



MATHEMATICS

TRIGONOMETRY FUNCTIONS

ACTIVITY 1

Consider the functions $f(x) = \cos 3x$ and $g(x) = \sin x$ for $x \in [-90^\circ; 180^\circ]$.

- 8.1 Solve for x if $f(x) = g(x)$. (8)
- 8.2 Sketch the graphs of f and g on the system of axes on DIAGRAM SHEET 2 for $x \in [-90^\circ; 180^\circ]$. (6)
- 8.3 Solve for x if $f(x) \leq g(x)$ where $x \in [-90^\circ; 0^\circ]$. (3)
- [17]

ACTIVITY 2

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

- 11.1 Calculate the points of intersection of the graphs f and g for $x \in [180^\circ; 360^\circ]$. (7)
- 11.2 Draw sketch graphs of f and g for $x \in [180^\circ; 360^\circ]$ on the same system of axes provided on DIAGRAM SHEET 3. (4)
- 11.3 For which values of x will $f(x) \leq g(x)$ for $x \in [180^\circ; 360^\circ]$? (3)
- [14]



ACTIVITY 3

Given: $f(x) = 2 \cos x$ and $g(x) = \tan 2x$

- 12.1 Sketch the graphs of f and g on the same system of axes provided on DIAGRAM SHEET 4, for $x \in [-90^\circ; 90^\circ]$. (6)
- 12.2 Solve for x if $2 \cos x = \tan 2x$ and $x \in [-90^\circ; 90^\circ]$. Show ALL working details. (8)
- 12.3 Use the graph to solve for x : $2 \cos x \cdot \tan 2x > 0$. (4)
- 12.4 Write down the period of $f\left(\frac{x}{2}\right)$. (2)
- 12.5 Write down the equations of the asymptotes of $g(x - 25^\circ)$, where $x \in [-90^\circ; 90^\circ]$. (2)
- [22]

ACTIVITY 4

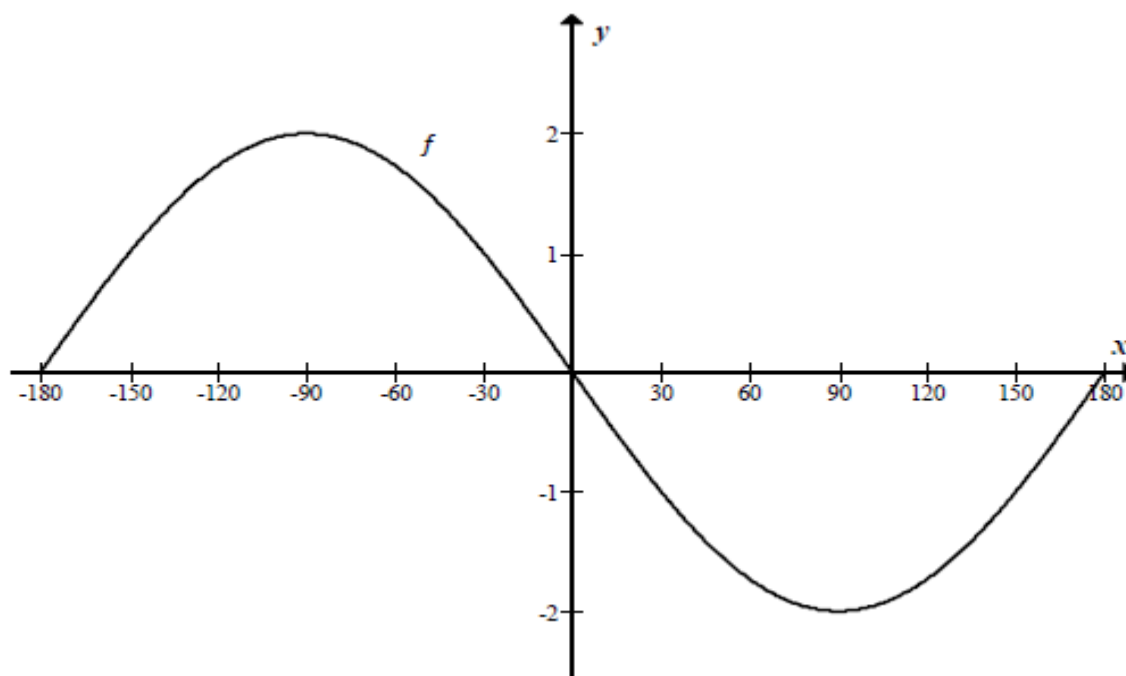
The graph of $f(x) = -2 \sin x$ is drawn below.





