

WTS TUTORING



2022 WTS CALCULUS

GRADE : 12

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2022 WTS TERM 03 CAMP

SUBJECTS	: MATHS / MATHS LIT & PHYSICS, ACCOUNTING, LIFE SCIENCES, ECONOMICS, BUSSNES STUDIES & NS
PROVINCE	: KWAZULU NATAL
VENUE	: DURBAN / ULUNDI
GRADE	: 08 TO 12
PRICE	: R2500 [FOOD +ACCOMMODATION + LESSONS]
DATE	: 10 JULY TO 17 JULY 2022
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➤ ALGEBRA

Polynomial:..... is a polynomial of degree.....

A linear Polynomial:..... is a polynomial of degree.....

A quadratic Polynomial:..... is a polynomial of degree.....

A cubic Polynomial:..... is a polynomial of degree.....

➤ Factorising a cubic polynomial

A. Sum and difference of two cubes

- Formula for sum : $(x^3 + a^3) = (x + a)(x^2 - ax + a^2)$
- Formula for difference : $(x^3 - a^3) = (x - a)(x^2 + ax + a^2)$

B. Grouping in pairs

- It is a factorising method that can be used when an expression has four or more terms and then therefore terms can be grouped in pairs
- Positive sign must be in between the brackets

C. Solving cubic using synthetic method

- there must be four terms and if one term is missing you must use zero instead
- ensure you take a coefficient with the sign

i. The remainder theorem

- If a polynomial $f(x)$ is divided by a linear polynomial $ax - b$, then the remainder is $f(\frac{b}{a})$
- Firstly substitute and equate to the remainder

ii. The factor theorem

- If $f(x)$ is a polynomial such as that $f\left(\frac{b}{a}\right) = 0$, then $ax - b$ is a factor of $f(x)$
- In factor theorem the remainder is zero

D. Solving cubic equations

- The standard form is: $f(x) = ax^3 + bx^2 + cx + c$
- Let $f(x) = 0$
- And you can use synthetic or inspection {smile method}

➤ CALULUS

A. Average gradient

The average gradient between two points is the gradient of a straight line drawn between the two points.

B. Gradient at a single point on a curve

- derivative is the gradient
- $f'(a) = m$ { a is the value of x at that point }

C. Finding the derivative

- Finding the derivative from first principles

The derivative of a function $f(x)$ is written as $f'(x)$ and is defined by:-----

STEPS:

- make it a point that you copy the formula as it is in the formula sheet
- simply substitute $(x + h)$ where there is x
- where there is a fraction you must find the LCD

D. Rules of derivative

- Finding the derivative using the rule for differentiating

Before you differentiate you might need to:

- Expand brackets because you have no rule for differentiate product.
- Rewrite terms which are square roots, cube roots, other roots as exponentials so that you can use the rule.
- Rewrite terms which are fractions , so that you can use the power rule
- Take note of notation we use for the derivative

E. Conclusion

Uses of the derivative:

- To find the equation of a tangent line
- To locate stationary points
- To find where a maximum or minimum value occur
- To describe rates of change
- To draw cubic polynomials

F. Tangent Equation

a) Finding the equation of a tangent line

The slope of the tangent line to the graph at a point is equal to the derivative of the function at that point. So to find the equation of the tangent line to $f(x)$ at $x = a$

- Take the derivative, and then
- Evaluate the derivative at $x = a$ i.e. to calculate $f'(a) = m$ to get the gradient of the tangent line,
- calculate the y-value at $x = a$ i.e. calculate $f(a)$ { point of contact }
- and lastly use the equation of the line: $y = mx + c$ or $y - y = m(x - x)$

b) Finding the unknown variables

- For two variables you need two points and hence equations
- For three variables you need to work with the first two variables and use one point for remaining variable
- For eq 1: $f'(a) = m$
- For eq 2: substitute the points of contact and if not given simple calculate $f(a)$
- Solve them simultaneously

➤ CUBIC FUNCTION

- **Finding the stationary points of a function**

- ✓ When we are drawing the graph or looking for the max. or min. values of a function:
- ✓ it is useful to identify the turning points; these points are where the gradient of the function is zero. We solve $f'(x) = 0$ and substitute the x values into original equation for the y values.

- **Stationary points on a cubic function**

There are 3 stationary points

- ✓ Local maximum
- ✓ Local minimum
- ✓ Inflection point

Take note:

Because solving $f'(x) = 0$ can help us identify local max. or min. Points, we often use the derivative in solving an applied problem where we need to find a max. or min. value.

- **Sketching cubic function**

a) Shape of the graph:

- $a > 0$: increase, decrease and increase
- $a < 0$: decrease, increase and decrease

b) Find intercepts:

- ✓ for the y-intercept by finding $f(0)$
- ✓ for the x-intercepts by finding where $f(x) = 0$

Note:

- ✓ We first need to identify one factor using the factor theorem.
- ✓ The factor theorem says if $f(a) = 0$ and then $x - a$ is a factor of $f(x)$

c) For the x -value of the turning point:

- ✓ make $f'(x) = 0$
- ✓ solve for x
- ✓ For the y -value of the turning point substitute the x -value of turning point into the original equation.

d) For the x -value of the point of inflection make:

- ✓ $f''(x) = 0$ and solve
- ✓ Average the x -value of turning points (midpoint formula)
- ✓ Average the x -intercepts of a curve

e) For the y -value:

- ✓ substitute the x -value to the original
- ✓ y -midpoint formula for turning points

NB: Show that the concavity of changes at $x = a$

For: $a > 0$

- The graph changes from concave down to concave up at $x = a$

For: $a < 0$

- The graph changes from concave up to concave down at $x = a$

Note:

- ✓ Use the number line to calculate the concavity
- ✓ $f'' > 0$: concave up
- ✓ $f'' < 0$: concave down

f) Draw a neat sketch

Follow these steps:

- ✓ Indicate the axes, both x and y intercepts
- ✓ Indicate the turning points
- ✓ Consider the shape with max and min points

➤ **Reading from the graph**

✓ **For which value(s) of x will:**

- | | |
|--------------------------------|--|
| a) $f'(x) > 0$ | {where the graph is increasing} |
| b) $f'(x) < 0$ | {where the graph is decreasing} |
| c) $f(x).g(x) > 0$ | {where both graphs are above or below} |
| d) $f(x).g(x) < 0$ | {One graph above and other one below the horizontal} |
| e) $f(x) > 0$ | {above the horizontal} |
| f) $f(x) < 0$ | {below the horizontal} |
| g) $g'(x) = 0$ | {x-value of a turning point} |
| h) $g'(x) > 0$ and $f'(x) > 0$ | {both graphs increasing} |
| i) $g'(x) < 0$ and $f'(x) < 0$ | {both graphs decreasing} |
| j) $g(x) - f(x) = 1$ | {graph $g(x)$ is above $f(x)$ and the distance is 1 unit} |
| k) $x.f(x) > 0$ | {+ve x-value and +ve y-value or -ve x-value and -ve y-value} |
| l) $f(x).f'(x) < 0$ | {+ve y-value and the decreasing curve} |

