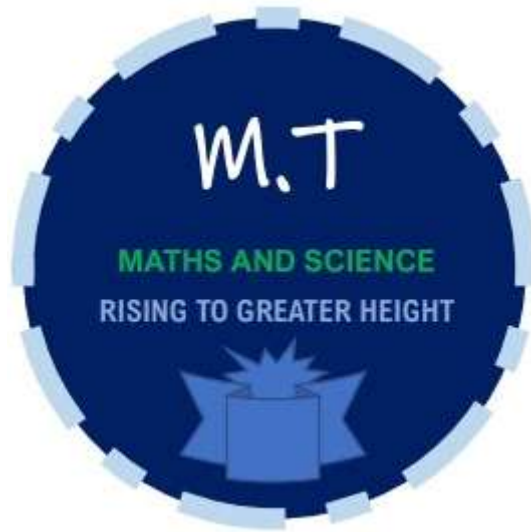




MATHEMATICS
GRADE 12



MATHEMATICS

GRADE 12

TOPIC:

TRIGONOMETRY



TRIGONOMETRY

ACTIVITY 1

5.1 If $5 \sin \beta - 4 = 0$ and $\beta \in (90^\circ ; 270^\circ)$, determine without the use of a calculator and with the aid of a diagram the values of:

5.1.1 $\cos \beta$ (3)

5.1.2 $\cos 2\beta$ (3)

5.1.3 $\sin 3\beta$ (4)

5.2 Simplify:

$$\frac{\sin(-180^\circ - \theta) \cdot \tan(180^\circ - \theta) \cdot \cos(-\theta)}{\cos^2(90^\circ + \theta) + 3\sin^2 \theta} \quad (6)$$

5.3 Prove the identity:

$$\frac{1}{8}(1 - \cos 4x) = \sin^2 x \cdot \cos^2 x \quad (5)$$

[21]

ACTIVITY 2

7.1 If $\sin 35^\circ = k$, determine the following in terms of k :

7.1.1 $\cos 55^\circ$ (3)

7.1.2 $\sin 70^\circ$ (3)

7.1.3 $\cos 80^\circ$ (3)

7.2 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)

7.3 Simplify to a single trigonometric ratio:

$$\frac{\tan(180^\circ + x) \cdot \cos(360^\circ - x)}{\sin(x - 180^\circ) \cos(90^\circ + x) + \cos(720^\circ + x) \cdot \cos(-x)} \quad (6)$$

7.4 Prove the identity:

$$\frac{\cos 2x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} = \frac{1}{\cos^2 x} \quad (4)$$

7.5 Determine the general solution of $2 \sin x \cos x - \cos^2 x = 0$ (7)

[31]



ACTIVITY 3

6.1 Given: $\tan x = \frac{3}{4}$, where $x \in [180^\circ; 270^\circ]$.

With the aid of a sketch, and **without the use of a calculator**, calculate:

6.1.1 $\sin x$ (3)

6.1.2 $2 - \sin 2x$ (3)

6.1.3 $\cos^2(90^\circ - x) - 1$ (3)

6.2 Evaluate: $\frac{-1 + \cos(180^\circ - \theta) \cdot \sin(\theta - 90^\circ)}{\cos(-\theta) \cdot \sin(90^\circ + \theta) \cdot \tan^2(540^\circ + \theta)}$ (7)

[16]

ACTIVITY 4

7.1 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

7.1.1 Use the above identity to deduce that $\sin(A + B) = \sin A \cos B + \cos A \sin B$. (3)

7.1.2 Hence determine the general solution of the equation
 $\sin(2x + 50^\circ) - \sin 15^\circ \cos 48^\circ = \sin 48^\circ \cos 15^\circ$. (4)

7.2 Given: $\frac{4\sin x \cos x}{2\sin^2 x - 1}$

7.2.1 Simplify $\frac{4\sin x \cos x}{2\sin^2 x - 1}$ to a single trigonometric ratio. (3)

7.2.2 For which value(s) of x in the interval $-90^\circ < x < 90^\circ$ will the above expression be undefined? (3)

7.2.3 Without using a calculator, determine the value of $\frac{4\sin 15^\circ \cos 15^\circ}{2\sin^2 15^\circ - 1}$. (2)

ACTIVITY 5

7.1 Given $\cos 40^\circ = t$, **without using a calculator**, determine each of the following in terms of t :

7.1.1 $\tan 40^\circ$ (2)

7.1.2 $\cos^2 130^\circ$ (3)

7.1.3 $\cos 220^\circ$ (3)

7.2 **Without using a calculator**, simplify the following expression:

$\sin 237^\circ \cdot \cos 147^\circ - \frac{\cos 213^\circ \cdot \cos 303^\circ}{\tan 237^\circ}$ (7)

[15]





ACTIVITY 6

6.1 Given: $\cos 17^\circ = \frac{k}{5}$

Without using a calculator, determine in terms of k , each of the following.

