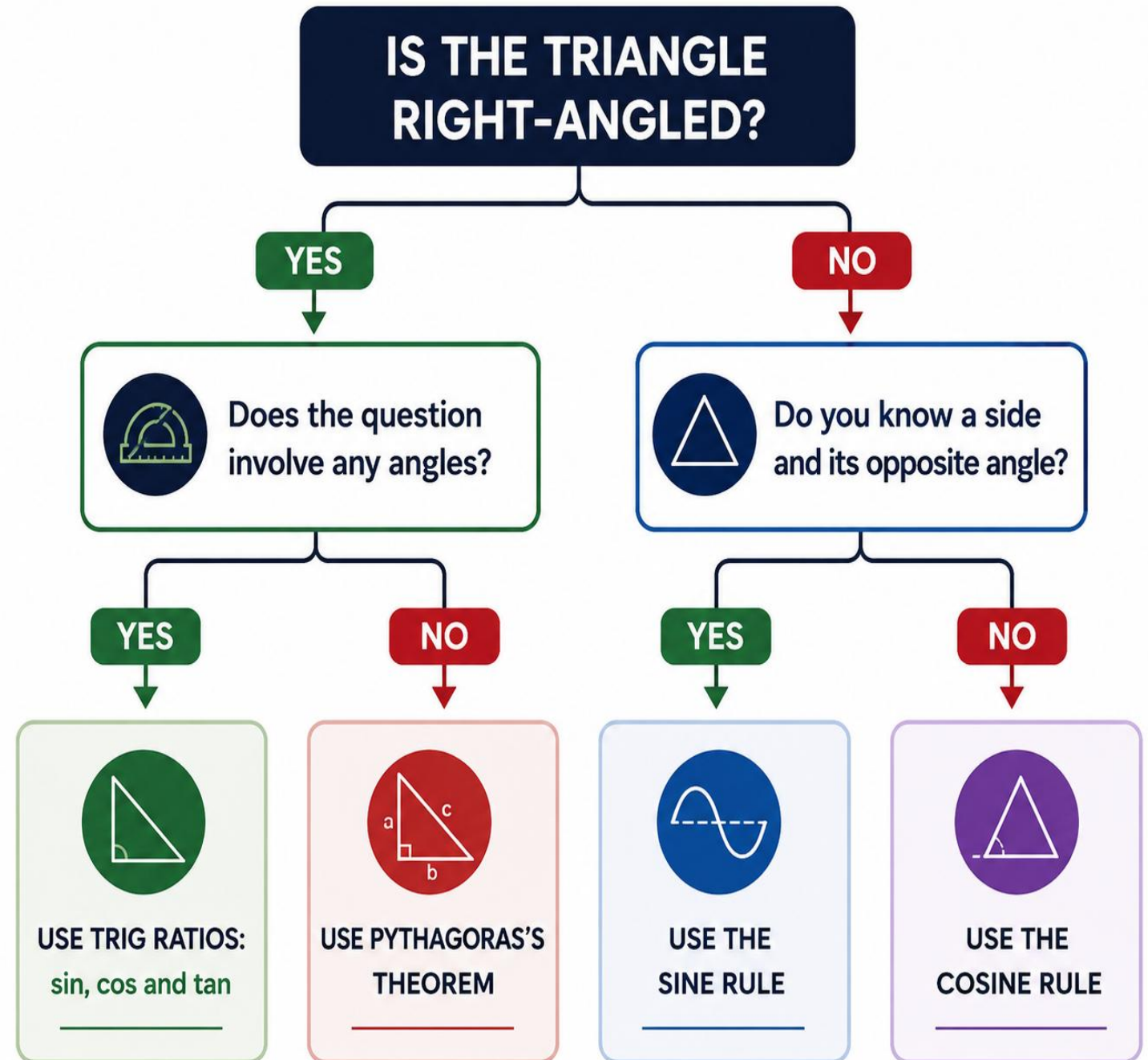




SINE, COSINE AND AREA RULES PART ONE



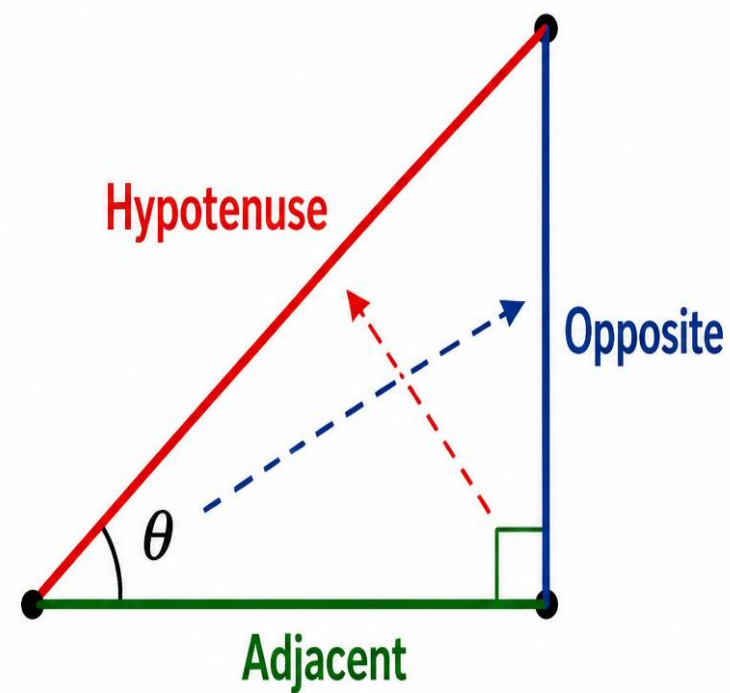
- All triangles have three angles and three sides.
- To **solve a triangle** is to find the :
 - ❖ lengths of each of its sides
 - ❖ its angles



Solving a right-angled triangle

- Finding the angles
- To solve a right-angle triangle, you need to find the missing lengths of its sides and the measures of its non-right angles.
- The trigonometric ratios of sine, cosine and tangent relate to **right angled triangles only** and are used to find the angles in a right-angled triangle

