

KING DAVID HIGH SCHOOL LINKSFIELD



**GRADE 12
ADVANCED PROGRAMME MATHEMATICS
PAPER 1 – ALGEBRA & CALCULUS
PRELIMINARY EXAMINATIONS
XX AUGUST 2020**

Total: 200 marks

Reading Time: 10 minutes

Writing Time: 2 hours

PLEASE WRITE YOUR NAME BELOW.

NAME: _____

This paper contains **30** pages and an information sheet of **(iv)** pages.
There are 3 pages of extra space (Pages 28-30). Should you use these please label your questions carefully and indicate you have used that you have used the extra space

Check that your paper is complete.

Please read the following instructions carefully:

1. Number all questions exactly as they appear on the question paper.
2. Pay careful attention to time management and mark allocation.
3. Write legibly and not in pencil.
4. Non programmable calculators may be used unless otherwise instructed.
5. All necessary calculations must be clearly shown. You will NOT receive full credit if you write down only the answers and show no working out.
6. **ENSURE THAT YOUR CALCULATOR IS IN RADIAN MODE.**

Q1 [16]	Q2 [18]	Q3 [6]	Q4 [15]	Q5 [17]	Q6 [19]	Q7 [6]	Q8 [25]	Q9 [10]	Q10 [41]	Q11 [17]	Q12 [10]	TOTAL [200]

QUESTION 1 (16 MARKS)

- (a) (i) Decompose $\frac{2x - 23}{2x^2 + 9x - 5}$ into its partial fractions. (6)

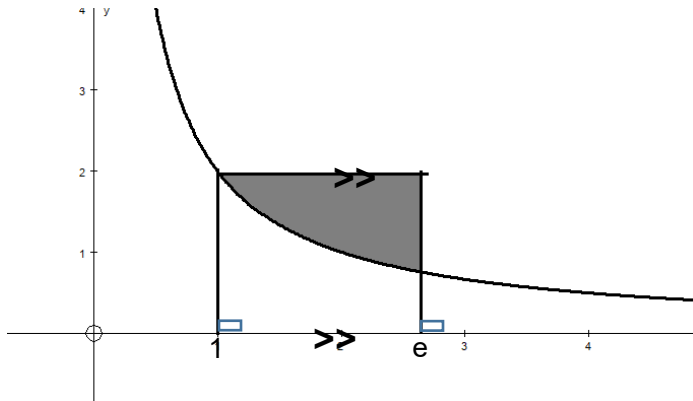
(ii) Hence determine $\int \frac{2x - 23}{2x^2 + 9x - 5} dx$. (4)

(b) In each case part of the graph of $y = \frac{2}{x}$ is shown.

It is also given that $\int_1^e \frac{1}{x} dx = 1$. Determine, in terms of e, the area

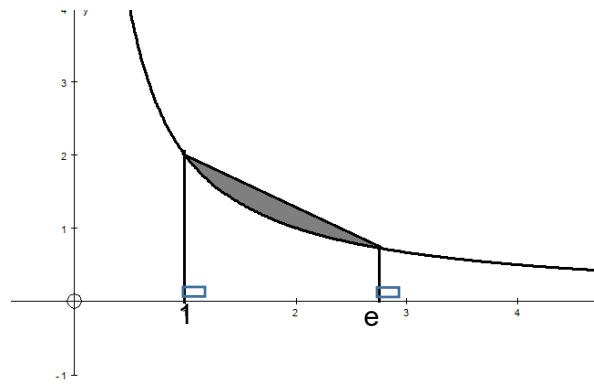
of the shaded region in each diagram.

(i)



(3)

(ii)



(3)

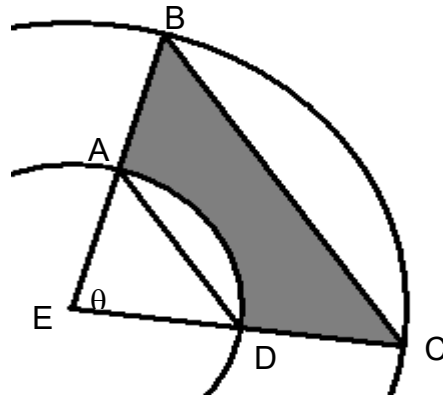
P.T.O.

