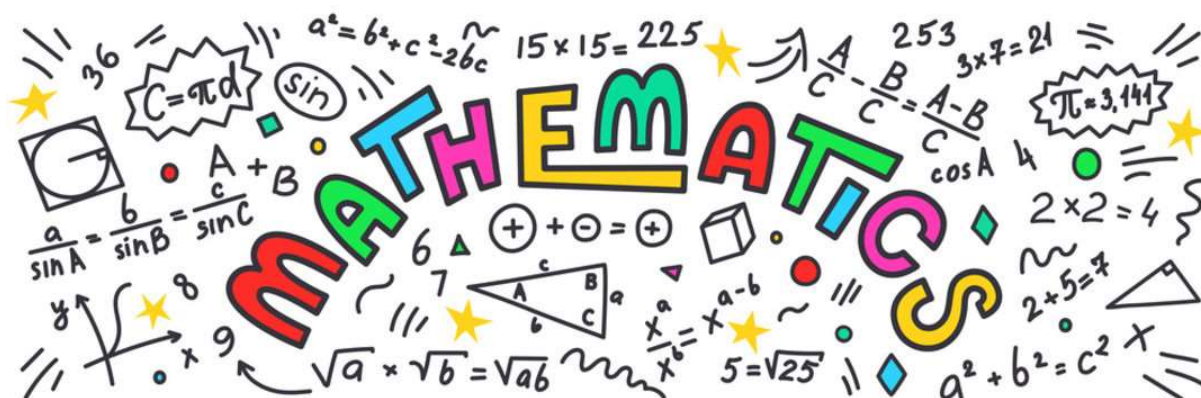




ST PETER'S
COLLEGE



PRELIMINARY EXAMINATION 2024
GRADE 12 FURTHER STUDIES MATHEMATICS
18th SEPTEMBER 2024

EXAMINER: N. LINDSAY

TIME: 2 HOURS

MODERATORS: B. DE BEER, R. OBERMEYER, S. DLAMINI

MARKS: 200

EXTERNAL MODERATOR: L. HEILBRONN

SESSION: ONE

NAME:

MATH TEACHER:

Read the following instructions carefully.

1. This paper consists of 23 pages (including the cover page) and 10 questions. A 2-page Information Sheet is also included. Please check that your paper is complete.
2. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
3. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
4. Diagrams are not necessarily drawn to scale.
5. If necessary, round off answers to **TWO** decimal places, unless otherwise stated.

Question	1	2	3	4	5	6	7	8	9	10	Total
Actual Mark											
Possible Mark	11	13	15	24	14	20	27	27	31	18	200
Marker's Signature											

Question 1**11 Marks**

Prove by mathematical induction that $\sum_{i=1}^n (2 \cdot 3^{i-1}) = 3^n - 1$ for each positive integer n . (11)

(a) By using the laws of logarithms, prove that: $\frac{3 + \ln x^3}{\ln(ex)} = 3$ (4)

(b) A radium isotope decays exponentially, and it reaches half of its original life at 1600 years. If A_0 represents the initial activity of radium in a sample and $A(t)$ represents the activity after t years, then $A(t) = A_0 e^{-kt}$.

(1) Find the value of k accurate to 6 decimal places. (5)

(2) Find the percentage of the original amount of radium that will be remaining after 4000 years. (4)

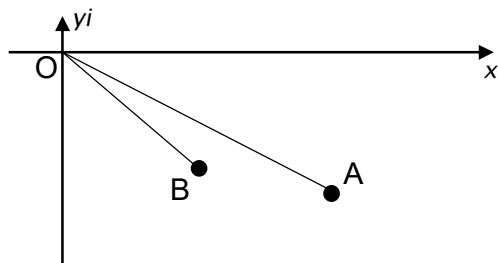


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Question 3

15 Marks

- (a) The complex numbers $z_1 = 2 - 2i$ and $z_2 = 1 - \sqrt{3}i$ are represented by the points A and B respectively on the Argand diagram below. O is the origin.



