



**MATHEMATICS
PREPARATORY EXAMINATION
GRADE 12**



MATHEMATICS P2

SEPTEMBER 2024

Stanmorephysics.com

**MARKS: 150
TIME: 3 hours**



**This question paper consists of 12 pages, 1 information sheet
and an answer book of 22 pages.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

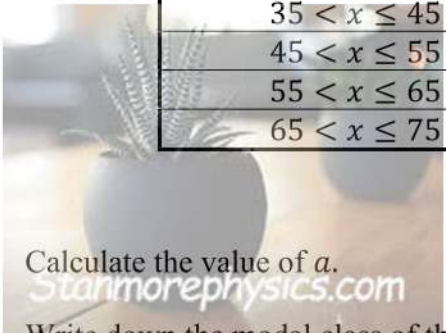


QUESTION 1

During a congress that took place in July, the ages of the 100 members who attended, were recorded in the following table:



AGE (x) (IN YEARS)	NUMBER OF PEOPLE
$15 < x \leq 25$	8
$25 < x \leq 35$	14
$35 < x \leq 45$	22
$45 < x \leq 55$	37
$55 < x \leq 65$	a
$65 < x \leq 75$	3



- 1.1 Calculate the value of a . (1)
- 1.2 Write down the modal class of the given data. (1)
- 1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)
- 1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)
- 1.5 The congress organisers discovered that a mistake was made, and that 3 people counted in the range $45 < x \leq 55$ should have been counted in the range $35 < x \leq 45$. Calculate the estimated average age of the people who attended the congress after the mistake was corrected. (2)

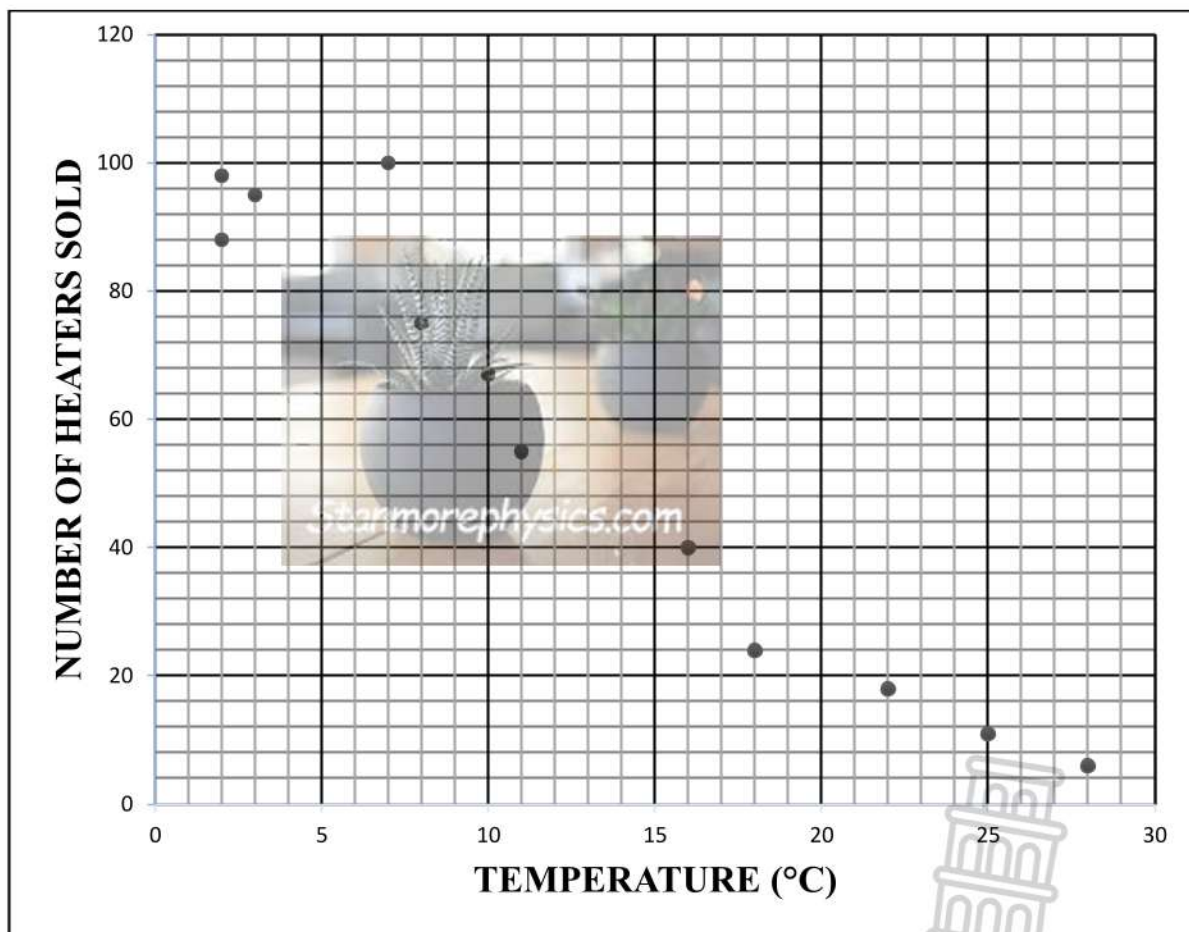
[9]

QUESTION 2

The scatter plot below shows the number of heaters a company sold per month and the average temperature of that month in a certain year.

Both the temperature in $^{\circ}\text{C}$ and the number of heaters sold, are given in the table below.

MONTH	J	F	M	A	M	J	J	A	S	O	N	D
TEMPERATURE ($^{\circ}\text{C}$)	2	7	8	10	18	22	28	25	16	11	2	3
NUMBER OF HEATERS SOLD	98	100	75	67	24	18	6	11	40	55	88	95



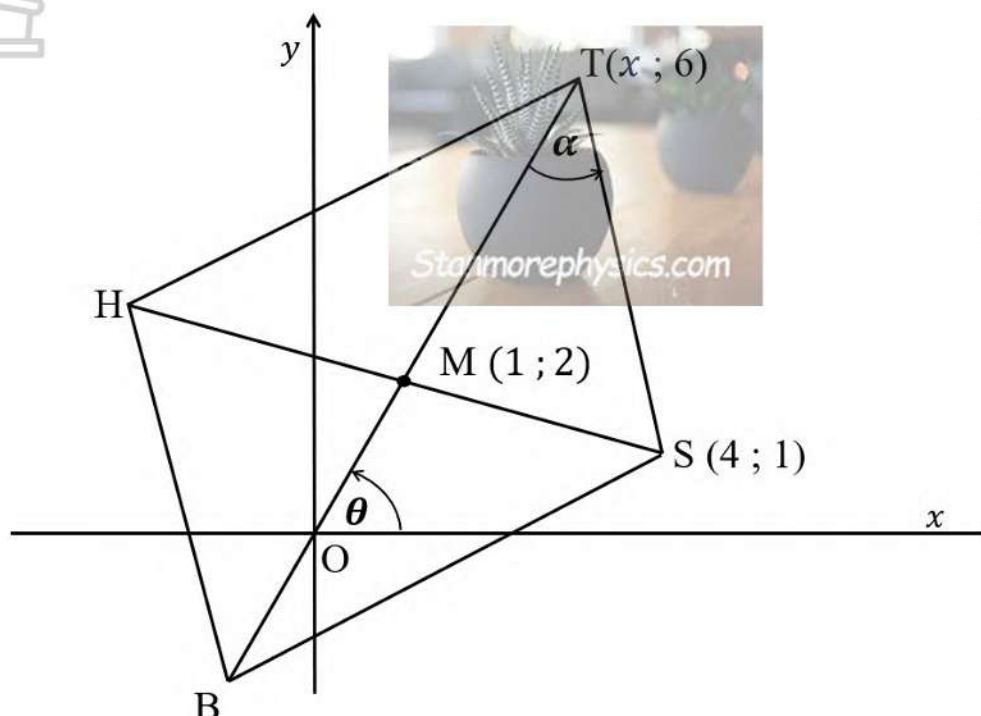
- 2.1 Describe the correlation between the number of heaters sold and the average temperature per month. Verify your answer by referring to the correlation coefficient. (2)
- 2.2 Determine the equation of the least squares regression line for the data. (3)
- 2.3 Predict the number of heaters sold for a month where the average temperature is 20°C . (2)
- 2.4 Draw the least squares regression line on the grid given in the ANSWER BOOK. (2)
- 2.5 Calculate the standard deviation of the number of heaters sold. (2)

[11]

QUESTION 3

In the diagram below HS and TB are the diagonals of parallelogram HTSB.

- HS and TB intersect at $M(1; 2)$.
- TB intersect the x - and y -axes at the origin.
- $T(x; 6)$ and $S(4; 1)$ are vertices of HTSB.
- $\widehat{MTS} = \alpha$ and the inclination angle of TB is θ .



3.1 Determine:

3.1.1 the equation of the line TB in the form $y = mx + c$. (2)

3.1.2 the size of θ . (2)

3.1.3 the coordinates of H. (3)

3.2 Show that the x -coordinate of T is 3 if it is further given that $TS = \sqrt{26}$. (3)

3.3 Calculate α , the size of $\widehat{MTS}$. (4)

3.4 Calculate the area of $\triangle BTS$. (4)

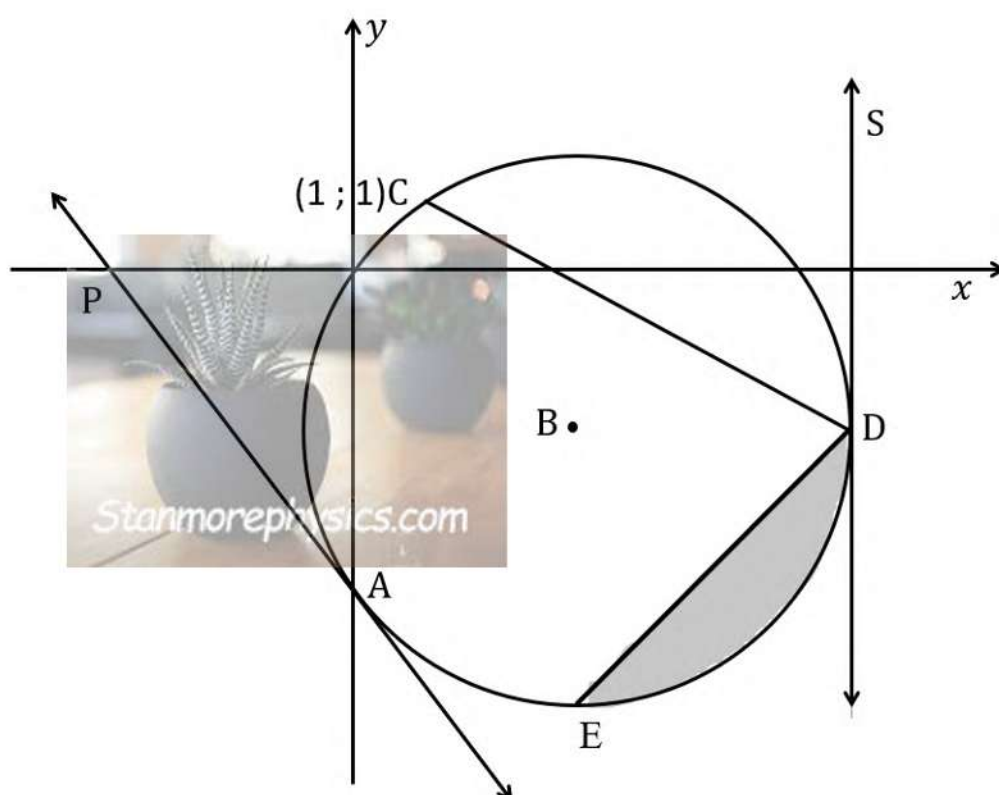
3.5 Determine the perpendicular height of the parallelogram if TS is the base. (3)

[21]

QUESTION 4

In the diagram, the equation of the circle centred at B is given as: $x^2 - 8x + y^2 + 6y = 0$

- DS and PA are tangents to the circle at D and A respectively.
- A is the y-intercept of the circle.
- DS is parallel to the y-axis.
- C(1; 1) is a point on the circumference of the circle.
- CD and DE are chords of the circle.
- BE, if drawn, is parallel to DS.
- The smaller segment of the circle on chord DE is shaded.