SOHCAHTOA



SOH

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

CAH

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

TOA

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

θ $\theta = \text{angle}$

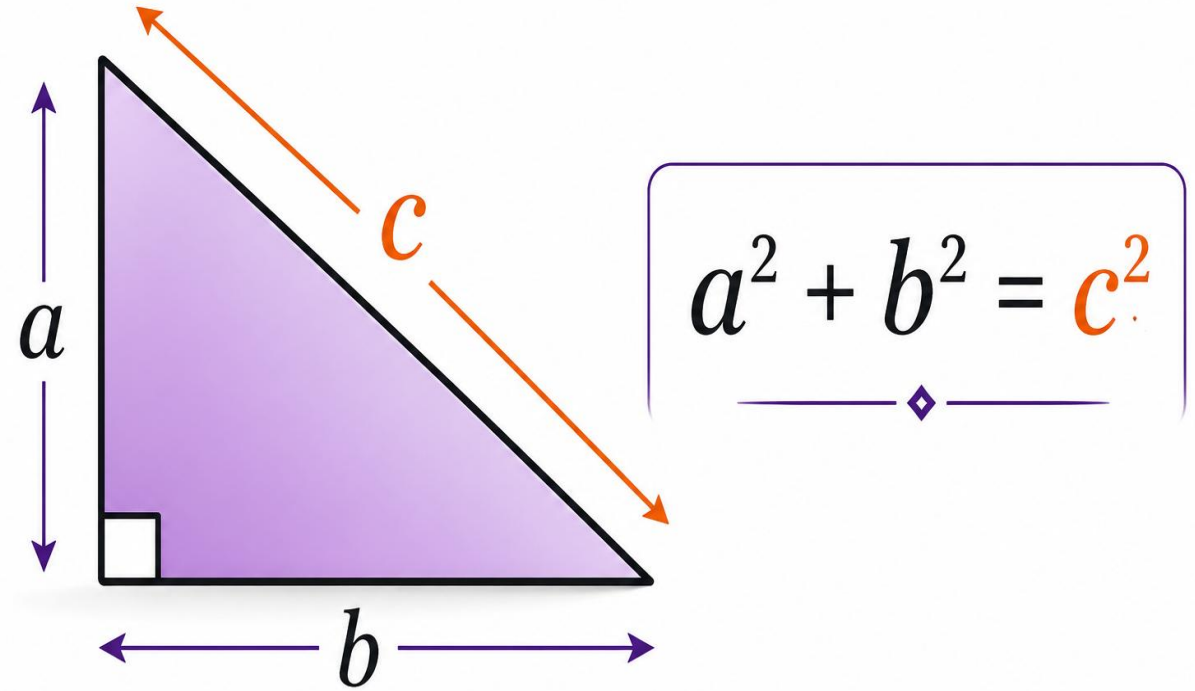
— Hypotenuse

— Opposite

— Adjacent

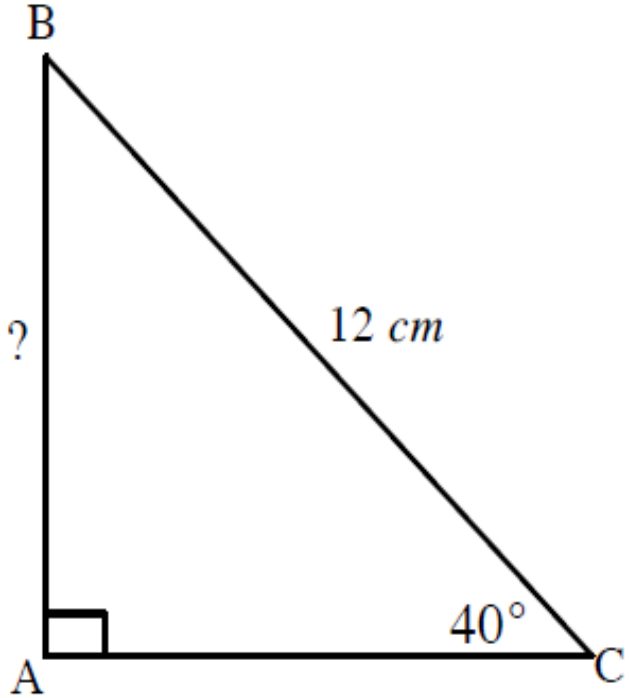
Solving a right-angled triangle

- Finding the side
- The Pythagoras theorem is used to find the sides of a right angled triangle



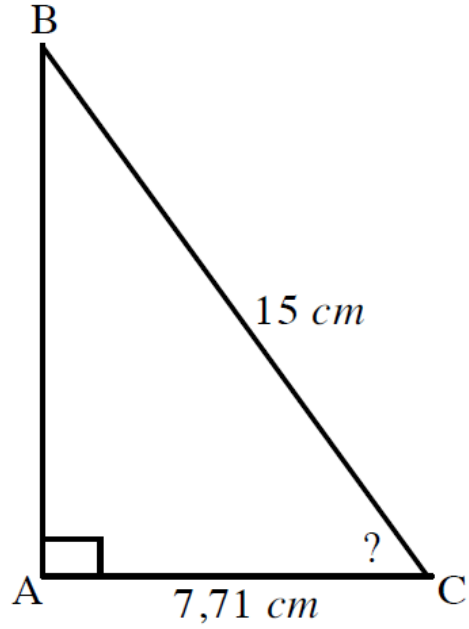
Example 1

$\triangle ABC$ is given with $\angle A = 90^\circ$, $\angle C = 40^\circ$ and $BC = 12\text{cm}$.
Determine the length of the side AB .



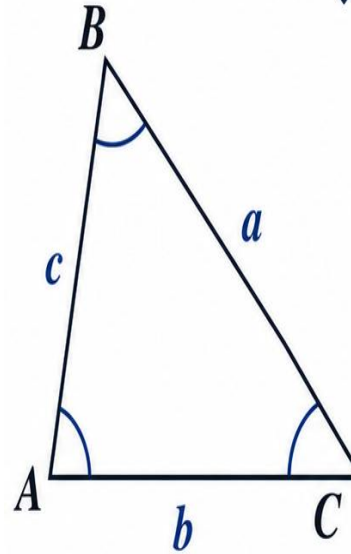
Example 2

$\triangle ABC$ is given with $\angle A = 90^\circ$, $AC = 7.71$ cm and $BC = 15$ cm.
Determine the size of $\angle C$



Solving a non-right angled triangle

- ❖ Solving a non-right-angled triangle means finding the lengths of all three sides and the measures of all three angles.
- ❖ Because Pythagoras' theorem and basic trigonometry (SOH CAH TOA) do not apply, you must use the **Sine Rule**, **Cosine Rule**, and the fact that all angles add up to 180°



SINE RULE

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

OR

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



FOR FINDING SIDES



FOR FINDING ANGLES

COSINE RULE

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

OR

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$



FOR FINDING SIDES



FOR FINDING ANGLES

a

Side opposite $\angle A$

b

Side opposite $\angle B$

c

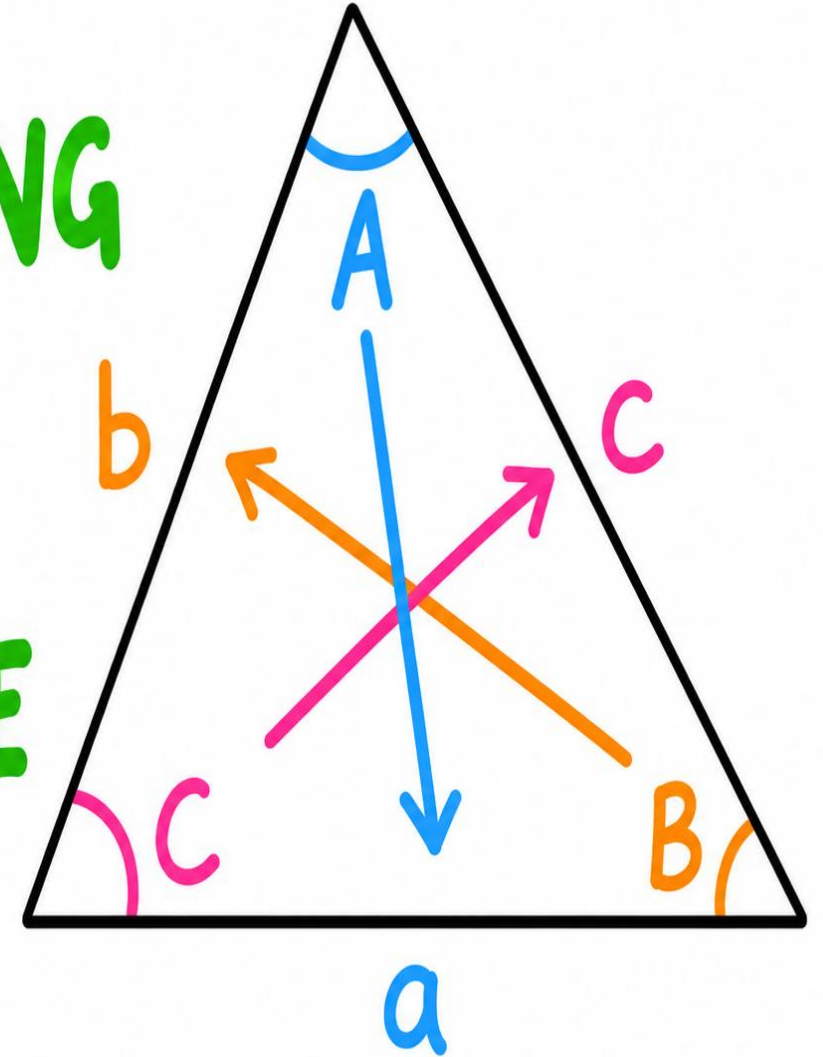
Side opposite $\angle C$

A, B, C are the angles
 a, b, c are the sides opposite
their respective angles.

Labelling a Triangle

- It is important to see how triangles are labelled.
- For sides we use lower case letters
- For angles we use upper case letters
- For example, angle **C** is opposite side **c**

