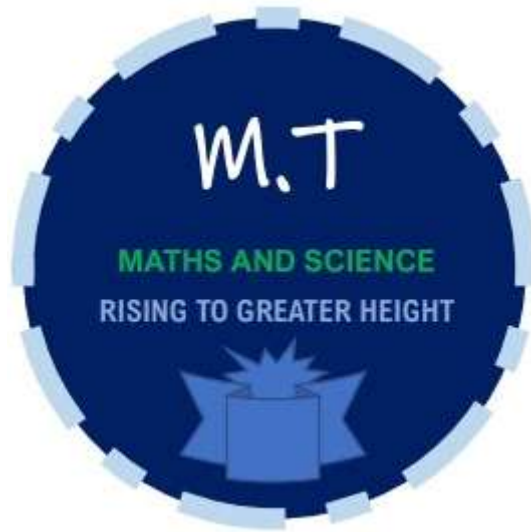




MATHEMATICS
GRADE 12



MATHEMATICS
GRADE 12

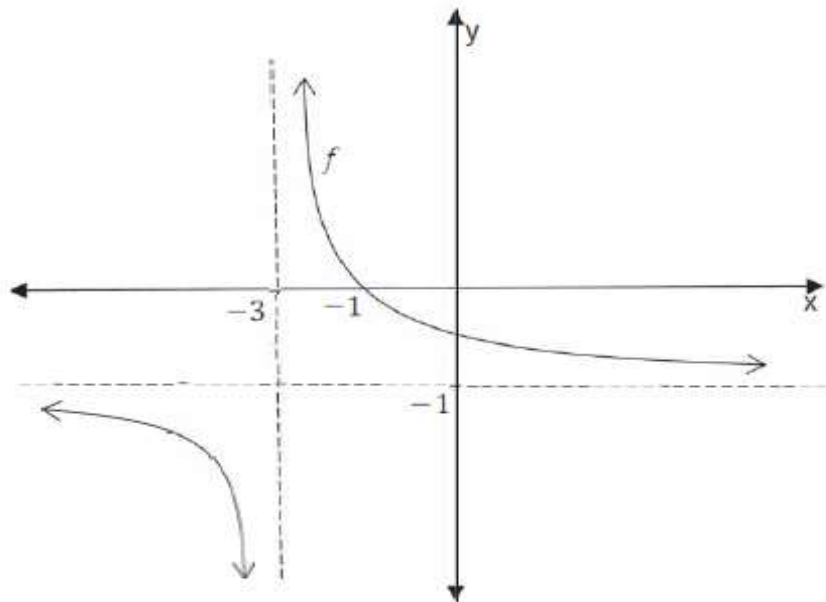
TOPIC:
FUNCTIONS



FUNCTIONS

ACTIVITY 1

The graph of $f(x) = \frac{2}{x+p} + q$ is sketched below:

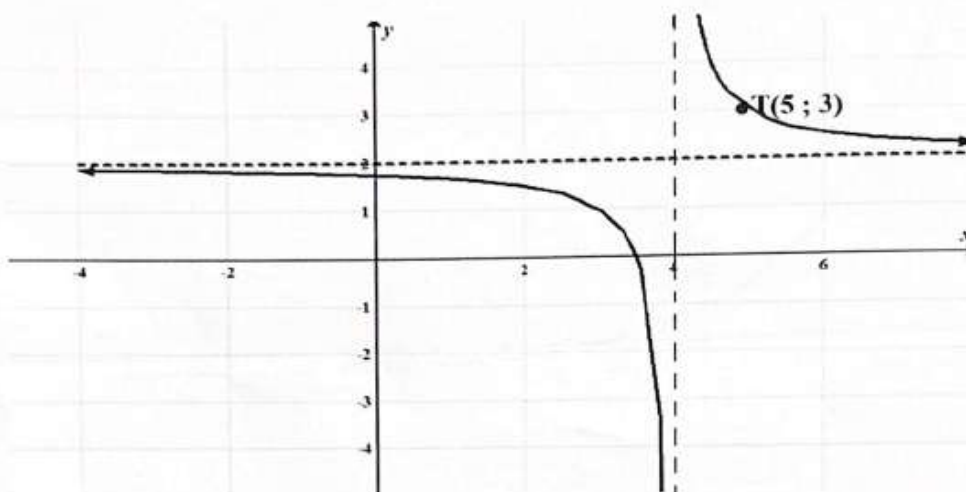


- 5.1 Write down the values of p and q (2)
- 5.2 The straight line $g(x) = -x + k$ is one of the axes of symmetry of the graph of f . Determine the value of k . (2)
- 5.3 If $h(x) = -2[g(x)]$, determine the equation of the inverse of h , h^{-1} , in the form $h^{-1}(x) = mx + c$. (3)
- 5.4 Draw a neat sketch of the graphs of h and h^{-1} on the same set of axes. Clearly show all intercepts with axes, point of intersection and the axis of symmetry. (4)

[11]

ACTIVITY 2

The diagram below represents the graph of $f(x) = \frac{a}{x-p} + q$. $T(5;3)$ is a point on f .



- 5.1 Determine the values of a , p and q . (4)
- 5.2 If the graph of f is reflected across the line having equation $y = -x + c$ and the new graph coincides with the graph of $y = f(x)$, determine the value of c . (2)
- [6]

ACTIVITY 3

Given: $f(x) = \frac{2}{x-5} + 3$

- 4.1 Write down the equations of the asymptotes of f . (2)
- 4.2 Write down the range of f . (1)
- 4.3 Determine the coordinates of the x - and y -intercepts of f . (3)
- 4.4 Sketch the graph of f , clearly showing all asymptotes and intercepts with the axes. (4)
- 4.5 Describe the transformation that the graph of f has to undergo to form the graph of h , where $h(x) = -\frac{2}{x-5} - 5$. (3)
- [13]



ACTIVITY 4

Given: $f(x) = \frac{2}{x-1} - 2$

- 4.1 Write down the equations of the asymptotes of f . (2)
- 4.2 Draw the graph of f . Clearly label ALL the intercepts with the axes and the asymptotes on your graph. (4)
- 4.3 Determine the equation of the line of symmetry of f for $m < 0$. (2)
- 4.4 For which values of x will $f(x) \leq -4$? (2)
- [10]

ACTIVITY 5

Consider the graphs of $g(x) = \frac{6}{x+3} - \frac{3}{2}$ and $h(x) = \frac{6}{x-3} + 2$.

- 4.1 Write down the domain of g . (1)
- 4.2 Write down the range of h . (1)
- 4.3 If the graph of g is shifted so that it coincides with the graph of h ,
- 4.3.1 how many units must the graph be shifted horizontally? (1)
- 4.3.2 how many units must the graph be shifted vertically? (1)
- 4.4 Write down the equations of the asymptotes of g . (2)
- 4.5 Calculate the x -intercept of g . (1)
- 4.6 Sketch the graph of g in your ANSWER BOOK.
Show clearly all asymptotes and intercepts with the axes. (3)
- 4.7 Determine the value of k if $h(x) = -x + k$ is an axis of symmetry of g . (2)
- 4.8 For which value(s) of x will $\frac{6}{x+3} - \frac{3}{2} > -x + k$? (1)
- 4.9 The graph of g is reflected in the x -axis.
Write down the new equation in the form $y = \dots$ (1)
- [14]





ACTIVITY 6

4.1 Given the function $p(x) = \left(\frac{1}{3}\right)^x$.

4.1.1 Is p an increasing or decreasing function? (1)

4.1.2 Determine p^{-1} , the inverse of p , in the form $y = \dots$ (2)

4.1.3 Write down the domain of p^{-1} . (1)

4.1.4 Write down the equation of the asymptote of $p(x) - 5$. (1)

4.2 Given: $f(x) = \frac{4}{x-1} + 2$

4.2.1 Write down the equations of the asymptotes of f . (2)

4.2.2 Calculate the x -intercept of f . (2)

4.2.3 Sketch the graph of f , label all asymptotes and indicate the intercepts with the axes. (4)

4.2.4 Use your graph to determine the values of x for which $\frac{4}{x-1} \geq -2$. (2)

4.2.5 Determine the equation of the axis of symmetry of $f(x-2)$, that has a negative gradient. (3)
[18]





ACTIVITY 7

Given the functions defined by: $f(x) = -\frac{2}{x} - 1$ and $g(x) = k^x$.
The point (1; 3) lies on g .

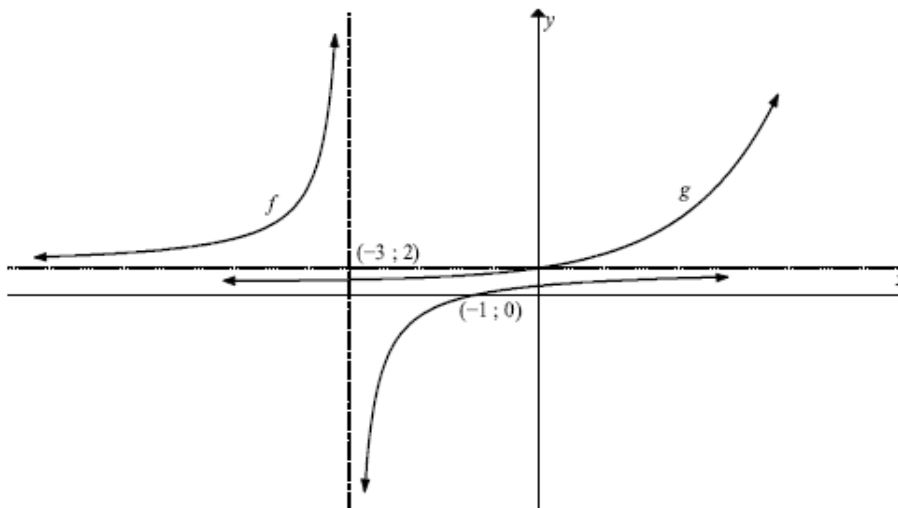
- 4.1 Determine the value of k . (2)
- 4.2 Write down the equations of the asymptotes of f . (2)
- 4.3 Write down the equation of g^{-1} , the inverse of g . (2)
- 4.4 Determine the x -intercept(s) of f . (2)
- 4.5 Draw neat sketches of f and g^{-1} on the same system of the axes, clearly indicating all asymptotes and intercepts with the axes. (5)
- 4.6 For which values of x will the axis of symmetry of f , which has a negative gradient, intersect with the graph of f ? (4)
- 4.7 Consider the graph of h , where $h(x) = g^{-1}(x+2)$. Determine the value(s) of x for which $g^{-1}(x+2) \leq 1$. (2)

[19]



ACTIVITY 8

The sketch below represents the graphs of $f(x) = \frac{a}{x+p} + q$ and $g(x) = 2^x + 1$. The point $A(-3; 2)$ is where the asymptotes of f intersect. The graph of f intersects the x -axis at $(-1; 0)$.