4.1 Show that the coordinates of B is (4 ; -3). Show all your calculations. (3)

4.2 Determine:

4.2.1 the equation of the tangent DS. (2)

4.2.2 the length of chord CD. (3)

4.2.3 the equation of the tangent PA in the form $y = mx + c$. (5)

4.2.4 the area of the shaded segment. (3)

4.3 Another circle with center M and equation $(x + 1)^2 + (y - 2)^2 = 8$ is drawn. Determine, with an explanation, whether the two circles will intersect or not. (3)

[19]

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression completely:

$$\cos(-\theta) \times \sin(90^\circ - \theta) \times (1 + \tan^2 \theta) \quad (5)$$

5.2 Given: $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

5.2.1 Use the formula for $\cos(A + B)$ to derive a formula for $\sin(A - B)$. (4)

5.2.2 **Without using a calculator**, show that:

$$\sin(x + 63^\circ) \cdot \cos(x + 378^\circ) + \cos(x + 63^\circ) \cdot \cos(x + 108^\circ) = \frac{1}{\sqrt{2}} \quad (5)$$

5.3 Given that $\sin 10^\circ = \sqrt{k}$

Without using a calculator, write each of the following in terms of k :

5.3.1 $\sin 190^\circ$ (2)

5.3.2 $\cos 20^\circ$ (3)

5.3.3 $\cos 50^\circ$ (4)

5.4 Given:

$$\frac{1}{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)} - \frac{(\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta)}$$

5.4.1 Simplify the expression above to a single trigonometry ratio. (5)

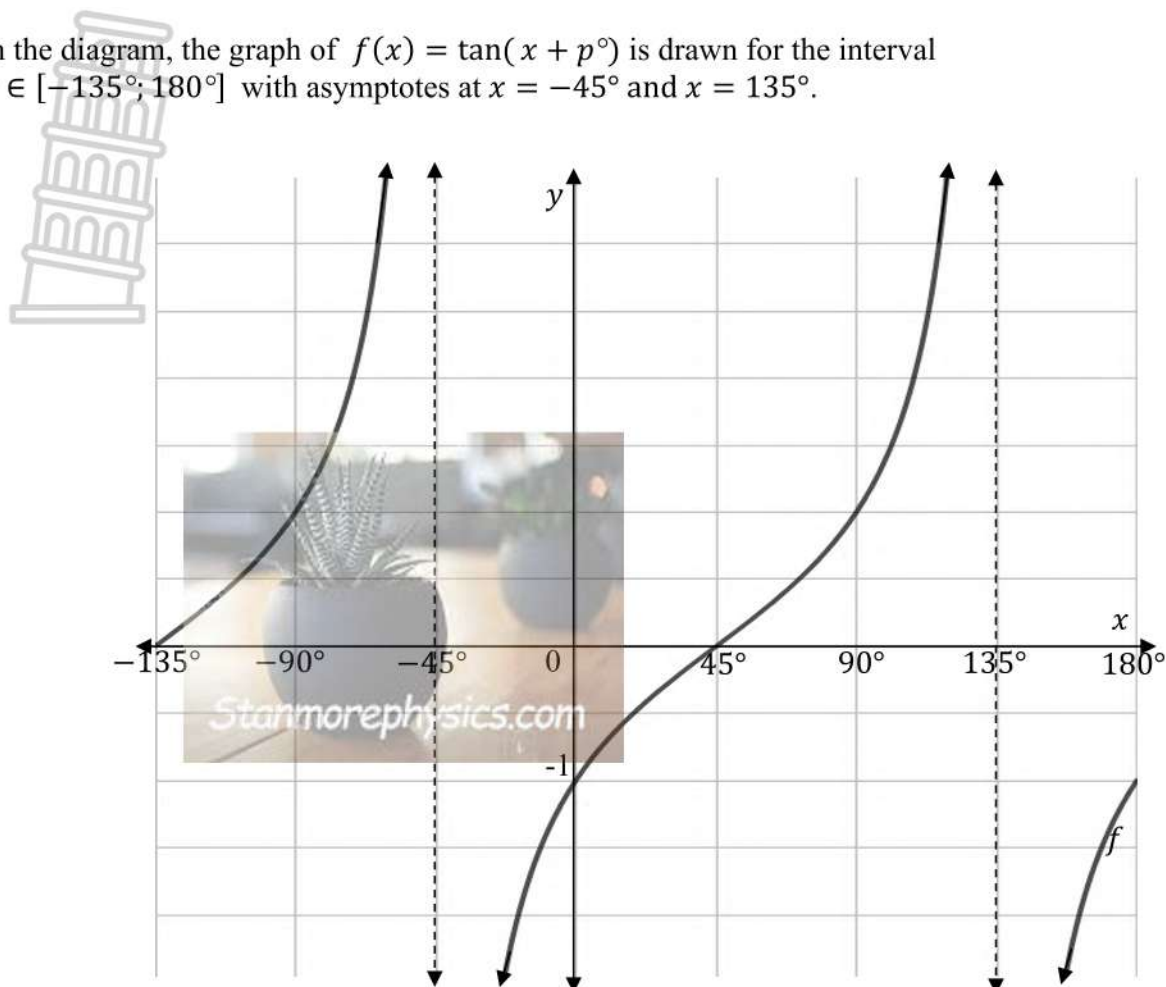
5.4.2 Determine the general solution of θ for which this expression will be undefined? (3)

[31]



QUESTION 6

In the diagram, the graph of $f(x) = \tan(x + p^\circ)$ is drawn for the interval $x \in [-135^\circ; 180^\circ]$ with asymptotes at $x = -45^\circ$ and $x = 135^\circ$.

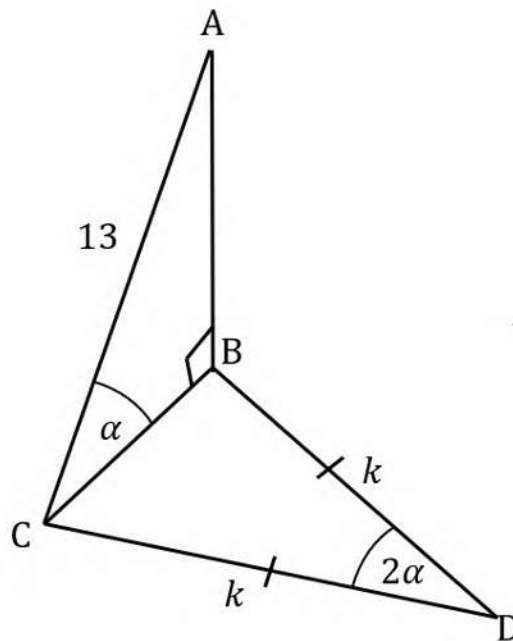


- 6.1 Write down the value of p . (1)
- 6.2 Draw the graph of $g(x) = \sin 2x$ for the interval $x \in [-135^\circ; 180^\circ]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with axes, as well as the minimum and maximum points on the graph. (3)
- 6.3 Write down the period of g . (1)
- 6.4 The graph of g is shifted 45° to the left to form the graph of h . Determine the equation of h in the simplest form. (2)
- 6.5 Use the graph to determine the values of x in the interval $x \in [-135^\circ; 0^\circ]$ for which:
 - 6.5.1 $f(x) \leq -1$ (2)
 - 6.5.2 $\sin x \cos x + 2 < 2$ (3)

[12]

QUESTION 7

In the diagram below, ABC is a vertical triangular wall on the floor CBD. $CA = 13$ meters, $CD = BD = k$ meters, $\widehat{ACB} = \alpha$ and $\widehat{BDC} = 2\alpha$



7.1 Show that:

7.1.1 $CB = 13 \cos \alpha$ (1)

7.1.2 $k = \frac{13}{2 \tan \alpha}$ (4)

7.2 Calculate the area of the floor $\triangle BCD$ if $\alpha = 26^\circ$ (2)
[7]

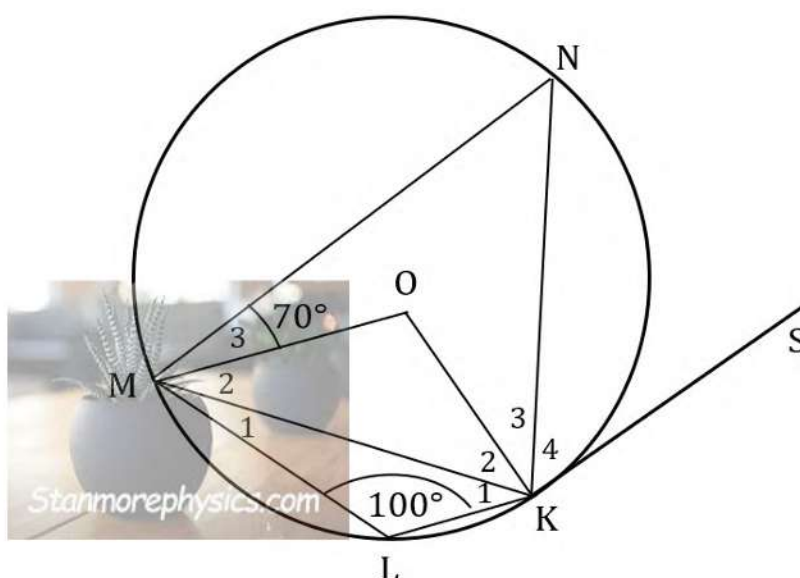
QUESTION 8

8.1 Complete the following theorem:

The angle at the centre of a circle is twice the angle _____ subtended by _____.

(2)

8.2 In the diagram below, O is the centre of the circle.
K, L, M and N are points on the circumference of the circle.
KS is a tangent to the circle at K and $\hat{L} = 100^\circ$



8.2.1 Determine, with reasons:

(a) the size of $\hat{N}$.

(2)

(b) the size of $\hat{K}_2$.

(3)

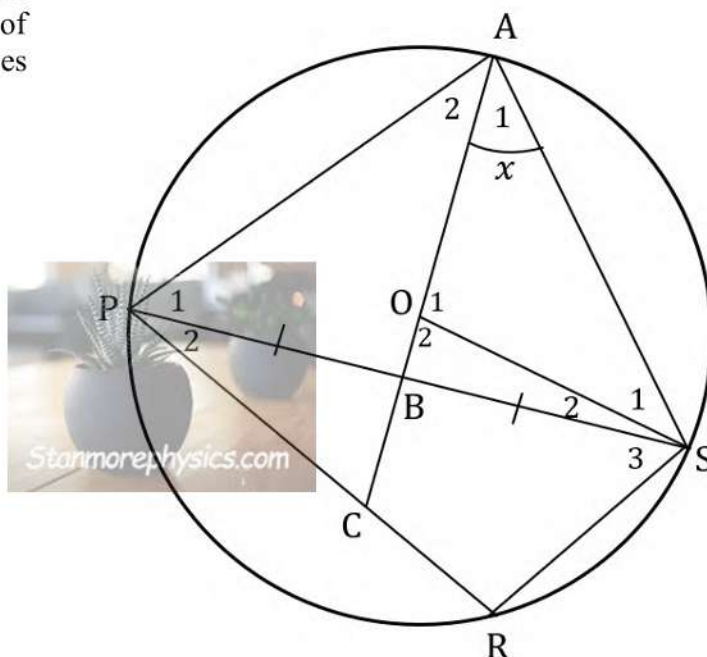
8.2.2 Prove, giving reasons, that: $MN \parallel KS$

(4)

[11]

QUESTION 9

In the diagram below, O is the centre of the circle. P, R, S and A are points on the circumference of the circle. AOB is a straight line such that B lies on PS with $PB = BS$ and $\widehat{A_1} = x$



9.1 Determine, with reasons, $\widehat{P_1}$ in terms of x . (4)

9.2 Prove, giving reasons, that:

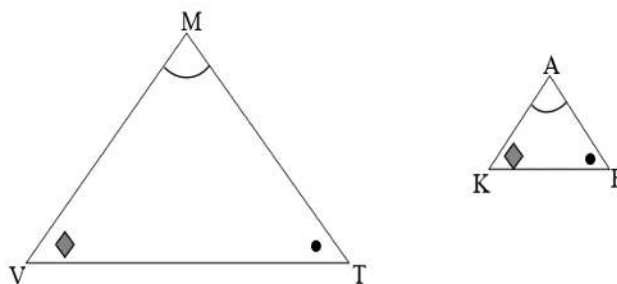
9.2.1 AC bisects $\widehat{PAS}$. (4)

9.2.2 COSR is a cyclic quadrilateral. (4)

[12]

QUESTION 10

10.1 In the diagram, $\triangle MVT$ and $\triangle AKF$ are drawn such that $\widehat{M} = \widehat{A}$, $\widehat{V} = \widehat{K}$ and $\widehat{T} = \widehat{F}$.

