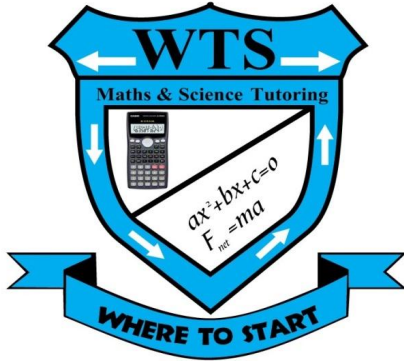


WHERE TO START IN MATHS AND SCIENCE TUTORING

WTS M&S TUTORING



WTS MATHS PAPER TWO

GRADE 12

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WTS IS FOR THE GLOBE

WHERE TO START MATHS & SCIENCE TUTORING	
FINAL EXAMINATION	
GRADE	12
SUBJECT	MATHEMATICS
PAPER	PAPER 2
DURATION OF THE PAPER	3 HOURS
TOTAL MARKS	150
NUMBER OF QUESTIONS	10 – 12
QUESTION PAPER FORMAT	LEVEL 1 Knowledge questions 20%
	LEVEL 2 Routine questions 35% procedures
	LEVEL 3 Complex questions 30% procedures
	LEVEL 4 Problem solving questions 15%
EXPECTED WORK COVERAGE	
STATISTICS	20±3 marks
ANALYTICAL GEOMETRY	40±3 marks
TRIGONOMETRY	50±3 marks
EUCLIDEAN GEOMETRY	40±3 marks

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.

DATA HANDING

1. Consider the Maths test out of 50 marks for WTS finishing School classes, A and B:

A	8	8	10	12	16	19	20	21	24	25	26	
B	8	8	10	12	16	19	20	21	24	24	26	50

NB: Choose one class to answer the questions below

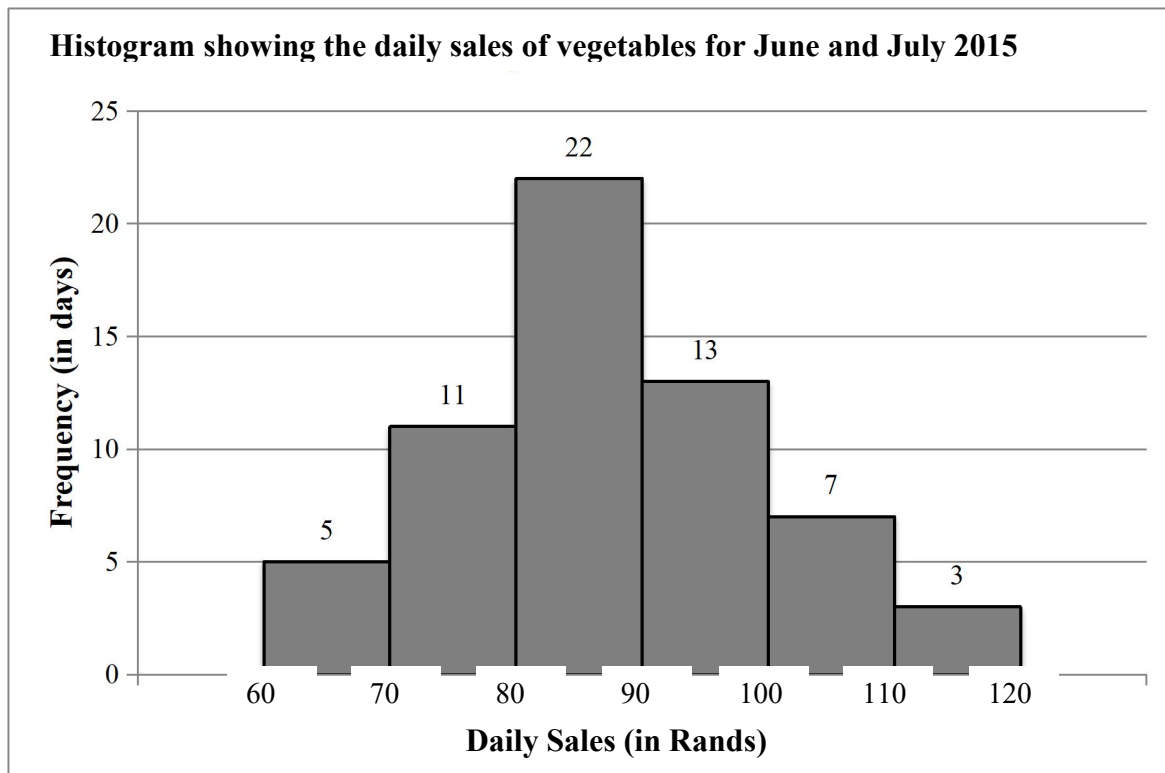
- a) Calculate the mean.
- b) Write down the five-number summary of the data.
- c) Draw a box and whisker diagram for the data.
- d) Refer to the box and whisker diagram and comment on the skewness of the data set.
- e) Calculate the standard deviation for this data set.
- f) How many learners and its percentage were:
 - (i) One standard deviation above the mean?
 - (ii) One standard deviation below the mean?
 - (iii) One standard deviation within the mean?
 - (v) One standard deviation outside the mean?
- g) Repeat QUESTION (f) if it is now two standard deviation of the mean
- h) Write down the range
- i) Write down the IQR
- j) Write down the semi-IQR
- k) Write down outlier interval
- l) Write down any outlier
- n) Arrange the above data in a steam and leaf

2. Eleven athletes took part in a javelin throwing competition. Their height, in *cm*, and their arm span, in *cm*, is shown in the table below.

Athletic	1	2	3	4	5	6	7	8	9	10	11
Height(<i>cm</i>)	156	173	181	174	167	170	169	174	177	168	110
Arm span	164	181	193	178	172	178	165	183	190	173	100

- Represent the height and arm span for each athlete on the scatter plot
- Determine the equation of the least squares regression line.
- Calculate the mean for the height
- Calculate the mean for the arm span
- Draw the least squares regression line on the scatter plot
- Use the equation in b) to estimate the arm span of an athlete whose height is 176 *cm*.
- Is (c) an extrapolation or interpolation? Explain
- Use the equation in b) to estimate the arm span of an athlete whose height is 200 *cm*.
- Is (e) an extrapolation or interpolation? Explain
- Use the equation in b) to estimate the height of an athlete whose arm span is 185 *cm*.
- Calculate the correlation coefficient
- Comment on the strength of relationship between the variables
- Identify an outlier in the data set
- Discuss the trend of the data collected

3. Mrs. Fuyane, a street vendor, has kept record of the daily sales of vegetables for the months of June and July 2015. The information is represented in the histogram below.



- Use the information provided in the histogram to complete the frequency and cumulative frequency columns in the table provided in the ANSWER BOOK.
- Draw an ogive (cumulative frequency graph)
- Use your cumulative frequency graph to estimate the following for the sales:
 - Lower quartile
 - Median
 - Upper quartile
- Hence, draw the box and whisker plot
- Identify the modal class of the data
- Using ogive curve estimate the interval of the upper 25% of the daily sales.
- Using ogive curve estimate the number of daily sale below R90
- Estimate the mean of daily sales

4. The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time(hours)	Midpoint(x)	Frequency(f)	Cumulative frequency	$f \cdot x$	Points($x; y$)
$0 \leq t < 20$			25		
$20 \leq t < 40$			69		
$40 \leq t < 60$			129		
$60 \leq t < 80$			157		
$80 \leq t < 100$			166		
$100 \leq t < 120$			172		

- Complete the above table
- Hence calculate the mean.
- Draw an ogive (cumulative frequency curve) to represent the above data.
- Draw the histogram graph.
- Hence, draw the frequency polygon.
- Write down the modal class of the data.
- Write down the median class interval.

5. Learners at WTS School travel from three different neighborhoods, A, B and C. The table below shows the number of learners from each, and their mean travelling times from home to school.

Neighborhood	A	B	C
Number of learners	135	225	200
Mean travelling time (in min.)	24	32	x

The mean travelling time for learners living in neighborhood C is the same as the mean travelling time for all 560 learners.

- a. Calculate the mean travelling time for neighborhood C.

6. In the grid below a, b, c, d, e, f and g represent values in a data set written in an increasing order.

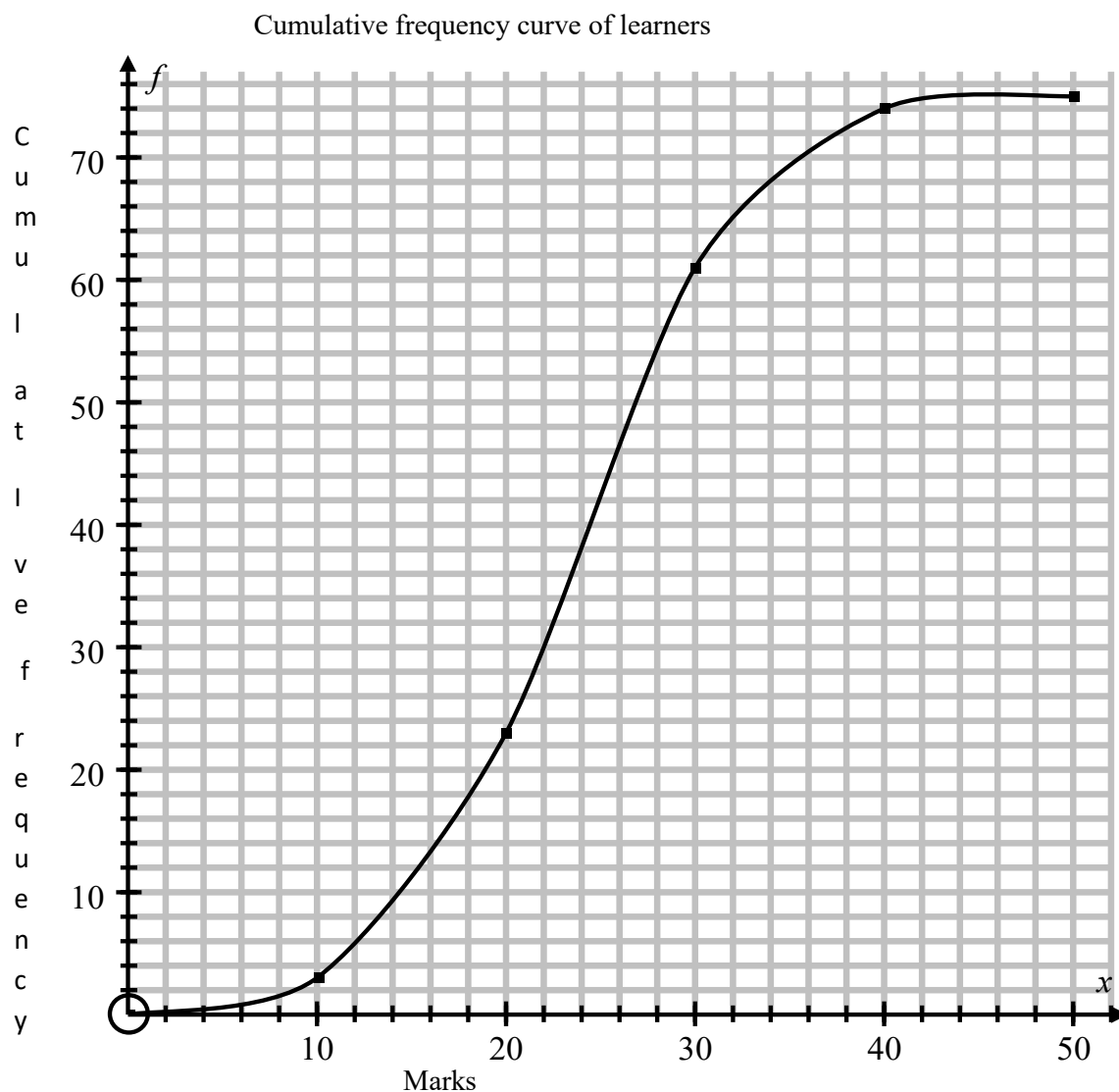
No values in a data set is represented.

a	b	c	d	e	f	g
-----	-----	-----	-----	-----	-----	-----

Determine the value of a, b, c, d, e, f and g if :

- ✓ The maximum value is 42.
- ✓ The range is 35.
- ✓ The median is 23.
- ✓ The difference between the median and the upper quartile is 14.
- ✓ The mean is 25

6. The cumulative frequency curve below represents the marks scored by 75 learners in a test of which the total mark was 50.

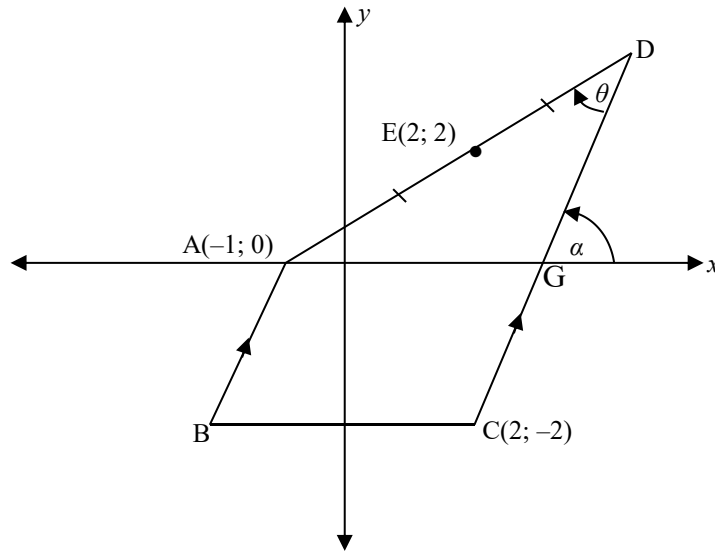


Use the graph to answer the following questions:

- How many learners scored a mark of higher than 35 out of 50?
- How many learners scored a mark between 15 and 30 out of 50?
- What was the median mark obtained by the learners?

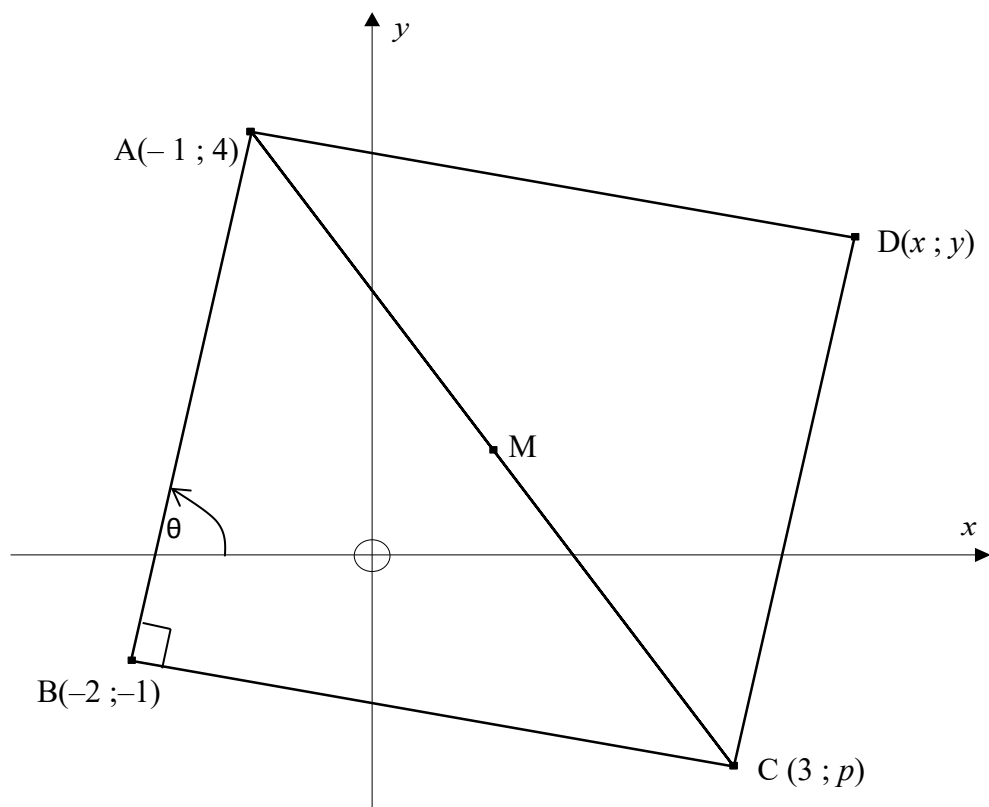
ANALYTICAL GEOMETRY

1. In the diagram below, A $(-1; 0)$, B, C $(2; -2)$ and D are the vertices of a trapezium having $AB \parallel DC$. The length of DC is three times the length of AB (i.e. $DC = 3AB$). $\hat{ADC} = \theta$. E $(2; 2)$ is the midpoint of AD. The angle of inclination of DC is α .



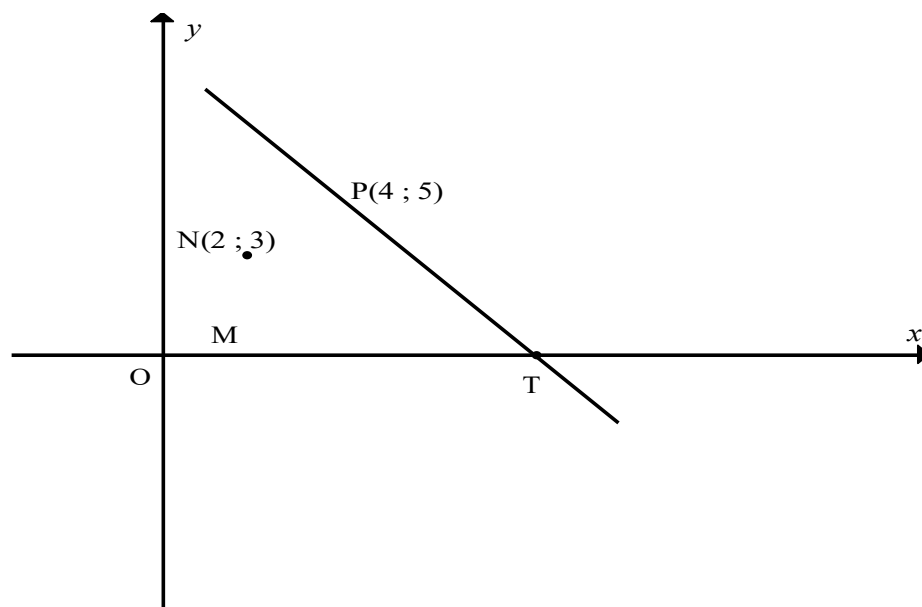
- a) Determine the coordinates of D.
- b) Calculate the size of α , correct to ONE decimal place.
- c) Calculate the size angle AGC
- d) Calculate the equation of CD
- e) Hence, the coordinate of G
- f) Determine the equation of AB in the form $y = mx + c$.
- g) Calculate the size of θ , correct to ONE decimal place.
- h) Calculate the coordinates of B if $BC \parallel x\text{-axis}$
- i) Calculate the coordinate of F if DCBF form a parallelogram
- j) Calculate the equation which is perpendicular to CD passing through point A
- k) Calculate the length of AB, BC and CD.
- l) Hence, calculate the area of ABCD, if $BC \perp CD$
- m) Prove that angle BCD is not a right angled triangle using the following:
 - i. Gradients
 - ii. Pythagoras theorem

2. In the diagram below, $A(-1 ; 4)$, $B(-2 ; -1)$, $C(3 ; p)$ and $D(x ; y)$ are four points in a Cartesian plane. M is the midpoint of AC , $\hat{B} = 90^\circ$ and the inclination of line AB is θ .