Key note:

- Remember to manipulate the new given equation into the original one
- Maximum and minimum values means the y values of the turning point

✓ Reading y-value

For which value(s) of k will $f(x)$ has:

- Only one real roots { horizontal line cuts the graph once}
- equal roots { horizontal line touches the turning points}
- Three distinct roots { horizontal line cuts the graph three times}

Note:

The line $y = k + c$ has to intersect the graph $f(x)$ at different places. Make sure the given equation is derived to the original equation and make y the subject of the formula

✓ The transformation of cubic graph

a) Translation

It affects the turning points

1. Horizontal translation: $f(x + c)$

Then x -value(s) of the turning point is translated c unit left (-ve) or right (+ve)

2. Vertical translation: $f(x) + q$

Then y -value of turning point will be translated q unit up (+ve) or down (-ve)

b) The reflection

It can affect the whole equation and the turning points. The reflection about axes

The reflections are as follows:

- reflected across x -axis $p(x) = -p(x)$
- reflected across y -axis $p(x) = p(-x)$
- reflected about the line $y = x$

c) The enlargement

- You simply multiply by the scale factor given
- Especial applies on the turning points

B. EQUATION OF CUBIC FUNCTIONS

✓ Finding the value of a, b, c, and d

i) 3- x-intercepts given, if *the value of a is given*

✓ Simple use: $y = a(x - x)(x - x)(x - x)$

ii) 3-x-intercepts and 1 point along the curve

✓ Simple use: $y = a(x - x)(x - x)(x - x)$

iii) Given only the turning point and or point

✓ derive $f(x) = 0$ and then substitute for two x –values of tuning point or

✓ substitute **x and y** direct to the equation $f(x)$

iv) Given the gradient of the tangent and one point

✓ $f'(x) = m$ and substitute given value of x { if not given equate the two equations }

✓ substitute x-value and y-value in $f(x)$ and then solve simultaneously

Take note: Given y-intercept, automatically you have c-value or d-value

C. DERIVED GRAPH

- ✓ Parabola is a derivative of cubic graph
- ✓ Straight is the second derivative of cubic graph

➤ **Given $y = f'(x)$ for parabola**

You must be able to find the following:

1. The equation of $f'(x)$

- ✓ if 3 points given: simple use $y = a(x - x_1)(x - x_2)(x - x_3)$
- ✓ if turning point and one point given: simple use: $y = a(x - p)^2 + q$

2. The equation of $f(x)$

- ✓ you first need to derivative $f(x)$
- ✓ And then equate a with the value of a for both equation, and for b and c also (equating co-efficient)
- ✓ lastly write the final equation

3. The stationary points

- ✓ average the x-intercepts of the graph or use the midpoint formula
- ✓ x-intercept of the parabola give us the turning point since $f'(x) = 0$

D.PROPERTIES GIVEN

Given properties

- If there is prime that indicate the turning points
- If there is no prime that indicate the axes
- $f'(x) > 0$: indicate the increasing curve:
- $f'(x) < 0$: indicate the decreasing curve:

E. APPLICATION OF CALCULUS

➤ TO SHAPE

Note:

- Perimeter: is the sum of all sides

Rectangle:.....

Circle:.....

- Area: is the product of two sides

Rectangle:.....

Circle:.....

- Prism Volume : is the area base times height

Rectangle:.....

Cylinder:.....

- Pyramid Volume: is one-third area of base times height

Rectangular pyramid:.....

Cone:.....

- Surface area: is the sum of all faces

Rectangle:.....

Cylinder:.....

- Diameter: is the line passing through the centre from circumference

Diameter:

Radius:.....

NB: you must be in the position to imagine the shape if it is not given

Key: To Application of Calculus

Maximum/ minimum/largest/smallest/at least/ at most and fixed etc. :

- Derive
- Let derivative be zero
- Solve
- Substitute into the original equation if necessary

➤ **TO GRAPH**

You must recall the following:

- Distance/ length formula:.....
- Pythagoras formula:.....
- Gradient formula:.....
- Midpoint formula:.....
- Line equation:.....

➤ **TO RATE**

Rate:

- derive
- substitute with given value

At rest/ stationary/initial:

- time is zero
- velocity is zero

Flow of water:

- rate is flow in minus flow out

Rate of maximum:

- is the second derivative

Velocity/ speed:

- is the derivative of distance or displacement

Acceleration:

- is the derivative of velocity or second derivative of distance

KWV QP: 01

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 5$ (5)

7.2 Determine $\frac{dy}{dx}$ if:

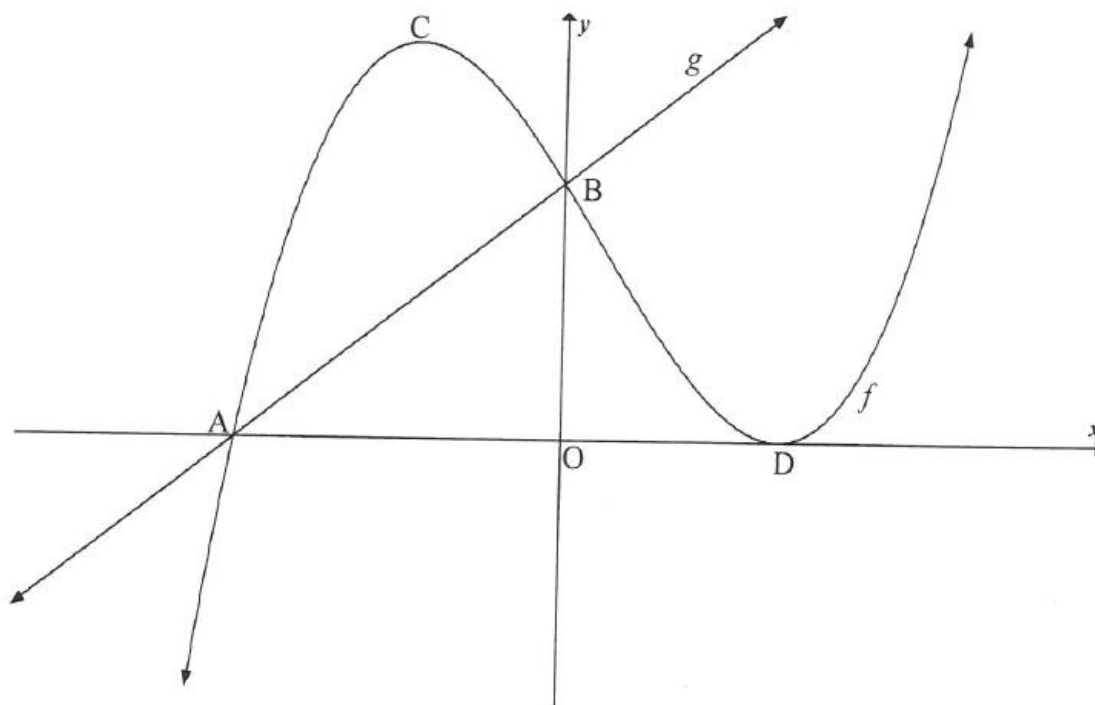
7.2.1 $y = 2x^5 + \frac{4}{x^3}$ (3)

7.2.2 $y = (\sqrt{x} - x^2)^2$ (4)
[12]

QUESTION 8

Sketched below are the graphs of $f(x) = (x-2)^2(x-k)$ and $g(x) = mx+12$

- A and D are the x -intercepts of f .
- B is the common y -intercept of f and g .
- C and D are turning points of f .
- The straight line g passes through A.