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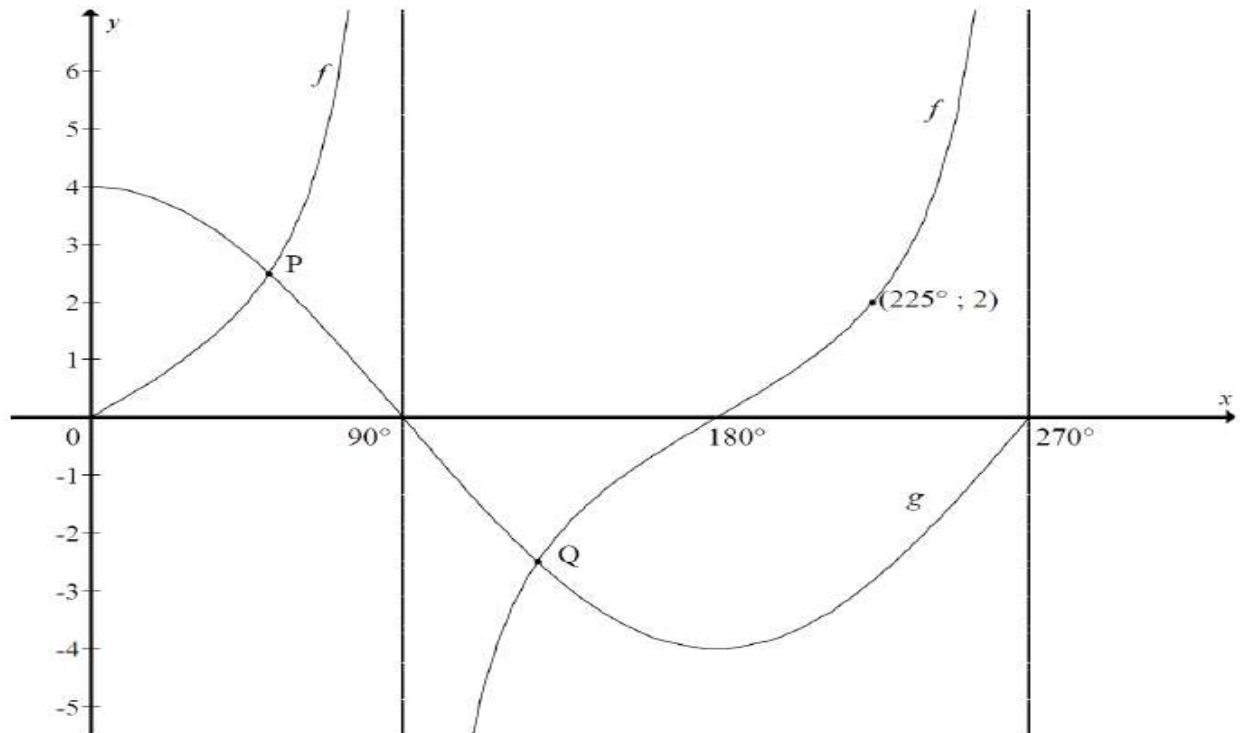


- 12.1 Write down the period of f . (1)
- 12.2 Write down the amplitude of h if $h(x) = \frac{f(x)}{4}$. (2)
- 12.3 Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $x \in [-180^\circ; 180^\circ]$ on the grid provided on DIAGRAM SHEET 5. (3)
- 12.4 Use the graph to determine the number of solutions for $-2 \sin x = \cos(x - 30^\circ)$, $x \in [-180^\circ; 180^\circ]$. (1)
- 12.5 For which values of x is $g(x) \geq 0$? (2)



ACTIVITY 5

The graphs of the functions $f(x) = a \tan x$ and $g(x) = b \cos x$ for $0^\circ \leq x \leq 270^\circ$ are shown in the diagram below. The point $(225^\circ; 2)$ lies on f . The graphs intersect at points P and Q.



- 10.1 Determine the numerical values of a and b . (4)
 - 10.2 Determine the minimum value of $g(x) + 2$. (2)
 - 10.3 Determine the period of $f\left(\frac{1}{2}x\right)$. (2)
 - 10.4 Show that, if the x -coordinate of P is θ , then the x -coordinate of Q is $(180^\circ - \theta)$. (4)
- [12]**

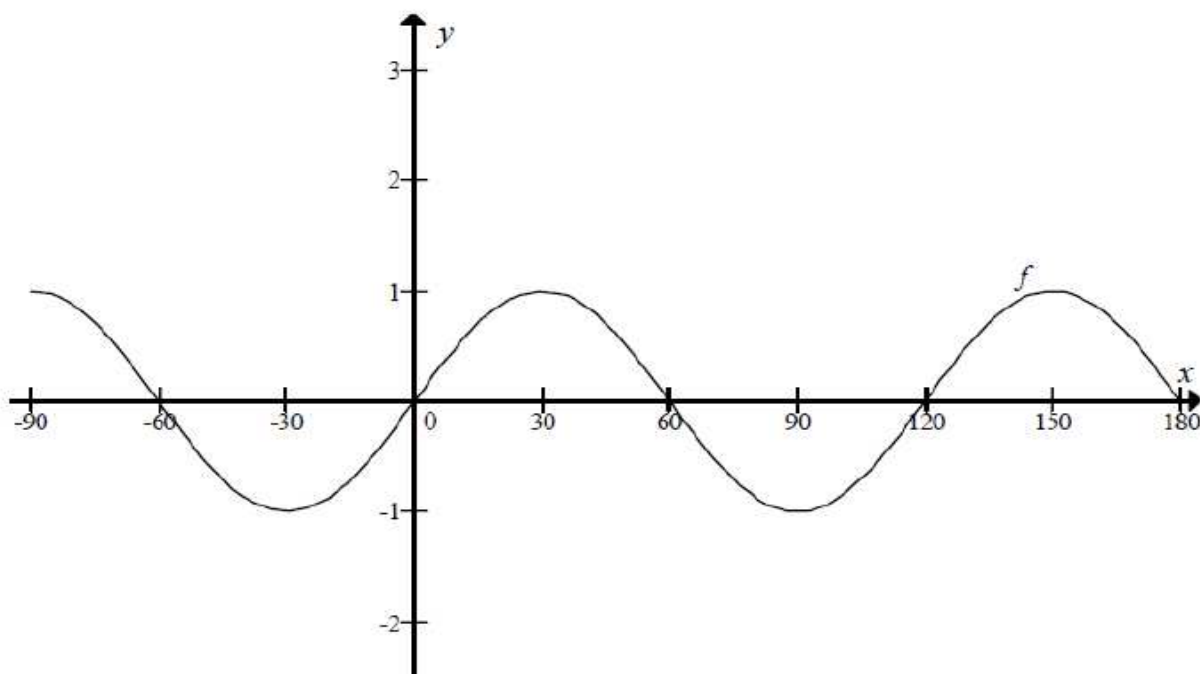


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ACTIVITY 6

The graph of $f(x) = \sin 3x$ is drawn below for $x \in [-90^\circ; 180^\circ]$.



