

PRELIMINARY EXAMINATION
FURTHER STUDIES MATHEMATICS: PAPER 1
MODULE 1 – ALGEBRA AND CALCULUS

August 2026

Name & Surname: _____

Time: 2 hours

Marks: 200 marks

Examiner: C van den Ber

Moderator: C Loots-Grevelink

Instructions

1. This paper consists of 10 pages.
 An information booklet of 4 pages is provided.
 Please check that your question paper is complete.
2. Check that your calculator is set on radians
3. Non-programmable and non-graphical calculators may be used, unless otherwise stated.
4. All necessary calculations must be clearly shown and writing should be legible.
5. Answers only will NOT necessarily be awarded full marks.
6. Diagrams have not been drawn to scale.
7. If applicable, calculations must be done in radians and answers given in radians.
8. Round off your answers to two decimal places, unless otherwise indicated.

QUESTION	1	2	3	4	5	6	7	8	9	10	11	12	TOT
MAX MARKS	18	9	21	10	10	14	18	21	19	27	13	20	200
ACTUAL													

QUESTION 1

- (a) Consider the following statements and state whether they are TRUE or FALSE or NOT NECESSARILY TRUE. Give reasons for your conclusion.

(1) If $\lim_{x \rightarrow -3} f(x)$ exists, then f is continuous at $x = -3$.

(2) If $\lim_{x \rightarrow 4} f(x) \neq f(4)$, there will be a jump continuity at $x = 4$.

(3) If f is continuous at $x = 4$, then $\lim_{x \rightarrow 4^+} f(x)$ (8)

(b) Given: $f(x) = \frac{x^2 - 5x + 6}{x^2 - 2x - 3}$

Discuss the type of discontinuity that exists at $x = 3$.

Explain your answer by showing calculations. (5)

- (c) Let f and g be differentiable functions such that:

$$\begin{array}{lll} f(1) = 2 & f'(1) = 3 & f'(2) = -4 \\ g(1) = 2 & g'(1) = -3 & g'(2) = 5 \end{array}$$

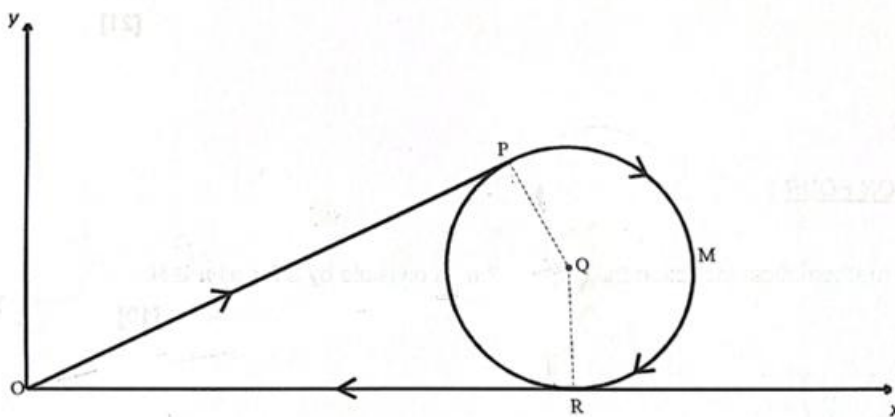
If $h(x) = f(x) \times g(x)$, determine the numerical value of $h'(1)$.

(5)

[18]

QUESTION 2

Shaun is planning the route for the annual cross-country run, for Harlequin's Running Club. He decides to start at O, the origin, and follow the arrows to P, then around arc PMR, which is part of a circle centred at Q. The final leg of the run will be from point R to O. OP and OR are tangents to the circle.



Circle Q is represented by the equation $(x-21)^2 + (y-5)^2 = 25$.

Calculate the total length of the run. All distances are in kilometres.

(9)

[9]

QUESTION 3

(a) Given $p(x) = \ln|x + 2e|$

Solve for x in terms of e if $|p(x)| = 1$

(8)

(b) Solve for a and b if:

$$\log_8 a + \log_4 b^2 = 5 \quad \text{and} \quad a^3 \cdot b = 2^{21}$$

(10)

(c) The number of red kangaroo in a particular area of Australia is predicted to decrease over time according to the model

$$k(t) = \frac{615}{5 - 125e^{-0,16t}}, \text{ where } k \text{ is the number of red kangaroo and } t \text{ is the time in years.}$$

Determine how many kangaroo will be in the area eventually.

(3)

[21]

3

QUESTION 4

Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 for all $n \in \mathbb{N}$.