6.1.1 $\sin 17^\circ$ (3)

6.1.2 $\tan 253^\circ$ (3)

6.1.3 $\sin 124^\circ$ (4)

6.2 Given: $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

Use the above identity to deduce that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. (3)

6.3.1 Simplify the following expression.

$\cos(60^\circ + x) + \sin(30^\circ - x)$ (3)

6.3.2 Hence, or otherwise, determine **without the use of a calculator**, the value of:

$\cos(60^\circ + x) + \sin(30^\circ - x)$ if $x = 45^\circ$ (2)

6.4 Prove that:

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \tan \theta$$
 (5)

6.5 Determine the general solution of the equation:

$\cos 2x + 5\sin x = -2$ (6)

[29]





ACTIVITY 7

NO CALCULATOR IS ALLOWED IN THIS QUESTION

- 5.1 If $4 \tan A = 3$ and $3 \sin B - 1 = 0$, use a sketch and determine the value of $5 \sin A - 6 \cos^2 B$, where $180^\circ \leq A \leq 360^\circ$ and $0^\circ \leq B \leq 90^\circ$. (5)
- 5.2 If $\cos 24^\circ = p$, determine the following in terms of p :
- 5.2.1 $\cos 336^\circ$ (2)
- 5.2.2 $\cos 48^\circ$ (3)
- 5.3 Prove that: $\tan A = \frac{1 - \cos 2A}{\sin 2A}$ (4)
- 5.4 Simplify without using a calculator:
- $$\sin 20^\circ \cdot \cos 320^\circ + \cos(-20^\circ) \cdot \sin 400^\circ$$
- (5)
- 5.5 Determine the general solution of $2 \cos^2 \theta + 5 \sin \theta + 1 = 0$ (7)
- [26]

ACTIVITY 8

- 5.1 Given: $\sin 2x = \frac{\sqrt{15}}{x}$ and $0^\circ \leq 2x \leq 90^\circ$.
Determine with the aid of a diagram and without using a calculator the value of $\cos x$. (5)
- 5.2 Simplify the following expression to one trigonometric ratio of θ :
- $$\frac{\sin(180^\circ - \theta) \cdot \sin(540^\circ - \theta) \cdot \cos(\theta - 90^\circ)}{\tan(-\theta) \cdot \sin^2(360^\circ - \theta)}$$
- (7)
- 5.3 Given the identity: $\frac{\sin 5x \cdot \cos 3x - \cos 5x \cdot \sin 3x}{\tan 2x} - 1 = -2 \sin^2 x$.
- 5.3.1 Prove the above identity. (4)
- 5.3.2 For which value(s) of x will the above identity be undefined for $0^\circ \leq x \leq 180^\circ$. (4)
- [20]





ACTIVITY 9

- 5.1 If $\sin A = -\frac{3}{7}$, where $A \in [90^\circ; 270^\circ]$ determine, using a diagram, without the use of a calculator, the value of $\sin(A + 30^\circ)$. (4)
- 5.2 Simplify fully to one trigonometric ratio of x :
 $-\sin^2(90^\circ - x) - \tan x \cdot \cos(-x) \cdot \sin(-x - 360^\circ)$ (6)
- 5.3 Show that, for any given value of A , the roots of the equation $x^2 - 2x \sin A = \cos^2 A$ are real and rational. (3)
- 5.4 Prove the identity:

$$\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = \frac{2}{\tan 2x}$$
 (3)
- 5.5 If $\cos 22^\circ = p$ and $\sin 38^\circ = q$, without the use of a calculator, determine the following in terms of p and q .
- 5.5.1 $\sin 68^\circ$ (2)
- 5.5.2 $\cos 16^\circ$ (4)
- [22]