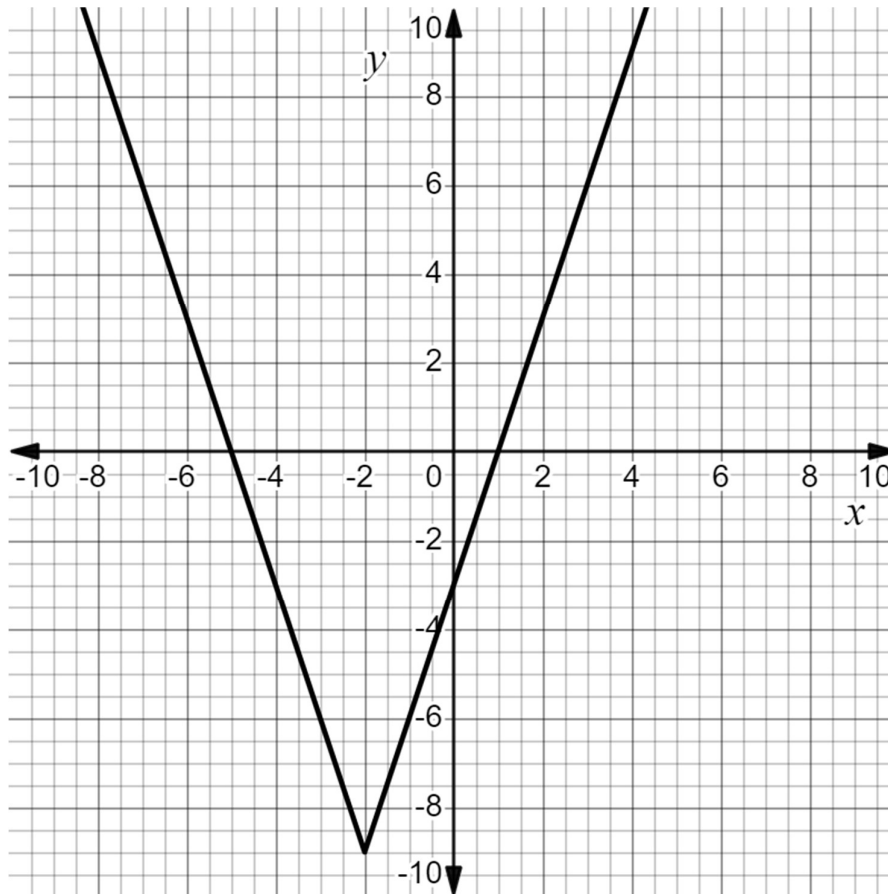
Calculate AB, giving your answer in the form $a\sqrt{b - \sqrt{3}}$, where $a, b \in \mathbb{Z}^+$. (7)

- (b) Given that $\frac{3w+7}{5} = \frac{p-4i}{3-i}$, where p is a real constant.

Express w in the form $a + bi$, where a and b are real constants. Give your answer in its simplest form in terms of p . (8)

(a) Solve for $x \in \mathbb{R}$: $x^2 + |x + 2| = -4x + 2$ (8)

(b) The function $f(x) = 3|x + 2| - 9$ is drawn below.

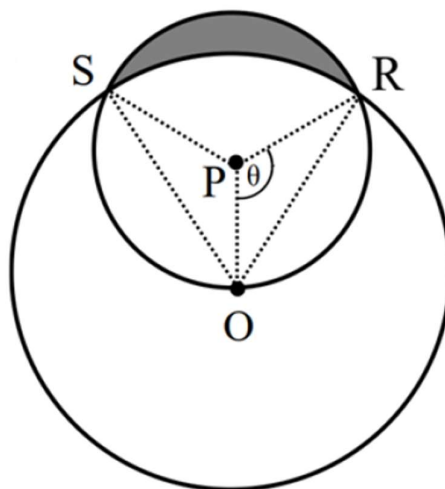


(1) On the same set of axes draw the function $g(x) = -|x - 2| + 3$. (6)

(2) Hence, or otherwise, solve $|x + 2| + \frac{|x - 2|}{3} \leq 4$? (6)

- (3) Write down the coordinates of the vertex point of $h(x)$, the graph of $f(x)$ after it has reflected in the y -axis and shifted 3 units down and 5 units right. (4)

In the diagram below are two circles. The larger circle, with centre O and radius $5\sqrt{3}$ cm, passes through S and R . The smaller circle, with centre P and radius 5 cm, passes through S , R and O .



