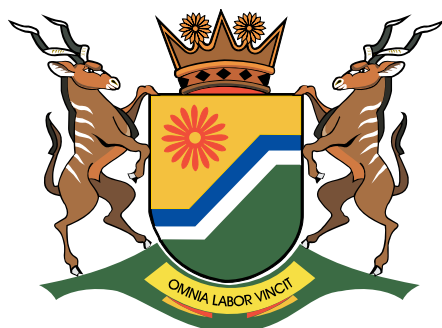




education

MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

MST ACADEMY SCHOOLS

**EUCLIDEAN GEOMETRY
REVISION MATERIAL**

EXAMINABLE THEOREMS

GRADE 12

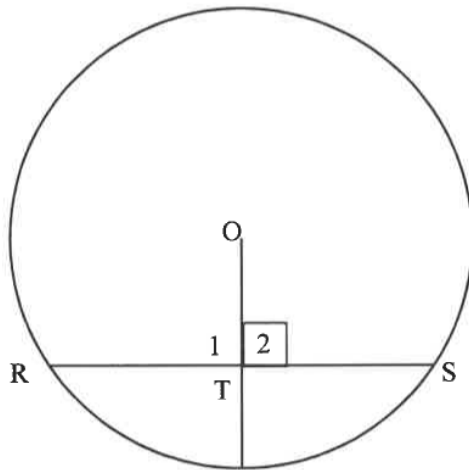
EUCLIDEAN GEOMETRY AND MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus and two- and three-dimensional trigonometry.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
 - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
 - The line drawn from the centre of a circle that bisects a chord is perpendicular to the chord;
 - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
 - The opposite angles of a cyclic quadrilateral are supplementary;
 - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
 - A line drawn parallel to one side of a triangle divides the other two sides proportionally;
 - Equiangular triangles are similar.

QUESTION 1

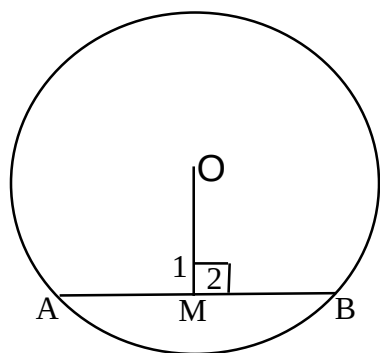
1.1

In the diagram below, O is the centre of the circle and point T lies on chord RS .
Prove the theorem which states that if $OT \perp RS$ then $RT = TS$.



(5)

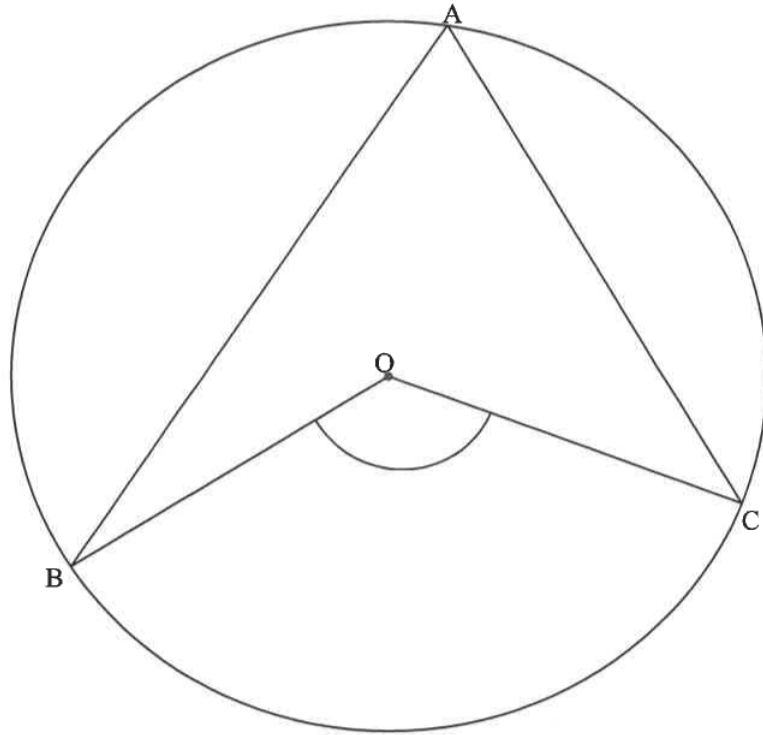
1.2



Use the diagram above to prove the theorem which states that if $AM = MB$, then $OM \perp AB$

QUESTION 2

In the diagram, O is the centre of the circle with A , B and C drawn on the circle.