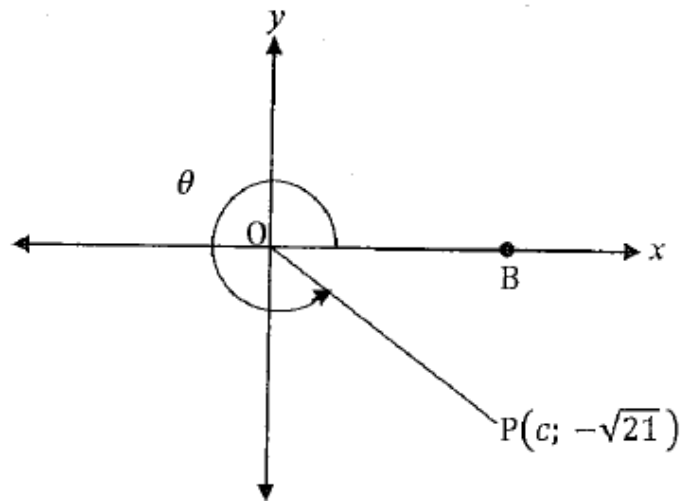
ACTIVITY 10

- 6.1 Prove the identity: $\frac{2\sin^2 x}{2\tan x - \sin 2x} = \frac{\cos x}{\sin x}$ (5)
- 6.2 Determine the general solution of $2 + 2\cos 2x = 0$ (4)
- [9]



ACTIVITY 11

- 5.1 In the diagram below, P is the point $(c; -\sqrt{21})$ such that $OP = 5$ units. Reflex $\widehat{BOP} = \theta$ as indicated.



- 5.1.1 Calculate the numerical value of c . (2)
- 5.1.2 Determine **without the use of calculator**, the numerical value of the following:
- a) $\cos \theta$ (1)
 - b) $\tan \theta + \sin^2 \theta$ (2)
 - c) $\sin 2\theta$ (2)
- 5.2 Simplify, **without the calculator**, to a single trigonometric ratio of x :

$$\frac{\sin(x-180^\circ) \cdot \tan x \cdot \cos 690^\circ}{\cos^2(x-90^\circ)}$$

(5)
[12]



ACTIVITY 12

5.1 Given:

$4 \tan \alpha + 5 = 0$, $\alpha \in (0^\circ; 180^\circ)$. Evaluate without using a calculator:

$$\sqrt{41} \cos \alpha - 4 \sin (-150^\circ) \cdot \cos 180^\circ \quad (5)$$

5.2 Simplify, without the use of a calculator.

$$5.2.1 \quad \frac{\cos 99^\circ}{\cos 33^\circ} - \frac{\sin 99^\circ}{\sin 33^\circ} \quad (6)$$

$$5.2.2 \quad \frac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 130^\circ + \cos(-\theta)} \quad (5)$$

5.3 Prove the identity:

$$\frac{2 \sin^2 x}{2 \tan x - \sin 2x} = \frac{\cos x}{\sin x} \quad (6)$$

5.4 Determine the general solution of the following equation:

$$8 \sin \theta \cos \theta = -2 \sqrt{3} \quad (7)$$

[29]



ACTIVITY 13

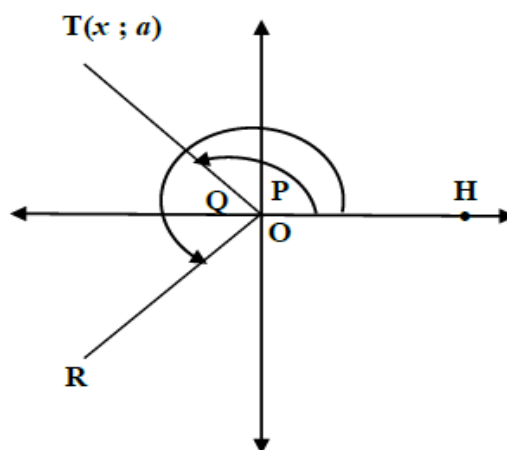
5.1 Simplify the following expression to a single trigonometric function.

$$\frac{\sin x \cdot \sin(90^\circ + y) - \cos x \cdot \sin(180^\circ + y)}{\cos x \cdot \cos(y - 360^\circ) + \sin(-x) \sin y} \quad (6)$$

5.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

5.2.1 Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ (2)

5.2.2 In the diagram, T is a point such that $\widehat{HOT} = P$ and $\sin P = a$. T is reflected about the x-axis to R such that $\widehat{HOR} = Q$