- Determine the size of θ .
- Show that $p = -2$.
- Calculate the coordinates of M , the midpoint of AC .
- Determine the equation of a circle, with centre M , and passes through the points A and C .
Write your answer in the form $y = (x - a)^2 + (y - b)^2 = r^2$.
- Hence, determine the co-ordinates of D if $ABCD$ is a rectangle.
- Determine the length of AB (give the answer in surd form).
- Hence, prove analytically that $ABCD$ is a square.

3. In the figure below, T is a point on the x-axis. A circle having T as its centre intersects a circle having N(2 ; 3) as its centre at P(4 ; 5) and M. TP is a tangent to the circle centre N at P.



- Determine the equation of circle centre N in the form: $(x - a)^2 + (y - b)^2 = r^2$
- Hence, rewrite the equation in the form of $Ax^2 + Bx + Cy^2 + Dy = E$
- Also hence, rewrite it back to the form of $(x - a)^2 + (y - b)^2 = r^2$ show all your working out
- Calculate the gradient of the tangent PT
- Hence, calculate the coordinates of T, the x-intercept of PT.
- Calculate the length of PT. Leave your answer in surd form.
- Hence, write down the equation of the circle with the center T
- K is a point on the circumference drawn from T to form a straight line and, then calculate the following:
 - Coordinate of K
 - Length of PK
 - Calculate the area of circle centered at T.
- If K (t ; -4) and the length of PK = $10\sqrt{2}$, calculate the value of t .
- Calculate the size of $\hat{NTP}$, correct to ONE decimal place.
- Prove that MNPT is a kite.
- Calculate the size of $\hat{MNP}$, correct to ONE decimal place.

- m) Calculate the y intercept of the smaller circle.
- n) Calculate the x intercept of the circle with center T.

4. Consider the following:

- a. Determine the centre and radius of the circle with $x^2 + y^2 + 8x + 4y - 38 = 0$.
- b. A second circle has the equation $(x - 4)^2 + (y - 6)^2 = 26$. Calculate the distance between the centres of the two circles.
- c. Hence, show that the circles described in a) and b) intersect each other.
- d. Show that the two circles intersect along the line $y = -x + 4$.
- e. Calculate the x and y intercept of the equation in a)
- f. Calculate the x and y intercept of the equation in b)
- g. Using equation in number a) check whether point $A(-2; 3)$ lies on the circle or not?

5. Given circle $x^2 - 2x + y^2 - 16y + 39 = 0$ with centre W and y -intercepts $B(0; r)$ and $C(0; t)$ where $r < t$.

- a) Calculate the values of r and t .
- b) Show that the point $E(2; 13)$ lies on the circumference of the circle.
- c) Prove that points B, W and E are collinear.

6. In addition to the circle with centre A and equation $(x - 3)^2 + (y + 2)^2 = 25$, you are given the circle $(x - 12)^2 + (y - 10)^2 = 100$ with centre B.

- a. Calculate the distance between the centres A and B.
- b. In how many points do these two circles intersect? Justify your answer.

➤ TRIGONOMETRY

➤ QUESTION ONE

1. Simplify to a single trigonometric ratio of A:

- a)
$$\frac{\tan (180^{\circ}+A) \cos (180^{\circ}-A) \sin (360^{\circ}-A)}{\cos (90^{\circ}-A)}$$
- b)
$$\frac{\tan (180^{\circ}+A) \cos (360^{\circ}-A)}{\sin (180^{\circ}-A) \cdot \cos (90^{\circ}+A)+\cos (540^{\circ}+A) \cdot \cos (-A)}$$
- c)
$$\cos (90^{\circ}-2 A) \tan (180^{\circ}+A) \sin ^2(A-360^{\circ})$$

2. Simplify the following without using a calculator:

- a)
$$\frac{\sin 10^{\circ} \cdot \cos 120^{\circ}}{\cos 80^{\circ} \cdot \sin 150^{\circ}}$$
- b)
$$\frac{\cos (A-360^{\circ}) \cdot \sin (-A)}{\tan A \cdot \cos 180^{\circ}}$$
- c)
$$\frac{2 \cos 105^{\circ} \cdot \cos 15^{\circ}}{\cos (45^{\circ}-x) \cdot \cos x - \sin (45^{\circ}-x) \cdot \sin x}$$
- d)
$$\frac{\tan 480^{\circ} \cdot \sin 300^{\circ} \cdot \cos 14^{\circ} \cdot \sin (-135^{\circ})}{\sin 104^{\circ} \cdot \cos 225^{\circ}}$$
- e)
$$\frac{\sin 104^{\circ}(2 \cos ^2 15^{\circ}-1)}{\tan 38^{\circ} \cdot \sin ^2 412^{\circ}}$$
- f)
$$\frac{\sin (90^{\circ}+\theta)+\cos 180^{\circ} \sin (-\theta)}{\sin 180^{\circ}-\tan 135^{\circ}}$$
- g)
$$\frac{\cos 330^{\circ} \sin 140^{\circ}}{\sin (-160^{\circ}) \tan 405^{\circ} \cdot \sin 290^{\circ}}$$
- h)
$$(\sqrt{2} \cos 75^{\circ}-1)(1+\sqrt{2} \cos 75^{\circ})$$
- i)
$$\frac{\sin 33^{\circ}}{\sin 11^{\circ}}-\frac{\cos 33^{\circ}}{\cos 11^{\circ}}$$

3. Consider the following identity:

a. $\frac{1-2\sin A \cos A}{\sin A - \cos A} = \sin A - \cos A$

i. prove the identity above

ii. For which values A of will the identity not be defined?

b. $\frac{1-\tan A}{1+\tan A} = \frac{\cos 2A}{1+\sin 2A}$

i. Calculate the values of A for which $\frac{1-\tan A}{1+\tan A}$ is undefined for $A \in [0^\circ; 360^\circ]$.

iii. Hence or otherwise, calculate the value of $\frac{1-\tan 22,5^\circ}{1+\tan 22,5^\circ}$.

4. Prove the following:

a) $\frac{\sin 2x}{\cos 2x + \sin^2 x} = 2 \tan x$

b) $\frac{\sin \theta}{1 - \cos \theta} = \frac{1}{\sin \theta}$

c) $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{1}{\cos x}$

d) $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$

e) $\frac{\sin(180^\circ + x) \tan(x - 360^\circ) \cos 38^\circ}{\tan(360^\circ - x) \cos 240^\circ \tan 225^\circ \sin 448^\circ} = -\frac{3}{2}$

f) $\frac{\cos^2(90^\circ + \theta)}{\cos(-\theta) + \sin(90^\circ - \theta) \cos \theta} = \frac{1}{\cos \theta} - 1$

g) $\cos(90^\circ - 2x) \tan(180^\circ + x) + \sin^2(360^\circ - x) = 3\sin^2 x$

h) $(\tan x - 1)(\sin 2x - 2\cos^2 x) = 2(1 - 2\sin x \cos x)$

i) $\sin(45^\circ + x) \cdot \sin(45^\circ - x) = \frac{1}{2} \cos x$

5. Simplify the following without using a calculator:

- a) $\cos 75^\circ$
- b) $\sin 15^\circ$
- c) $\cos 105^\circ$
- d) $\tan 15^\circ$
- e) $\cos 25^\circ \cdot \cos 20^\circ - \sin 25^\circ \cdot \sin 20^\circ$
- f) $\sin 140^\circ \cdot \cos 80^\circ - \cos 40^\circ \cos 10^\circ$
- g) $\cos (x - 300^\circ) - \sin (x - 150^\circ)$

6. It is given that $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$:

a) Similarly, write down a formula for $\cos \alpha$...

b) Hence show that $\cos \alpha = \frac{1 - \tan^2 \frac{1}{2} \alpha}{1 + \tan^2 \frac{1}{2} \alpha}$.

7. Use $\cos (A - B) = \cos A \cdot \cos B + \sin A \sin B$, to prove the ff:

- a. $\sin (A - B) = \sin A \cdot \cos B - \cos A \sin B$
- b. $\sin (A + B) = \sin A \cdot \cos B + \cos A \sin B$

8. DOUBLE ANGLES

a) Consider: $\sin (A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$, let $B = A$, and then simplify

Note: $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

b) Consider: $\cos (A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$, let $B = A$ and then simplify

Note: $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

c) Use $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$, Prove the following:

- i. $\cos 2\alpha = 2\cos^2 \alpha - 1$
- ii. $\cos 2\alpha = 1 - 2\sin^2 \alpha$

➤ QUESTION TWO

1. If $\cos 26^\circ = k$, determine the following in terms of k , in its simplest form:

- a) Calculate the value x, y and r
- b) Indicate all angles of a triangle
- c) $\cos 154^\circ$
- d) $\cos 206^\circ$
- e) $\sin 334^\circ$
- f) $\sin 386^\circ$
- g) $\tan 514^\circ$
- h) $\tan 566^\circ$
- i) $\sin 26^\circ$
- j) $\sin 64^\circ$
- k) $\tan 64^\circ$
- l) $\cos 64^\circ$
- m) $\sin (-26^\circ)$
- n) $\cos (-26^\circ)$
- o) $\tan (-64^\circ)$
- p) $\cos (-154^\circ)$
- q) $\sin 52^\circ$
- r) $\cos 52^\circ$
- s) $\tan 52^\circ$
- t) $2\cos^2 13^\circ - 1$
- u) $2\cos^2 26^\circ - 1$

- v) $1 - 2 \cos^2 13^\circ$
- w) $1 - 2 \sin^2 13^\circ$
- x) $2 \sin^2 13^\circ - 1$
- y) $\cos^2 13^\circ - \sin^2 13^\circ$
- z) $\cos 56^\circ$
- aa) $\sin 71^\circ$
- ab) $\cos 86^\circ$
- ac) $\sin 13^\circ \cdot \cos 13^\circ$
- ad) $\cos 13^\circ$
- ae) $\sin 13^\circ$

1.2 Repeat all above questions considering the following

- (a) $\sin 154^\circ = k$
- (b) $\sin 26^\circ = \sqrt{k}$

2. Given that $\tan x = p$, in any right-angled triangle:

Show that $\sin 2x = \frac{2p}{p^2+1}$

3. Given that: $17 \sin x - 8 = 0$, where $90^\circ \leq x \leq 270^\circ$

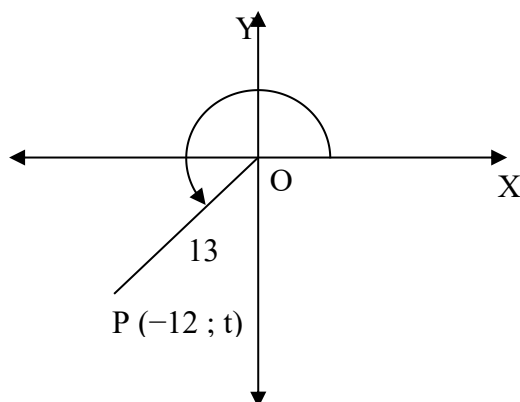
- a. $\sin 2x$
- b. $\cos 2x$
- c. $\tan 2x$
- d. $\cos (90^\circ - x)$
- f. $\sin (x - 45^\circ)$
- g. $\cos (x + 60^\circ)$
- h. $\tan (90^\circ + x)$

4. if $\tan x = 2,4$ and $90^\circ \leq x \leq 270^\circ$ and then evaluate $\tan 2x$

5. It is known that $13\sin\alpha - 5 = 0$ and $4\tan\beta + 3 = 0$ where $\alpha \in [90^\circ; 270^\circ]$ and $\beta \in [90^\circ; 270^\circ]$, without using a calculator, determine the value:

- $\cos(\alpha + \beta)$
- $\sin(\beta + \alpha)$
- $\cos(\alpha - \beta)$
- $\sin(\beta - \alpha)$
- $\tan(\beta - \alpha)$
- $\sin(\alpha - 300^\circ) - \cos(\beta - 150^\circ)$

6. In the diagram below, reflex angle $\widehat{XOP} = \theta$. P is the point $(-12; t)$ and $OP = 13$ units.

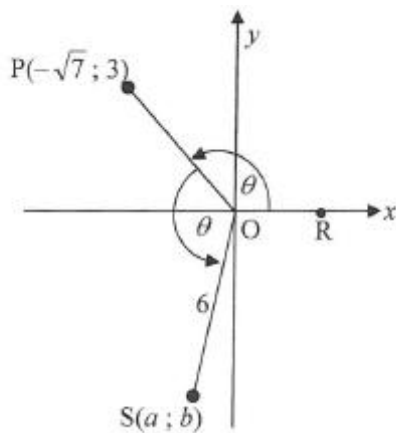


Calculate, without using a calculator:

- The value of t
- $\sin 2\theta$
- $\frac{1 - \cos 2\theta}{\tan 2\theta}$

4. $P(-\sqrt{7}; 3)$ and $S(a; b)$ are points on the Cartesian plane, as show in the diagram below.

$\widehat{POR} = \widehat{POS} = \theta$ and $OS = 6$.



Determine, without using a calculator, the value of:

- a. $\tan \theta$
- b. $\sin(-\theta)$
- c. a
- d. b
- e. θ

5. If $\sin 26^\circ \cos 16^\circ = k$ and $\cos 26^\circ \sin 16^\circ = w$, determine interms of k and w the value of

- a. $\sin 42^\circ$
- b. $\sin 10^\circ$
- c. $\cos 10^\circ$

6. If $\cos 2A = k$ and $\sin 2B = w$ and then determine the following in terms of k and w

- i. $\cos A$
- ii. $\sin A$
- iii. $\sin B \cos B$

7. Without using a calculator, determine the following in terms of $\sin 36^\circ$;

- a. $\sin 324^\circ$
- b. $\cos 72^\circ$

8. Simplify the following:

- a. If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ without using a calculator.
- b. Given that $\sin \alpha = \frac{1}{3}$, calculate the numerical value of $\sin 3\alpha$, without using a calculator.
- c. If $x = 3 \sin \alpha$ and $y = 3 \cos \alpha$, determine the value of $x^2 + y^2$

9. Write down the maximum and minimum of the following:

- i. $f(x) = \sin x$
- ii. $f(x) = \cos 2x$
- iii. $f(x) = 2 \sin x$
- iv. $f(x) = 1 + \sin x$
- v. $f(x) = \sin x \cos x$

2. Consider $\sin x + \cos x = \sqrt{1+k}$

- i. Determine the value of k
- ii. Hence, calculate maximum value of f

➤ QUESTION THREE

1. Determine the general solution of following:

a) $\cos x = 0.5$

b) $\sin 2x = 0.5$

c) $\tan \frac{1}{2}x = 0.5$

d) $2\cos x - 1 = 0$

e) $3\cos x = \sqrt{2}\sin x$

f) $2\sin x \cos x = 0.5$

g) $\sin 2x \cos 2x = \frac{1}{4}$

h) $2\cos^2 3x - 1 = 0,5$

i) $1 - 2\sin^2 2x = 0,5$

j) $\cos^2 x - \sin^2 x = 0,5$

k) $1 - 2\cos^2(x - 10^\circ) = 0,5$

l) $4\sin x \cos x = 2$

m) $\cos 2x - 4 \sin x + 5 = 0$

n) $\sin^2 x + \cos 2x - \cos x = 0$

o) $\sin x + 2\cos^2 x = 1$

p) $6\cos x - 5 = \frac{4}{\cos x} \quad ; \quad \cos x \neq 0$

q) $\cos 3x = \sin x$

r) $\cos 2x = \sin (x - 30^\circ) .$

s) $1 - \frac{\tan^2 \frac{1}{2}x}{1 + \tan^2 \frac{1}{2}x} = \frac{1}{4}$

t) $\sin (x + 64^\circ) \cos (x + 379^\circ) + \sin (x + 19^\circ) \cos (x + 244^\circ) = \frac{1}{\sqrt{2}}$

2. Calculate the values of x where $x \in [-180^\circ; 360^\circ]$

i. $\sin x = \cos 2x$

ii. $\sin x = \cos (x + 30^\circ)$

➤ **QUESTION FOUR**

1.1 On the set of axes provided in the answer book, draw the sketch graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x - 30^\circ)$ for $x \in [-180^\circ; 90^\circ]$. Clearly indicate the coordinates of the turning points and the intercepts with the axes.

1.2 Write down range for the following:

- a) $f(x)$ and $g(x)$
- b) $h(x) = -f(x)$
- c) $h(x) = f(x) + 2$

1.3 Write down the period for the following:

- a) $f(x)$ and $g(x)$
- b) $h(x) = f\left(\frac{2}{3}x\right)$

1.4 Write down the amplitude for the following:

- a) $f(x)$ and $g(x)$
- b) $h(x) = 2f(x)$
- c) $h(x) = -2f(x)$

1.5 Determine graphically the value(s) of x for which

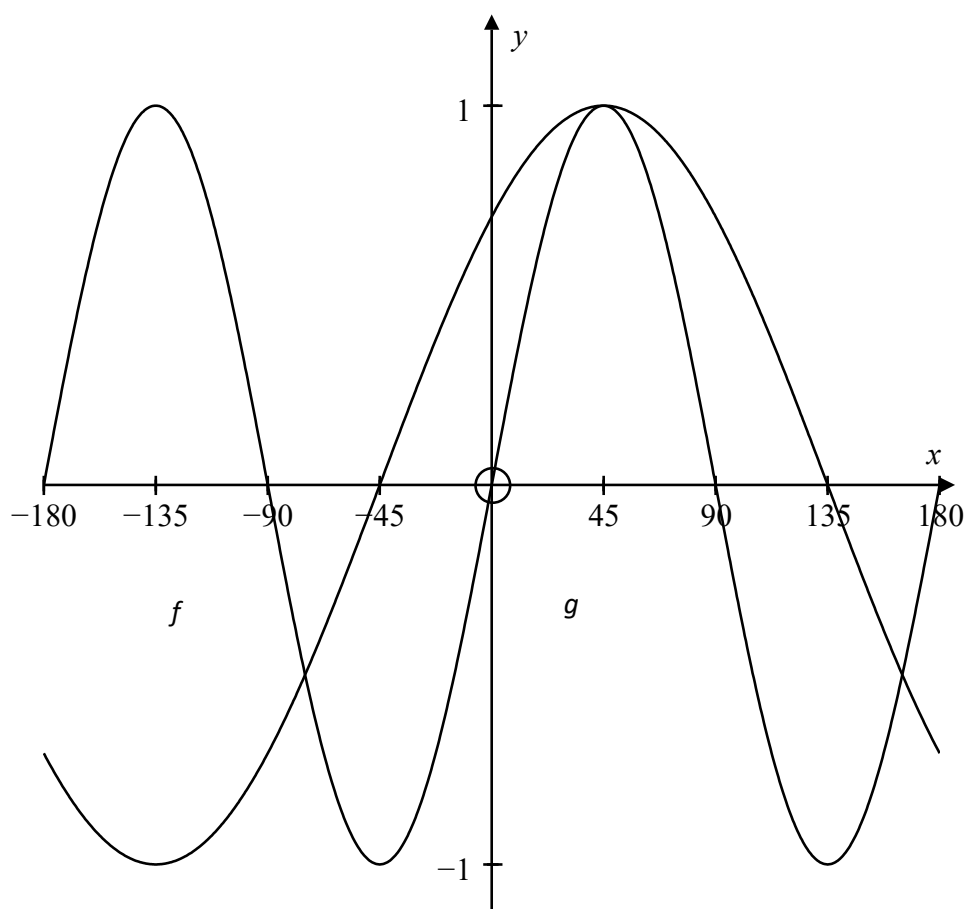
- a) $f(x) = g(x)$
- b) $f(x) < g(x)$
- c) $f(x) > g(x)$
- d) $f(x) \geq g(x)$
- e) $f(x) \cdot g(x) \leq 0$
- f) $f(x) \cdot g(x) \geq 0$
- h) $f(x) = 0$
- i) $f(x) \geq 0$

- j) $g(x) = 0$
- k) $g(x) < 0$
- l) $g'(x) > 0$ And $f(x) > 0$
- m) $g(x) - f(x) = 1.5$
- n) the graph of f and g decreasing
- o) $g(x) \geq -1$
- p) the graph of f and g increasing
- q) $\frac{f(x)}{g(x)} > 0$
- r) $\frac{g(x)}{f(x)} \leq 0$

2. Given: $f(x) = 2\cos x$ and $g(x) = \tan 2x$

- a. Sketch the graph of f and g on the same set of axes, for $x \in [-90^\circ; 90^\circ]$.
- b. Solve for x if $2\cos x = \tan 2x$ and $x \in [-90; 90]$. Show all working details.
- c. Use the graph to solve for x : $2\cos x \cdot \tan 2x > 0$.
- d. Write down the equation of the asymptotes of $g(x - 25^\circ)$, where $x \in [-90^\circ; 90]$.

