_K = \frac{GmM}{2R}$$

$$V_g = \frac{-GM}{R}$$

$$E_p = \frac{-GmM}{R}$$

$$E_T = E_K + E_p$$

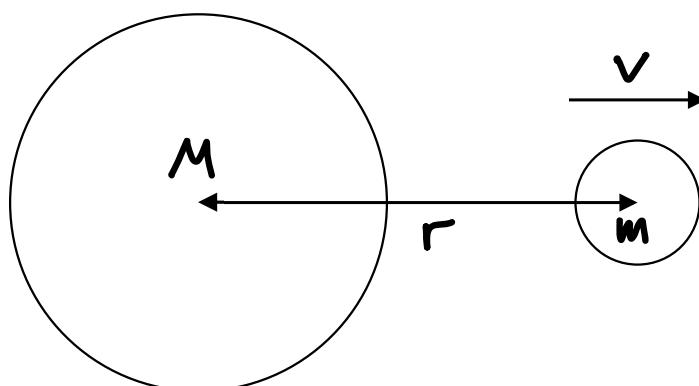
$$E_T = \frac{GmM}{2R} + \frac{(-)GmM}{R}$$

$$E_T = \frac{-GmM}{2R}$$

Derivation | Escape Velocity

Derive the expression for the escape velocity from a planet:

$$v = \sqrt{\frac{2GM}{r}}$$



$$E_k = E_p$$
$$E_k = \frac{1}{2}mv^2$$
$$E_p = \frac{GmM}{r}$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{G\cancel{m}M}{r}$$

$$\frac{v^2}{2} = \frac{GM}{r}$$

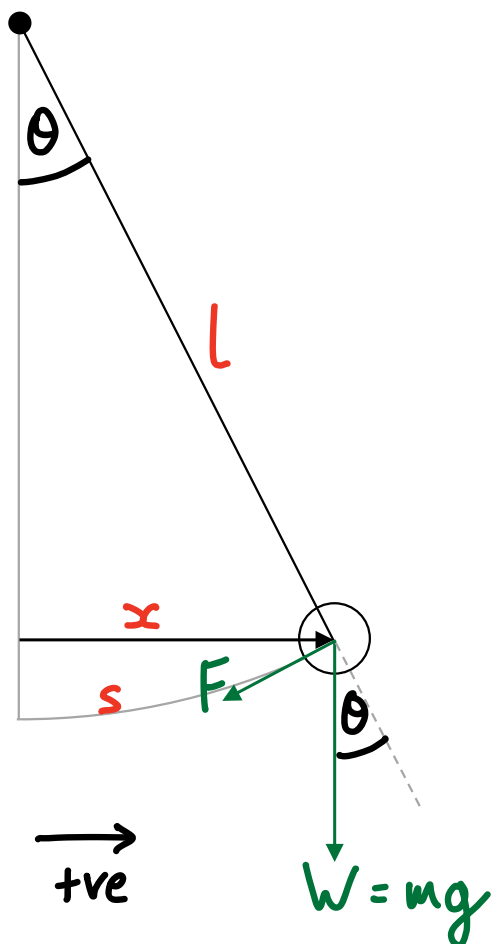
$$v^2 = \frac{2GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

Derivation | Period of a Pendulum

Derive the expression for time period for a pendulum undergoing SHM:

$$T = 2\pi \sqrt{\frac{l}{g}}$$



$$F = -mg \sin \theta \quad F = ma$$

$$\cancel{-mg} \sin \theta = \cancel{ma}$$

$$-g \sin \theta = a$$

$\sin \theta \approx \theta$ as θ is small

$$-g \theta = a$$

$$\theta = \frac{s}{l} \quad s \approx x \quad \theta = \frac{x}{l}$$

$$-\frac{gx}{l} = a \quad \frac{a}{x} = -\frac{g}{l}$$

$$a = -(2\pi f)^2 x$$

$$\frac{a}{x} = -4\pi^2 f^2$$

$$\frac{a}{x} = \frac{-4\pi^2}{T^2}$$

$$\frac{+g}{l} = \frac{+4\pi^2}{T^2}$$

$$T^2 \frac{g}{l} = 4\pi^2$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

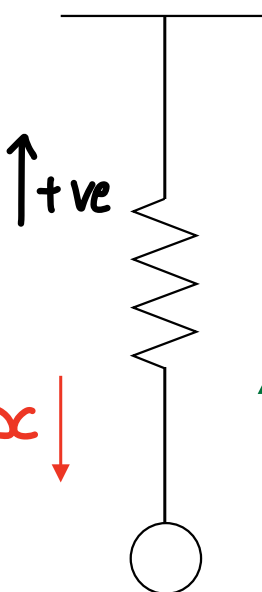
$$T = 2\pi \sqrt{\frac{l}{g}}$$



Derivation | Period of a Spring-Mass

Derive the expression for period for a spring-mass system undergoing SHM:

$$T = 2\pi \sqrt{\frac{m}{k}}$$



$F = -kx$ x F $F = ma$

$$-kx = ma$$
$$\frac{a}{x} = \frac{-k}{m}$$
$$a = -(2\pi f)^2 x$$
$$\frac{a}{x} = -4\pi^2 f^2$$
$$\frac{a}{x} = \frac{-4\pi^2}{T^2}$$
$$\frac{-k}{m} = \frac{-4\pi^2}{T^2}$$
$$T^2 \frac{k}{m} = 4\pi^2$$
$$T^2 = 4\pi^2 \frac{m}{k}$$
$$T = 2\pi \sqrt{\frac{m}{k}}$$



