

WTS TUTORING



2022 WTS FUCTIONS & INVERSES

GRADE : 12

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WHERE TO START MATHS & SCIENCE IS FOR THE NATION



**WHERE TO START
MATHS & SCIENCE TUTORING**
REGISTRATION NUMBER: 149 – 892 NPO

2022 WTS TERM 03 CAMP

SUBJECTS	: MATHS / MATHS LIT & PHYSCS, ACCOUNTING, LIFE SCIENCES, ECONOMICS, BUSSNES STUDIES & NS
PROVINCE	: KWAZULU NATAL
VENUE	: DURBAN / ULUNDI
GRADE	: 08 TO 12
PRICE	: R2500 [FOOD +ACCOMMODATION + LESSONS]
DATE	: 10 JULY TO 17 JULY 2022
TIME	: 08:00 TO 22:00 DAILY
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- ✓ A **relation** is just a set of ordered pairs. There is absolutely nothing special at all about the data that are in a relation.
- ✓ In other words, any bunch of data is a relation so long as these numbers comes in pairs.
- ✓ A set of all the starting point is called “**the domain**” and a set of all ending points is called a range. In other words the domain is what you start with, and the range is what you end with (i.e. x values and y-values respectively).
- ✓ A function is a relationship between x and y, where for every x-value there is only one y-value.
- ✓ One way to decide whether or not a graph represents a function is to use the vertical line test.
- ✓ If any line drawn parallel to the y-axis cuts the graph only once, then the graph represents a function.
- ✓ We use function notation $f(x)$ to show that each y-value is a function of an x-value.

➤ TYPES OF MAPPINGS

❖ FUNCTIONS

- ✓ One to one mapping- x and y values are not repeated.
- ✓ Many to one mapping- x values are not repeated but y values are repeated.

❖ NON-FUNCTIONS

- ✓ One to many mapping- x values repeat y values do not.
- ✓ Many to many mapping- x and y values repeat.

KEY WORDS:

- ✓ Domain: the set of possible x-values
- ✓ Range: the set of possible y-values
- ✓ Axis of symmetry: an imaginary line that divides a graph into two mirror images of each other.
- ✓ Maximum: the highest possible y-value of a function.
- ✓ Minimum: the lowest possible y-value of a function.
- ✓ Asymptote: an imaginary line that a graph approaches but never touches.
- ✓ Turning point: The point at which a graph reaches its maximum or minimum value and changes direction.
- ✓ Increasing- as x increases the y values increase.(positive gradient)
- ✓ Decreasing- as x increases the y values decrease. (negative gradient)

KEY NOTES:

- ✓ $m > 0$ [kwv right-hand rule : both x and y increases]
- ✓ $m < 0$ [kwv left-hand rule : as x increases ,y decreases]
- ✓ $m = 0$ [horizontal line]
- ✓ $m = \text{undefined}$ [vertical line]
- ✓ $m = \frac{y_2 - y_1}{x_2 - x_1}$ [average gradient]
- ✓ $m_1 = m_2$ [parallel lines]
- ✓ $m_1 \cdot m_2 = -1$ [perpendicular lines]
- ✓ $[\leq / \geq]$ [value included]
- ✓ $(< / >)$ [value excluded]
- ✓ The point is given by: [*smallest : largest*]

➤ INVERSE FUNCTIONS

- ✓ The term inverse means: reflection across the line $y = x$ (**kvv right-hand rule**)
- ✓ Swop x and y and hence make y be the subject of the formula
- ✓ The notation for the inverse of a function is f^{-1}

- ✓ For the linear : $y = ax + c$ to $y = ax + c$
- ✓ For exponential: $y = a^x$ to $y = \log_a x$ **NB: base remains the same**
- ✓ For logarithm: $y = \log_a x$ to $y = a^x$ **NB: base remains the same**
- ✓ For parabola : $y = a^x$ to $y = \pm\sqrt{2a}$
- ✓ Only use the log for exponential graph or if the variable is an exponent
- ✓ To get rid of square root: simple square both side.
- ✓ NB: Inverse of parabola is not a function
- ✓ Restriction of parabolic inverse to be a function:
 1. For Domain: $x \geq 0$ and for the Range: $y \geq 0$
 2. For Domain: $x \leq 0$ and for the Range: $y \leq 0$
- ✓ The inverse of a parabola is not a function.
- ✓ However, if the domain of the original function is restricted then the inverse may be a function too.
- ✓ For example $f(x) = x^2$, then the inverse of f , $f^{-1}(x) = \pm\sqrt{x}$, is not a function.
But if $f(x)$ has a restricted domain $x \geq 0$ or $x \leq 0$, then the inverse will be a function.
- ✓ The inverse function is formed by reflecting the function across the line $y=x$

➤ READING FROM THE GRAPH

❖ For which value(s) of x will:

- a. $f'(x) > 0$ {where the graph is increasing/ positive gradient}
- b. $f'(x) < 0$ {where the graph is decreasing/negative gradient}
- c. $f(x).g(x) > 0$ {where both graphs are above or below}
- d. $f(x).g(x) < 0$ {One graph above and other one below the horizontal}
- e. $f(x) > 0$ {above the horizontal/ x-axis}
- f. $f(x) < 0$ {below the horizontal/ x-axis}
- g. $g'(x) = 0$ {x-value of a turning point}
- h. $g'(x) > 0$ and $f'(x) > 0$ {both graphs increasing}
- i. $g'(x) < 0$ and $f'(x) < 0$ {both graphs decreasing}
- j. $g(x) - f(x) = 1$ {graph $g(x)$ is above $f(x)$ and the distance is 1 unit}
- k. $x.f(x) > 0$ {+ve x- & +ve y- value or -ve x- & -ve y-value}
- l. $f(x).f'(x) < 0$ {+ve y-value and the decreasing curve}

KEY NOTE:

- Remember to manipulate the new given equation into the original one
- Maximum and minimum values means the y values of the turning point

➤ THE TRANSFORMATION OF CUBIC GRAPH

✓ Translation

It affects the turning points

1. Horizontal translation: $f(x + c)$

Then x-value(s) of the turning point is translated c unit left (-ve) or right (+ve)

2. Vertical translation: $f(x) + q$

Then y-value of turning point will be translated q unit up (+ve) or down (-ve)

✓ The reflection

It can affect the whole equation and the turning point. The reflection about axes

The reflections are as follows:

- reflected across x-axis $p(x) = -p(x)$
- reflected across y-axis $p(x) = p(-x)$
- reflected about the line $y = x$
- reflected about the line $y = -x$

✓ The enlargement

- You simply multiply by the scale factor given
- Especially applies on the turning points

➤ THE LINEAR FUNCTION

✓ The forms:

1. $y = c$ [horizontal line]
2. $x = c$ [vertical line]
3. $y = mx$ [table method]
4. $y = mx + c$ [dual intercept method]

➤ Consider: $k(x) = mx + c$

❖ For shape:

If $a > 0$ [*kwv right – hand rule*] and $a < 0$ [*kwv left – hand rule*]

✓ Domain and Range: are all real numbers.

✓ Drawing a neat sketch:

- Determine the x-intercept (let $y = 0$)
- Determine the y-intercept (let $x = 0$)
- Plot these two points and draw a straight line through them.