8.1 Write down the y -coordinate of B. (1)

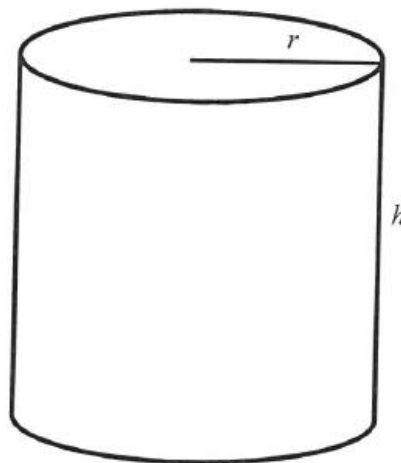
8.2 Calculate the x -coordinate of A. (3)

8.3 If $k = -3$, calculate the coordinates of C. (6)

8.4 For which values of x will f be concave down? (3)
[13]

QUESTION 9

A 340 ml can with height h cm and radius r cm is shown below.



$1 \text{ ml} = 1 \text{ cm}^3$

- 9.1 Determine the height of the can in terms of the radius r . (3)
- 9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)
- [9]

KWV MEMO: 01

QUESTION/VRAG 7

7.1	$f(x+h) = 3(x+h)^2 - 5 = 3(x^2 + 2xh + h^2) - 5$ $= 3x^2 + 6xh + 3h^2 - 5$ $f(x+h) - f(x) = 3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5$ $= 6xh + 3h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$ $= \lim_{h \rightarrow 0} (6x + 3h)$ $= 6x$ OR/OF $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$ $= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 5 - 3x^2 + 5}{h}$ $= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$ $= \lim_{h \rightarrow 0} (6x + 3h)$ $= 6x$	$\checkmark 3x^2 + 6xh + 3h^2 - 5$ $\checkmark 6xh + 3h^2$ $\checkmark \frac{f(x+h) - f(x)}{h}$ $\checkmark \text{common factor/ } (6x + 3h)$ $\checkmark \text{answer}$ (5)
7.2.1	$y = 2x^5 + \frac{4}{x^3}$ $y = 2x^5 + 4x^{-3}$ $\frac{dy}{dx} = 10x^4 - 12x^{-4}$	$\checkmark 2x^5 + 4x^{-3}$ $\checkmark 10x^4$ $\checkmark -12x^{-4}$ (3)

7.2.2	$y = (\sqrt{x} - x^2)^2$ $y = \left(x^{\frac{1}{2}} - x^2\right)^2$ $= x - 2x^{\frac{5}{2}} + x^4$ $\frac{dy}{dx} = 1 - 5x^{\frac{3}{2}} + 4x^3$	$\checkmark x - 2x^{\frac{5}{2}} + x^4$ $\checkmark 1$ $\checkmark -5x^{\frac{3}{2}}$ $\checkmark 4x^3$ (4) [12]
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QUESTION/VRAAG 8

8.1	$y = 12$	✓ answer (1)
8.2	$12 = (0-2)^2(0-k)$ $k = -3$ $(x-2)^2(x+3) = 0$ $x = -3$ OR/OF $y = 0$ $(x-2)^2(x-k) = 0$ $(x^2 - 4x + 4)(x-k) = 0$ $x^3 - kx^2 - 4x^2 + 4kx + 4x - 4k = 0$ But $-4k$ is the y - intercept <i>Maar $-4k$ is die y-afsnit</i> $-4k = 12$ $k = -3$ $x = -3$	✓ substituting (0;12) ✓ $k = -3$ ✓ $x = -3$ (3) ✓ $-4k$ ✓ $-4k = 12$ or $k = -3$ ✓ $x = -3$ (3)
8.3	$f(x) = x^3 + 3x^2 - 4x^2 - 12x + 4x + 12$ $f(x) = x^3 - x^2 - 8x + 12$ $f'(x) = 3x^2 - 2x - 8$ $3x^2 - 2x - 8 = 0$ $(3x + 4)(x - 2) = 0$ $x = -\frac{4}{3} \text{ or } x = 2$ $y = \frac{500}{27} \text{ or } 18,52 \text{ or } 18\frac{14}{27}$ $C\left(-\frac{4}{3}; 18,52\right)$	✓ $f(x) = x^3 - x^2 - 8x + 12$ ✓ derivative ✓ derivative equal to 0 ✓ factors or formula ✓ $x = -\frac{4}{3}$ ✓ $y = \frac{500}{27}$ or 18,52 or $18\frac{14}{27}$ (6)

8.4	$f''(x) = 6x - 2$ $6x - 2 < 0$ $x < \frac{1}{3}$ f is concave down when $x < \frac{1}{3}$ f is konkaf na onder vir $x < \frac{1}{3}$ OR/OF $f''(x) = 6x - 2$ $6x - 2 = 0$ $x = \frac{1}{3}$ f is concave down when $x < \frac{1}{3}$ f is konkaf na onder vir $x < \frac{1}{3}$ OR/OF $x = \frac{x_c + x_d}{2}$ $= \frac{-\frac{4}{3} + 2}{2}$ $= \frac{1}{3}$ f is concave down when $x < \frac{1}{3}$ f is konkaf na onder vir $x < \frac{1}{3}$	$\checkmark 6x - 2$ $\checkmark\checkmark x < \frac{1}{3}$ (3) $\checkmark 6x - 2$ $\checkmark\checkmark x < \frac{1}{3}$ (3) $\checkmark \frac{-\frac{4}{3} + 2}{2} \text{ or } -\frac{-1}{3(1)}$ $\checkmark\checkmark x < \frac{1}{3}$ (3) [13]
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QUESTION/VRAAG 9

9.1	$V = \pi r^2 h$ $\pi r^2 h = 340$ $h = \frac{340}{\pi r^2}$	✓ formula ✓ equating to 340 ✓ $h = \frac{340}{\pi r^2}$ (3)
9.2	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r \left(\frac{340}{\pi r^2} \right)$ $= 2\pi r^2 + \frac{680}{r}$ $A'(r) = 4\pi r - \frac{680}{r^2}$ $A'(r) = 0$ for minimum surface area/ <i>vir minimum buite-oppervlakte</i> $4\pi r - \frac{680}{r^2} = 0$ $r^3 = \frac{680}{4\pi} = \frac{170}{\pi}$ $= 54,11268$ $r = 3,78 \text{ cm}$	✓ $2\pi r^2 + 2\pi rh$ ✓ substituting h ✓ $4\pi r - \frac{680}{r^2}$ ✓ $A'(r) = 0$ ✓ $r^3 = \frac{680}{4\pi}$ ✓ answer (6) [9]