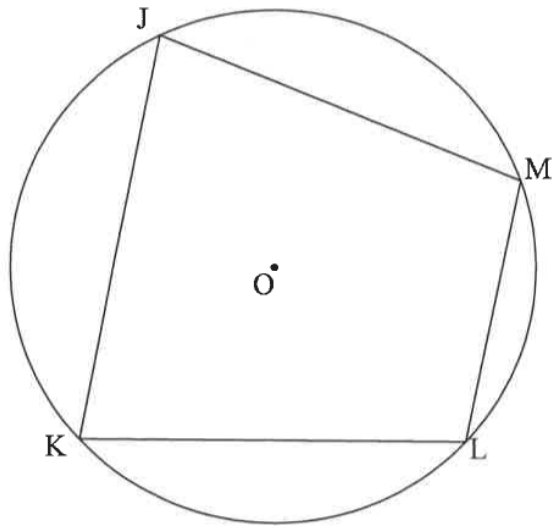


Prove the theorem which states that $\angle BOC = 2\angle A$. (5)

QUESTION 3

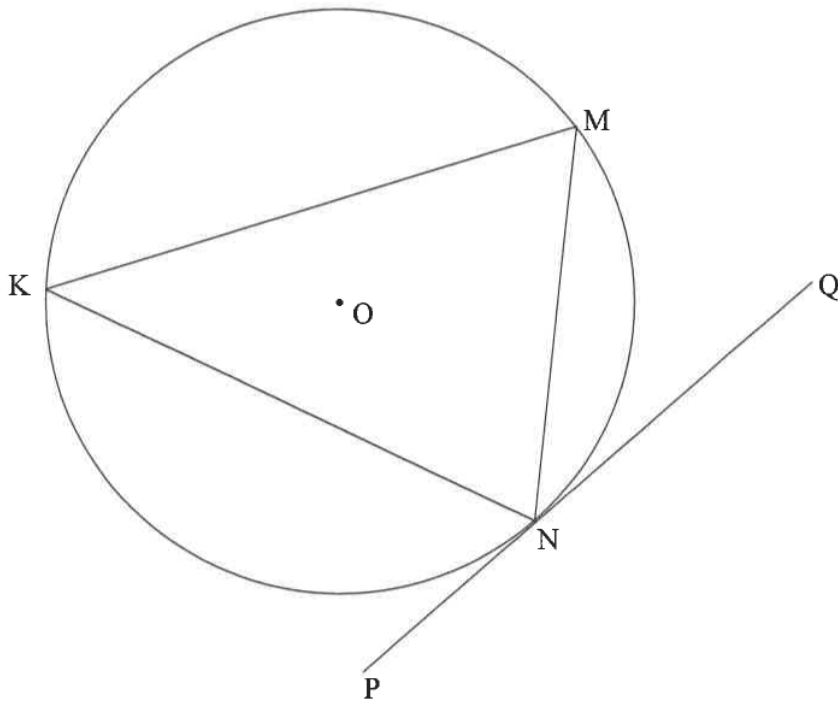
In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.
Prove the theorem which states that $\hat{J} + \hat{L} = 180^\circ$.

(5)



QUESTION 4

In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.

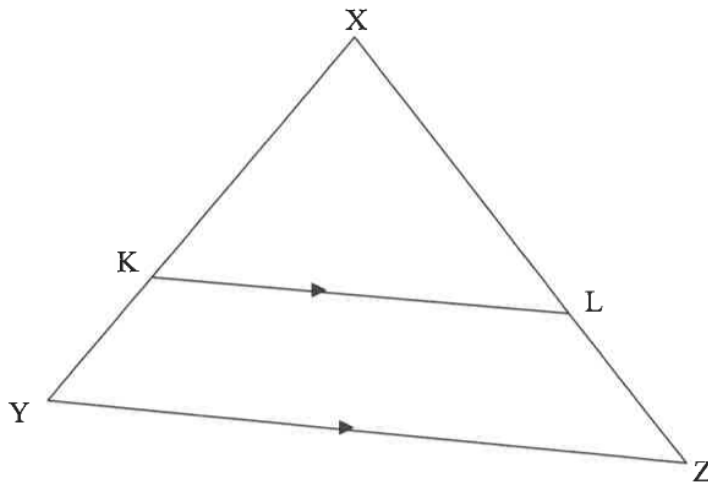


Prove the theorem which states that $\hat{MNQ} = \hat{K}$.