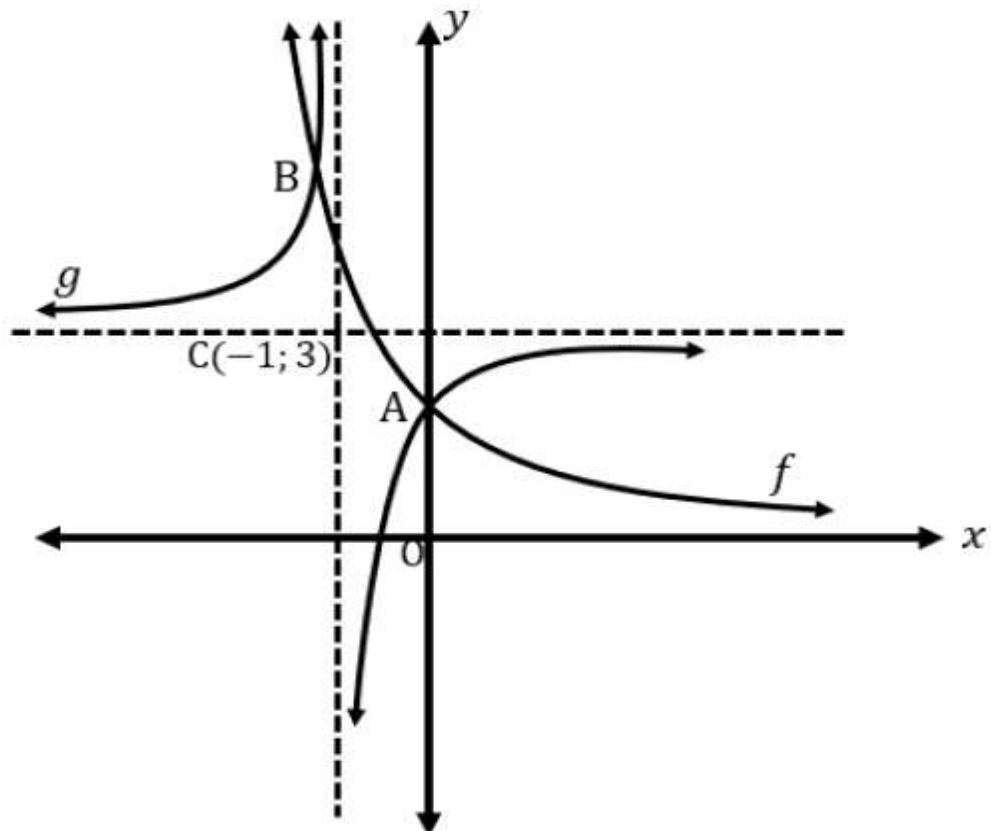


- 3.1 For which value(s) of x is $f(x) = g(x)$. (1)
 - 3.2 Write down the equation of the asymptotes of f . (2)
 - 3.3 Hence determine the equation of g . (4)
 - 3.4 Write down the range of f . (2)
 - 3.5 Determine the equation of k if $k(x)$ is the reflection of g along the x -axis followed by a translation 4 units down. (2)
 - 3.6 Determine the equation of $p(x)$, the axis of symmetry of f , if $p'(x) < 0$. (3)
 - 3.7 For which values of x is $x \cdot g'(x) \geq 0$? (2)
- [16]**

ACTIVITY 9

In the diagram below, the graphs of $f(x) = \left(\frac{1}{5}\right)^x$ and $g(x) = \frac{a}{x+p} + q$ are sketched.

The two graphs intersect at points A and B as shown on the sketch. The point A is a point where both graphs cross the y-axis. The asymptotes of graph g meet at point $C(-1; 3)$.

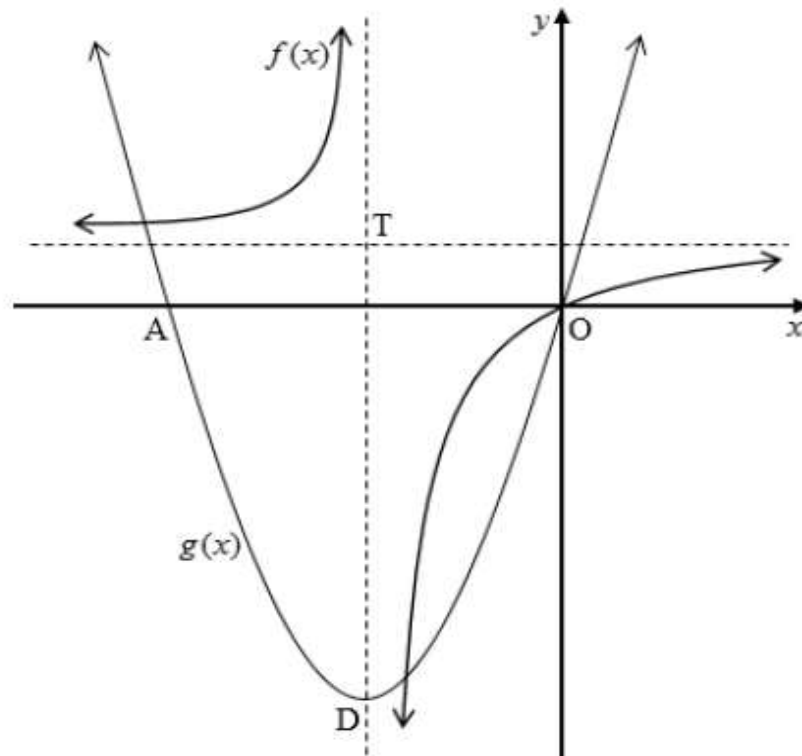


- 4.1 Determine the coordinates of A. (1)
- 4.2 Determine the equation of g by finding the values of a ; p and q . (4)
- 4.3 Determine the $f^{-1}(x)$ and write it in the form $y = \dots$ (3)
- 4.4 Sketch the graph of $f^{-1}(x)$ and clearly show any two points on the graph. (3)
- 4.5 Determine the values of x for which $f^{-1}(x) \geq -2$ (2)
- 4.6 Graph $h(x)$ is produced by shifting graph g 1 unit to the right and 2 units vertically downwards. Write down the equation of graph h . (2)

ACTIVITY 10

The diagram below shows the graphs of $f(x) = \frac{2}{x+2} + 1$ and $g(x) = a(x+2)^2 - 8$.

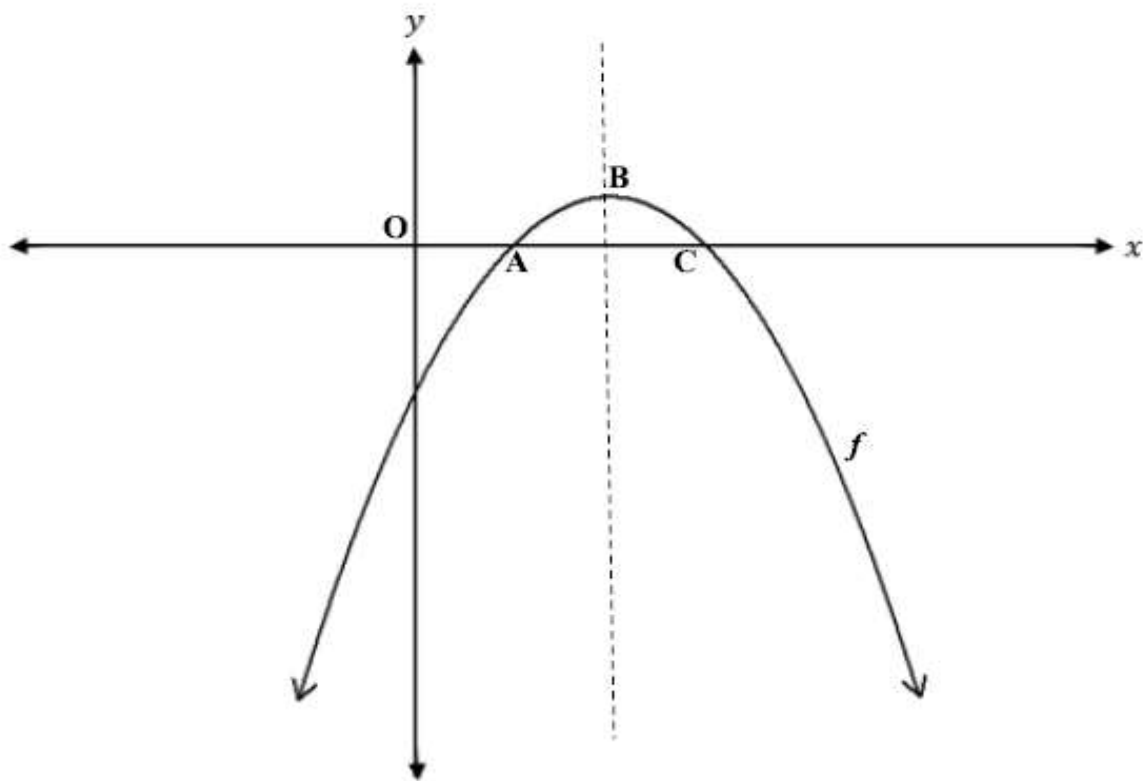
Both graphs pass through the origin, O. The vertical asymptote of f passes through D, the turning point of g . The asymptotes of f intersect at T. A is the other x-intercept of g .



- 4.1 Write down the coordinates of D, the turning point of g . (1)
- 4.2 Write down the equations of the asymptotes of f . (2)
- 4.3 Determine:
 - 4.3.1 The value of a (2)
 - 4.3.2 The length OA (3)
 - 4.3.3 The range of f (1)
 - 4.3.4 The equation of the axis of symmetry of f with a negative gradient (2)
- 4.4 For which values of x will:
 - 4.4.1 $g(x) < 0$? (2)
 - 4.4.2 $g(x) \cdot f(x) \geq 0$? (2)

ACTIVITY 11

5.1 Sketched below is the parabola f , with the equation $f(x) = -(x-2)^2 + 1$.



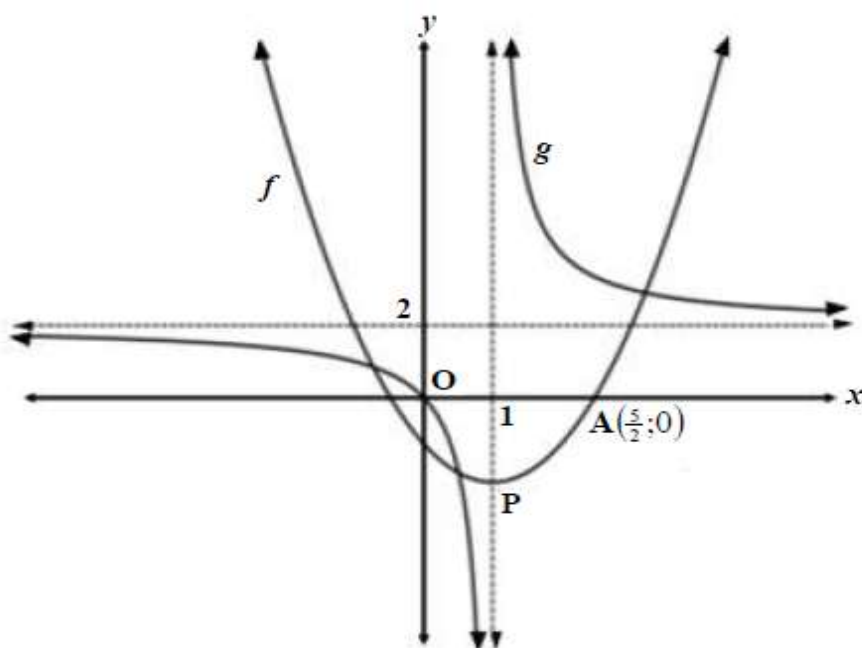
- 5.1 Write down the coordinates of B. (1)
- 5.2 Write down the equation of the axis of symmetry. (1)
- 5.3 Determine the coordinates of A and C. (3)
- 5.4 For which value(s) of x will $f(x) \leq 0$. (2)
- 5.5 Determine the average gradient between A and B. (2)
- 5.6 The graph of $g(x)$ is obtained by shifting the graph of f , 2 units to right, 1 unit downwards, write down the equation of $g(x)$, and then sketch the graph of $g(x)$ on the same set of axis. (3)

[12]

ACTIVITY 12

The graphs of $f(x) = (x+p)^2 + q$ and $g(x) = \frac{a}{x+b} + c$ are drawn in the sketch below and have the following properties:

- $A\left(\frac{5}{2}; 0\right)$ is a point on the graph of f .
- P is the turning point of f and lies on the vertical asymptote of g .
- The vertical asymptote passes through $(1; 0)$.
- The horizontal asymptote passes through $(0; 2)$.
- Graph g passes through the origin.

