



education

Department of
Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

TIME: 3 HOURS

SEPTEMBER 2024

MARKS: 150

This question paper consists of 14 pages, 1 information sheet
and an answer book of 22 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. Unless stated otherwise, you may use an approved scientific calculator (non-programmable and non-graphical).
6. If necessary, round off answers to TWO decimal places unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.



QUESTION 1

A sample of residents were asked, how many litres of water they use per week in their household.

The results were summarised in the table below.

Number of litres used	Frequency
$50 \leq x < 100$	20
$100 \leq x < 150$	30
$150 \leq x < 200$	50
$200 \leq x < 250$	100
$250 \leq x < 300$	80
$300 \leq x < 350$	70
$350 \leq x < 400$	50

- 1.1 Complete the table in the answer book provided. (3)
- 1.2 Calculate the estimate of the mean litres of water used by each household per week. (2)
- 1.3 Draw an ogive of the above data on the grid provided in the answer book. (2)
- 1.4 Use the ogive to determine the median of the data set. (1)
- 1.5 Comment on the skewness of the data. Give a reason for your answer. (2)
- 1.6 The municipality placed a restriction on the usage of water due to water shortages. The residents may not use more than 300 litres of water per household per week. How will this affect the standard deviation? Explain your answer. (2)

[12]

QUESTION 2

The leg strength and the number of leg presses done per day by a random sample of ten, eighteen-year-old boys were recorded as shown in the table below.

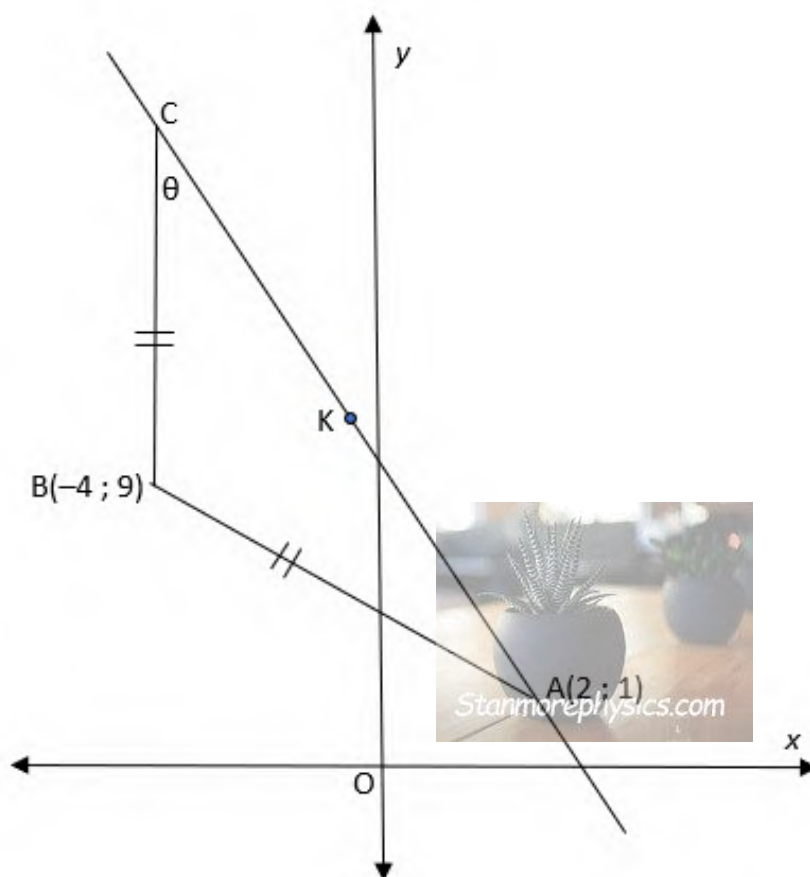
Number of leg presses done per day (x)	36	136	51	126	90	43	77	68	103	124
Strength of upper leg (y)	0.2	0.85	0.35	0.91	0.73	0.34	0.61	0.59	0.78	0.90

- 2.1 Determine the equation of the regression line and write your answer in the form.
 $y = \dots$ (3)
 - 2.2 Use the regression line to predict the leg strength of the eighteen-year-old boy if he does 110 leg-presses per day. (2)
 - 2.3 An eighteen-year-old boy does 250 leg presses per day. Can your regression line formula predict the strength of his legs? (Explain you answer). (2)
 - 2.4 Calculate the correlation coefficient of the above data set. (2)
 - 2.5 Use your answer in 2.4 to describe the relationship between the number of leg presses and the leg strength of an eighteen-year-old boy. (2)
- [11]



QUESTION 3

In the diagram below, ABC is an isosceles triangle with $A(2 ; 1)$ and $B(-4 ; 9)$. $AB = BC$ and BC is parallel to the y-axis.



3.1 Calculate:

3.1.1 The length of AB.

(2)

3.1.2 The coordinates of C.

(2)

3.1.3 The coordinates of K, the midpoint of AC.

(2)

3.1.4 The equation of AC in the form $y = mx + c$.

(3)

3.1.5 The size of θ .

(3)

3.1.6 The area of triangle ABC.

(4)

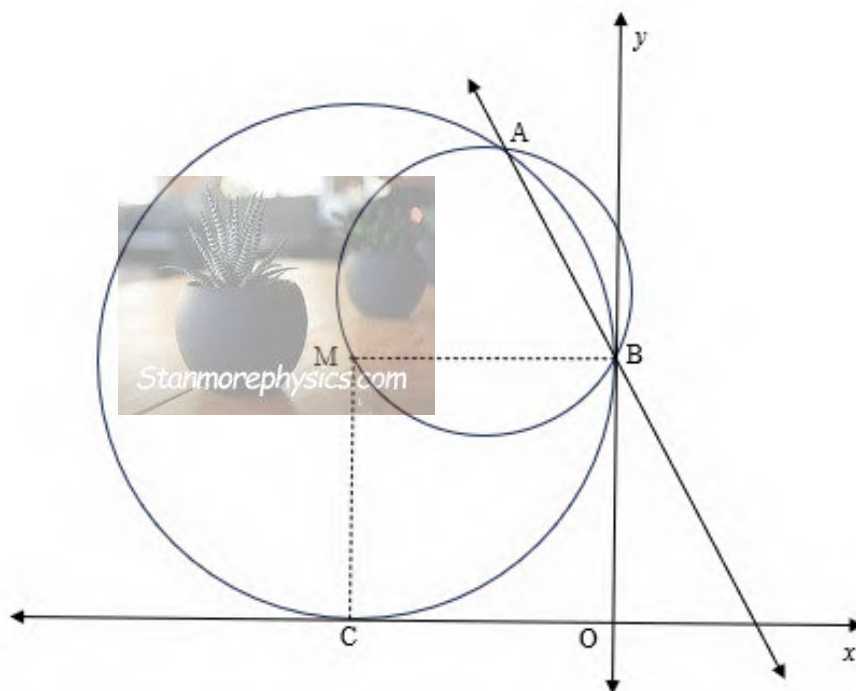
3.1.7 The coordinates of D if ABCD is a rhombus.

(2)

[18]

QUESTION 4

In the diagram, a circle centred at M touches the x -axis at C and the y -axis at point B. A second circle with equation $x^2 + y^2 + x - 3y + 2 = 0$ passes through A and M and intersects the circle M at A. The equation of the common chord AB is given by $y = -x + 1$.



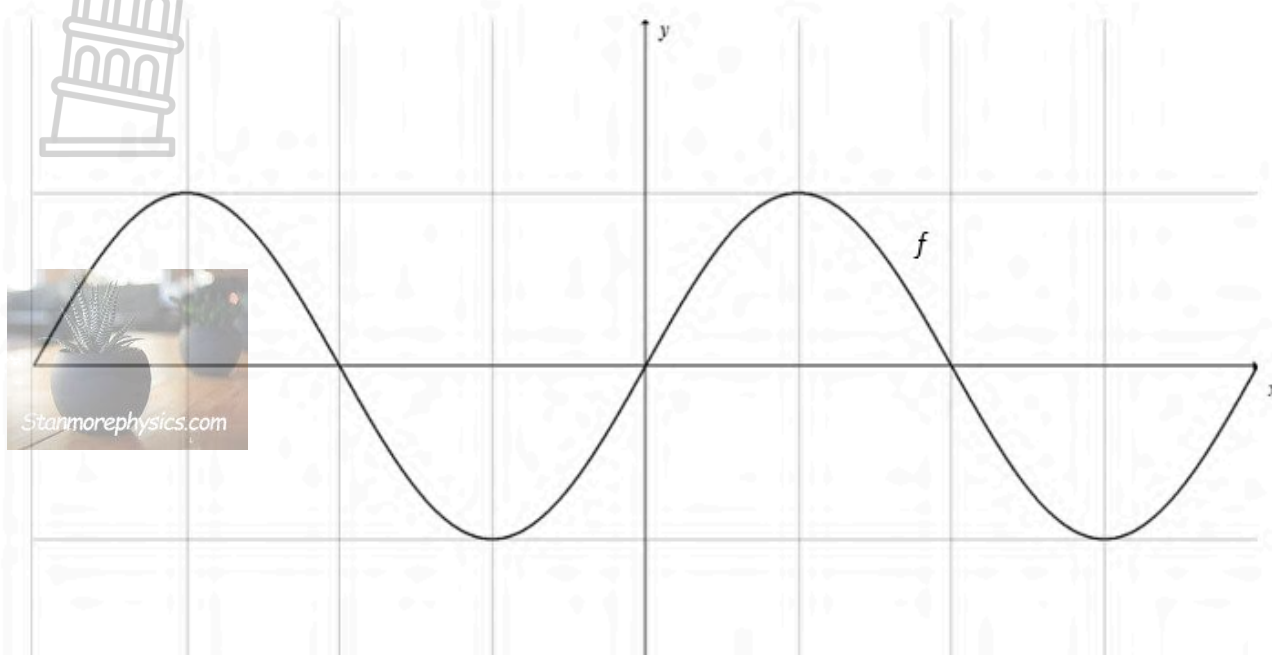
- 4.1 Show that the equation of the circle centred at M, is $x^2 + y^2 + 2x - 2y + 1 = 0$ (5)
- 4.2 Determine the coordinates of the centre and the radius of the circle which passes through B, M and A. (4)
- 4.3 Calculate the coordinates of A. (5)
- 4.4 The straight line with equation $y = -x + k$ is a tangent to the circle with centre M.
 - 4.4.1 Show that this equation can be written as:

$$2x^2 + (4 - 2k)x + (k^2 - 2k + 1) = 0$$
 (3)
 - 4.4.2 Calculate the numerical value(s) of k . (5)

[22]

QUESTION 5

In the diagram below, the graph of $f(x) = \sin 2x$ is drawn for the interval $x \in [-180^\circ; 180^\circ]$.