(5)

QUESTION 5

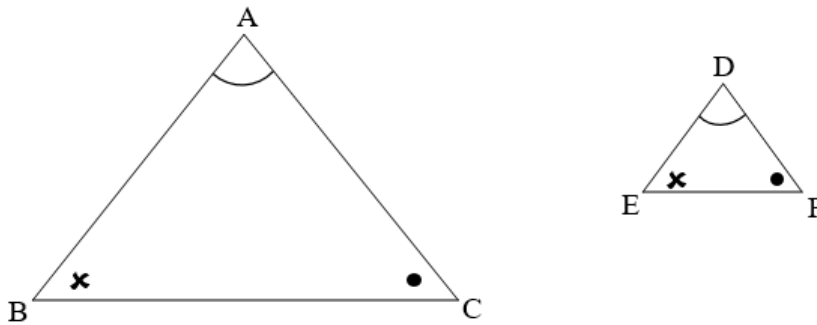
Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY} = \frac{XL}{LZ}$.



(5)

QUESTION 6

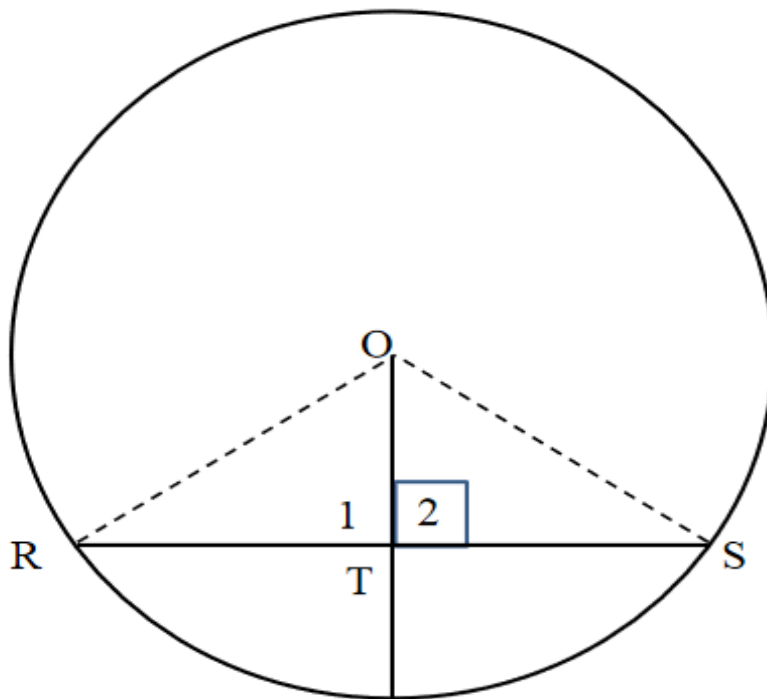
In the diagram, $\triangle ABC$ and $\triangle DEF$ are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$.

(6)

QUESTION 1



Construction/*Konstr*: Draw/*trek* radii OR and/en OS

In $\triangle OTR$ and/ *en* $\triangle OTS$

$OR = OS$ (radii)

$OT = OT$ (common side/ *gemene sy*)

$\hat{T}_1 = \hat{T}_2 = 90^\circ$ ($\angle^s$ on straight line/ *op 'n reguit lyn*)

$\triangle OTR \equiv \triangle OTS$ (90° HS)

$\therefore RT = TS$ ($\equiv \Delta^s$)

✓ *Constr/Konstr*

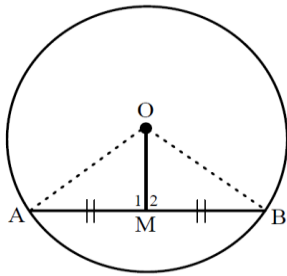
✓ S (OT is
common/*gemeen*)

✓ S/R

✓ R

✓ S

(5)



Join OA and OB

In $\triangle OAM$ and $\triangle OBM$:

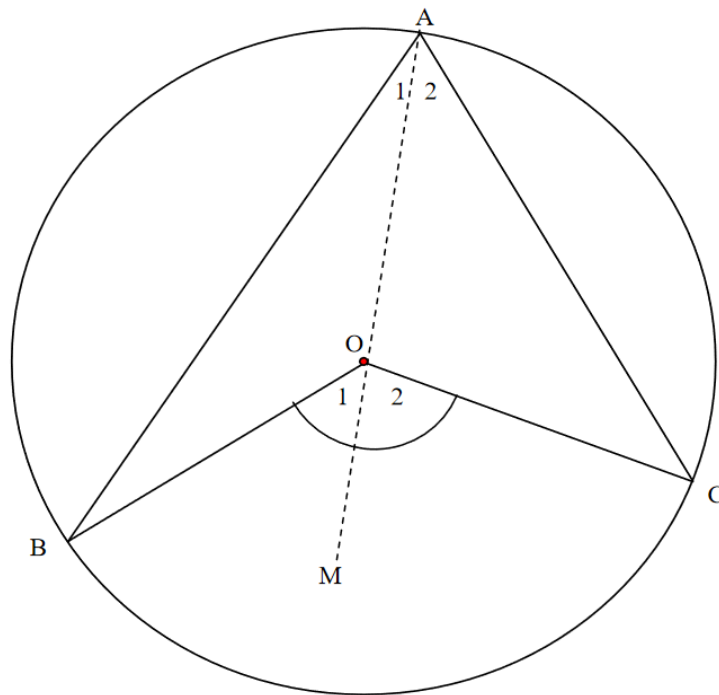
- (a) $OA = OB$ radii
 - (b) $AM = BM$ given
 - (c) $OM = OM$ common
- $\therefore \triangle OAM \equiv \triangle OBM$ SSS

$$\therefore \hat{M}_1 = \hat{M}_2$$

But AMB is a straight line

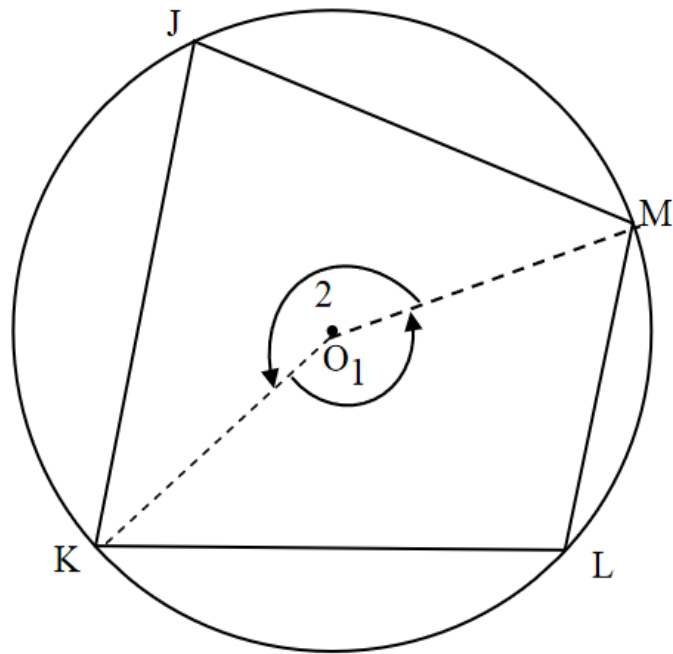
$$\therefore \hat{M}_1 = \hat{M}_2 = 90^\circ \quad \text{..... Adjacent supplementary } \angle s$$