➤ DETERMINING THE EQUATION

- ✓ Determine the gradient of the function.
- ✓ Substitute the value of the gradient into the general formula for the linear function.
- ✓ Solve for c .
- ✓ Write the equation in the form: $y = mx + c$

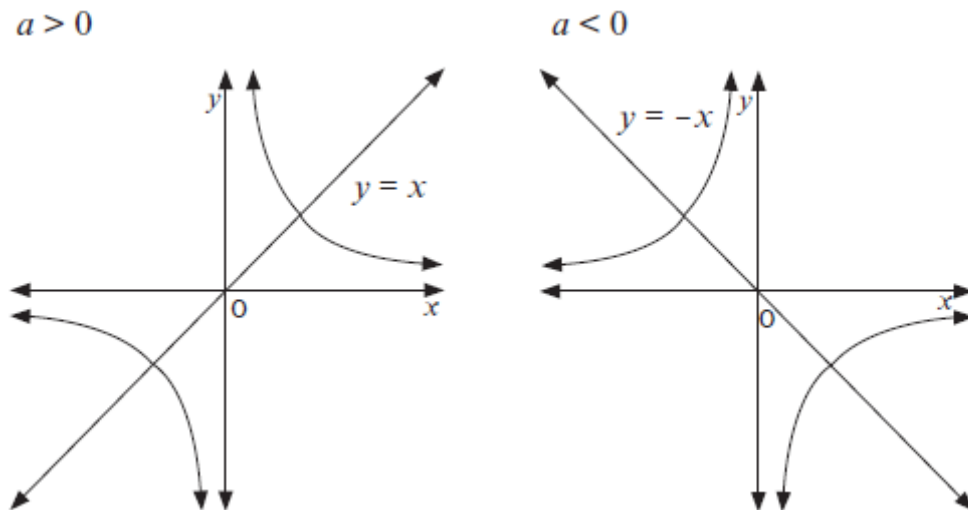
➤ HYPERBOLA FUNCTION

✓ Forms

1. $k(x) = \frac{a}{x}$ [Table method]
2. $w(x) = \frac{a}{x} + q$ [Table method but take note of y asymptote: $y = q$]
3. $v(x) = \frac{a}{x+p}$ [Table method but take note of x asymptote: $x - p = 0$]
4. $s(x) = \frac{a}{x+p} + q$ [Dual-intercepts method and both asymptotes]

➤ Consider: $s(x) = \frac{a}{x+p} + q$

✓ For shape: if $a > 0$ [1st and 3rd quad] & if $a < 0$ [2nd and 4th quad]



- ✓ Two asymptotes; $y = q$ & $x - p = 0$
- ✓ For x -intercepts let $y = 0$
- ✓ For y -intercepts let $x = 0$
- ✓ For domain and range: all real numbers excepts the value of p and q

- ✓ Draw a neat sketch
- ✓ Follow these steps:
 - Indicate the asymptotes
 - Indicate the axes, both x and y intercepts
 - Consider the shape

➤ EQUATIONS OF AXIS OF SYMMETRY

1. For increasing function/ $m > 0$ / positive gradient: $y = x + c$
2. For decreasing function/ $m < 0$ / negative gradient : $y = -x + c$

NB: for the value of c use asymptotes the point of intersection.

➤ DETERMINING THE EQUATION

- ✓ Always start with the asymptotes if given in the graph or domain/ range form
- ✓ And hence substitute with any point along the curve for the value of a
- ✓ Take note for the two unknowns: two points are needed and solve the two equations simultaneously

➤ PARABOLA GRAPH

✓ Forms

1. $k(x) = ax^2$ [Table method]
2. $w(x) = ax^2 + q$ [Table method but take note of turning point: $(0; q)$]
3. $v(x) = ax^2 + bx + c$ [Dual-intercepts method]
4. $s(x) = a(x - p)^2 + q$ [Dual-intercepts method and turning point: $(p; q)$]

KEYS NOTE:

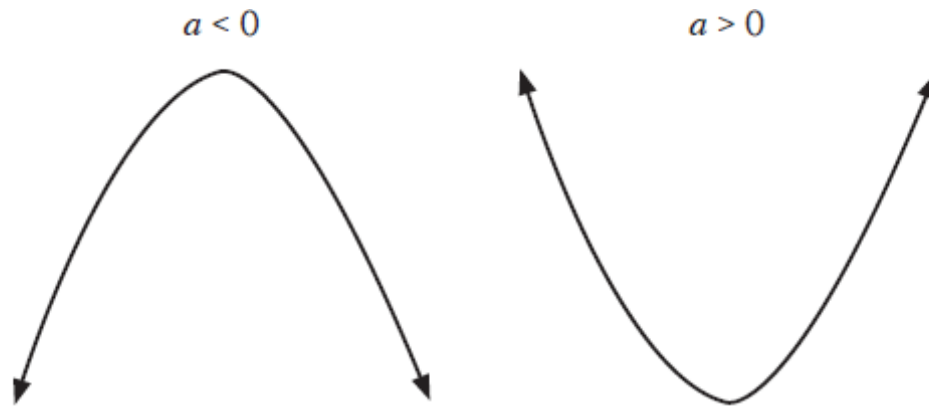
- ✓ The graph of a quadratic function is a parabola.
- ✓ A parabola can open up or down.
- ✓ If the parabola opens up, the lowest point is called the vertex.
- ✓ If the parabola opens down, the vertex is the highest point
- ✓ NOTE: if the parabola opened left or right it would not be a function.
- ✓ The standard form of a quadratic function is $g(x) = ax^2 + bx + c$
- ✓ The parabola will open up when the value of a is positive.
- ✓ The parabola will open down when the value of a is negative.

➤ LINE OF SYMMETRY

- ✓ Parabolas have a symmetric property to them.
- ✓ If we drew a line down the middle of the parabola, we could fold the parabola in half.
- ✓ We call this line the line of symmetry.
- ✓ Or, if we graphed one side of the parabola, we could “fold” (or REFLECT) it over, the line of symmetry to graph the other side.

➤ Consider: $f(x) = ax^2 + bx + c$

✓ For shape: if $a > 0$ [concave up] & if $a < 0$ [concave down]



✓ Axis of symmetry ; $x = -\frac{b}{2a}$

✓ Maximum or minimum $y = \text{substitute the } x \text{ value of the turning point into original}$

✓ Know how to complete the square from: $f(x) = ax^2 + bx + c$ to $f(x) = a(x - p)^2 + q$

✓ For x -intercepts let $y = 0$

✓ For y -intercepts let $x = 0$

✓ For domain: all real numbers

✓ For range: $[-\infty; y]$ or $[y; \infty)$

✓ Draw a neat sketch:

1. Intercepts
2. Shape
3. Turning point

➤ **PARABOLAS OF THE FORM $y = a(x + p)^2 + q$**

- ✓ The value of a tells us if the graph is concave ($a > 0$) or convex ($a < 0$).
- ✓ The equation of the axis of symmetry of the graph is obtained by putting the expression $x + p = 0$ and solving for x .
- ✓ The axis of symmetry passes through the x -coordinate of turning point of the parabola.
- ✓ The graph of $y = a(x + p)^2 + q$ is obtained by shifting the graph of $y = ax^2$ by p units to the left or right and then q units up or down:
 - If $p > 0$, the shift is left
 - If $p < 0$, the shift is right.
 - If $q > 0$, the shift is upwards.
 - If $q < 0$, the shift is downwards.
- ✓ The y -coordinate of the turning point is q .
- ✓ The y -intercept of the graph can be determined by putting $x = 0$.
- ✓ The x -intercept(s) of the graph can be determined by putting $y = 0$.

➤ **DETERMINING THE EQUATION**

- ✓ Always start with substituting the turning if given into : $s(x) = a(x - p)^2 + q$
- ✓ And hence substitute with any point along the curve for the value of a
- ✓ If intercepts $[x_1 : x_2]$ and one point $(x; y)$ on the curve given and then substitute into:
$$y = a(x - x_1)(x - x_2)$$
- ✓ Take note for the two unknowns: two points are needed and solve the two equations simultaneously
- ✓ Any point along the y -axis is given by : $y = y_f - y_i$
- ✓ Any point along the x -axis is given by : $x = x_f - x_i$
- ✓ Length is given by: $d = f(x) - g(x)$ if f is above g and for the maximum;
substitute with the x value turning point

➤ READING Y-VALUE

➤ For which value(s) of k will $f(x)$ has:

- ✓ The roots of a quadratic equation are the x -values when the equation is zero.
- ✓ The roots are the x -intercepts of the graph.
- ✓ When you are asked to ‘determine the nature of the roots of an equation:
“you are NOT asked to solve the equation”.

✓ **Summary:**

To find the nature of the roots of a quadratic equation: $ax^2 + bx + c = 0$, $\Delta = b^2 - 4ac$ the discriminant.