Use the graphs to answer the following questions.

- 5.1 Sketch the graph of $g(x) = \cos(x - 45^\circ)$ on the same set of axes in the answer book provided. (3)
- 5.2 Determine the values of x in the interval $x \in [0^\circ; 180^\circ]$ for which.
 - 5.2.1 $f(x) = g(x)$ (7)
 - 5.2.2 $f(x + 30^\circ) = g(x + 30^\circ)$ (2)
 - 5.2.3 $f(x) > g(x)$ (2)
- 5.3 Write down the period of g . (1)
- 5.4 Show how the graphs of f and g can be used to solve:
 $\sqrt{2} \sin 2x = \cos x + \sin x$. (3)

[18]

QUESTION 6

6.1 If $\sqrt{5} \sin \theta + 2 = 0$ and $\theta \in [90^\circ; 270^\circ]$.

Determine without the use of a calculator the value of the following:

6.1.1 $\tan \theta$ (2)

6.1.2 $\cos 2\theta$ (2)

6.2 Simplify the following expression to ONE trigonometric ratio without the use of a calculator:

$$2 \cos^2 15^\circ - 1 + \frac{2 \sin 140^\circ}{\cos 310^\circ} \quad (5)$$

6.3 If $\sin \frac{x}{2} = p$, express $\sin x - 1$ in terms of p . (4)

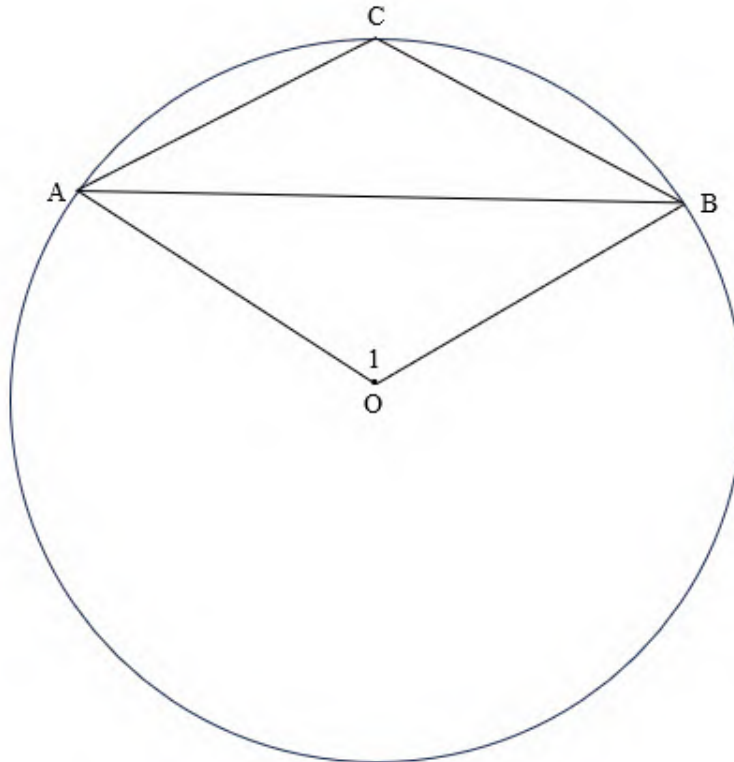
6.4 Prove the following:

$$\frac{3 \sin x + 2 \sin 2x}{2 + 3 \cos x + 2 \cos 2x} = \tan x \quad (5)$$

6.5 Show that: $\frac{\sin x + \cos x}{\sin x - \cos x} = \frac{p+t}{p-t}$ if $\tan x = \frac{p}{t}$ (5)
[23]

QUESTION 7

In the diagram below, O is the centre of the circle with radius equal to r units.
 $AC = CB = t$ units and $\angle AOB = 2\theta$.



7.1 Determine AB in terms of r and θ (3)

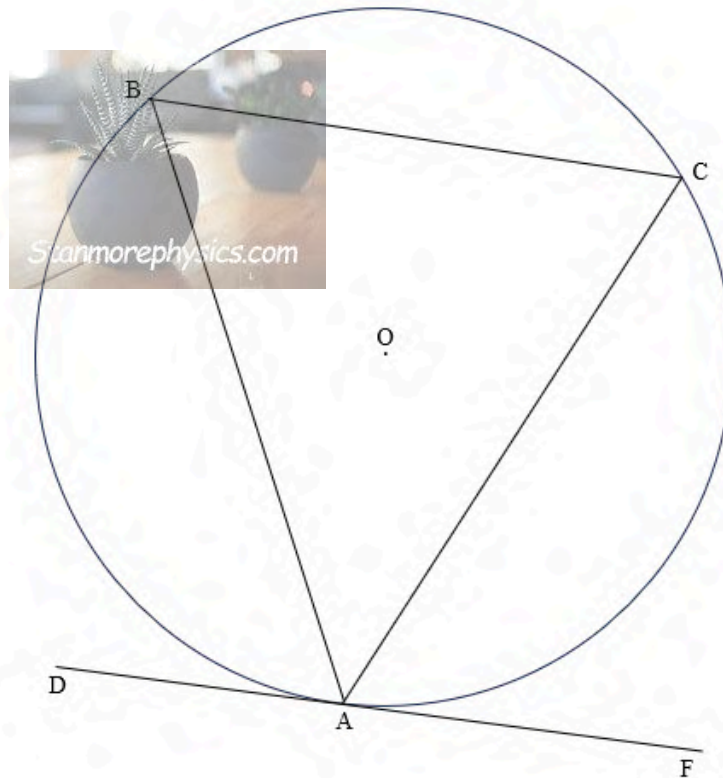
7.2 Show that $\frac{r}{t} = \sqrt{2(1 - \cos \theta)}$ (4)

7.3 If $\theta = 30^\circ$, determine the area of $\triangle AOB$, in terms of r (2)
[9]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

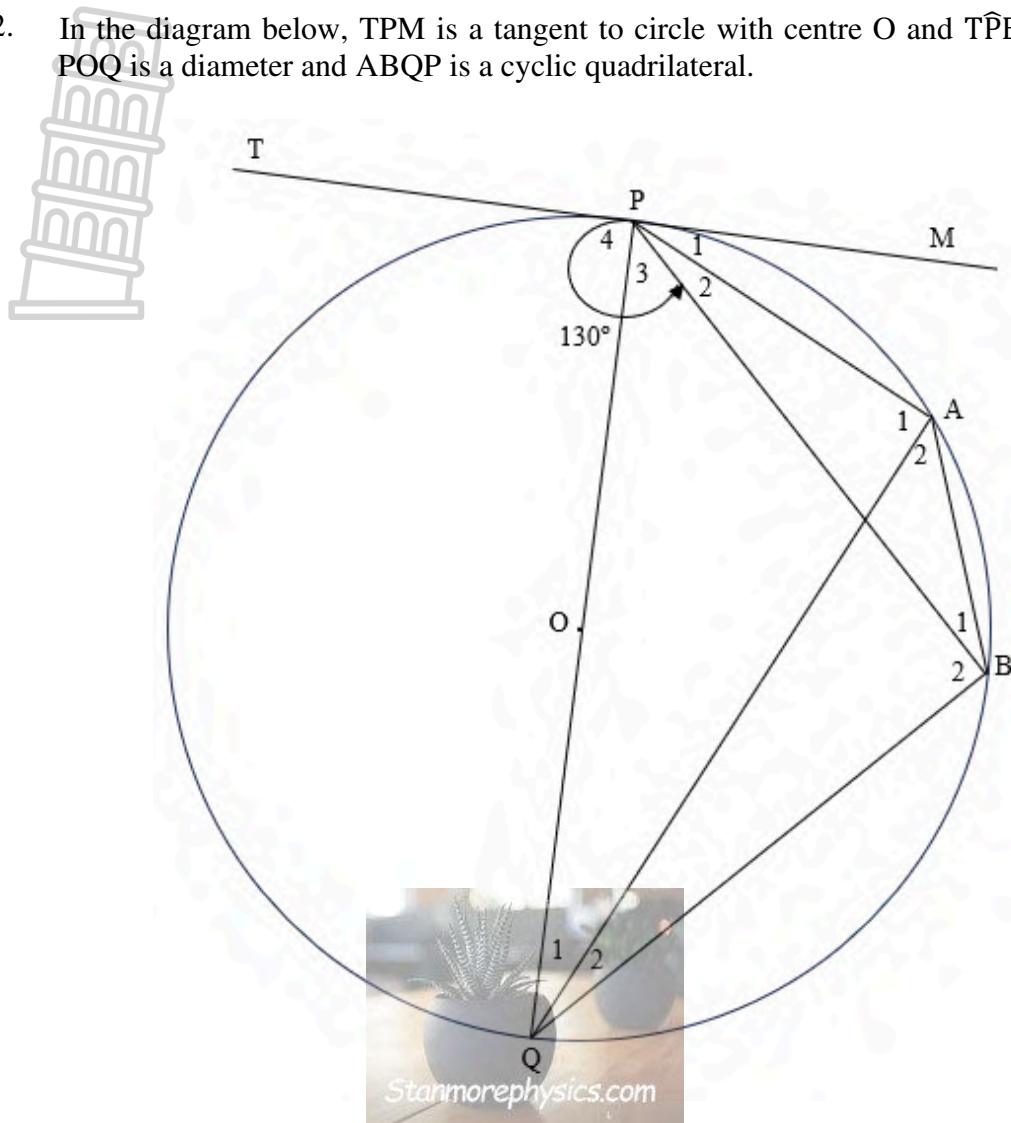
8.1 In the diagram, chords AB, BC and CA are drawn in the circle with centre O. DAF is a tangent to the circle at A,



Prove the theorem which states that $\hat{C}\hat{A}\hat{F} = \hat{A}\hat{B}\hat{C}$ (5)



- 8.2. In the diagram below, TPM is a tangent to circle with centre O and $\widehat{TPB} = 130^\circ$. POQ is a diameter and ABQP is a cyclic quadrilateral.