**LABELLING
ANY
TRIANGLE**

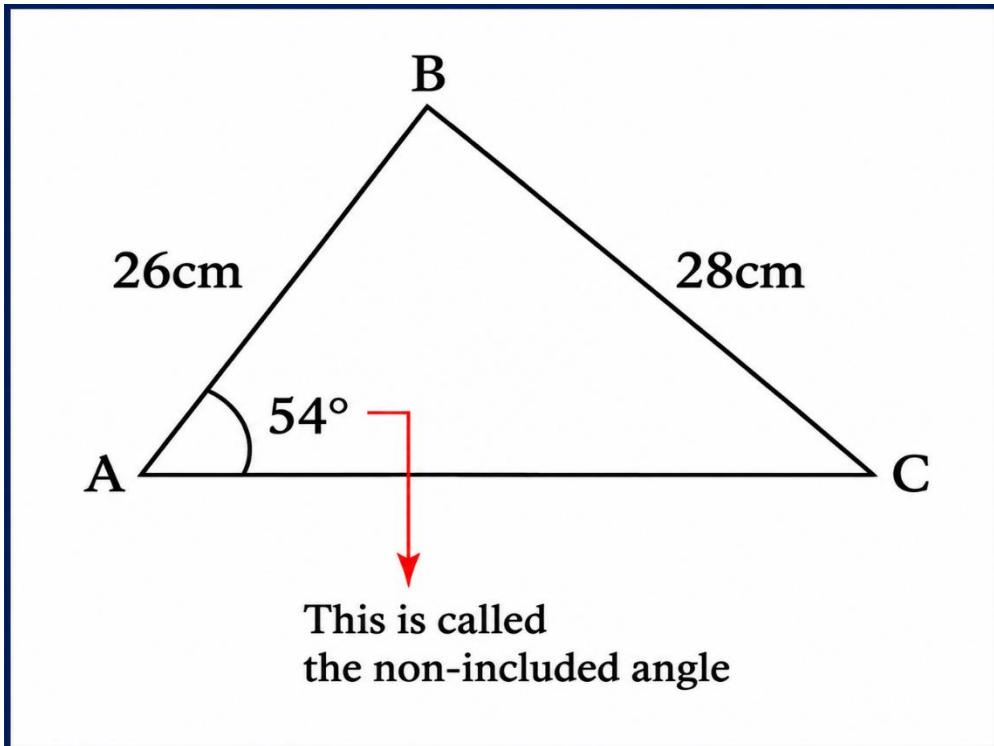


SINE RULE

- The ratio of the sine of the angle divided by the side opposite that angle is the same for all three pairs of sides and angles.

The Sine Rule is used when given either:

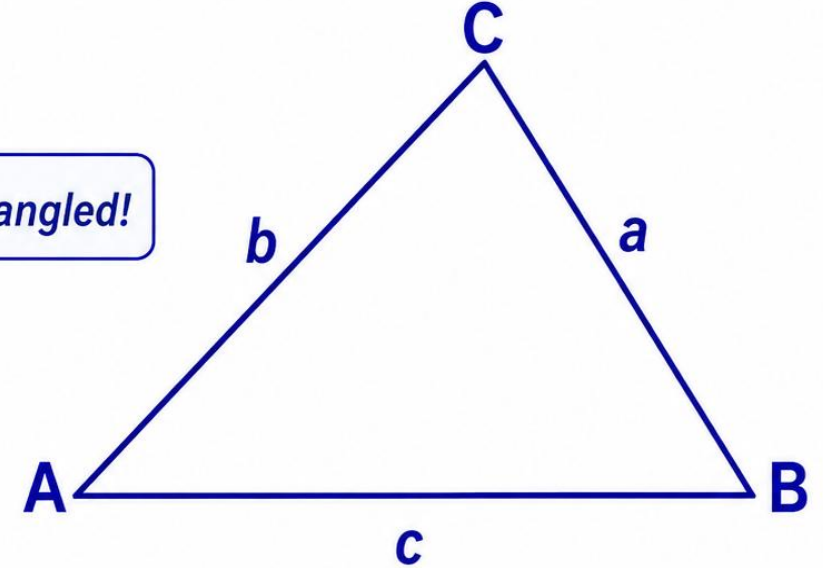
- ❖ two angles and one side
- ❖ or two sides and a non-included angle.



THE SINE RULE:



Not right-angled!



In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

OR



or

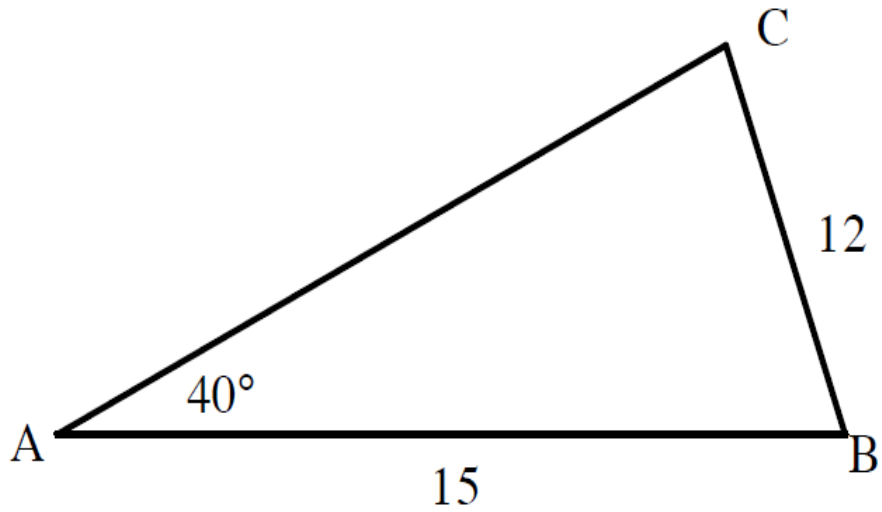
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Examples on the Sine Rule

Example 3

In $\triangle ABC$ $\angle A = 40^\circ$, $BC = 12$ units and $AB = 15$ units $\angle C = 40^\circ$.

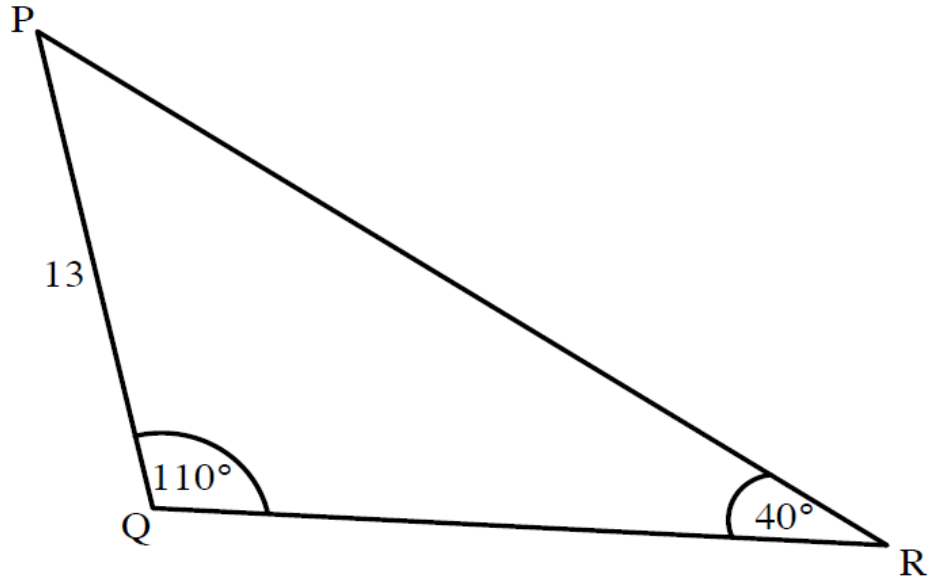
Determine the other angles of the triangle.
Assume that $\angle C$ is acute.



Example 4

In $\triangle PQR$, $\angle Q = 110^\circ$, $\angle R = 40^\circ$ and $PQ = 13$ units .

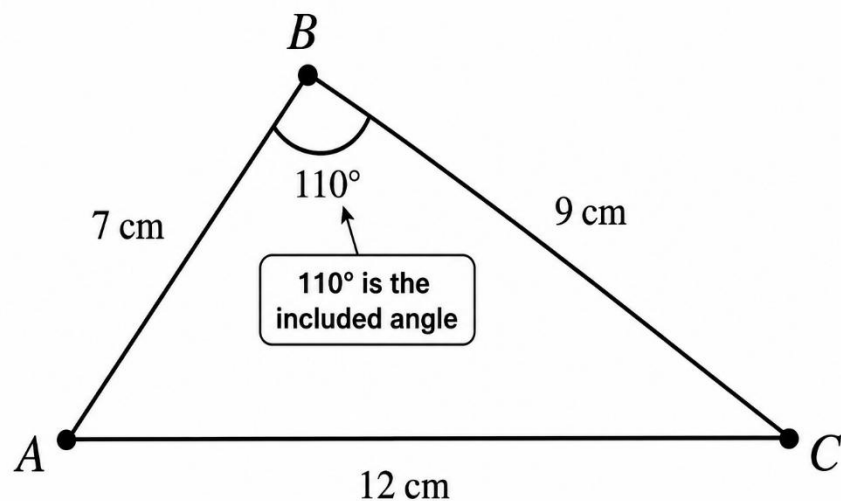
Calculate the lengths of the sides.