- (a) Determine the coordinates of T in terms of a. (2)
- (b) Write down the coordinates of R in terms of a. (2)
- (c) Calculate $\cos(P + Q)$. (2)
- (d) Hence, show that $P + Q = 360^\circ$. (1)

5.3 Given: $\cos \theta = d$

5.3.1 Write down the values of d such that $\cos \theta$ is defined. (2)

5.3.2 Determine the general solution for θ if:

$$\cos \theta = \frac{1}{\cos \theta} + \frac{5}{6} \quad (6)$$

[23]



ACTIVITY 14

5.1 Given: $\cos 16^\circ = \frac{\sqrt{k}}{t}$.

Without using a calculator, determine in the simplest form, the value of each of the following in terms of k and t :

5.1.1 $\sin 106^\circ$ (2)

5.1.2 $\sin 16^\circ$ (3)

5.1.3 $\cos 8^\circ$ (4)

5.2 Prove that:

$$\frac{\sqrt{4(1-\cos\theta)(1+\cos\theta)}}{\sin 2\theta} = \frac{1}{\cos\theta} \quad (4)$$

5.3 It is given that $\sin p + \sqrt{3} \cos p = 1$.

5.3.1 Show that the equation can be written as $\sin(60^\circ + p) = \frac{1}{2}$ (3)

5.3.2 Hence, determine the general solution of $\sin p + \sqrt{3} \cos p = 1$. (4)

5.4 **Without using a calculator**, determine the value of:

$$\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \dots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ + 2 \quad (3)$$

[23]

ACTIVITY 15

5.1 **Without using a calculator**, write the following expressions in terms of $\sin 11^\circ$:

5.1.1 $\sin 191^\circ$ (1)

5.1.2 $\cos 22^\circ$ (1)

5.2 Simplify $\cos(x-180^\circ) + \sqrt{2} \sin(x+45^\circ)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90^\circ$

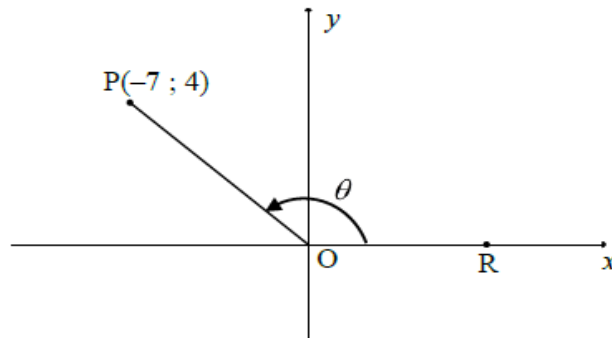
Without using a calculator, determine the value of $\sin 2P$.

(5)

[12]

ACTIVITY 16

- 5.1 In the diagram below, $P(-7 ; 4)$ is a point in the Cartesian plane. R is a point on the positive x -axis such that obtuse $\widehat{POR} = \theta$.



Calculate, **without using a calculator**, the:

- 5.1.1 Length OP (2)
- 5.1.2 Value of:
- (a) $\tan \theta$ (1)
- (b) $\cos(\theta - 180^\circ)$ (2)
- 5.2 Determine the general solution of: $\sin x \cos x + \sin x = 3 \cos^2 x + 3 \cos x$ (7)
- 5.3 Given the identity: $\frac{\sin 3x}{1 - \cos 3x} = \frac{1 + \cos 3x}{\sin 3x}$
- 5.3.1 Prove the identity given above. (3)
- 5.3.2 Determine the values of x , in the interval $x \in [0^\circ ; 60^\circ]$, for which the identity will be undefined. (3)

[18]



ACTIVITY 17

- 5.1 In $\triangle MNP$, $\hat{N} = 90^\circ$ and $\sin M = \frac{15}{17}$.
Determine, **without using a calculator**:
- 5.1.1 $\tan M$ (3)
- 5.1.2 The length of NP if $MP = 51$ (2)
- 5.2 Simplify to a single term: $\cos(x - 360^\circ) \cdot \sin(90^\circ + x) + \cos^2(-x) - 1$ (4)
- 5.3 Consider: $\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ)$
- 5.3.1 Write as a single trigonometric term in its simplest form. (2)
- 5.3.2 Determine the general solution of the following equation:
 $\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ) = \cos(2x - 20^\circ)$ (7)
[18]