Determine with reasons the size of the following.

8.2.1 $\widehat{QPB}$

(3)

8.2.2 $\widehat{PQB}$

(3)

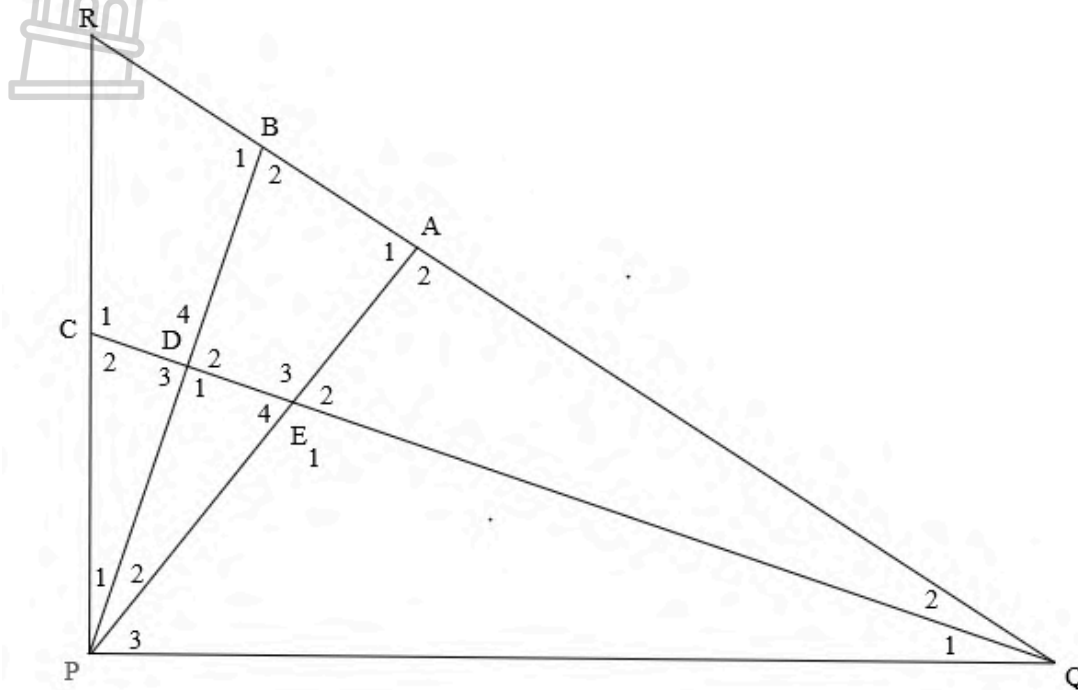
8.2.3 $\widehat{PAB}$

(2)

[13]

QUESTION 9

9.1 In the diagram below, triangle QRP is drawn with $RP \perp QP$ and $PA \perp QR$. QC intersects PA and PB at E and D respectively. $Q_1 = Q_2$ and $P_1 = P_2$



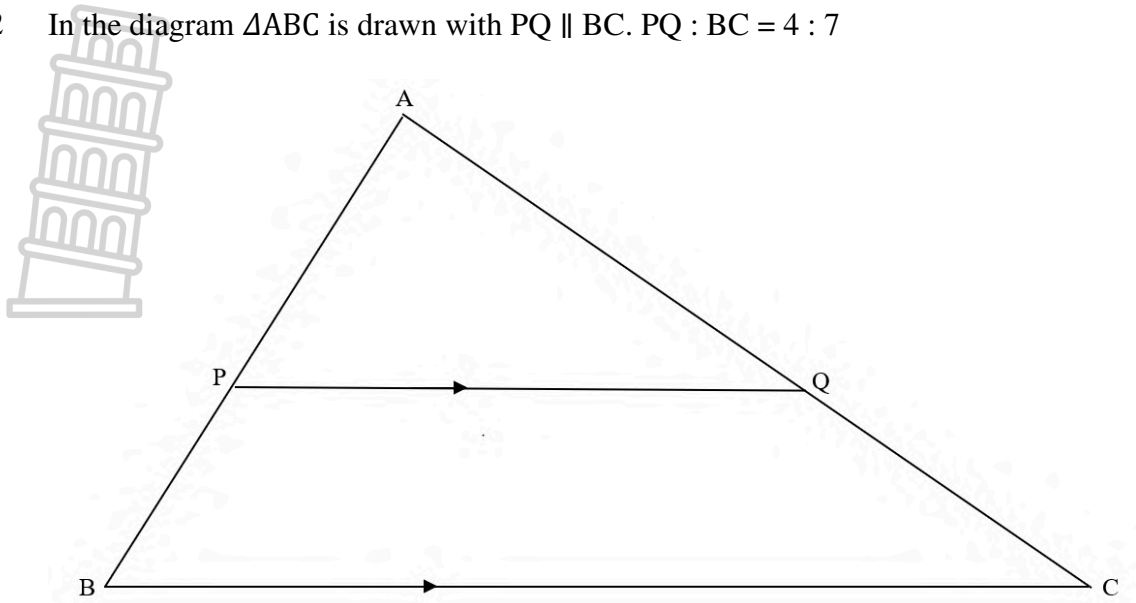
Prove:

9.1.1 BP is a tangent to circle PQE. (6)

9.1.2 ADPQ is a cyclic quadrilateral (3)



9.2 In the diagram $\triangle ABC$ is drawn with $PQ \parallel BC$. $PQ : BC = 4 : 7$

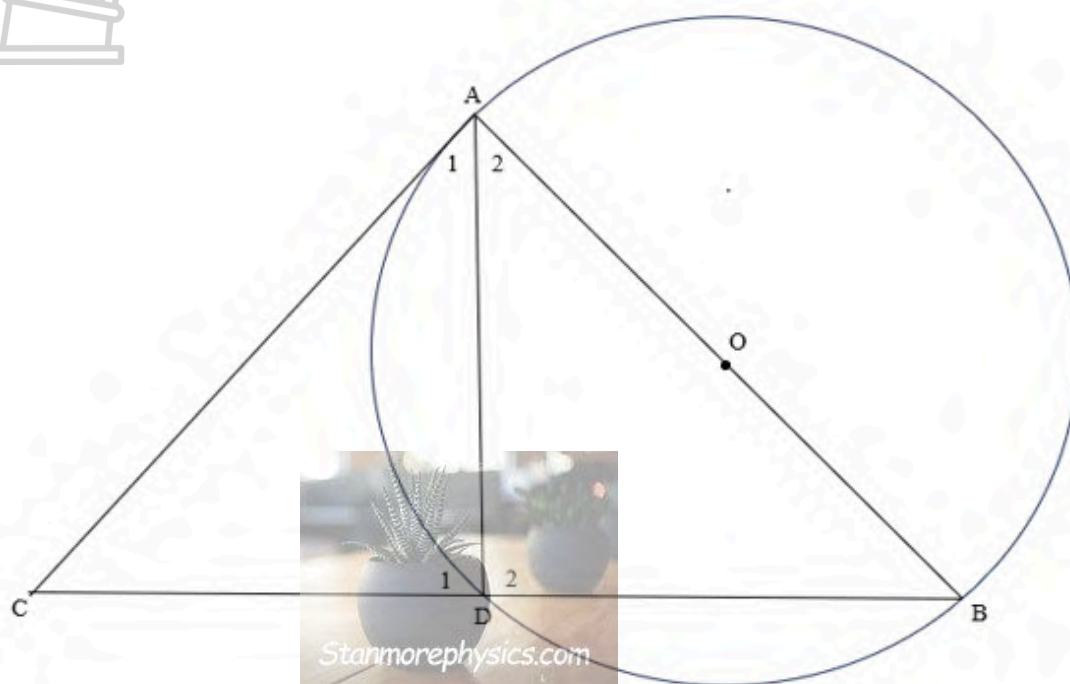


Prove $\triangle APQ = \frac{16}{49} \triangle ABC$

(5)
[14]

QUESTION 10

In the diagram below, CA is a tangent to the circle O with diameter AOB. Points A, D and B are points on the circumference of the circle. BD is produced to C and AD intersects CB at D. It is further given that $CD : DB = 2 : 3$.