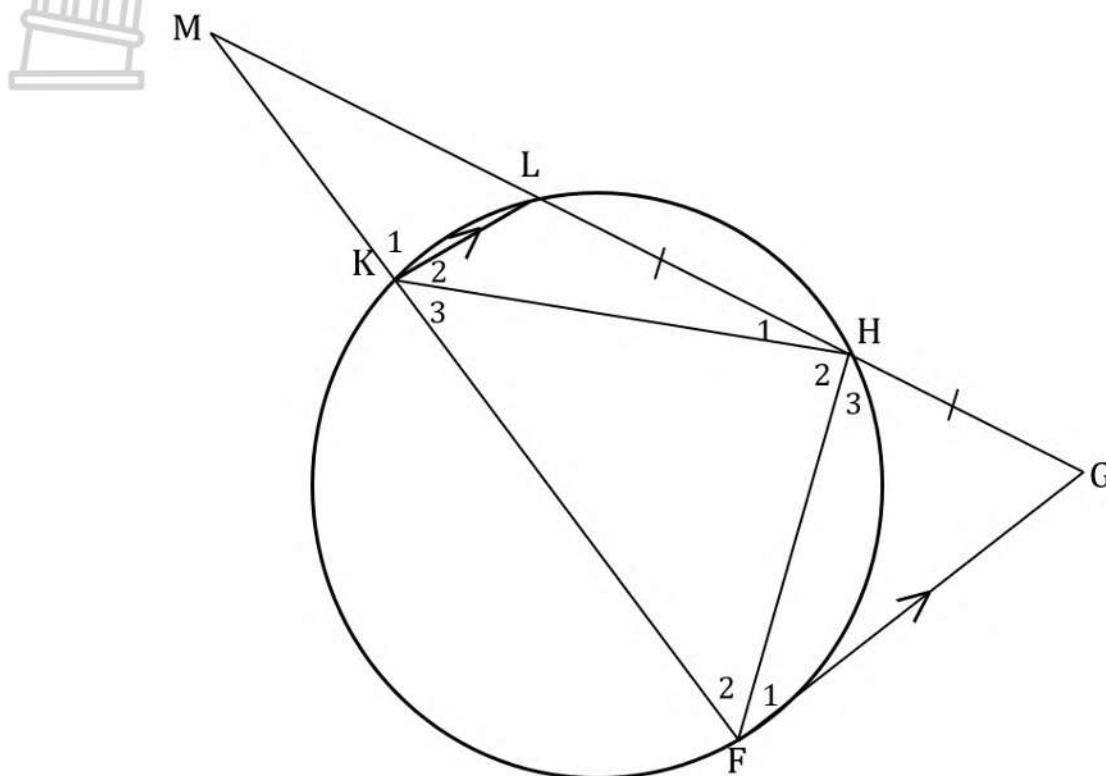


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is prove

$$\text{that } \frac{MV}{AK} = \frac{MT}{AF}.$$

(7)

- 10.2 In the diagram, HLKF is a cyclic quadrilateral. The chords HL and FK are produced to intersect at M. The line FG parallel to KL, meets MH produced at G. $\frac{MK}{KF} = \frac{1}{2}$ and $LH = HG$.



10.2.1 Give a reason why $\widehat{GFM} = \widehat{LKM}$. (1)

10.2.2 Prove, giving reasons, that:

(a) $\triangle MFH \parallel \triangle MGF$ (4)

(b) $\frac{MF}{ML} = \sqrt{6}$ (5)

[17]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

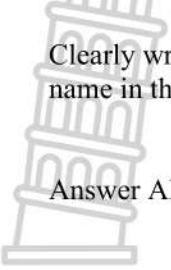
Answer Book
Antwoordboek

MATHEMATICS P2 / WISKUNDE V2
GR 12
SEPTEMBER 2024

NAME NAAM					BOOK NUMBER BOEKNUMMER		OF VAN		BOOKS BOEKE
SURNAME VAN					PAPER NUMBER VRAESTELNUMMER	2			
DATE DATUM									
TEACHER ONDERWYSER									
SUBJECT NAME VAKNAAM	MATHEMATICS/ <i>WISKUNDE</i>								

MARKER/NASIENER					MODERATOR			
Question Vraag	Marks Punte			Marker's Initials Nasiener se Voorletters	Marks Punte			Moderator's Initials Moderator se Voorletters
1								
2								
3								
4								
5								
6								
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This answer book consists of 22 pages./Hierdie antwoordeboek bestaan uit 22 bladsye.

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
 - Clearly write your name and your teacher's name in the space provided. - Answer ALL questions in the spaces provided. - NO pages may be torn from this answer book. - Candidates may NOT retain an answer book or remove it from the examination room. - Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins. - If you require additional space for your answers: - Use the additional space provided at the end of the answer book. - When answering a question in the additional space, clearly indicate the question number in the column on the LHS. - Rule off after each answer. - Draw a neat line through any work/rough work that must not be marked.	- Skryf jou naam en jou onderwyser se naam in die ruimtes verskaf. - Beantwoord ALLE vrae in die ruimtes wat verskaf is. - GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE. - GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE. - Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop. - In geval jy bykomende ruimte benodig vir jou antwoorde: - Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word. - As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan. - Trek 'n lyn na elke antwoord. - Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.



QUESTION/VRAAG 1

	Solution/Oplissing	Marks Punte
1.1		(1)
1.2		
		(1)

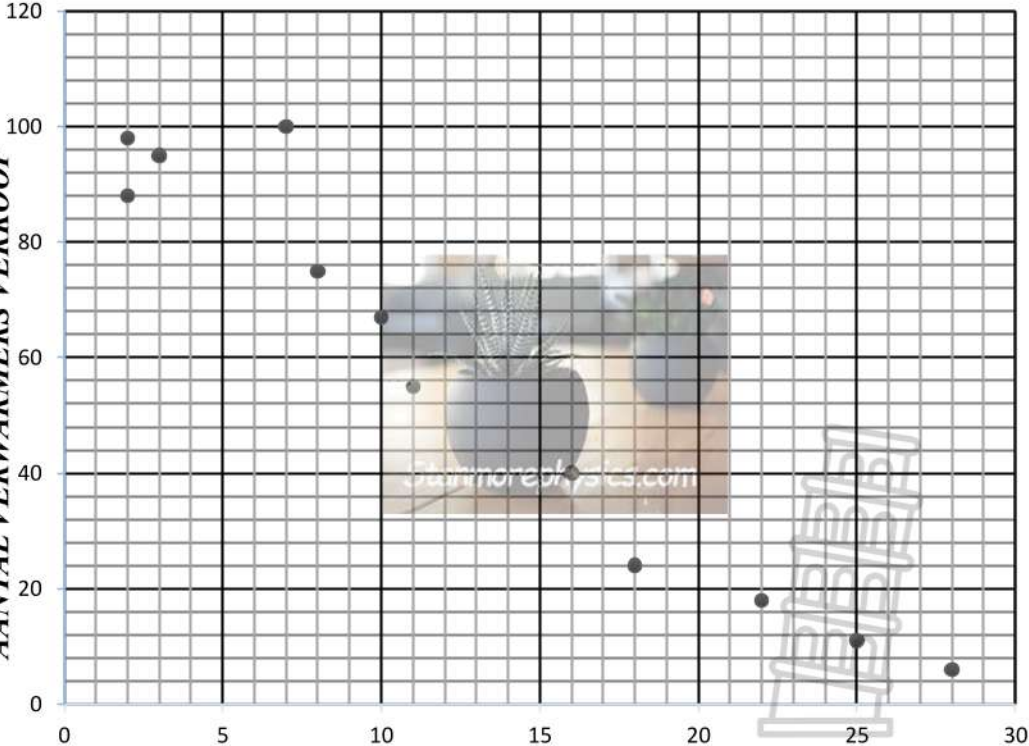
1.3	<table border="1"> <thead> <tr> <th>Age / Ouderdom (x) (in years /jare)</th><th>Number of people Aantal mense</th><th>Cumulative frequency Kumulatiewe frekwensie</th></tr> </thead> <tbody> <tr> <td>$15 < x \leq 25$</td><td>8</td><td></td></tr> <tr> <td>$25 < x \leq 35$</td><td>14</td><td></td></tr> <tr> <td>$35 < x \leq 45$</td><td>22</td><td></td></tr> <tr> <td>$45 < x \leq 55$</td><td>37</td><td></td></tr> <tr> <td>$55 < x \leq 65$</td><td>a</td><td></td></tr> <tr> <td>$65 < x \leq 75$</td><td>3</td><td></td></tr> </tbody> </table>	Age / Ouderdom (x) (in years /jare)	Number of people Aantal mense	Cumulative frequency Kumulatiewe frekwensie	$15 < x \leq 25$	8		$25 < x \leq 35$	14		$35 < x \leq 45$	22		$45 < x \leq 55$	37		$55 < x \leq 65$	a		$65 < x \leq 75$	3		(2)
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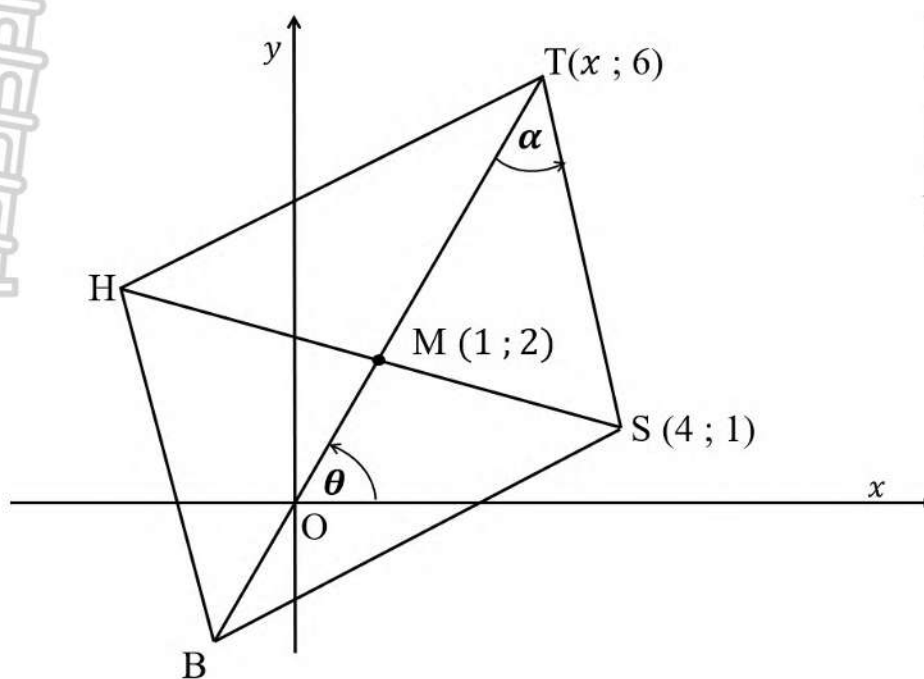
1.5		(2)
		[9]

QUESTION/VRAAG 2

MONTH/ MAAND	J	F	M	A	M	J	J	A	S	O	N	D
TEMPERATURE/ TEMPRATUUR (°C)	2	7	8	10	18	22	28	25	16	11	2	3
NUMBER OF HEATERS SOLD/ AANTAL VERWARMERS VERKOOP	98	100	75	67	24	18	6	11	40	55	88	95