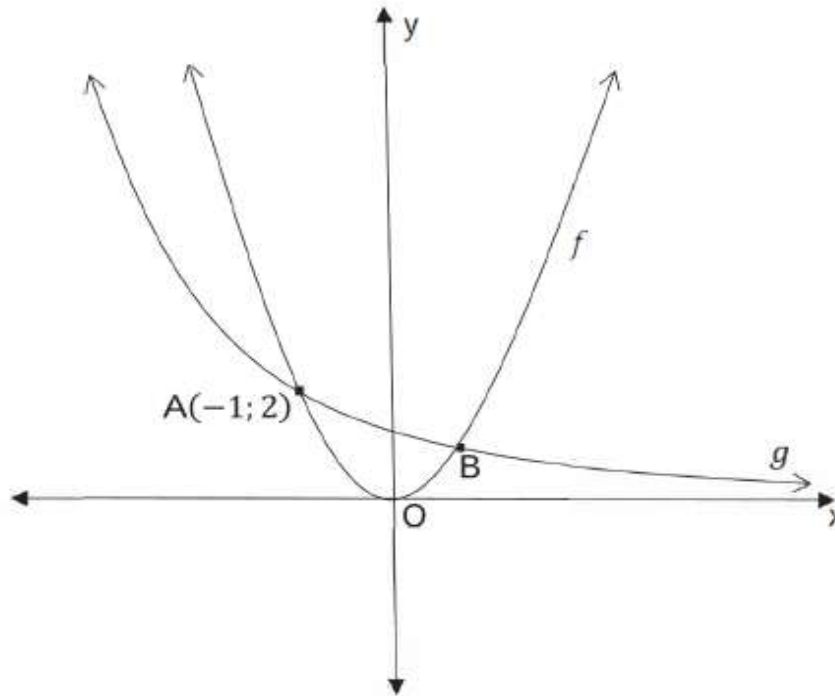


- 4.1 Show that the equation of g can be written as $y = \frac{2}{x-1} + 2$. (3)
- 4.2 Calculate the coordinates of P. (3)
- 4.3 Write down the equation of the vertical asymptote of p if $p(x) = g(x-1)$. (1)
- 4.4 Determine the equation of the axis of symmetry of g in the form $y = mx + c$, if $m < 0$. (2)
- 4.5 Write down the equation of k , if k is the reflection of g about the x -axis. (2)
- 4.6 For which values of x is $f'(x) \cdot g(x) > 0$? (2)
- 4.7 For which values of k will the equation $g(x) = x + k$ have two real roots of opposite signs? (1)

(1)
[14]

ACTIVITY 13

The graphs of $f(x) = ax^2$ and $g(x) = b^x$ are sketched on the same set of axes. Points $A(-1; 2)$ and B are points of intersection of f and g . The graph of f has the turning point at the origin:



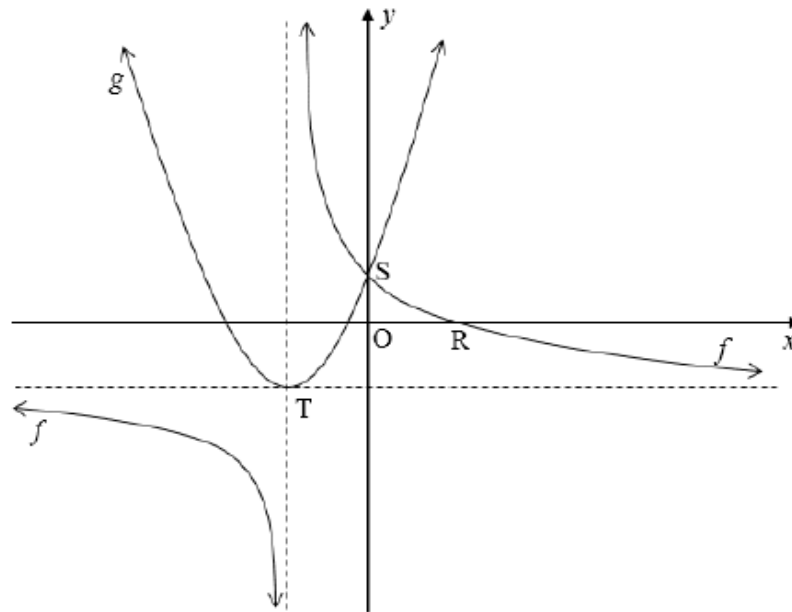
- 6.1 Calculate the values of a and b . (2)
- 6.2 The inverse of f is NOT a function. Write down at least one condition which can be used to restrict the domain of f such that its inverse will be a function. (1)
- 6.3 For which value(s) of x , where $x \in (-\infty; 0]$, will $g(x) \leq f(x)$? (2)
- 6.4 If $h(x) = g(x + 3)$, write down the coordinates of ...
 - 6.4.1 A' , the new coordinates of A on the graph of h . (1)
 - 6.4.2 A'' , the new coordinates of A on the graph of h^{-1} , the inverse of h (2)

[8]

ACTIVITY 14

The diagram below shows the graphs of $f(x) = \frac{5}{x+p} + q$ and $g(x) = 5x^2 + 10x + 3$.

The two graphs intersect at S, the y-intercept of both graphs. R is the x-intercept of f . The asymptotes of f cut at T, the turning point of g .

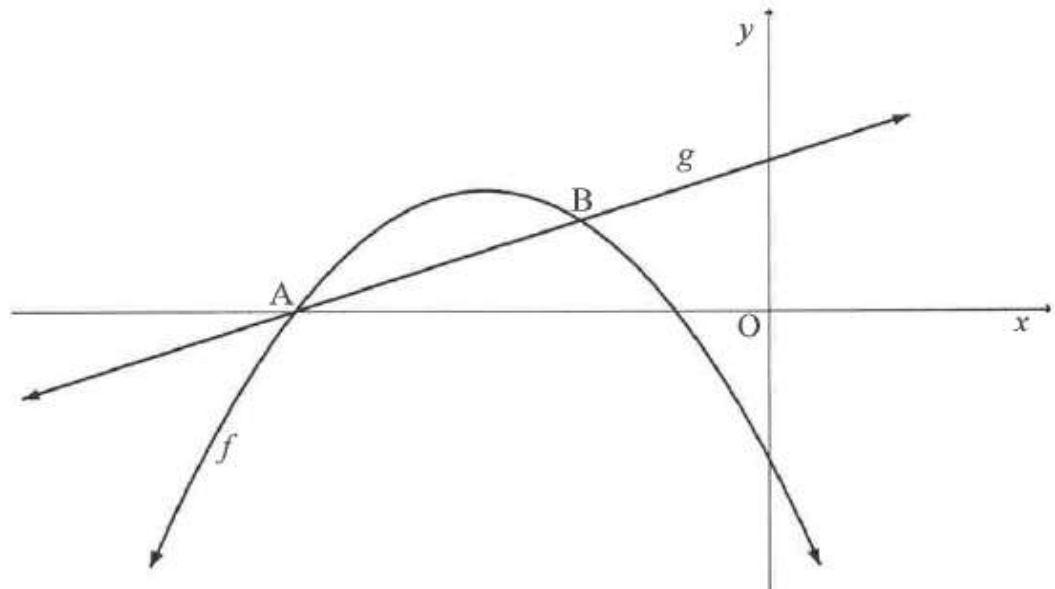


- 4.1 Write down the coordinates of S. (2)
- 4.2 Determine:
 - 4.2.1 The coordinates of T (4)
 - 4.2.2 The values of p and q (2)
 - 4.2.3 The length of OR (2)
 - 4.2.4 The range of g (2)
- 4.3 Determine the equation of:
 - 4.3.1 The tangent to g at S (3)
 - 4.3.2 The axis of symmetry of f , with a positive gradient (2)
- 4.4 For which values of x will $g'(x) \cdot f(x) \leq 0$? (2)

[19]

ACTIVITY 15

The graphs of the functions $f(x) = -(x+3)^2 + 4$ and $g(x) = x + 5$ are drawn below. The graphs intersect at A and B.

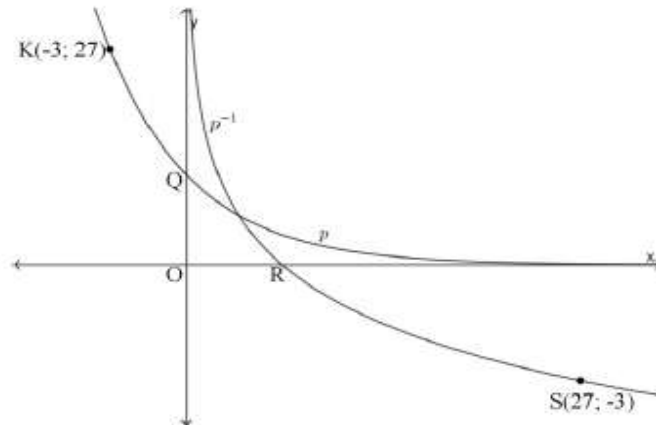


- 5.1 Write down the coordinates of the turning point of f . (2)
- 5.2 Write down the range of f . (1)
- 5.3 Show that the x -coordinates of A and B are -5 and -2 respectively. (4)
- 5.4 Hence, determine the values of c for which the equation $-(x+c+3)^2 + 4 = (x+c) + 5$ has ONE negative and ONE positive root. (2)
- 5.5 The maximum distance between f and g in the interval $x_A < x < x_B$ is k .
If $h(x) = g(x) + k$, determine the equation of h in the form $h(x) = \dots$ (5)
[14]

ACTIVITY 16

The graphs of $p(x) = b^x$ and $p^{-1}(x)$ are sketched below.

- $K(-3; 27)$ lies on the graph of p and $S(27; -3)$ lies on the graph of p^{-1} .
- Q is the y -intercept of p and R is the x -intercept of p^{-1} .