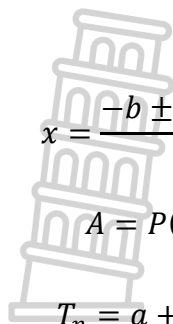
Prove

$$10.1 \quad AD^2 = 6x^2 \quad (6)$$

$$10.2 \quad \sqrt{AD^2 + AC^2 + AB^2} = x\sqrt{31} \quad (4)$$

[10]

TOTAL: 150



INFORMATION SHEET:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta$$

$$- \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta$$

$$+ \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha$$

$$= 2\sin \alpha \cos \alpha$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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**PREPARATORY EXAMINATION/
VOORBEREIDENDE EKSAMEN**

GRADE/GRAAD 12

**MATHEMATICS P2/
WISKUNDE V2**

SEPTEMBER 2024

Stanmorephysics.com

MARKS/PUNTE: 150

**MARKING GUIDELINES/
NASIENRIGLYNE**

**These marking guidelines consist of 22 pages./
Hierdie nasienriglyne bestaan uit 22 bladsye.**

NOTE:

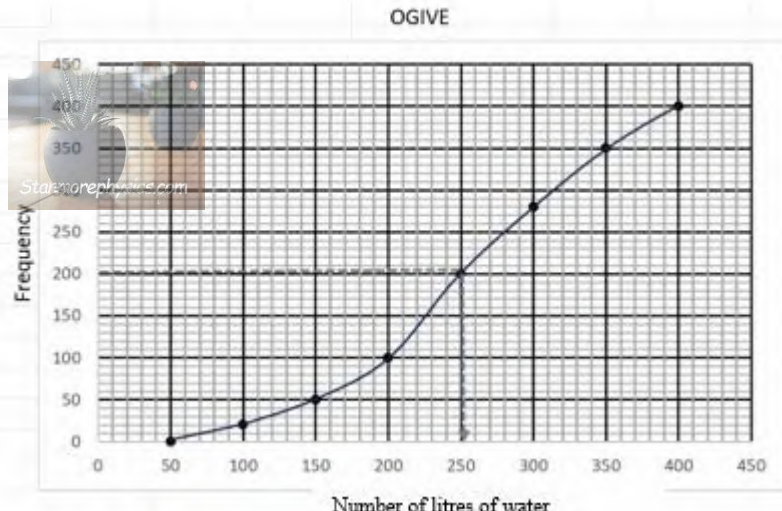
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.



QUESTION/VRAAG 1

1.1	<table><thead><tr><th>Number of litres used</th><th>Frequency (f)</th><th>Midpoint of interval (x)</th><th>Cumulative frequency</th><th>f.(x)</th></tr></thead><tbody><tr><td>$50 \leq x < 100$</td><td>20</td><td>75</td><td>20</td><td>1500</td></tr><tr><td>$100 \leq x < 150$</td><td>30</td><td>125</td><td>50</td><td>3750</td></tr><tr><td>$150 \leq x < 200$</td><td>50</td><td>175</td><td>100</td><td>8750</td></tr><tr><td>$200 \leq x < 250$</td><td>100</td><td>225</td><td>200</td><td>22500</td></tr><tr><td>$250 \leq x < 300$</td><td>80</td><td>275</td><td>280</td><td>22000</td></tr><tr><td>$300 \leq x < 350$</td><td>70</td><td>325</td><td>350</td><td>22750</td></tr><tr><td>$350 \leq x < 400$</td><td>50</td><td>375</td><td>400</td><td>18750</td></tr><tr><td colspan="4">TOTAL</td><td>100 000</td></tr></tbody></table> ✓ midpoint of interval column/<i>middelpunt van intervalekolom</i> ✓ cumulative frequency column/<i>kummulatiewe frekwensiekolom</i> ✓ f.(x) column/<i>f.(x) kolom</i> (3)	Number of litres used	Frequency (f)	Midpoint of interval (x)	Cumulative frequency	f.(x)	$50 \leq x < 100$	20	75	20	1500	$100 \leq x < 150$	30	125	50	3750	$150 \leq x < 200$	50	175	100	8750	$200 \leq x < 250$	100	225	200	22500	$250 \leq x < 300$	80	275	280	22000	$300 \leq x < 350$	70	325	350	22750	$350 \leq x < 400$	50	375	400	18750	TOTAL				100 000	
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TOTAL				100 000																																											
1.2	$\bar{x} = \frac{100000}{400}$ $= 250$	✓ substitute/<i>substitusie</i> ✓ answer/<i>antwoord</i> (2)																																													
1.3		✓ plotting points correctly/<i>stip punte korrek</i> ✓ shape/<i>vorm</i> (2)																																													
1.4	Median = 250	✓ accept/<i>aanvaar</i> (249–251) (1)																																													
1.5	Symmetrical. Mean = Median/ <i>Simmetries. Gemiddeldeld =Mediaan</i>	✓ answer/<i>antwoord</i> ✓ reason/<i>rede</i> (2)																																													

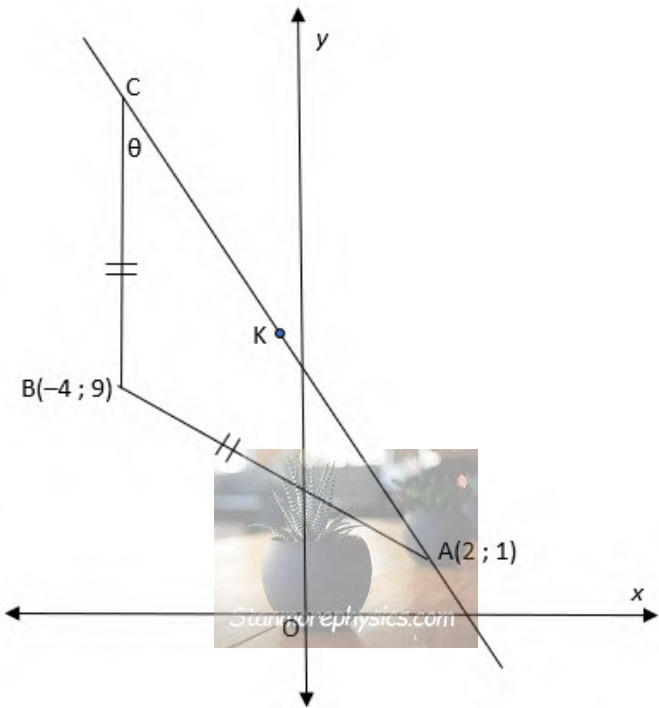
1.6	Standard deviation will increase/ <i>Standaard afwyking sal toeneem</i> The values will be further from the new mean./ <i>Die waardes sal verder wees vanaf die nuwe gemiddelde.</i>	✓ increase/vermeerder ✓ reason/rede (2)
		[12]

QUESTION/VRAAG 2

Number of leg presses done per day (x)	36	136	51	126	90	43	77	68	103	124
Strength of upper leg (y)	0,2	0,85	0,35	0,91	0,73	0,34	0,61	0,59	0,78	0,90

2.1	$a = 0,0478072\dots$ $b = 0,0067704\dots$ $\hat{y} = 0,05 + 0,01x$	Answer only: full marks, but if a and b are swapped only 1/3 marks/ <i>Slegs antwoord : volpunte, maar as a en b omgeruil is, slegs 1/3</i>	✓ a ✓ b ✓ equation/vergelyking (3)
2.2	$\hat{y} = 0,0478072 + 0,0067704(110)$ $= 0,79255196$ $= 0,79$ OR $\hat{y} = 0,05 + 0,01(110)$ $= 1,15$	Answer only: full marks/ <i>Slegs antwoord : volpunte</i> Accept 1,15/Aanvaar 1,15	✓ substitute 110 into eq./vervang 110 in vgl. ✓ answer/antwoord (2)
2.3	No. This is extrapolation./ <i>Nee. Dit is ekstrapolasie.</i>		✓ answer/antwoord ✓ reason/rede (2)
2.4	$r = 0,97$	No penalty for rounding/ <i>Geen penalisasie vir afronding</i>	✓ ✓ $r = 0,97$ (2)
2.5	There is a positive strong correlation between the number of leg presses and leg strength of an eighteen-year-old boy./ <i>Daar is 'n positief sterk korrelasie tussen beendrukke en beenkrag van 'n agtienjarige seun.</i>		✓ positive/positief ✓ strong/sterk (2)
			[11]

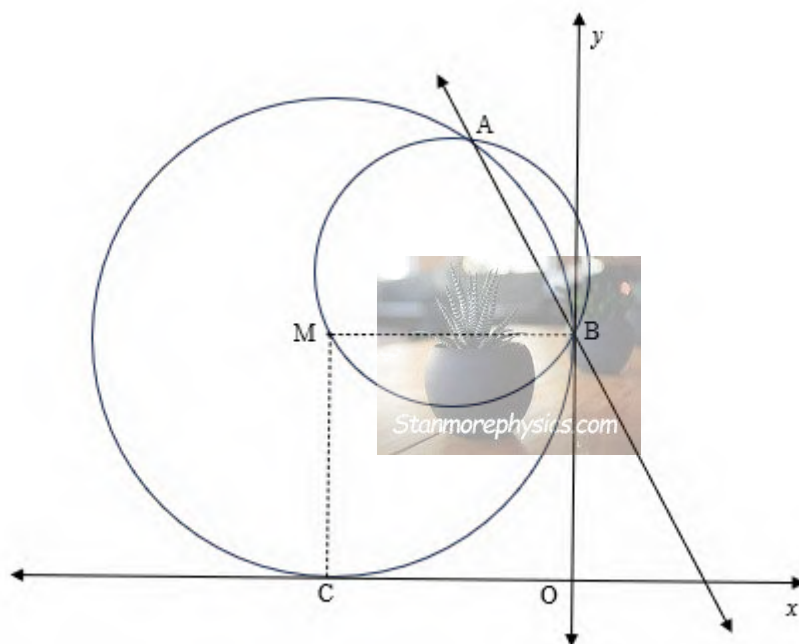
QUESTION/VRAAG 3



3.1	$AB = \sqrt{(2 + 4)^2 + (1 - 9)^2}$ $= \sqrt{100}$ $= 10$	✓subst./vervang ✓answer/antwoord (2)
3.2	$AB = BC = 10$ y-value $9 + 10$ $C (-4 ; 19)$ BC to y – axis Answer only: full marks/ Slegs antwoord: volpunte	✓$x = -4$ ✓$y = 19$ (2)
3.3	$K = \left(\frac{2 + (-4)}{2} ; \frac{1 + 19}{2} \right)$$= (-1 ; 10)$ Answer only: full marks./ Slegs antwoord: volpunte	✓subst./vervang ✓answer/antwoord (2)
3.4	$m_{AC} = \frac{19 - 1}{-4 - 2}$$= \frac{18}{-6}$$= -3$ through/deur(2; 1) $y - 1 = -3(x - 2)$ $\therefore y = -3x + 7$	✓$m_{AC} = -3$ ✓subst./vervang (2; 1) ✓equation/vergelyking (3)