3. The graphs of $f(x) = \{(x; y)/y = \sin ax\}$ and $g(x) = \{(x; y)/y = \cos(x - b)\}$ for the domain $x \in [-180^\circ; 180^\circ]$ are shown in the diagram below



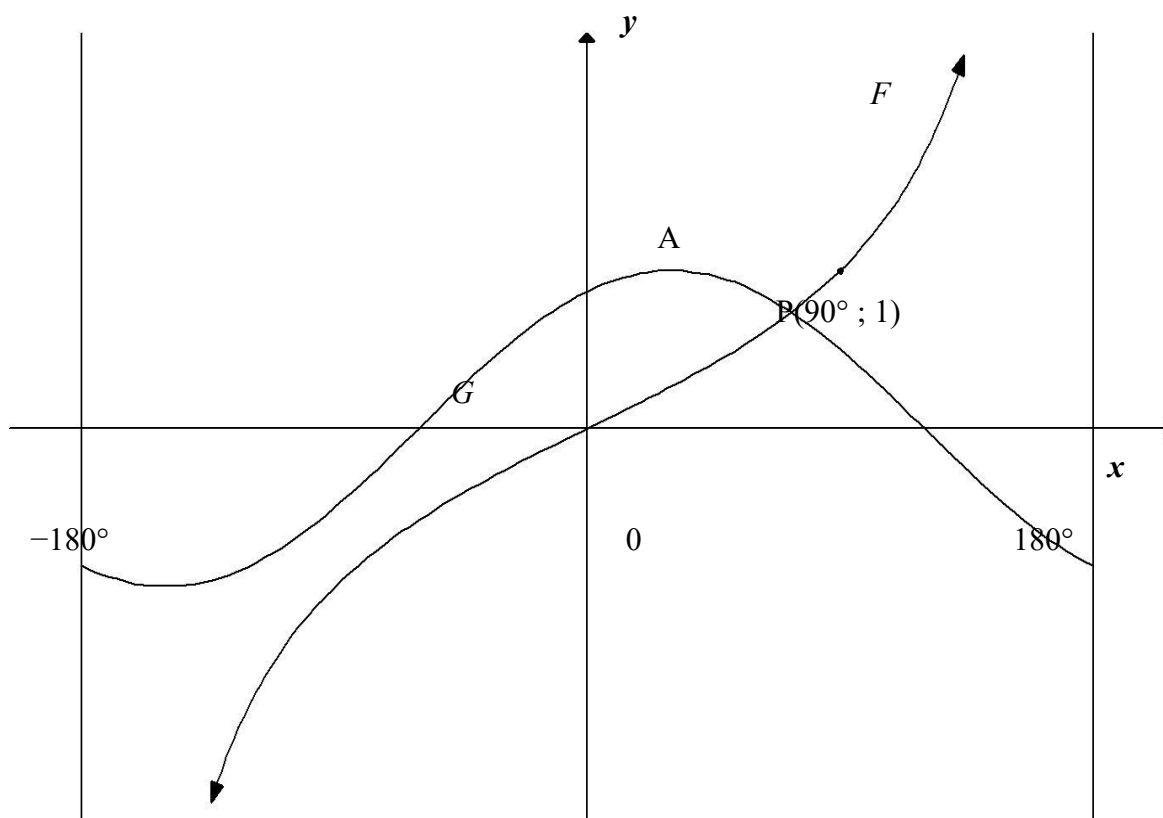
- Write down the values of a and b .
- Write down the period of g .
- Determine the equation of h if $h(x) = f(x - 30^\circ)$.
- Explain how you would use the graphs to solve the equation:

$$\sqrt{2} \sin 2x = \cos x + \sin x.$$

- write down the amplitude of :

- f
- $h(x) = f\left(\frac{2}{3}x\right)$
- $k(x) = \frac{f(x)}{2}$

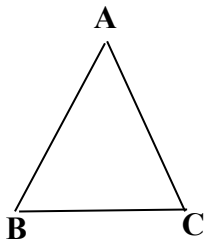
4. In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30^\circ)$ are drawn on the same system of axes for $-180^\circ \leq x \leq 180^\circ$. The point P (90° ; 1) lies on f . Use the diagram to answer the following questions.



- Determine the value of b .
- Write down the coordinates of A, a turning point of g .
- Write down the equation of the asymptote(s) of $y = \tan b(x + 20^\circ)$ for $x \in [-180^\circ; 180^\circ]$.
- Determine the range of h if $h(x) = 2g(x) + 1$.
- Write down the new equation of g if it is shifted 60 units to the right
- Write down the new equation of g if it is shifted 60 units to the left
- Write down the equation of $h(x) = f(x - 30^\circ) + 2$
- Write down the new asymptote of f if it is shifted 30 units to the right
- Write down the new equation for $k(x) = f(x + 60^\circ)$

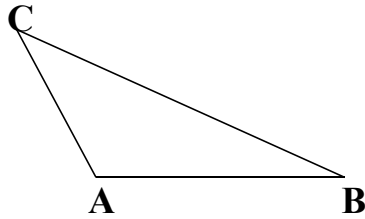
➤ **QUESTION FIVE**

1. Use the diagram below to prove that in any $\triangle ABC$.



$$\text{Area of } \triangle ABC = \frac{1}{2} a b \sin C$$

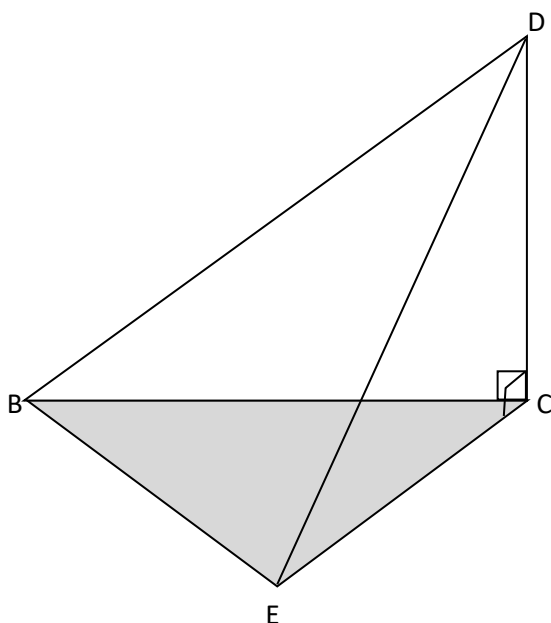
2. In $\triangle ABC$, $\hat{A}$ is obtuse.



Prove that $\frac{a}{\sin A} = \frac{b}{\sin B}$

3. Use the diagram in 2. to prove cosine rule

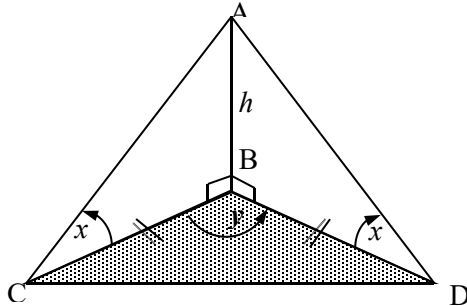
4. In the figure below, CD is a vertical mast. The points B, C and E are in the same horizontal plane. BD and ED are cables joining the top of the mast to pegs on the ground. $DE = 28,1$ m and $BC = 20,7$ m. The angle of elevation of D from B is $43,6^\circ$. $\hat{CBE} = 63^\circ$ and $\hat{BDE} = 35,7^\circ$.



Give your answers correct to ONE decimal place in each of the following questions:

- Calculate the length of BD.
- Calculate the length of BE.
- Calculate the area of $\triangle BEC$.
- Calculate the length of DC
- Calculate the length of DE
- Hence, calculate angle DEC
- Calculate the perimeter of BDCE

5. In the diagram, C and D represent two ships in the same horizontal plane as B, the base of a lighthouse AB which is h metres high. Each ship is the same distance away from the base of the lighthouse. The angles of elevation from C and D to A are both x . $\hat{C}BD = y$.



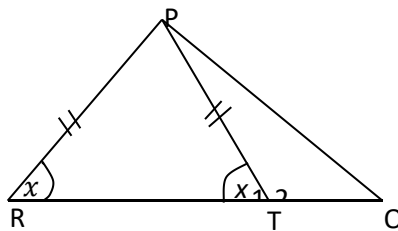
- Write down BD in terms of h and a trigonometric ratio of x .
- Write down the size of angle CDB
- Hence, or otherwise, prove that $CD^2 = \frac{2h^2(1 - \cos y)}{\tan^2 x}$
- Calculate CD if $h = 23 \text{ cm}$, $y = 150^\circ$ and $x = 30^\circ$

6. In the figure alongside

$\triangle PRT$, has $PR = PT$.

If $TO = \frac{1}{2} RT = a$

and $\hat{R} = \hat{T}_1 = x$,

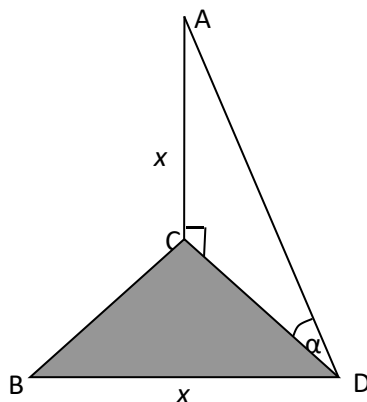


- Prove that $PT = \frac{a}{\cos x}$.
- Show that the area of $\triangle POT = \frac{1}{2} a^2 \tan x$

7. In the figure B and D are points in the same horizontal plane as C, the foot of a vertical tower AC.

If $AC = BD = x$ and

$$\angle BDC = 90^\circ - \alpha$$



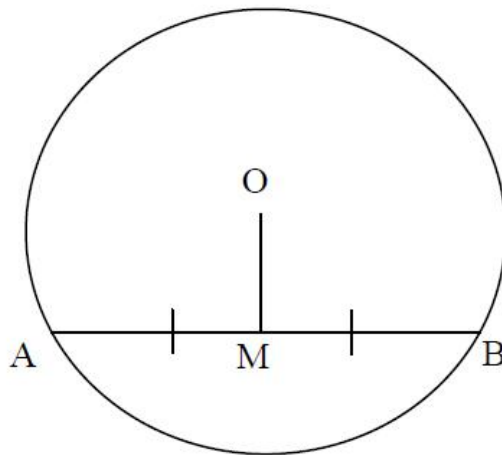
- Prove that $CD = \frac{x}{\tan \alpha}$.
- Show that $BC = \frac{x}{\tan \alpha} \sqrt{\tan^2 \alpha - 2 \sin \alpha \cdot \tan \alpha + 1}$

EUCLIDEAN GEOMETRY

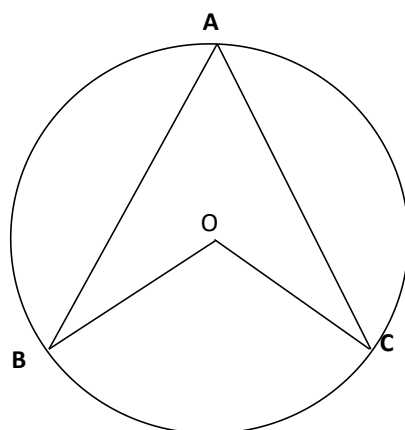
1. Complete the following statement:

- a. The line drawn from the centre of a circle perpendicular to a chord.....
- b. The opposite angles of a cyclic quadrilateral are
- c. The angle between the tangent and a chord is
- d. If two triangle are equiangular, the corresponding sides are.....
- e. The opposite angles of a cyclic quadrilateral are
- f. The angle subtended by an arc at the centre of the circle is
- g. The angle between a tangent to a circle and a chord is h.
The angle subtended by a chord at the centre of a circle is
- i. If two triangles are equiangular, then the corresponding sides are

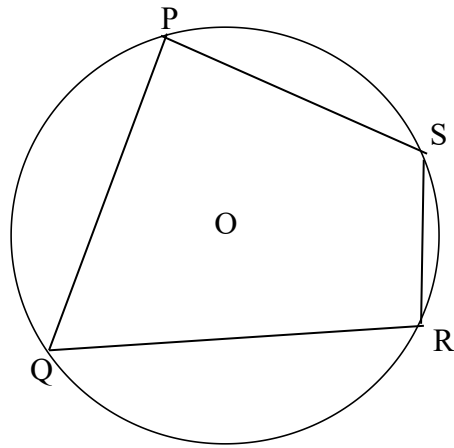
2. In the diagram below, AB is a chord of the circle with centre O. M is the midpoint of AB. Prove the theorem that states $OM \perp AB$.



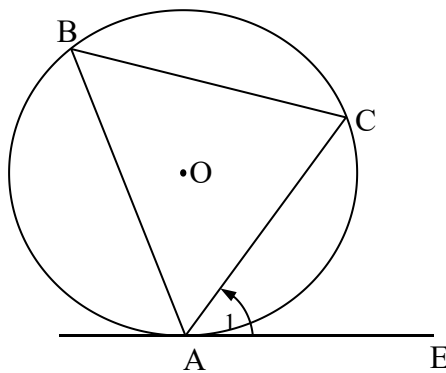
3. In the figure Below o is the centre of the circle and A,B and C are three points on the circumference of the circle. Use the figure and prove the theorem that states that $\angle BOC = 2\angle A$



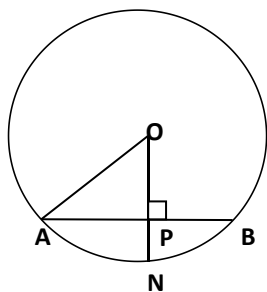
4. In the diagram below, O is the centre of the circle. Use the diagram to prove the theorem which states that: If PQRS is a cyclic quadrilateral then $\hat{PQR} + \hat{PSR} = 180^\circ$.



5. In the diagram below, O is the centre of the circle passing through A, B and C. EA is a tangent to the circle at A. Use this diagram to prove the theorem which states that $\hat{EAC} = \hat{ABC}$.



6.



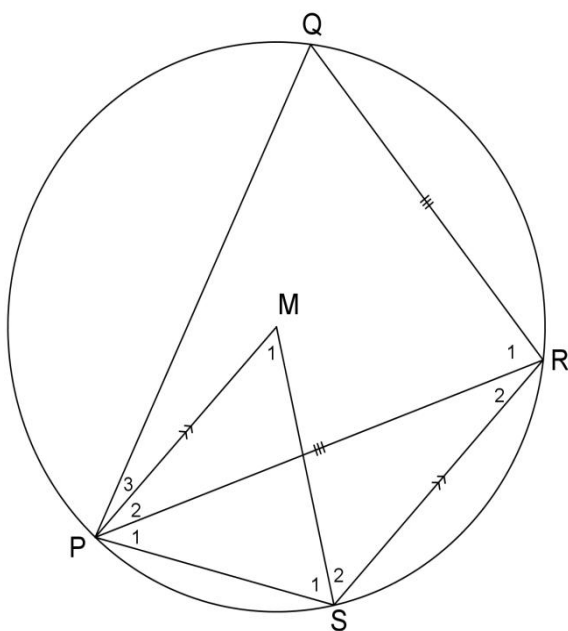
O is the centre of the circle. AB is a chord and

OP is extended and intersects the circle at N.

AB = 16 cm and PN = 2 cm.