QUESTION 2 (18 marks)

E is the centre of two concentric circles.

$EA = AB = ED = DC = r$ and $\angle AED = \theta$ where $\theta \in (0; \frac{\pi}{2})$.

See the sketch below.



(a) Determine the perimeter of the shaded region in terms of r and θ

(6)

(b) Determine the area of the shaded region in terms of r and θ (4)

- (c) If $r = 1$, determine the maximum area of the shaded region correct to 2 decimal places. (8)

QUESTION 3 (6 marks)

$g(x)$ is continuous at $x = a$ and is defined as follows:

$$g(x) = \begin{cases} x - 3 & \text{if } x \leq a \\ ax^2 - 2x - 1 & \text{if } x > a \end{cases}$$

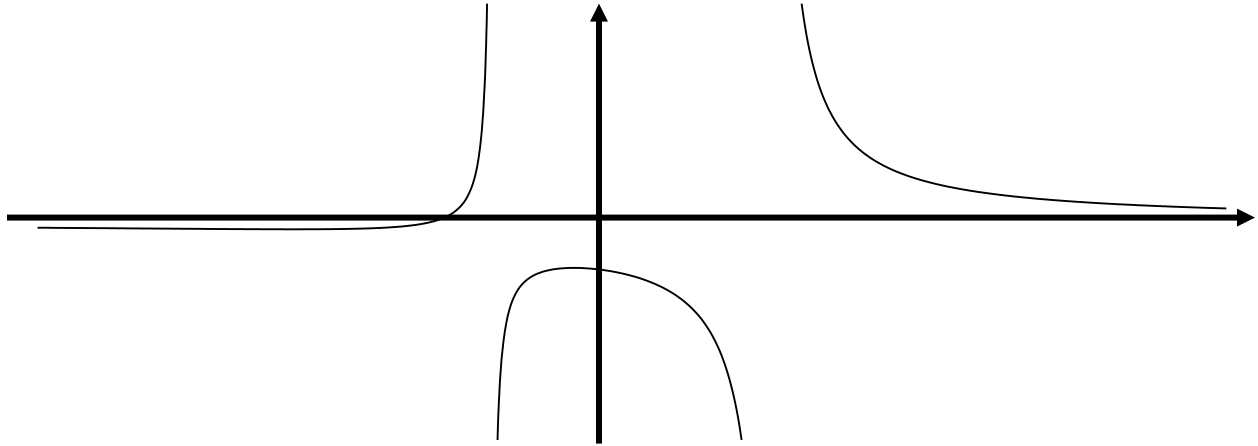
Determine the values of a .

(6)

P.T.O.

QUESTION 4 (15 marks)

Given: $h(x) = \frac{x^3 + 2x^2 + 2x + 4}{x^4 - 4}$. The graph of $y = h(x)$ is sketched below:



(a) Show that, when simplified, $h(x) = \frac{x+2}{x^2-2}$ for $x \neq \pm\sqrt{2}$. (2)

(b) Write down the equation(s) of any asymptotes of $y = h(x)$. (4)

(c) Show that the stationary points occur when $x = -2 \pm \sqrt{2}$. (6)

(d) Determine the x and y intercepts of $y = h(x)$. (3)

QUESTION 5 (17 marks)

(a) Prove that $\log_x e = \frac{1}{\ln x}$. (2)