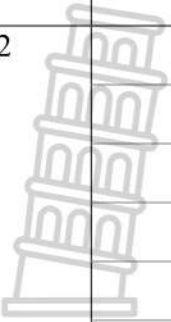
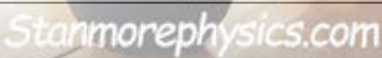
Solution/Oplissing		Marks Punte																										
2.1		(2)																										
2.2		(3)																										
2.3		(2)																										
2.4	NUMBER OF HEATERS SOLD/ AANTAL VERWARMERS VERKOOP <table border="1"><thead><tr><th>TEMPERATURE/ TEMPRATUUR (°C)</th><th>NUMBER OF HEATERS SOLD/ AANTAL VERWARMERS VERKOOP</th></tr></thead><tbody><tr><td>2</td><td>98</td></tr><tr><td>2</td><td>88</td></tr><tr><td>3</td><td>95</td></tr><tr><td>7</td><td>100</td></tr><tr><td>8</td><td>75</td></tr><tr><td>10</td><td>67</td></tr><tr><td>11</td><td>55</td></tr><tr><td>16</td><td>40</td></tr><tr><td>18</td><td>24</td></tr><tr><td>22</td><td>18</td></tr><tr><td>25</td><td>10</td></tr><tr><td>28</td><td>5</td></tr></tbody></table> TEMPERATURE/ TEMPRATUUR (°C)	TEMPERATURE/ TEMPRATUUR (°C)	NUMBER OF HEATERS SOLD/ AANTAL VERWARMERS VERKOOP	2	98	2	88	3	95	7	100	8	75	10	67	11	55	16	40	18	24	22	18	25	10	28	5	(2)
TEMPERATURE/ TEMPRATUUR (°C)	NUMBER OF HEATERS SOLD/ AANTAL VERWARMERS VERKOOP																											
2	98																											
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18	24																											
22	18																											
25	10																											
28	5																											
2.5		(2)																										
		[11]																										

QUESTION/VRAAG 3

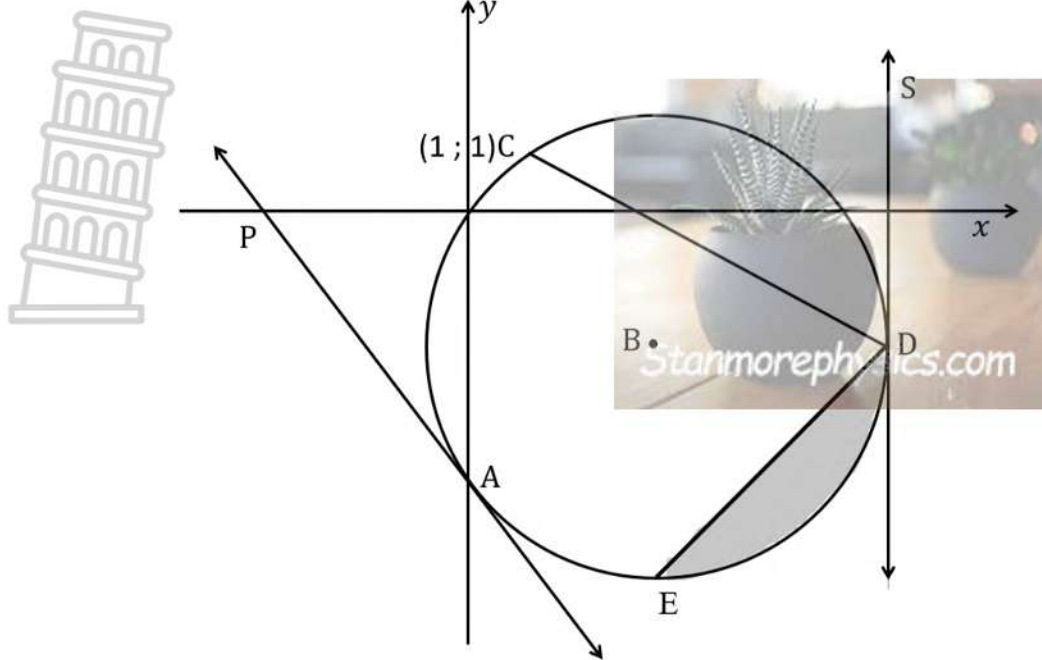


	Solution/Oplissing	Marks Punte
3.1.1		(2)
3.1.2		(2)
3.1.3		(3)

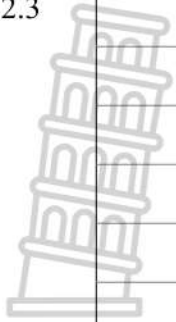


	Solution/Oplissing	Marks Punte
3.2	 	(3)
3.3		(4)
3.4		(4)
3.5		(3)
		[21]

QUESTION/VRAAG 4

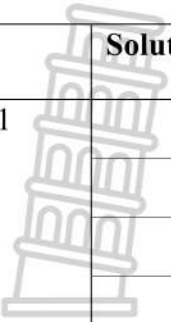


	Solution/Oplossing	Marks Punte
4.1		(3)
4.2.1		(2)
4.2.2		(3)

4.2.3 	Handwritten answer area for question 4.2.3, consisting of 15 horizontal lines.	(5)
4.2.4	 Handwritten answer area for question 4.2.4, consisting of 15 horizontal lines.	(3)



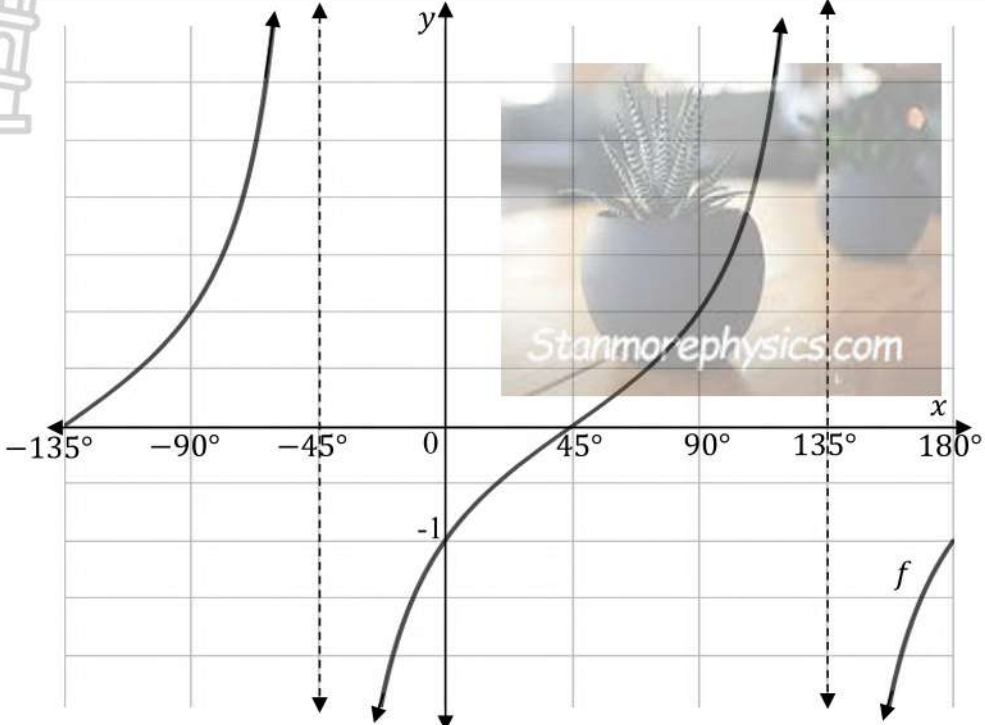
QUESTION/VRAAG 5

	Solution/Oplissing	Marks Punte
5.1		(5)
5.2.1	 Stanmorephysics.com	(4)
5.2.2		(5)

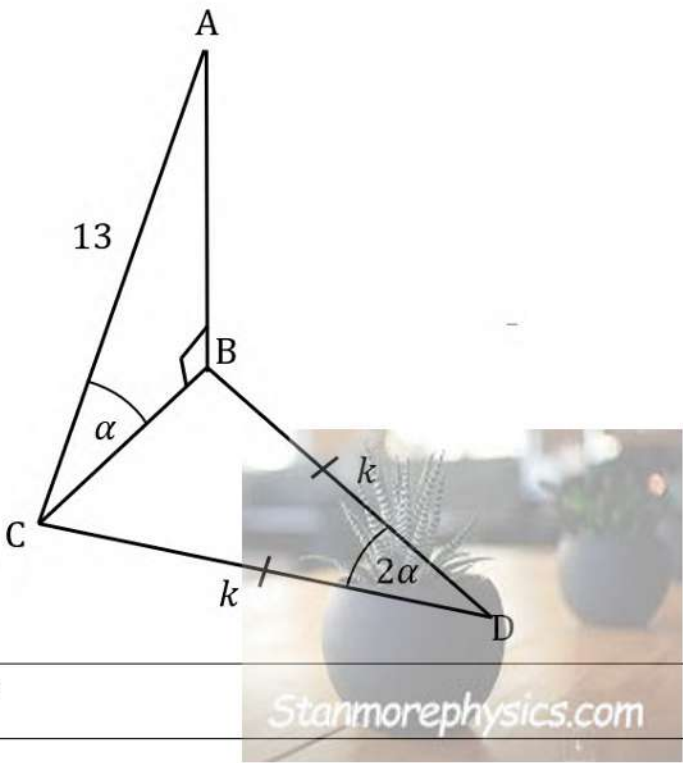
	Solution/Oplossing	Marks Punte
5.3.1		(2)
5.3.2		(3)
5.3.3		(4)

	Solution/Oplissing	Marks Punte
5.4.1		
5.4.2		(5)
		(3)
		[31]

QUESTION/VRAAG 6

	Solution/Oplissing	Marks Punte
6.1		
		(1)
6.2		(3)
6.3		
		(1)
6.4		(2)
6.5.1		
		(2)
6.5.2		(3)
		[12]

QUESTION/VRAAG 7



	Solution/Oplossing	Marks Punte
7.1.1		(1)
7.1.2		(4)

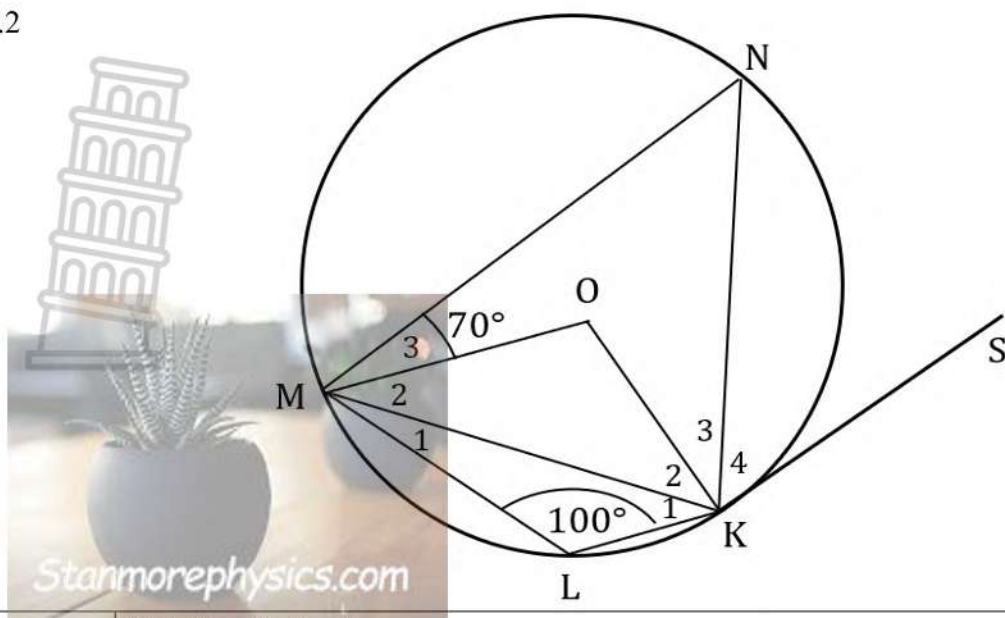




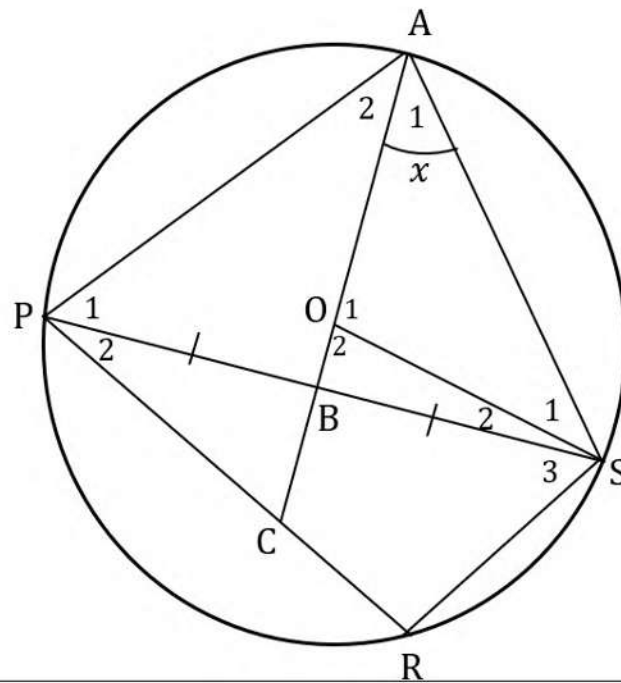
- **Gee redes vir jou bewerings in VRAAG 8, 9 en 10.**