	OR/OF $m_{AC} = \frac{19 - 1}{-4 - 2}$ $= \frac{18}{-6}$ $= -3$ through/deur $(-4; 19)$ $y - 19 = -3(x - (-4))$ $-19 = -3x - 12$ $\therefore y = -3x + 7$	✓ $m_{AC} = -3$ ✓ subst./vervang $(-4; 19)$ ✓ equation/vergelyking (3)
3.5	$\tan \beta = m_{AC} = -3$ $\text{ref angle} = 71,57^\circ$ $\beta = 108,43^\circ$ $\theta = 90^\circ - 71,57^\circ$ $= 18,43^\circ$ OR/OF In a right angled triangle $\tan \theta = \frac{6}{18}$ $\theta = 18,43^\circ$	✓ $\tan \beta = m_{AC} = -3$ ✓ method/metode ✓ answer/antwoord (3) ✓ value of 6 ✓ value of 18 ✓ answer/antwoord (3)
3.6	$AC = 6\sqrt{10}$ $\perp h = \sqrt{10}$ $\text{Area } \Delta ABC = \frac{1}{2} \cdot b \cdot \perp h$ $= \frac{1}{2} \cdot 6\sqrt{10} \cdot \sqrt{10}$ $= 30 \text{ units}^2 / \text{eenhede}^2$ OR/OF $AC = 6\sqrt{10}$ $BC = 10$ $\text{Area } \Delta ABC = \frac{1}{2} \cdot BC \cdot AC \cdot \sin C$ $= \frac{1}{2} \cdot 10 \cdot 6\sqrt{10} \cdot \sin 18,43^\circ$ $= 29,99 \text{ units}^2 / \text{eenhede}^2$	✓ $AC = 6\sqrt{10}$ ✓ $\perp h = \sqrt{10}$ ✓ substitution into correct formula/substitusie in korrek formule ✓ answer/antwoord (4) ✓ $AC = 6\sqrt{10}$ ✓ $BC = 10$ ✓ substitution into correct formula/substitusie in korrek formule ✓ answer/antwoord (4)
3.7	D (2 ; 11)	✓ x - value/waarde ✓ y - value/waarde (2)
		[18]

QUESTION/VRAAG 4



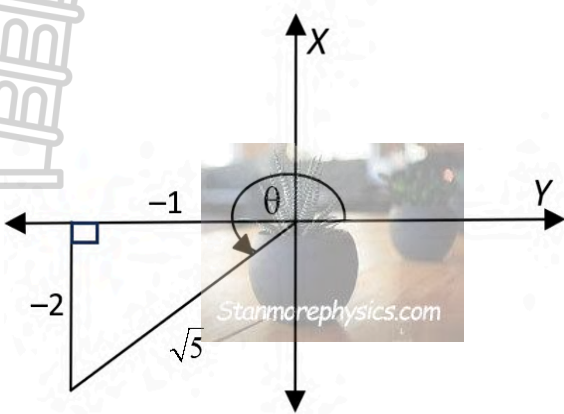
4.1	$x^2 + x + y^2 - 3y = -2$ $x^2 + x + \left(\frac{1}{2}\right)^2 + y^2 - 3y + \left(\frac{-3}{2}\right)^2 = -2 + \frac{1}{4} + \frac{9}{4}$ $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{1}{2}$ centre $\left(\frac{-1}{2}; \frac{3}{2}\right)$ radius $= \sqrt{\frac{1}{2}}$	✓ LHS/LK ✓ RHS/RK ✓ centre/middelpunt ✓ radius/radius (4)
4.2	$y = -x + 1 \dots (1)$ $x^2 + x + y^2 - 3y + 2 = 0 \dots (2)$ substitute (1) into (2) $x^2 + x + (-x + 1)^2 - 3(-x + 1) + 2 = 0$ $x^2 + x + x^2 - 2x + 1 + 3x - 3 + 2 = 0$ $2x^2 + 2x = 0$ $2x(x + 1) = 0$ $x = 0$ or $x = -1$ $A(-1; 2)$ OR/OF	✓ substitute <i>substitusie</i> ✓ standard form/standard vorm ✓ factors/faktore ✓ value(s) of x/waardes van x ✓ coordinates of A/koördinate van A (5)

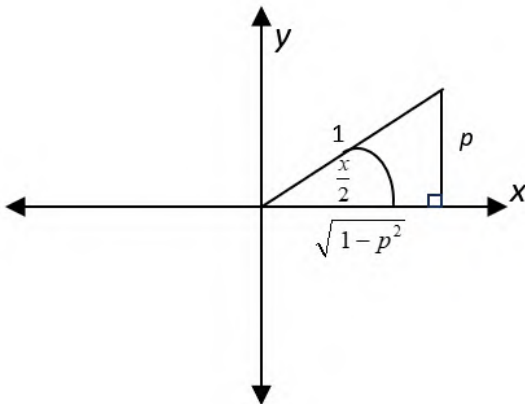
	$x = -y + 1 \dots (1)$ $x^2 + 2x + y^2 - 2y + 1 = 0 \dots \dots (2)$ substitute(1) into (2) $(-y + 1)^2 + 2(-y + 1) + y^2 - 2y + 1 = 0$ $y^2 - 2y + 1 - 2y + 2 + y^2 - 2y + 1 = 0$ $2y^2 - 6y + 4 = 0$ $2(y^2 - 3y + 2) = 0$ $(y - 2)(y - 1) = 0$ $y = 2$ or/of $y = 1$ $A(-1; 2)$	✓ substitute <i>substitusie</i> ✓ standard form/ <i>standaard vorm</i> ✓ factors/ <i>faktore</i> ✓ value(s) of <i>y/waardes van y</i> ✓ coordinates of <i>A/koördinate van A</i> (5)
4.3	y – intercept $x = 0$ y – intercept $x = 0$ $(0)^2 + y^2 + (0) - 3y + 2 = 0$ $y^2 - 3y + 2 = 0$ $(y - 1)(y - 2) = 0$ $y = 2$ or $y = 1$ Centre $M(-1; 1)$ passes through $(0; 1)$ $(x + 1)^2 + (y - 1)^2 = r^2$ substitute $(0; 1)$ $(0 + 1)^2 + (1 - 1)^2 = r^2$ $r^2 = 1$ $(x + 1)^2 + (y - 1)^2 = 1$ $x^2 + 2x + 1 + y^2 - 2y + 1 - 1 = 0$ $x^2 + 2x + y^2 - 2y + 1 = 0$ OR/OF $y = -x + 1$ y – intercept $x = 0$ y - intercept $(0; 1)$ Centre $M(-1; 1)$ passes through $(0; 1)$ $(x + 1)^2 + (y - 1)^2 = r^2$ substitute $(0; 1)$ $(0 + 1)^2 + (1 - 1)^2 = r^2$ $r^2 = 1$ $(x + 1)^2 + (y - 1)^2 = 1$ $x^2 + 2x + 1 + y^2 - 2y + 1 - 1 = 0$ $x^2 + 2x + y^2 - 2y + 1 = 0$	✓ substitute/ <i>substitusie</i> $x = 0$ ✓ factors/ <i>faktore</i> ✓ both <i>y-values/beide y-waardes</i> ✓ substitute centre M and point/ <i>substitusie in middelpunt M en punt</i> ✓ equation/ <i>vergelyking</i> (5) ✓ substitute/ <i>substitusie</i> $x = 0$ ✓ point / <i>punt</i> $(0; 1)$ ✓ substitute centre M and point/ <i>substitusie van middelpunt M en punt</i> ✓ $r^2 = 1$ ✓ equation/ <i>vergelyking</i> (5)
4.4.1	$y = -x + k \dots (1)$ $x^2 + y^2 + 2x - 2y + 1 = 0 \dots \dots (2)$ substitute(1) into (2) $x^2 + (-x + k)^2 + 2x - 2(-x + k) + 1 = 0$ $x^2 + x^2 - 2kx + k^2 + 2x + 2x - 2k + 1 = 0$ $2x^2 - 2kx + 4x + k^2 - 2k + 1 = 0$ $2x^2 + (4 - 2k)x + (k^2 - 2k + 1) = 0$	✓ substitute/ <i>substitusie</i> ✓ simplify/ <i>vereenvoudig</i> ✓ standard form/ <i>standaard vorm</i> (3)

4.4.2	Equal roots $\Delta = 0$ $b^2 - 4ac = 0$ $(4 - 2k)^2 - 4(2)(k^2 - 2k + 1) = 0$ $16 - 16k + 4k^2 - 8k^2 + 16k - 8 = 0$ $-4k^2 + 8 = 0$ $-4k^2 = -8$ $k^2 = 2$ $k = \pm\sqrt{2}$	$\checkmark (4 - 2k)^2 - 4(2)(k^2 - 2k + 1) = 0$ $\checkmark$ simplify/vereenvoudig $\checkmark$ standard form/standaard vorm $\checkmark k = \sqrt{2}$ or/of 1,41 $\checkmark k = -\sqrt{2}$ or/of -1,41 (5)
		[22]