ACTIVITY 18

- 5.1 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180^\circ; 270^\circ]$, calculate, **without using a calculator**, the values in simplest form of:
- 5.1.1 $\sin 2\theta$ (4)
- 5.1.2 $\sin^2 \theta$ (3)
- 5.2 Simplify $\sin(180^\circ - x) \cdot \cos(-x) + \cos(90^\circ + x) \cdot \cos(x - 180^\circ)$ to a single trigonometric ratio. (6)
- 5.3 Determine the value of $\sin 3x \cdot \cos y + \cos 3x \cdot \sin y$ if $3x + y = 270^\circ$. (2)
- 5.4 Given: $2\cos x = 3\tan x$
- 5.4.1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$. (3)
- 5.4.2 Determine the general solution of x if $2\cos x = 3\tan x$. (5)
- 5.4.3 Hence, determine two values of y , $144^\circ \leq y \leq 216^\circ$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

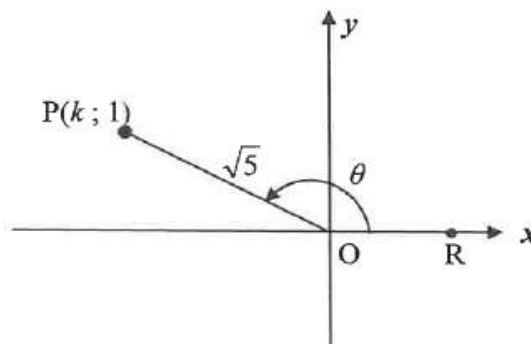


ACTIVITY 19

- 6.1 If $\cos 26^\circ = \frac{1}{p}$. Determine the following in terms of p .
- 6.1.1 $\sin 26^\circ$ (3)
- 6.1.2 $\cos 52^\circ$ (3)
- 6.1.3 $\tan^2 64^\circ \times (p + 1)$ (4)
- 6.2 Simplify: $\frac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ}$ (5)
- 6.3 Determine the value of p , correct to two decimal places if $\theta = 82^\circ$ and $2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta)$ (3)
- 6.4 Prove the identity: $4 \sin \theta \cdot \cos^3 \theta - 4 \cos \theta \cdot \sin^3 \theta = \sin 4\theta$ (6)
- [24]

ACTIVITY 20

- 5.1 In the diagram, $P(k; 1)$ is a point in the 2nd quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x -axis and obtuse $\hat{R\hat{O}P} = \theta$.



- 5.1.1 Calculate the value of k . (2)
- 5.1.2 Without using a calculator, calculate the value of:
- (a) $\tan \theta$ (1)
- (b) $\cos(180^\circ + \theta)$ (2)
- (c) $\sin(\theta + 60^\circ)$ in the form $\frac{a+b}{\sqrt{20}}$ (5)

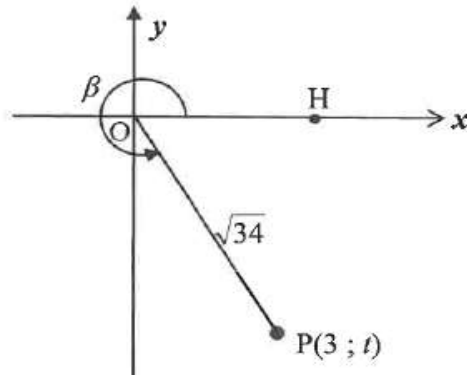
ACTIVITY 21

5.1 Given: $\frac{\sin(A - 360^\circ) \cdot \cos(90^\circ + A)}{\cos(90^\circ - A) \cdot \tan(-A)}$

Simplify the expression to a single trigonometric ratio.

(6)

5.2 In the diagram, $P(3 ; t)$ is a point in the Cartesian plane. $OP = \sqrt{34}$ and $\widehat{HOP} = \beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan \beta$ (1)

5.2.3 $\cos 2\beta$ (4)

5.3 Prove:

5.3.1 $\sin(A + B) - \sin(A - B) = 2 \cos A \cdot \sin B$ (2)

5.3.2 Without using a calculator, that $\sin 77^\circ - \sin 43^\circ = \sin 17^\circ$ (4)
[19]



ACTIVITY 22

- 5.1 Without using a calculator, simplify the following expression to a single trigonometry ratio:

$$\frac{1 - \sin(-\theta)\cos(90^\circ + \theta)}{\cos(\theta - 360^\circ)} \quad (5)$$