COSINE RULE

The Cosine rule is used when given either:

- ☐ all three sides
- ☐ or two sides and the included angle



THE COSINE RULE

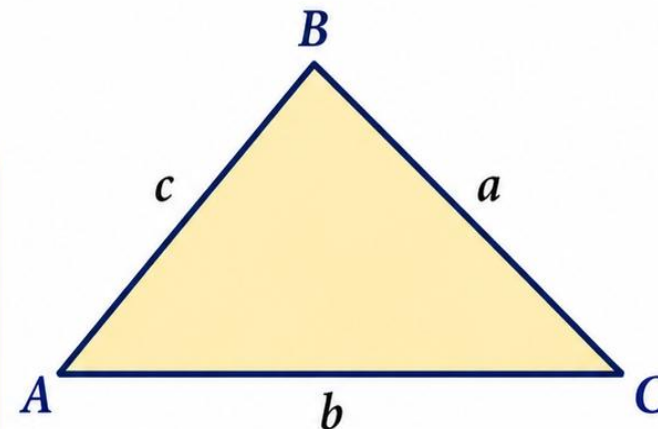
There are two forms.

1 To find the length of a side.

$$a^2 = b^2 + c^2 - (2bc \cos A)$$

or $b^2 = c^2 + a^2 - (2ac \cos B)$

or $c^2 = a^2 + b^2 - (2ab \cos C)$



2 To find an angle when given all three sides.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

or $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

or $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

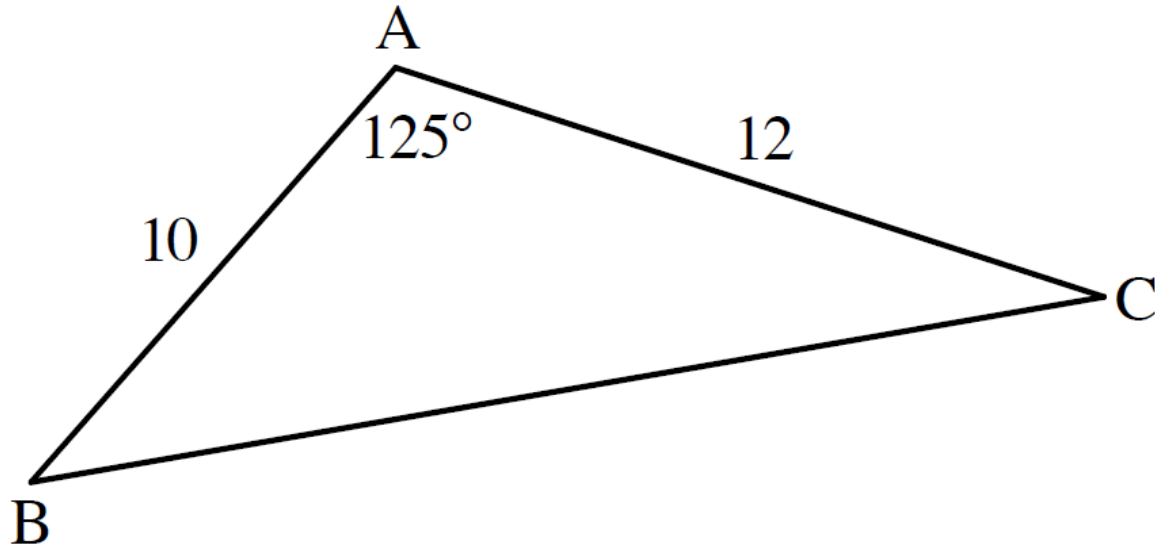
Notes:

- Sides a , b , c are opposite angles A , B , C respectively.
- The cosine rule applies to any triangle.
- Ensure your calculator is in degree mode.

Examples on the Cosine Rule

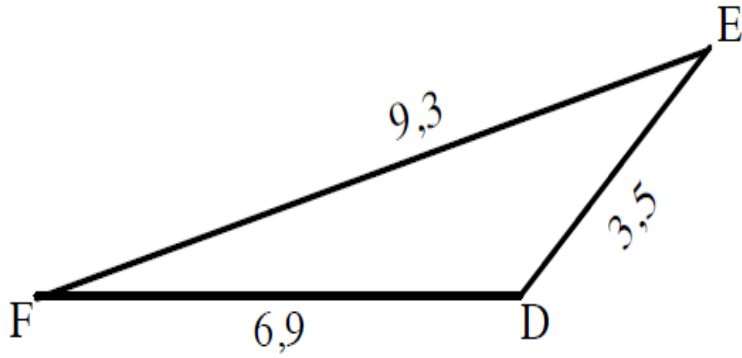
Example 5

In $\triangle ABC$, $AC = 12$ units, $AB = 10$ units and $\angle A = 125^\circ$.
Calculate the length of BC .



Example 6

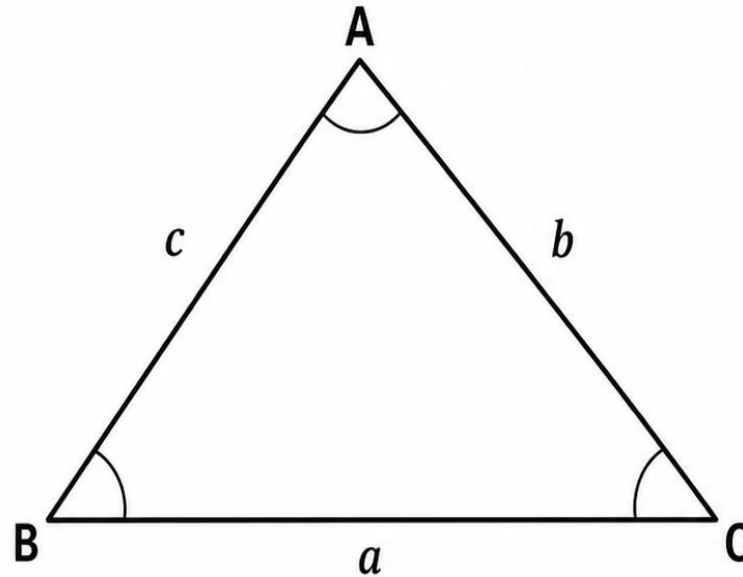
In the diagram below, $\triangle DEF$, with $ED = 3,5$ units, $EF = 9,3$ units and $FD = 6,9$ units. Calculate the obtuse angle $\angle D$?



AREA RULE

- The Area Rule is used to find the area of any triangle when at **least two sides and an included angle** are known.

AREA RULE FOR A NON-RIGHT ANGLED TRIANGLE



AREA RULE:

$$\text{Area } \triangle ABC = \frac{1}{2} bc \sin A$$

Where:

- $b = AC$
- $c = AB$
- A = included angle between sides b and c (at A)

Other forms:

$$\text{Area} = \frac{1}{2} ac \sin B$$

(using sides a and c
with included angle B at B)

$$\text{Area} = \frac{1}{2} ab \sin C$$

(using sides a and b
with included angle C at C)

$a = BC$ (opposite A)

$b = AC$ (opposite B)

$c = AB$ (opposite C)

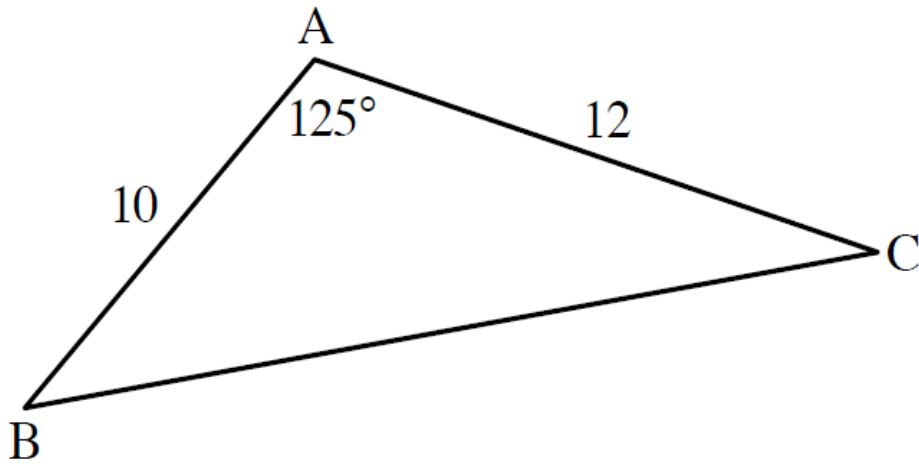


Remember: The area of a triangle = *one half* $\times$ *product of two sides* $\times$ *sine of the included angle*.