- Calculate the length of AP.
- Calculate the length of the radius of the circle.
- Hence calculate the length of ON

7. In the diagram alongside, M is the centre of circle PQRS. $PM \parallel RS$, $QR = PR$ and $\hat{R}_2 = 28^\circ$

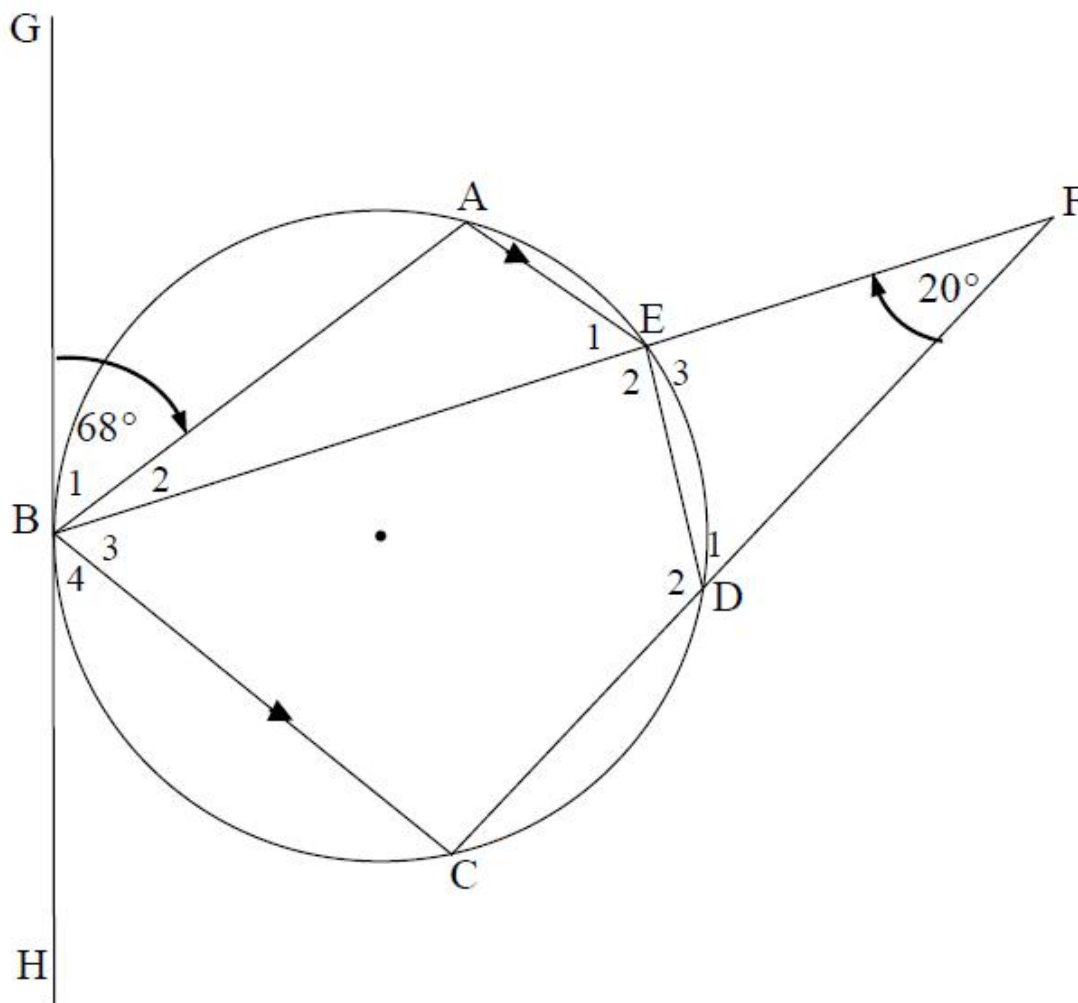


Determine, giving reasons, the size of the following angles:

- $\hat{S}_2$
- $\hat{PSR}$
- $\hat{Q}$
- $\hat{P}_3$

8. In the diagram, A, B, C, D and E are points on the circumference of the circle such that $AE \parallel BC$. BE and CD produced meet in F. GBH is a tangent to the circle at B.

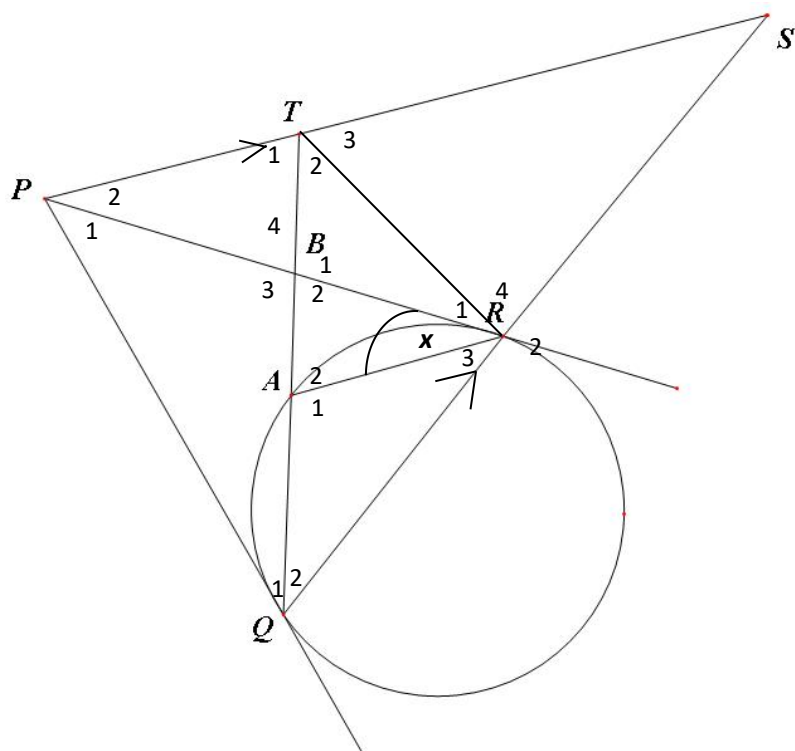
$\angle B_1 = 68^\circ$ and $\angle F = 20^\circ$.



Determine the size of each of the following:

- $\angle E_1$
- $\angle B_3$
- $\angle D_1$
- $\angle E_2$
- $\angle C$

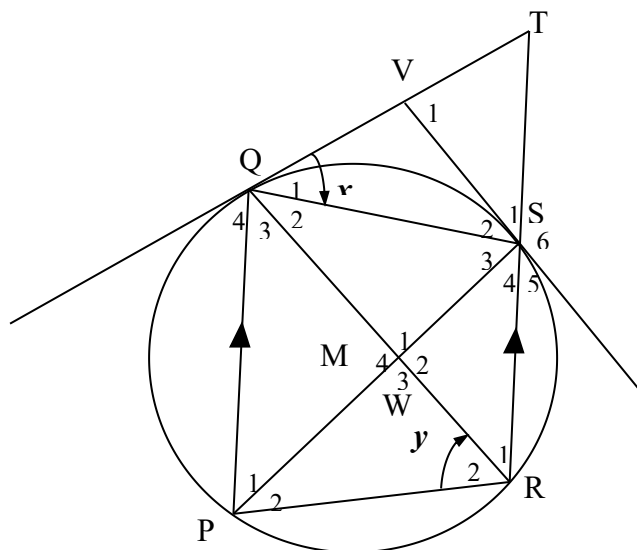
9. In the figure PR and PQ are two tangents drawn from point P to circle AQR. The straight line drawn through P parallel to AR meets QR produced at S, and QA produced at T. The tangent PR cuts QT at B.



Let $\hat{R}_2 = x$

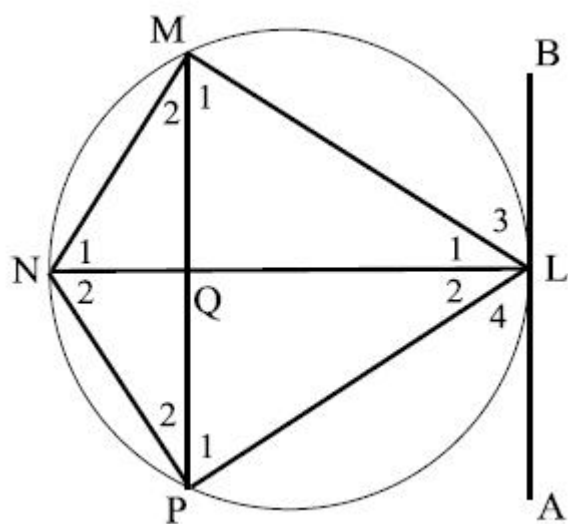
- a. Prove that PTRQ is a cyclic quadrilateral.
- b. If it is further given that $QA = RA$, prove that:
 - i) $\hat{S} = x$
 - ii) $PQ = RS$
 - iii) PTS is a tangent to circle TAR.

10. In the diagram below, PQ and RS are chords of the circle such that $PQ \parallel RS$. The tangent to the circle at Q meets RS produced at T and the tangent at S meets QT at V. PS and QR intersect at W. QS and PR are drawn. Let $\hat{Q}_1 = x$ and $\hat{R}_2 = y$.



- a) Write down a reason why $QV = VS$.
- b) Write down the following angles in terms of x :
 - (i) $\hat{S}_2$
 - (ii) $\hat{R}_1$
 - (iii) $\hat{V}_1$
- c) Show that $\hat{R}_1 = \hat{S}_4$
- d) Prove that QVSW is a cyclic quadrilateral.
- e) Write down the following angles in terms of y :
 - (i) $\hat{Q}_4$
 - (ii) $\hat{T}$

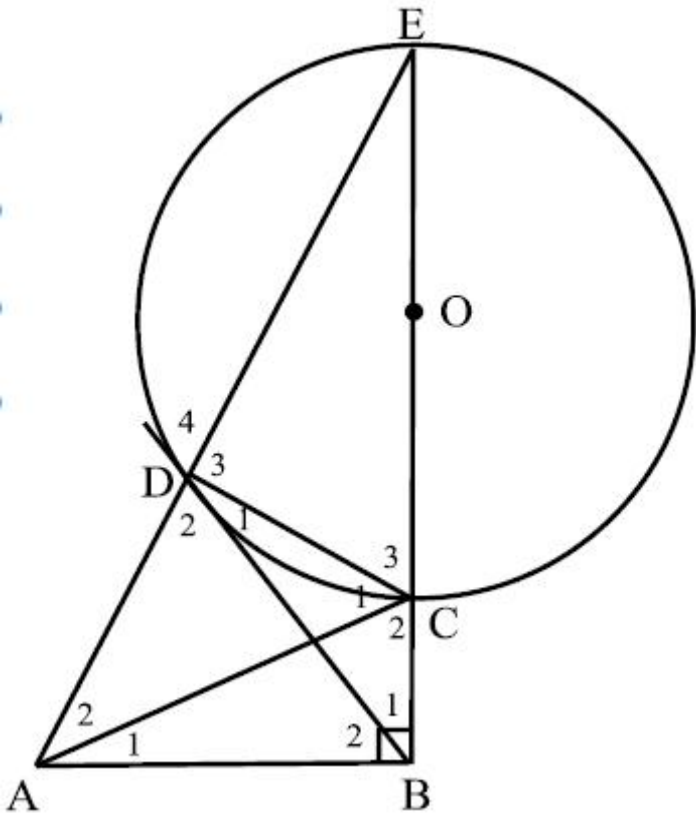
11. ALB is a tangent to circle LMNP. $ALB \parallel MP$.



Prove that:

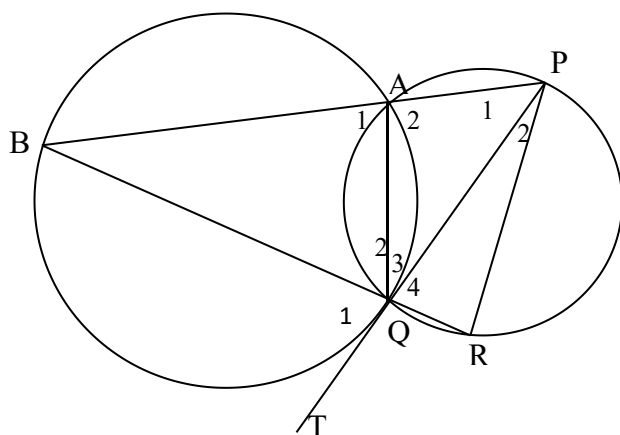
- $LM = LP$
- LN bisects angle MNP
- LM is a tangent to circle MNQ

12. EC is a diameter of circle DEC. EC is produced to B. BD is a tangent at D. ED is produced to A and $AB \perp BE$.



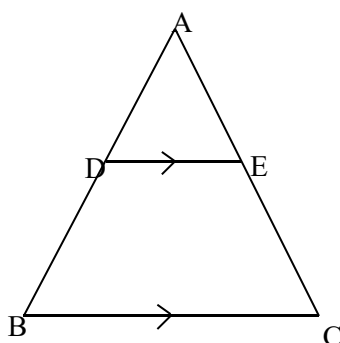
- Prove that:
- a. ABCD is a cyclic quadrilateral.
 - b. $\widehat{A_1} = \widehat{E}$
 - c. $BD = BA$
 - d. $\widehat{C_2} = \widehat{C_3}$

14. In the diagram below, PQT is a tangent to the larger circle ABQ at Q. A smaller circle intersects the larger circle at A and Q. BAP and BQR are straight lines with P and R on the smaller circle. AQ and PR are drawn.



- Prove that $PQ = PR$
- Prove that $\triangle PBQ \parallel \triangle PQA$.
- Prove that the lengths of PA, PR and PB (in this order) form a geometric sequence.

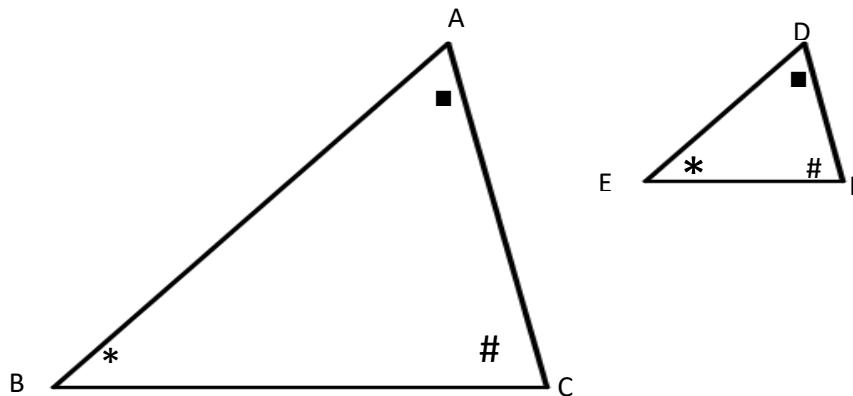
15. Given $\triangle ABC$ with $DE \parallel BC$ as shown in the figure below:



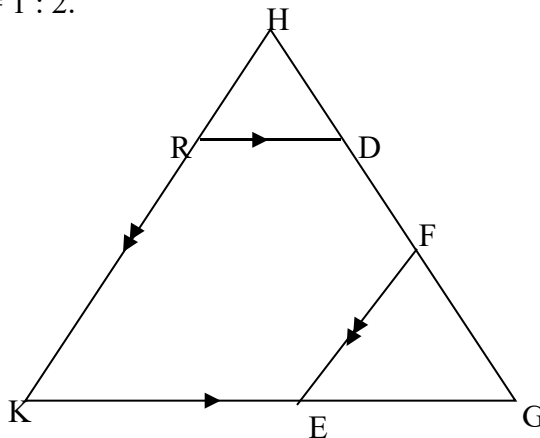
Prove that: $\frac{AD}{DB} = \frac{AE}{EC}$

16. In the diagram below, ABC and DEF are given with. Use the diagram in the

ANSWER BOOK to prove the theorem that states that $\frac{DE}{AB} = \frac{DF}{AC}$.



17. In the diagram below, $\triangle GHK$ is drawn having the point R on KH and the points D and F on GH such that $RD \parallel KG$ and $EF \parallel KH$. It is also given that $RH = 3$ units, $RK = 9$ units, $HD = 2$ units and $GE : EK = 1 : 2$.



Calculate the length of:

- $\frac{GF}{HF}$
- $\frac{GF}{GH}$
- $\frac{GH}{HF}$
- $\frac{HD}{GD}$
- $\frac{HG}{DG}$
- DG

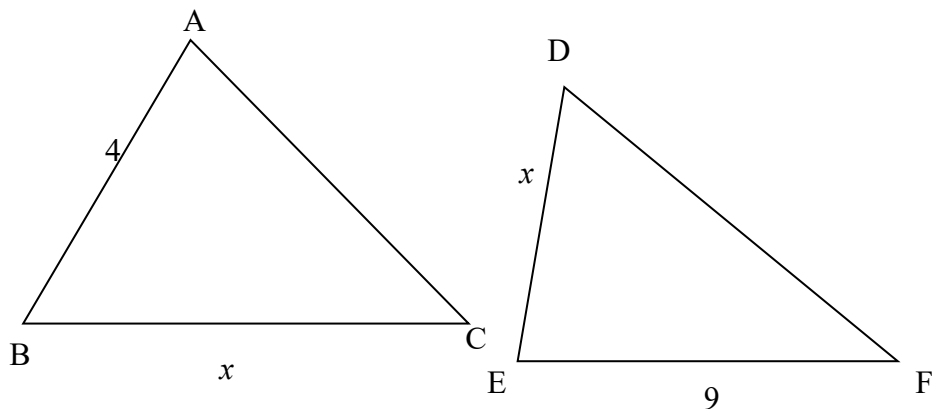
g. FD

h. Calculate the value of $\frac{\text{area of } \triangle KHG}{\text{area of } \triangle EFG}$

i. Calculate the value of $\frac{\text{area of } \triangle RHD}{\text{area of } \triangle KHG}$

j. Calculate the ratio of $\text{area of } \triangle RHD : \text{area of } \triangle KHG$

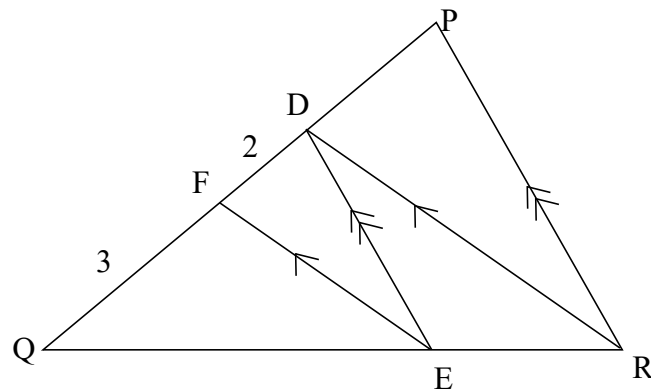
18. In the diagram below, $\triangle ABC$ and $\triangle DEF$ are drawn. $AB = 4$ units, $BC = (x + 9)$ units, $DE = x$ units and $EF = 9$ units.



a) If $\triangle ABC \parallel \triangle DEF$, calculate the value of x .

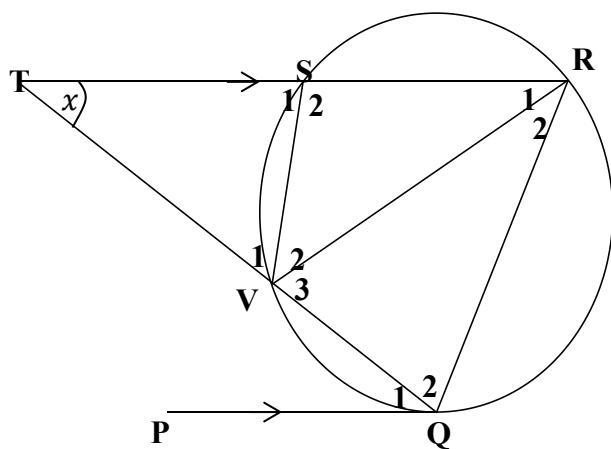
b) Hence, write down the length of BC

19. In the diagram below, $DE \parallel PR$, $FE \parallel DR$, $QF = 3$ cm, $FD = 2$ cm.



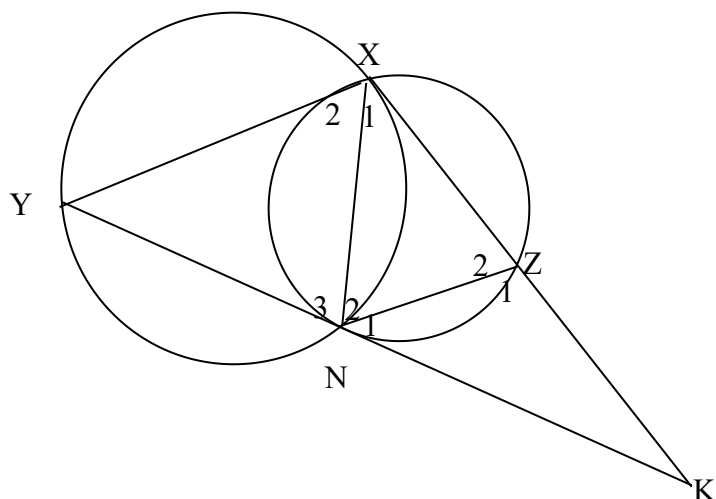
- a. Determine the value of $\frac{QE}{QR}$.
- b. Calculate the length of DP.
- c. Determine the value of $\frac{QE}{ER}$.

