



MATHEMATICS

GRADE 12

OPTIMISATION

ACTIVITY 1

A ball is thrown vertically upwards. The height of the ball in meters, after t seconds is given by the equation $h(t) = 9t - 2t^2$.

- 9.1 Determine the height of the ball after 2 seconds. (1)
 - 9.2 Determine the speed of the ball after 2 seconds. (3)
 - 9.3 After how many seconds will the ball reach the ground again? (2)
- [6]

ACTIVITY 2

A closed rectangular box, with a rectangle as base, has a length $(2x)$ cm and width (x) cm. The total surface area (all 6 sides) is 243 cm^2 .

- 10.1 Show that the height, h , is equal to $\left(\frac{81}{2x} - \frac{2x}{3}\right) \text{ cm}$. (2)
 - 10.2 Show that the volume of the box, in terms of x , is given by the formula:
$$V = 81x - \frac{4}{3}x^3$$
 (2)
 - 10.3 Calculate the value of x if the volume of the box is a maximum. (3)
- [7]

ACTIVITY 3

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

- 10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)
 - 10.2 Calculate the distance between town P and town T. (5)
- [8]

ACTIVITY 4

A hotel has 72 rooms to let to clients. The daily rent per room is R500. If the rent is increased by R100 per day, 2 rooms can be left vacant daily.

The rent is Rx per room per day, where $x > 500$.

- 10.1 Show that the total daily income (I) is given by $I = 82x - \frac{x^2}{50}$. (5)
 - 10.2 Calculate the maximum daily income per room. (2)
- [7]



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ACTIVITY 5

The profit, $W(x)$, earned by a company to manufacture x bicycles per week, is given by:

$$W(x) = -\frac{x^3}{150} + 3x^2 - 250x - 2700.$$

What should the weekly bicycle production be to maximise profit?

[8]

ACTIVITY 6

At a price of R400 per bag, 200 bags are sold. For every R20 increase in the price of the bags, four fewer bags are sold. At what price must the bags be sold to maximise the profit?

[7]

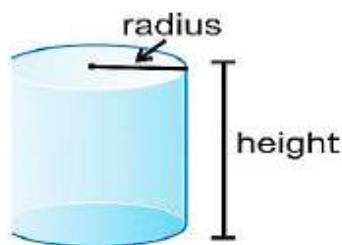
ACTIVITY 7

The sum of two numbers is 25. Determine the two numbers such that the square of the one number plus three times the square of the other number is a minimum.

[7]

ACTIVITY 8

A cylinder closed at both ends is to have a volume of $2000\pi \text{ m}^3$. What should its dimensions be if the surface area of the cylinder is to be as small as possible?

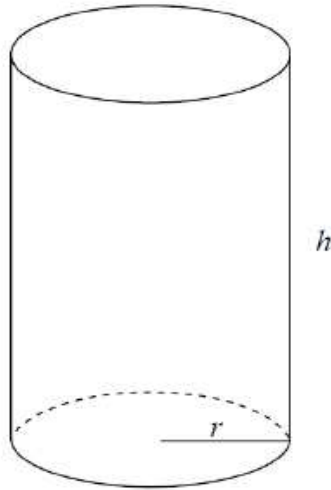


[7]



ACTIVITY 9

A drinking glass, in the shape of a cylinder, must hold 300 cm^3 of liquid when full. The height of the glass is h cm and the radius of the base is r cm.



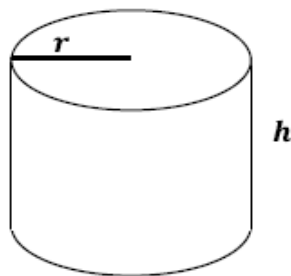
9.1 Show that the height of the glass, h , can be expressed as $h = \frac{300}{\pi r^2}$. (1)

9.2 Hence, determine the value of r for which the total surface area of the glass will be a minimum. (5)

[6]

ACTIVITY 10

A cylinder with radius r and height h is given. The dimensions of the cylinder are such that the sum of the diameter and the height is equal to 24 units.