KWV QP: 02

QUESTION 8

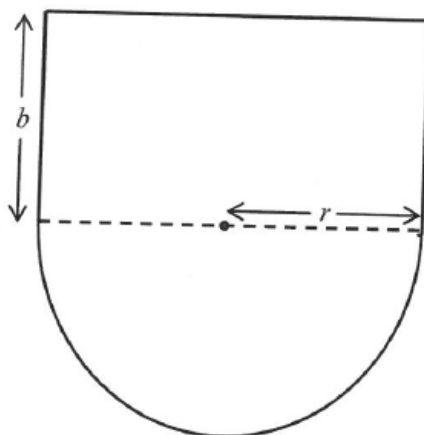
- 8.1 Given $f(x) = 3 - 2x^2$. Determine $f'(x)$, using first principles. (5)
- 8.2 Determine $\frac{dy}{dx}$ if $y = \frac{12x^2 + 2x + 1}{6x}$. (4)
- 8.3 The function $f(x) = x^3 + bx^2 + cx - 4$ has a point of inflection at (2 ; 4). Calculate the values of b and c . (7)
[16]

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

- 9.1 Write down the coordinates of the y -intercept of f . (1)
- 9.2 Calculate the coordinates of the x -intercepts of f . (5)
- 9.3 Calculate the coordinates of the turning points of f . (6)
- 9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)
- 9.5 Write down the values of x for which $f'(x) < 0$. (2)
[17]

QUESTION 10



The figure above shows the design of a theatre stage which is in the shape of a semicircle attached to a rectangle. The semicircle has a radius r and the rectangle has a breadth b . The perimeter of the stage is 60 m.

- 10.1 Determine an expression for b in terms of r . (2)
- 10.2 For which value of r will the area of the stage be a maximum? (6)
- [8]

KWV MEMO: 02

QUESTION/VRAAG 8

8.1	$f(x+h) = 3 - 2(x+h)^2$ $= 3 - 2x^2 - 4xh - 2h^2$ $f(x+h) - f(x) = 3 - 2x^2 - 4xh - 2h^2 - 3 + 2x^2$ $= -4xh - 2h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \rightarrow 0} (-4x - 2h)$ $= -4x$ OR/OF $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{3 - 2(x+h)^2 - (3 - 2x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{3 - 2x^2 - 4xh - 2h^2 - 3 + 2x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \rightarrow 0} (-4x - 2h)$ $= -4x$	$\checkmark 3 - 2x^2 - 4xh - 2h^2$ $\checkmark -4xh - 2h^2$ $\checkmark f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\checkmark \lim_{h \rightarrow 0} (-4x - 2h)$ $\checkmark -4x \quad (5)$ $\checkmark f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\checkmark 3 - 2x^2 - 4xh - 2h^2$ $\checkmark -4xh - 2h^2$ $\checkmark \lim_{h \rightarrow 0} (-4x - 2h)$ $\checkmark -4x \quad (5)$
8.2	$y = \frac{12x^2 + 2x + 1}{6x}$ $= 2x + \frac{1}{3} + \frac{1}{6x}$ $= 2x + \frac{1}{3} + \frac{1}{6}x^{-1}$ $\frac{dy}{dx} = 2 - \frac{1}{6}x^{-2}$ $= 2 - \frac{1}{6x^2}$	$\checkmark \frac{12x^2}{6x} + \frac{2x}{6x} + \frac{1}{6x}$ $\checkmark \frac{1}{6}x^{-1}$ $\checkmark 2$ $\checkmark -\frac{1}{6}x^{-2}$ (4)

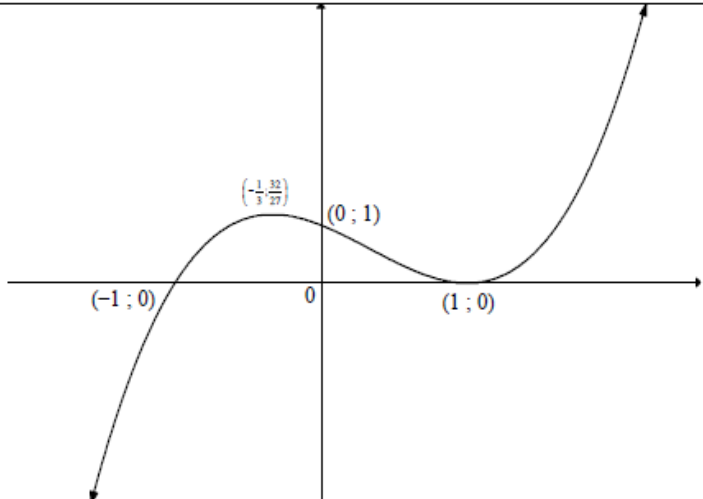
8.3	$y = x^3 + bx^2 + cx - 4$ $y' = 3x^2 + 2bx + c$ $y'' = 6x + 2b$ At point of inflection: $y'' = 6x + 2b = 0$ Substitute $x = 2$: $6(2) + 2b = 0$ $2b = -12$ $b = -6$ $y = x^3 - 6x^2 + cx - 4$ Substitute $(2; 4)$: $4 = 2^3 - 6(2)^2 + c(2) - 4$ $2c = 24$ $c = 12$ $y = x^3 - 6x^2 + 12x - 4$	$\checkmark y' = 3x^2 + 2bx + c$ $\checkmark y'' = 6x + 2b$ $\checkmark y'' = 0$ $\checkmark \text{sub } x = 2 \text{ into } y'' = 0$ $\checkmark \text{value of } b$ $\checkmark \text{substitute } (2; 4)$ $\checkmark \text{value of } c$
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(7)

[16]

QUESTION/VRAAG 9

9.1	(0 ; 1)	$\checkmark$ answer (1)
9.2	$f(x) = x^3 - x^2 - x + 1$ $f(x) = x^2(x - 1) - (x - 1)$ $f(x) = (x - 1)(x^2 - 1)$ $f(x) = (x - 1)(x - 1)(x + 1)$ $f(x) = 0$ $(x - 1)(x - 1)(x + 1) = 0$ x-intercepts: $(-1; 0); (1; 0)$ OR $f(x) = x^3 - x^2 - x + 1$ $f(x) = (x - 1)(x^2 - 1)$ $f(x) = (x - 1)(x - 1)(x + 1)$ $f(x) = 0$ $(x - 1)(x - 1)(x + 1) = 0$ x-intercepts: $(-1; 0); (1; 0)$ OR	$\checkmark (x - 1)$ $\checkmark (x^2 - 1)$ $\checkmark (x - 1)(x - 1)(x + 1)$ $\checkmark (-1; 0)$ $\checkmark (1; 0)$ (5) $\checkmark (x - 1)$ $\checkmark (x^2 - 1)$ $\checkmark (x - 1)(x - 1)(x + 1)$ $\checkmark (-1; 0)$ $\checkmark (1; 0)$ (5)