Examples on the Area Rule

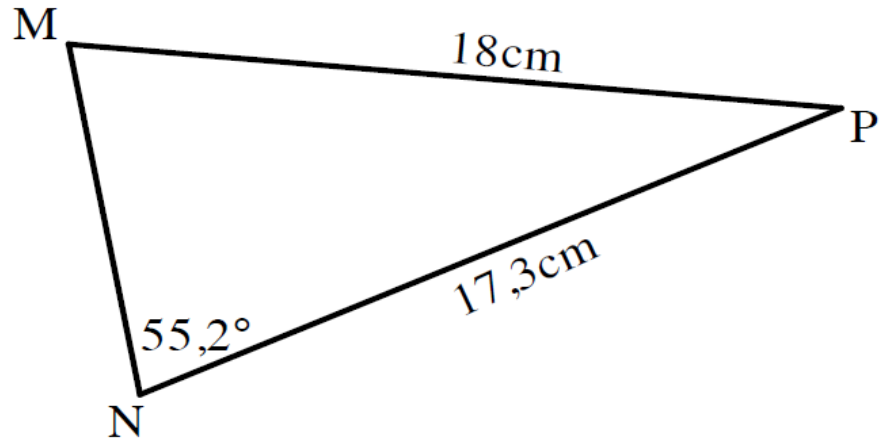
Example 7

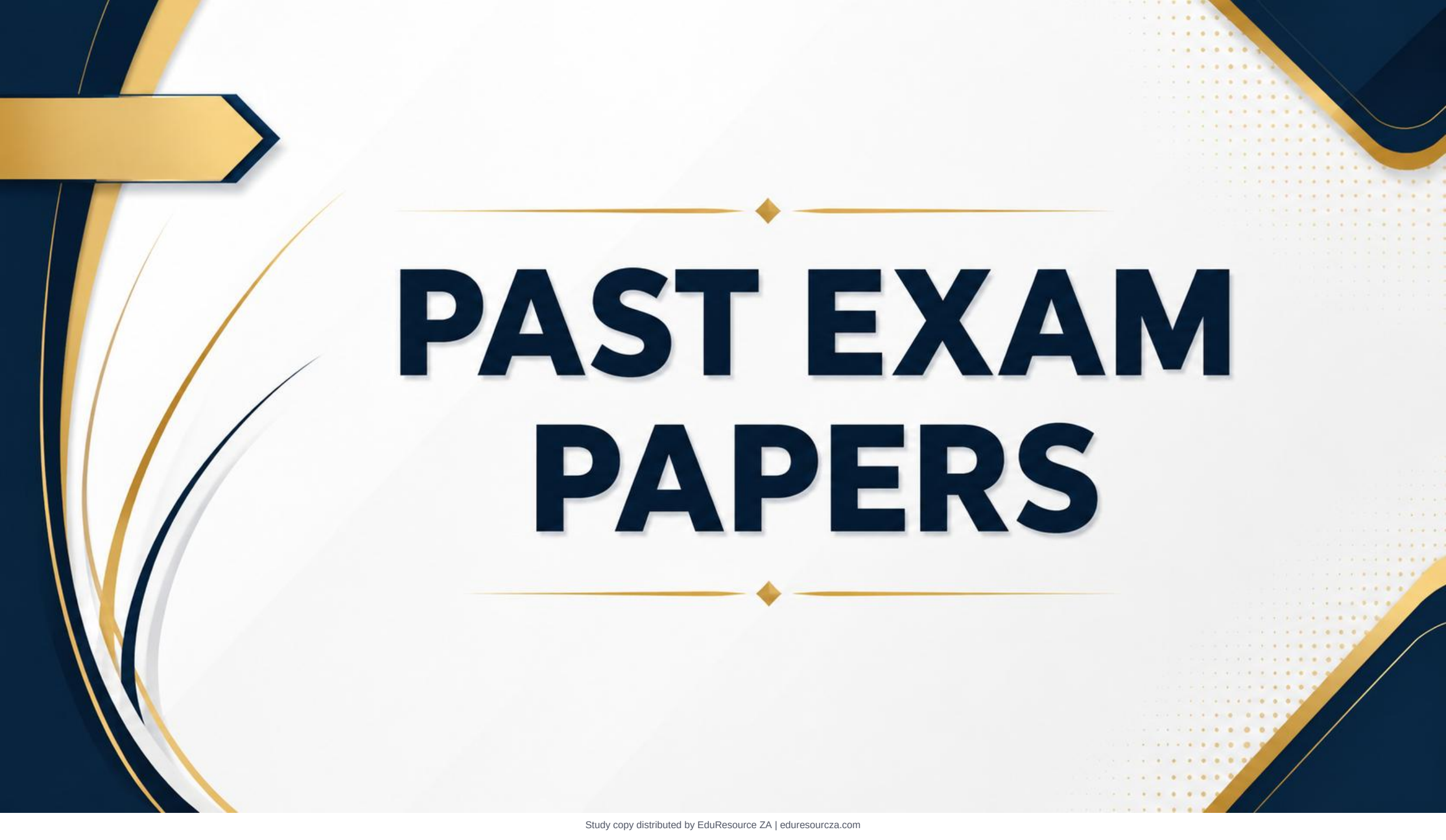
In the diagram below, $\triangle ABC$, with $AC = 12$ cm, $AB = 10$ cm and $\angle A = 125^\circ$. Calculate the area of $\triangle ABC$.



Example 8

In the diagram below, $\triangle MNP$, with $MP = 18\text{ cm}$, $PN = 17,3\text{ cm}$ and $\angle N = 55,2^\circ$. Calculate the area of $\triangle MNP$

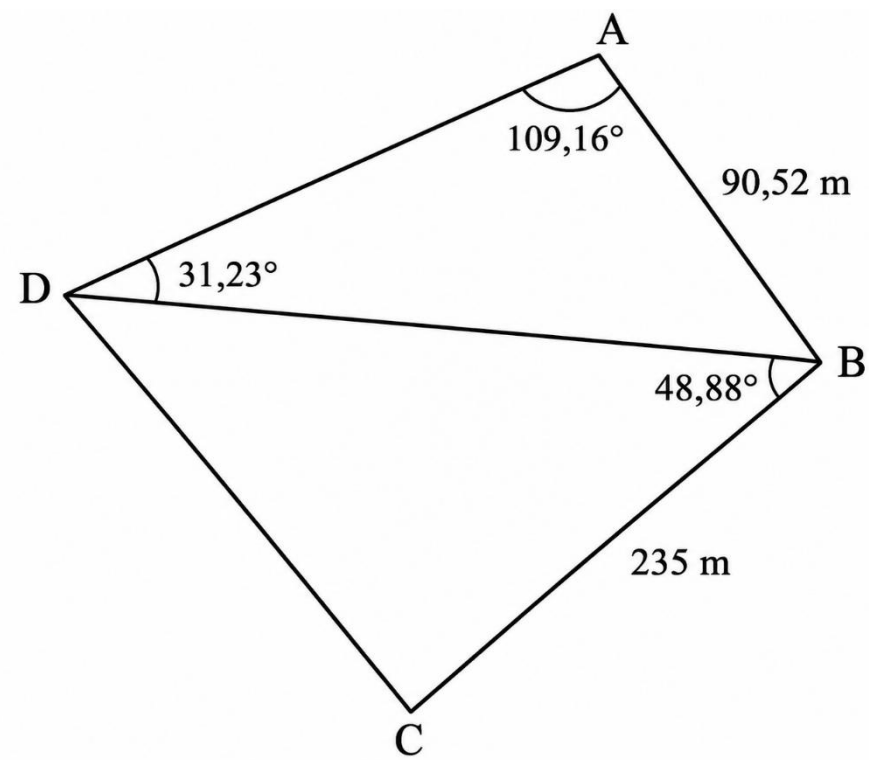




PAST EXAM PAPERS

2016 November Paper 2 Q 7.2

Quadrilateral ABCD is drawn with $BC = 235\text{ m}$ and $AB = 90,52\text{ m}$. It is also given that $\angle ADB = 31,23^\circ$, $\angle DAB = 109,16^\circ$, and $\angle CBD = 48,88^\circ$.