- 10.1 Write down the period of f . (1)
- 10.2 Write down the solutions for $\sin 3x = -1$ on the interval $[-90^\circ; 180^\circ]$. (2)
- 10.3 Give the maximum value of h if $h(x) = f(x) - 1$. (2)
- 10.4 Draw the graph of $g(x) = 3 \cos x$ for $x \in [-90^\circ; 180^\circ]$ on the grid on DIAGRAM SHEET 4. (3)
- 10.5 Use the graphs to determine how many solutions there are to the equation $\frac{\sin 3x}{3} - \cos x = 0$ on the interval $[-90^\circ; 180^\circ]$. (2)
- 10.6 Use the graphs to solve: $f(x) \cdot g(x) < 0$. (4)
- [14]**



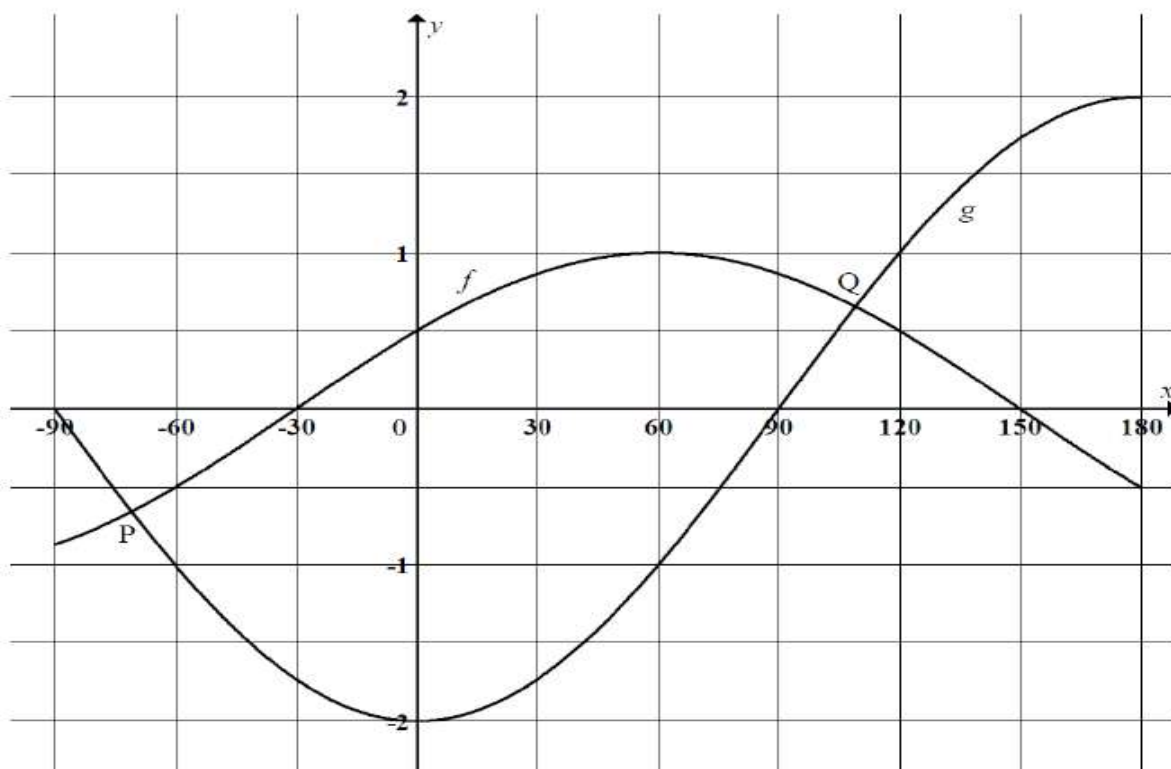


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ACTIVITY 7

The graphs of $f(x) = \sin(x + 30^\circ)$ and $g(x) = -2\cos x$ for $-90^\circ \leq x \leq 180^\circ$ are given below. The graphs intersect at point P and point Q.



10.1 Calculate $f(0) - g(0)$. (1)

10.2 Calculate the x -coordinates of point P and point Q. (7)

10.3 For which values of x will $f(x) \geq g(x)$? (2)

10.4 Graph h is obtained by the following transformation of f : $h(x) = 2f(x + 60^\circ)$. Describe the relationship between g and h . (2)
[12]



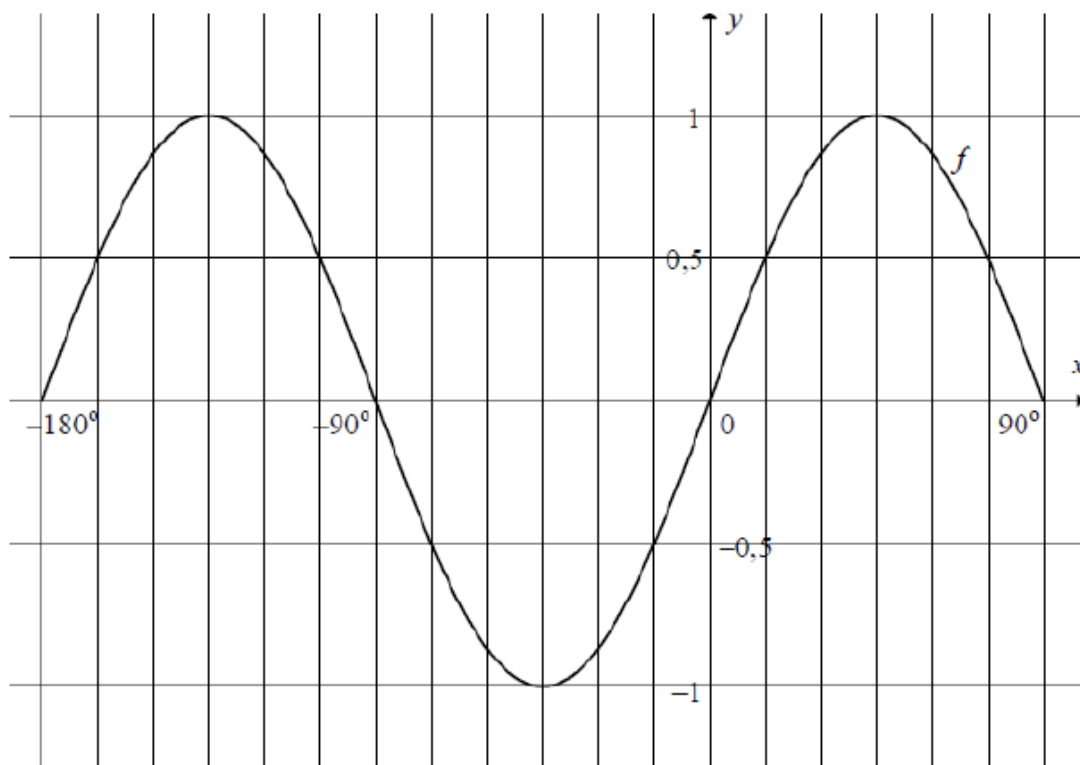


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ACTIVITY 6

The graph of $f(x) = \sin 2x$ for $-180^\circ \leq x \leq 90^\circ$ is shown in the sketch below.