[10]

QUESTION 5

Given $z = \frac{p+qi}{p-qi}$ in the complex plane.

(a) Express z in the form $a + bi$. (6)

(b) If $\text{Im}(z) = 1$, prove that $p = q$. (4)

[10]

QUESTION 6

(a) Determine whether the following statements are TRUE or FALSE.

If FALSE, provide a valid reason for your conclusion.

(1) If a function f has a gradient of zero at its stationary point and the point is either a maximum local maximum or minimum point on the graph. (3)

(2) A point of inflection can be described as a stationary point that is not a turning point. (3)

(b) The function of f with equation $f(x) = 2x^4 - 4x^3 - 1$ is given.

Showing all calculations, determine the nature of the stationary point(s) of f . (8)

[14]

QUESTION 7

(a) Given: $g'(x) = -3(2+3x)^{-2}$
 $g''(x) = 18(2+3x)^{-3}$
 $g'''(x) = -162(2+3x)^{-4}$

Write down the formula for $g^{(n)}(x)$, the n th derivative of g . (8)

(b) Given the curve with equation $(y-1)\sin(xy) = x^2 - 4$

Use implicit differentiation to determine the gradient of the curve at the point $(2; 0)$ (10)

[18]

QUESTION 8

For each of the questions below, five alternatives are given. Choose the correct alternative

And write down only the letter of your choice next to the question number.

(a) The solution for the inequality $x > |3x - 1|$ is the interval

A: $\left(\frac{1}{2}; \frac{2}{3}\right)$

B: $\left(\frac{1}{3}; \frac{1}{2}\right)$

C: $\left[\frac{1}{4}; \frac{1}{3}\right]$

D: $\left(\frac{1}{4}; \frac{1}{2}\right)$

E: $\left[\frac{1}{3}; \frac{2}{3}\right]$

(b) If $y = 3\tan^2(2x)$, then $\frac{dy}{dx}$ is equal to:

A: $12\tan(2x) \cdot \sec^2(2x)$

B: $12\tan^2(2x) \cdot \sec(2x)$

C: $6\tan(2x) \cdot \sec^2(2x)$

D: $6\tan(2x)$

E: $12\tan(2x)$

(c) The inverse function of $f(x) = e^{3x-5}$ is:

A: $f^{-1}(x) = \ln(3x + 5)$

B: $f^{-1}(x) = \log_3(x + 5)$

C: $f^{-1}(x) = \frac{1}{3}\ln(x + 5)$

D: $f^{-1}(x) = \frac{1}{3}\ln x + 5$

E: $f^{-1}(x) = \frac{1}{e^{3x-5}}$

(d) The domain of the function $f(x) = \sqrt{\ln\sqrt{x-1}}$ is:

A: $(-\infty; 1]$

B: $[2; 0]$

C: $[1; \infty]$

D: $[2; e)$

E: $[e; \infty)$

(e) The equation $x^5 + x + 30 = 0$ has

A: no solution

B: exactly one solution

C: exactly two solutions

D: exactly three solutions

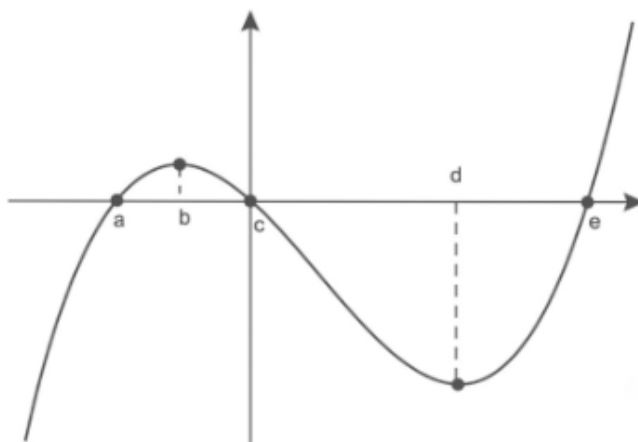
E: four or more solutions

(f) Given: $p(x) = \frac{x^2-9}{x-2}$

Which statement about the function of p is TRUE?