20. In the diagram below, PQ is a tangent to the circle at Q. TSR is a line which cuts the circle at S such that $TR \parallel PQ$. QV is produced to meet RST at T. $\hat{T} = x$.



- Write, down with reasons, TWO other angles each equal to x .
- Prove that $TSV \parallel RQV$.
- Prove that $TS \cdot TR = TV \cdot TQ$

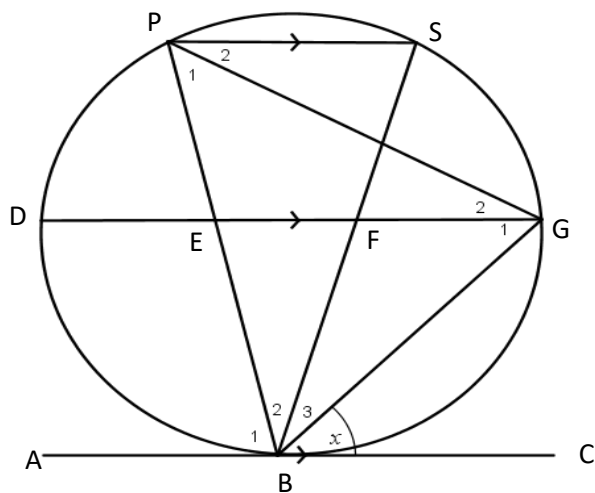
21. Two circles intersect at X and N. KX is a tangent to the larger circle XYN at X and KX cuts the smaller circle at Z. KNY is a tangent to the circle XZN at N. XN, XY and NZ are drawn.



- Prove that $YX \parallel NZ$.
- Write down two triangles similar to $\triangle XYK$.
- Prove that $ZK = \frac{XK^3}{KY^2}$.

22. In the diagram, P, S, G, B and D are points on the circumference of the circle such that

$PS \parallel DG \parallel AC$. ABC is a tangent to the circle at B.



a. Give a reason why $\widehat{G_1} = x$

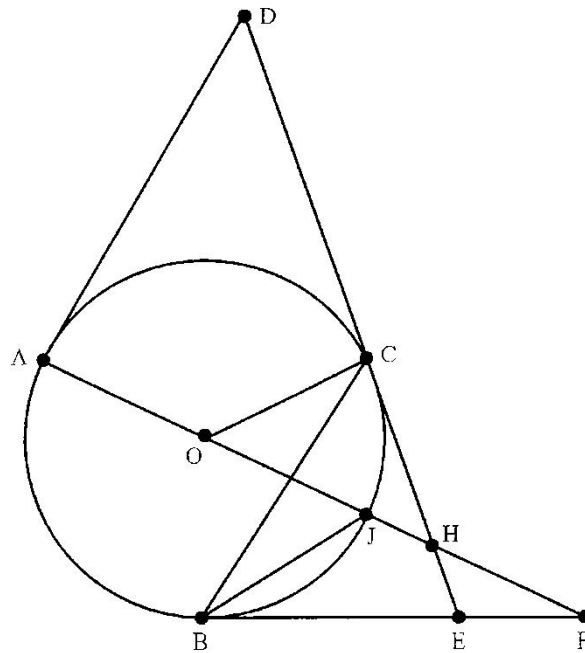
b. Prove that:

i. $BE = \frac{BP \cdot BF}{BS}$

ii. $\triangle BGP \parallel \triangle BEG$

iii. $\frac{BG^2}{BP^2} = \frac{BF}{BS}$

23. In the figure, AD, DC and BE are tangents to the circle. CO is a radius and chord BC is drawn. Radius AO is drawn and extended to cut the circle at J and BE is extended at F.

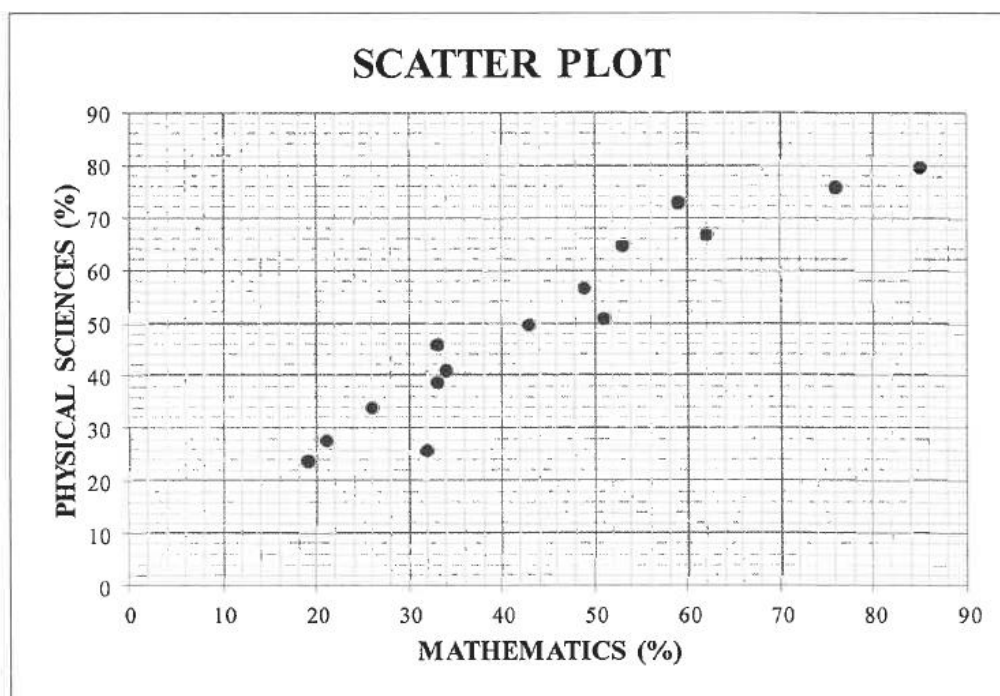


- Prove $\triangle DAH \parallel \triangle OCH$
- Prove $OH = \frac{AO \cdot DH}{DC}$
- Prove $\triangle JBF \parallel \triangle BAF$
- Prove $BF^2 = JF \cdot AF$

QUESTION 1

A Mathematics teacher was curious to establish if her learners' Mathematics marks influenced their Physical Sciences marks. In the table below, the Mathematics and Physical Sciences marks of 15 learners in her class are given as percentages (%).

MATHEMATICS (AS %)	26	62	21	33	53	76	32	59	43	33	49	51	19	34	85
PHYSICAL SCIENCES (AS %)	34	67	28	46	65	76	26	73	50	39	57	51	24	41	80



- 1.1 Determine the equation of the least squares regression line for the data. (3)
 - 1.2 Draw the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)
 - 1.3 Predict the Physical Sciences mark of a learner who achieved 69% for Mathematics. (2)
 - 1.4 Write down the correlation coefficient between the Mathematics and Physical Sciences marks for the data. (1)
 - 1.5 Comment on the strength of the correlation between the Mathematics and Physical Sciences marks for the data. (1)
 - 1.6 What trend did the teacher observe between the results of the two subjects? (1)
- [10]**

QUESTION 1

The table below shows the monthly income (in rands) of 6 different people and the amount (in rands) that each person spends on the monthly repayment of a motor vehicle.

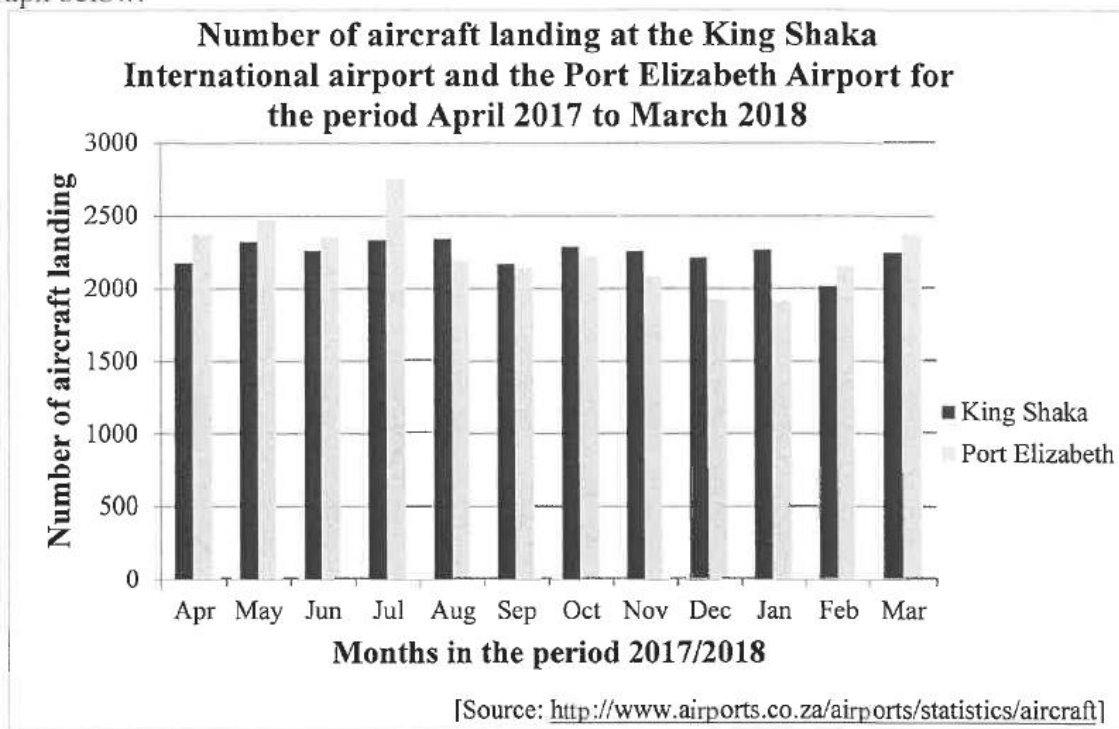
MONTHLY INCOME (IN RANDS)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS)	2 000	3 000	3 500	5 200	5 500	6 000

- 1.1 Determine the equation of the least squares regression line for the data. (3)
- 1.2 If a person earns R14 000 per month, predict the monthly repayment that the person could make towards a motor vehicle. (2)
- 1.3 Determine the correlation coefficient between the monthly income and the monthly repayment of a motor vehicle. (1)
- 1.4 A person who earns R18 000 per month has to decide whether to spend R9 000 as a monthly repayment of a motor vehicle, or not. If the above information is a true representation of the population data, which of the following would the person most likely decide on:
- A Spend R9 000 per month because there is a very strong positive correlation between the amount earned and the monthly repayment.
- B NOT to spend R9 000 per month because there is a very weak positive correlation between the amount earned and the monthly repayment.
- C Spend R9 000 per month because the point (18 000 ; 9 000) lies very near to the least squares regression line.
- D NOT to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. (2)

[8]

QUESTION 2

The number of aircraft landing at the King Shaka International Airport and the Port Elizabeth Airport for the period starting in April 2017 and ending in March 2018, is shown in the double bar graph below.



- 2.1 The number of aircraft landing at the Port Elizabeth Airport exceeds the number of aircraft landing at the King Shaka International Airport during some months of the given period. During which month is this difference the greatest? (1)
- 2.2 The number of aircraft landing at the King Shaka International Airport during these months are:
- | | | | | | |
|-------|-------|-------|-------|-------|-------|
| 2 182 | 2 323 | 2 267 | 2 334 | 2 346 | 2 175 |
| 2 293 | 2 263 | 2 215 | 2 271 | 2 018 | 2 254 |
- Calculate the mean for the data. (2)
- 2.3 Calculate the standard deviation for the number of aircraft landing at the King Shaka International Airport for the given period. (2)
- 2.4 Determine the number of months in which the number of aircraft landing at the King Shaka International Airport were within one standard deviation of the mean. (3)
- 2.5 Which ONE of the following statements is CORRECT?
- A. During December and January, there were more landings at the Port Elizabeth Airport than at the King Shaka International Airport.
 - B. There was a greater variation in the number of aircraft landing at the King Shaka International Airport than at the Port Elizabeth Airport for the given period.
 - C. The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of landings at the King Shaka International Airport.
- (1)
[9]

QUESTION 2

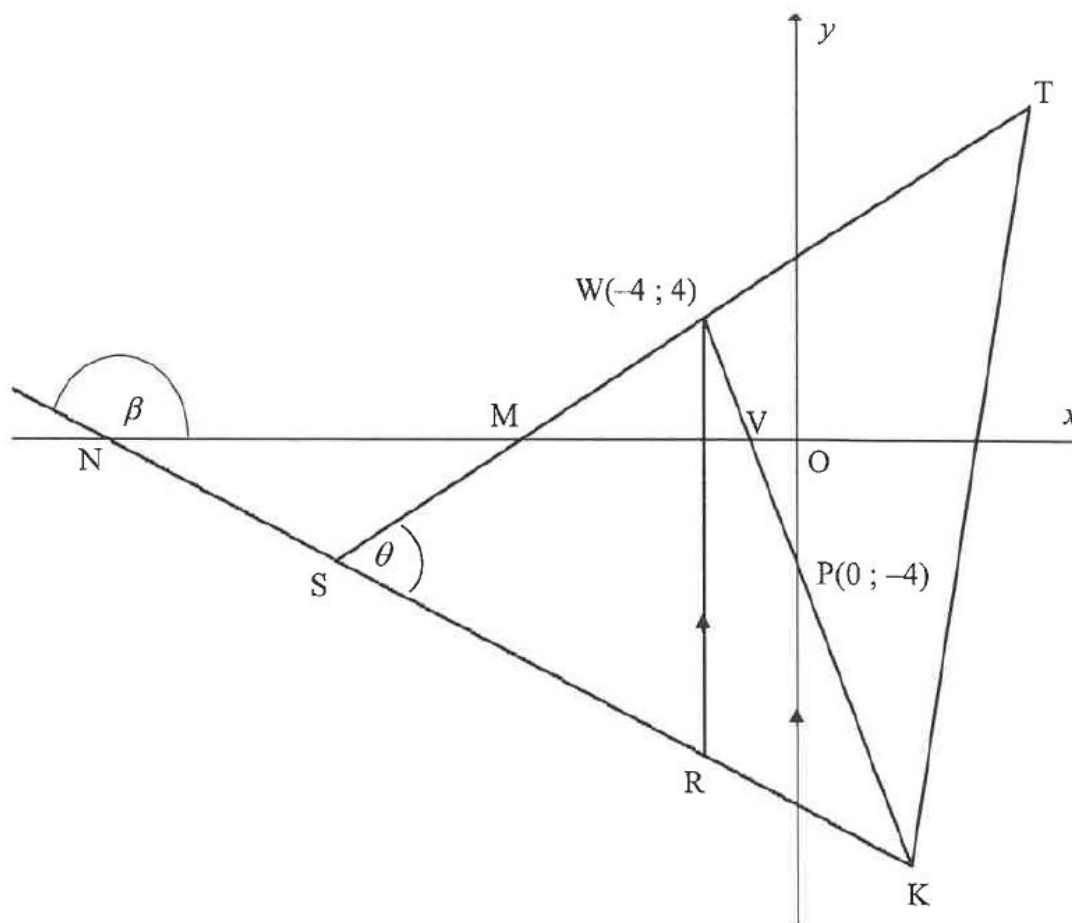
A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

- 2.1 How many people paid R200 or less on their monthly cellphone contracts? (1)
- 2.2 Use the information above to show that $a = 24$ and $b = 16$. (5)
- 2.3 Write down the modal class for the data. (1)
- 2.4 On the grid provided in the ANSWER BOOK, draw an ogive (cumulative frequency graph) to represent the data. (4)
- 2.5 Determine how many people paid more than R420 per month for their cellphone contracts. (2)
- [13]**

QUESTION 3

$\triangle TSK$ is drawn. The equation of ST is $y = \frac{1}{2}x + 6$ and ST cuts the x -axis at M . $W(-4; 4)$ lies on ST and R lies on SK such that WR is parallel to the y -axis. WK cuts the x -axis at V and the y -axis at $P(0; -4)$. KS produced cuts the x -axis at N . $\hat{TSK} = \theta$.