- If $\Delta < 0$: The roots are non-real/no real roots.
- If $\Delta = 0$: There are two equal, real and rational roots.
- If $\Delta > 0$: There are two real roots which may be rational or irrational.
- If Δ is a perfect square: the roots are rational.
- If Δ is not a perfect square: then the roots are irrational
 - The nature of the roots also tells us about the x -intercepts of the graph of the quadratic equation

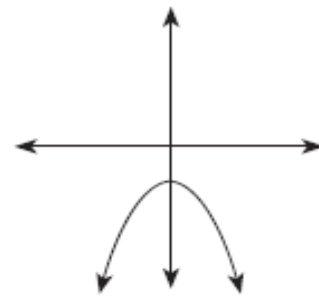
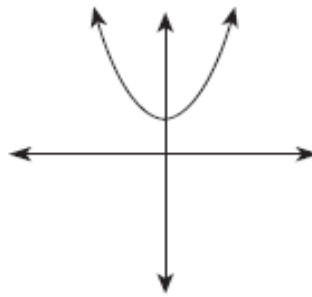
Nature of roots

$$\Delta < 0$$

Roots are non-real.

There are no x -intercepts.

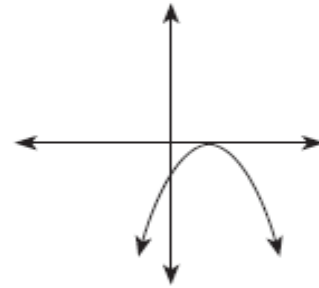
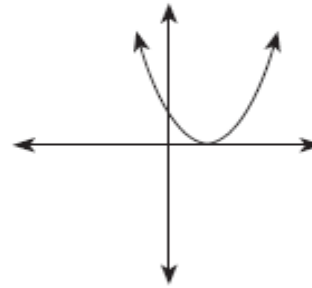
Graphs



$$\Delta = 0$$

Roots are real and equal.

There is only one x -intercept and it is at the turning point of the graph.

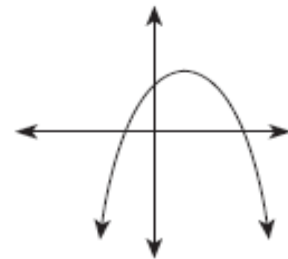
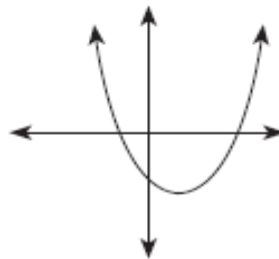


$$\Delta > 0$$

Roots are real and unequal (two roots):

If Δ is a squared rational number, roots are rational.

If Δ is not a squared number, the roots are irrational.



Note: The line $y = k + c$ has to intersect the graph $f(x)$ at different places.

Make sure the given equation is derived to the original equation and make y the subject of the formula

➤ EXPONENTIAL FUNCTION

✓ Forms

1. $k(x) = ab^x$ [Table method]
2. $w(x) = ab^x + q$ [Table method but take note of y asymptote: $y = q$]
3. $v(x) = ab^{x-p}$ [Table method but take note of shift: $x - p = 0$]
4. $s(x) = ab^{x+p} + q$ [Dual-intercepts method and the y- asymptote : $y = q$]

➤ Consider: $k(x) = ab^x$

✓ $b > 1$:increasing function: as x increases y also increases

✓ Kwv- Right -hand rule

➤ Consider: $k(x) = ab^{-x}$

✓ $0 < b < 1$:decreasing function: as x increases y decreases

✓ Kwv- left -hand rule

➤ Consider: $w(x) = ab^{x+p} + q$

✓ Asymptote; $y = q$

✓ If $p < 0$, the graph shift p units to the right.

✓ If $p > 0$, the graph shift p units to the left.

✓ If $q > 0$, the graph shift q units upwards.

- ✓ If $q < 0$, the graph shift q units downwards.
- ✓ NB: p is for the translation not for asymptote.
- ✓ For x –intercepts let $y = 0$
- ✓ For y –intercepts let $x = 0$
- ✓ For domain all real numbers
- ✓ For the range: $y > q$ or $y < q$
- ✓ Drawing a neat sketch
- **FOLLOW THESE STEPS:**
- ✓ Indicate the asymptote :Only One asymptote
- ✓ Indicate the axes, both x and y intercepts
- ✓ Consider the shape

➤ DETERMINING THE EQUATION

- ✓ Always start with the asymptote if given in the graph or range form
- ✓ And hence substitute with any point along the curve for the value of a
- ✓ Take note for the two unknowns: two points are needed and solve the two equations simultaneously

➤ LOGARITHM FUNCTION

❖ **Form: $y = \log_a x$**

- ✓ Work with log as it is or change it to exponential form: $x = a^y$
- ✓ And hence use the table method to draw the graph as x dependent

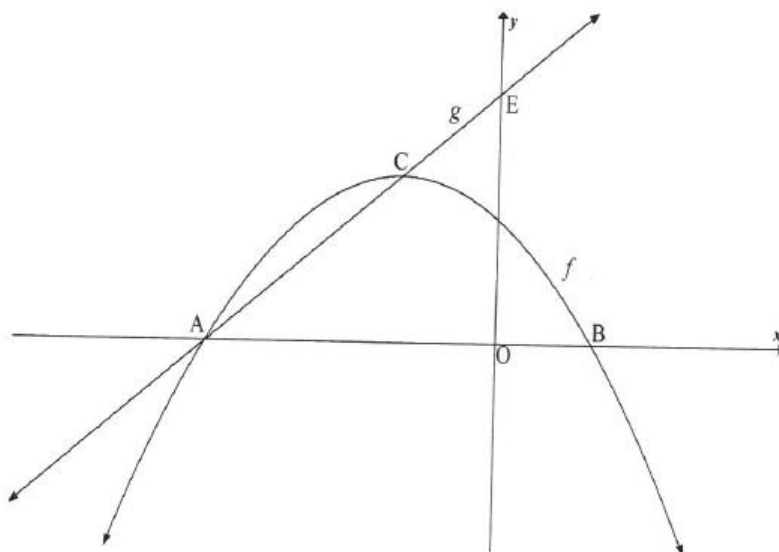
➤ DETERMINING THE EQUATION

❖ **Form: $y = \log_a x$**

- ✓ Substitute with any point along the curve for the value of a

QUESTION 4

The sketch below shows the graphs of $f(x) = -x^2 - 2x + 3$ and $g(x) = mx + q$. Graph f has x -intercepts at A and B(1 ; 0) and a turning point at C. The straight line g , passing through A and C, cuts the y -axis at E.



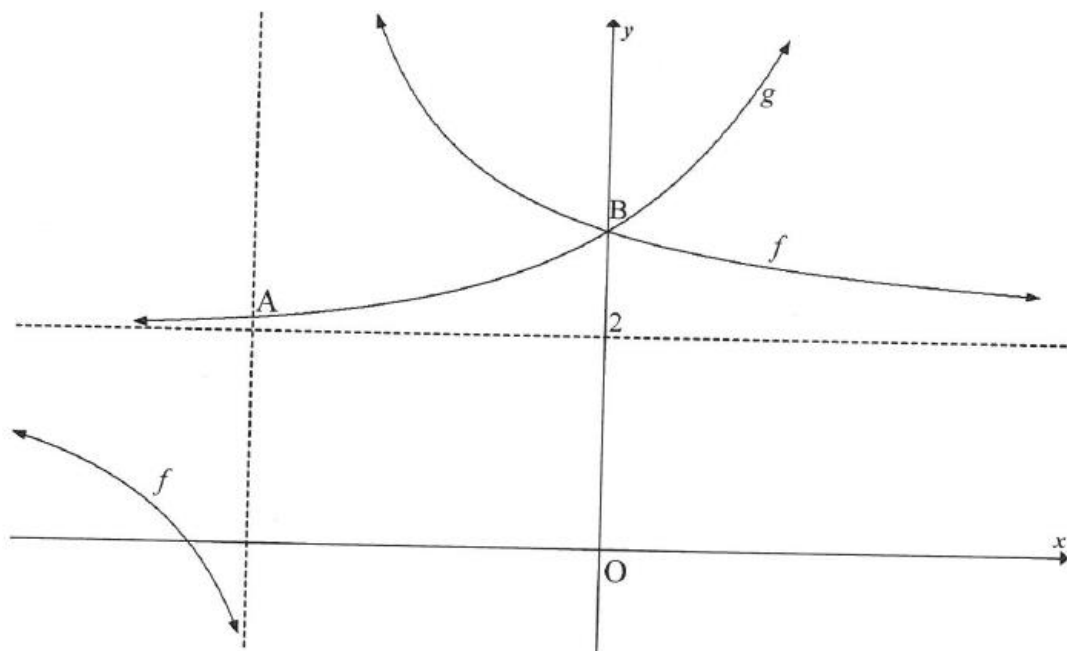
- 4.1 Write down the coordinates of the y -intercept of f . (1)
- 4.2 Show that the coordinates of C are $(-1; 4)$. (3)
- 4.3 Write down the coordinates of A. (1)
- 4.4 Calculate the length of CE. (6)
- 4.5 Determine the value of k if $h(x) = 2x + k$ is a tangent to the graph of f . (5)
- 4.6 Determine the equation of g^{-1} , the inverse of g , in the form $y = \dots$ (2)
- 4.7 For which value(s) of x is $g(x) \geq g^{-1}(x)$? (3)