- A: p has vertical asymptotes at $x = 2$ and intersects the oblique asymptote $y = x + 2$
- B: p has vertical asymptotes at $x = 2$ and does not intersect the oblique asymptote $y = x + 2$
- C: p has vertical asymptotes at $x = -2$ and does not intersect the oblique asymptote $y = x + 2$
- D: p has vertical asymptotes at $x = -2$ and intersects the oblique asymptote $y = x + 2$
- E: p has vertical asymptotes at $y = 3$ and $y = -3$ and a vertical asymptote at $x = 2$

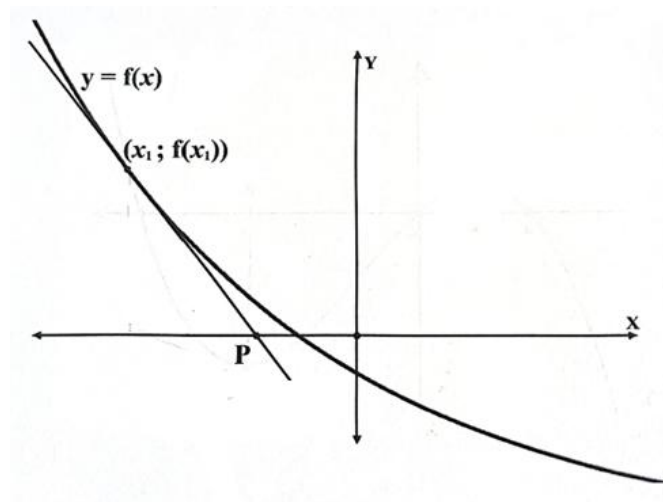
- (g) The graph of a polynomial f is shown below. If f' is the derivative of f ,
The remainder of the division of $f'(x)$ by $(x-b)$ is more likely to be equal to;



- A: $f(b)$ B: 1 C: 0
- D: 2 E: -1

[7x 3=21]

QUESTION 9



- (a) A curve with no stationary points has equation $y = f(x)$. The equation $f(x) = 0$ has one real root. The tangent to the curve at point $(x_1; f(x_1))$ meets the x -axis at P.

Write down the x -coordinate of P in terms of $x_1; f(x_1)$ and $f'(x_1)$. (5)

- (b) The graph of h with equation $h(x) = x^5 - x^2 - 6x + 5$ has a stationary point between $x = 1$ and $x = 2$.

- (1) Show, using the Newton-Raphson formula, that

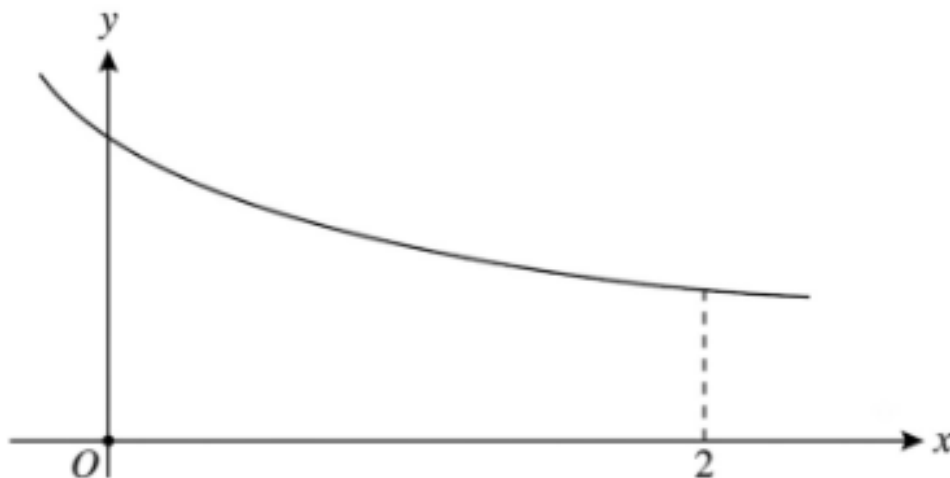
$$x_{n+1} = \frac{15(x_n)^4 + 6}{20(x_n)^3 - 2} \quad (9)$$

- (2) Hence, determine the x -coordinate of this stationary point correct to 5 decimal places. (5)

[19]

QUESTION 10

- (a) The diagram below shows the curve with equation $y = \frac{1}{\sqrt{3x+2}}$



A region is bounded by the curve and the lines $x = 0$ and $x = 2$.