QUESTION 2



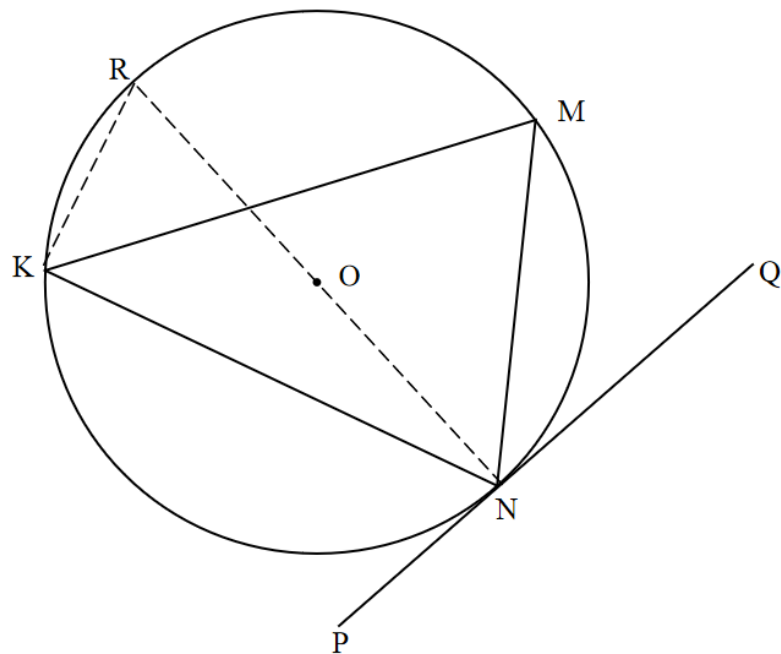
Construction: AO is drawn and produced to M	✓ Constr
$\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext $\angle$ of Δ /buite $\angle$ van Δ]	✓ S/R
But $\hat{A}_1 = \hat{B}$ [$\angle$ s opp = radii/ $\angle$ e teenoor = radii]	✓ S/R
$\therefore \hat{O}_1 = 2\hat{A}_1$	✓ S
Similarly/ <i>Netso</i> : $\hat{O}_2 = 2\hat{A}_2$	
$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$	✓ S
$\quad\quad\quad = 2(\hat{A}_1 + \hat{A}_2)$	
$\quad\quad\quad \hat{BOC} = 2\hat{BAC}$	
	(5)

QUESTION 3



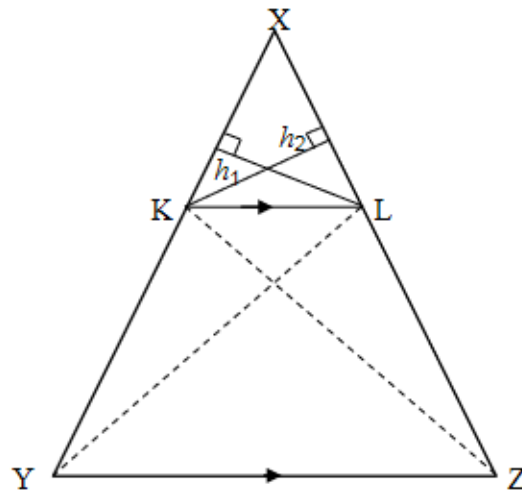
Constr/<i>Konstr.</i>: Draw KO and MO/<i>Trek KO en MO</i> Proof: $\hat{O}_1 = 2\hat{J}$ [∠ at centre = twice ∠ at circumference] <i>[midpts∠ = 2 × omtreks∠]</i> $\hat{O}_2 = 2\hat{L}$ [∠ at centre = twice ∠ at circumference] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [∠s around a point / <i>∠e om 'n punt</i>] $\therefore 2\hat{J} + 2\hat{L} = 360^\circ$ $\therefore 2(\hat{J} + \hat{L}) = 360^\circ$ $\therefore \hat{J} + \hat{L} = 180^\circ$ OR/OF Constr/<i>Konstr.</i>: Draw KO and MO/<i>Trek KO en MO</i> Proof: Let $\hat{J} = x$ $\hat{O}_1 = 2x$ [∠ at centre = twice ∠ at circumference] <i>[midpts∠ = 2 × omtreks∠]</i> $\hat{O}_2 = 360^\circ - 2x$ [∠s around a point / <i>∠e om 'n punt</i>] $\therefore \hat{L} = 180^\circ - x$ [∠ at centre = twice ∠ at circumference] $\therefore \hat{J} + \hat{L} = 180^\circ$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5) ✓ construction ✓ S ✓ R ✓ S/R ✓ S (5)
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QUESTION 4