- 7.1 Determine the value of b . (2)
 - 7.2 Determine the equation of p^{-1} in the form $y = . . .$ (2)
 - 7.3 Write down the coordinates of Q and R . (2)
 - 7.4 For which value(s) of x will:
 - 7.4.1 $0 < p(x) \leq 1$ (1)
 - 7.4.2 $\log_{\frac{1}{3}} x < -3$ (1)
 - 7.5 T is a point in the first quadrant where TQ is parallel to the x -axis and TS is parallel to the y -axis. Calculate the area of ΔQTS . (4)
- [12]**

ACTIVITY 17

3.1 Given: $f(x) = \frac{2x+3}{-x-3}$

3.1.1 Show that $f(x)$ can also be written as $f(x) = \frac{3}{x+3} - 2$. (2)

3.1.2 Hence, determine the coordinates of the intercepts of f with the axes. (3)

3.1.3 Sketch the graph of f . Clearly indicate the intercepts with both axes, as well as the asymptotes. (3)

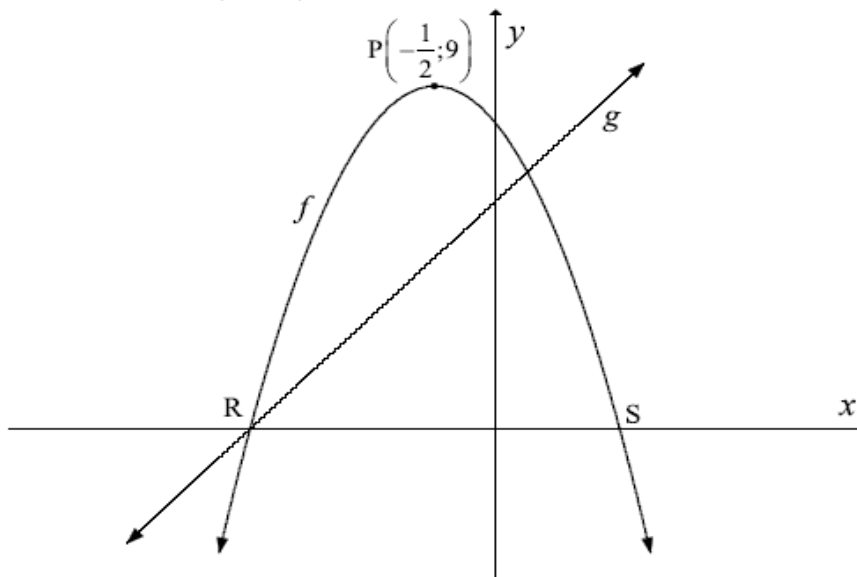
3.1.4 Determine the equation of the axis of symmetry of f , which is a decreasing function. (2)

3.1.5 Determine the value(s) of x for which:

(a) f is decreasing. (2)

(b) $f(x) \geq 0$. (2)

3.2 Sketched below is the parabola f and the straight line $g(x) = 3x + 6$. R and S are the x -intercepts of f , and $P\left(-\frac{1}{2}; 9\right)$ is its turning point. g has its x -intercept at R.

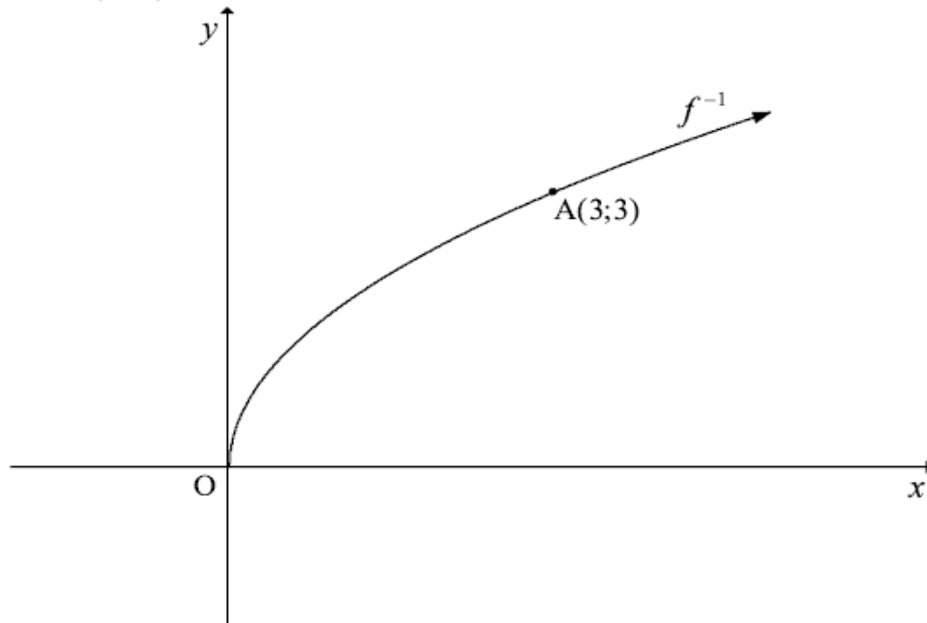


3.2.1 Calculate the coordinates of R. (2)

3.2.2 Determine the equation of the parabola. (4)

ACTIVITY 18

- 4.1 The graph of $f^{-1}(x) = \sqrt{3x}$, $x \geq 0$ is drawn in the sketch below. f^{-1} passes through the point $A(3;3)$.



- 4.1.1 Determine the equation of f in the form $y = \dots\dots\dots$ (3)
- 4.1.2 For which values of x will $f(x) \leq f^{-1}(x)$? (2)
- 4.2 Consider $g(x) = \left(\frac{1}{3}\right)^x$.
- 4.2.1 Determine the equation of g^{-1} in the form $y = \dots\dots\dots$ (2)
- 4.2.2 Draw a sketch graph of g^{-1} , indicating any intercepts with the axes as well as one more point on the graph. (3)
- 4.2.3 The graph of $h(x) = a\left(\frac{1}{3}\right)^x + 7$ passes through the point $(-2; 10)$. Calculate the value of a . (2)
- 4.2.4 Describe the translation from h to g . (3)

[15]



ACTIVITY 19

The graph of $g(x) = a\left(\frac{1}{5}\right)^x - 5$ passes through $A(-2; -4)$.

5.1 Show that $a = \frac{1}{25}$. (2)

5.2 Determine the coordinates of the x -intercept of g . (3)

5.3 Given: $h(x) = \left(\frac{1}{5}\right)^x$.

5.3.1 Determine the equation of $h^{-1}(x)$, the inverse of h , in the form $y = \dots$ (2)

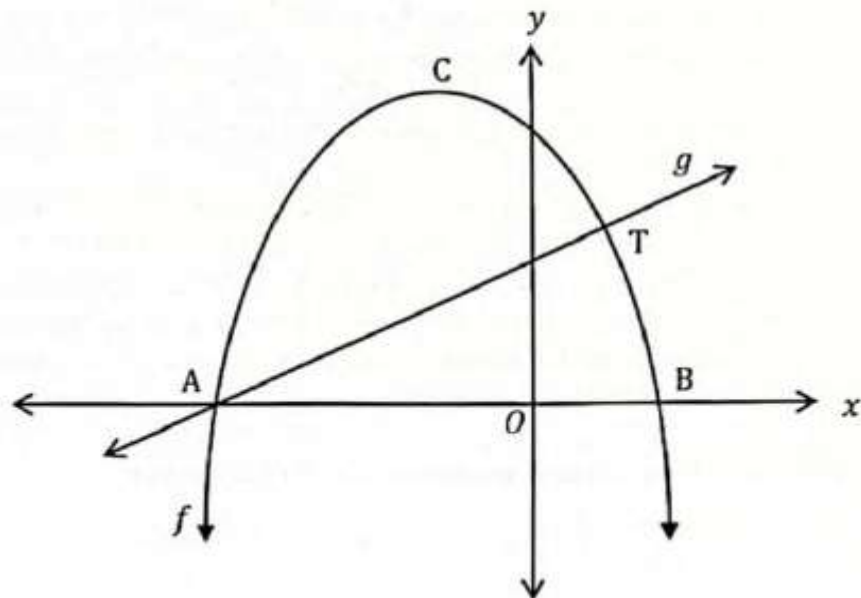
5.3.2 Describe the translation from g to h . (2)
[9]



ACTIVITY 20

The sketch shows the graphs of the functions $f(x) = -x^2 - x + 12$ and $g(x) = x + 4$.

A and B are the x -intercepts of f , while C is the turning point of f . The functions intersect at A and T.

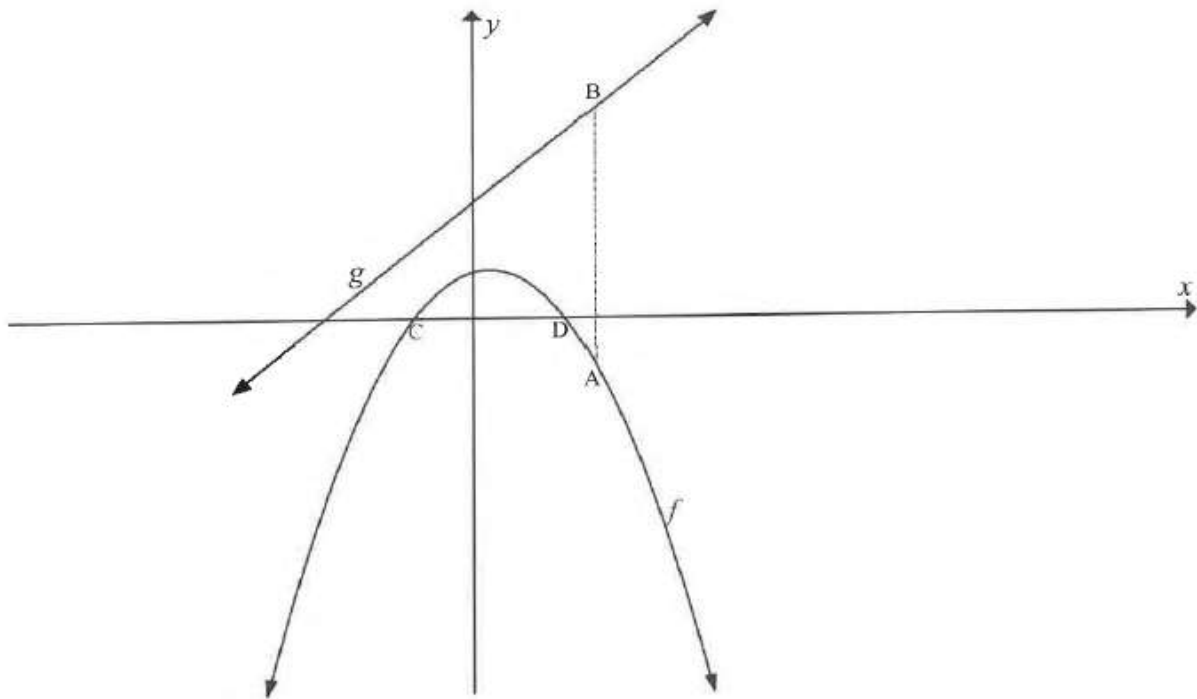


- 5.1 Determine the x -coordinates of A and B. (3)
- 5.2 Determine the range of $f(x)$. (3)
- 5.3 Determine the maximum value of $k(x) = 3^{f(x)-12}$ (2)
- 5.4 Determine for which value(s) of x will $x \cdot f(x) > 0$. (2)
- 5.5 For which real value(s) of k will $-x^2 - x + 12 = k$ have two negative unequal roots? (2)
- 5.6 Write down the equation of h in the form $h(x) = a(x + p)^2 + q$ if h is the reflection of f in the straight line $x = 1$. (2)

[14]

ACTIVITY 21

The graphs of $f(x) = -x^2 + x + 6$ and $g(x) = 3x + 10$ are drawn below. C and D are the x -intercepts of f . A is a point on f and B is a point on g such that AB is parallel to the y -axis.