QUESTION/VRAAG 5

5.1.1	 $\tan \theta = \frac{-2}{-1}$ $= 2$	✓ $x = -1$ ✓ answer/antwoord (2)
5.1.2	$\cos 2\theta$ $= 2 \cos^2 \theta - 1$ $= 2 \left(\frac{-1}{\sqrt{5}} \right)^2 - 1$ $= \frac{-3}{5}$ OR/OF $1 - 2 \sin^2 \theta$ $= 1 - 2 \left(\frac{-2}{\sqrt{5}} \right)^2$ $= \frac{-3}{5}$ OR/OF $\cos^2 \theta - \sin^2 \theta$ $= \left(\frac{-1}{\sqrt{5}} \right)^2 - \left(\frac{-2}{\sqrt{5}} \right)^2$ $= \frac{-3}{5}$	✓ double angle identity/dubbel hoek identiteit ✓ substitution into correct formula/substitusie in korrekte formule ✓ answer/antwoord (3) ✓ double angle identity/dubbel hoek identiteit ✓ substitution into correct formula/substitusie in korrekte formule ✓ answer/antwoord (3) ✓ double angle identity dubbel hoek identiteit ✓ substitution into correct formula/substitusie in korrekte formule ✓ answer/antwoord (3)

5.2	$2 \cos^2 15^\circ - 1 + \frac{2 \sin 140^\circ}{\cos 310^\circ}$ $= \cos 30^\circ + \frac{(2 \sin 40^\circ)}{(\cos 50^\circ)}$ $= \frac{\sqrt{3}}{2} + \frac{2 \sin 40^\circ}{\sin 40^\circ}$ $= \frac{\sqrt{3} + 4}{2}$	✓ $\cos 30^\circ$ ✓ $\sin 40^\circ$ ✓ $\cos 50^\circ$ ✓ $\sin 40^\circ$ ✓ answer/antwoord (5)
5.3	 $\sin x - 1$ $= 2 \sin \frac{x}{2} \cos \frac{x}{2} - 1$ $= 2 \left(\frac{p}{1} \right) \left(\frac{\sqrt{1-p^2}}{1} \right) - 1$ $= 2p\sqrt{1-p^2} - 1$	✓ $x = \sqrt{1-p^2}$ ✓ double angle/dubbelhoek ✓ substitution/substitusie ✓ answer/antwoord (4)
5.4	$\frac{3 \sin x + 2(2 \sin x \cos x)}{2 + 3 \cos x + 2(2 \cos^2 x - 1)}$ $= \frac{\sin x \cdot (3 + 4 \cos x)}{4 \cos^2 x + 3 \cos x}$ $= \frac{\sin x \cdot (3 + 4 \cos x)}{\cos x \cdot (4 \cos x + 3)}$ $= \tan x$	✓ $2 \sin x \cos x$ ✓ $2 \cos^2 x - 1$ ✓ $\sin x (3 + 4 \cos x)$ ✓ $\cos x (4 \cos x + 3)$ (4)

5.5	$\frac{\sin x + \cos x}{\sin x - \cos x}$ $= \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}}$ $= \frac{\tan x + 1}{\tan x - 1}$ $= \frac{\frac{p}{t} + 1}{\frac{p}{t} - 1}$ $= \frac{p + t}{p - t} \div \frac{p - t}{t}$ $= \frac{p + t}{p - t} \times \frac{t}{p - t}$ $= \frac{p + t}{p - t}$	✓divide numerator and denominator by cos x/deel noemer en teller met cos x ✓$\frac{\tan x+1}{\tan x-1}$ ✓$\frac{\frac{p}{t}+1}{\frac{p}{t}-1}$ ✓$\frac{p+t}{t} \div \frac{p-t}{t}$ ✓$\frac{p+t}{t} \times \frac{t}{p-t}$ (5)
		[23]

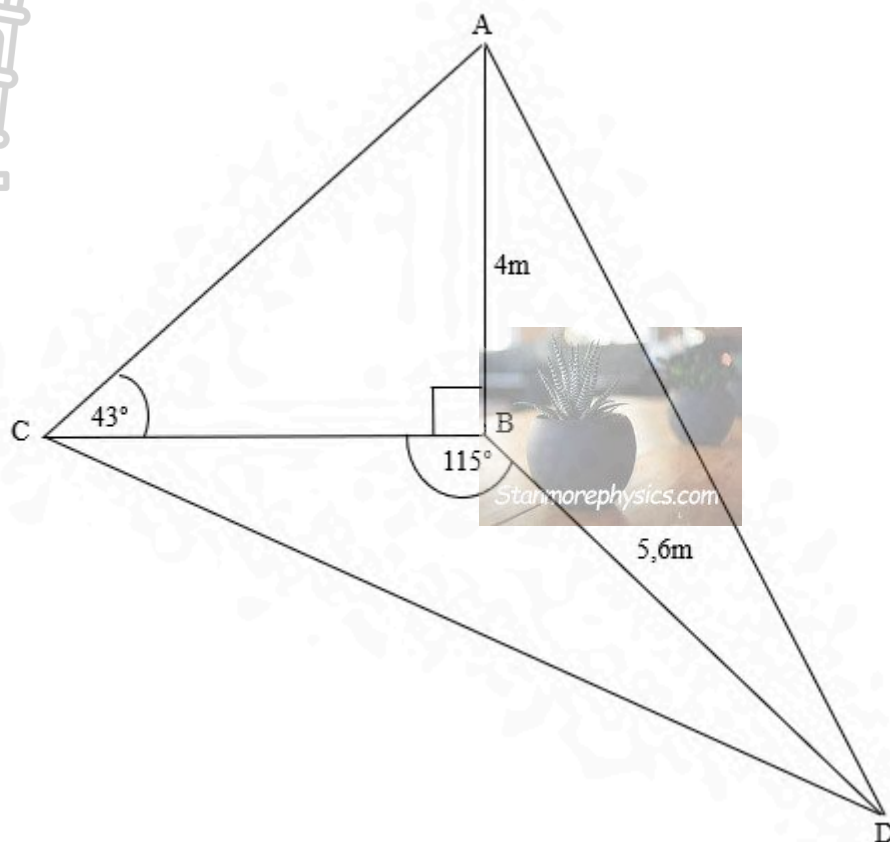
QUESTION/VRAAG 6

6.1	✓ turning points/draaipunte ✓ x-intercepts/x-afsnitte ✓ end points/eindpunte
6.2.1	$\sin 2x = \cos(x - 45^\circ)$ $\sin 2x = \sin[90^\circ - (x - 45^\circ)]$ $\sin 2x = \sin(135^\circ - x)$ $2x = 135^\circ - x + k \cdot 360^\circ; k \in \mathbb{Z}$ $3x = 135^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 45^\circ + k \cdot 120^\circ; k \in \mathbb{Z}$ OR/OF $2x = 180^\circ - (135^\circ - x) + k \cdot 360^\circ; k \in \mathbb{Z}$ $2x = 45^\circ + x + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 45^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x \in \{45^\circ; 165^\circ\}$ OR/OF $\sin 2x = \cos(x - 45^\circ)$ $\cos(90^\circ - 2x) = \cos(x - 45^\circ)$ $90^\circ - 2x = x - 45^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $-3x = -135^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 45^\circ + k \cdot 120^\circ; k \in \mathbb{Z}$

	OR/OF $90^\circ - 2x = 360^\circ - (x - 45^\circ) + k \cdot 360^\circ; k \in \mathbb{Z}$ $90^\circ - 2x = 360^\circ - x + 45^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $-x = 315^\circ + k \cdot 360; k \in \mathbb{Z}$ $x = -315^\circ + k \cdot 360; k \in \mathbb{Z}$ $x \in \{45^\circ; 165^\circ\}$	(7)
6.2.2	$x \in \{15^\circ; 135^\circ\}$	✓ $x = 15^\circ$ ✓ $x = 135^\circ$
6.2.3	$x \in (165^\circ; 180^\circ)$ OR/OF $165^\circ < x < 180^\circ$	✓✓ critical values & notation/kritiese waardes en notasie (2)
6.3	360°	✓ answer/antwoord (1)
6.4	$\sqrt{2} \sin 2x = \cos x + \sin x$ $\sin 2x = \frac{\cos x + \sin x}{\sqrt{2}}$ $\sin 2x = \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$ $\sin 2x = \cos x \cos 45^\circ + \sin x \sin 45^\circ$ $\sin 2x = \cos (x - 45^\circ)$ $f(x) = g(x)$	✓ $\sin 2x = \frac{\cos x + \sin x}{\sqrt{2}}$ ✓ $\sin 2x = \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$ ✓ $\sin 2x = \cos x \cos 45^\circ + \sin x \sin 45^\circ$ (3)
		[18]



QUESTION/VRAAG 7

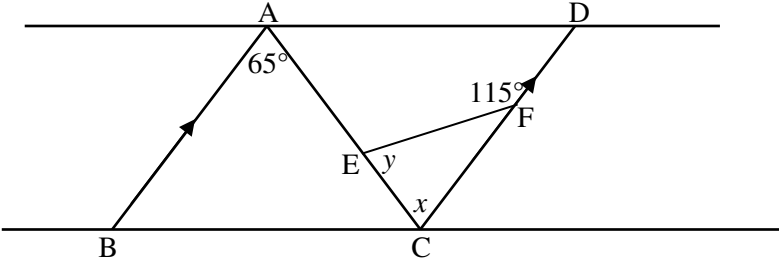


7.1	$\tan 43^\circ = \frac{4}{CB}$ $CB = \frac{4}{\tan 43^\circ}$ $= 4,29 \text{ m}$ OR/OF $\frac{CB}{\sin 47^\circ} = \frac{4}{\sin 43^\circ}$ $CB = \frac{4 \sin 47^\circ}{\sin 43^\circ}$ $= 4,29 \text{ m}$	✓ substitution into the correct formula/substitusie in korrek formule ✓ CB subject of formula ✓ answer/antwoord (3)
	$\frac{CB}{\sin 47^\circ} = \frac{4}{\sin 43^\circ}$ $CB = \frac{4 \sin 47^\circ}{\sin 43^\circ}$ $= 4,29 \text{ m}$	✓ substitution into the correct formula/substitusie in korrek formule ✓ CB subject of formula ✓ answer/antwoord (3)