- 5.2 Given that $\cos 20^\circ = p$

Without using a calculator, write EACH of the following in terms p :

5.2.1 $\cos 200^\circ$ (2)

5.2.2 $\sin(-70^\circ)$ (2)

5.2.3 $\sin 10^\circ$ (3)

- 5.3 Determine, **without using a calculator**, the value of:

$$\cos(A + 55^\circ)\cos(A + 10^\circ) + \sin(A + 55^\circ)\sin(A + 10^\circ) \quad (3)$$

- 5.4 Consider: $\frac{\cos 2x + \sin 2x - \cos^2 x}{\sin x - 2\cos x} = -\sin x$

5.4.1 Prove the above identity. (3)

5.4.2 Determine the value of $\frac{\cos 2x + \sin 2x - \cos^2 x}{-3\sin^2 x + 6\sin x \cos x}$ (3)

- 5.5 Given: $3 \tan 4x = -2 \cos 4x$

5.5.1 Without using a calculator, show that $\sin 4x = -0,5$ is the only solution to the above equation. (4)

5.5.2 Hence, determine the general solution of x in the equation $3 \tan 4x = -2 \cos 4x$ (3)
[28]





ACTIVITY 23

- 5.1 If $5 \cos \theta - 3 = 0$; $180^\circ < \theta < 360^\circ$ and $17 \sin \alpha = 8$; $90^\circ < \alpha < 270^\circ$, determine, without the use of a calculator, the value of $\tan \alpha + \tan \theta$. (6)
- 5.2 If $\cos 42^\circ = p$, determine each of the following in terms of p :
- 5.2.1 $\sin 48^\circ$ (2)
- 5.2.2 $\sin(-2202^\circ)$ (2)
- 5.2.3 $\cos 84^\circ$ (2)
- 5.3 Determine the value of the expression without the use of a calculator:
- $$\frac{\tan 300^\circ + \cos(90^\circ + x)}{\sin(180^\circ - x) + 2 \cos(-30^\circ)} \quad (6)$$
- 5.4 Prove the following identity: $\frac{1 - \sin 2x}{\cos 2x} = \frac{\cos x - \sin x}{\cos x + \sin x}$ (5)
- 5.5 Determine the general solution of $\cos x - \sin x = \sqrt{2}$. (5)
- [28]

ACTIVITY 24

- 5.1 If $\sin 54^\circ = p$, express each of the following in terms of p , **without the use of a calculator**.
- 5.1.1 $\sin 594^\circ$ (2)
- 5.1.2 $\cos 36^\circ$ (2)
- 5.1.3 $\cos 18^\circ$ (4)
- 5.2 Simplify the following **without the use of a calculator**.
- $$\frac{\cos 140^\circ - \sin(90 - \theta)}{\sin 410^\circ + \cos(-\theta)} \quad (6)$$





MATHEMATICS GRADE 12

5.4 Determine the general solution of: $\cos^2 x - \sin^2 x = \frac{1}{2}$ (4)

5.5 Given the identity:

$$\frac{\sin 2\theta \cdot \tan \theta}{\cos 2\theta + 1} = \tan^2 \theta$$

5.5.1 Prove the identity (4)

5.5.2 Determine the values of θ for which the identity is undefined if $0^\circ \leq \theta \leq 180^\circ$. (4)
[30]

ACTIVITY 25

5.1 If θ is a reflex angle, and $\tan \theta = -\frac{3}{4}$, determine without the use of a calculator and with the aid of a sketch, the value of:

5.1.1 $\sin \theta$ (2)

5.1.2 $\cos 2\theta$ (3)

5.1.3 $\cos(\theta + 30^\circ)$ (3)

5.2 If $x = 4 \sin \alpha$ and $y = 4 \cos \alpha$, calculate the value of $x^2 + y^2$. (2)

5.3 Simplify the expression to a single trigonometric ratio:
 $\sin(90^\circ - x) \cdot \cos(-x) - \sin(x - 180^\circ) \cdot \sin(90^\circ + x)$ (6)

5.4 Given the following the identity: $\frac{\sin 7x + \sin x}{2 \cos 3x} = \sin 4x$

5.4.1 Prove the identity. (4)

5.4.2 For which values is the identity above, undefined. Determine the general solution of x for which the identity is undefined. (3)

5.5 Calculate the general solution of x if $2 \sin(3x + 20^\circ) = 2 \cos x$ (6)

[29]