Calculate the length of the radius such that the volume of the cylinder is a maximum. [6]



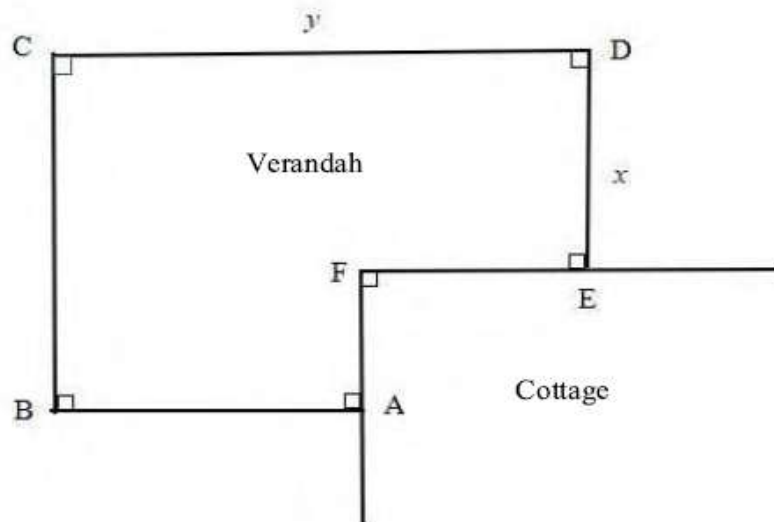
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ACTIVITY 11

The diagram below shows the plan for a verandah which is to be built onto the corner of a cottage. A railing ABCDE is to be constructed around the four edges of the verandah. It is given that $AB = DE = x$ and $BC = CD = y$, and the length of the railing must be 30 metres. Calculate the value of x and y for which the veranda will have a maximum area.



[8]

ACTIVITY 12

In February 2024, Bafana Bafana played in the semi-finals of the AFCON tournament. During the match, their striker Tebogo Mokoena kicked the soccer ball vertically upwards into the air and its motion was represented by the equation: $h(t) = 1 + 20t - 5t^2$, where h is the height of the ball above the ground in metres, and t is the time in seconds after the ball was kicked.

- 9.1 Determine the maximum height of the ball above the ground. (5)
- 9.2 How long will it take for the ball to hit the ground? (3)
- 9.3 Determine the velocity of the ball 1,5 seconds after he has kicked it. (2)

[10]





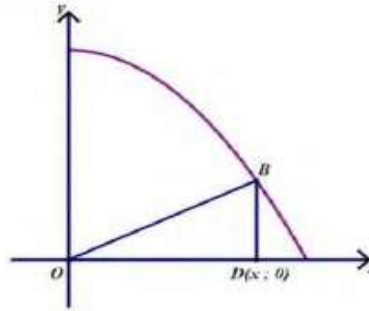
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ACTIVITY 13

Refer to the figure showing the parabola given by $f(x) = 4 - \frac{x^2}{4}$ with $0 \leq x \leq 4$. D is the point $(x; 0)$ and DB is parallel to the y -axis, with B on the graph of f .



- 9.1 Write down the coordinates of B in terms of x . (2)
- 9.2 Show that the area, A, of $\triangle OBD$ is given by: $A = 2x - \frac{x^2}{8}$. (3)
- 9.3 Determine how far D should be from O in order that the area of $\triangle OBD$ is as large possible. (4)

[9]

ACTIVITY 14

The radius of the base of a cylindrical cold drink can is x cm, and its volume 440 cm^3 .

$$A = \pi r^2, C = 2\pi r, V = \pi r^2 h$$

- 8.1 Determine the height of the can in terms of x . (2)
- 8.2 Show that the area of the material needed to manufacture the can is $2\pi x^2 + \frac{880}{x}$. (2)
- 8.3 Determine the value of x (correct to two decimals) for which the least amount of material is needed to manufacture such a can. (4)

[8]





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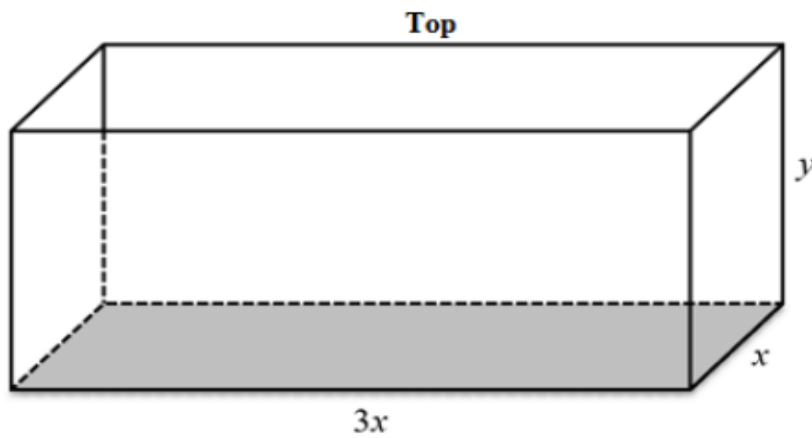
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ACTIVITY 15

The wooden box in the diagram is a rectangular prism and it is open at the top. The dimensions of the base are $3x$ metres by x metres and the height is y metres.