	$f(x) = x^3 - x^2 - x + 1$ $f(x) = (x+1)(x^2 - 2x + 1)$ $f(x) = (x+1)(x-1)(x-1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ $x\text{-intercepts: } (-1; 0); (1; 0)$	$\checkmark (x+1)$ $\checkmark (x^2 - 2x + 1)$ $\checkmark (x-1)(x-1)(x+1)$ $\checkmark (-1; 0)$ $\checkmark (1; 0)$ (5)
9.3	$f(x) = x^3 - x^2 - x + 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(x) = 0$ $(3x+1)(x-1) = 0$ $x = -\frac{1}{3} \quad \text{or} \quad x = 1$ $y = \frac{32}{27} \quad y = 0$ $\left(-\frac{1}{3}; \frac{32}{27}\right) (1; 0)$	$\checkmark f'(x) = 3x^2 - 2x - 1$ $\checkmark f'(x) = 0$ $\checkmark \text{factorisation}$ $\checkmark x \text{ value}$ $\checkmark x \text{ value}$ $\checkmark y = \frac{32}{27}$ (6)
9.4		$\checkmark y\text{- and } x\text{-intercepts}$ $\checkmark \text{shape}$ $\checkmark \text{turning points}$ (3)
9.5	$f'(x) < 0$ $-\frac{1}{3} < x < 1$ OR/OF $\left(-\frac{1}{3}; 1\right)$	$\checkmark x > -\frac{1}{3}$ $\checkmark x < 1$ (2) $\checkmark \left(-\frac{1}{3}\right)$ $\checkmark 1$ (2) [17]

QUESTION/VRAAG 10

10.1	$60 = 2b + 2r + \frac{1}{2}(2\pi r)$ $2b = 60 - 2r - \pi r$ $b = 30 - r - \frac{1}{2}\pi r$	$\checkmark 60 = 2b + 2r + \frac{1}{2}(2\pi r)$ $\checkmark b = 30 - r - \frac{1}{2}\pi r$
10.2	Area = area of rectangle + area of semicircle $A(r) = \text{length} \times \text{breadth} + \frac{1}{2}(\text{area of circle})$ $= (2r)\left(30 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$ $= 60r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $= 60r - 2r^2 - \frac{1}{2}\pi r^2$ $= 60r - \left(2 + \frac{1}{2}\pi\right)r^2$ For a maximum, $A'(r) = 0$ $60 - 2\left(2 + \frac{1}{2}\pi\right)r = 0$ $60 - (4 + \pi)r = 0$ $r = \frac{60}{4 + \pi}$ $= 8,40 \text{ m}$	$\checkmark (2r)\left(30 - r - \frac{1}{2}\pi r\right)$ $\checkmark \frac{1}{2}(\pi r^2)$ $\checkmark 60r - 2r^2 - \frac{1}{2}\pi r^2$ $\checkmark A'(r) = 0$ $\checkmark 60 - 2\left(2 + \frac{1}{2}\pi\right)r$ $\checkmark \text{answer}$

(2)

(6)

[8]

KWV QP: 03

QUESTION 7

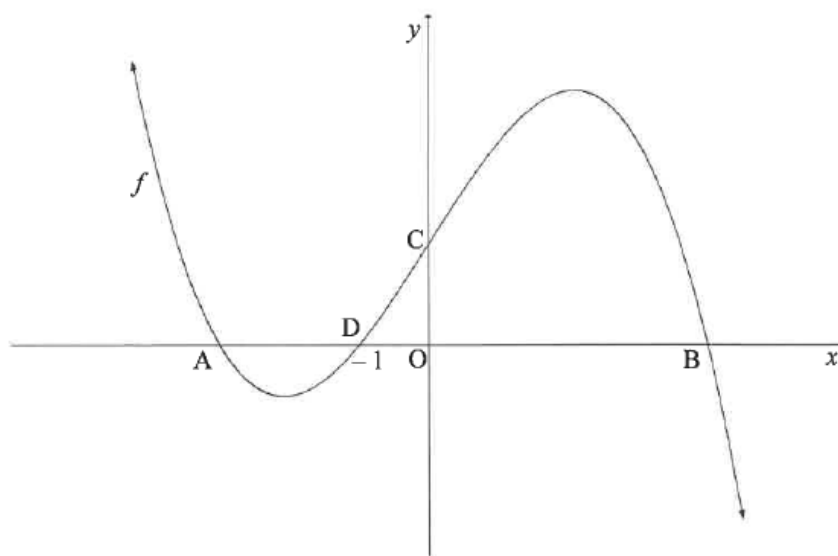
- 7.1 Given: $f(x) = 2 - 3x^2$
Determine $f'(x)$ from first principles. (5)
- 7.2 Determine:
- 7.2.1 $D_x[(4x+5)^2]$ (3)
- 7.2.2 $\frac{dy}{dx}$ if $y = \sqrt[4]{x} + \frac{x^2-8}{x^2}$ (4)
- [12]

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.

A, B and D(-1 ; 0) are the x -intercepts of f .

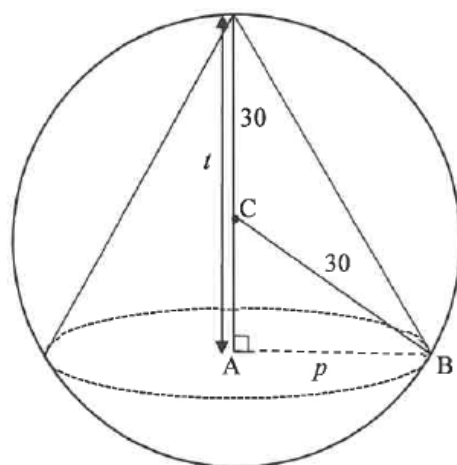
C is the y -intercept of f .



- 8.1 Write down the coordinates of C. (1)
- 8.2 Calculate the coordinates of A and B. (5)
- 8.3 Determine the point of inflection of g if it is given that $g(x) = -f(x)$. (4)
- 8.4 Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$. (4)
- [14]

QUESTION 9

A right circular cone with radius p and height t is machined (cut out) from a solid sphere (with centre C) with a radius of 30 cm, as shown in the sketch.



$$\text{Sphere: } V = \frac{4}{3} \pi r^3$$

$$\text{Cone: } V = \frac{1}{3} \pi r^2 h$$

9.1 From the given information, express the following:

9.1.1 AC in terms of t . (1)

9.1.2 p^2 , in its simplest form, in terms of t . (3)

9.2 Show that the volume of the cone can be written as $V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$. (1)

9.3 Calculate the value of t for which the volume of the cone will be a maximum. (3)

9.4 What percentage of the sphere was used to obtain this cone having maximum volume? (4)
[12]

KWV MEMO: 03

QUESTION/VRAAG 7

Penalize 1 mark for incorrect notation in the whole question.