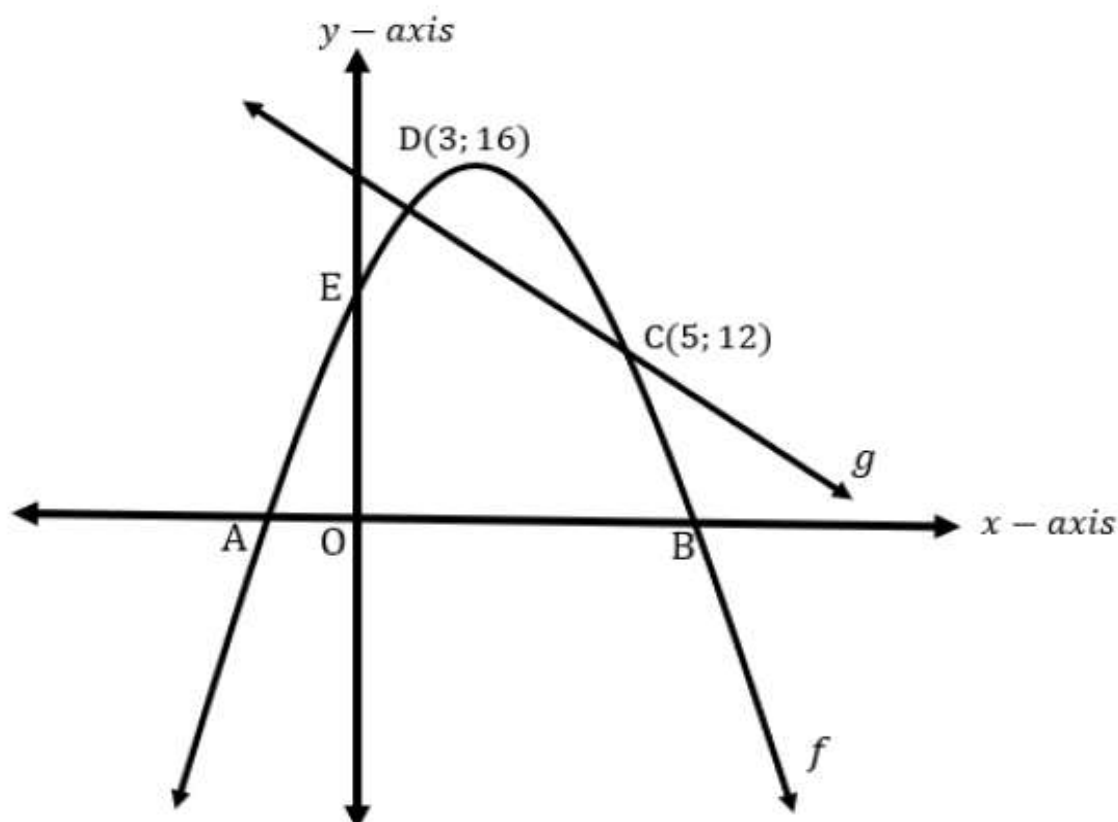


- 4.1 Calculate the coordinates of the turning point of f . (3)
- 4.2 Calculate the distance of CD. (2)
- 4.3 Calculate the vertical distance of AB in terms of x . (2)
- 4.4 Calculate the smallest distance of AB between f and g . (4)
- 4.5 Write the values of x for which $f(x) \cdot g(x) > 0$. (3)
- 4.6 If $f(x) + 2 = k$ has two distinct roots, determine the value(s) of k . (3)

[17]

ACTIVITY 22

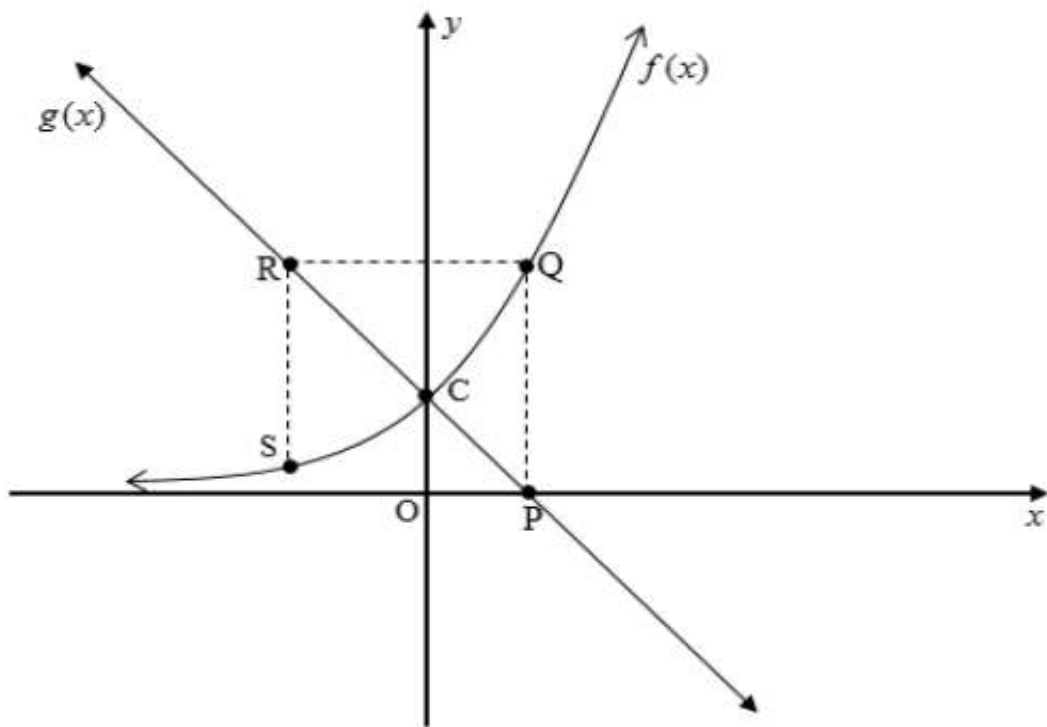
The functions $f(x) = a(x - p)^2 + q$ and $g(x) = -\frac{1}{5}x + c$ are drawn below, with f passing through A, B, C, D and E. A and B are the x -intercepts of f . D(3; 16) is the turning point of graph f . C(5; 12) is the point of intersection of graphs f and g .



- 5.1 Determine the value of c in graph g . (2)
- 5.2 Determine the equation of graph f by finding the values of a , p and q . (3)
- 5.3 Determine the equation of $k(x)$, a line perpendicular to graph g and passing through point E. (3)
- 5.4 Determine the values of x for which $f(x) \geq k(x)$ (3)
- 5.5 It is further given that graph $h(x) = 2^{-f(x)}$, determine the minimum value of the graph h . (2)
- 5.6 Determine the average gradient of graph f between $x = -3$ and $x = 1$ (4)

ACTIVITY 23

In the diagram below, the graphs of $f(x) = 3^x$ and $g(x) = -x + 1$ are given.

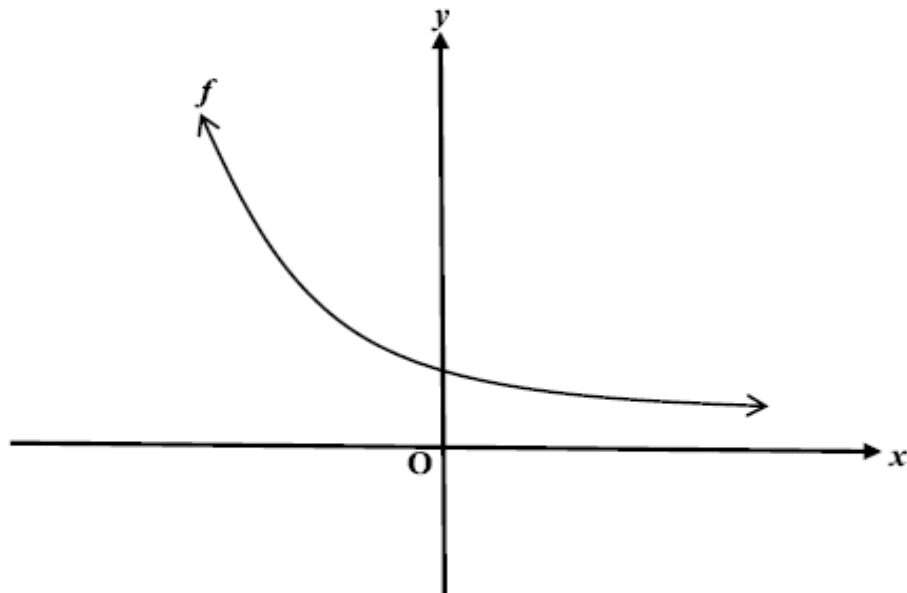


- 5.1 Write down the coordinates of C. (1)
- 5.2 Write down the range of $f(x)$. (1)
- 5.3 Determine the equation of $f^{-1}(x)$, in the form $y = \dots$ (2)
- 5.4 For which values of x is $f^{-1}(x) < -1$? (2)
- 5.5 If $PQ \parallel SR \parallel y\text{-axis}$ and $QR \parallel x\text{-axis}$, determine the coordinates of S. (4)
- 5.6 Describe the translation(s) of $f(x)$ to $p(x) = 3(3^x) - 2$ (2)

[12]

ACTIVITY 24

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $\left(2; \frac{1}{9}\right)$ lies on f .