11.1	Construction: Draw diameter NR and draw RK <i>Konstruksie: Trek middellyn NR en verbind RK</i> $\hat{M}\hat{N}Q + \hat{R}\hat{N}M = 90^\circ$ [radius $\perp$ tangent/<i>raaklyn</i>] $\hat{N}\hat{K}R = 90^\circ$ [$\angle$ in semicircle/<i>semi-sirkel</i>] $\therefore \hat{M}\hat{K}N = 90^\circ - \hat{R}\hat{K}M$ $= 90^\circ - \hat{R}\hat{N}M$ [$\angle$s same segment/<i>∠e dieselfde segment</i>] $\therefore \hat{M}\hat{N}Q = \hat{K}$	✓ construction ✓ S / R ✓ S / R ✓ S ✓ S / R (5)
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QUESTION 5



Constr: Join KZ and LY and draw h_1 from $K \perp XL$ and h_2 from $L \perp XK$

Konstr: Verbind KZ en LY en trek h_1 vanaf $K \perp XL$ en h_2 vanaf $L \perp XK$

Proof / Bewys:

$$\frac{\text{area } \triangle XKL}{\text{area } \triangle LYK} = \frac{\frac{1}{2} \text{XK} \times h_1}{\frac{1}{2} \text{KY} \times h_1} = \frac{\text{XK}}{\text{KY}}$$

$$\frac{\text{area } \triangle XKL}{\text{area } \triangle KLZ} = \frac{\frac{1}{2}XL \times h_2}{\frac{1}{2}LZ \times h_2} = \frac{XL}{LZ}$$

area $\triangle XKL$ = area $\triangle XKL$ [common / *gemeenskaplik*]

But area $\triangle LYK$ = area $\triangle KLZ$ [same base & height ; $LK \parallel YZ$ /
dies basis & hoogte ; $LK \parallel YZ$]

$$\therefore \frac{\text{area } \triangle XKL}{\text{area } \triangle LYK} = \frac{\text{area } \triangle XKL}{\text{area } \triangle KLZ}$$

$$\therefore \frac{XK}{KY} = \frac{XL}{LZ}$$

✓ constr / <i>konstr</i>

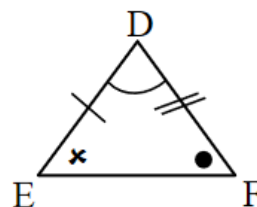
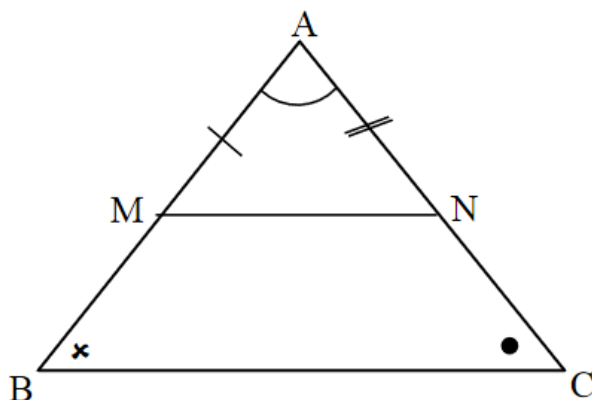
$$\begin{aligned} & \frac{\text{area } \triangle XKL}{\text{area } \triangle LYK} \\ &= \frac{\frac{1}{2} \text{XK} \times h_1}{\frac{1}{2} \text{KY} \times h_1} \end{aligned}$$

✓ S ✓ R

✓ S

(5)

QUESTION 6



Constr: Let M and N lie on AB and AC respectively such that $AM = DE$ and $AN = DF$. Draw MN.

Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat $AM = DE$ en $AN = DF$. Verbind MN.

Proof:

In $\triangle AMN$ and $\triangle DEF$

$AM = DE$ [Constr]

$AN = DF$ [Constr]

$\hat{A} = \hat{D}$ [Given]

$\therefore \triangle AMN \equiv \triangle DEF$ (SAS)

$\therefore \hat{AMN} = \hat{E} = \hat{B}$

$MN \parallel BC$ [corresp $\angle$'s are equal/ooreenkomstige $\angle$ e =]

$\frac{AB}{AM} = \frac{AC}{AN}$ [line $\parallel$ one side of $\triangle$ **OR** prop theorem; $MN \parallel BC$]

$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]

✓ Constr / Konst

✓ $\triangle AMN \equiv \triangle DEF$

✓ SAS

✓ $MN \parallel BC$ and R

✓ $\frac{AB}{AM} = \frac{AC}{AN}$ ✓ R

(6)