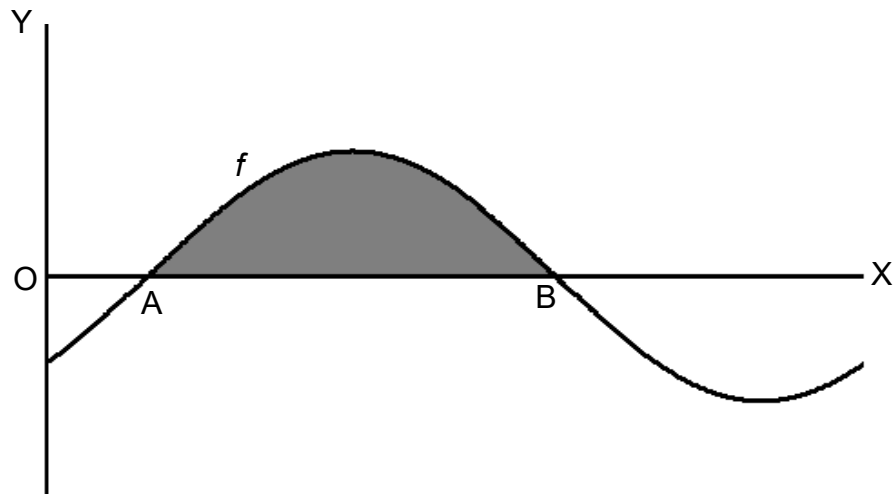
(b) Hence solve for x if $\ln x - 4\log_x e = 3$ (7)

(c) If $f(x) = \ln(x - 1) + 2$ and $g(x) = e^{x-2} + 1$ determine $(g \circ f)(x)$ and state its domain and range. (8)

P.T.O.

QUESTION 6 (19 marks)

A sketch of the graph of $f(x) = \sin x - \cos x$ for $x \in [0; 2\pi]$ is shown below. The area bounded by the graph of f and the x – axis between the x – intercepts marked A and B has been shaded.



- (a) Determine the coordinates of A and B, leaving your answers in terms of π . (4)

(b) Determine the area of the shaded region.

(6)

- (c) The volume generated when the graph of f rotates around the x -axis between $x = 0$ and $x = k$ is equal to $\pi(k - \frac{1}{4})$ units³. Calculate the smallest possible value of k if $k > 0$. (9)

P.T.O.

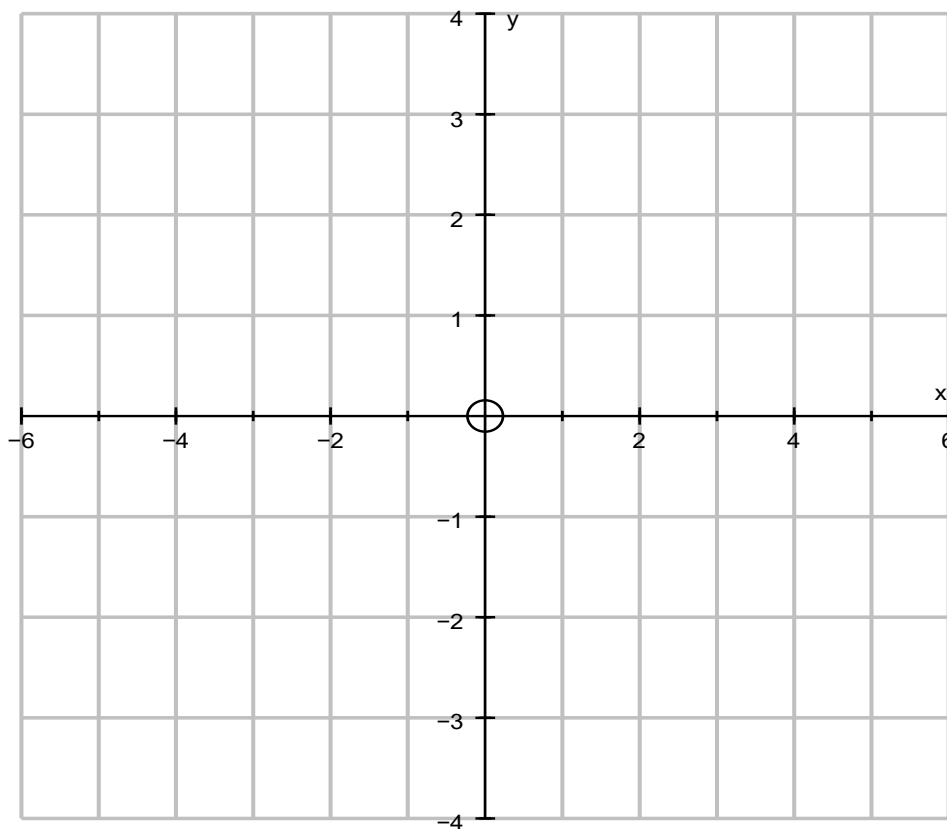
QUESTION 7 (6 marks)