QUESTION/VRAAG 8

8.2

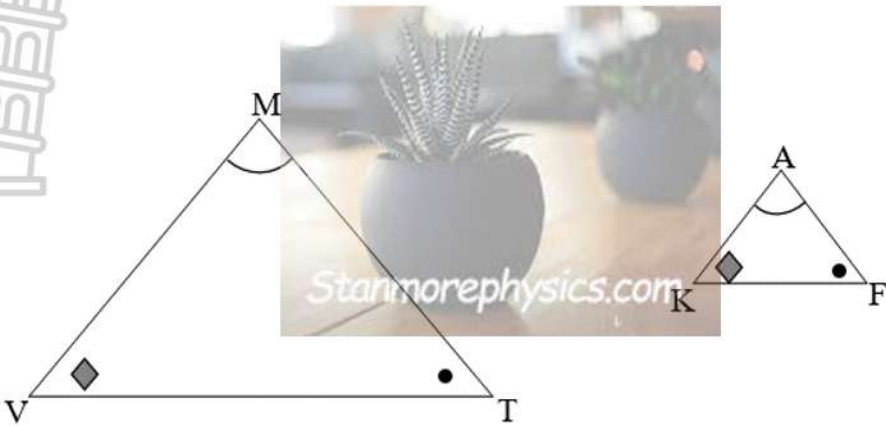


	Solution/Oplissing	Marks Punte
8.2.1(a)		(2)
8.2.1(b)		(3)
8.2.2		(4)
		[11]



QUESTION/VRAAG 10

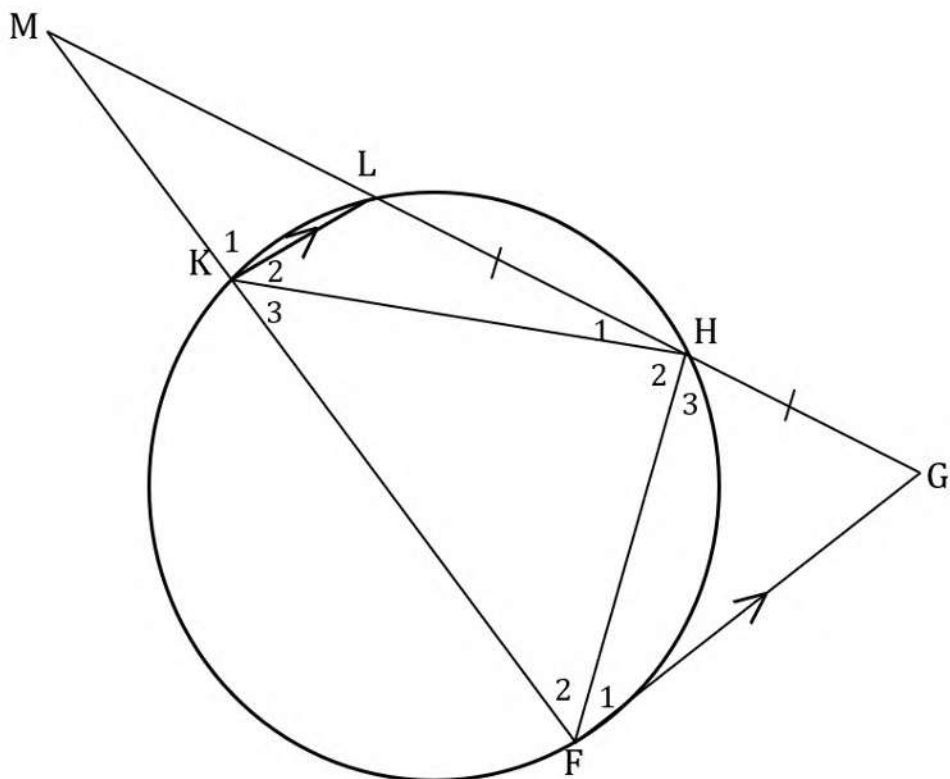
10.1



	Solution/Oplossing	Marks Punte
10.1		
		(7)



10.2



	Solution/Oplissing	Marks Punte
10.2.1		(1)
10.2.2(a)		(4)



	Solution/Oplissing	Marks Punte
10.2.2(b)		
		(5)
		[17]

TOTAL/TOTAAL: 150

Please turn over/Blaai om asseblief

MATHEMATICS GR12

FINAL MARKING GUIDELINE

MATHEMATICS P2

SEPTEMBER 2024

Stanmorephysics.com

MARKS: 150

TIME: 3 hours

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if statement AND reason are both correct
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION 1 / VRAAG 1

1.1	$a = 16$	✓ 16	(1)
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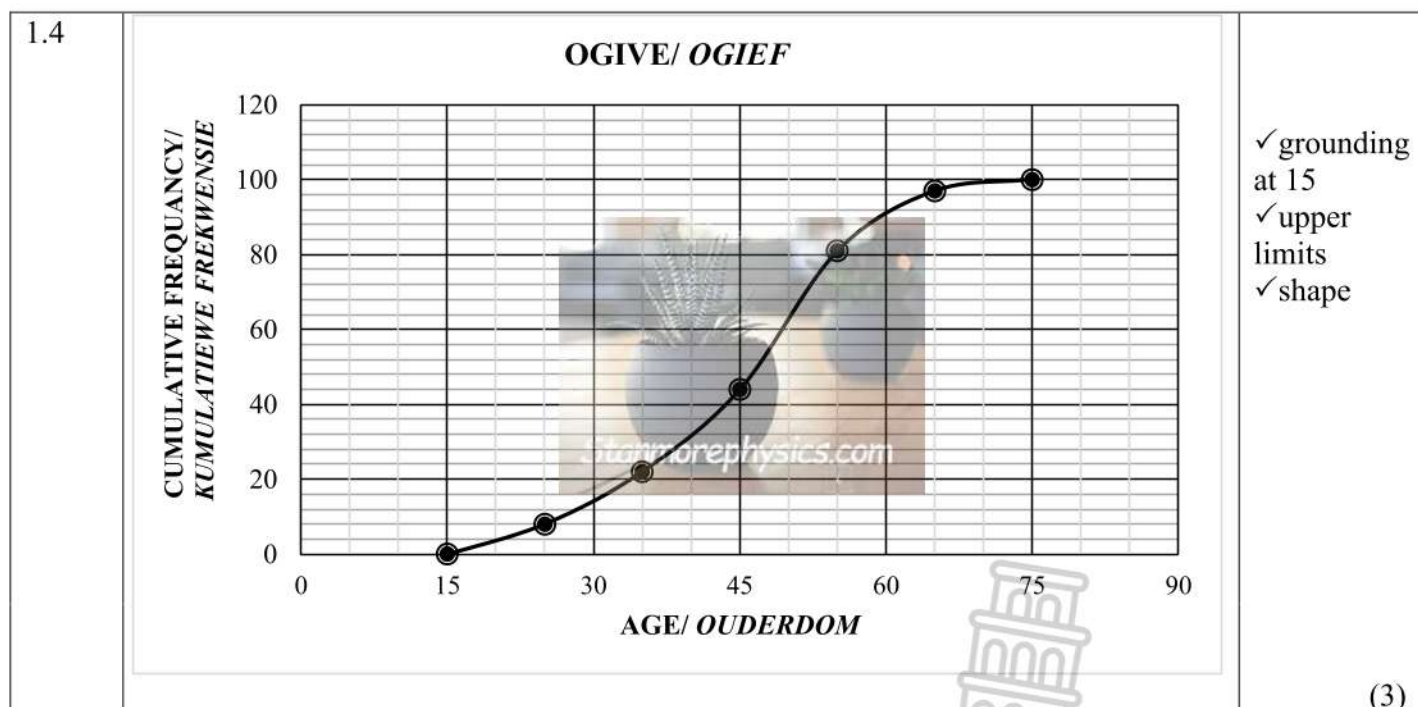
1.2	$45 < x \leq 55$ or accept 45 - 55	✓ $45 < x \leq 55$	(1)
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1.3

AGE (x) (IN YEARS)	NUMBER OF PEOPLE	Cumulative frequency <i>Kumulatiewe frekwensie</i>
$15 < x \leq 25$	8	8
$25 < x \leq 35$	14	22
$35 < x \leq 45$	22	44
$45 < x \leq 55$	37	81
$55 < x \leq 65$	$a=16$	97
$65 < x \leq 75$	3	100

✓ 8 and 22
✓ others

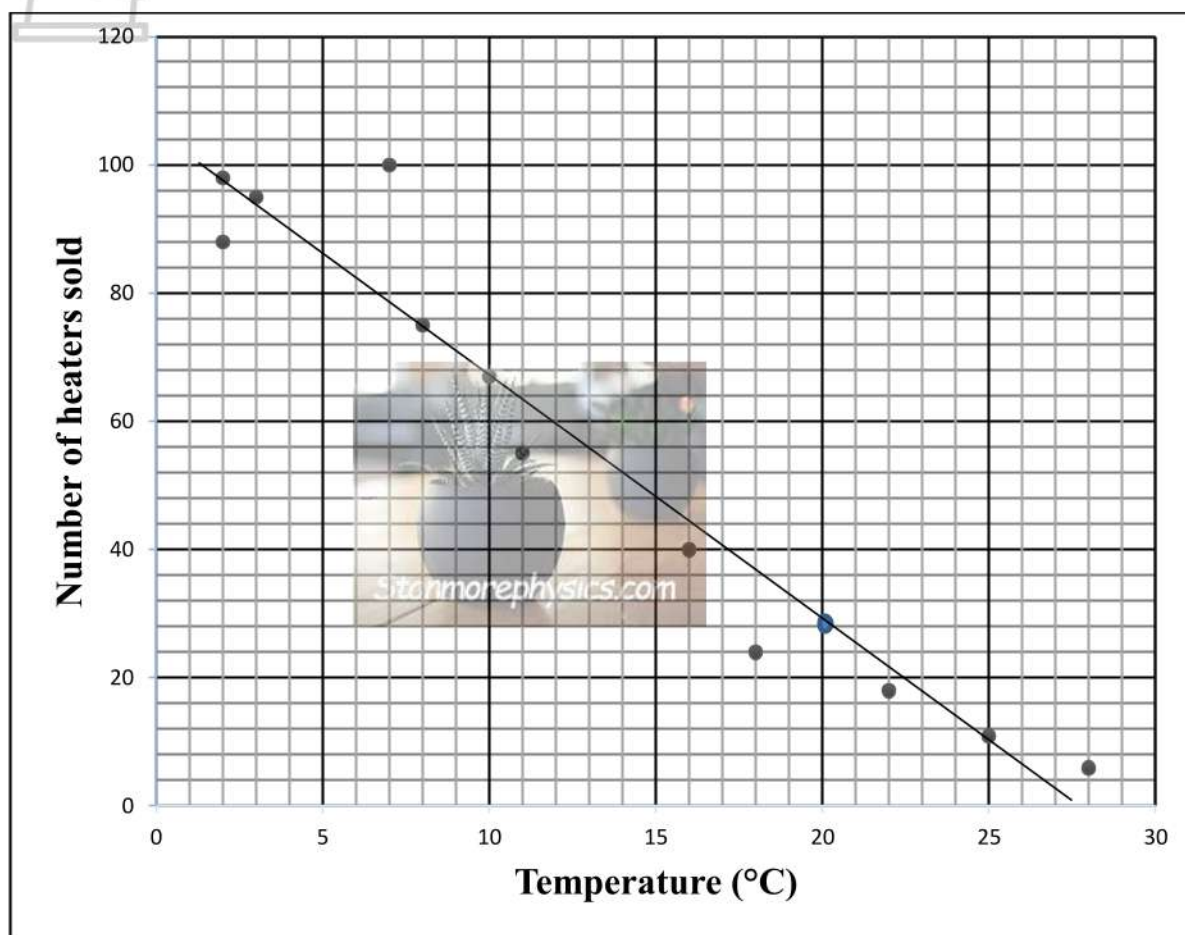
(2)