- (a) Show that $\theta = \frac{2\pi}{3}$ radians. (3)

(b) Determine the arc length SR that lies on the circumference of circle with centre O . (3)

(c) Calculate the area of the shaded region. (8)

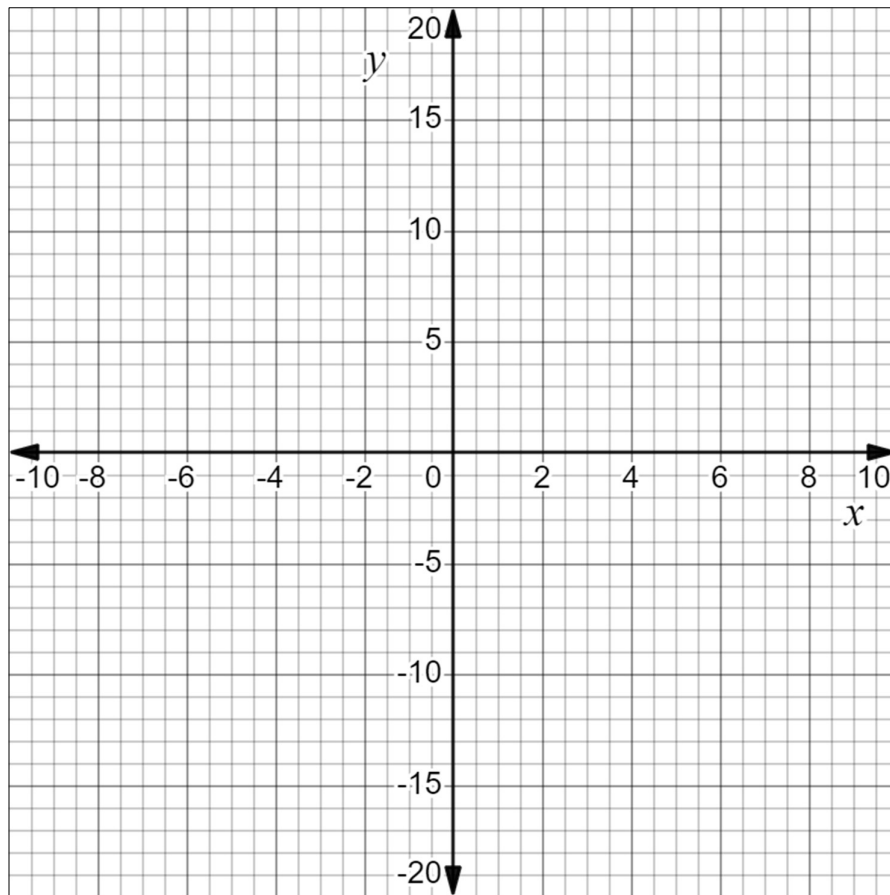
Question 6**20 Marks**

Consider the function $f(x) = \frac{x^3 - 3x^2 + 3x - 1}{x^2 + x - 2}$.

(a) Determine, with classification, the equations of any asymptotes. (6)

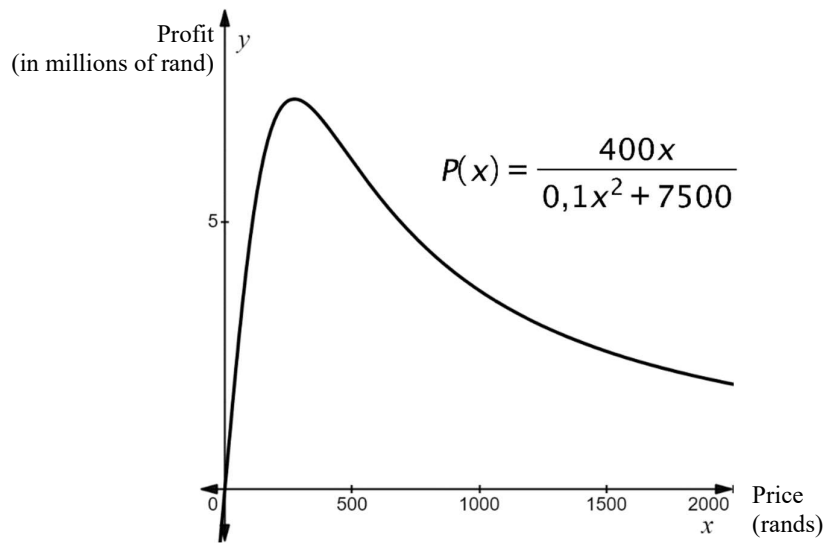
(b) Determine the coordinates of any stationary points. (7)

