



REDHILL HIGH SCHOOL

GRADE 12

MATHEMATICS Preliminary examination Paper 1

5 September 2022

Examiner: V van Rooy

Moderator: A Gomes

Time: 3 hours

Marks: 150

NAME: _____

Teacher: AG / JO / VvR

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and a separate FORMULAE SHEET. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper.
4. You may use an approved calculator, unless otherwise stated.
6. All necessary calculations must be clearly shown. Answers only, will not necessarily be awarded full marks.
7. It is in your own interest to write legibly and to present your work neatly.
8. Unless otherwise stated in the question, all numerical answers must be given correct to TWO decimal places.

SECTION A							
Q1	Q2	Q3	Q4	Q5	TOTAL /150		
17	15	17	13	13			
SECTION B							
Q6	Q7	Q8	Q9	Q10	Q11	Q12	
15	16	15	9	6	6	8	
SUBTOTAL SECTION A:			/75	SUBTOTAL SECTION B:			/75

SECTION A**[75 marks]****Question 1****[17 marks]**

a) Solve for x:

$$(1) \quad \sqrt{x^2 - 5} = 2\sqrt{5} \quad (3)$$

$$(2) \quad \frac{5^{2x} - 1}{5^x + 1} = 4 \quad (3)$$

$$(3) \quad (x - 1)^2 > 9 \quad (4)$$

b) Consider the equation: $x^2 + 5xy + 6y^2 = 0$

(1) Calculate the values of the ratio $\frac{x}{y}$ (3)

(2) Hence, calculate the values of x and y if $x + y = 8$ (4)

Question 2

[15 marks]

- a) Sarah's investment earns interest at 11% p.a. compounded semi-annually. Mary's investment earns an effective interest rate at 11,42% p.a. Whose investment, Sarah's or Mary's, earns a higher rate of interest per annum. (3)
- b) Buhle decided to start saving before retirement. She makes payments of R10 000 monthly into an account yielding 7,72% p.a. compounded monthly, starting on 1 November 2016 with a final payment on 1 April 2026.
- (1) Calculate how much will be in the savings account immediately after the last deposit is made. (4)

- (2) At the end of the investment period Buhle re-invested the full amount for her to be able to draw a monthly pension from the fund. She re-invested the money at an interest rate of 10% p.a. compounded monthly. If she draws an amount of R30 000 per month from the investment, for how many full months will she be able to receive R30 000. (4)

- (3) After withdrawing R30 000 for 20 months Buhle requires R 1 500 000. Determine whether she can access this amount of money from the annuity. (4)

Question 3**[17 marks]**

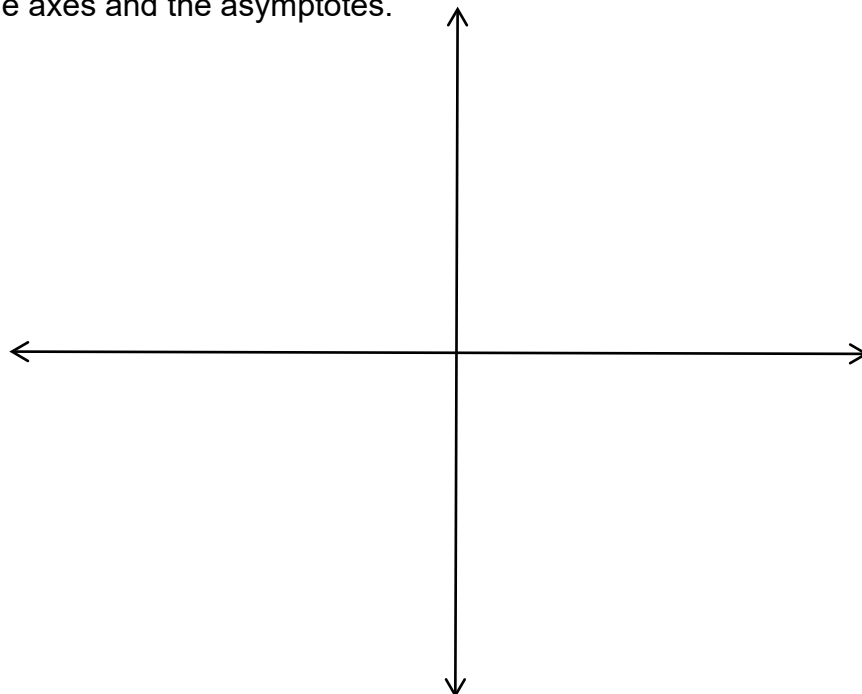
- a) The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function

$$f(x) = \frac{-2}{x+p} + q$$

- (1) Show that $p = 4$ and $q = -3$ (3)