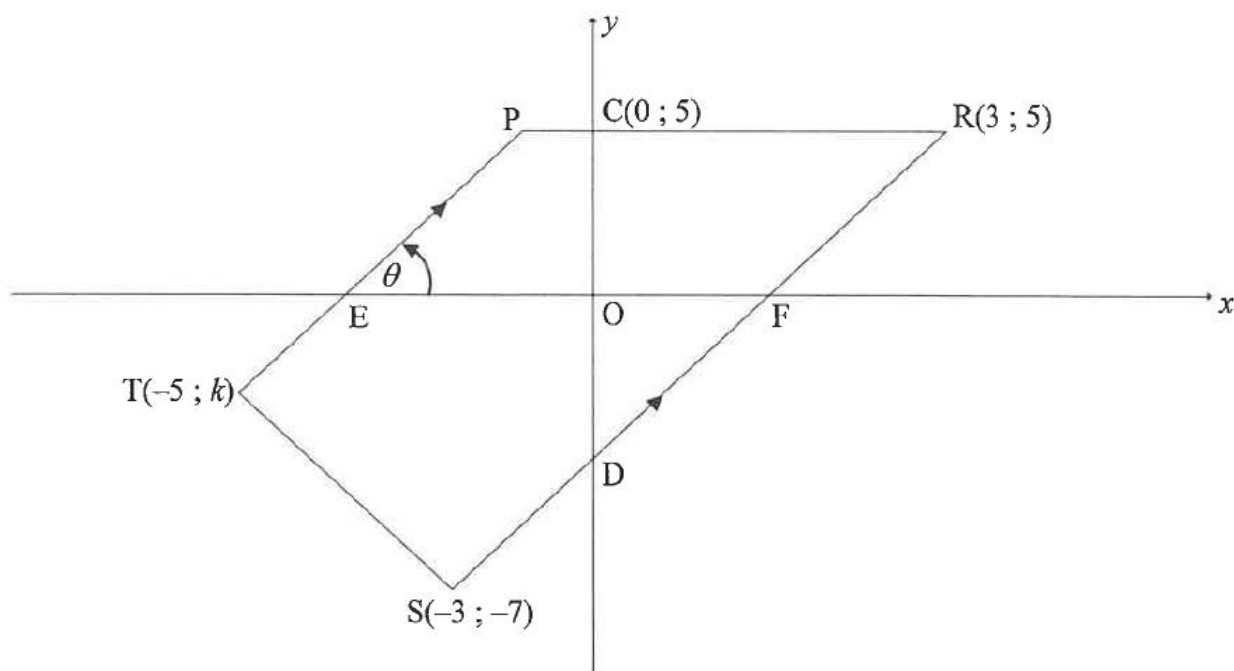


- 3.1 Calculate the gradient of WP . (2)
- 3.2 Show that $WP \perp ST$. (2)
- 3.3 If the equation of SK is given as $5y + 2x + 60 = 0$, calculate the coordinates of S . (4)
- 3.4 Calculate the length of WR . (4)
- 3.5 Calculate the size of θ . (5)
- 3.6 Let L be a point in the third quadrant such that $SWRL$, in that order, forms a parallelogram. Calculate the area of $SWRL$. (4)

[21]

QUESTION 3

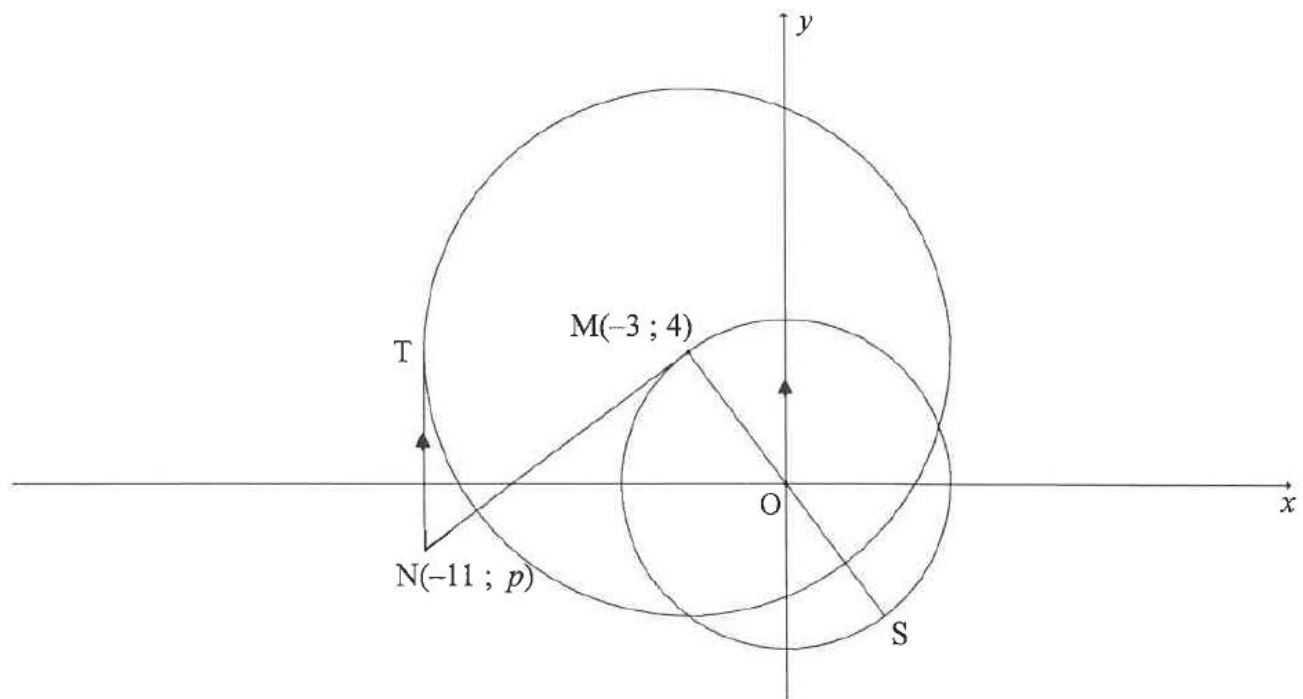
In the diagram, P, R(3 ; 5), S(-3 ; -7) and T(-5 ; k) are vertices of trapezium PRST and $PT \parallel RS$. RS and PR cut the y-axis at D and C(0 ; 5) respectively. PT and RS cut the x-axis at E and F respectively. $\hat{PEF} = \theta$.



- 3.1 Write down the equation of PR. (1)
 - 3.2 Calculate the:
 - 3.2.1 Gradient of RS (2)
 - 3.2.2 Size of θ (3)
 - 3.2.3 Coordinates of D (3)
 - 3.3 If it is given that $TS = 2\sqrt{5}$, calculate the value of k . (4)
 - 3.4 Parallelogram TDNS, with N in the 4th quadrant, is drawn. Calculate the coordinates of N. (3)
 - 3.5 $\triangle PRD$ is reflected about the y-axis to form $\triangle P'R'D'$. Calculate the size of $\hat{R'D'R'}$. (3)
- [19]**

QUESTION 4

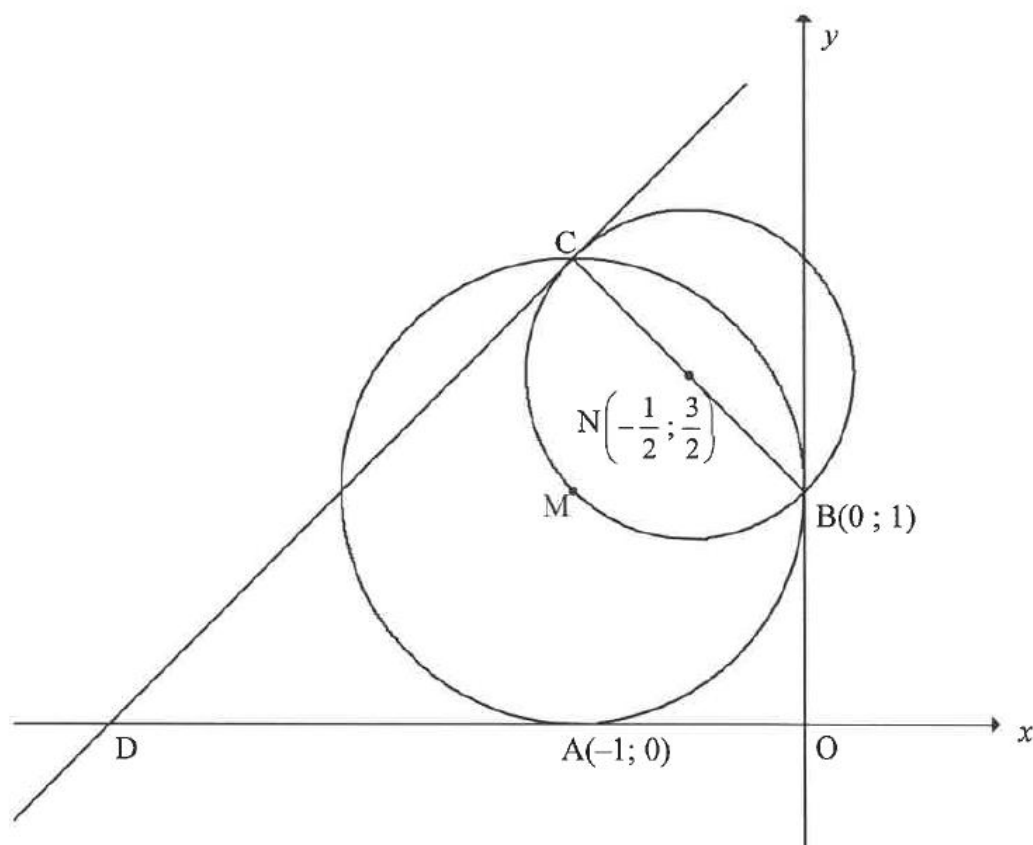
$M(-3 ; 4)$ is the centre of the large circle and a point on the small circle having centre $O(0 ; 0)$. From $N(-11 ; p)$, a tangent is drawn to touch the large circle at T with NT is parallel to the y -axis. NM is a tangent to the smaller circle at M with MOS a diameter.



- 4.1 Determine the equation of the small circle. (2)
 - 4.2 Determine the equation of the circle centred at M in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)
 - 4.3 Determine the equation of NM in the form $y = mx + c$ (4)
 - 4.4 Calculate the length of SN . (5)
 - 4.5 If another circle with centre $B(-2 ; 5)$ and radius k touches the circle centred at M , determine the value(s) of k , correct to ONE decimal place. (5)
- [19]

QUESTION 4

In the diagram, a circle having centre M touches the x -axis at $A(-1; 0)$ and the y -axis at $B(0; 1)$. A smaller circle, centred at $N\left(-\frac{1}{2}; \frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C . BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C , cuts the x -axis at D .

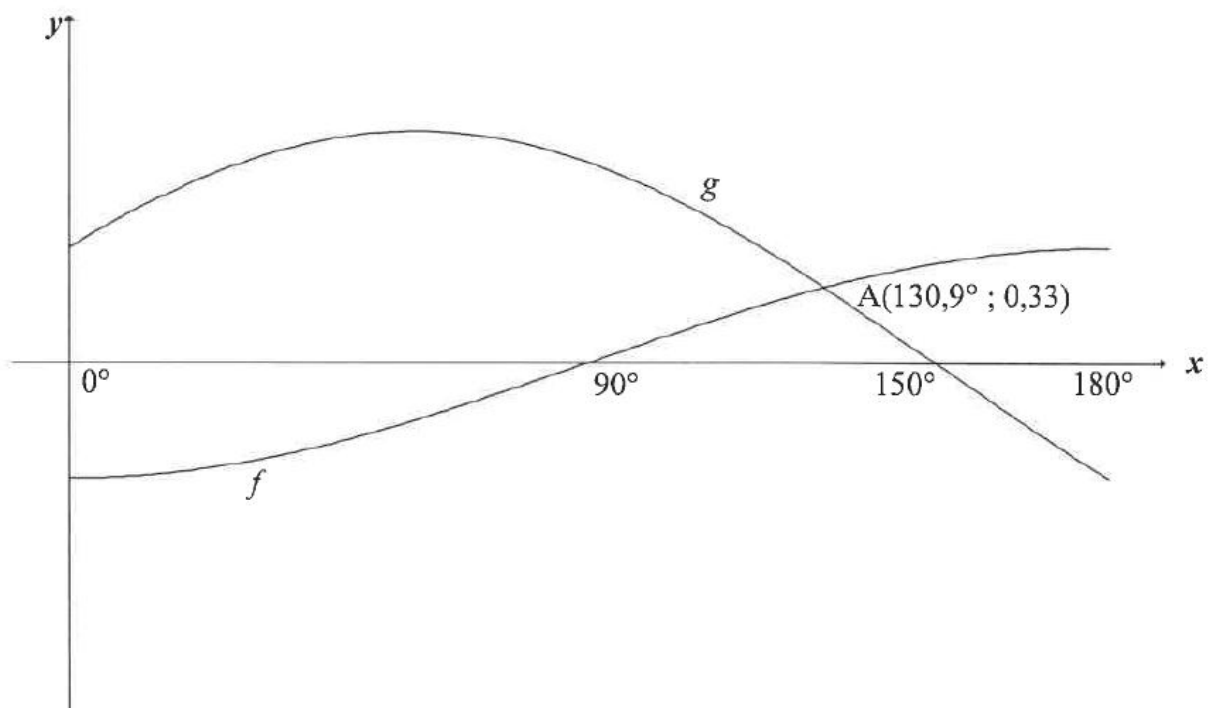


- 4.1 Determine the equation of the circle centred at M in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)
- 4.2 Calculate the coordinates of C . (2)
- 4.3 Show that the equation of the tangent CD is $y - x = 3$. (4)
- 4.4 Determine the values of t for which the line $y = x + t$ will NOT touch or cut the smaller circle. (3)
- 4.5 The smaller circle centred at N is transformed such that point C is translated along the tangent to D . Calculate the coordinates of E , the new centre of the smaller circle. (3)
- 4.6 If it is given that the area of quadrilateral $OBCD$ is $2a^2$ square units and $a > 0$, show that $a = \frac{\sqrt{7}}{2}$ units. (5)

[20]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}\cos x$ and $g(x) = \sin(x + 30^\circ)$, for the interval $x \in [0^\circ; 180^\circ]$, are drawn below. $A(130,9^\circ; 0,33)$ is the approximate point of intersection of the two graphs.



- 5.1 Write down the period of g . (1)
- 5.2 Write down the amplitude of f . (1)
- 5.3 Determine the value of $f(180^\circ) - g(180^\circ)$. (1)
- 5.4 Use the graphs to determine the values of x , in the interval $x \in [0^\circ; 180^\circ]$, for which:
- 5.4.1 $f(x - 10^\circ) = g(x - 10^\circ)$ (1)
- 5.4.2 $\sqrt{3} \sin x + \cos x \geq 1$ (4)
- [8]

QUESTION 5

5.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x \cdot \tan x} + \sin(180^\circ + x) \cos(90^\circ - x) \quad (5)$$

5.2 **Without using a calculator**, determine the value of: $\frac{\sin^2 35^\circ - \cos^2 35^\circ}{4 \sin 10^\circ \cos 10^\circ}$ (4)

5.3 Given: $\cos 26^\circ = m$

Without using a calculator, determine $2 \sin^2 77^\circ$ in terms of m . (4)

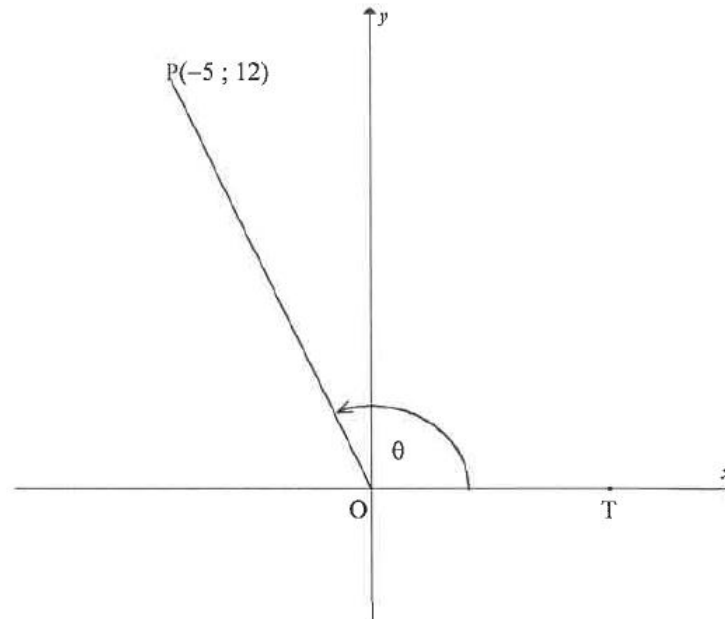
5.4 Consider: $f(x) = \sin(x + 25^\circ) \cos 15^\circ - \cos(x + 25^\circ) \sin 15^\circ$

5.4.1 Determine the general solution of $f(x) = \tan 165^\circ$ (6)

5.4.2 Determine the value(s) of x in the interval $x \in [0^\circ; 360^\circ]$ for which $f(x)$ will have a minimum value. (3)
[22]

QUESTION 6

- 6.1 In the diagram, $P(-5 ; 12)$ and T lies on the positive x -axis. $\widehat{POT} = \theta$



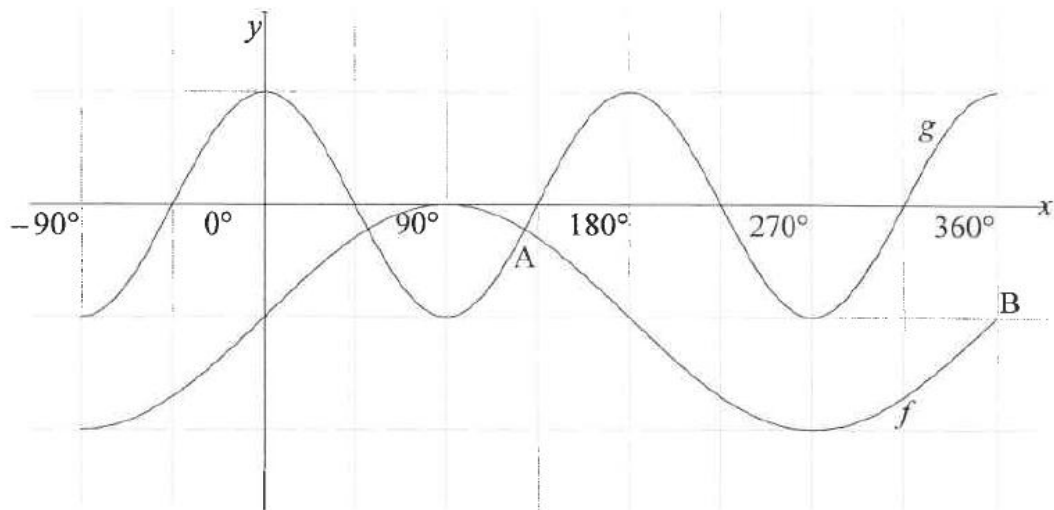
Answer the following **without using a calculator**:

- 6.1.1 Write down the value of $\tan \theta$ (1)
- 6.1.2 Calculate the value of $\cos \theta$ (3)
- 6.1.3 $S(a ; b)$ is a point in the third quadrant such that $\widehat{TOS} = \theta + 90^\circ$ and $OS = 6,5$ units. Calculate the value of b . (4)
- 6.2 Determine, **without using a calculator**, the value of the following trigonometric expression:
- $$\frac{\sin 2x \cdot \cos(-x) + \cos 2x \cdot \sin(360^\circ - x)}{\sin(180^\circ + x)} \quad (5)$$
- 6.3 Determine the general solution of the following equation:
- $$6\sin^2 x + 7\cos x - 3 = 0 \quad (6)$$
- 6.4 Given: $x + \frac{1}{x} = 3\cos A$ and $x^2 + \frac{1}{x^2} = 2$

Determine the value of $\cos 2A$ **without using a calculator**. (5)
[24]

QUESTION 6

In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90^\circ; 360^\circ]$. Graphs f and g intersect at A. $B(360^\circ; -1)$ is a point on f .

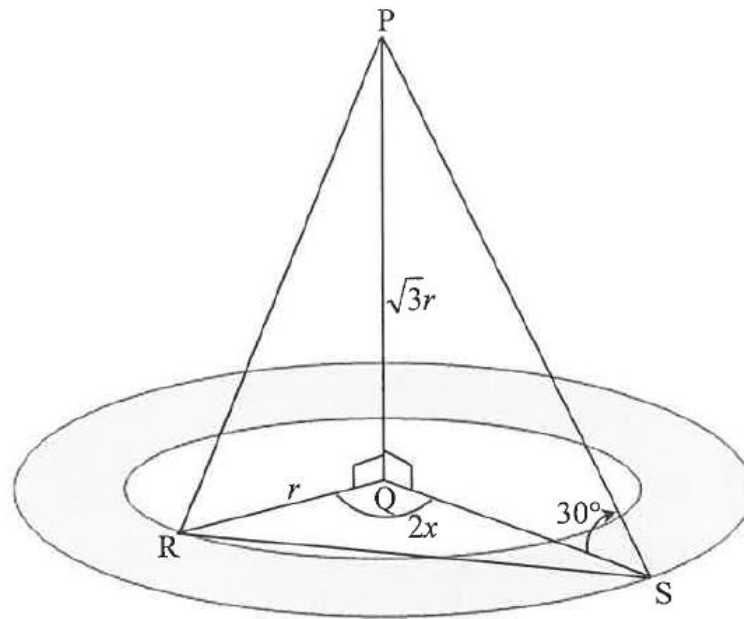


- 6.1 Write down the range of f . (2)
- 6.2 Write down the values of x in the interval $x \in [-90^\circ; 360^\circ]$ for which graph f is decreasing. (2)
- 6.3 P and Q are points on graphs g and f respectively such that PQ is parallel to the y -axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum. (6)
- [10]

QUESTION 7

A landscape artist plans to plant flowers within two concentric circles around a vertical light pole PQ . R is a point on the inner circle and S is a point on the outer circle. R , Q and S lie in the same horizontal plane. RS is a pipe used for the irrigation system in the garden.