- 10.1 Write down the range of f . (2)
- 10.2 Determine the period of $f\left(\frac{3}{2}x\right)$. (2)
- 10.3 Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $-180^\circ \leq x \leq 90^\circ$ on the system of axes on DIAGRAM SHEET 3. Clearly label ALL x -intercepts and turning points. (4)
- 10.4 Hence, or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x) \cdot g(x) < 0$. (4)
- 10.5 Describe the transformation that graph f has to undergo to form $y = \sin(2x + 60^\circ)$. (2)
- 10.6 Determine the general solution of $\sin 2x = \cos(x - 30^\circ)$. (6)
- [20]





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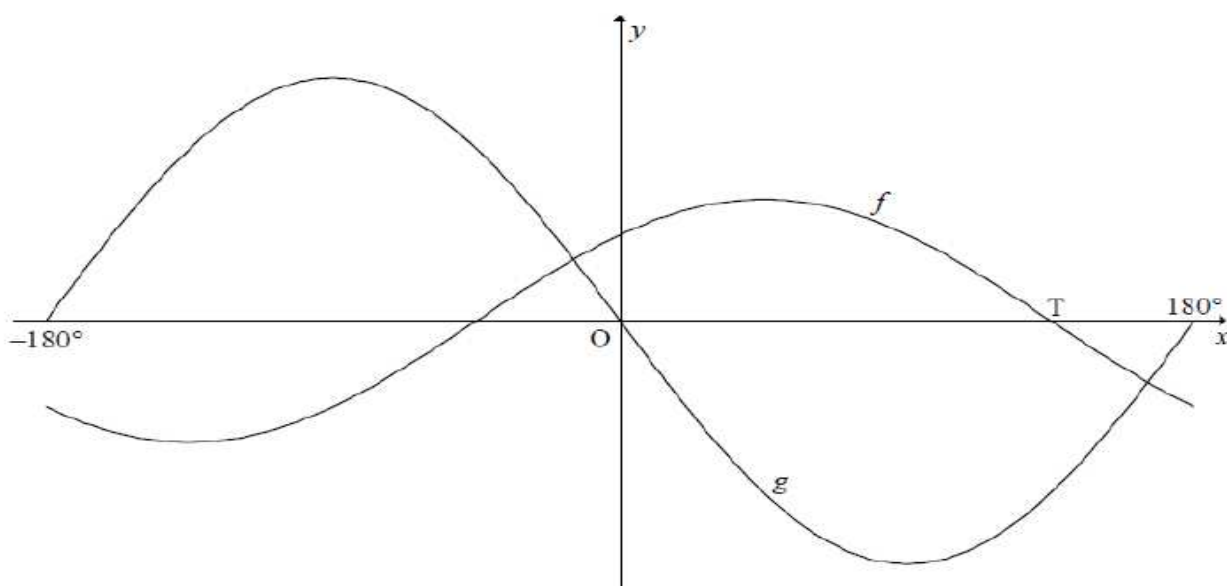
TRIGONOMETRY FUNCTIONS

ACTIVITY 7

- 12.1 Draw the graphs of $f(x) = \tan x + 1$ and $g(x) = \cos 2x$ for $x \in [-180^\circ; 180^\circ]$ on the same system of axes provided on DIAGRAM SHEET 2. Clearly show all intercepts with the axes, turning points and asymptotes. (6)
- 12.2 Write down the period of g . (1)
- 12.3 If $h(x) = -\cos 2(x + 10^\circ)$, describe fully, in words, the transformation from g to h . (2)
- 12.4 For which values of x , where $x > 0$, will $f'(x)g(x) > 0$? (4)
- [13]

ACTIVITY 8

The graphs of $f(x) = \cos(x - 45^\circ)$ and $g(x) = -2 \sin x$ are drawn below for $x \in [-180^\circ; 180^\circ]$. The point T is an x-intercept of f as indicated on the diagram.