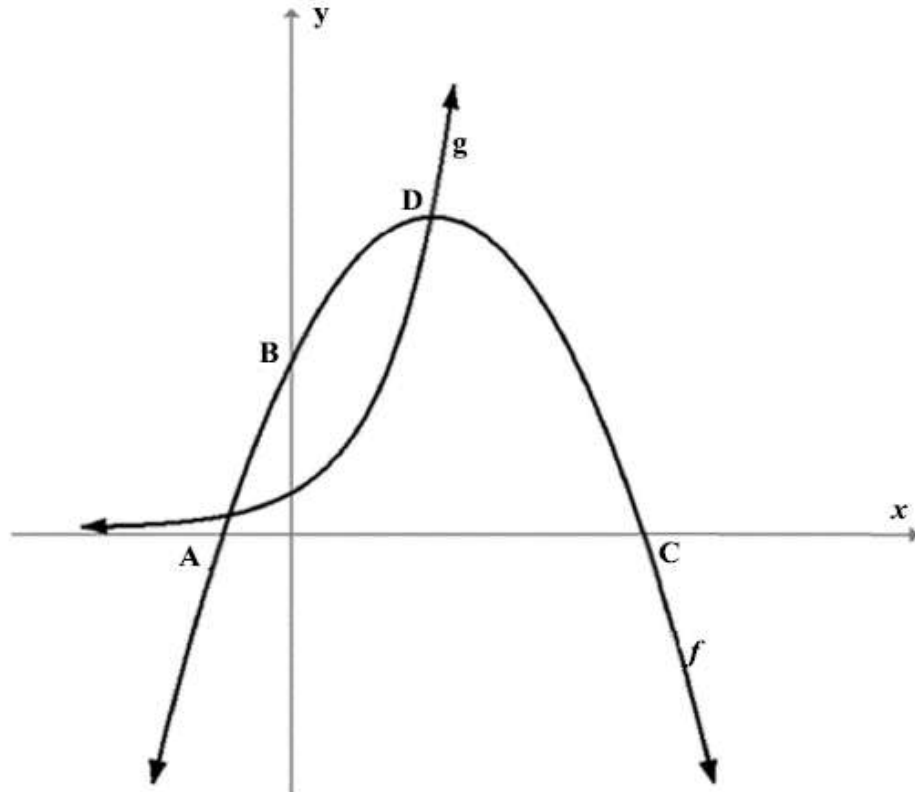


- 7.1 Determine the value of k . (2)
- 7.2 Write down the range of f . (1)
- 7.3 Explain the transformation of f to f^{-1} . (1)
- 7.4 Determine the equation of f^{-1} in the form $y = \dots$ (2)
- 7.5 Sketch the graph of f^{-1} . Indicate on your graph the coordinates of ONE point. (3)
- 7.6 Prove that $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$. (3)

[12]

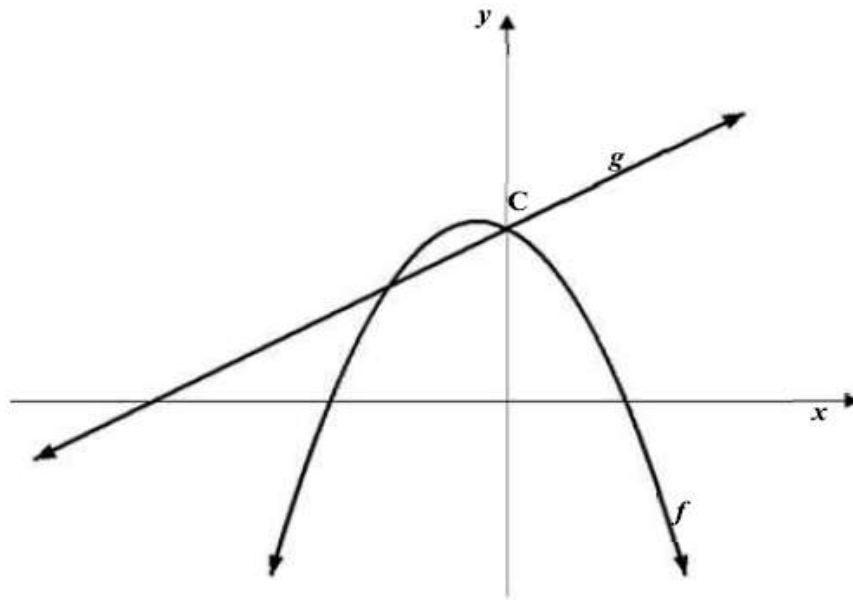
ACTIVITY 25

- 5.1 The sketch below, shows the graphs of $f(x) = -(x - 2)^2 + 9$ and $g(x) = b^x$ where b is a constant. D is the turning point of f and a point of intersection of f and g . B is the y -intercept and A and C, the x -intercepts of f .



- | | | |
|-------|--|-----|
| 5.1.1 | Determine the length of AC. | (4) |
| 5.1.2 | Determine the value of b . | (2) |
| 5.1.3 | Determine the values of x for which $g(x) \geq 9$. | (1) |
| 5.1.4 | Write down the equation of h if $h(x) = f(x + 2) - 9$. | (2) |
| 5.1.5 | How can the domain of h be restricted so that h^{-1} will be a function? | (1) |
| 5.1.6 | Show, algebraically, that $g\left(x + \frac{1}{2}\right) = \sqrt{3}g(x)$. | (2) |

5.2 Given: $f(x) = ax^2 + bx + c$ and $g(x) = mx + c$



If given $f(x)$, $g(x) < 0$ for all values of x when $-6 < x < -3$ or $x > 2$, determine the value of 'a' in terms of m (show all workings).

(5)
[17]

ACTIVITY 26

Consider the function $f(x) = \frac{-2x+5}{x-1}$

4.1 Show, using necessary calculations, that f can be written in the form (4)

$$f(x) = \frac{a}{x+p} + q$$

4.2 write down the equations of the asymptotes of f , If $f(x) = \frac{3}{x-1} - 2$. (2)

4.3 Calculate the intercepts of the graph of f with the axes. (3)

4.4 Sketch the graph of f . (3)

4.5 Write down the range of $y = -f(x)$. (1)

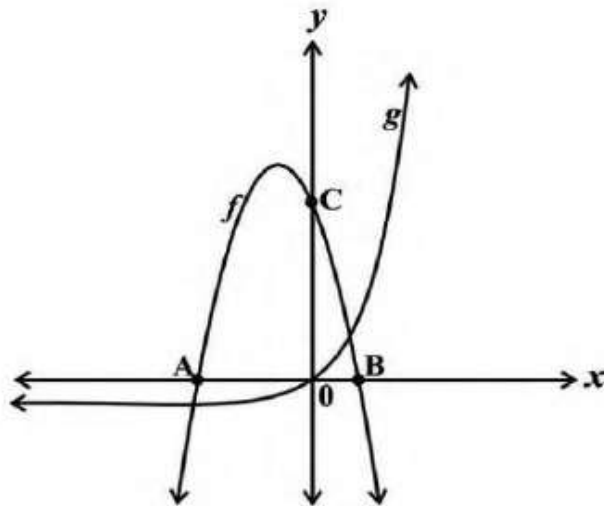
4.6 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$ (2)

[15]

ACTIVITY 27

Sketched below is the graph of the functions $f(x) = ax^2 + bx + c$ and $g(x) = d^x + q$.

A(-3;0) and B(1;0) are the x -intercepts, and C(0;6) is the y -intercepts of f . The graph of g passes through the origin and point (1;2)

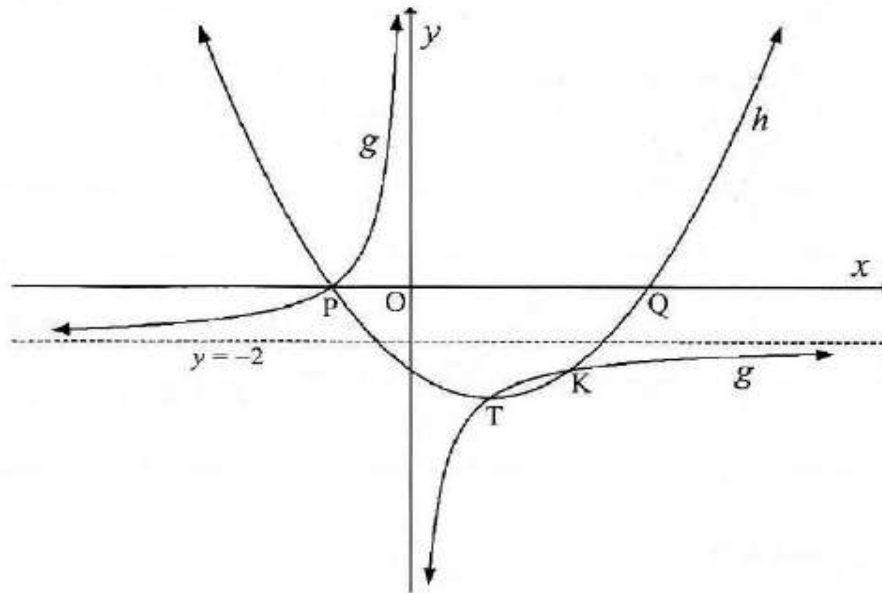


- 5.1 Use the graphs to determine the values of x where $f'(x) \cdot g(x) < 0$ (4)
- 5.2 Determine d and q . (4)
- 5.3 Determine g^{-1} , the inverse of g , in the form $y = \dots$ (3)
- 5.4 State the domain of g^{-1} . (2)
- 5.5 Determine a , b and c . (4)
- 5.6 Draw the graph of f^{-1} , the inverse of f . Show the turning point and the intercepts with the axes. (4)
- 5.7 Determine the values of k for which $f(x) + k = g(x)$ has two roots that are opposite in sign. (2)

[23]

ACTIVITY 28

The graphs of $g(x) = \frac{a}{x} + q$ and $h(x) = x^2 - 2x - 3$ are drawn. The graph of h cuts the x -axis at P and Q. The two graphs intersect at points P, T and K. T is the turning point of h . The line $y = -2$ is the asymptote of g .



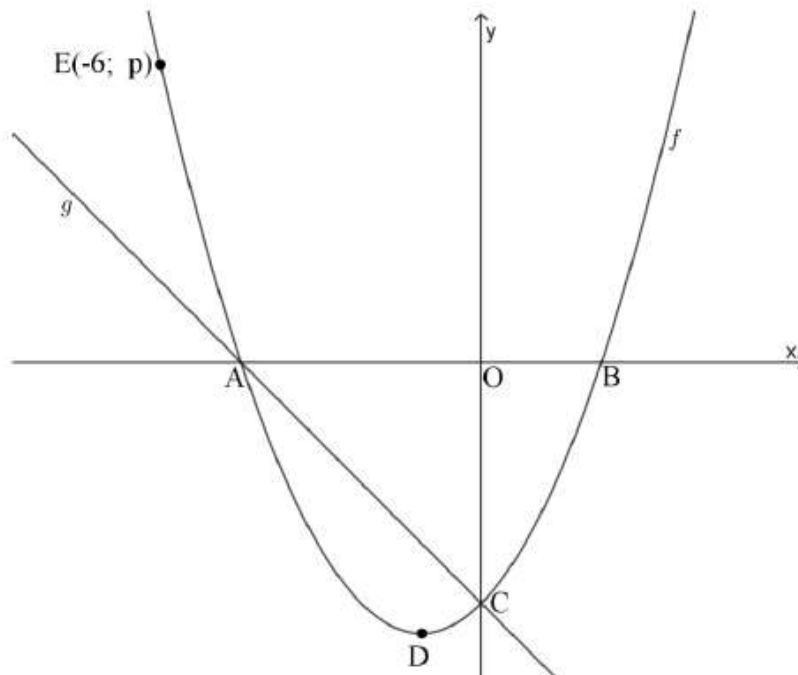
- 4.1 Calculate the coordinates of P and Q. (3)
- 4.2 Calculate the coordinates of T. (3)
- 4.3 Write down the equation of the vertical asymptote of g . (1)
- 4.4 Determine the equation of g . (3)
- 4.5 Is g a function? Motivate your answer. (2)
- 4.6 Calculate the coordinates of K. (5)
- 4.7 For which values of x will $g(x) > h(x)$? (3)

[20]

ACTIVITY 29

The graphs of $f(x) = x^2 + 2x - 8$ and $g(x) = mx + c$ are sketched below.