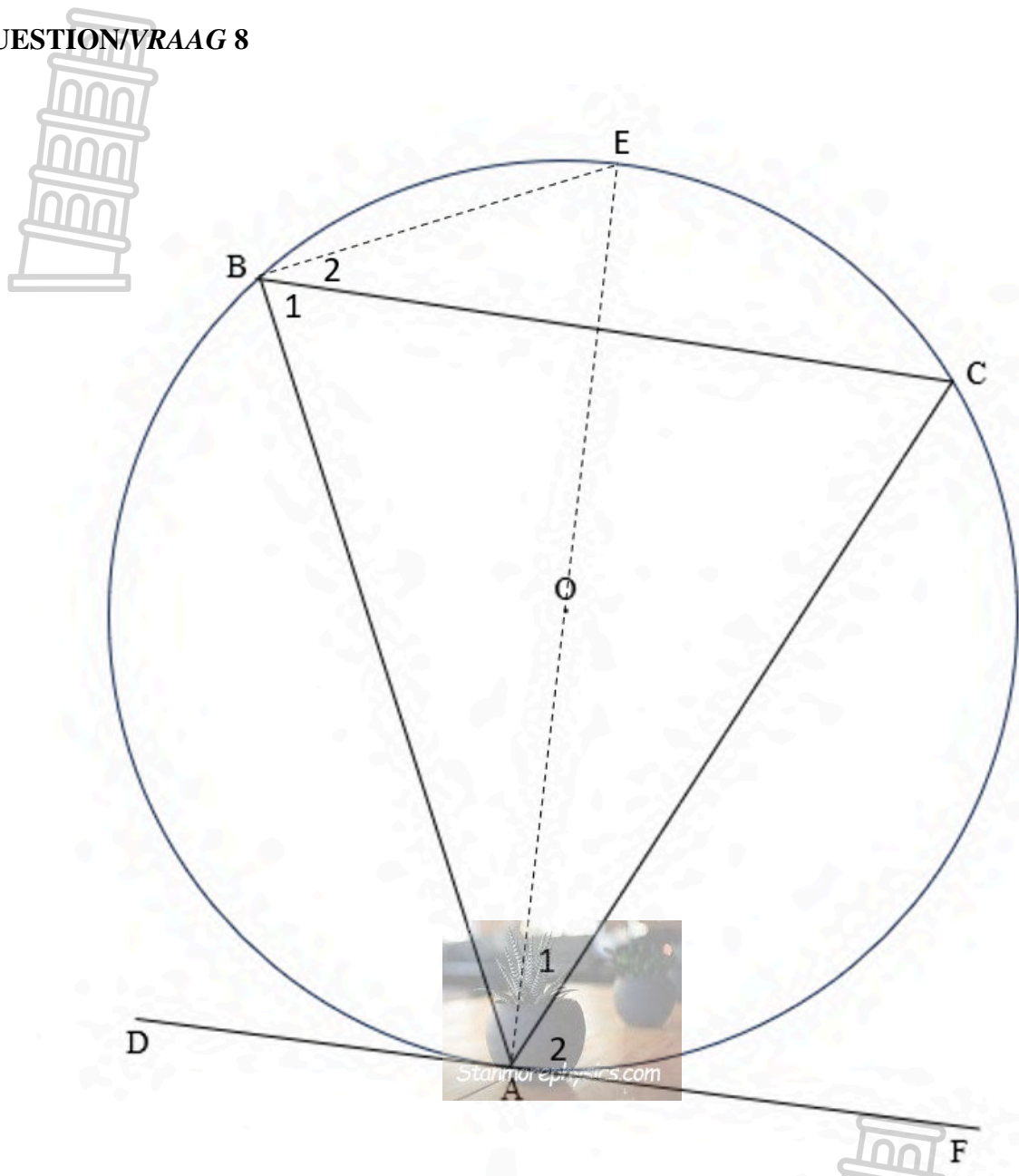
7.2	$CD^2 = CB^2 + BD^2 - 2CB \cdot BD \cdot \cos 115^\circ$ $= (4,29)^2 + (5,6)^2 - 2(4,29)(5,6) \cos 115^\circ$ $= 70,07006$ $CD = 8,37 \text{ m}$	✓ substitution into correct formula/substitusie in korrekte formule ✓ simplification/vereenvoudig ✓ answer/antwoord (3)
7.3	$\text{Area } \Delta BCD = \frac{1}{2} CB \cdot BD \cdot \sin B$ $= \frac{1}{2} (4,29)(5,6) \sin 115^\circ$ $= 10,89m^2$	✓ formula/ formule ✓ substitution into correct formula/substitusie in korrekte formule ✓ answer/antwoord (3)
		[9]

GEOMETRY/MEETKUNDE

Please read carefully through the following table before marking **QUESTION 8–10/**
*Lees asseblief sorgvuldig deur die volgende tabel alvorens **VRAAG 8–10** nagesien word.*

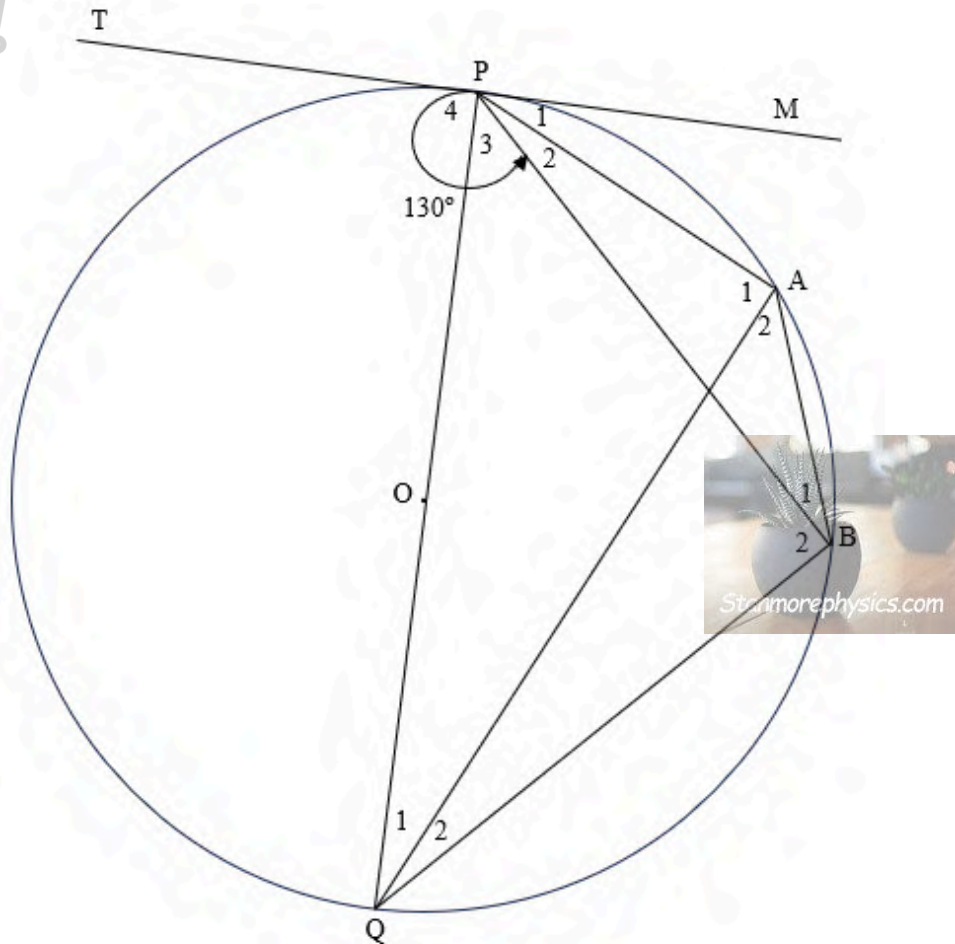
	The order in which the candidate answers a geometry question must follow logically/<i>Die volgorde waarin 'n kandidaat 'n meetkundevraag beantwoord moet logies volg.</i> Example/Voorbeeld Given/<i>Gegee</i> $AB \parallel CD$ and/<i>en</i> $\widehat{EFD} = 115^\circ$  The candidate first needs to calculate x BEFORE he/she can calculate y/<i>Die kandidaat moet eerste vir x bereken VOORDAT hy/sy vir y kan bereken.</i>
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering</i> <i>('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede</i> <i>('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if the statement AND reason are both correct (Both MUST be correct to get one mark)
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i> <i>(Beide MOET korrek wees om een punt te kry)</i>