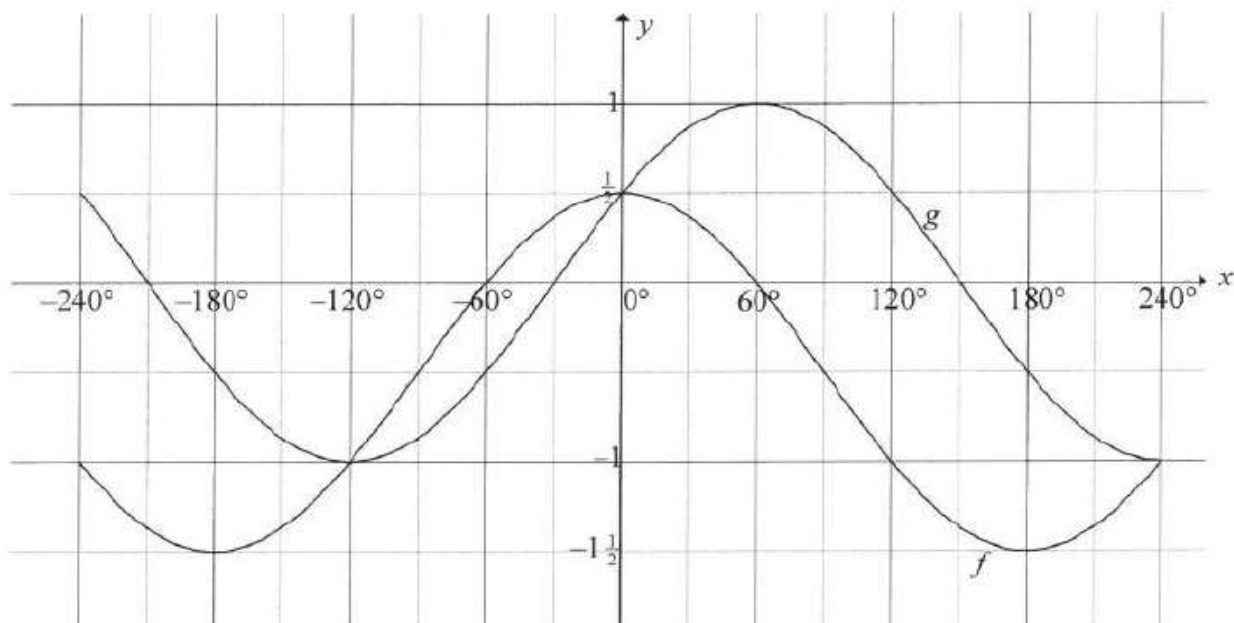
- 9.1 Show that $\cos(x - 45^\circ) = -2 \sin x$ can be written as $\tan x = -0,2612$. (4)
- 9.2 Solve the equation: $\cos(x - 45^\circ) = -2 \sin x$ for $x \in [-180^\circ; 180^\circ]$. (3)
- 9.3 Write down the coordinates of point T. (2)
- 9.4 Write down the interval for which $f(x) \geq g(x)$. (2)
- 9.5 Write down the interval for which both f and g are strictly increasing. (3)
- 9.6 The graph h is obtained when the graph f is shifted 45° to the right. Write down the equation of h in its simplest form. (2)

[16]



ACTIVITY 9

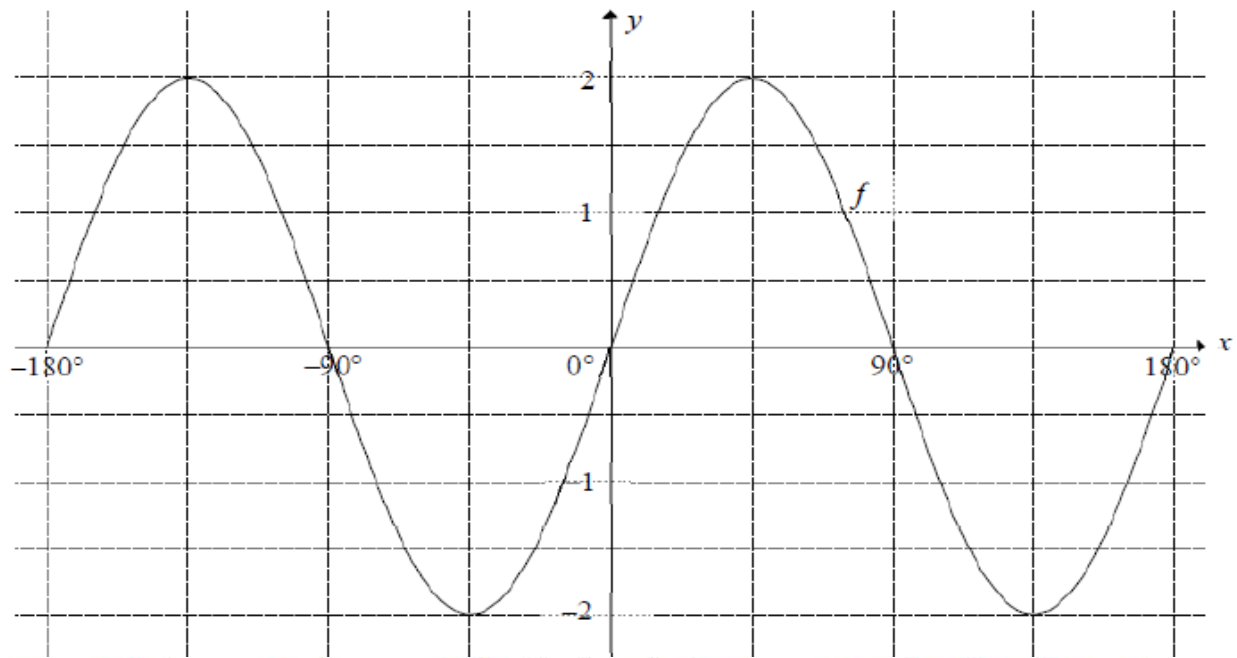
In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $\left(0^\circ; \frac{1}{2}\right)$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.



- 6.1 Determine the values of p and q . (4)
 - 6.2 Determine the values of x in the interval $-240^\circ \leq x \leq 240^\circ$ for which $f(x) > g(x)$. (2)
 - 6.3 Describe a transformation that the graph of g has to undergo to form the graph of h , where $h(x) = -\cos x$. (2)
- [8]**