- A and B are the x -intercepts of f
- C is the y -intercept for f and g
- D is the turning point of f
- $E(-6; p)$ is a point on f



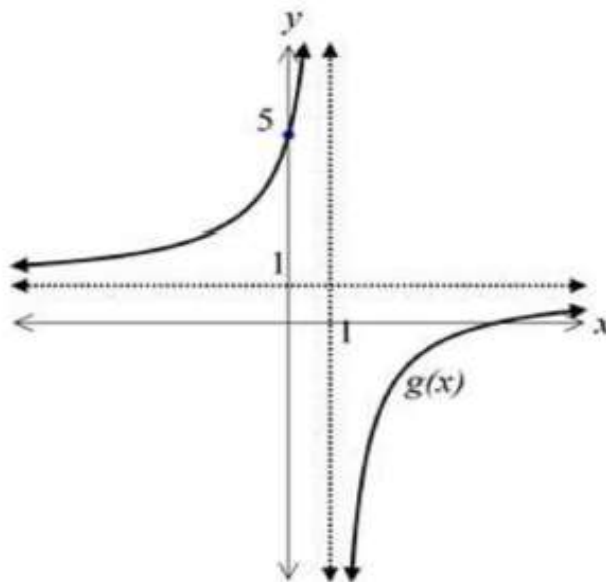
- 5.1 Write down the value of p . (1)
- 5.2 Determine the coordinates of A and B. (4)
- 5.3 Determine the coordinates of D. (2)
- 5.4 Determine the values of m and c . (2)
- 5.5 Determine the equation of the tangent to f at point C. (3)
- 5.6 Write down the range for $-f(x)$. (2)
- 5.7 For which values of x will $f(x) - g(x) > 0$? (2)
- 5.8 For which values of k will $x^2 + 2x + k = 0$ have no real roots? (2)

[18]

ACTIVITY 30

In the diagram, the hyperbola of $g(x) = \frac{-4}{x+r} + t$ is drawn.

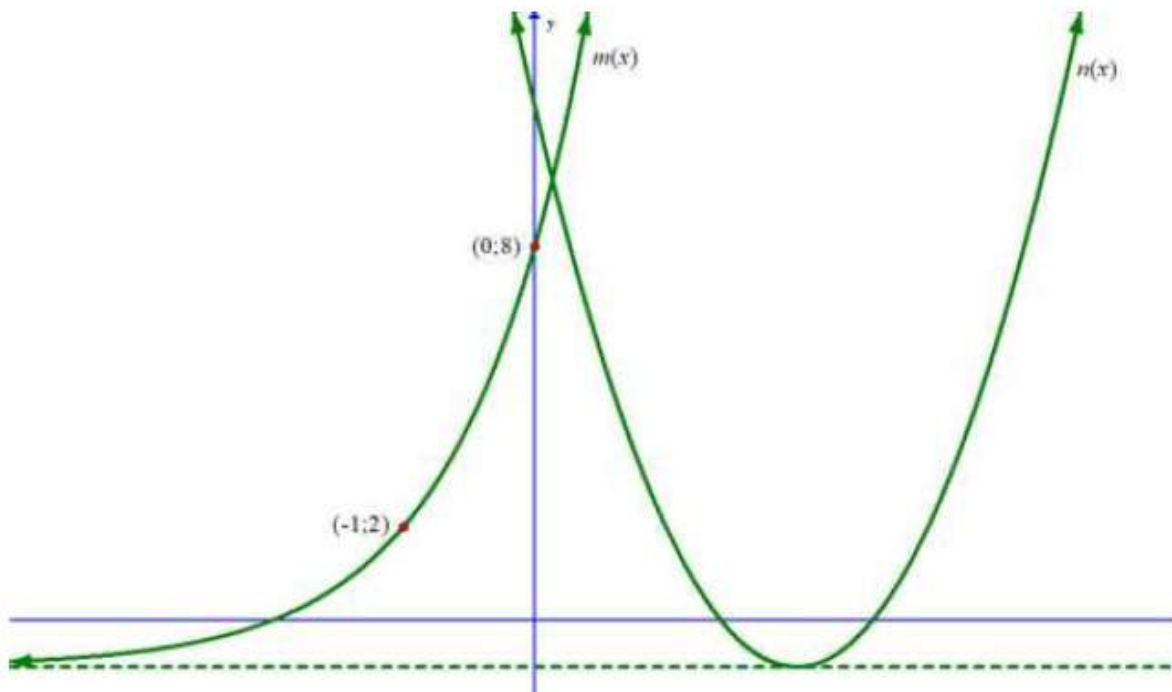
The asymptotes of g cut both the x - and y -axes at 1. The y -intercept of g is 5.



- 3.1 Is $g(x)$ an increasing or decreasing function? (1)
 - 3.2 Write down the values of r and t . (2)
 - 3.3 Write down the range of $g(x)$. (2)
 - 3.4 Write down the equation of the axis of symmetry with a negative gradient. (2)
 - 3.5 Determine the value(s) of x for which:
 - 3.5.1 $g(x) \geq 5$ (2)
 - 3.5.2 $g(x) < 0$ (3)
 - 3.6 Write down the equation of the vertical asymptote of $g(x+4)$. (1)
- [13]**

ACTIVITY 31

Drawn below are the functions $m(x) = ab^{x+2} + q$ and $n(x) = 3x^2 - 12x + 11$.
 $(0; 8)$ and $(-1; 2)$ are on $m(x)$.



- 4.1 Calculate the x -coordinate of the turning point of $n(x)$. Show ALL calculations. (2)
 - 4.2 If the graph of $p(x)$ is obtained by reflecting graph $n(x)$ over the y -axis, determine the equation of $p(x)$. (2)
 - 4.3 The turning point of $n(x)$ is given as $(2; -1)$. Determine the equation of $m(x)$. (7)
 - 4.4 For which value(s) of x will $x \cdot n(x) < 0$? (4)
- [15]**

ACTIVITY 32

Consider the graph of $g(x) = 4^{-x}$.

- 5.1 Give the equation of $g^{-1}(x)$ in the form $y = \dots$ (2)
 - 5.2 Sketch the graphs of g and g^{-1} on the same set of axes, showing the intercepts with the axes, another coordinate and asymptotes. Clearly label the graphs g and g^{-1} . (4)
 - 5.3 Write down the domain of $g^{-1}(x)$. (1)
 - 5.4 Use the graphs to solve for x : $\frac{g^{-1}(x)}{g(x)} \leq 0$. (2)
- [9]**



ACTIVITY 33

3.1

Given : $f(x) = (x-1)^2 - 9$

- 3.1.1 Determine the turning point of f . (2)
- 3.1.2 Write down the equation of axis of symmetry of f . (1)
- 3.1.3 Write down the range of f . (1)
- 3.1.4 Calculate the y – intercept of f . (2)

3.2 Consider the function $g(x) = x^2$.

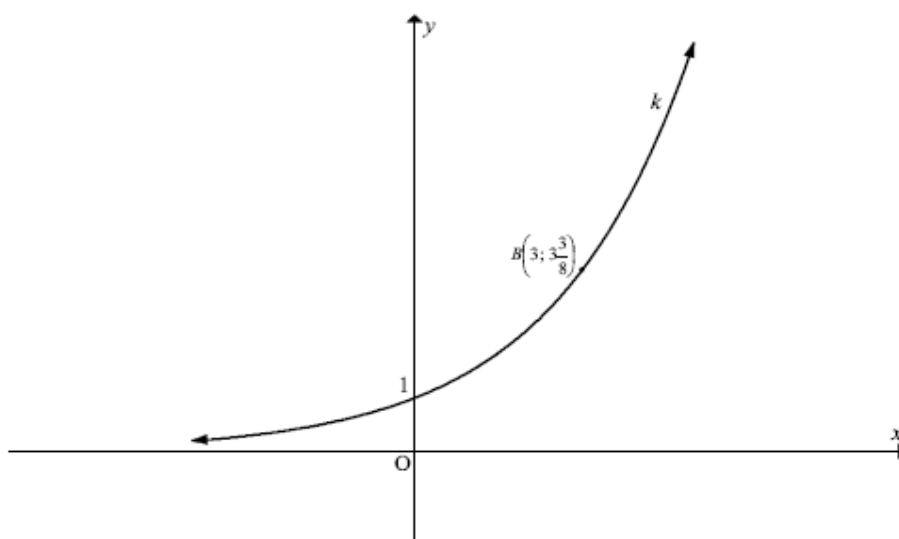
- 3.2.1 What transformation did the graph of $f(x) = (x-1)^2 - 9$ undergo in order to obtain the graph of g . (2)
- 3.2.2 Restrict the graph of g such that the inverse of this graph is a function. (1)
- 3.2.3 Determine the inverse of g in the form $y = \dots$ (include all restrictions) (3)

[12]



ACTIVITY 34

The diagram below shows the graph of $k(x) = a^x$, point $B\left(3; 3\frac{3}{8}\right)$ lies on k .



- 4.1 Show that $a = \frac{3}{2}$ (2)
- 4.2 Draw the graph of h , the reflection of k in the line $y = x$ showing all the intercepts with the axes and the coordinates of another point on the graph. (3)
- 4.3 Write down the equation of h . (2)
- 4.4 Write down the value(s) of x for which $h < 3$? (1)

[8]



ACTIVITY 35

Given: $h(x) = a^x$; $a > 0$ and $a \neq 0$. $B\left(-1 ; \frac{1}{2}\right)$ is a point that lies on h , the graph of $h(x)$.

$(4; -2)$ is a point on the graph.

6.1 Determine the value of a . (2)

6.2 Write down the equation of h^{-1} in the form $y = \dots$. (2)

6.3 Sketch the graphs of h and h^{-1} on the same set of axes. Clearly show all intercepts with the axes. (4)

6.4 Write down the domain of h^{-1} . (1)

6.5 Determine the value(s) of for which $h^{-1}(x) > 1$. (1)

6.6 If it is given that $t(x) = \left(\frac{1}{2}\right)^x - 1$.

6.6.1 Describe the transformation from h to t . (2)

6.6.2 Determine the equation of the asymptote of t . (1)

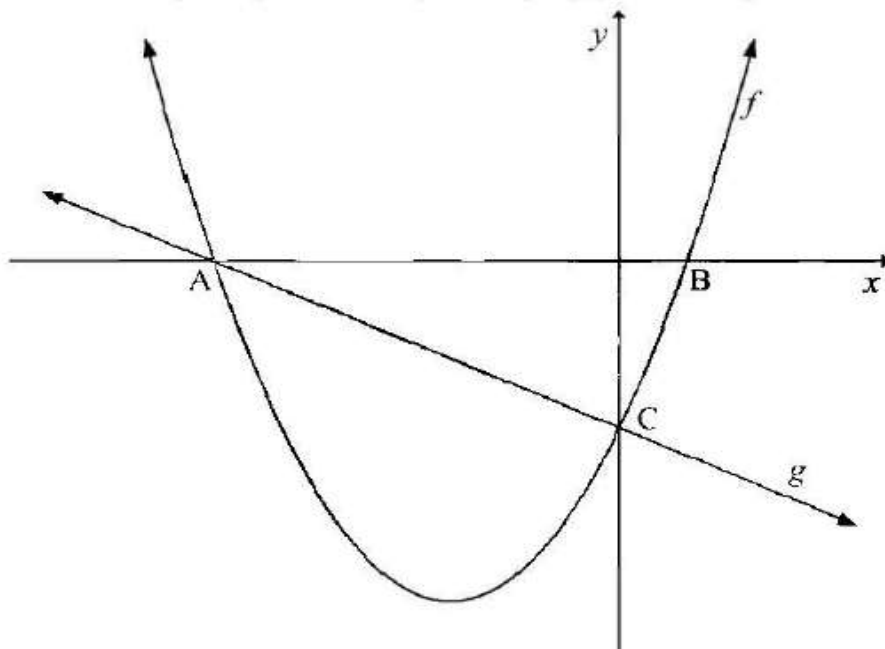
[13]



ACTIVITY 36

The graphs of $f(x) = x^2 + 5x - 6$ and $g(x) = mx + c$ are drawn below.