Derivation | Ideal Gas Equation

Derive the ideal gas equation:

$$pV = nRT$$

$$pV = NkT$$

$$p \propto T$$

$$\frac{p}{T} = k_1$$

$$p \propto \frac{1}{V}$$

$$pV = k_2$$

$$V \propto T$$

$$\frac{V}{T} = k_3$$

$$\frac{pV}{T} = k$$

$$pV = kT$$

$$pV = nRT$$

$$pV = NkT$$

$$(nR = Nk)$$



Derivation | E_k and Temperature

Derive the expression relating the kinetic energy of a particle to the temperature of an ideal gas:

$$E_k = \frac{3}{2} kT$$

$$pV = \frac{1}{3} Nm \bar{c}^2$$

$$pV = NkT$$

$$\cancel{\frac{1}{3} Nm \bar{c}^2} = \cancel{NkT}$$

$\times 3$

$$m \bar{c}^2 = 3kT$$

$\div 2$

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} kT$$

$$E_k = \frac{3}{2} kT$$

$$(E_k \propto T)$$



Derivation | Kinetic Theory

Derive the kinetic theory formula for an ideal gas:

$$pV = \frac{1}{3} Nm \bar{c}^2$$



Cube of side 'L'

$$A = L^2$$

$$V = L^3$$

$$F = \frac{\Delta p}{t}$$

$$\Delta p = mv - mu = 2mv$$

$$F = \frac{2mv}{t}$$

$$v = \frac{2L}{t}$$

$$t = \frac{2L}{v}$$

$$F = \frac{2mv}{2L/v}$$

In x direction, n molecules at $\bar{c}$

$$F = \frac{mv^2}{L}$$

$$F_T = \frac{nm\bar{c}^2}{L}$$

$$p = \frac{F_T}{A} = \frac{nm\bar{c}^2/L}{L^2} = \frac{nm\bar{c}^2}{L^3} = \frac{nm\bar{c}^2}{V}$$

$$N = 3n(x, y, z)$$

$$n = \frac{N}{3}$$

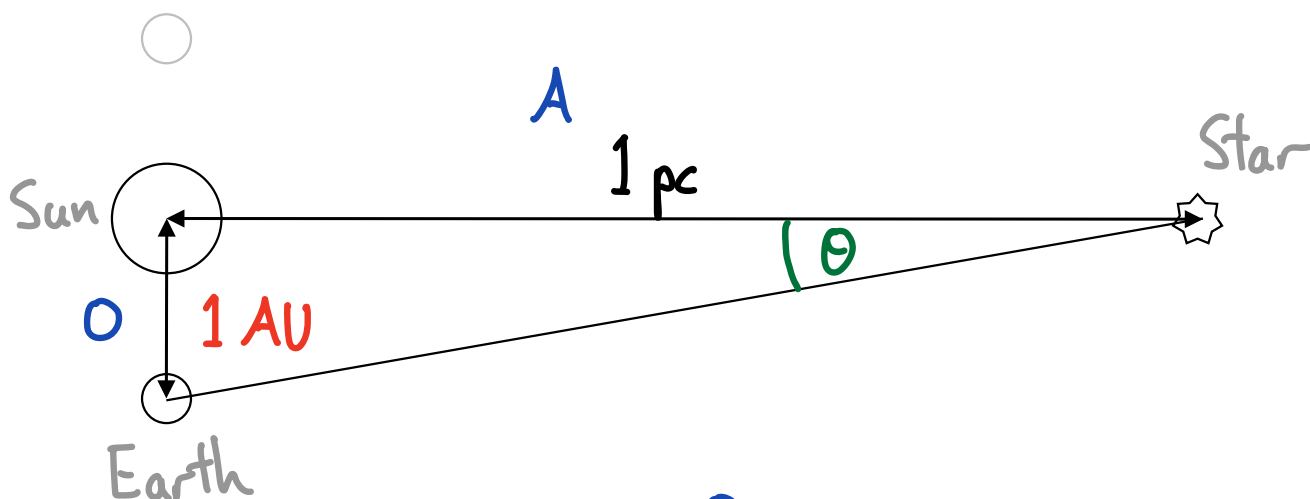
$$p = \frac{Nm\bar{c}^2}{3V}$$

$$pV = \frac{1}{3} Nm \bar{c}^2$$

Derivation | The Parsec

Derive the parsec and show that:

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$$



$$\tan \theta = \frac{O}{A}$$

$$A = \frac{O}{\tan \theta}$$

$$\text{If } O = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$\text{and } \theta = 1 \text{ second of arc} = \frac{1^\circ}{60 \times 60}$$

$$A = \frac{1.5 \times 10^{11}}{\tan \left(\frac{1}{3600} \right)}$$

$$A = 3.1 \times 10^{16} \text{ m} = 1 \text{ pc}$$

Derivation | Specific Charge on an e⁻

Derive the expression for the specific charge on an electron:

$$\frac{e}{m} = \frac{2V}{B^2 r^2}$$

$$r = \frac{mv}{Be}$$

$$\frac{1}{2}mv^2 = eV$$

$$r^2 = \frac{m^2 v^2}{B^2 e^2}$$

$$v^2 = \frac{2eV}{m}$$

$$r^2 = \frac{m^{\cancel{2}} 2eV}{B^2 e^{\cancel{2}} m^{\cancel{1}}}$$

$$r^2 = \frac{m 2V}{B^2 e}$$

$$e = \frac{m 2V}{B^2 r^2}$$

$$\frac{e}{m} = \frac{2V}{B^2 r^2}$$