The total surface area is 147 m^2 .

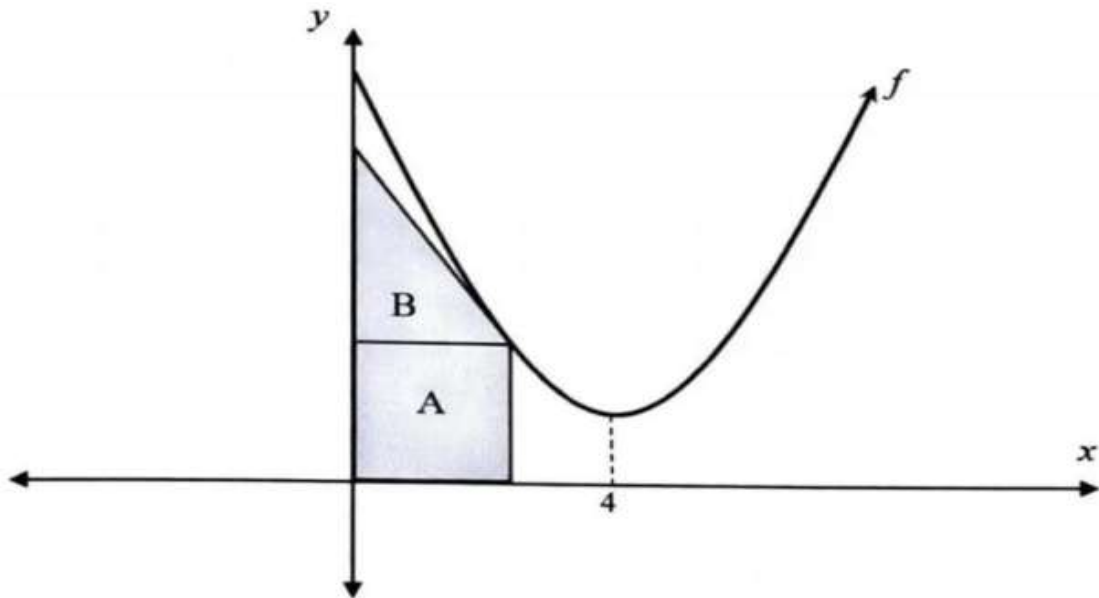


- 10.1 Show that $y = \frac{147 - 3x^2}{8x}$. (2)
- 10.2 Calculate the value of x for which the volume of the box is a maximum. (5)
[7]



ACTIVITY 16

A river boards the farm of a farmer, which is represented by the equation $f(x) = x^2 - 8x + 17$. A tarred road is represented by the x - and y -axes and a border fence at $x = 4$.



- 9.1 Show that the area of the rectangular field (shaded area A), is given by:
- $$A(x) = x^3 - 8x^2 + 17x.$$
- 9.2 Determine the area of the largest rectangular field that the farmer can fence in (shaded area A).
- 9.3 He decided to include an additional triangular field (shaded area B). Determine the largest area of the triangular field, if the base of area B is the same as the base of area A.
(Note: The fence should only touch the river and not cross it.)
- (1)
(5)
(4)
[10]



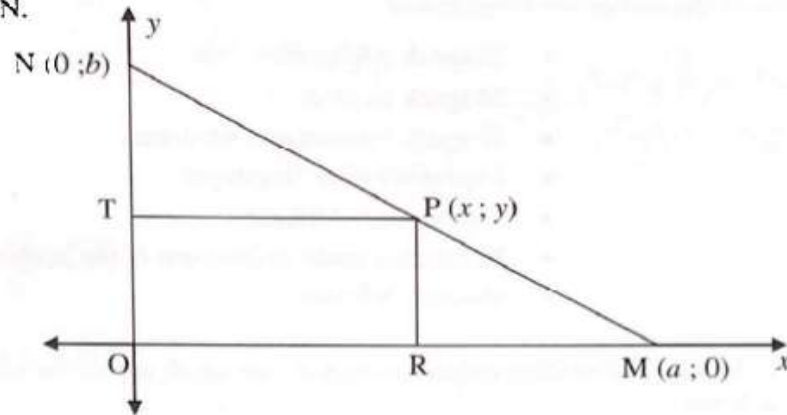
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ACTIVITY 17