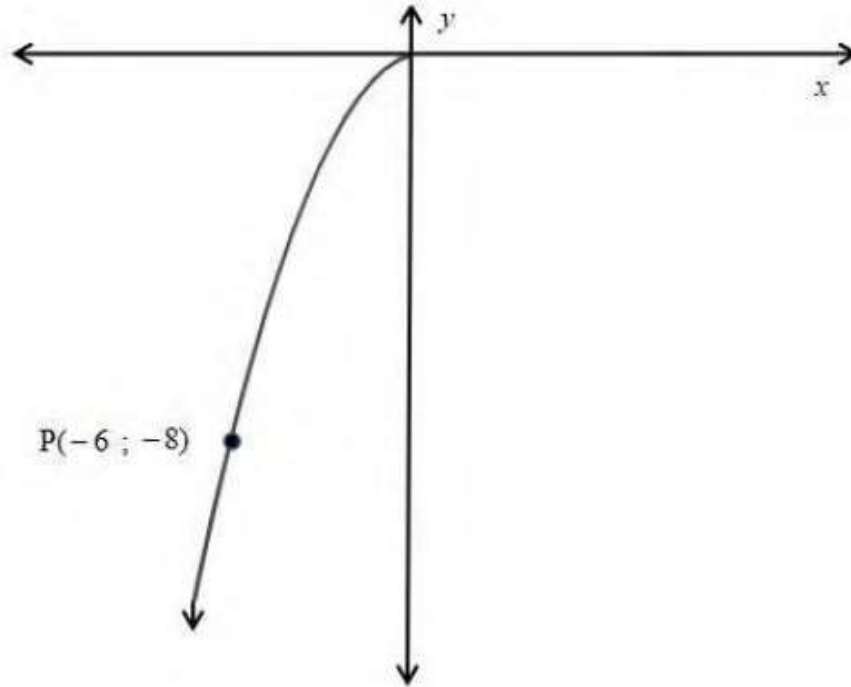
A and B are the x -intercepts of f and C, the y -intercept. g passes through A and C.



- 6.1 Calculate the coordinates of A and B. (3)
- 6.2 Determine the equation of g . (3)
- 6.3 If $h(x) = f(x) + k$, determine the values of k for which g and h will not intersect. (5)

ACTIVITY 37

The graph of $f(x) = ax^2$, $x \leq 0$, is sketched below. The point $P(-6 ; -8)$ lies on the graph of f .

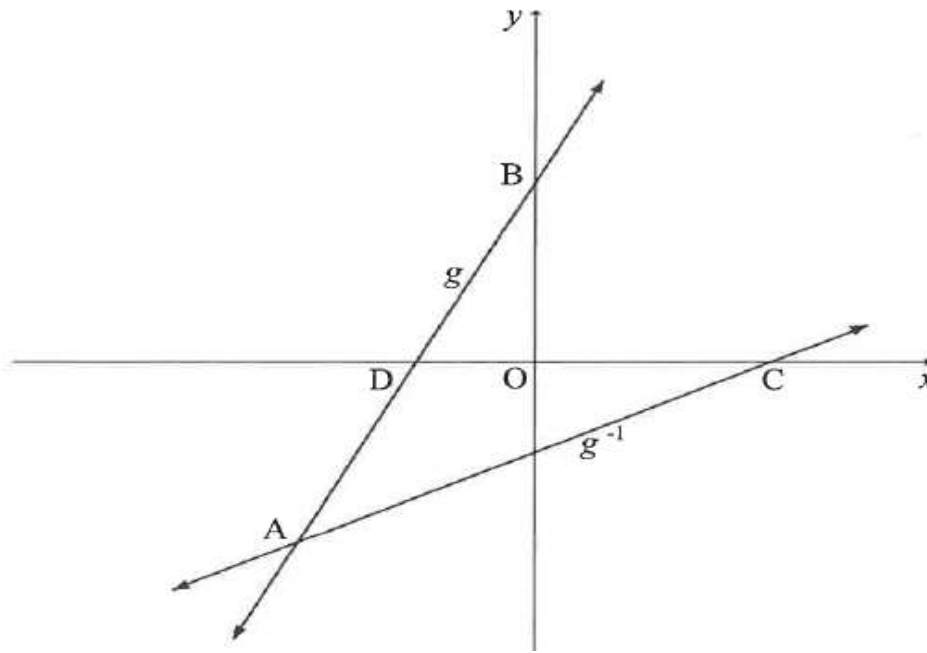


- 4.1 Calculate the value of a . (2)
 - 4.2 Determine the equation of f^{-1} , in the form $y = \dots$ (2)
 - 4.3 Write down the range of f^{-1} . (1)
 - 4.4 Sketch the graph of f^{-1} . Indicate the coordinates of any point on the graph different to $(0 ; 0)$. (2)
 - 4.5 The graph of f is reflected across the line $y = x$, and thereafter it is reflected across the x -axis.
Determine the equation of the new function in the form $y = \dots$ (2)
- [9]**

ACTIVITY 38

The graphs of $g(x) = 2x + 6$ and g^{-1} , the inverse of g , are shown in the diagram below.

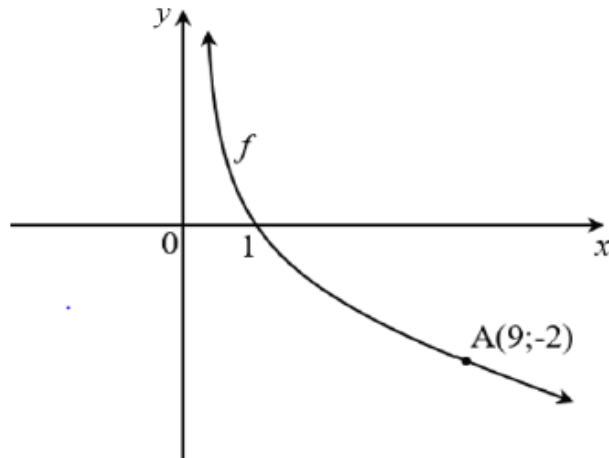
- D and B are the x - and y -intercepts respectively of g .
- C is the x -intercept of g^{-1} .
- The graphs of g and g^{-1} intersect at A.



- 5.1 Write down the y -coordinate of B. (1)
 - 5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)
 - 5.3 Determine the coordinates of A. (3)
 - 5.4 Calculate the length of AB. (2)
 - 5.5 Calculate the area of $\triangle ABC$. (5)
- [13]**

ACTIVITY 39

The diagram below shows the graph of $f(x) = \log_b x$, where b is a constant. f passes through the point $A(9; -2)$.

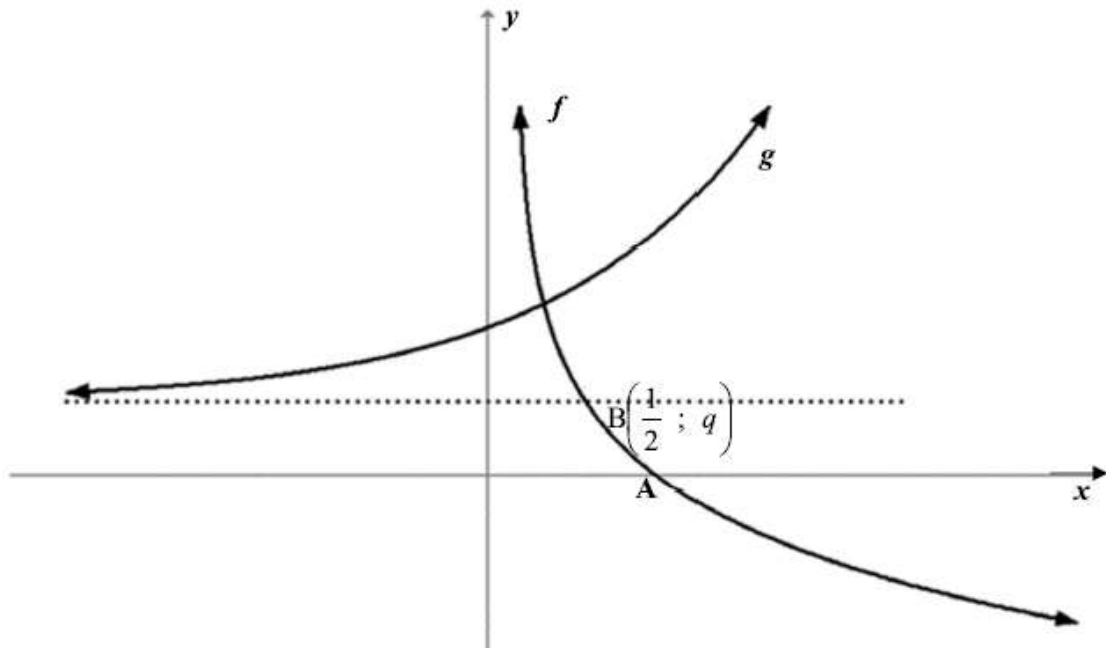


- 5.1 Show that $b = \frac{1}{3}$. (2)
 - 5.2 Determine the equation of f^{-1} , the inverse of f , in the form $y = \dots$ (2)
 - 5.3 For which values of x is $f(x) \geq 0$? (2)
 - 5.4 Write down the equation of the asymptote of g , if $g(x) = f^{-1}(x+1)$. (2)
- [8]**

ACTIVITY 40

The sketch below shows the graphs of $g(x) = 2^x + q$ and $f(x) = \log_{\frac{1}{2}} x$.

Graph f and the asymptote of g intersect at $B\left(\frac{1}{2}; q\right)$.



- 6.1 Write down the coordinates of A, the x -intercept of f . (1)
- 6.2 Determine the domain of f . (1)
- 6.3 Determine the equation of f^{-1} in the form of $y = \dots$ (2)
- 6.4 Sketch the graph of f^{-1} . Indicate on your graph the intercept(s) with the axes and the coordinates of one other point on the graph. (3)
- 6.5 Determine the equation of the asymptote of g . (1)
- 6.6 Describe in words the transformation of g to f^{-1} . (2)

[10]