The function $f(x)$ has the following properties:

- $f(-2) = 0$
- $f'(x) > 0$ if $x \in (-\infty; 1)$
- $f'(x) < 0$ if $x \in (1; \infty)$
- $f'(x) = 0$ if $x = 1$
- $f''(x) > 0$ if $x \in (-\infty; -2)$ or $x \in (2; \infty)$
- $f''(x) < 0$ if $x \in (-2; 2)$
- $\lim_{x \rightarrow -\infty} f(x) = -3$
- $\lim_{x \rightarrow \infty} f(x) = 0$

Draw a neat sketch of $y = f(x)$ using the information above.

(6)



P.T.O.

QUESTION 8 (25 marks)

(a) Given : $f(x) = \cos x$ and $g(x) = \frac{2}{e^{2x}}$

Determine $\frac{d}{dx}(f(g(x)))$ (8)

(b) Determine the equation of the tangent to the curve

$2xy + \pi \sin y = 2\pi$ at the point $(1; \frac{\pi}{2})$ (8)

(c) Determine (i) $\frac{d}{dx} \ln(\sec x + \tan x)$ (4)

(ii) $D_x [e^{-x} \sqrt{2x^3}]$ (5)

QUESTION 9 (10 marks)

$$f(x) = 1 + x - 2 \tan x.$$

(a) Show that $f(x)$ has a zero in the interval $[0 ; 1]$. (4)

(b) Use the Newton – Raphson method to calculate this zero correct to 4 decimal places. **All working must be clearly shown.** (6)

QUESTION 10 (41 marks)

(a) (i) Prove that $\tan^2 x + \cos^2 x = \sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}$. (5)

(ii) Hence determine $\int_0^{\frac{\pi}{4}} (\tan x + \cos x)^2 dx$. (8)

(b) Integrate the following:

(i) $\int \cos 5x \sin 3x \, dx$

(5)

(ii) $\int x^2 \sin x^3 \, dx$

(5)

$$(iii) \int (x^2 + 2x) \sqrt{x^3 + 3x^2 - 4} \, dx \quad (6)$$

$$(iv) \int_{\sqrt{2}}^2 \frac{x}{x^2 - 1} \, dx \quad (6)$$

$$(v) \int_1^2 x^3 \ln x \, dx \quad (6)$$

QUESTION 11 (17 marks)

(a) (i) If $\sqrt{x+iy} = a+bi$ show that $x = a^2 - b^2$ and $y = 2ab$. (5)

(ii) Hence find x and y if $\sqrt{x+iy} = 5i - 12$. (4)

- (b) When proving $6^n + 4$ is divisible by 5 using mathematical induction, it is necessary to first prove the statement is true for $n = 1$ and then assume the statement is true for $n = k$.

You are only required to prove that $6^n + 4$ is divisible by 5, if $n \in \mathbb{N}$ and $n = k + 1$. (8)

QUESTION 12 (10 marks)

The area under the curve of $f(x)$ from $x = 1$ to $x = b$ can be calculated by constructing n rectangles of equal width and then using a Riemann sum as follows:

$$\begin{aligned}\text{Area} &= \frac{4}{n} \sum_{i=1}^n \left[\left(1 + \frac{4i}{n}\right)^2 - 4\left(1 + \frac{4i}{n}\right) \right] \\ &= \frac{64}{3} + \frac{32}{n} - \frac{32}{3n^2}\end{aligned}$$

(a) What is the function $f(x)$? (1)

(b) What is the value of b ? (2)

(c) If the region is divided into 8 rectangles, what is the estimated area? (3)

(d) What is the value of $\int_1^b f(x)dx$? (2)

(e) Is your answer to (c) bigger than or less than your answer in (d)?
Briefly explain your answer. (2)

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