1.5	$\bar{x} = \frac{20(8) + 30(14) + 40(25) + 50(34) + 60(16) + 70(3)}{100}$ $= \frac{4450}{100}$ $= 44,5$	✓ or 4450 ✓ answer AO full marks	(2)
[9]			

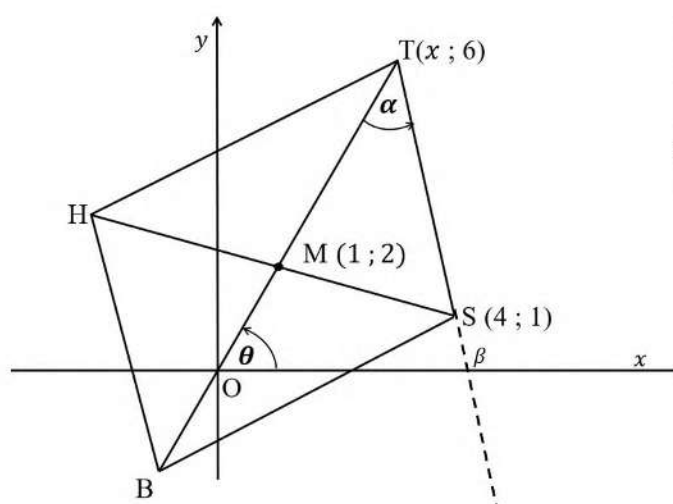
QUESTION 2 / VRAAG 2

MONTH	J	F	M	A	M	J	J	A	S	O	N	D
TEMPERATURE (°C)	2	7	8	10	18	22	28	25	16	11	2	3
NUMBER OF HEATERS SOLD	98	100	75	67	24	18	6	11	40	55	88	95

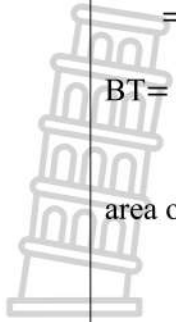
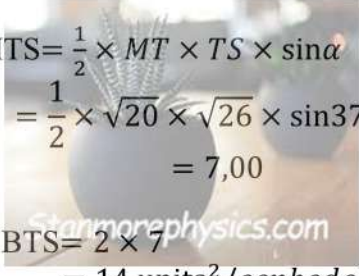


2.1	strong , negative $r = -0,97$	✓ strong, negative ✓ $r = -0,97$ (2)
2.2	$a = 104,80$ $b = -3,82$ $y = 104,80 - 3,82x$	✓ a ✓ b ✓ equation (3)
2.3	$y = 104,80 - 3,82(20)$ $= 28,4 \approx 28$	✓ subst ✓ answer (2)
2.4	On sketch	✓ line within domain ✓ through 2 points (2)
2.5	$\sigma y = 34,13$	✓✓ answer (2)
[11]		

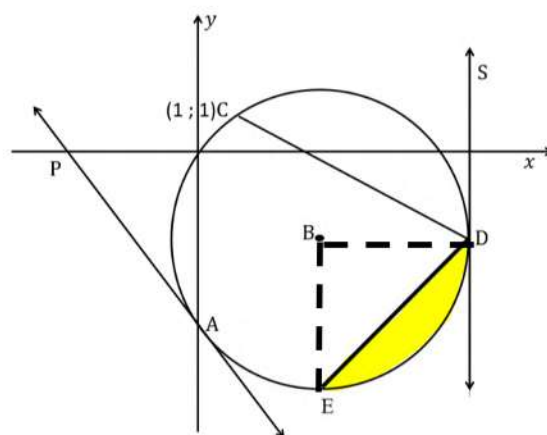
QUESTION 3 / VRAAG 3



3.1.1	$m_{TB} = \frac{2-0}{1-0} = \frac{2}{1}$ $y = 2x + 0$	✓ $m = 2$ ✓ $c = 0$ AO-full marks (2)
3.1.2	$\tan \theta = 2$ $\therefore \theta = 63,43^\circ$	✓ $\tan \theta = 2$ ✓ $\theta = 63,43^\circ$ (2)
3.1.3	$\frac{x+4}{2} = 1 \quad \frac{y+1}{2} = 2$ $x = -2 \quad y = 3$ $H(-2; 3)$	✓ Method ✓ $x = -2$ ✓ $y = 3$ AO Full marks (3)
3.2	$TS = \sqrt{26}$ $\sqrt{(x-4)^2 + (6-1)^2} = \sqrt{26}$ $x^2 - 8x + 16 + 25 = 26$ $x^2 - 8x + 15 = 0$ $(x-5)(x-3) = 0$ $x \neq 5; x = 3$	✓ sub ✓ standard form ✓ factors / formula Answer given (3)
3.3	$m_{TS} = \frac{6-1}{3-4} = -5$ $\tan \beta = -5$ $\beta = 180^\circ - 78,69 \dots = 101,31^\circ$ $\widehat{MTS} = \alpha = 101,31^\circ - 63,43^\circ$ $\alpha = 37,88^\circ$	✓ sub ✓ $\tan \beta = -5$ ✓ $101,31^\circ$ ✓ answer α (4)

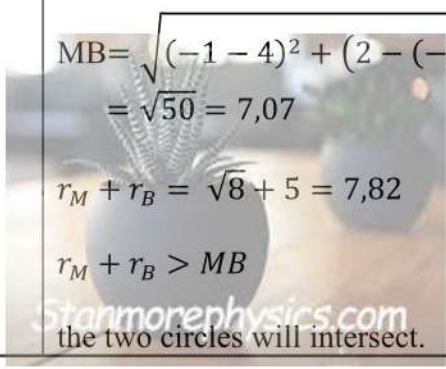
3.4	 $MT = \sqrt{(3-1)^2 + (6-2)^2}$ $= \sqrt{20}$ BT = 2MT diagonals of parallelogram bisect each other / hoeklyne van parallelogram halveer mekaar $\text{area of } \Delta BTS = \frac{1}{2} \times BT \times TS \times \sin \alpha$ $= \frac{1}{2} \times 2\sqrt{20} \times \sqrt{26} \times \sin 37,88^\circ$ $= 14 \text{ units}^2 / \text{eenhede}^2$ OR $MT = \sqrt{(3-1)^2 + (6-2)^2}$ $= \sqrt{20}$  $\text{area of } \Delta MTS = \frac{1}{2} \times MT \times TS \times \sin \alpha$ $= \frac{1}{2} \times \sqrt{20} \times \sqrt{26} \times \sin 37,88^\circ$ $= 7,00$ $\therefore$ area of $\Delta BTS = 2 \times 7$ $= 14 \text{ units}^2 / \text{eenhede}^2$	✓ subst. ✓ Method ✓ subst. ✓ answer OR ✓ subst. ✓ subst. ✓ answer ✓ Method /answer (4)
3.5	Area of parallelogram = $2 \times 14 = 28 \text{ units}^2$ Area of parallelogram = $TS \times h$ $28 = \sqrt{26} \times h$ $h = 5,49 \text{ units/eenhede}$	✓ area of parm ✓ subst. ✓ answer (3)
[21]		

QUESTION 4 / VRAAG 4



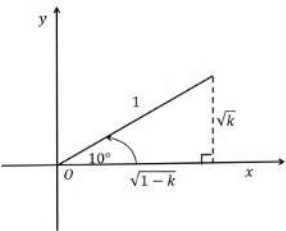
4.1	$x^2 - 8x + y^2 + 6y = 0$ $x^2 - 8x + (4)^2 + y^2 + 6y + (3)^2 = 0 + 16 + 9$ $(x - 4)^2 + (y + 3)^2 = 25$ $B(4; -3)$	$\checkmark +16 + 9$ $\checkmark (x - 4)^2 \text{ and } (y + 3)^2$ $\checkmark \text{ answer}$	(3)
4.2.1	$D(9; -3)$ $DS: x = 9$	$\checkmark \checkmark \text{ answer}$	(2)
4.2.2	$D(9; -3)$ $C(1; 1)$ $CD = \sqrt{(9 - 1)^2 + (-3 - 1)^2}$ $CD = \sqrt{80} = 4\sqrt{5} = 8,94$	$\checkmark \text{ coordinates of D}$ $\checkmark \text{ Subst}$ $\checkmark \text{ answer}$	(3)
4.2.3	$x^2 - 8x + y^2 + 6y = 0$ $(0)^2 - 8(0) + y^2 + 6y = 0$ $y^2 + 6y = 0$ $y(y + 6) = 0$ $\therefore y = -6$ $A(0; -6)$ $B(4; -3)$ $m_{BA} = \frac{-3 - (-6)}{4 - 0}$ $m_{BA} = \frac{3}{4}$ $m_{PA} = -\frac{4}{3} \quad (\text{radius} \perp \text{tangent/}$ $\text{radius} \perp \text{raaklyn})$ $y = -\frac{4}{3}x - 6$	$\checkmark x = 0$ $\checkmark y = -6$ $\checkmark m_{BA} = \frac{3}{4}$ $\checkmark m_{PA} = -\frac{4}{3}$ $\checkmark c = -6$	(5)

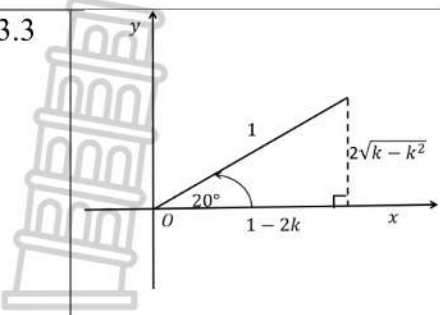
MARKING GUIDELINE

4.2.4 	Area of a quarter circle = $\frac{1}{4} \pi(5)^2$ $= 19,63$ Area of $\triangle BDE = \frac{1}{2} \times 5 \times 5$ $= 12,5$ Area of shaded area = $19,63 - 12,5 = 7,13$	✓ answer ✓ answer ✓ answer (3)
4.3	M(-1 ; 2) radius = $\sqrt{8}$ B(4 ; -3) radius = 5 $MB = \sqrt{(-1 - 4)^2 + (2 - (-3))^2}$ $= \sqrt{50} = 7,07$ $r_M + r_B = \sqrt{8} + 5 = 7,82$ $r_M + r_B > MB$ the two circles will intersect. 	✓ length of MB ✓ $r_M + r_B$ ✓ conclusion and reason (3)
[19]		