A farmer has a piece of land in the shape of a right-angled triangle OMN , as shown in the figure below. He allocates a rectangular piece of land $PTOR$ to his daughter, giving her the freedom to choose P anywhere along the boundary MN . Let $OM = a$, $ON = b$ and $P(x; y)$ be any point on MN .



- 9.1 Determine an equation for MN in terms of a and b . (2)
- 9.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN . (6)

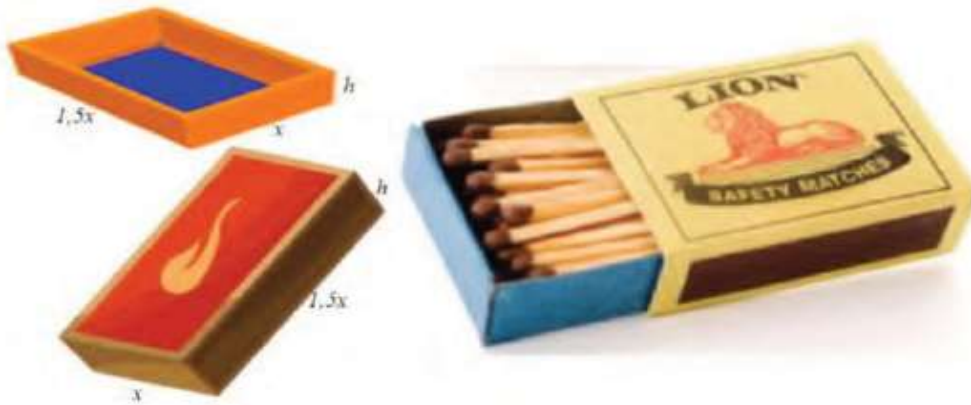
[8]



ACTIVITY 18

A match box consists of an outer cover, open at both ends, into which slides a rectangular box without a top.

- The length of the box is one and a half times its breadth.
- The thickness of the material is negligible.
- The volume of the box is 25cm^3 .



If the breadth of the box is x cm:

11.1 Write down the height of the box in terms of x . (2)

11.2 Show that the amount of the material used for the box is given by: (2)

$$A = \frac{100}{x} + 4,5x^2 + \frac{50}{1,5x}$$

11.3 Hence determine the breadth of the box such that the material used is a minimum. (3)

[7]

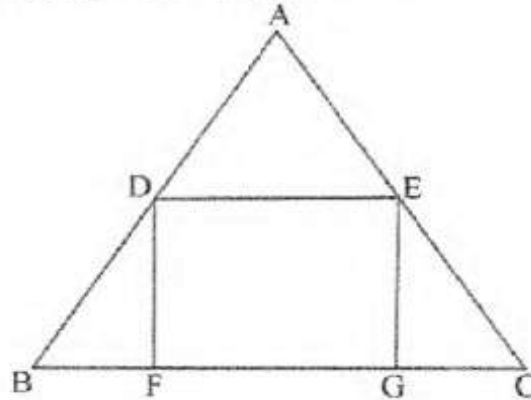
ACTIVITY 19

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10 ; 2)$ and a point on f . (8)

ACTIVITY 20

In the sketch below, $\triangle ABC$ is an equilateral triangle with side $AB = m$ units. $DEGF$ is a rectangle with $BF = GC = x$ units.



10.1 Prove that the area of the rectangle $DEGF$ is $\sqrt{3}x(m-2x)$ (3)

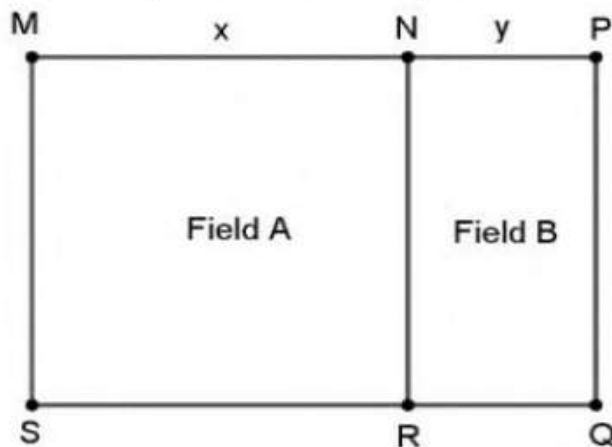
10.2 Determine, in terms of m , the maximum area of the rectangle. (4)
[7]



ACTIVITY 21

The diagram below, shows a drawing of two adjacent fields that need to be fenced by a certain farmer.