7.1	$f(x+h) = 2 - 3(x+h)^2$ $= 2 - 3(x^2 + 2xh + h^2)$ $= 2 - 3x^2 - 6xh - 3h^2$ $f(x+h) - f(x) = 2 - 3x^2 - 6xh - 3h^2 - (2 - 3x^2)$ $= -6xh - 3h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= \lim_{h \rightarrow 0} (-6x - 3h)$ $= -6x$ OR/OF $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2 - 3(x+h)^2 - (2 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{2 - 3x^2 - 6xh - 3h^2 - (2 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= \lim_{h \rightarrow 0} (-6x - 3h)$ $= -6x$	$\checkmark 2 - 3x^2 - 6xh - 3h^2$ $\checkmark -6xh - 3h^2$ $\checkmark$ subst. into formula $\checkmark$ factorisation $\checkmark$ answer (5) OR/OF $\checkmark$ subst. into formula $\checkmark$ simplification $\checkmark -6xh - 3h^2$ $\checkmark$ common factor $\checkmark$ answer (5)
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7.2.1	$D_x[(4x+5)^2]$ $= D_x(16x^2 + 40x + 25)$ $= 32x + 40$	$\checkmark 16x^2 + 40x + 25$ $\checkmark 32x$ $\checkmark + 40$ (3)
7.2.2	$y = \sqrt[4]{x} + \frac{x^2 - 8}{x^2}$ $y = x^{\frac{1}{4}} + 1 - 8x^{-2}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}} + 16x^{-3}$	$\checkmark x^{\frac{1}{4}}$ $\checkmark 1 - 8x^{-2}$ $\checkmark \frac{1}{4}x^{-\frac{3}{4}}$ $\checkmark 16x^{-3}$ (4) [12]

QUESTION/VR4AG 8

8.1	$C(0;12)$	✓ $C(0;12)$ (1)
8.2	$-x^3 + 13x + 12 = 0$ $x^3 - 13x - 12 = 0$ $(x+1)(x^2 - x - 12) = 0$ $(x+1)(x-4)(x+3) = 0$ $A(-3;0)$ $B(4;0)$	✓ $f(x) = 0$ ✓ $(x+1)$ ✓ $(x^2 - x - 12)$ ✓ $x = -3$ or 4 ✓ clearly indicating A and B (5)
8.3	$f'(x) = -3x^2 + 13$ $f''(x) = -6x$ $-6x = 0$ $x = 0$ For $f(x)$, point of inflection will be at $(0; 12)$. <i>Vir $f(x)$, sal buigpunt wees by $(0; 12)$</i> For $g(x)$, point of inflection will be at $(0; -12)$. <i>Vir $g(x)$, sal buigpunt wees by $(0; -12)$.</i> OR/OF $g(x) = x^3 - 13x - 12$ $g'(x) = 3x^2 - 13$ $g''(x) = 6x$ $6x = 0$ $x = 0$ $(0; -12)$ OR/OF $f'(x) = -3x^2 + 13$ TP's where $-3x^2 + 13 = 0$ $x^2 = \frac{13}{3}$ $x = \pm \sqrt{\frac{13}{3}}$ $= \pm 2,08$ x-value of point of inflection: $\frac{-2,08 + 2,08}{2} = 0$ For $f(x)$, point of inflection will be at $(0; 12)$. <i>Vir $f(x)$, sal buigpunt wees by $(0; 12)$</i> For $g(x)$, point of inflection will be at $(0; -12)$. <i>Vir $g(x)$, sal buigpunt wees by $(0; -12)$.</i>	✓ $f'(x) = -3x^2 + 13$ ✓ $f''(x) = -6x$ ✓ equating to zero ✓ $(0; -12)$ (4) OF/OR ✓ $g'(x) = 3x^2 - 13$ ✓ $g''(x) = 6x$ ✓ equating to zero ✓ $(0; -12)$ (4) OR/OF ✓ $f'(x) = -3x^2 + 13$ ✓ $-3x^2 + 13 = 0$ ✓ x-values of TP's ✓ $(0; -12)$ (4)

8.4	$f'(x) = -3x^2 + 13$ $-3x^2 + 13 = -14$ $-3x^2 = -27$ $x^2 = 9$ $x = 3$ or $x = -3$	✓ equating derivative to -14 ✓ simplification ✓ ✓ answers (4) [14]
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QUESTION/VRAAG 9

9.1.1	$AC = t - 30$	✓ answer (1)
9.1.2	$30^2 = (t - 30)^2 + p^2$ [Pythagoras] $p^2 = 900 - (t - 30)^2$ $p^2 = 900 - (t^2 - 60t + 900)$ $p^2 = 900 - t^2 + 60t - 900$ $p^2 = 60t - t^2$	$\checkmark p^2 = 900 - (t - 30)^2$ $\checkmark (t^2 - 60t + 900)$ $\checkmark p^2 = 60t - t^2$ (3)
9.2	$V(t) = \frac{1}{3} \pi r^2 t$ $= \frac{1}{3} \pi (60t - t^2) t$ $= 20\pi t^2 - \frac{1}{3} \pi t^3$	$\checkmark$ substitution (1)
9.3	$V(t) = 20\pi t^2 - \frac{1}{3} \pi t^3$ $V'(t) = 40\pi t - \pi t^2$ $40\pi t - \pi t^2 = 0$ $t(40\pi - t\pi) = 0$ $t = 0$ OR $t = 40$ cm N/A	$\checkmark 40\pi t$ $\checkmark -\pi t^2$ $\checkmark$ answer with selection (3)
9.4	Volume of cone/keël $= 20(\pi)(40)^2 - \frac{1}{3} \pi (40)^3$ $= 10\,666,67\pi$ or $33510,33211$	$\checkmark$ volume of cone
	Volume of sphere/sfeer $= \frac{4}{3} \pi r^3$ $= \frac{4}{3} \pi (30)^3$ $= 36000\pi$ or $113097,3355$	$\checkmark$ volume of sphere
	$\frac{10\,666,67\pi}{36000\pi}$ $= 0,296296$ $\approx 29,63\%$	$\checkmark \frac{10\,666,67\pi}{36000\pi}$ $\checkmark$ % cut out (4)

[12]

KWV QP : 04

QUESTION 7

7.1 Given $f(x) = x^2 + 2$.

Determine $f'(x)$ from first principles. (4)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 4x^3 + \frac{2}{x}$ (3)

7.2.2 $y = 4\sqrt[3]{x} + (3x^3)^2$ (4)

7.3 If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g in the form $y = \dots$ (3)

[14]

QUESTION 8

A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x -axis at $(-3; 0)$; $\left(-\frac{3}{2}; 0\right)$ and $(1; 0)$.

8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)

8.2 Calculate the x -coordinates of the turning points of h . (3)

8.3 Determine the value(s) of x for which h will be decreasing. (2)

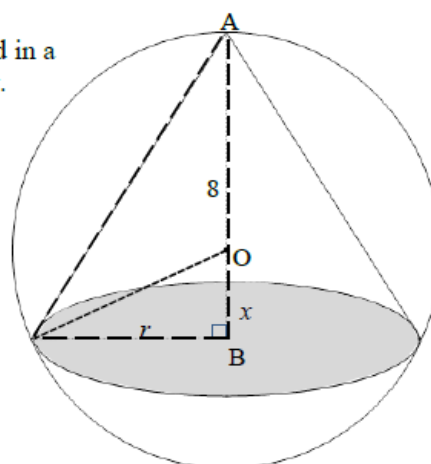
8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)

[12]

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. $OB = x$.