- (2) Calculate the x-intercept of $f(x)$ (2)

- (3) Sketch the graph of $f(x)$ on the axes below. Clearly label ALL intercepts with the axes and the asymptotes. (4)



b) **Given:** $g(x) = 3^x$

(1) Write down the equation of $g^{-1}(x)$ in the form $y = \dots$. (2)

(2) Point $P(6; 11)$ lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g . Write down the coordinates of the image of P on $g(x)$. (2)

c) $B(k; 2)$ is a point on $h(x) = \log_4 x$.

(1) Calculate the value of k . (2)

(2) Determine the values of x for which $-1 \leq h(x) \leq 2$ (2)

Question 4**[13 marks]**

a) Determine $f'(x)$ from first principles if $f(x) = -2x^2 + 6x$ (4)

b) Determine:

(1) $\frac{dy}{dx}$ if $y = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

$$(2) \quad f'(x) \text{ if } f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4} \quad (3)$$

- c) The tangent to the curve of $y = 2x^2 - 3x - 5$ is drawn at the point $(2; -3)$.
A straight line, parallel to the tangent, passing through the y-intercept of the curve is then drawn. Determine the equation of the straight line. (4)

Question 5

[13 marks]

Consider the quadratic sequence: $72 ; 100 ; 120 ; 132 ; \dots$

- a) Determine T_n , the n^{th} term of the quadratic sequence. (4)
- b) A term in the quadratic sequence is equal to the twelfth term of the sequence of the first differences. Determine the position of this term in the quadratic sequence. (5)

c) Determine the maximum value of the quadratic sequence. (3)

d) Hence, determine the maximum value of the following quadratic sequence:
-23 ; 5 ; 25 ; 37 ; (1)

SECTION B

Question 6

[15 marks]

- a) A bag contains 7 yellow balls, 3 red balls and 2 blue balls. A ball is chosen at random from the bag and not replaced. A second ball is then chosen. Determine the probability that of the two balls chosen, one is red and the other is blue.

(4)

- b) Learners at a hostel may choose a meal and a drink for lunch. Their selections on a certain day were recorded and shown in the partially completed table below.

	Sandwich (S)	Pasta (P)	TOTAL
Bottled Water (W)			
Fruit Juice (F)	a	30	b
TOTAL	200		250

The probability of a learner choosing fruit juice and a sandwich on that day is 0,48.

- (1) Calculate the number of learners who chose fruit juice and a sandwich for lunch on that day.

(1)

- (2) Is the choice of fruit juice independent of the choice of a sandwich for lunch on that day? Show ALL calculations to motivate your answer. (4)

- c) Two learners from each grade at a high school (Grades 8, 9, 10, 11 and 12) are elected to form a sports committee.

- (1) In how many different ways can the chairperson and the deputy chairperson of the sports committee be elected if there is no restrictions on who may be elected? (2)

A photographer wants to take a photograph of the sports committee. In how many different ways can the members be arranged in a straight line if:

- (2) Any member may stand in any position? (1)

- (3) Members from the same grade must stand next to each other and the Grade 12 members must be in the centre. (3)

Question 7**[16 marks]**

- a) In a convergent geometric series, $S_2 = 90$ and $S_\infty = \frac{375}{4}$. Determine the first term and its common ratio. (6)

b) The first three terms of an arithmetic sequence are:

$$2p + 3 ; p + 6 ; p - 2 ; \dots$$

(1) Show that $p = 11$

(2)