- Field A is a square of width x .
- Field B is rectangular in shape of width y .
- Area of Field A + Area of Field B = 1000 units².
- NR is the only fence that is shared by both Field A and Field B.



10.1 Express y in terms of x . (2)

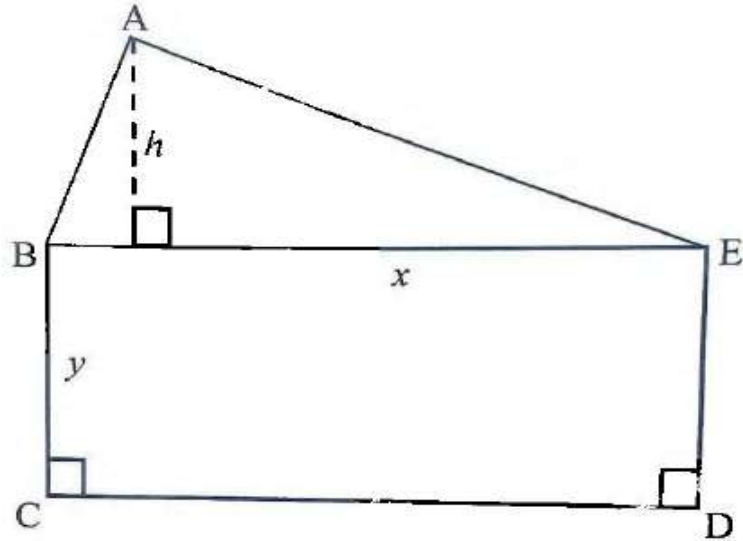
10.2 Show that the length of the fence needed is given by $P = 3x + \frac{2000}{x}$. (2)

10.3 Calculate the value of x such that the length of the fence required is a minimum. (3)

[7]

ACTIVITY 22

In the figure below, $\triangle ABE$ is mounted on a rectangle $BCDE$. $\triangle ABE$ has a base of length x metres. The base and the perpendicular height of the triangle add up to 10 metres. y is the breadth of rectangle $BCDE$. The perimeter of the rectangle is 32m.

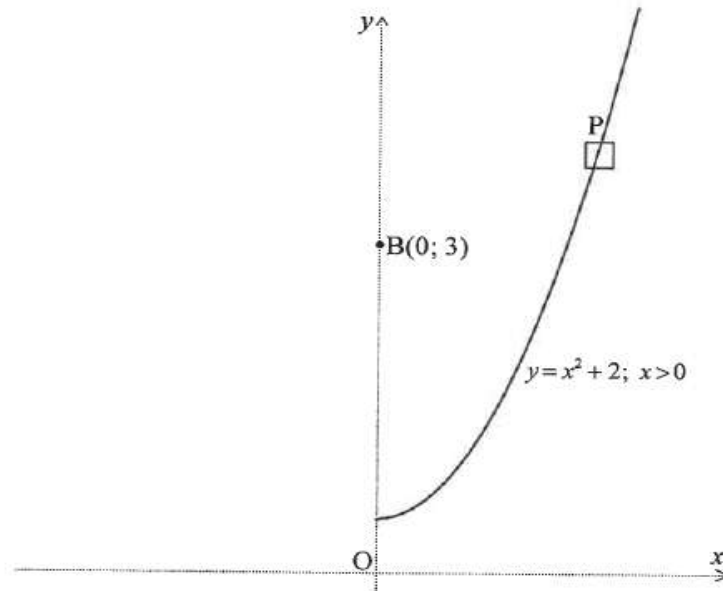


- 9.1 Show that the Area of the figure $ABCDE$ is $A(x) = -\frac{x}{2}(3x - 42) m^2$ (4)
 - 9.2 Calculate the value of x for which $ABCDE$ has a maximum area. (2)
 - 9.3 Calculate the maximum area of $ABCDE$. (2)
- [8]

ACTIVITY 23

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point $B(0; 3)$ and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny.