[21]

QUESTION 5

The sketch below shows the graphs of $f(x) = \frac{3}{x-p} + q$ and $g(x) = 2^x + r$

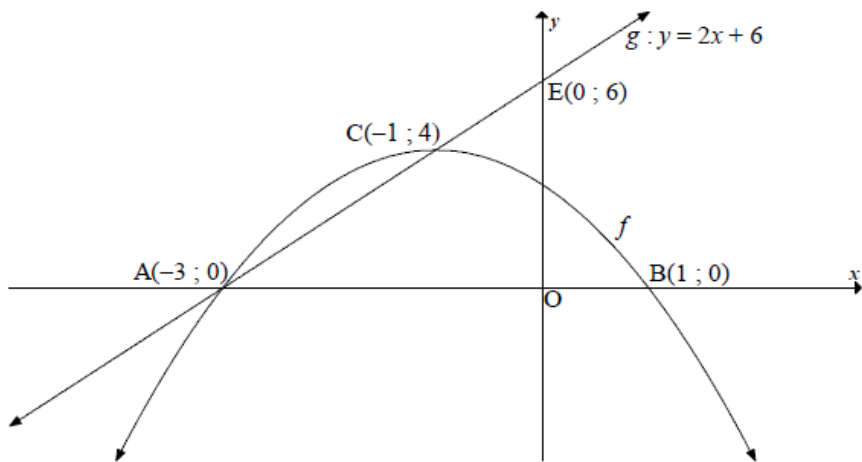
- g intersects the vertical asymptote of f at A .
- B is the common y -intercept of f and g .
- $y = 2$ is the common horizontal asymptote of f and g .



- 5.1 Write down the value of r . (1)
 - 5.2 Determine the value of p . (4)
 - 5.3 Determine the coordinates of A . (3)
 - 5.4 For which value(s) of x is $f(x) - g(x) \geq 0$? (2)
 - 5.5 If $h(x) = f(x-2)$, write down the equation of h . (2)
- [12]**

MEMO: 01

QUESTION/VR4AG 4



4.1	(0 ; 3)	✓ (0 ; 3) (1)
4.2	$x = -\frac{b}{2a} \quad \text{or} \quad -2x - 2 = 0$ $= -\frac{(-2)}{2(-1)} \quad \therefore x = -1$ $= -1$ $y = -(-1)^2 - 2(-1) + 3 \quad \text{or} \quad y = \frac{4ac - b^2}{4a}$ $= 4 \quad \quad \quad = \frac{4(-1)(3) - (-2)^2}{4(-1)}$ $C(-1 ; 4)$	$\checkmark x = -\frac{(-2)}{2(-1)} \quad \text{or} \quad -2x - 2 = 0$ ✓ simplification ✓ in the context of a turning point $-(-1)^2 - 2(-1) + 3$ $\frac{4(-1)(3) - (-2)^2}{4(-1)}$ (3)

4.3	B(1 ; 0) By symmetry/<i>Deur simmetrie</i> A(-3 ; 0) OR/OF $x^2 + 2x - 3 = 0$ $(x + 3)(x - 1) = 0$ $x = -3 \text{ or } x = 1$ A(-3 ; 0)	✓ A(-3 ; 0) (1) ✓ A(-3 ; 0) (1)
4.4	Equation of g: $m = \frac{4 - 0}{-1 + 3}$ $= 2$ $y = 2x + q$ OR/OF $y - 0 = 2(x + 3)$ $0 = 2(-3) + q \text{ or } 4 = 2(-1) + q$ $q = 6$ $y = 2x + 6$ E(0 ; 6) C(-1 ; 4) $CE = \sqrt{(0 + 1)^2 + (6 - 4)^2}$ $= \sqrt{5} \text{ units/2,24 units}$	✓ $m = 2$ ✓ subs of A(-3;0) or C(-1;4) ✓ $y = 2x + 6$ ✓ E(0 ; 6) ✓ substitution into distance formula ✓ answer (6)
4.5	$f'(x) = -2x - 2$. But $m_{\tan} = 2$ $-2x - 2 = 2$ $x = -2$ $f(-2) = 3$ $y = 2x + k$ $3 = 2(-2) + k$ $k = 7$ OR/OF $-x^2 - 2x + 3 = 2x + k$ $-x^2 - 4x + 3 - k = 0$ $x^2 + 4x - 3 + k = 0$ For equal roots: $\Delta = b^2 - 4ac = 0$ $(-4)^2 - 4(-1)(3 - k) = 0 \quad (4)^2 - 4(1)(k - 3) = 0$ $16 + 12 - 4k = 0 \text{ or } 16 - 4k + 12 = 0$ $k = 7 \quad k = 7$	✓ $-2x - 2$ ✓ $-2x - 2 = 2$ ✓ $x = -2$ ✓ $y = 3$ ✓ answer (5) ✓ $-x^2 - 2x + 3 = 2x + k$ ✓ standard form ✓ $b^2 - 4ac = 0$ ✓ substitution ✓ answer (5)

4.6	$g: y = 2x + 6$ $g^{-1}: x = 2y + 6$ $2y = x - 6$ $y = \frac{x-6}{2} \text{ or } y = \frac{x}{2} - 3$	$\checkmark x = 2y + 6$ $\checkmark y = \frac{x-6}{2} \text{ or } y = \frac{x}{2} - 3$ (2)
4.7	$g(x) \geq g^{-1}(x)$ $2x + 6 \geq \frac{x-6}{2}$ $4x + 12 \geq x - 6$ $3x \geq -18$ $x \geq -6$	$\checkmark 2x + 6 \geq \frac{x-6}{2}$ $\checkmark 4x + 12 \geq x - 6$ $\checkmark x \geq -6$ (3) [21]

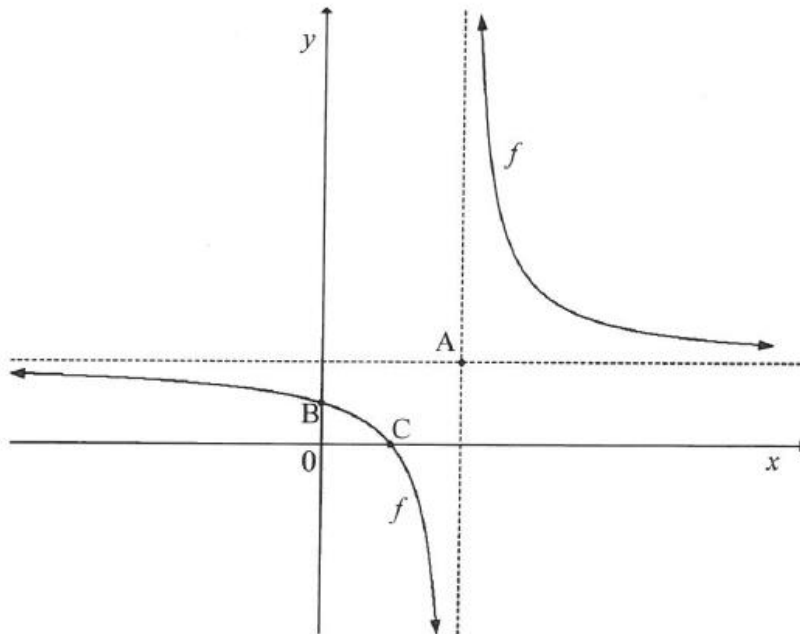
QUESTION/VRAAG 5

5.1	$r = 2$	$\checkmark r = 2$ (1)
5.2	$g(x) = 2^x + 2$ $g(0) = 2^0 + 2 = 3$ $B(0; 3)$ $3 = \frac{3}{0-p} + 2$ $p = -3$	$\checkmark g(0) = 2^0 + 2$ $\checkmark y = 3$ $\checkmark$ substitute $B(0; 3)$ and $q = 2$ $\checkmark p = -3$ (4)
5.3	at A: $x = -3$ $y = 2^{-3} + 2 = 2\frac{1}{8}$ $A\left(-3; 2\frac{1}{8}\right)$ or $A\left(-3; \frac{17}{8}\right)$ or $A(-3; 2,125)$	$\checkmark$ at A : $x = -3$ (p-value) $\checkmark$ substitute $x = -3$ into exponential equation $\checkmark$ y-value (3)
5.4	$-3 < x \leq 0$ OR/ OF $(-3; 0]$	$\checkmark -3 < x$ $\checkmark x \leq 0$ (2)
5.5	$f(x) = \frac{3}{x+3} + 2$ $f(x-2) = \frac{3}{x-2+3} + 2$ $h(x) = \frac{3}{x+1} + 2$	$\checkmark$ substitution of $x - 2$ $\checkmark h(x) = \frac{3}{x+1} + 2$ (2) [12]

KWV 02

QUESTION 4

The sketch below shows the graph of $f(x) = \frac{6}{x-4} + 3$. The asymptotes of f intersect at A. The graph f intersects the x -axis and y -axis at C and B respectively.