$\text{Volume of sphere} = \frac{4}{3}\pi r^3$ $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$



9.1 Calculate the volume of the sphere. (1)

9.2 Show that $r^2 = 64 - x^2$. (1)

9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)

[9]

KWV MEMO: 04

QUESTION/VRAAG 7

7.1	$f(x) = x^2 + 2$ $f(x+h) = (x+h)^2 + 2$ $= x^2 + 2xh + h^2 + 2$ $f(x+h) - f(x) = x^2 + 2xh + h^2 + 2 - (x^2 + 2)$ $= 2xh + h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $= \lim_{h \rightarrow 0} (2x + h)$ $= 2x$ OR/OF $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2 - (x^2 + 2)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $= \lim_{h \rightarrow 0} (2x + h)$ $= 2x$	$\checkmark x^2 + 2xh + h^2 + 2$ $\checkmark \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $\checkmark \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $\checkmark \text{ answer}$ (4) OR/OF $\checkmark x^2 + 2xh + h^2 + 2$ $\checkmark \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $\checkmark \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $\checkmark \text{ answer}$ (4)
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7.2.1	$y = 4x^3 + 2x^{-1}$ $\frac{dy}{dx} = 12x^2 - 2x^{-2}$	$\checkmark + 2x^{-1}$ $\checkmark 12x^2$ $\checkmark - 2x^{-2}$ (3)
7.2.2	$y = 4\sqrt[3]{x} + (3x^3)^2$ $= 4x^{\frac{1}{3}} + 9x^6$ $\frac{dy}{dx} = \frac{4}{3}x^{-\frac{2}{3}} + 54x^5$	$\checkmark 4x^{\frac{1}{3}}$ $\checkmark 9x^6$ $\checkmark \frac{4}{3}x^{-\frac{2}{3}}$ $\checkmark 54x^5$ (4)
7.3	Point of contact: (1 ; 5) $m = 2$ $y - y_1 = m(x - x_1)$ or $y = 2x + c$ $y - 5 = 2(x - 1)$ $5 = 2 + c$ $c = 3$ $y = 2x + 3$ $y = 2x + 3$	$\checkmark m = 2$ $\checkmark$ substitution of (1 ; 5) $\checkmark$ answer (3) [14]

QUESTION/VR44G 8

8.1	$h(x) = -2(x + \frac{3}{2})(x - 1)(x + 3)$ $h(x) = -(2x + 3)(x^2 + 2x - 3)$ $h(x) = -2x^3 - 7x^2 + 9$ OR/OF $h(x) = -(2x + 3)(x - 1)(x + 3)$ $h(x) = -(2x + 3)(x^2 + 2x - 3)$ $h(x) = -2x^3 - 7x^2 + 9$	$\checkmark \checkmark -2(x + \frac{3}{2})(x - 1)(x + 3)$ $\checkmark$ correct simplification (3) OR/OF $\checkmark \checkmark -(2x + 3)(x - 1)(x + 3)$ $\checkmark$ correct simplification (3)
8.2	$h'(x) = -6x^2 - 14x$ $-6x^2 - 14x = 0$ $-2x(3x + 7) = 0$ $x = 0$ or $x = -\frac{7}{3}$	$\checkmark$ first derivative $\checkmark = 0$ $\checkmark$ both answers (3)
8.3	$x < -\frac{7}{3}$ or $x > 0$ OR/OF $x \in \left(-\infty; -\frac{7}{3}\right) \cup (0; \infty)$	$\checkmark \checkmark$ answer (2) OR/OF $\checkmark \checkmark$ answer (2)

8.4	$y = 4x + 7$ $-6x^2 - 14x = 4$ $0 = 6x^2 + 14x + 4$ $0 = 3x^2 + 7x + 2$ $0 = (3x + 1)(x + 2)$ $x = -\frac{1}{3}$ or $x = -2$	$\checkmark y = 4x + 7$ $\checkmark h'(x) = 4$ $\checkmark$ standard form $\checkmark$ both answers (4) [12]
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QUESTION/VRAAG 9

9.1	Volume of Sphere $= \frac{4}{3}\pi(8)^3$ or $= \frac{2048\pi}{3}$ or $= 2144,66$	$\checkmark$ answer (1)
9.2	$r^2 + x^2 = 8^2$ (Pythagoras) $r^2 = 64 - x^2$	$\checkmark$ substitution or reason Pythagoras (1)
9.3	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(64 - x^2)(8 + x)$ $= \frac{\pi}{3}(512 + 64x - 8x^2 - x^3)$ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \frac{8}{3}$ $x \neq -8$ $\frac{V_{cone}}{V_{sphere}} = \frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $= \frac{8}{27} = 0,3$	$\checkmark h = 8 + x$ $\checkmark \frac{1}{3}\pi(64 - x^2)(8 + x)$ $\checkmark$ expansion $\checkmark \frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $\checkmark x = \frac{8}{3}$ $\checkmark$ volume of the cone $\checkmark \frac{8}{27}$ or 0,3 (7) [9]

KWV QP 05

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

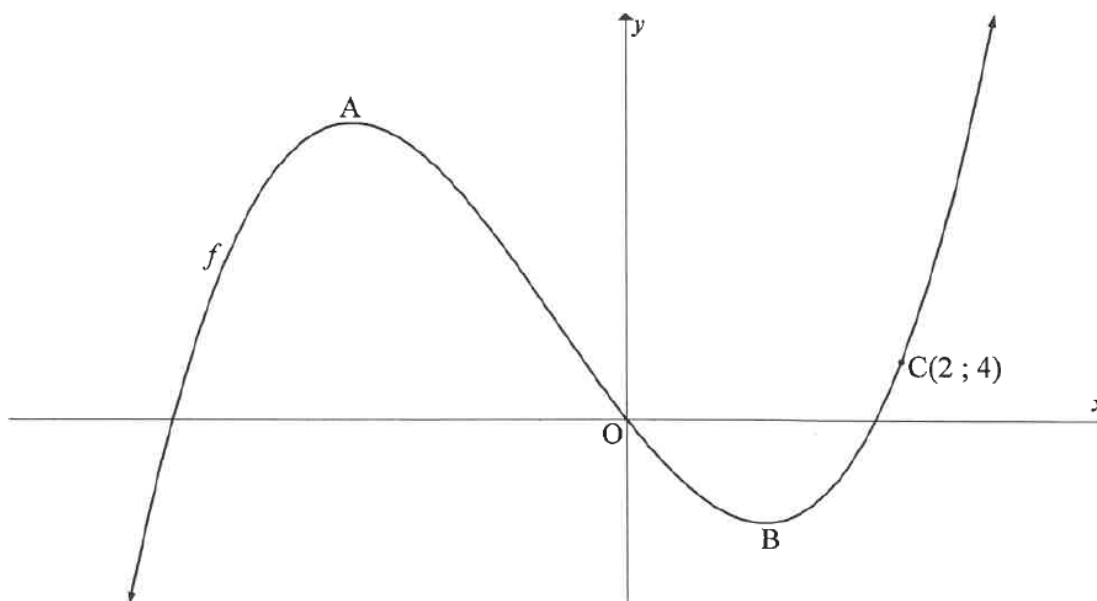
8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = (\sqrt{x} + 3)(\sqrt{x} - 1)$ (4)
[12]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.