[7]

ACTIVITY 24

A large cruise ship uses fuel at a cost of $4x^2$ rand per hour, where x is the speed of the ship in km/h .

Other operating costs, including labour, amount to R1 000 per hour.

[Hint: distance = speed $\times$ time: $s = vt$]

9.1 Show that the total cost for a trip of 500 km is given by, $C(x) = 2\,000x + \frac{500\,000}{x}$. (3)

9.2 At what speed should the ship travel on this 500 km trip to keep the total cost as low as possible? (5)

[8]

ACTIVITY 25

During an experiment, the temperature T (in degrees Celsius), varies with time t (in hours), according to the formula $T(t) = 30 + 4t - \frac{1}{2}t^2$, $t \in [0; 10]$.

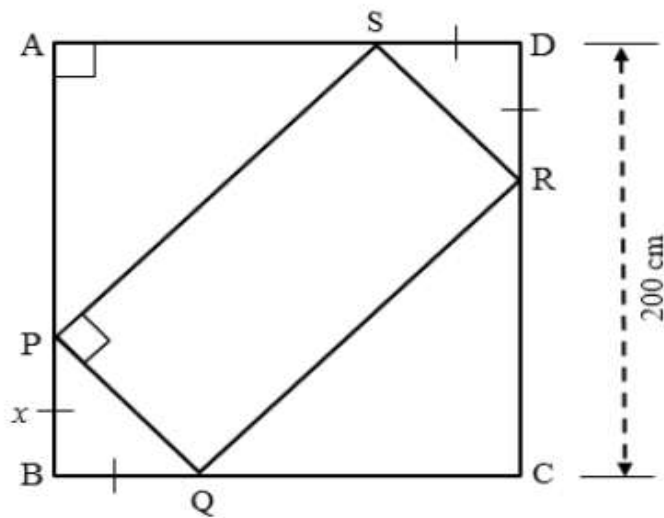
8.1 Determine an expression for the rate of change of temperature with time. (2)

8.2 During which time interval was the temperature decreasing? (4)

[6]

ACTIVITY 26

In the diagram below, ABCD is a square with side length $CD = 200$ cm. PQRS is a rectangle with vertices on the sides of the square. $PB = BQ = SD = DR = x$ cm.

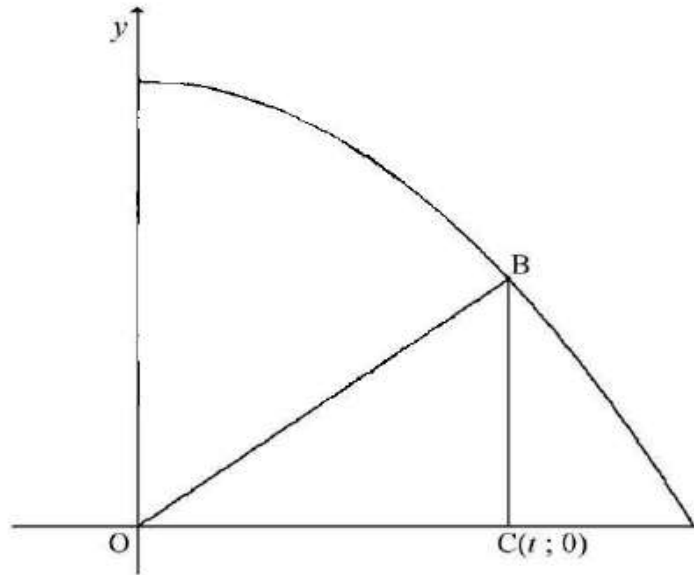


- 9.1 Show that the area of the rectangle is given by, $A = 2(200x - x^2)$. (3)
 - 9.2 Determine the value of x for which the area of the rectangle will be a maximum. (3)
 - 9.3 What is the ratio of the maximum area of PQRS : area of ABCD? (3)
- [9]

ACTIVITY 27

The diagram shows the graph of the parabola with equation $f(x) = 9 - \frac{x^2}{9}$; $x \in [0; 9]$.

BC is a line parallel to the y -axis, with $C(t; 0)$ a point on the x -axis and B on the graph of f . OB is drawn.