ACTIVITY 10

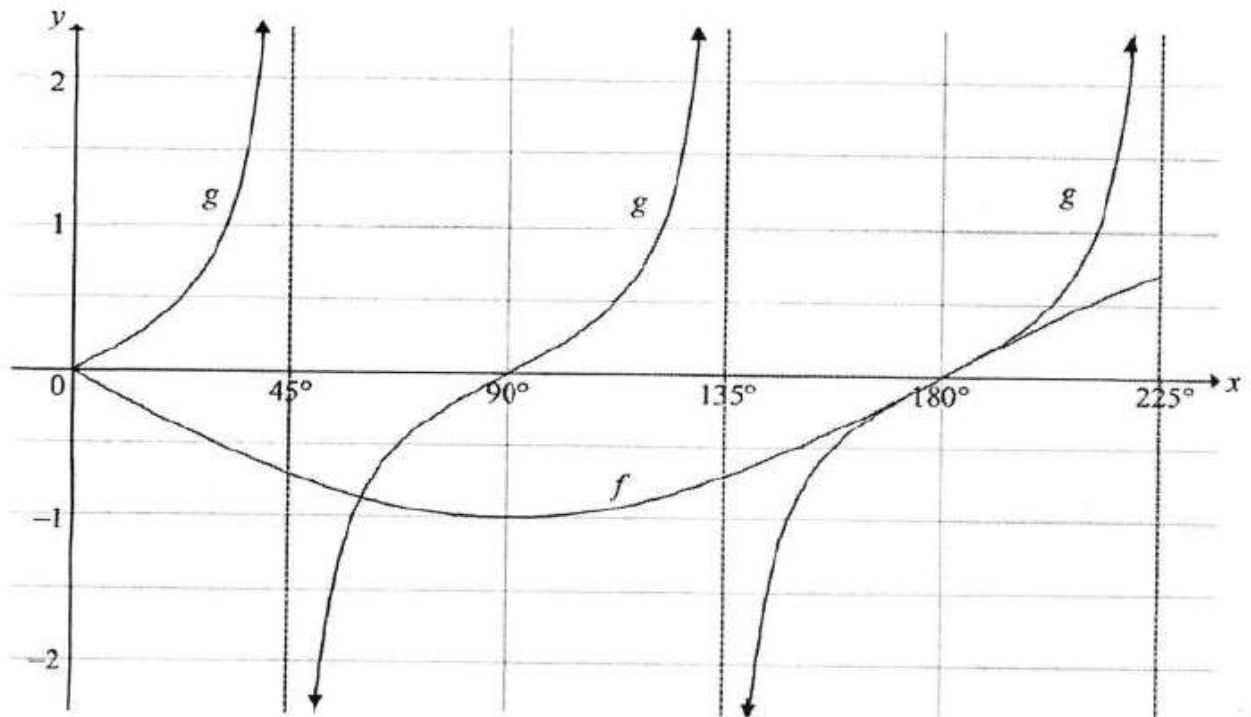
In the diagram the graph of $f(x) = 2 \sin 2x$ is drawn for the interval $x \in [-180^\circ; 180^\circ]$.



- 6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180^\circ; 180^\circ]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)
 - 6.2 Write down the maximum value of $f(x) - 3$. (2)
 - 6.3 Determine the general solution of $f(x) = g(x)$. (4)
 - 6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180^\circ; 0^\circ]$. (3)
- [12]**

ACTIVITY 11

In the diagram, the graphs of the functions $f(x) = a \sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0^\circ \leq x \leq 225^\circ$.



- 5.1 Write down the values of a and b . (2)
 - 5.2 Write down the period of $f(3x)$. (2)
 - 5.3 Determine the values of x in the interval $90^\circ \leq x \leq 225^\circ$ for which $f(x) \cdot g(x) \leq 0$. (3)
- [7]**

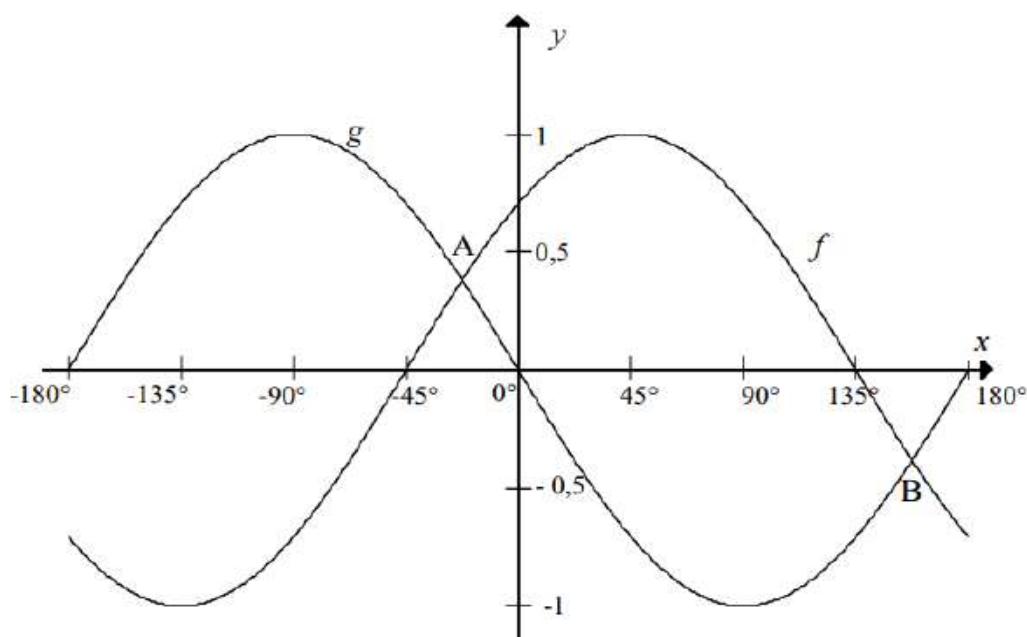


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ACTIVITY 12

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = q \sin x$ are shown for the interval $-180^\circ \leq x \leq 180^\circ$.



- 6.1 Determine the values of p and q . (2)
- 6.2 The graphs intersect at $A(-22,5^\circ ; 0,38)$ and B . Determine the coordinates of B . (2)
- 6.3 Determine the value(s) of x in the interval $-180^\circ \leq x \leq 180^\circ$ for which $f(x) - g(x) < 0$. (2)
- 6.4 The graph f is shifted 30° to the left to obtain a new graph h .
 - 6.4.1 Write down the equation of h in its simplest form. (2)
 - 6.4.2 Write down the value of x for which h has a minimum in the interval $-180^\circ \leq x \leq 180^\circ$. (1)

[9]