- 4.1 Write down the coordinates of A. (1)
 - 4.2 Calculate the coordinates of B. (2)
 - 4.3 Calculate the coordinates of C. (2)
 - 4.4 Calculate the average gradient of f between B and C. (2)
 - 4.5 Determine the equation of a line of symmetry of f which has a positive y -intercept. (2)
- [9]

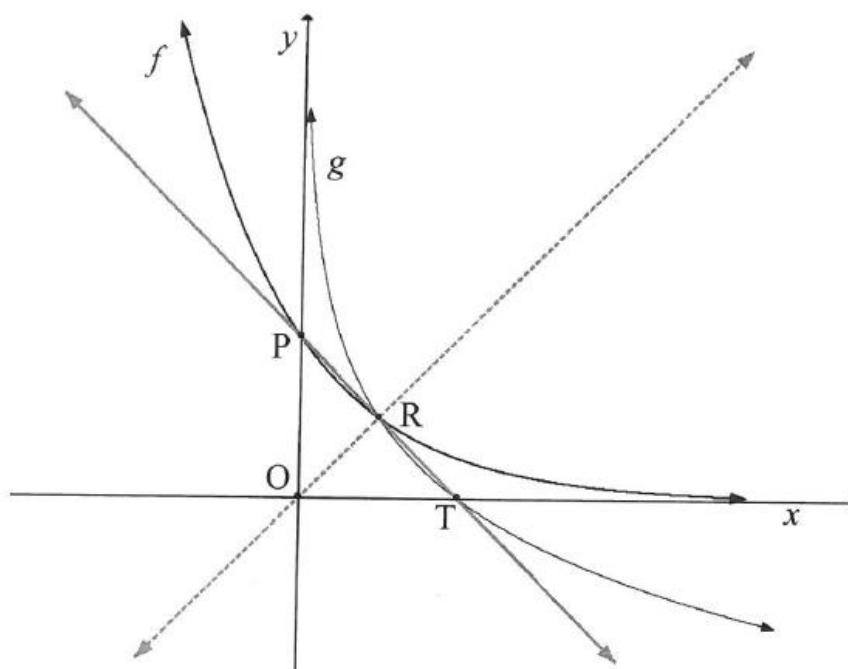
QUESTION 5

Given: $f(x) = x^2 - 5x - 14$ and $g(x) = 2x - 14$

- 5.1 On the same set of axes, sketch the graphs of f and g . Clearly indicate all intercepts with the axes and turning points. (6)
- 5.2 Determine the equation of the tangent to f at $x = 2\frac{1}{2}$. (2)
- 5.3 Determine the value(s) of k for which $f(x) = k$ will have two unequal positive real roots. (2)
- 5.4 A new graph h is obtained by first reflecting g in the x -axis and then translating it 7 units to the left. Write down the equation of h in the form $h(x) = mx + c$. (2)
- [12]

QUESTION 6

In the sketch below, P is the y -intercept of the graph of $f(x) = b^x$. T is the x -intercept of graph g , the inverse of f . R is the point of intersection of f and g . Straight lines are drawn through O and R and through P and T.



- | | | |
|-----|--|-------------|
| 6.1 | Determine the equation of g (in terms of b) in the form $y = \dots$ | (2) |
| 6.2 | Write down the equation of the line passing through O and R. | (1) |
| 6.3 | Write down the coordinates of point P. | (1) |
| 6.4 | Determine the equation of the line passing through P and T. | (2) |
| 6.5 | Calculate the value of b . | (5) |
| | | [11] |

MEMO:02

QUESTION/VR4AG 4

4.1	A (4; 3)	✓(4; 3) (1)
4.2	$y = \frac{6}{-4} + 3$ $= \frac{3}{2}$ $B\left(0; \frac{3}{2}\right)$	✓ $x = 0$ ✓ $y = \frac{3}{2}$ (2)
4.3	$0 = \frac{6}{x-4} + 3$ $-3 = \frac{6}{x-4}$ $-3(x-4) = 6$ $-3x + 12 = 6$ $x = 2$ C(2 ; 0)	✓ $y = 0$ ✓ $x = 2$ (2)
4.4	Average gradient = $\frac{0 - \frac{3}{2}}{2 - 0}$ $= -\frac{3}{4}$	$\frac{0 - \frac{3}{2}}{2 - 0}$ ✓ $-\frac{3}{4}$ (2)
4.5	$y = -x + 7$ OR/OF $m = -1$ $\therefore y - 3 = -(x - 4)$ $y = -x + 7$	✓ $m = -1$ ✓ $y = -x + 7$ OR/OF ✓ $m = -1$ ✓ $y = -x + 7$ (2) [9]

QUESTION/VRAAG 5

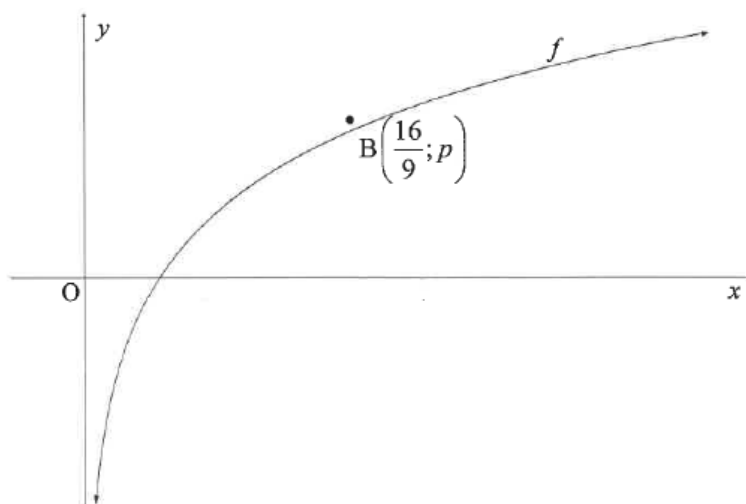
5.1		<i>f</i>: ✓ x-intercepts ✓ y-intercept ✓ shape ✓ TP <i>g</i>: ✓ x-intercept and y-intercept ✓ shape (6)
5.2	$y = -20\frac{1}{4}$	✓✓ $y = -20\frac{1}{4} / -\frac{81}{4}$ (2)
5.3	$-20\frac{1}{4} < k < -14$	✓ $-20\frac{1}{4} < k$ ✓ $k < -14$ (2)
5.4	Reflecting in the x-axis: $y = -2x + 14$ $y = -2(x + 7) + 14$ Shifting 7 units to the left: $= -2x - 14 + 14$ $= -2x$	✓ $y = -2x + 14$ ✓ $y = -2x$ (2) [12]

QUESTION/VRAAG 6

6.1	$f : y = b^x$ $f^{-1} : x = b^y$ $y = \log_b x$	✓ interchange x and y ✓ answer (2)
6.2	$y = x$	✓ answer (1)
6.3	P(0; 1)	✓ answer (1)
6.4	T(1; 0) $y = mx + c$ $y = -x + 1$	✓ coordinates of T ✓ $y = -x + 1$ (2)
6.5	At point R, PT and OR intersect: $-x + 1 = x$ $2x = 1$ $x = \frac{1}{2}$ $y = \frac{1}{2}$ Substitute $\left(\frac{1}{2}; \frac{1}{2}\right)$ into the equation of f : $y = b^x$ $\frac{1}{2} = b^{\frac{1}{2}}$ $b = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ OR/OF At point R, PT and OR intersect: $-x + 1 = x$ $2x = 1$ $x = \frac{1}{2}$ $y = \frac{1}{2}$ Substitute $\left(\frac{1}{2}; \frac{1}{2}\right)$ into the equation of g : $y = \log_b x$ $\frac{1}{2} = \log_b \left(\frac{1}{2}\right)$ $b^{\frac{1}{2}} = \frac{1}{2}$ $b = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	✓ $-x + 1 = x$ ✓ $x = \frac{1}{2}$ ✓ $y = \frac{1}{2}$ ✓ substitution ✓ $b = \frac{1}{4}$ (5) ✓ substitution ✓ $b = \frac{1}{4}$ (5) [11]

QUESTION 4

The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9}; p\right)$ is a point on f .