- 10.1 Write down the coordinates of B in terms of t . (2)
- 10.2 Show that the area of $\triangle OBC$ can be given by: $A(t) = \frac{9}{2}t - \frac{t^3}{18}$. (2)
- 10.3 Determine the maximum area of $\triangle OBC$. (5)

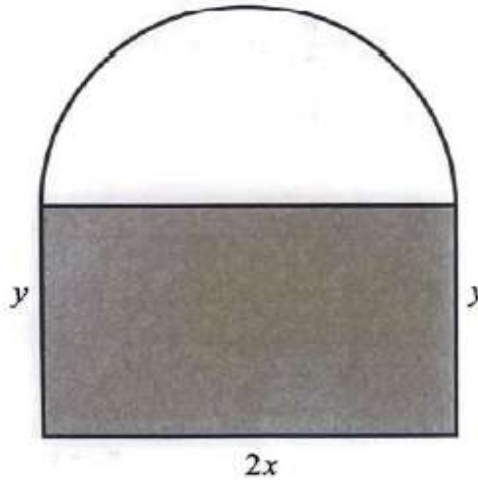
ACTIVITY 28

A wire, 12 metres long, is cut into two pieces. One part is bent to form an equilateral triangle and the other a square. A side of the triangle has a length of $2x$ metres.

- 9.1 Write down the length of a side of the square in terms of x . (2)
- 9.2 If this square is now used as the base of a rectangular prism with a height of $4x$ metres, determine the maximum volume of the rectangular prism. (7)
(9)

ACTIVITY 29

The figure represents a large stained glass window frame. The upper part of the frame is a semi-circle. The lower part of the frame is a rectangle with sides of $2x$ metres and y metres as shown. The perimeter of the window frame is 30 metres.



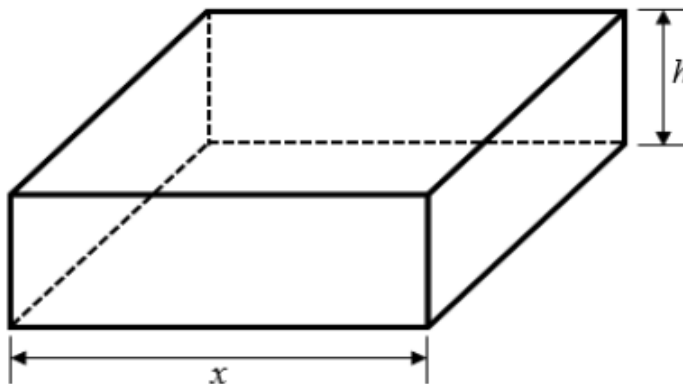
8.1 Show that: $y = \frac{30 - x(2 + \pi)}{2}$ (4)

8.2 Calculate the value of x for which the area of the window frame will be a maximum.

(Leave answer in terms of π) (5)
[9]

ACTIVITY 30

An open box with a square base is to be made out of a given quantity of pre-cut cardboard sheet. The total surface area of the outside of the box is 300 cm^2 .



Determine the value of x for which the volume of the box will be a maximum, and hence the maximum volume of the box.

[7]



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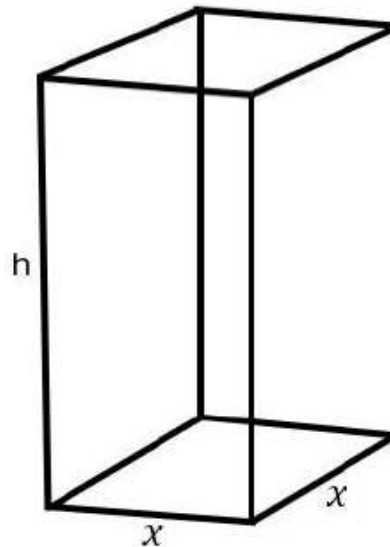
GRADE 12

OPTIMISATION

ACTIVITY 31

An industrial chemical is stored in rectangular containers with a square base of x centimetres. The height of each container is h centimetres as shown on the figure below.

The volume of each container is $2\,160\,000\text{ cm}^3$.



9.1 Determine the height of the container in terms of x . (2)

9.2 For safety reasons during the transportation of the containers, a triple layer of material is needed at the base of the container.

(Ignore the thickness of the material.)

Show that the total surface area of the material used for the container is given by:

$$A(x) = 4x^2 + \frac{8\,640\,000}{x} \quad (3)$$

9.3 Determine the dimensions of the container that will ensure that the minimum amount of material is used when making the containers. (5)
[10]