A and B are the turning points of f . $C(2; 4)$ is a point on f .



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at $C(2; 4)$. (3)
[11]

QUESTION 10

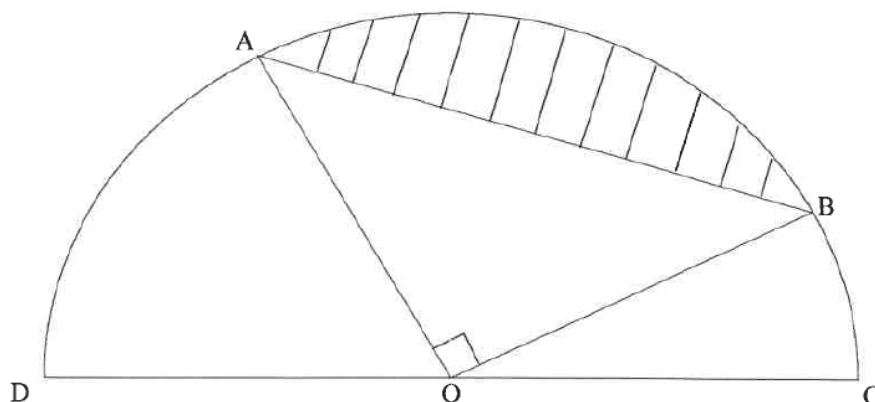
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x -axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f , use this information to draw a sketch graph of f , only indicating the x -coordinates of the x -intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $(x - x^2)$ units for $0 < x < 1$. $\triangle AOB$ is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

$$\text{Area} = \left(\frac{\pi - 2}{4}\right)(x^4 - 2x^3 + x^2) \quad (5)$$

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)
[13]

KWV MEMO 0.5

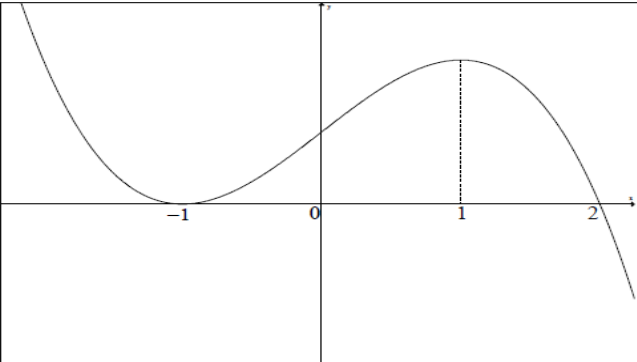
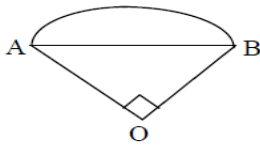
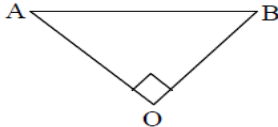
QUESTION/VRAAG 8

8.1	$f(x) = 3x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$ $= 6x$	✓ substitution ✓ expansion ✓ simplification ✓ $\lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$ ✓ $6x$
8.2.1	$f(x) = x^2 - 3 + 9x^{-2}$ $f'(x) = 2x - 18x^{-3}$	✓ $9x^{-2}$ ✓ $2x$ ✓ $-18x^{-3}$
8.2.2	$g(x) = (\sqrt{x} + 3)(\sqrt{x} - 1)$ $g(x) = x + 2x^{\frac{1}{2}} - 3$ $g'(x) = 1 + x^{-\frac{1}{2}}$	✓ x ✓ $2x^{\frac{1}{2}}$ ✓ 1 ✓ $x^{-\frac{1}{2}}$
		[12]

QUESTION/VRAAG 9

9.1	$f'(x) = 6x^2 + 6x - 12$ $6x^2 + 6x - 12 = 0$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $x = -2$ or $x = 1$ $y = 20$ or $y = -7$ $\therefore A(-2 ; 20)$ and $B(1 ; -7)$	$\checkmark 6x^2 + 6x - 12$ $\checkmark = 0$ $\checkmark$ factors $\checkmark x$ -values $\checkmark y$ -values (5)
9.2	$f''(x) = 12x + 6$ $12x + 6 > 0$ $12x > -6$ $x > -\frac{1}{2}$ OR/OF $x = \frac{-2+1}{2} = -\frac{1}{2}$ $\therefore x > -\frac{1}{2}$	$\checkmark 12x + 6$ $\checkmark f''(x) > 0$ $\checkmark x > -\frac{1}{2}$ (3) OR/OF $\checkmark x = -\frac{1}{2}$ $\checkmark \checkmark x > -\frac{1}{2}$ (3)
9.3	$f'(2) = 24$ Equation of the tangent: $y - 4 = 24(x - 2)$ $y = 24x - 44$	$\checkmark f'(2)$ $\checkmark 24$ $\checkmark$ equation (3)
		[11]

QUESTION/VR4AG 10

10.1		✓ $x = -1$ and $x = 2$ ✓ TP at $x = -1$ ✓ TP at $x = 1$ ✓ shape (4)
10.2.1	 Area of segment = $\frac{1}{4}$ Area of big circle $= \frac{1}{4} \pi (x - x^2)^2$  Area triangle ABO counted $= \text{Area } \Delta = \frac{1}{2} (x - x^2)^2$ Area of shaded region $= \frac{1}{4} \pi (x - x^2)^2 - \frac{1}{2} (x - x^2)^2$ $= \frac{\pi - 2}{4} (x - x^2)^2$ $= \left(\frac{\pi - 2}{4} \right) (x^2 - 2x^3 + x^4)$	✓ ✓ $\frac{1}{4} \pi (x - x^2)^2$ ✓ Area $\Delta = \frac{1}{2} (x - x^2)^2$ ✓ subtract areas ✓ common factor (5)
10.2.2	Area of shaded region $= \frac{(\pi - 2)}{4} (x^4 - 2x^3 + x^2)$ $\frac{dA}{dx} = \left(\frac{\pi - 2}{4} \right) (4x^3 - 6x^2 + 2x)$ $4x^3 - 6x^2 + 2x = 0$ $x(2x^2 - 3x + 1) = 0$ $x(2x - 1)(x - 1) = 0$ $x \neq 0 \quad \text{or} \quad x = \frac{1}{2} \quad \text{or} \quad x \neq 1$	✓ $\left(\frac{\pi - 2}{4} \right) (4x^3 - 6x^2 + 2x)$ ✓ factors ✓ $x = 0; x = 1; x = \frac{1}{2}$ ✓ $x = \frac{1}{2}$ (4)

[13]

MERCY!!!!!!

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