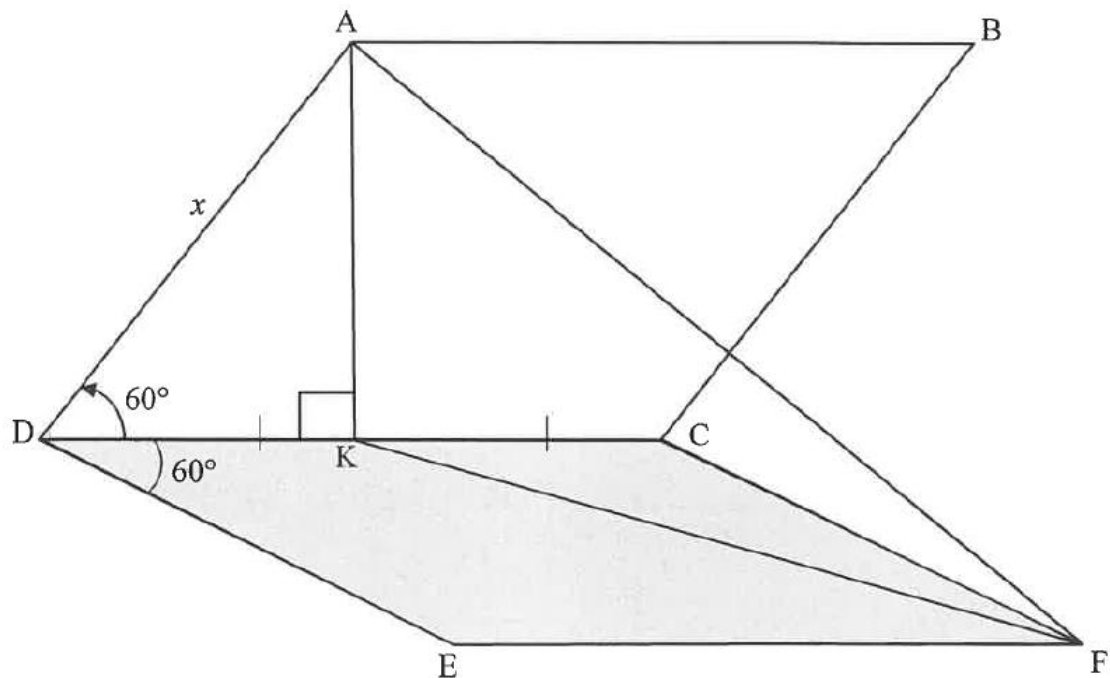
- The radius of the inner circle is r units and the radius of the outer circle is QS .
- The angle of elevation from S to P is 30° .
- $\angle RQS = 2x$ and $PQ = \sqrt{3}r$



- 7.1 Show that $QS = 3r$ (3)
- 7.2 Determine, in terms of r , the area of the flower garden. (2)
- 7.3 Show that $RS = r\sqrt{10 - 6\cos 2x}$ (3)
- 7.4 If $r = 10$ metres and $x = 56^\circ$, calculate RS . (2)
- [10]

QUESTION 7

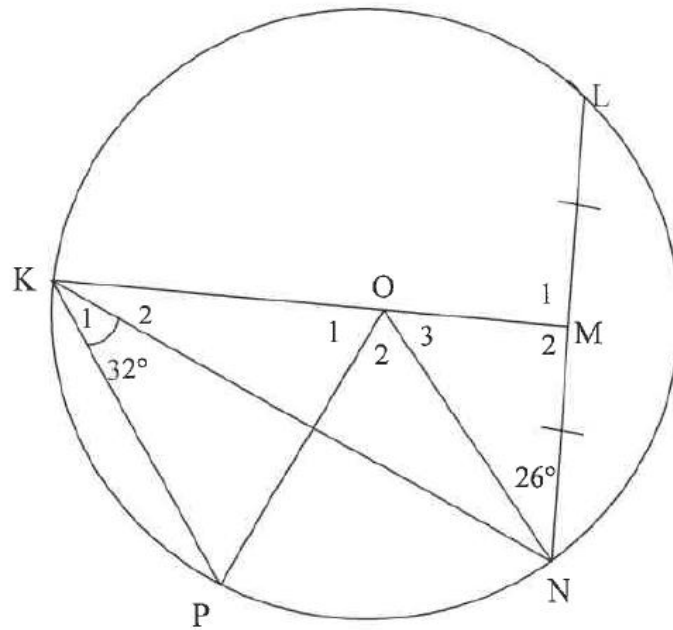
The diagram below shows a solar panel, $ABCD$, which is fixed to a flat piece of concrete slab $EFCD$. $ABCD$ and $EFCD$ are two identical rhombuses. K is a point on DC such that $DK = KC$ and $AK \perp DC$. AF and KF are drawn. $\hat{ADC} = \hat{CDE} = 60^\circ$ and $AD = x$ units.



- 7.1 Determine AK in terms of x . (2)
 - 7.2 Write down the size of $\hat{KCF}$. (1)
 - 7.3 It is further given that $\hat{AKF}$, the angle between the solar panel and the concrete slab, is y . Determine the area of $\triangle AKF$ in terms of x and y . (7)
- [10]

QUESTION 8

- 8.1 O is the centre of the circle.. KOM bisects chord LN and $\hat{MNO} = 26^\circ$. K and P are points on the circle with $\hat{NKP} = 32^\circ$. OP is drawn.



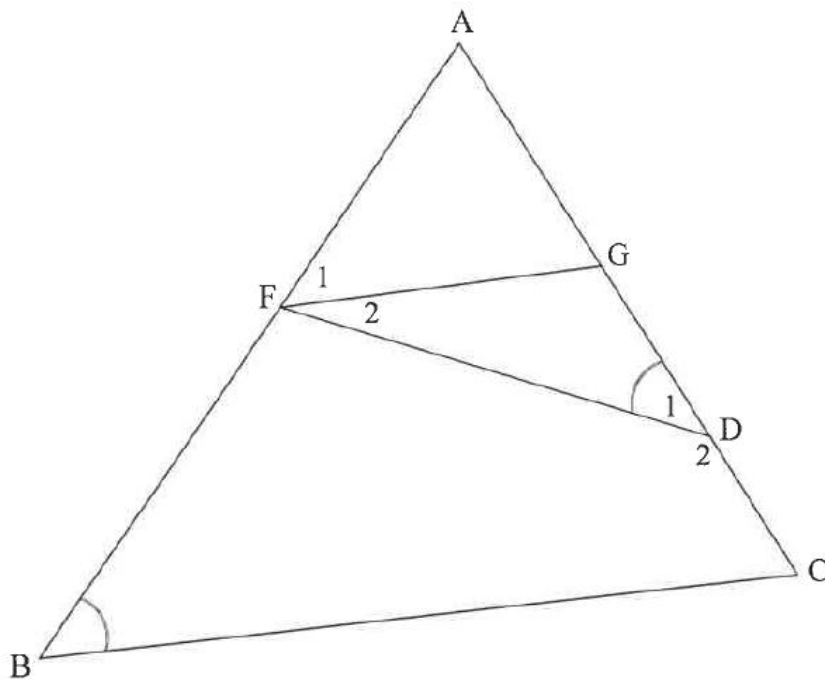
- 8.1.1 Determine, giving reasons, the size of:

(a) $\hat{O}_2$ (2)

(b) $\hat{O}_1$ (4)

- 8.1.2 Prove, giving reasons, that KN bisects $\hat{OKP}$. (3)

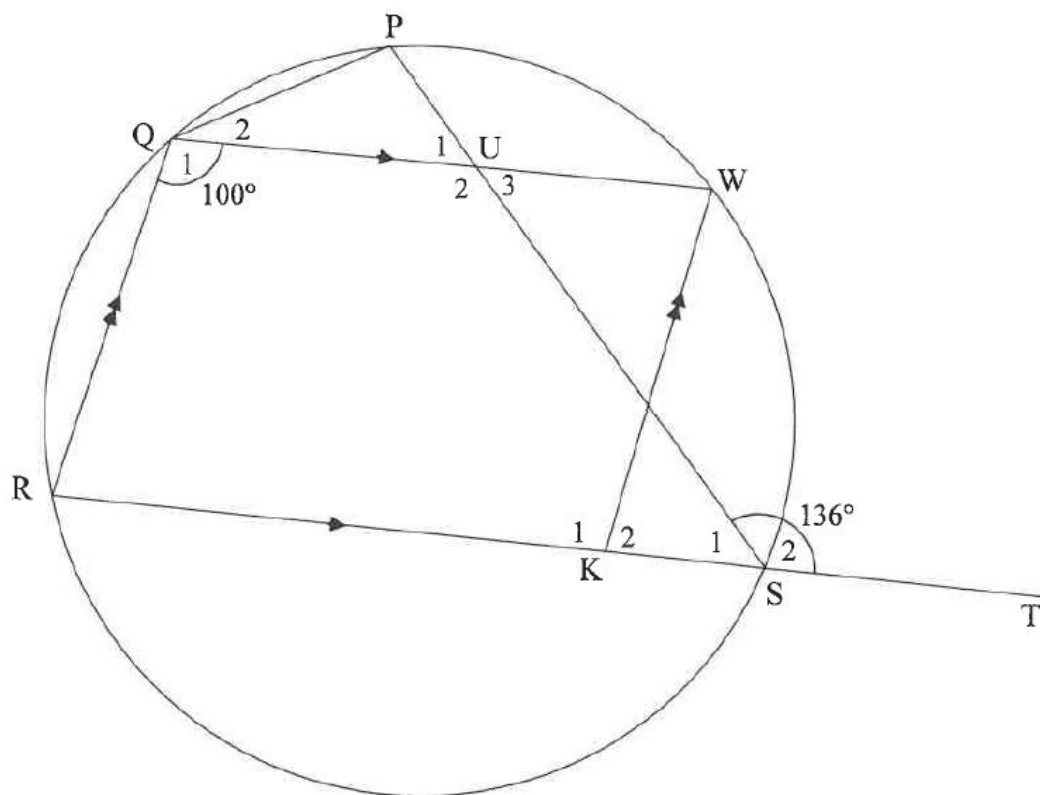
- 8.2 In $\triangle ABC$, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.



- 8.2.1 If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that $FG \parallel BC$. (4)
- 8.2.2 If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x . (4)
- [17]

QUESTION 8

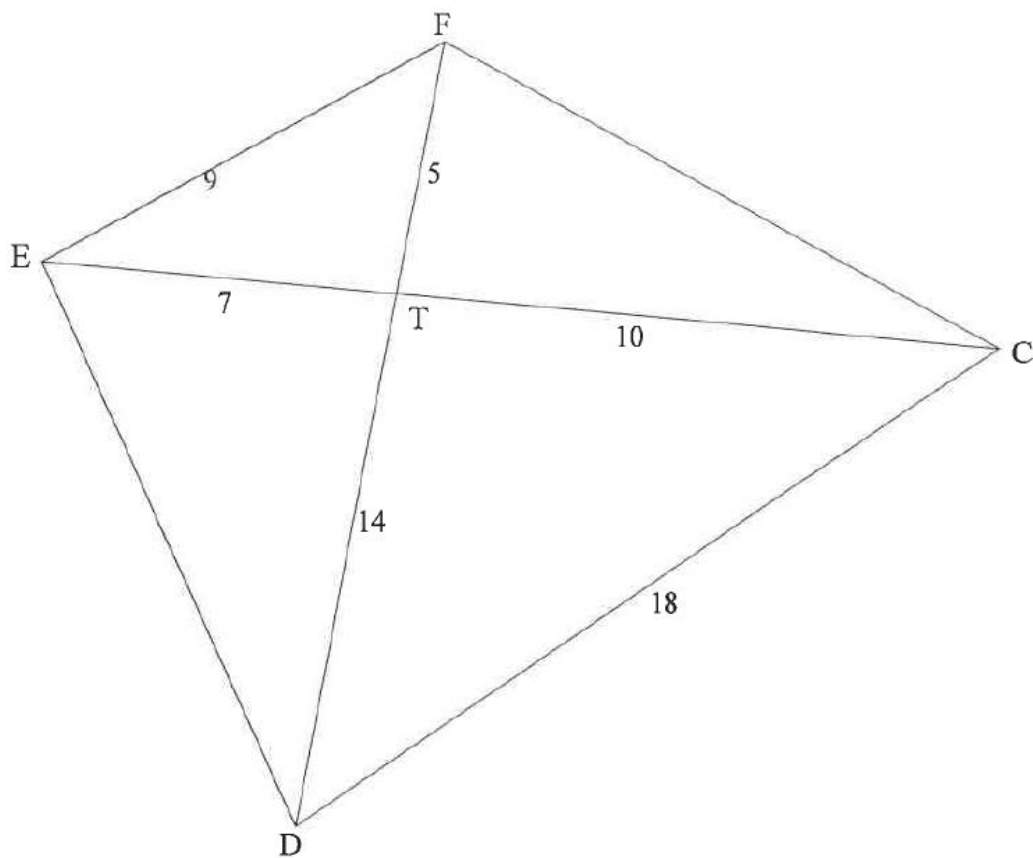
- 8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $\angle PST = 136^\circ$ and $\hat{Q}_1 = 100^\circ$. PS and QW intersect at U.



Determine, with reasons, the size of:

- | | | |
|-------|--------------|-----|
| 8.1.1 | $\hat{R}$ | (2) |
| 8.1.2 | $\hat{P}$ | (2) |
| 8.1.3 | $\angle PQW$ | (3) |
| 8.1.4 | $\hat{U}_2$ | (2) |

- 8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.
 EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and
 TD = 14 units.

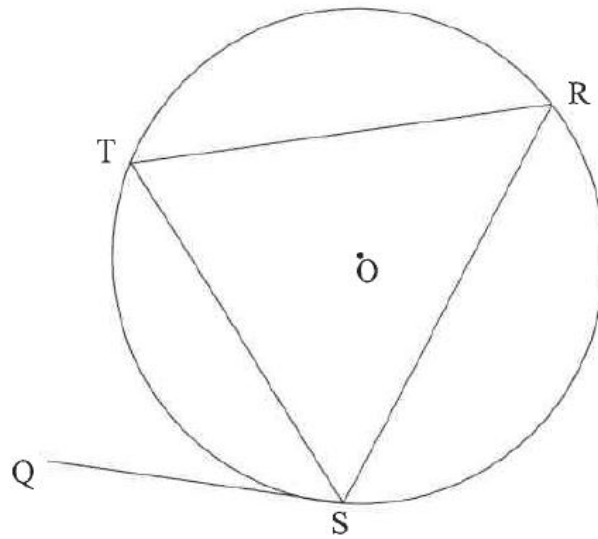


Prove, with reasons, that:

- 8.2.1 $\hat{EFD} = \hat{ECD}$ (4)
- 8.2.2 $\hat{DFC} = \hat{DEC}$ (3)
- [16]

QUESTION 9

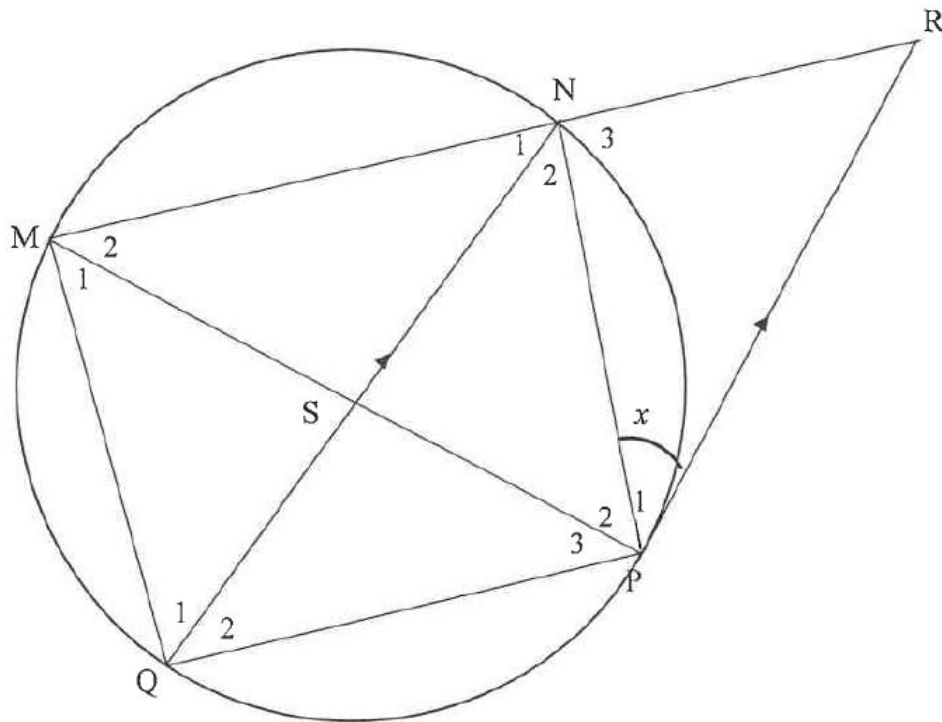
- 9.1 In the diagram, O is the centre of the circle. Points S , T and R lie on the circle. Chords ST , SR and TR are drawn in the circle. QS is a tangent to the circle at S .



Use the diagram to prove the theorem which states that $\hat{QST} = \hat{R}$.

(5)

- 9.2 Chord QN bisects $\hat{MNP}$ and intersects chord MP at S. The tangent at P meets MN produced at R such that $QN \parallel PR$. Let $\hat{P}_1 = x$.