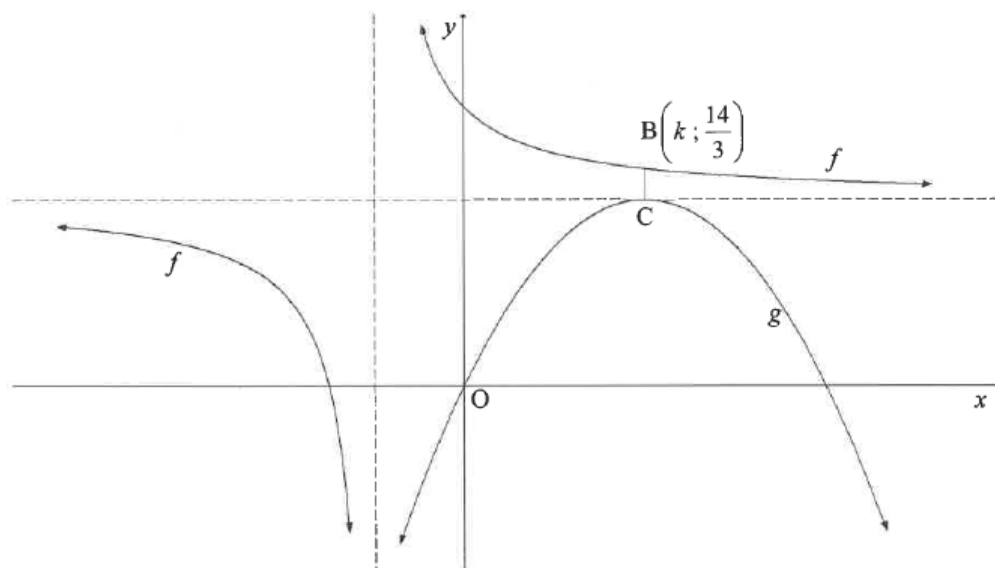
- 4.1 For which value(s) of x is $\log_{\frac{4}{3}} x \leq 0$? (2)
- 4.2 Determine the value of p , without the use of a calculator. (3)
- 4.3 Write down the equation of the inverse of f in the form $y = \dots$ (2)
- 4.4 Write down the range of $y = f^{-1}(x)$. (2)
- 4.5 The function $h(x) = \left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f .
Write down the coordinates of B'' , the image of B on h . (2)

[11]

QUESTION 5

The graphs of $f(x) = \frac{2}{x+1} + 4$ and parabola g are drawn below.

- C, the turning point of g , lies on the horizontal asymptote of f .
- The graph of g passes through the origin.
- B $\left(k; \frac{14}{3}\right)$ is a point on f such that BC is parallel to the y -axis.



- 5.1 Write down the domain of f . (2)
- 5.2 Determine the x -intercept of f . (2)
- 5.3 Calculate the value of k . (3)
- 5.4 Write down the coordinates of C. (2)
- 5.5 Determine the equation of g in the form $y = a(x + p)^2 + q$. (3)
- 5.6 For which value(s) of x will $\frac{f(x)}{g(x)} \leq 0$? (4)
- 5.7 Use the graphs of f and g to determine the number of real roots of $\frac{2}{x} - 5 = -(x - 3)^2 - 5$. Give reasons for your answer. (4)

[20]

MEMO:03

QUESTION/VR4AG 4

4.1	$0 < x \leq 1$ or $(0 ; 1]$	✓✓ answer (2)
4.2	$p = \log_{\frac{4}{3}} \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \left(\frac{4}{3}\right)^2$ $p = 2$	✓ substitution ✓ $\left(\frac{4}{3}\right)^2$ ✓ answer (3)
4.3	$f : y = \log_{\frac{4}{3}} x$ $f^{-1} : x = \log_{\frac{4}{3}} y$ $y = \left(\frac{4}{3}\right)^x$	✓ $x = \log_{\frac{4}{3}} y$ ✓ $y = \left(\frac{4}{3}\right)^x$ (2)
4.4	$y > 0$ or $y \in (0; \infty)$	✓✓ answer (2)
4.5	$\left(-2; \frac{16}{9}\right)$	✓ -2 ✓ $\frac{16}{9}$ (2) [11]

QUESTION/VRAAG 5

5.1	$x \in R; x \neq -1$	$\checkmark x \in R$ $\checkmark x \neq -1$ (2)
5.2	x -intercept of f : $0 = \frac{2}{x+1} + 4$ $\frac{2}{x+1} = -4$ $2 = -4x - 4$ $4x = -6$ $x = -\frac{3}{2}$	$\checkmark$ equating to 0 $\checkmark$ answer (2)
5.3	$y = \frac{2}{x+1} + 4$ $\frac{14}{3} = \frac{2}{k+1} + 4$ $\frac{2}{k+1} = \frac{14}{3} - 4$ $\frac{2}{k+1} = \frac{2}{3}$ $2k + 2 = 6$ $k + 1 = 3$ $k = 2$	$\checkmark$ substitution $\checkmark$ simplification $\checkmark$ answer (3)
5.4	$C(2; 4)$	$\checkmark 2$ $\checkmark 4$ (2)
5.5	$y = a(x + p)^2 + q$ $= a(x - 2)^2 + 4$ Substitute $(0; 0)$: $0 = a(0 - 2)^2 + 4$ $0 = 4a + 4$ $a = -1$ $y = -(x - 2)^2 + 4$	$\checkmark a(x - 2)^2 + 4$ $\checkmark$ Substitute $(0; 0)$ $\checkmark a = -1$ (3)
5.6	$x \leq -\frac{3}{2}$ or $-1 < x < 0$ or $x > 4$	$\checkmark x \leq -\frac{3}{2}$ $\checkmark -1 < x < 0$ $\checkmark x > 4$ (4)

5.7	$\frac{2}{x} - 5$: f shifted 1 unit to the right and 9 units down. f is 1 eenheid na regs en 9 eenhede afgeskuif. $-(x - 3)^2 - 5$: g shifted 1 unit to the right and 9 units down. g is 1 eenheid na regs en 9 eenhede afgeskuif. Therefore the shift of both graphs took place relative to each other/Dus het die skuif van die grafieke relatief tot mekaar plaasgevind. They only intersect in the third quadrant. Hulle sny mekaar slegs in die derde kwadrant. Therefore there is only one point of intersection. Daar is dus slegs een snypunt.	$\checkmark$ both shifted 1 unit to the right $\checkmark$ both shifted 9 units down $\checkmark$ relative shift $\checkmark$ one real root (4) [20]
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QUESTION 4

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

- 4.1 Write down the range of g . (1)
- 4.2 Determine the equation of g^{-1} in the form $y = \dots$ (2)
- 4.3 Is g^{-1} a function? Justify your answer. (2)
- 4.4 The point $M(a ; 2)$ lies on g^{-1} .
- 4.4.1 Calculate the value of a . (2)
- 4.4.2 M' , the image of M , lies on g . Write down the coordinates of M' . (1)
- 4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of M' on h . (3)
- [11]**