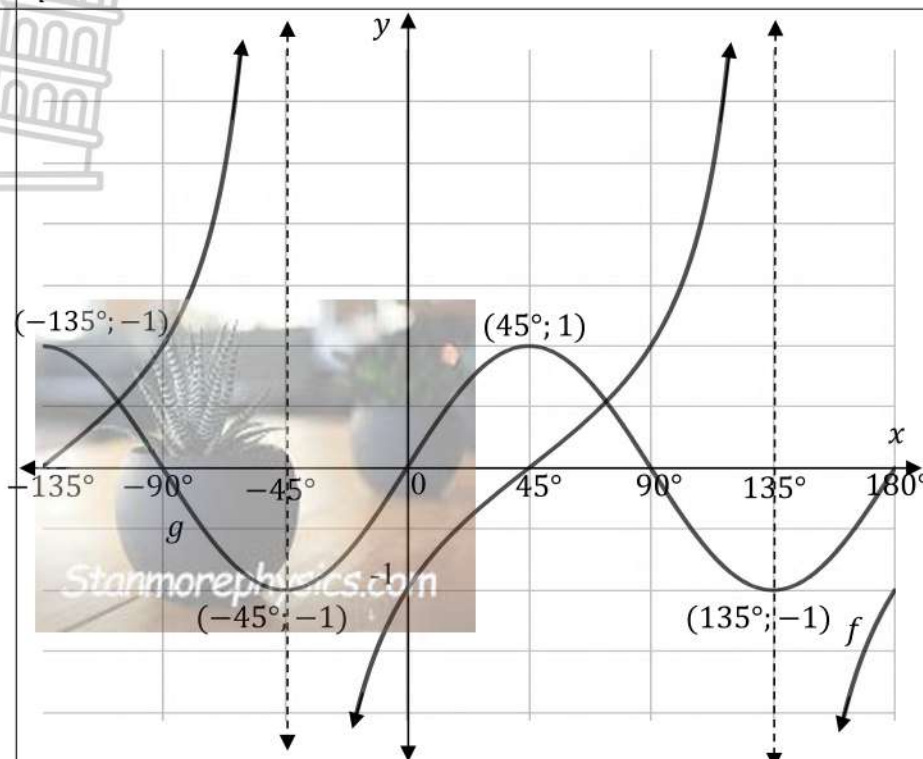


QUESTION 5 / VRAAG 5

5.1	$\cos(-\theta) \times \sin(90^\circ - \theta) \times (1 + \tan^2 \theta)$ $= \cos \theta \times \cos \theta \times \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right)$ $= \cos^2 \theta \times \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right)$ $= \cos^2 \theta + \sin^2 \theta$ $= 1$	✓ neg angle ✓ co-function ✓ $\frac{\sin^2 \theta}{\cos^2 \theta}$ ✓ S ✓ answer (5)
5.2.1	$\sin(A - B) = \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ - A) + B]$ $= \cos(90^\circ - A) \cdot \cos B - \sin(90^\circ - A) \cdot \sin B$ $= \sin A \cdot \cos B - \cos A \cdot \sin B$	✓ co-function ✓ re-group ✓ compound angle ✓ co-functions (4)
5.2.2	$LHS = \sin(x + 63^\circ) \cdot \cos(x + 37^\circ) + \cos(x + 63^\circ) \cdot \cos(x + 108^\circ)$ $= \sin(x + 63^\circ) \cdot \cos(x + 18^\circ) + \cos(x + 63^\circ) \cdot \cos(x + 90^\circ + 18^\circ)$ $= \sin(x + 63^\circ) \cdot \cos(x + 18^\circ) - \cos(x + 63^\circ) \cdot \sin(x + 18^\circ)$ $= \sin[(x + 63^\circ) - (x + 18^\circ)]$ $= \sin 45^\circ$ $= \frac{1}{\sqrt{2}}$ $\therefore LHS = RHS$	✓ $\cos(x + 18^\circ)$ ✓ $\cos(x + 90^\circ + 18^\circ)$ ✓ $-\sin(x + 18^\circ)$ ✓ compound formula ✓ $\sin 45^\circ$ (5)
5.3.1	$\sin 190^\circ$ $= \sin(180^\circ + 10^\circ)$ $= -\sin 10^\circ$ $= -\sqrt{k}$	✓ $-\sin 10^\circ$ ✓ answer (2)
5.3.2	$\cos 20^\circ$ $= \cos 2(10^\circ)$ $= 1 - 2 \sin^2(10^\circ)$ $= 1 - 2k$	✓ $\cos 2(10^\circ)$ ✓ double angle ✓ $1 - 2k$ (3)
5.3.3	 $\cos 50^\circ$ $= \cos(60^\circ - 10^\circ)$ $= \cos 60^\circ \cdot \cos 10^\circ + \sin 60^\circ \cdot \sin 10^\circ$ $= \frac{1}{2} \times \frac{\sqrt{1-k}}{1} + \frac{\sqrt{3}}{2} \times \sqrt{k}$ $= \frac{\sqrt{1-k} + \sqrt{3k}}{2}$ OR	✓ compound angle ✓ sketch ✓ subst. special angles ✓ subst - cos 10° & sin 10° (4) OR

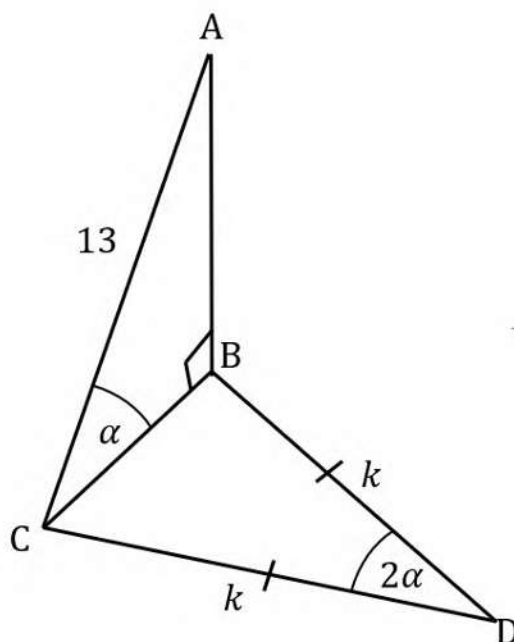
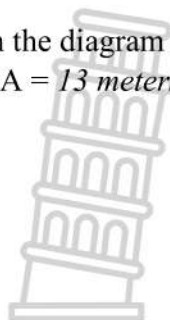
5.3.3 	$\begin{aligned}\cos 50^\circ &= \cos(30^\circ + 20^\circ) \\ &= \cos 30^\circ \cdot \cos 20^\circ - \sin 30^\circ \cdot \sin 20^\circ \\ &= \frac{\sqrt{3}}{2} \times (1 - 2k) - \frac{1}{2} \times (\sqrt{k - k^2}) \\ &= \frac{\sqrt{3}(1 - 2k) - \sqrt{k - k^2}}{2}\end{aligned}$	✓ compound angle ✓ sketch ✓ subst. special angles ✓ subst cos 10° & sin 10° (4)
5.4.1	$\begin{aligned}&\frac{1}{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)} - \frac{(\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta)} \\ &= \frac{1 - (\cos \theta + \sin \theta)^2}{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)} \\ &= \frac{1 - (\cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta)}{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)} \\ &= \frac{1 - (1 + \sin 2\theta)}{(\cos^2 \theta - \sin^2 \theta)} \\ &= \frac{-\sin 2\theta}{\cos 2\theta} \\ &= -\tan 2\theta\end{aligned}$	✓ Simplify ✓ $\sin^2 \theta + \cos^2 \theta = 1$ ✓ $\sin 2\theta$ ✓ $\cos 2\theta$ ✓ answer (5)
5.4.2	$\begin{aligned}\cos \theta &= \sin \theta & \text{or} & & \cos \theta &= -\sin \theta \\ \tan \theta &= 1 & & & \tan \theta &= -1 \\ \theta &= 45^\circ + 180^\circ k & & & \theta &= 135^\circ + 180^\circ k; k \in \mathbb{Z}\end{aligned}$ OR $\begin{aligned}\cos 2\theta &= 0 \\ 2\theta &= 90^\circ + 180^\circ k \\ \theta &= 45^\circ + 90^\circ k; k \in \mathbb{Z}\end{aligned}$ OR $\begin{aligned}\cos 2\theta &= 0 \\ 2\theta &= 90^\circ + 360^\circ k & \text{or} & & 2\theta &= 270^\circ + 360^\circ k \\ \theta &= 45^\circ + 180^\circ k & & & \theta &= 135^\circ + 180^\circ k; k \in \mathbb{Z}\end{aligned}$	✓ $\tan \theta = \pm 1$ ✓ 45° & 135° ✓ $+180^\circ k; k \in \mathbb{Z}$ OR ✓ $\cos 2\theta = 0$ ✓ $2\theta = 90^\circ + 180^\circ$ ✓ $\theta = 45^\circ + 90^\circ k; k \in \mathbb{Z}$ OR ✓ $\cos 2\theta = 0$ ✓ 45° & 135° ✓ $+180^\circ k; k \in \mathbb{Z}$ (3)

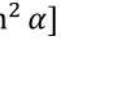
QUESTION 6 / VRAAG 6

6.1	$p = -45^\circ$	✓ 45°	(1)
6.2		✓ x-intercepts ✓ TPs ✓ shape	(3)
6.3	180°	✓ 180°	(1)
6.4	$g(x) = \sin 2x$ $h(x) = \sin 2(x + 45^\circ)$ $h(x) = \sin(2x + 90^\circ)$ $h(x) = \cos 2x$	✓ $\sin 2(x + 45^\circ)$ ✓ $h(x) = \cos 2x$	(2)
6.5.1	$-45^\circ < x \leq 0^\circ$	✓ CV ✓ inequality	(2)
6.5.2	$\sin x \cos x + 2 < 2$ $\sin x \cos x < 0$ $2\sin x \cos x < 0$ $\sin 2x < 0$ $-90^\circ < x < 0^\circ$	✓ $\sin 2x < 0$ ✓ CV ✓ inequality	(3)
			[12]

QUESTION 7 / VRAAG 7

In the diagram below, ABC is a fixed triangular wall on floor CBD. $CA = 13 \text{ meters}$, $CD = BD = k \text{ meters}$, $\widehat{ACB} = \alpha$ and $\widehat{BDC} = 2\alpha$



7.1.1 $\frac{CB}{13} = \cos\alpha \quad \text{or} \quad \frac{CB}{\sin(90^\circ - \alpha)} = \frac{13}{\sin 90^\circ}$ $CB = 13 \cos \alpha$	✓ Method (1)
7.1.2 $CB^2 = k^2 + k^2 - 2(k)(k) \cos 2\alpha$ $(13 \cos \alpha)^2 = 2k^2 - 2k^2 \cos 2\alpha$ $169 \cos^2 \alpha = 2k^2(1 - \cos 2\alpha)$ $169 \cos^2 \alpha = 2k^2[1 - (1 - 2 \sin^2 \alpha)]$ $169 \cos^2 \alpha = 2k^2[2 \sin^2 \alpha]$ $\frac{169 \cos^2 \alpha}{4 \sin^2 \alpha} = k^2$ $k = \frac{13}{2 \tan \alpha}$ OR $\frac{CB}{\sin 2\alpha} = \frac{k}{\sin(90^\circ - \alpha)}$ $\frac{13 \cos \alpha}{\sin 2\alpha} = \frac{k}{\cos \alpha}$ $\frac{13 \cos \alpha \cos \alpha}{2 \sin \alpha \cos \alpha} = k$ $k = \frac{13}{2 \tan \alpha}$	✓ subst. ✓ factorisation ✓ double angle ✓ S  (4) ✓ subst. ✓ co – function ✓ double angle ✓ tan α (4)

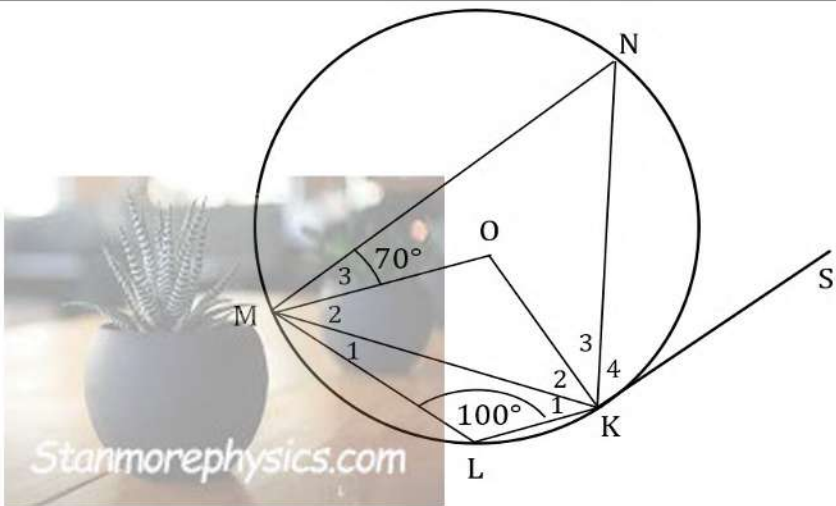
MARKING GUIDELINE

7.2	$\text{Area } \triangle BCD = \frac{1}{2} CD BD \sin 2\alpha$ $= \frac{1}{2} \times k \times k \times \sin 2(26^\circ)$ $= \frac{1}{2} \times \frac{13}{2 \tan 26^\circ} \times \frac{13}{2 \tan 26^\circ} \times \sin 52^\circ$ $= 69,98 \text{ unit}^2 / \text{eenhede}^2$	✓ subst. ✓ 69,98 (2)
[7]		