QUESTION/VRAAG 8



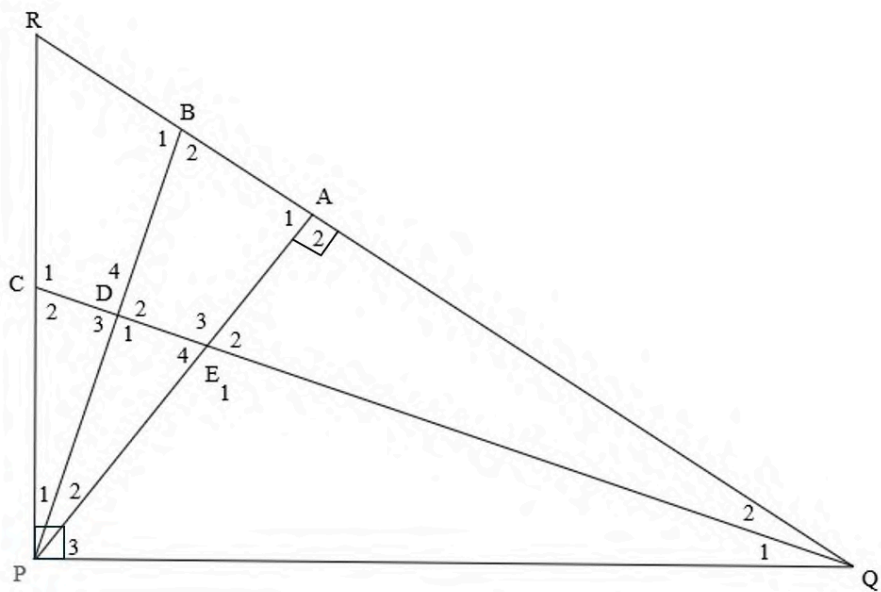
8.1	Construction/<i>Konstruksie</i>: Draw AE through O and join BE/<i>Trek AE deur O en verbind BE.</i> $\hat{A}_1 + \hat{A}_2 = 90^\circ$ radius $\perp$ tangent/<i>radius $\perp$ raaklyn</i> $\hat{B}_1 + \hat{B}_2 = 90^\circ$ angles in a semi circle/<i>hoeke in halfsirkel</i> $\hat{A}_1 = \hat{B}_2$ subtended by the same arc/<i>onderspan deur dieselfde boog</i> $\hat{A}_2 = \hat{B}_1$	✓ constr./konstr. ✓ S/R ✓ S/R ✓ S/R ✓ S (5)
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8.2

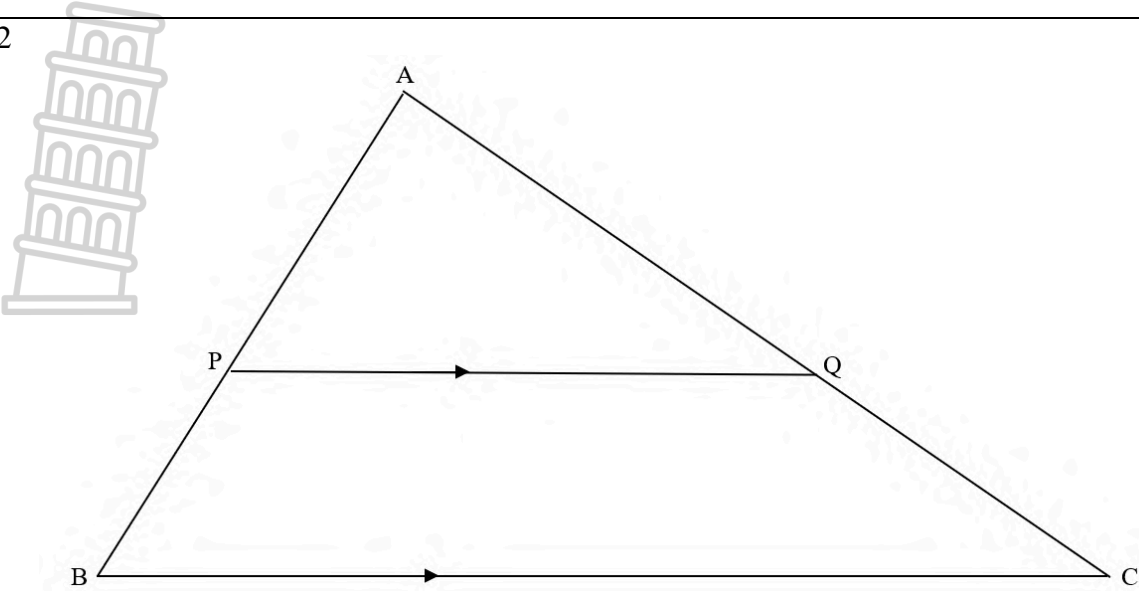


8.2.1	$\widehat{P}_4 = 90^\circ$ radius $\perp$ tangent/ <i>radius $\perp$ raaklyn</i> $\widehat{P}_3 = 130^\circ - 90^\circ$ $= 40^\circ$	$\checkmark S \checkmark R$ $\checkmark S$ (3)
8.2.2	$\widehat{B}_2 = 90^\circ$ angles in semi-circle/ <i>hoeke in halfsirkel</i> $\widehat{Q}_1 + \widehat{Q}_2 = 50^\circ$ sum of interior angles of Δ / <i>binnehoeke van Δ</i>	$\checkmark S \checkmark R$ $\checkmark S/R$ (3)
8.2.3	$\widehat{PAB} = 130^\circ$ opp. angles of cyclic quad OR tan chord theorem/ <i>teenoorst. hoek van koordevierhoek OF tan koordstelling</i>	$\checkmark S \checkmark R$ (2)
		[13]

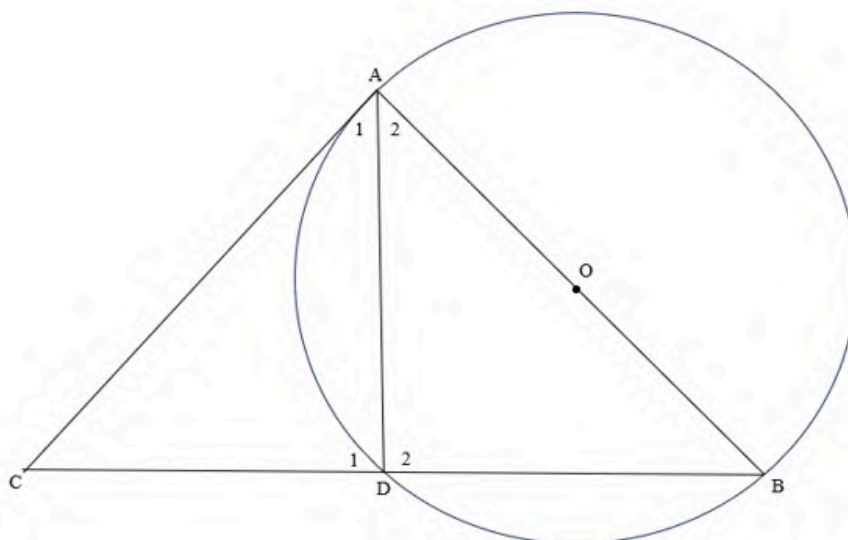
QUESTION/VRAAG 9



9.1.1	$P_1 + P_2 + P_3 = 90^\circ$ given/gegee $\widehat{Q}_1 + \widehat{Q}_2 + \widehat{P}_3 = 90^\circ$ sum int $\angle$'s of Δ /som van binne $\angle$ 'e van Δ $\widehat{Q}_1 + \widehat{Q}_2 = \widehat{P}_1 + \widehat{P}_2$ $\widehat{Q}_1 = \widehat{Q}_2$ and $\widehat{P}_1 = \widehat{P}_2$ given/gegee $2\widehat{Q}_1 = 2\widehat{P}_2$ $\widehat{Q}_1 = \widehat{P}_2$ PB is a tangent angle between line and chord/ <i>hoek tussen lyn en koord</i>	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ R
9.1.2	$\widehat{Q}_1 + \widehat{Q}_2 = \widehat{P}_1 + \widehat{P}_2$ given/gegee $2\widehat{Q}_2 = 2\widehat{P}_2$ $\widehat{Q}_2 = \widehat{P}_2$ ADPQ is a cyclic quad AD subtends equal $\angle$'s/ <i>AD onderspan gelyke $\angle$'e</i>	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R

9.2		
9.2	$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} = \frac{4}{7}$ proportionality theorem $PQ \parallel BC$ / <i>eweredigheidsstelling $PQ \parallel BC$</i> $\frac{Area \Delta APQ}{Area \Delta ABC} = \frac{\frac{1}{2} AP \cdot AQ \cdot \sin A}{\frac{1}{2} AB \cdot AC \cdot \sin A}$$= \frac{AP \cdot AQ}{AB \cdot AC}$$= \frac{4 \cdot 4}{7 \cdot 7}$$= \frac{16}{49}$	✓S✓R ✓correct formula/korrekte formule ✓substitution/substitusie ✓answer/antwoord (5) [14]

QUESTION/VRAAG 10



10.1	$\hat{D}_2 = 90^\circ$ In $\triangle ADC$ and $\triangle BDA$ $\hat{A}_1 = \hat{B}$ $\hat{D}_1 = \hat{D}_2 = 90^\circ$ $\hat{C} = \hat{A}_2$ $\triangle ADC \text{ } \text{//} \triangle BDA$ $\frac{AD}{BD} = \frac{DC}{DA} = \frac{AC}{BA}$ $AD^2 = BD \cdot DC$ $= 3x \cdot 2x$ $= 6x^2$	angle in semi-circle/ <i>hoek in halfsirkel</i> tan chord theorem/ <i>tankoord stelling</i> angles on str. line/ <i>hoeke op reguitlyn</i> sum of int $\angle$'s of $\triangle$ / <i>som van binne$\angle$'e van $\triangle$</i> $[\angle\angle\angle]$ similar $\triangle$'s/ <i>gelykvormige $\triangle$'e</i>	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S $\checkmark$ S/R $\checkmark$ S (6)
10.2	$\hat{A}_1 + \hat{A}_2 = 90^\circ$ $CB^2 = AC^2 + AB^2$ $(5x)^2 = AC^2 + AB^2$ $25x^2 = AC^2 + AB^2$ $AC^2 + AB^2 + AD^2$ $= 25x^2 + 6x^2$ $= 31x^2$ $\sqrt{AC^2 + AB^2 + AD^2} = \sqrt{31x^2}$ $= x\sqrt{31}$	radius $\perp$ tangent/ <i>radius $\perp$ raaklyn</i> pythagoras' theorem/ <i>stelling</i>	$\checkmark$ S $\checkmark$ S $\checkmark$ Simplify/ <i>vereenv</i> $\checkmark$ S (4)
			[10]

TOTAL/TOTAAL: 150