(2) Calculate the smallest value of n for which $T_n < -55$

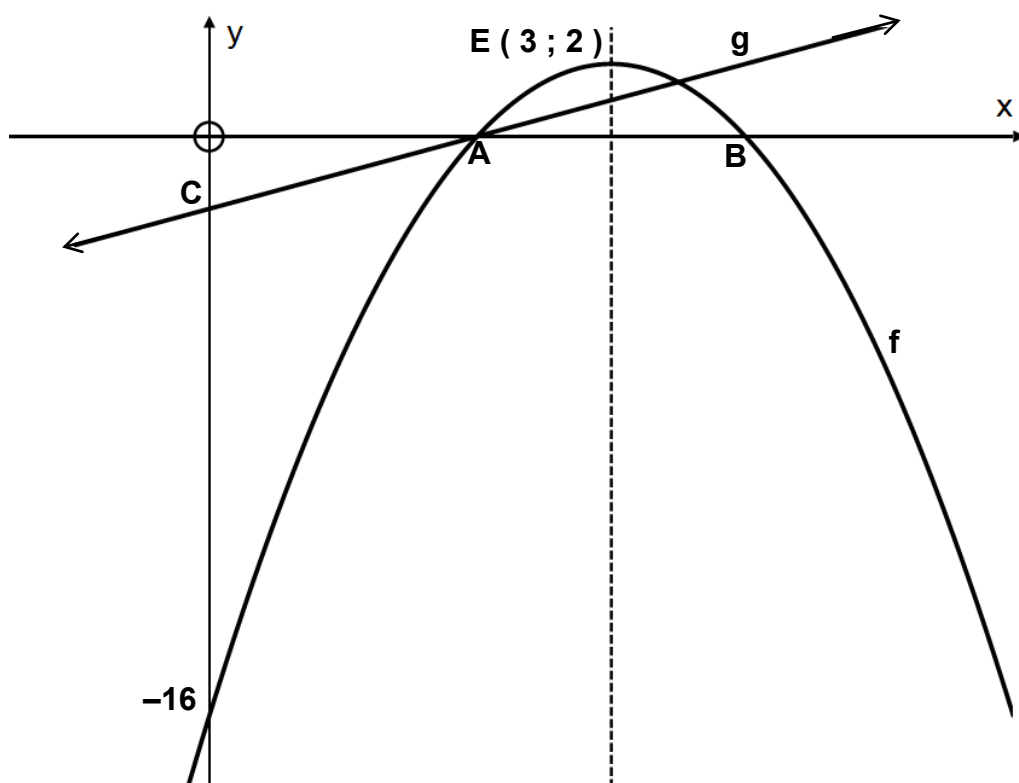
(3)

c) Given that $\sum_{k=1}^6 (x - 3k) = \sum_{k=1}^9 (x - 3k)$, prove that $\sum_{k=1}^{15} (x - 3k) = 0$

(5)

Question 8**[15 marks]**

In the sketch the curve of $f(x)$ is a parabola with turning point $E(3;2)$. The y-intercept is -16 and the x-intercepts are A and B. C is a point on the y-axis symmetrical to A with respect to the line $y = -x$. The graph of $g(x)$ is a line passing through A and C.



Determine:

- a) The equation of the curve of f in the form $f(x) = ax^2 + bx + c$. (4)

b) The coordinates of A, B and C. (3)

c) The equation of g. (2)

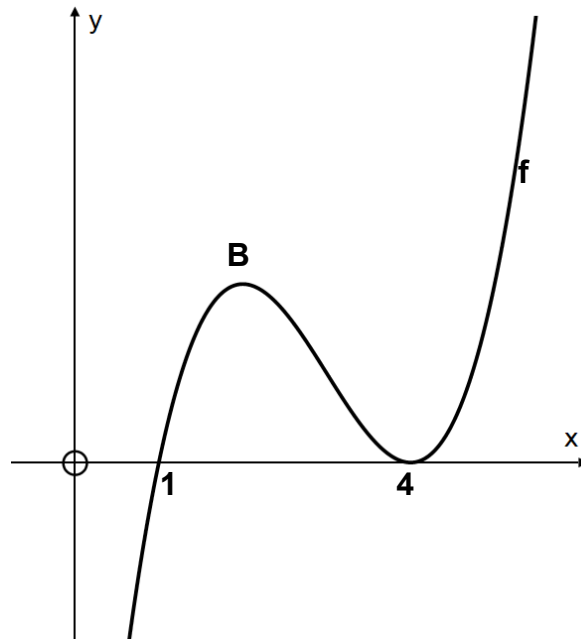
d) The graph of $h(x) = f(x + p) + q$ has a maximum value of 7 at $x = -1$.
Determine the values of p and q. (2)

e) If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will not intersect. (4)

Question 9**[9 marks]**

The graph of the cubic function with equation $f(x) = x^3 + ax^2 + bx + c$ is drawn below.