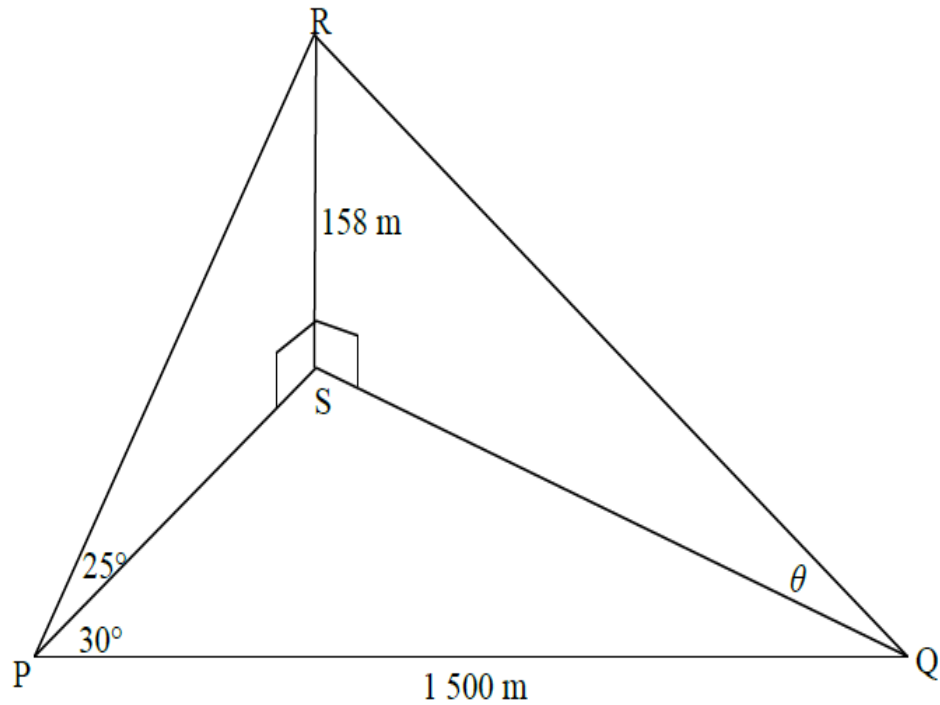
Determine the length of:

7.2.1 BD (3)

7.2.2 CD (3)

2015 Eastern Cape November Paper 2 Q 6.2

In the diagram below, PQ is a straight line 1 500 m long. RS is a vertical tower 158 m high with P, Q and S points in the same horizontal plane. The angles of elevation of R from P and Q are 25° and θ . $\angle SPQ = 30^\circ$



6.2.1 Determine the length of PS. (3)

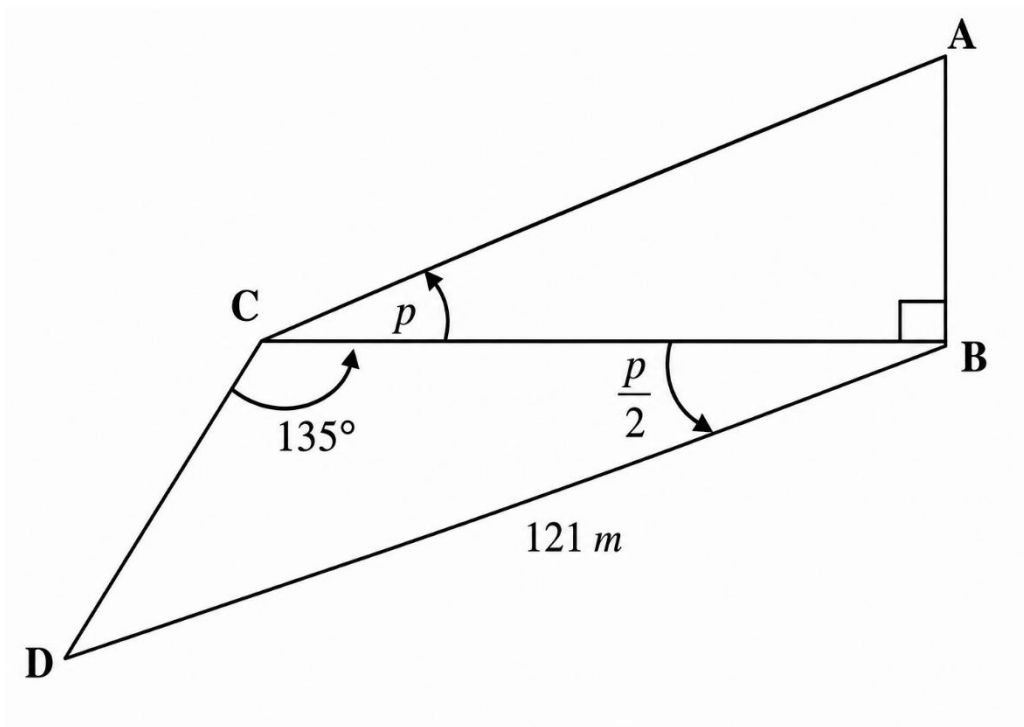
6.2.2 Determine the length of SQ. (3)

6.2.3 Hence, find the value of θ . (3)

6.2.4 Determine the area of $\triangle SPQ$. (4)

2023 Gauteng November Paper 2 Q 7

In the figure below, $\triangle ABC$ is drawn where $AB \perp BC$ and $\angle ACB = p$, with $CB = 3AB$. $\triangle DCB$ is drawn such that $\angle DCB = 135^\circ$ and $BD = 121 \text{ m}$

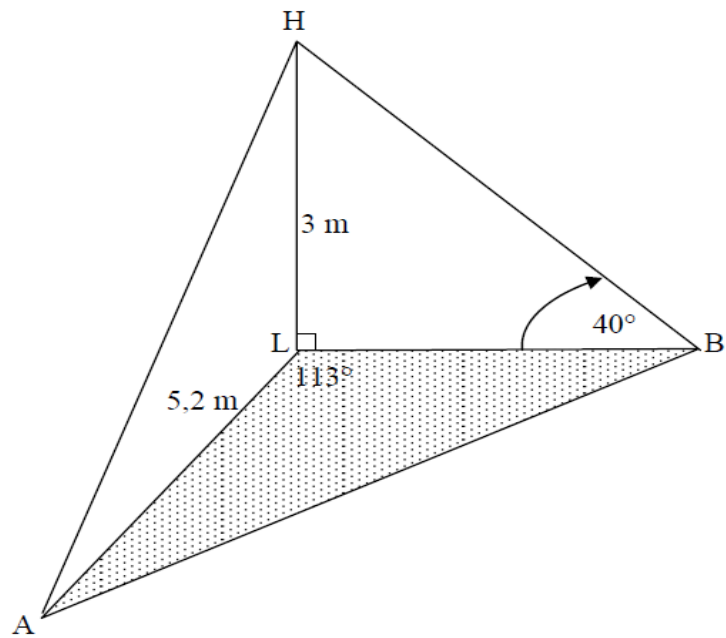


7.1 Determine the value of p , correct to 3 decimal places. (2)

7.2 If $p = 18,4^\circ$, determine the length of CD. (3)

2008 November Paper 2 Q 7

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, $AL = 5,2\text{m}$, the angle $\angle ALB = 113^\circ$ and the angle of elevation of H from B is 40° .

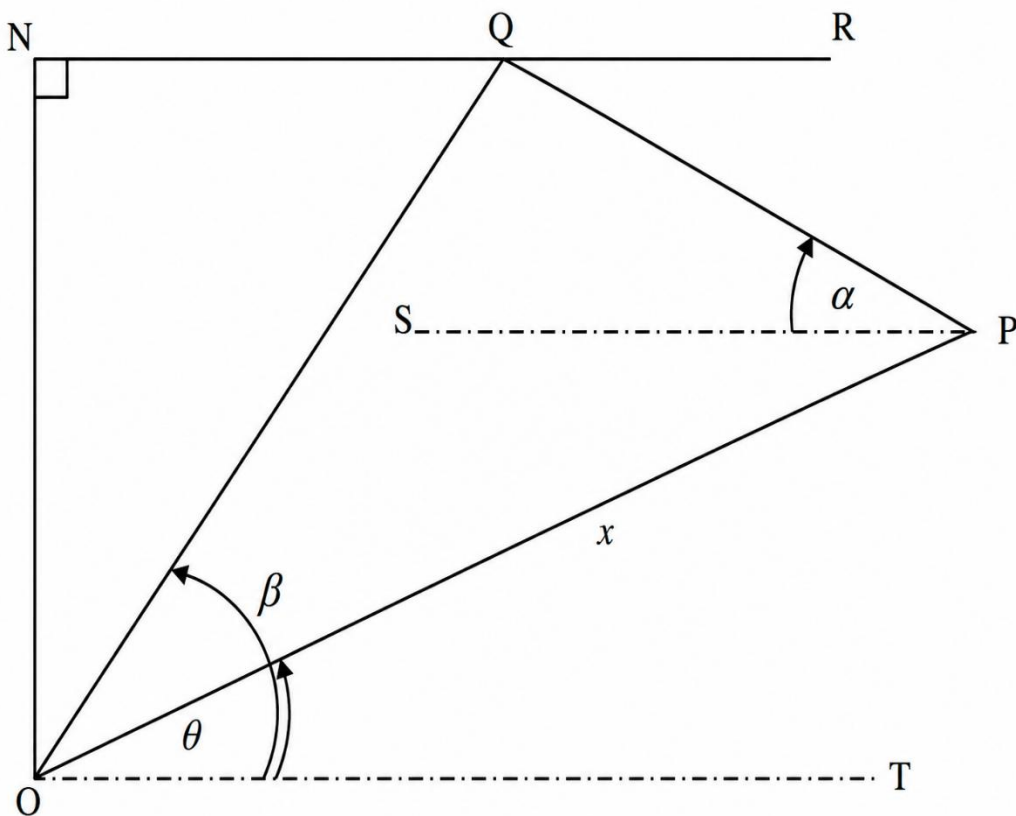


- 7.1 Calculate the length of LB. (2)
- 7.2 Hence, or otherwise, calculate the length of AB. (4)
- 7.3 Determine the area of $\triangle ABL$. (4)

2025 Eastern Cape November Paper 2 Q 6

A man standing on a horizontal plane at point O observes two helicopters positioned at points P and Q. The lines of sight from O to each helicopter make angles of elevation θ and β , respectively, with the horizontal plane. Furthermore, a second observer in the helicopter at P observes the helicopter at Q. The line of sight PQ makes an angle of elevation α with the horizontal plane through P.