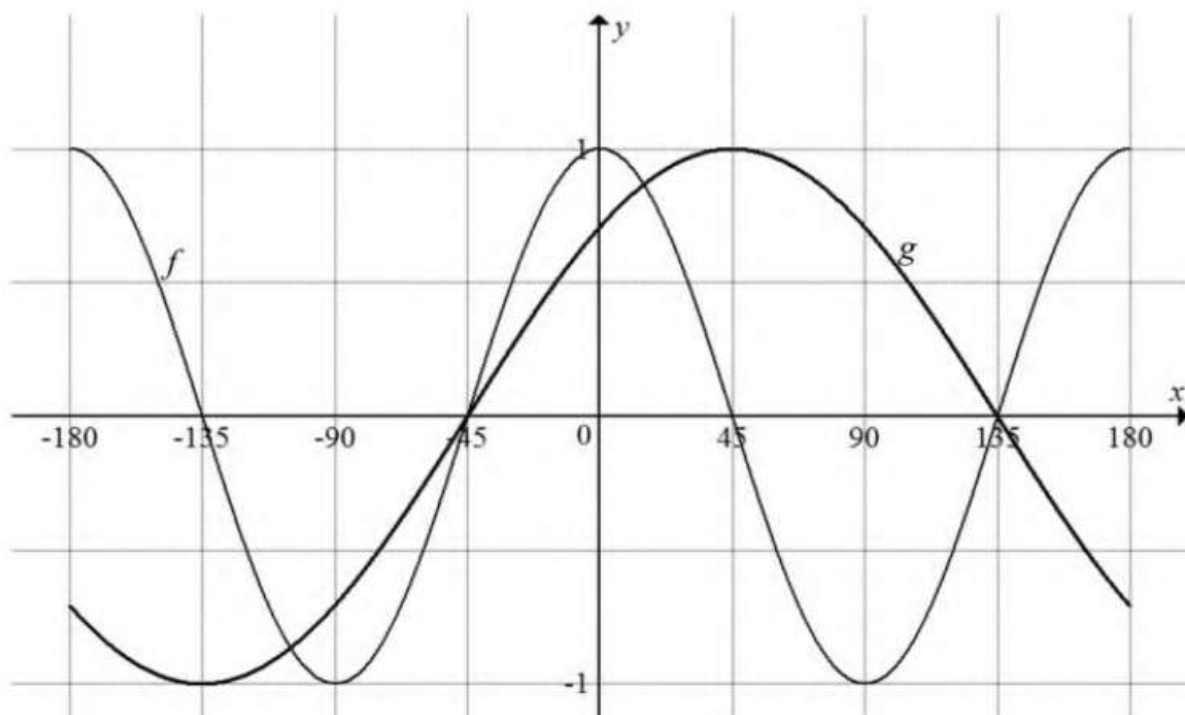
ACTIVITY 13

- 6.1 Use the system of axes provided on DIAGRAM SHEET 1 to draw the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x + 1$ for the interval $-180^\circ \leq x \leq 180^\circ$. Show clearly ALL intercepts with the axes, turning points and end points. (6)
- 6.2 Write down the period of f . (1)
- 6.3 For which value(s) of x in the interval $-180^\circ \leq x \leq 180^\circ$ will $g(x) - f(x)$ be a maximum? (1)
- 6.4 The graph f is shifted 45° to the right to obtain a new graph h . Write down the equation of h in its simplest form. (2)

[10]

ACTIVITY 14

In the diagram below the graphs of $f(x) = a \cos bx$ and $g(x) = \sin(x + p)$ are drawn for $x \in [-180^\circ; 180^\circ]$.

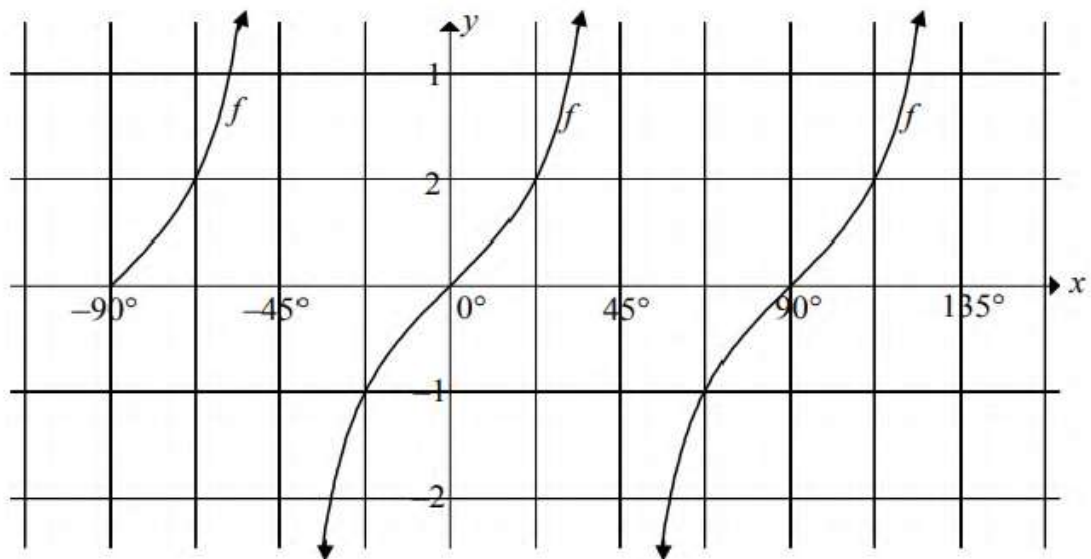


- 6.1 Write down the values of a , b and p . (3)
- 6.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2)
- 6.3 Write down the period of $f(2x)$. (2)
- 6.4 Determine the minimum value of h if $h(x) = 3f(x) - 1$. (2)
- 6.5 Describe how the graph g must be transformed to form the graph k , where $k(x) = -\cos x$. (2)

[11]