$f(1) = f(4) = 0$ and f has a local maximum at point B and a local minimum at $x = 4$



- a) Show that $a = -9$, $b = 24$ and $c = -16$ (3)

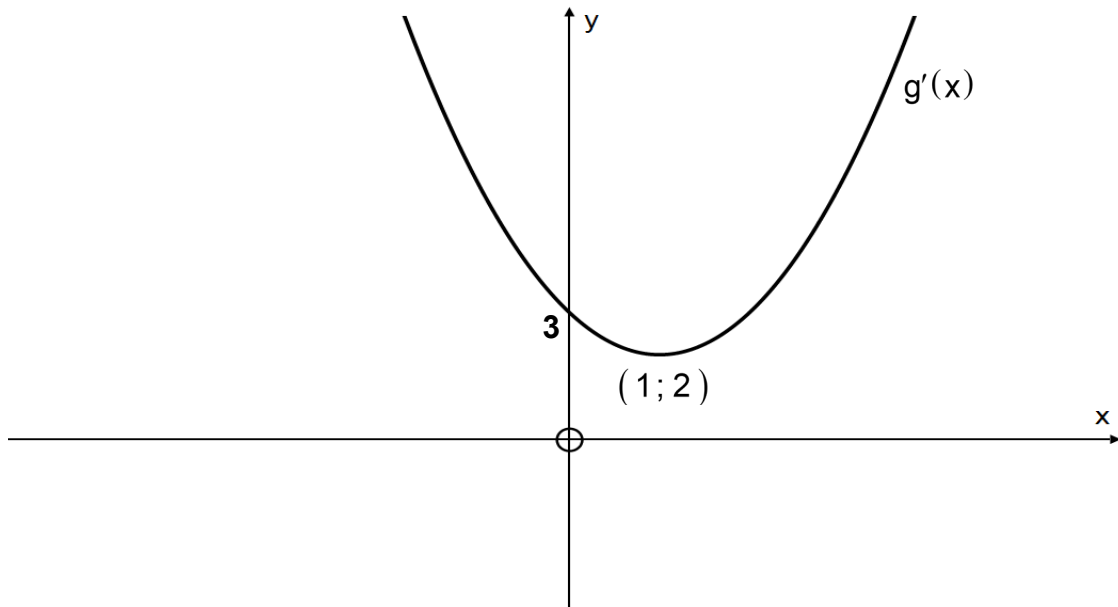
b) Calculate the coordinates of the stationary point B. (3)

c) Determine the value(s) of k for which $f(x) = k$ has only negative roots. (1)

d) Determine the value(s) of x for which $f(x)$ is concave up. (2)

Question 10**[6 marks]**

The sketch shows the graph of $y = g'(x)$. The turning point of $g'(x)$ is $(1; 2)$ and the y-intercept is $(0; 3)$



- a) For which values of x is $g(x)$ increasing? (1)
- b) What is the gradient of the tangent to $g(x)$ at the point where $x = 1$? (1)
- c) Sketch the graph of $g(x)$ on the same set of axes above if it is given that $g(0) = -2$ and $g\left(\frac{1}{2}\right) = 0$. Label all x-intercepts, y-intercept, stationary points and point of inflection, if any. (4)

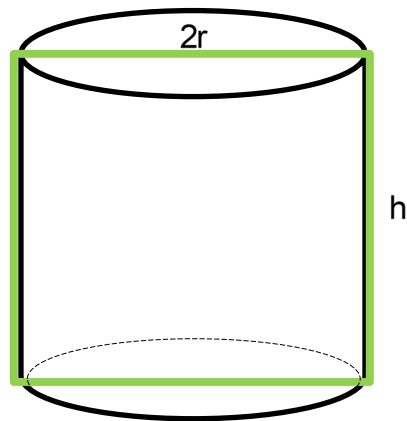
Question 11**[6 marks]**

a) Determine p if $x + p$ is a factor of $f(x) = 2x^3 + p^2x + 81$ (3)

b) Write $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{6}}$ in the form $\frac{\sqrt{2}}{a} + \frac{\sqrt{3}}{b}$, where $a \& b \in \mathbb{Z}$ (3)

Question 12**[8 marks]**

Cannan has a 16 m steel cable to wrap around a cylindrical tank to strengthen (reinforce) the tank as shown in the diagram.



a) Show that the height can be written as $h = 8 - 2r$. (2)

b) Write the volume of the tank in terms of r . (2)

- c) What must the radius and the height of the tank be so that the volume of the tank will be a maximum? (4)

SPACE FOR EXTRA WORKING