9.2.1 Determine the following angles in terms of x . Give reasons

(a) $\hat{N}_2$ (2)

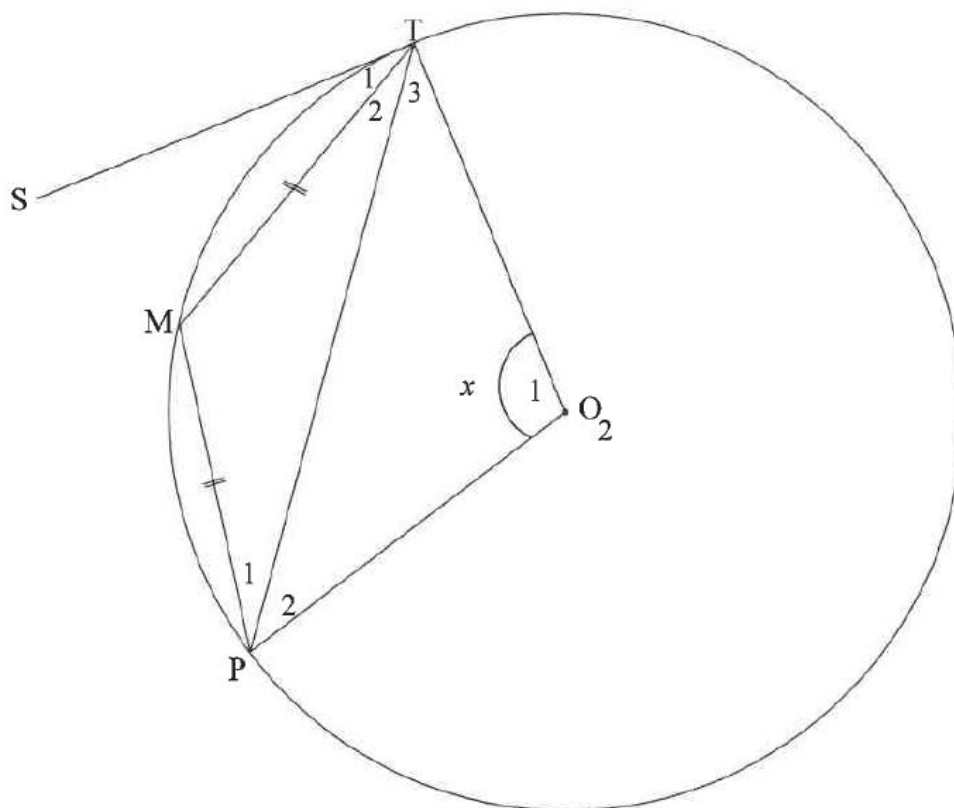
(b) $\hat{Q}_2$ (2)

9.2.2 Prove, giving reasons, that $\frac{MN}{NR} = \frac{MS}{SQ}$ (6)

[15]

QUESTION 9

In the diagram, O is the centre of the circle. ST is a tangent to the circle at T . M and P are points on the circle such that $TM = MP$. OT , OP and TP are drawn. Let $\hat{O}_1 = x$.

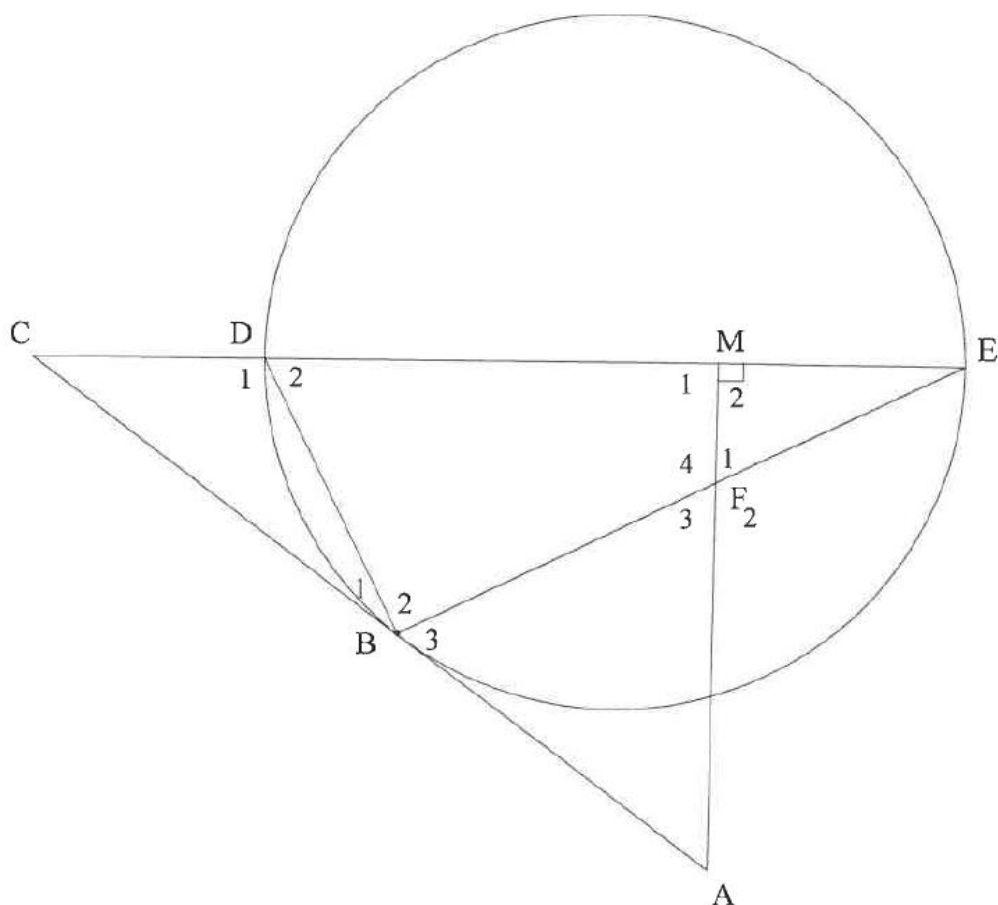


Prove, with reasons, that $\hat{STM} = \frac{1}{4}x$.

[7]

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that $AM \perp DE$. AM and chord BE intersect at F. AM and chord BE intersect at F.



10.1 Prove, giving reasons, that:

10.1.1 FBDM is a cyclic quadrilateral (3)

10.1.2 $\hat{B}_3 = \hat{F}_1$ (4)

10.1.3 $\triangle CDB \parallel \triangle CBE$ (3)

10.2 If it is further given that $CD = 2$ units and $DE = 6$ units, calculate the length of:

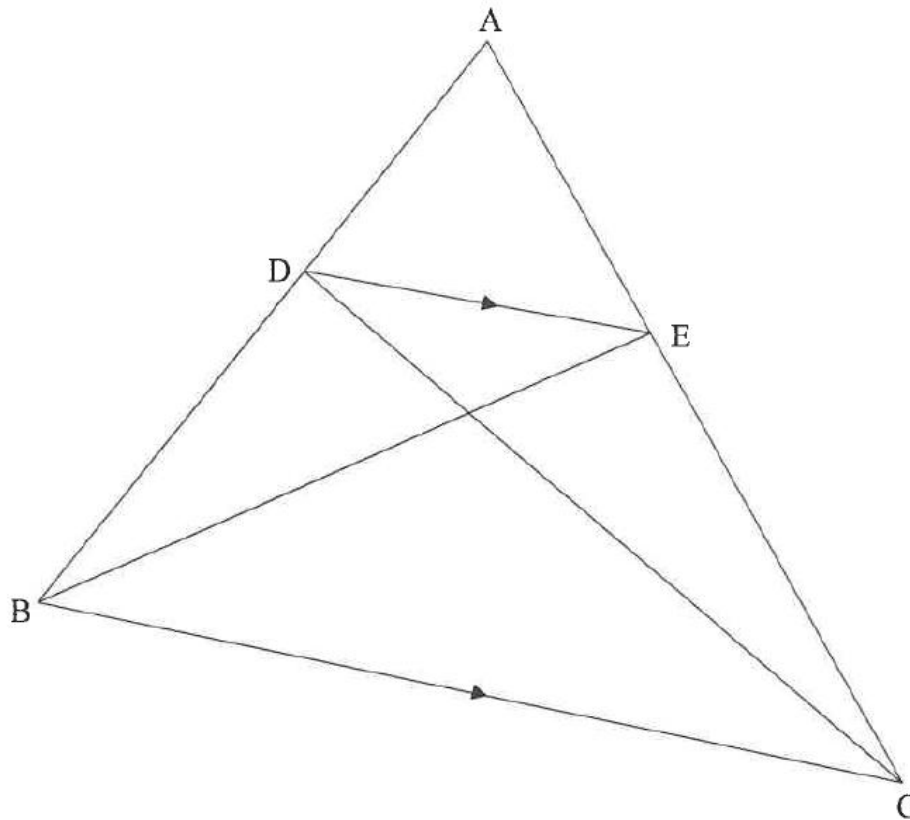
10.2.1 BC (3)

10.2.2 DB (4)

[17]

QUESTION 10

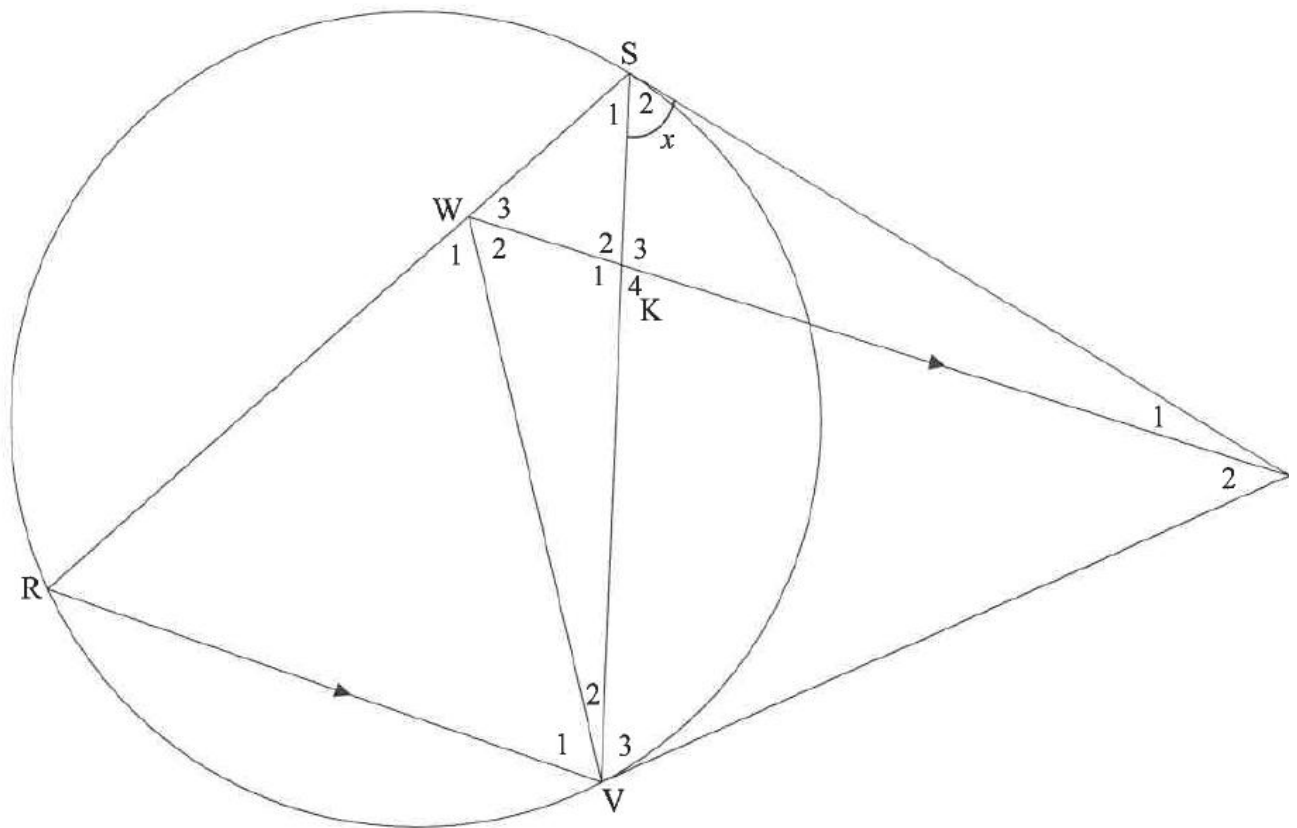
- 10.1 In the diagram, $\triangle ABC$ is drawn. D is a point on AB and E is a point on AC such that $DE \parallel BC$. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$

(6)

- 10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV . SV and WV are drawn. WT intersects SV at K . Let $\hat{S}_2 = x$.



- 10.2.1 Write down, with reasons, THREE other angles EACH equal to x . (6)
- 10.2.2 Prove, with reasons, that:
- (a) $WSTV$ is a cyclic quadrilateral (2)
 - (b) $\triangle WRV$ is isosceles (4)
 - (c) $\triangle WRV \parallel \triangle TSV$ (3)
 - (d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

EXAM GUIDELINES FOR PAPER TWO

➤ EUCLIDEAN GEOMETRY & MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
 - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
 - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
 - The opposite angles of a cyclic quadrilateral are supplementary;
 - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
 - A line drawn parallel to one side of a triangle divides the other two sides proportionally;
 - Equiangular triangles are similar.
4. Corollaries derived from the theorems and axioms are necessary in solving riders:
 - Angles in a semi-circle
 - Equal chords subtend equal angles at the circumference
 - Equal chords subtend equal angles at the centre
 - In equal circles, equal chords subtend equal angles at the circumference
 - In equal circles, equal chords subtend equal angles at the centre.
 - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
 - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.

- Tangents drawn from a common point outside the circle are equal in length.

5. The theory of quadrilaterals will be integrated into questions in the examination.

6. Concurrency theory is excluded.

➤ TRIGONOMETRY

1. The reciprocal ratios cosec θ , sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested. 2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

➤ ANALYTICAL GEOMETRY

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinearity must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

➤ STATISTICS

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line. 2. The interpretation of standard deviation in terms of normal distribution is not examinable. 3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram. In the case of the box and whisker diagram, observations that lie outside the interval (lower quartile – 1,5 IQR ; upper quartile + 1,5 IQR) are considered to be outliers. However, candidates will not be penalised if they did not make use of this formula in identifying outliers

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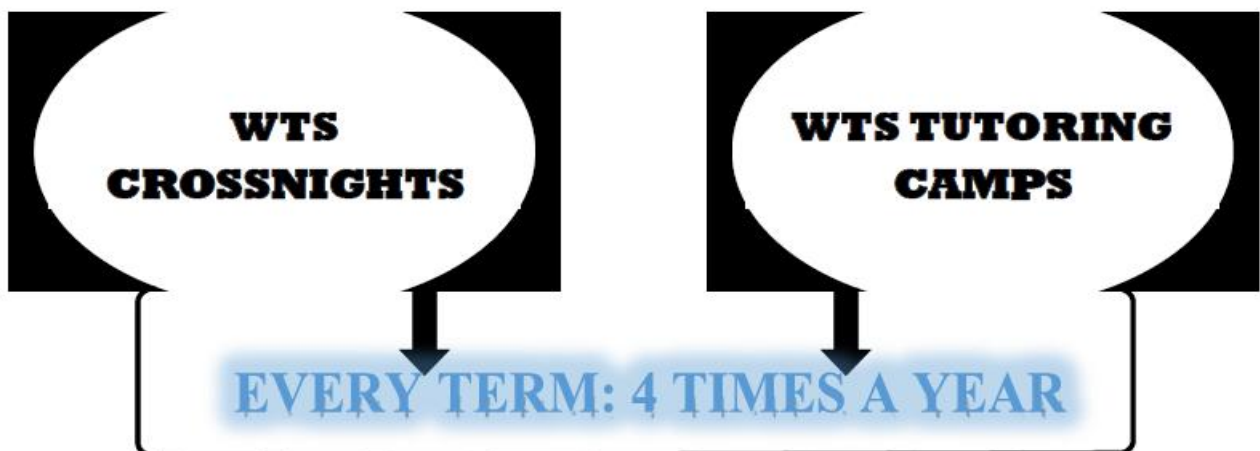
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- 2017 SA Man of the Year Award in Education

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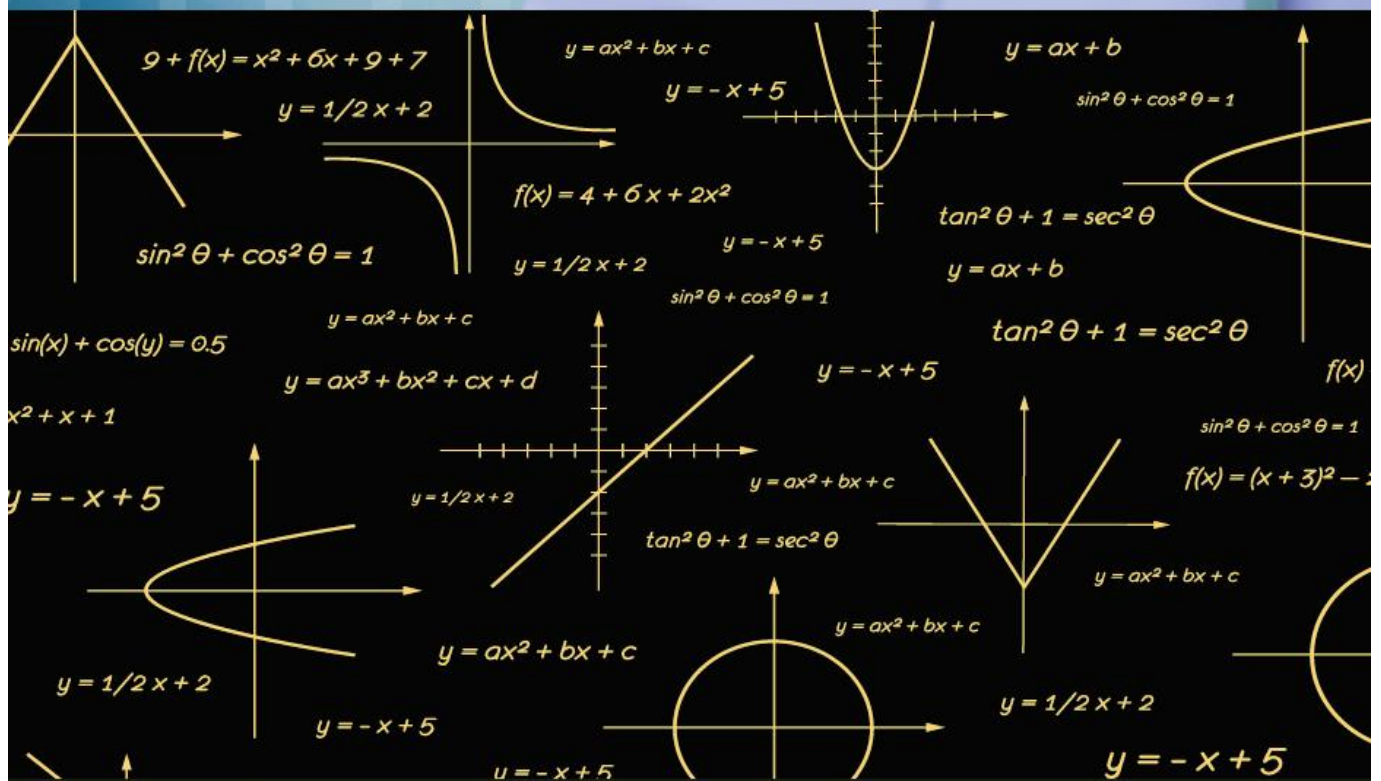
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