Given that $ON \perp NR$ and $OP = x$.



6.1 Show that: $\angle OQP = 180^\circ - (\beta + \alpha)$ (2)

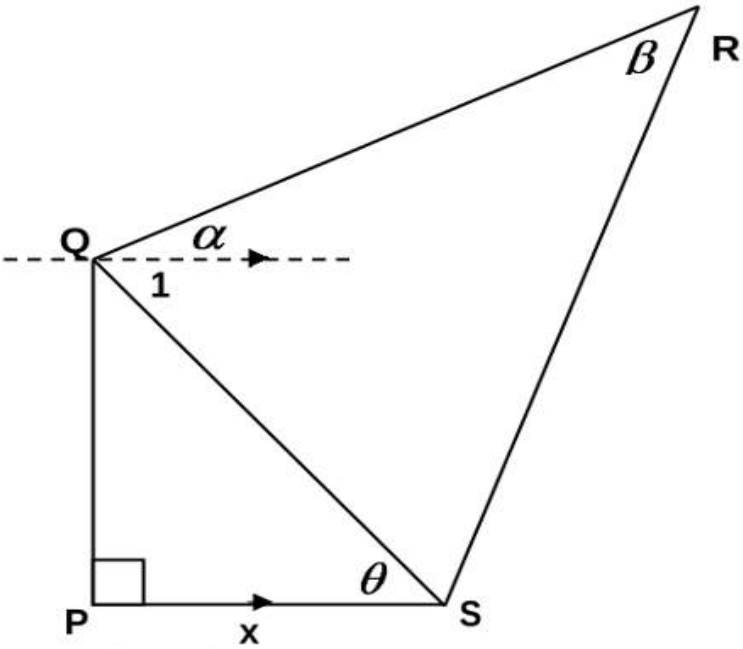
6.2 Show that:

$$OQ = \frac{x \sin(\alpha + \theta)}{\sin(\beta + \alpha)} \quad (3)$$

6.3 Determine the value of x if: $\theta = 15^\circ$, $\beta = 60^\circ$, $\alpha = 30^\circ$, and $OQ = 5\sqrt{2}$. (2)

2023 KwaZulu Natal November Paper 2 Q 7.2

In the diagram below, P, Q, R and S are points in the same vertical plane. The angle of elevation of R from Q is α and the angle of elevation of Q from S is θ . PS = x units and $\angle QRS = \beta$.



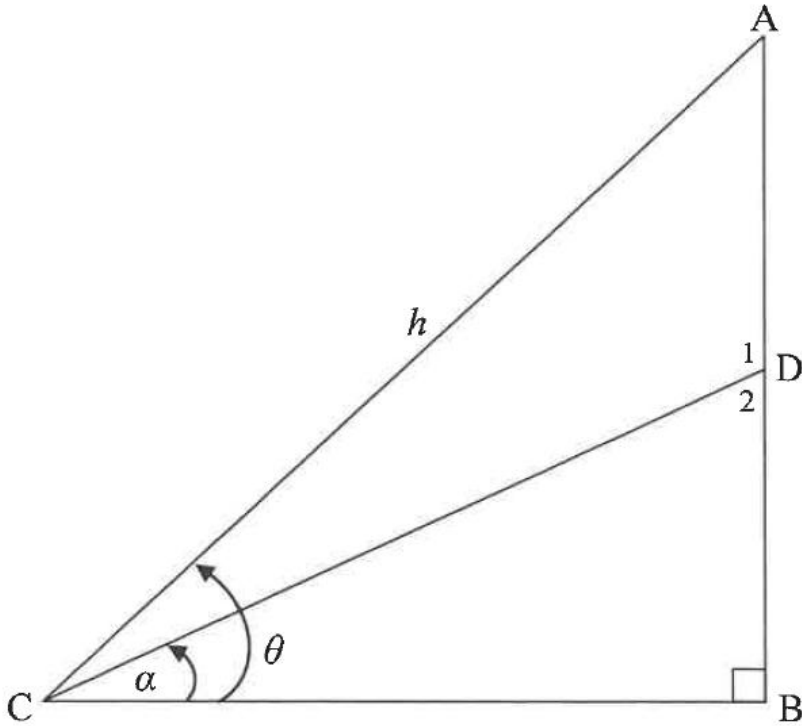
Prove that:

$$SR = \frac{x \cdot \sin(\alpha + \theta)}{\cos \theta \cdot \sin \beta}$$

(6)

2018 November Paper 2 Q 7.2

7.2 In the diagram below, $\angle DCB = \alpha$, $AC = h$ units
 $\angle ACB = \theta$.



7.2.1 Determine the size $\angle ACD$ in terms of θ and α (1)

7.2.2 Prove that:

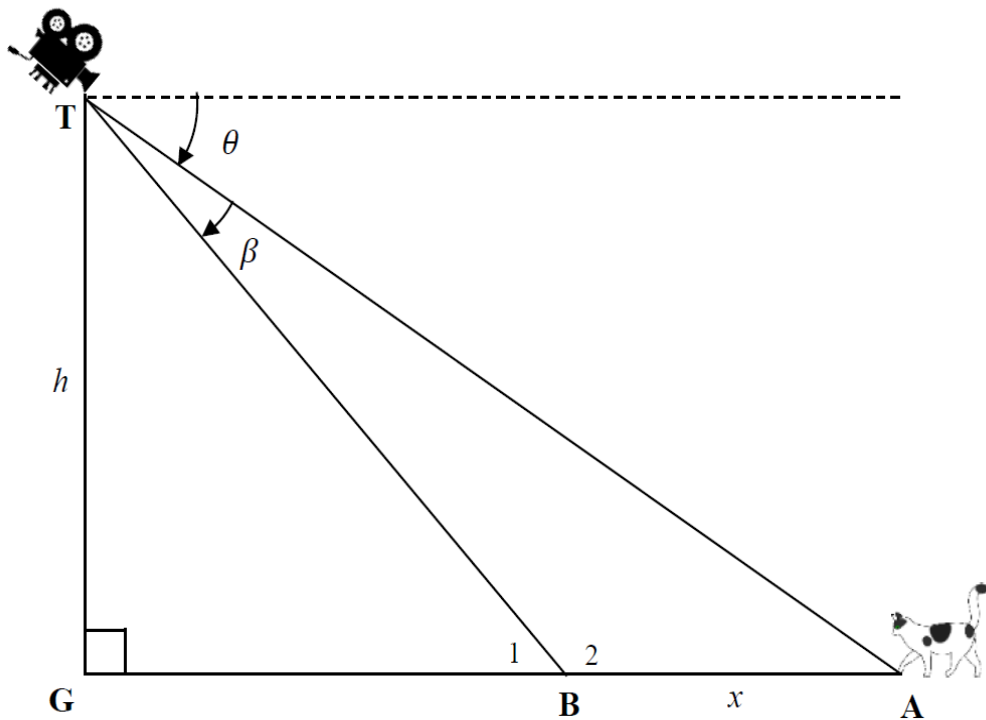
$$AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha} \quad (4)$$

7.2.3 Determine the length of AD if $h = 17$ units,
 $\theta = 58^\circ$ and $\alpha = 23^\circ$ (2)

7.2.4 Calculate the area of $\triangle ADC$ (3)

2022 Gauteng November Paper 2 Q 7

A camera person is stationed at the top of a vertical tower (TG) where $TG = h$. She films a cat starting at A, where the angle of depression of T after A is θ . The camera follows the cat as it walks (in the same vertical plane) a distance of x metres horizontally to B, creating the angle β .