QUESTION 5

5.1 Given: $f(x) = \frac{1}{x+2} + 3$

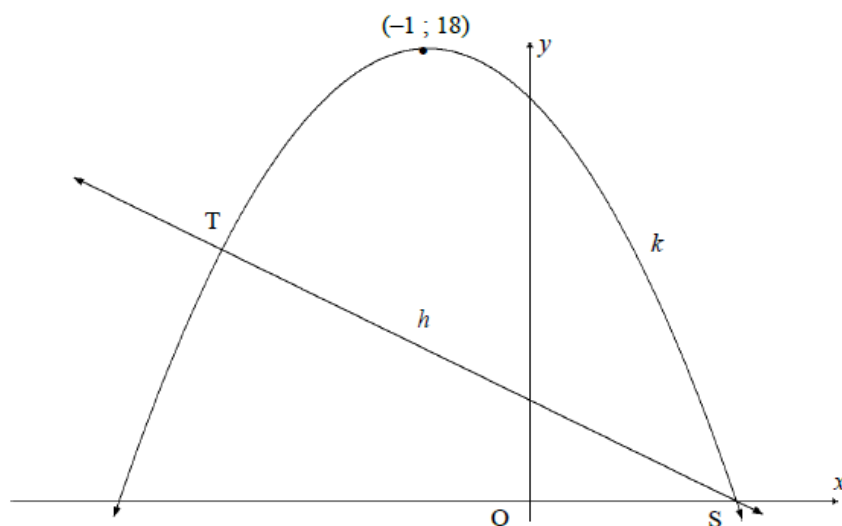
5.1.1 Determine the equations of the asymptotes of f . (2)

5.1.2 Write down the y -intercept of f . (1)

5.1.3 Calculate the x -intercept of f . (2)

5.1.4 Sketch the graph of f . Clearly label ALL intercepts with the axes and any asymptotes. (3)

5.2 Sketched below are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$. Graph k has a turning point at $(-1 ; 18)$. S is the x -intercept of h and k . Graphs h and k also intersect at T.



5.2.1 Calculate the coordinates of S. (2)

5.2.2 Determine the equation of k in the form $y = a(x + p)^2 + q$. (3)

5.2.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T. (5)

5.2.4 Determine the value(s) of x for which $k(x) < h(x)$. (2)

5.2.5 It is further given that k is the graph of $g'(x)$.

(a) For which values of x will the graph of g be concave up? (2)

(b) Sketch the graph of g , showing clearly the x -values of the turning points and the point of inflection. (3)

[25]

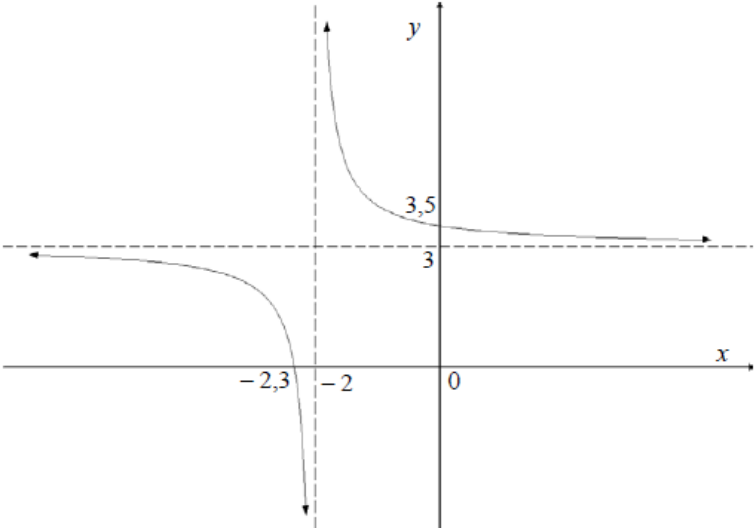
MEMO: 04

QUESTION/VRAG 4

4.1	$y > 0$ OR/OF $y \in (0 ; \infty)$	✓ answer (1) OR/OF ✓ answer (1)
4.2	$g: y = \left(\frac{1}{2}\right)^x$ $g^{-1}: x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$ or $y = -\log_2 x$ or $y = \log_2 \frac{1}{x}$	✓ $x = \left(\frac{1}{2}\right)^y$ ✓ equation (2)
4.3	Yes. The vertical line test cuts g^{-1} once <i>Ja. Die vertikale lyn toets sny g^{-1} slegs eenkeer.</i> OR/OF Yes. For every x-value there is a unique y-value <i>Ja. Vir elke x-waarde is daar 'n unieke y-waarde</i> OR/OF Yes. g is a one-to-one function / <i>Ja. g is 'n een-tot-een funksie</i> OR/OF Yes. The horizontal line cuts g only once <i>Ja. Die horisontale lyn sny g slegs een keer</i>	✓ yes ✓ valid reason (2) OR/OF ✓ yes ✓ valid reason (2) OR/OF ✓ yes ✓ valid reason (2) OR/OF ✓ yes ✓ valid reason (2)

4.4.1	$y = -\log_2 x$ $2 = -\log_2 a$ $a = 2^{-2} = \frac{1}{4}$ or $a = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	✓ correct subst into correct formula ($a ; 2$) ✓ answer (2)
4.4.2	$M'\left(2; \frac{1}{4}\right)$ or $M'(2; a)$	✓ answer (1)
4.5	$M''\left(-1; \frac{9}{4}\right)$	✓ -1 ✓✓ $\frac{9}{4}$ (3)
		[11]

QUESTION/VRAAG 5

5.1.1	$x = -2$ $y = 3$	✓ answer ✓ answer (2)
5.1.2	$\left(0; \frac{7}{2}\right)$	✓ answer (1)
5.1.3	$\frac{1}{x+2} + 3 = 0$ $1 + 3(x+2) = 0$ $3x = -7$ $x = -\frac{7}{3}$ x-intercept $\left(-\frac{7}{3}; 0\right)$	✓ $y = 0$ ✓ answer (2)
5.1.4		✓ asymptotes at $y = 3$ and $x = -2$ ✓ intercepts at $y = 3,5$ and $x = -2,3$ ✓ shape (reasonable representation in correct quadrants) (3)

5.2.1	$-2x + 4 = 0$ $2x = 4$ $x = 2$ $\therefore S(2 ; 0)$	$\checkmark y = 0$ $\checkmark x = 2$ (2)
5.2.2	Equation of k : $y = a(x+1)^2 + 18$ $0 = a(2+1)^2 + 18$ or $0 = a(-4+1)^2 + 18$ $9a = -18$ $a = -2$ $y = -2(x+1)^2 + 18$	$\checkmark y = a(x+1)^2 + 18$ $\checkmark$ substitute $(2 ; 0)$ or $(-4 ; 0)$ $\checkmark a$ (3)
5.2.3	$-2x^2 - 4x + 16 = -2x + 4$ $-2x^2 - 2x + 12 = 0$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3$ or $x = 2$ $y = -2(-3) + 4 = 10$ $T(-3 ; 10)$	$\checkmark$ equating $\checkmark$ standard form $\checkmark$ factors $\checkmark$ choosing $x = -3$ $\checkmark$ answer (5)
5.2.4	$x < -3$ or $x > 2$ OR/OF $(-\infty ; -3) \cup (2 ; \infty)$	$\checkmark\checkmark$ answer (2) OR/OF $\checkmark\checkmark$ answer (2)
5.2.5(a)	$x < -1$ OR/OF $(-\infty ; -1)$	$\checkmark\checkmark$ answer (2) OR/OF $\checkmark\checkmark$ answer (2)
5.2.5(b)		$\checkmark$ shape of cubic with local min tp moving to local max tp $\checkmark$ turning points at $x = 2$ and $x = -4$ $\checkmark$ point of inflection at $x = -1$ (3) [25]

KWV QP 05

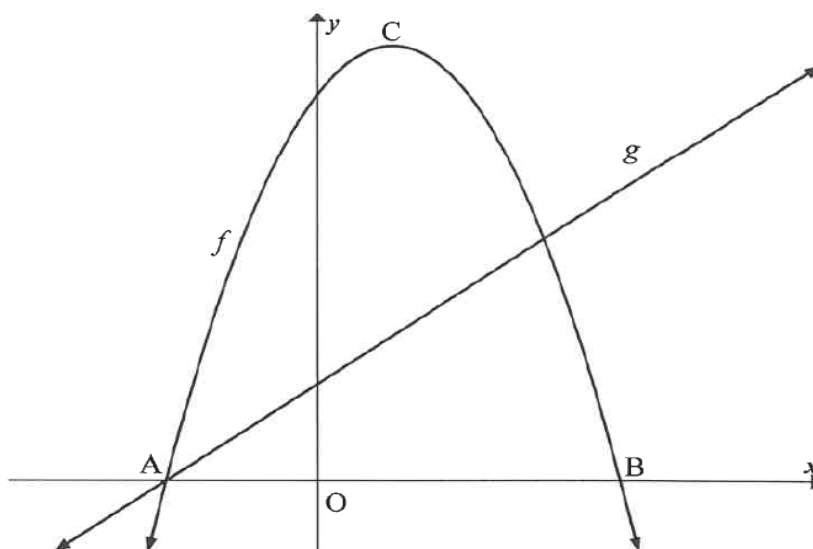
QUESTION 4

The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function $f(x) = \frac{-2}{x+p} + q$.