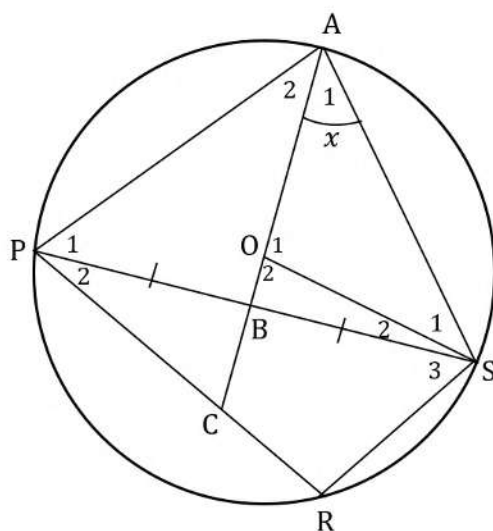


QUESTION 8 / VRAAG 8

8.1	At the circumference / by die omtrek The same chord (bow) / dieselfde koord (boog)	✓ ✓	(2)
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8.2			
8.2.1(a)	$\hat{N} = 80^\circ$ [opp $\angle$ s of cyclic quad/teenoorst $\angle$ e v koordevh]	✓ S ✓ R	(2)
8.2.1(b)	$\hat{O} = 160^\circ$ [$\angle$ at centre = $2 \times \angle$ at circumf/middelpts $\angle = 2 \text{ omtreks } \angle$] $\hat{M}_2 = \hat{K}_2 = 10^\circ$ [sum of $\angle$ s of Δ] / som v $\angle$ e v Δ [$\angle$'s opp = sides/ $\angle$ e teenoor = sye]	✓ S ✓ R ✓ S	(3)
8.2.2	$\hat{K}_4 = 80^\circ$ [tan-chord theorem/ raakl koordst] $\hat{N} = 80^\circ$ [proven/reeds bewys] $\therefore \hat{K}_4 = \hat{N}$ $\therefore MN \parallel KS$ [alternate angles = / verwis hoeke gelyk]	✓ S ✓ R ✓ S ✓ R	(4)
			[11]

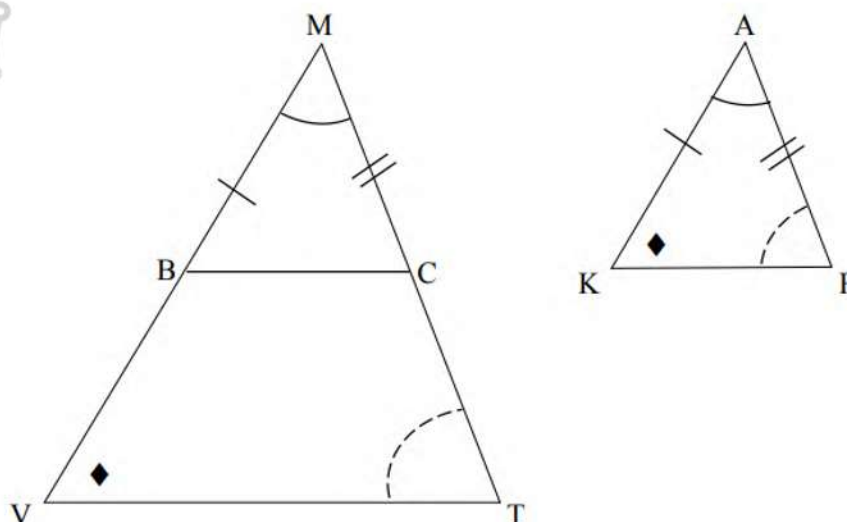
QUESTION 9/ VRAAG 9



9.1	$\hat{S}_1 = x$ [∠'s opp = radii/∠e teenoor = radiusse] $\hat{O}_1 = 180^\circ - 2x$ [sum of ∠s of Δ] / som v ∠e v Δ] $\hat{P}_1 = 90^\circ - x$ [∠ at centre = $2 \times$ ∠ at circumf/middelpts ∠ = 2omtreks ∠]	✓ S / R ✓ S ✓ S ✓ R (4)
9.2.1	$\hat{O}BP = 90^\circ$ [line from centre to midpt chord/lyn v midpt na midpt kd] $\hat{A}_2 = x$ [sum of ∠s of Δ] / som v ∠e v Δ] $\hat{A}_1 = \hat{A}_2 = x$ $\therefore$ CA bisects PĀS OR In ΔAPB and ΔASB: 1) $\hat{O}BP = \hat{A}BS = 90^\circ$ [line from centre to midpt chord / lyn v midpt na midpt kd] 2) PB = BS [given / gegee] 3) AB = AB [common side / gemene sy] $\therefore \Delta APB \cong \Delta ASB$ (s; ∠; s) $\therefore \hat{A}_1 = \hat{A}_2 = x$ $\therefore$ CA bisects PĀS	✓ S ✓ R ✓ S / R ✓ conclusion OR ✓ S ✓ R ✓ both sides ✓ conclusion (4)
9.2.2	$\hat{R} = 180^\circ - 2x$ [opp ∠s of cyclic quad/ teenoorst ∠e v koordevh] $\hat{O}_1 = 180^\circ - 2x$ [proven/ bewys] $\therefore \hat{R} = \hat{O}_1$ COSR is a cyclic quad / is 'n kvh [converse: ext ∠ of cyclic quad/ omgekeerde buite ∠ v koordevh]	✓ S ✓ R ✓ $\hat{R} = \hat{O}_1$ ✓ R (4)

[12]

QUESTION 10 / VRAAG 10

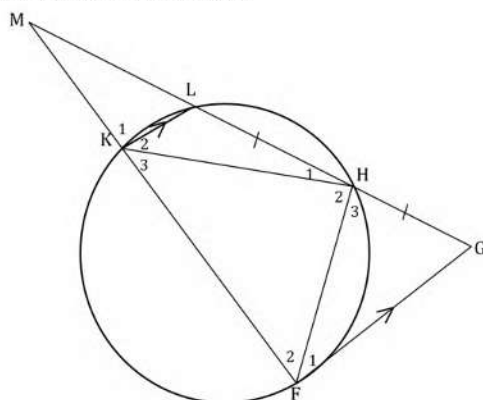


10.1	Constr/Konstr : Draw line BC such that MB = AK and MC = AF Trek lyn BC sodat MB = AK en MC = AF Proof/Bewys : In ΔBMC and/en ΔKAF MB = AK [constr/konstr] $\hat{M} = \hat{A}$ [given/gegee] MC = AF [constr/konstr] ΔBMC ≡ ΔKAF [s s s] ∴ $\hat{MBC} = \hat{AKF}$ or $\hat{MCB} = \hat{AFK}$ [$\equiv \Delta$] but /maar $\hat{V} = \hat{K}$ or $\hat{T} = \hat{F}$ [given/gegee] ∴ $\hat{MBC} = \hat{V}$ or $\hat{MCB} = \hat{T}$ But these are corresponding $\angle$s/maar hulle is ooreenk $\angle$e ∴ BC VT [corr $\angle$s = /ooreenk $\angle$e =] ∴ $\frac{MV}{MB} = \frac{MT}{MC}$ [prop theorem/eweredighst; BC VT] but /maar MB = AK and MC = AF [constr/konstr] ∴ $\frac{MV}{AK} = \frac{MT}{AF}$	✓ constr/konstr ✓ S / R ✓ S ✓ S ✓ S / R ✓ S ✓ R (7)
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10.2



MARKING GUIDELINE



10.2.1	[corresp $\angle$ s / ooreenkomstige $\angle$ e ; $KL \parallel FG$]	✓ R (1)
10.2.2(a)	In ΔMFH and ΔMGF: 1) $\widehat{GFM} = \widehat{K}_1$ [proven/bewys] $\widehat{K}_1 = \widehat{LHF}$ [ext $\angle$ of cyclic quad / buite $\angle$ v koordevh] $\therefore \widehat{LHF} = \widehat{GFM}$ 2) $\widehat{M} = \widehat{M}$ [common/gemeen] $\therefore \Delta MFH \parallel \Delta MGF$ [$\angle, \angle, \angle$] OR In ΔMFH and ΔMGF: 1) $\widehat{GFM} = \widehat{K}_1$ [proven/ bewys] $\widehat{K}_1 = \widehat{LHF}$ [ext $\angle$ of cyclic quad / buite $\angle$ v koordevh] $\therefore \widehat{LHF} = \widehat{GFM}$ 2) $\widehat{M} = \widehat{M}$ [common/bewys] 3) $\widehat{F}_2 = \widehat{G}$ [sum of $\angle$s of Δ] / som v $\angle$e v Δ] $\therefore \Delta MFH \parallel \Delta MGF$ [$\angle, \angle, \angle$] OR In ΔMFH and ΔMGF: 1) $\widehat{MLK} = \widehat{G}$ [corresp $\angle$s / ooreenkomstige $\angle$e ; $KL \parallel FG$] $\widehat{MLK} = \widehat{MFH}$ [ext $\angle$ of cyclic quad / buite $\angle$ v koordevh] $\therefore \widehat{G} = \widehat{MFH}$ 2) $\widehat{M} = \widehat{M}$ [common/ bewys] $\therefore \Delta MFH \parallel \Delta MGF$ [$\angle, \angle, \angle$] OR In ΔMFH and ΔMGF: 1) $\widehat{MLK} = \widehat{G}$ [corresp $\angle$s / ooreenkomstige $\angle$e ; $KL \parallel FG$] $\widehat{MLK} = \widehat{MFH}$ [ext $\angle$ of cyclic quad / buite $\angle$ v koordevh] $\therefore \widehat{G} = \widehat{MFH}$ 2) $\widehat{M} = \widehat{M}$ [common/ bewys] 3) $\widehat{LHF} = \widehat{GFM}$ [sum of $\angle$s of Δ] / som v $\angle$e v Δ] $\therefore \Delta MFH \parallel \Delta MGF$ [$\angle, \angle, \angle$]	✓ S / R ✓ $\widehat{LHF} = \widehat{GFM}$ ✓ S ✓ $\angle, \angle, \angle$ OR ✓ S / R ✓ $\widehat{LHF} = \widehat{GFM}$ ✓ S ✓ S ✓ S / R ✓ S / R ✓ S ✓ $\angle, \angle, \angle$ OR ✓ S / R ✓ S / R ✓ S ✓ S or $\angle, \angle, \angle$ (4)

10.2.2(b)	$\frac{MK}{KF} = \frac{ML}{LG}$ $= \frac{1}{2} \quad [\text{line } \parallel \text{ one side of } \Delta \text{ OR prop theorem; } KL \parallel FG]$ $LG = 2ML$ $LH + HG = 2ML$ But $LH = HG$ [given] $\therefore LH = HG = ML$ $\frac{MH}{MF} = \frac{MF}{MG} \quad [\Delta MFH \sim \Delta MGF]$ $MF^2 = MH \cdot MG$ $MF^2 = 2ML \times 3ML$ $MF^2 = 6ML^2$ $\frac{MF^2}{ML^2} = 6$ $\frac{MF}{ML} = \sqrt{6}$	✓ R ✓ S ✓ R ✓ S ✓ $MF^2 = 6ML^2$ (5)
TOTAL = 150		
		[17]



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$