Determine the exact area of the region WITHOUT USING A CALCULATOR. (7)

- (b) Determine: $\int 2x^3\sqrt{x^2+1} dx$ (Hint: Let $u = x^2 + 1$) (8)

- (c) (1) Show that $\int \sec^3 x dx = \sec x \cdot \tan x - \int \sec x \cdot \tan^2 x dx$ (6)

- (2) If $\int \sec x dx = \ln|\sec x + \tan x|$, then show that

$$\int \sec^3 x dx = \frac{1}{2} \sec x \cdot \tan x + \frac{1}{2} \ln|\sec x + \tan x| \quad (6)$$

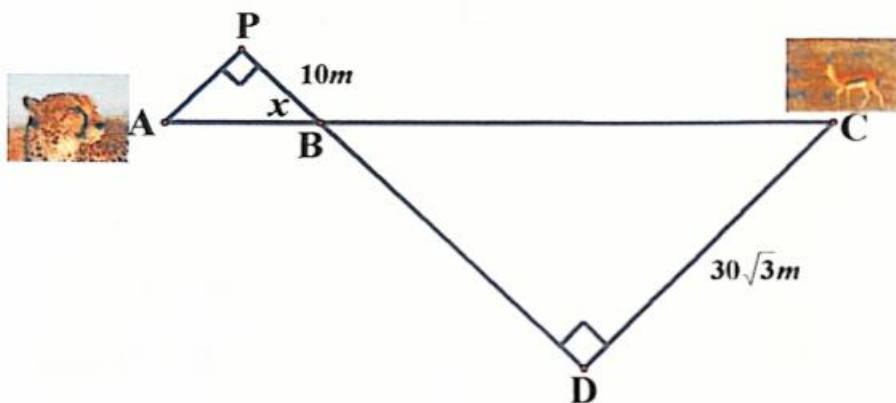
[27]

QUESTION 11

ABC and PBD are straight lines such that $\triangle PAB$ and $\triangle BDC$ are right-angled triangles.

$\angle PBA = x$, $PB = 10$ metres and $DC = 30\sqrt{3}$ metres. (Diagram not drawn to scale).

A cheetah at point P observes a gazelle walking from point D to point C . The cheetah then moves stealthy to a suitable point A in order to keep the gazelle in sight.



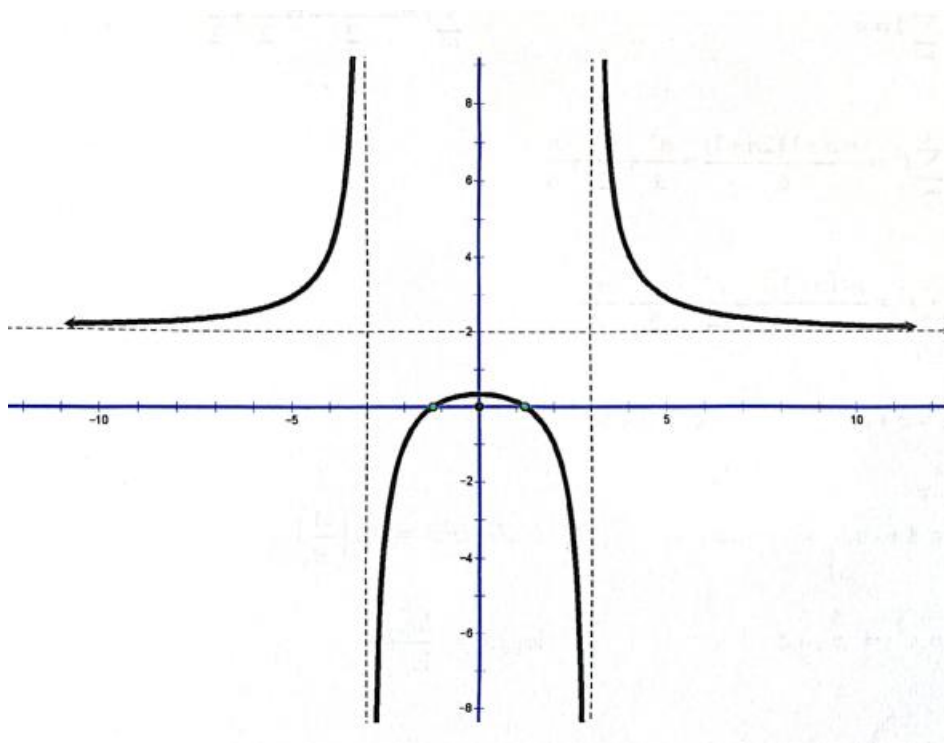
- (a) Show that $AC = 10\sec x + 30\sqrt{3}\csc x$ (4)
- (b) Determine the value of x for which the cheetah is closest to the gazelle. (9)

[13]

QUESTION 12

Given: $f(x) = \frac{2x^2-3}{x^2-\alpha}$, where α is a constant.

The graph below is a representation of f when $\alpha = 9$.



Show clearly any asymptotes and x- intercepts, draw a neat sketch of f if:

(a) $\alpha = 4$ (6)

(b) $\alpha = \frac{3}{2}$ (6)

(c) $0 < \alpha < \frac{3}{2}$ (4)

(d) $\alpha < 0$ (4)

[20]