ACTIVITY 15

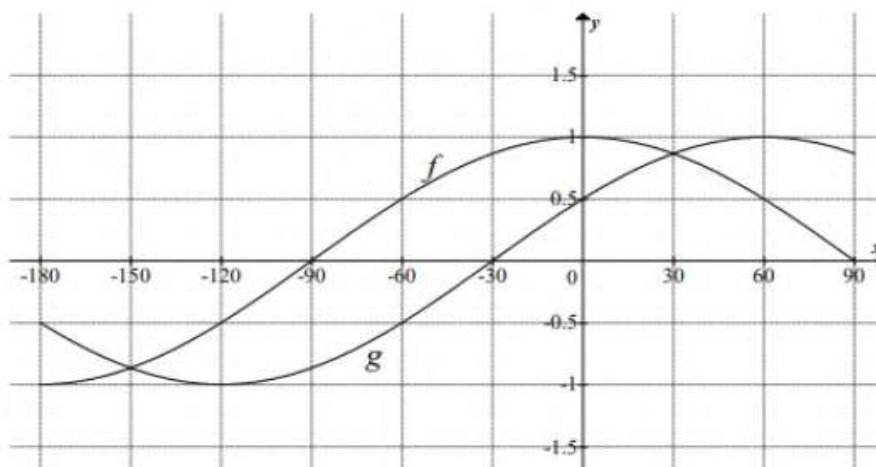
6.1 In the diagram, the graph of $f(x) = \tan bx$ is drawn for the interval $-90^\circ \leq x \leq 135^\circ$.



- 6.1.1 Determine the value of b . (1)
- 6.1.2 Determine the values of x in the interval $0^\circ \leq x \leq 135^\circ$ for which $f(x) \leq -1$. (2)
- 6.1.3 Graph h is defined as $h(x) = \tan b(x + 55^\circ)$. Write down the equations of the asymptotes of h in the interval $-90^\circ \leq x \leq 135^\circ$. (2)

ACTIVITY 16

In the diagram the graphs of $f(x) = \cos x$ and $g(x) = \sin(x+b)$ are drawn for the interval $-180^\circ \leq x \leq 90^\circ$.



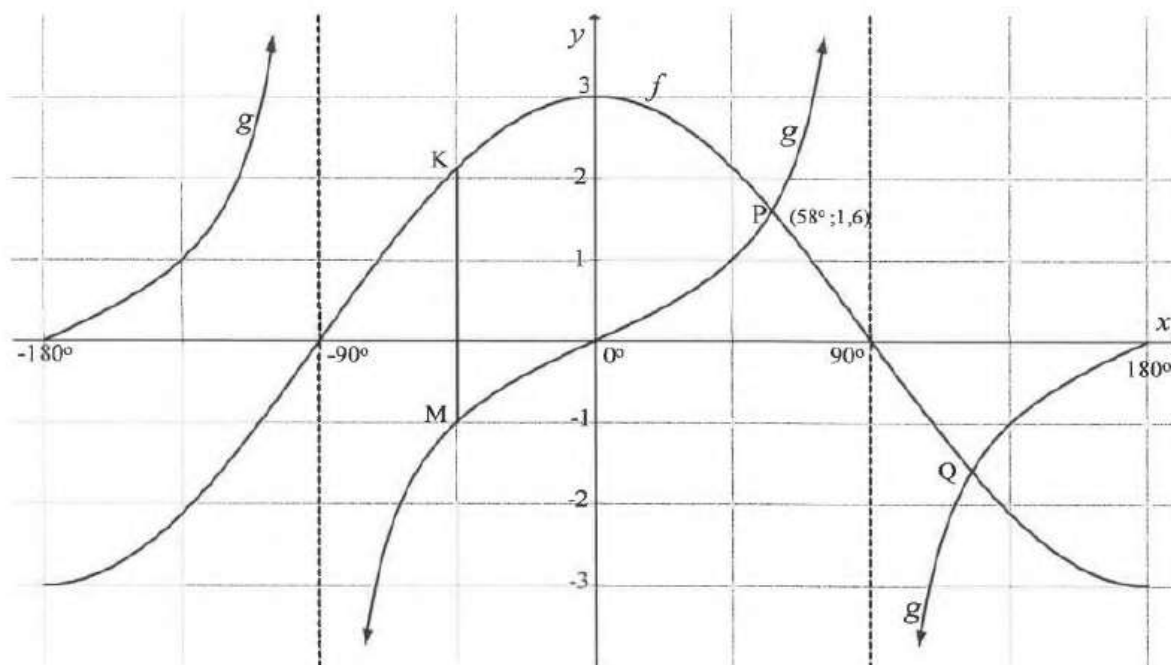
- 6.1 Write down the value of b . (1)
 - 6.2 Write down the period of g . (1)
 - 6.3 Write down the value(s) of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x) - g(x) = 0$. (2)
 - 6.4 For which values of x in the interval $-180^\circ \leq x \leq 90^\circ$ is $\sin(90^\circ - x) > g(x)$? (3)
 - 6.5 The graph of h is obtained by shifting f 3 units upwards. Determine the range of h . (2)
- [9]**

ACTIVITY 17

- 6.1 Determine the general solution for $\sin(x-30^\circ) = \cos 2x$ (5)
- 6.2 Consider the functions $f(x) = \sin(x-30^\circ)$ and $g(x) = \cos 2x$
 - 6.2.1 Write down the period of g . (1)
 - 6.2.2 State the range of f . (2)
 - 6.2.3 On the grid provided in the ANSWER BOOK, draw the graphs of f and g for $x \in [-90^\circ; 180^\circ]$. Clearly show ALL intercepts with the axes, turning points and end points. (5)
 - 6.2.4 Write down the x -coordinates of the points of intersection of f and g in the interval $x \in [-90^\circ; 180^\circ]$. (3)

ACTIVITY 18

The graphs of the functions $f(x) = a \cos b\theta$ and $g(x) = c \tan \theta$ for $x \in [-180^\circ; 180^\circ]$ are sketched below. The graphs intersect at $P(58^\circ; 1,6)$ and Q .



- 6.1 Write down the range of f . (2)
- 6.2 If $M(-45^\circ; -1)$ lies on g , determine the value of c . (1)
- 6.3 Write down the values of a and b . (2)
- 6.4 Determine the coordinates of Q . (2)
- 6.5 K lies on f such that KM is parallel to the y -axis. Calculate the length of KM . (2)
- 6.6 If the system of axes is shifted 45° to the left and the graphs remain fixed, write down the equation that is now represented by graph f . (2)

[11]