- (c) Draw the graph of f on the axes provided showing all points of interest and any asymptotes. (7)



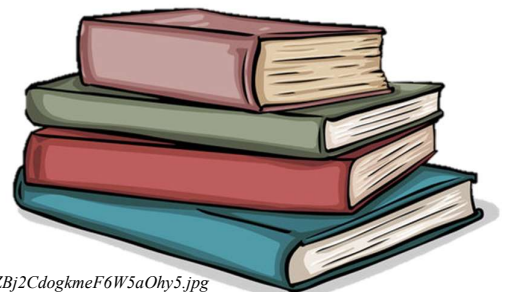
- (a) Determine $f'(x)$ from first principles if $f(x) = \sqrt{3x - 5}$. (8)

- (b) A book publisher finds that their profits, in millions of rand, are related to the price they choose to charge their customers, x , by the following function:

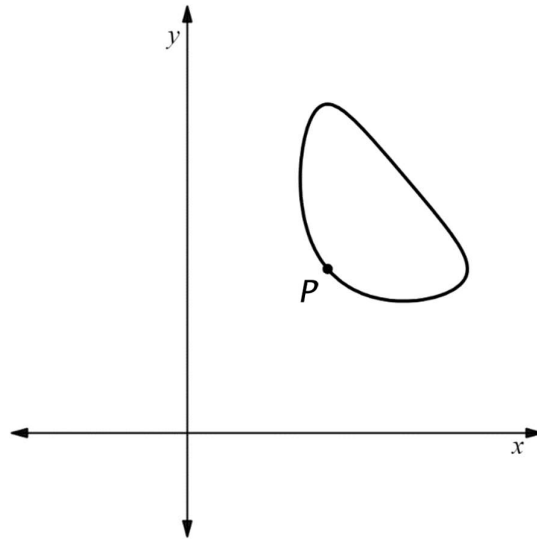


- (1) Determine the price, in rands and cents, that the publisher must charge, to maximise the profits. (8)

- (2) Hence, find this maximum profit (to the nearest rand). (2)



- (c) A portion of the graph of the implicitly defined relationship $x \sin y + y \sin x = \pi$ is shown below. The point $P\left(\frac{\pi}{2}; \frac{\pi}{2}\right)$ lies on the graph.



- (1) Find the slope of the tangent line through P . (7)

- (2) Write the equation for the tangent line through P . (2)

(a) Consider:

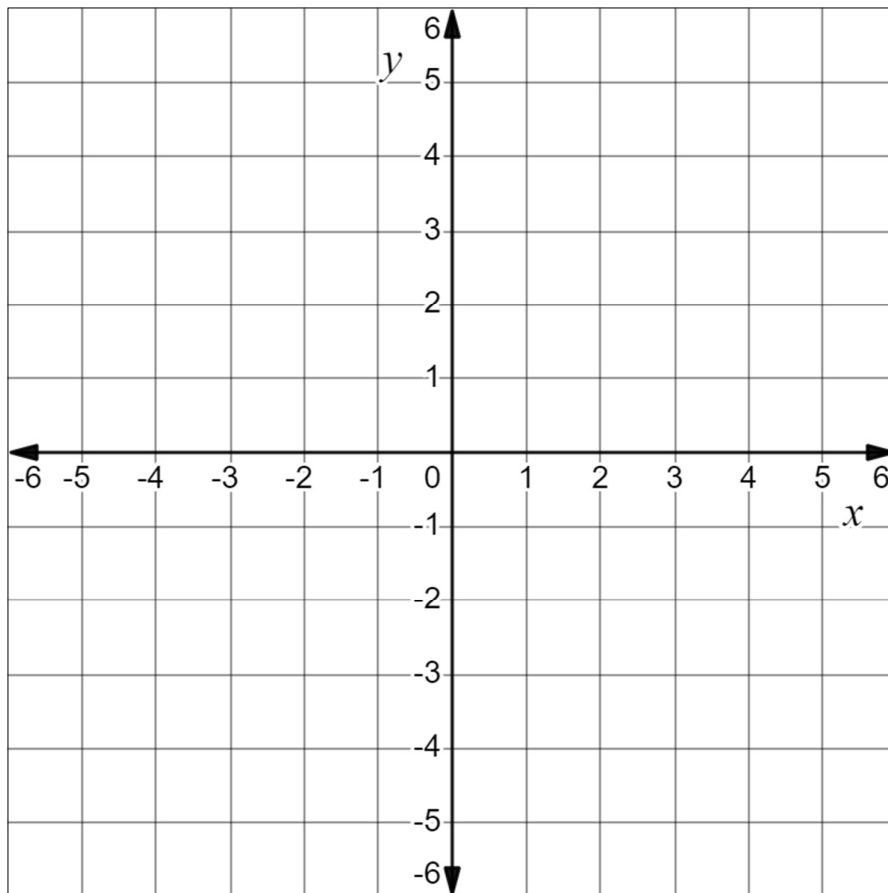
$$f(x) = \begin{cases} m \cdot \ln(x+2) & \text{for } x > -1 \\ e^{x+1} + k & \text{for } x \leq -1 \end{cases}$$

If f is differentiable for all values of x , determine the values of m and k . (8)

(b) On the axes provided draw a function g which satisfies the following:

- g is continuous for all values of x except at $x = -3$ and $x = 1$
- $g(-3) = -3$ and $\lim_{x \rightarrow -3^+} g(x) = -3$ but there is a jump discontinuity at $x = -3$
- $g(1) = 4$ and $\lim_{x \rightarrow 1} g(x)$ exists
- g is also not differentiable at $x = -2$

(8)



(c) Given: $y = \frac{\ln x^2 - 6}{2}$ and $y = x - 5$

- (1) Show that the function that can be used to find the x -value of the point of intersection of the two graphs can be written as $h(x) = x - \ln x - 2$. (3)

- (2) Show that $h(x) = 0$ has a solution in the interval $[3; 4]$. (3)

- (3) Use the Newton-Raphson method to find the x -value of the point of intersection of the two graphs to 5 decimal places. (5)

(a) Determine the following integrals:

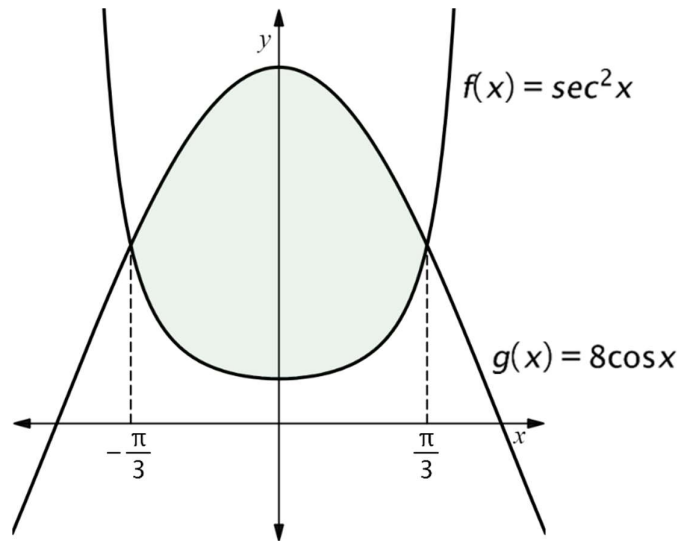
(1) $\int \cos^2 x \, dx$ (7)

(2) $\int x e^{-2x} dx$ (8)