- 4.1 Show that $p = 4$ and $q = -3$. (4)
- 4.2 Calculate the x -intercept of f . (2)
- 4.3 Sketch the graph of f . Clearly label ALL intercepts with the axes and the asymptotes. (4)
- [10]**

QUESTION 5

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x -intercepts of f . C is the turning point of f .



- 5.1 Calculate the coordinates of A and B. (3)
- 5.2 Determine the coordinates of C, the turning point of f . (2)
- 5.3 Write down the range of f . (1)
- 5.4 The graph of $h(x) = f(x+p) + q$ has a maximum value of 15 at $x = 2$. Determine the values of p and q . (3)
- 5.5 Determine the equation of g^{-1} , the inverse of g , in the form $y = \dots$ (2)
- 5.6 For which value(s) of x will $g^{-1}(x) \cdot g(x) = 0$? (2)
- 5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect. (5)
- [18]**

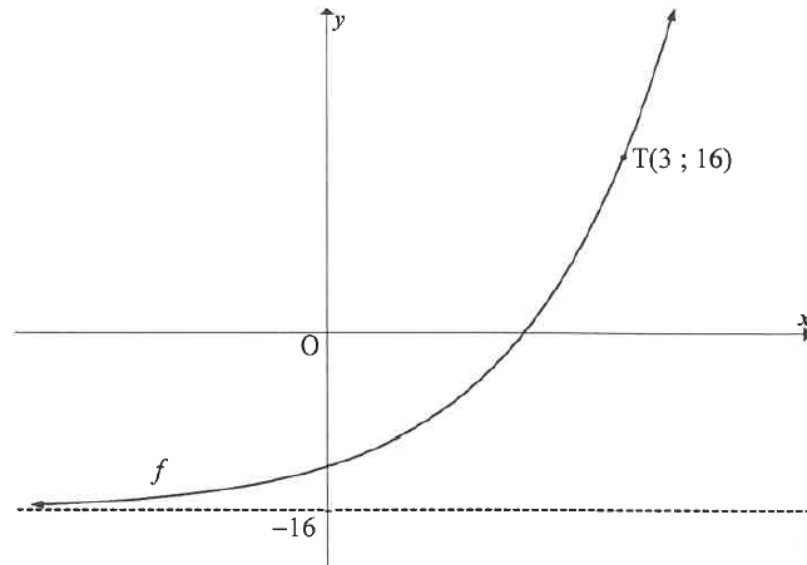
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \dots$ (2)

6.1.2 Point $P(6 ; 11)$ lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g . Write down the coordinates of the image of P on g . (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. $T(3 ; 16)$ is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q .

(4)
[8]

MEMO : 05

QUESTION/VRAAG 4

4.1	$x+1 = -x-7$ $2x = -8$ $x = -4$ $\therefore y = -3$ $\therefore f(x) = \frac{-2}{x+4} - 3$ $\therefore p = 4 \text{ and } q = -3$ OR/OF $p + q = 1 \dots\dots(1)$ $-p + q = -7$ $q = p - 7 \dots\dots(2)$ subs. (2) into (1) $p + p - 7 = 1$ $2p = 8$ $p = 4$ $q = -3$	$\checkmark x+1 = -x-7$ $\checkmark 2x = -8$ $\checkmark x = -4$ $\checkmark y = -3$ (4) OR/OF $\checkmark p + q = 1$ $\checkmark q = p - 7$ $\checkmark$ substitution $\checkmark$ simplification (4)
4.2	$y = \frac{-2}{x+4} - 3$ $0 = \frac{-2}{x+4} - 3$ $-2 - 3(x+4) = 0$ $-3x - 14 = 0$ $\therefore x = -\frac{14}{3}$	$\checkmark y = 0$ $\checkmark x = -\frac{14}{3}$ (2)
4.3		$\checkmark$ horizontal asymptote $\checkmark$ vertical asymptote $\checkmark$ y intercept $\checkmark$ shape (4)
		[10]

QUESTION/VRAAG 5

5.1	$-2x^2 + 4x + 16 = 0$ $x^2 - 2x - 8 = 0$ $(x - 4)(x + 2) = 0$ $x = 4$ or $x = -2$ $\therefore A(-2;0)$ and $B(4;0)$	$\checkmark$ factors $\checkmark x = -2$ $\checkmark x = 4$ (3)
5.2	$f(x) = -2x^2 + 4x + 16$ $-\frac{b}{2a} = -\frac{-4}{-2(2)} = 1$ $f(1) = -2(1)^2 + 4(1) + 16 = 18$ $\therefore C(1;18)$ OR/OF $f(x) = -2x^2 + 4x + 16$ $f'(x) = -4x + 4$ $-4x + 4 = 0$ $x = 1$ $f(1) = -2(1)^2 + 4(1) + 16 = 18$ $\therefore C(1;18)$	$\checkmark 1$ $\checkmark 18$ OR/OF $\checkmark 1$ $\checkmark 18$ (2)
5.3	$y \leq 18$ OR/OF $y \in (-\infty; 18]$	$\checkmark y \leq 18$ OR/OF $\checkmark y \in (-\infty; 18]$ (1)
5.4	TP (1 ; 18) for f TP (2 ; 15) for h $\therefore p = -1$ $q = -3$	$\checkmark$ TP for h at (2 ; 15) $\checkmark p = -1$ $\checkmark q = -3$ (3)
5.5	$y = 2x + 4$ $x = 2y + 4$ $\therefore y = \frac{1}{2}x - 2$	$\checkmark$ swop x and y $\checkmark y = \frac{1}{2}x - 2$ (2)
5.6	$g(x) = 0$ or $g^{-1}(x) = 0$ $x = 4$ or $x = -2$ (product 0 at x -intercepts)	$\checkmark x = 4$ $\checkmark x = -2$ (2)

5.7	$-2x^2 + 4x + 16 + k = 2x + 4$ $-2x^2 + 2x + 12 + k = 0$ $b^2 - 4ac < 0$ $(2)^2 - 4(-2)(12 + k) < 0$ $4 + 8(12 + k) < 0$ $100 + 8k < 0$ $k < -12,5$ OR/OF $g'(x) = 2$ $f'(x) = -4x + 4 = 2$ $x = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = 17,5$ $g\left(\frac{1}{2}\right) = 5$ $\therefore k < -12,5$	$\checkmark$ equating $\checkmark$ standard form $\checkmark b^2 - 4ac < 0$ $\checkmark$ substitution $\checkmark$ answer (5) OR/OF $\checkmark g'(x) = 2$ $\checkmark f'(x) = -4x + 4$ $\checkmark f\left(\frac{1}{2}\right) = 17,5$ $\checkmark g\left(\frac{1}{2}\right) = 5$ $\checkmark$ answer (5)
		[18]

QUESTION/VRAAG 6

6.1.1	$y = 3^x$ $x = 3^y$ $y = \log_3 x$	$\checkmark$ swop x and y $\checkmark$ equation (2)
6.1.2	$h(x) = 3^{x-4} + 2$ Transformation: 4 units left, 2 units down $P'(2;9)$	$\checkmark x = 2$ (A) $\checkmark y = 9$ (A) (2)
6.2	$f(x) = 2^{x+p} + q$ $q = -16$ $16 = 2^{p+3} - 16$ $2^{p+3} = 32$ $2^{p+3} = 2^5$ $\therefore p + 3 = 5$ $p = 2$	$\checkmark q = -16$ $\checkmark$ substitute (3 ; 16) $\checkmark 2^{p+3} = 2^5$ or $p + 3 = \log_2 32$ $\checkmark p = 2$ (4)
		[8]

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