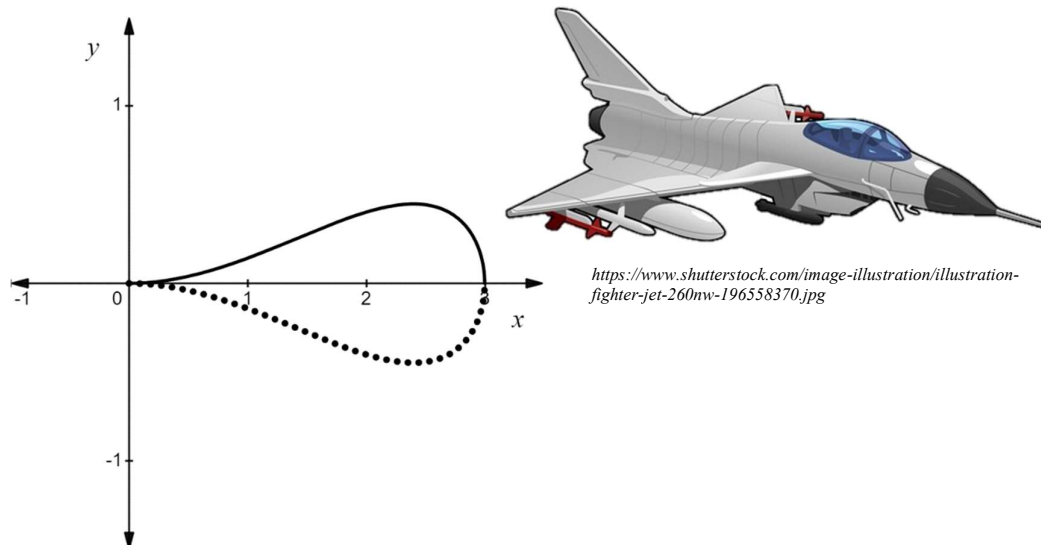
(b) (1) Resolve $\frac{6x+1}{6x^2-7x+2}$ into partial fractions. (5)

(2) Hence, determine $\int \frac{6x+1}{6x^2-7x+2} dx$ (5)

- (c) The sketch below shows the graphs of $f(x) = \sec^2 x$ and $g(x) = 8\cos x$. The graphs intersect at $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$. Calculate the shaded area. (6)



- (a) The fuel tank on the wing of a jet is formed by rotating the region bounded by the curve $y = \frac{1}{10}x^2 \cdot \sqrt{3-x}$ about the x -axis between $x = 0$ and $x = 3$, where the units are measured in metres. Determine the volume of the fuel tank in m^3 . (10)

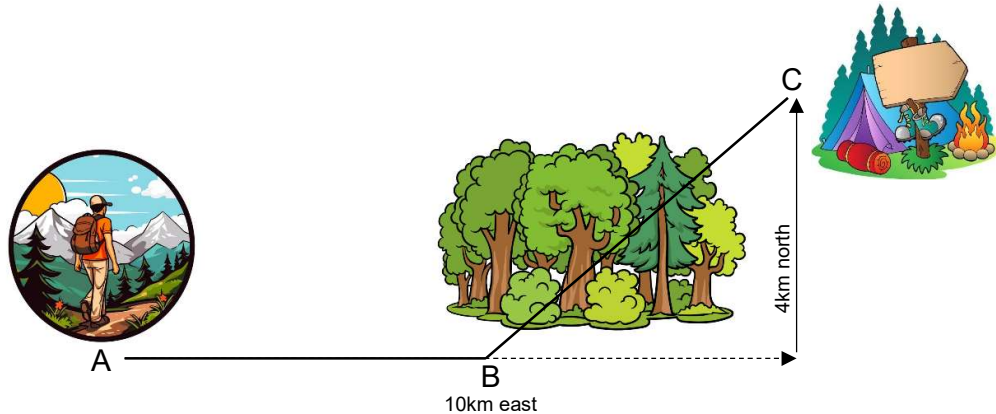


- (b) Ms Theron and the True Adventures students are hiking in Romania. They start at point A and head along a trail towards point B. At some point they will need to cross through the forest to get to their campsite at C.

While on the trail they can walk at a speed of 5km/hr but through the forest they slow down to 3 km/hr.

The campsite is 10km east and 4km north of point A. After how many kilometres on the trail should the group cross the forest to minimise the time taken between points A and C.

(8)



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TOTAL: 200 MARKS

ADDITIONAL WORKING SPACE