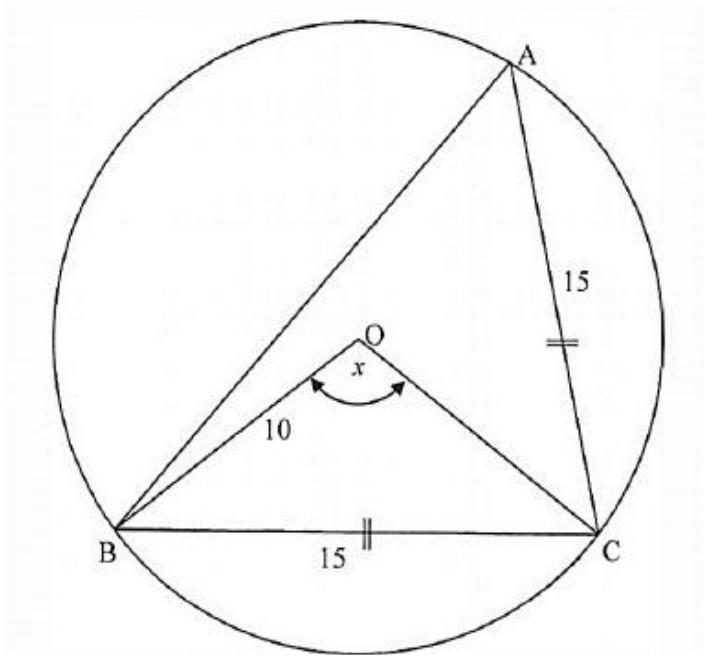
7.1 Determine $\angle B_1$ in terms of θ and β . (1)

7.2 Show that $x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$ (5)

7.3 Determine the distance (x) the cat walked from A to B if the tower was 20 m high, $\theta = 65^\circ$ and $\beta = 37^\circ$ (2)

2024 Free State June Paper 2 Q 5.1

In the diagram below, a circle with center O passes through A, B and C. $BC = AC = 15$ units. BO and OC . $OB = 10$ units and $\angle BOC = x$.

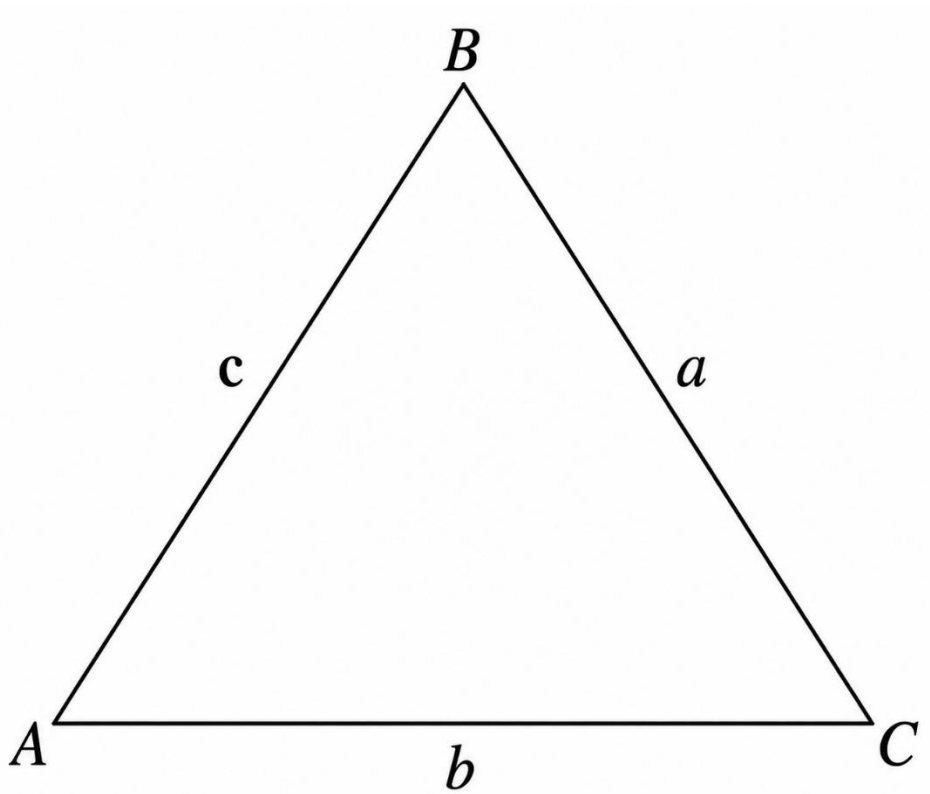


5.1.1 Calculate the size of x . (3)

5.1.2 Calculate the area of triangle ABC (4)

2009 November Paper 2 Q 11.2

Triangle ABC is isosceles with $AB = BC$



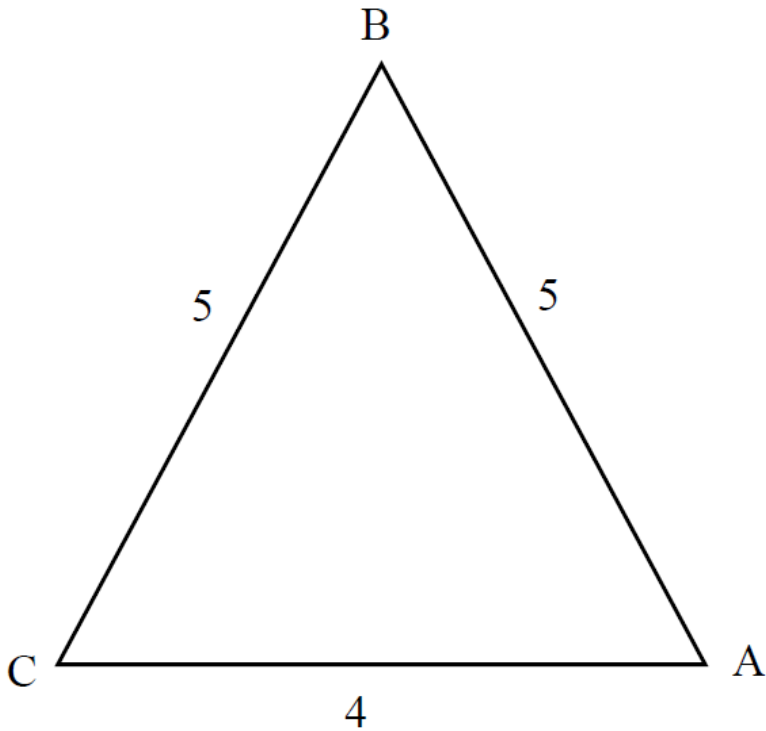
Prove that

$$\cos B = 1 - \frac{b^2}{2a^2}$$

(4)

2021 Eastern Cape June Paper 2 Q 7

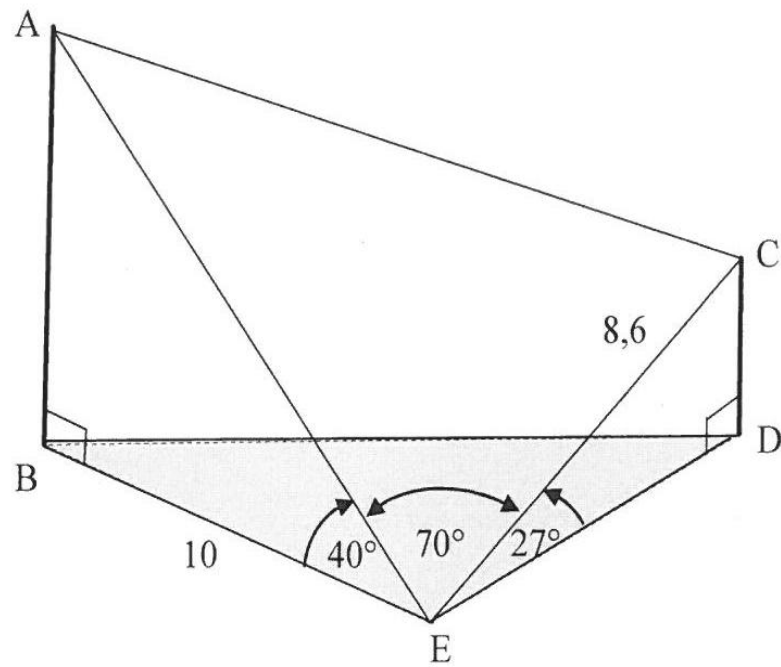
The diagram below shows $\triangle ABC$ with lengths 5,5 and 4 units.



Determine the numerical value of $\cos A - \cos B$ (5)

2017 March Paper 2 Q 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. $CE = 8,6$ m, $BE = 10$ m, $\angle AEB = 40^\circ$, $\angle AEC = 70^\circ$ and $\angle CED = 27^\circ$.

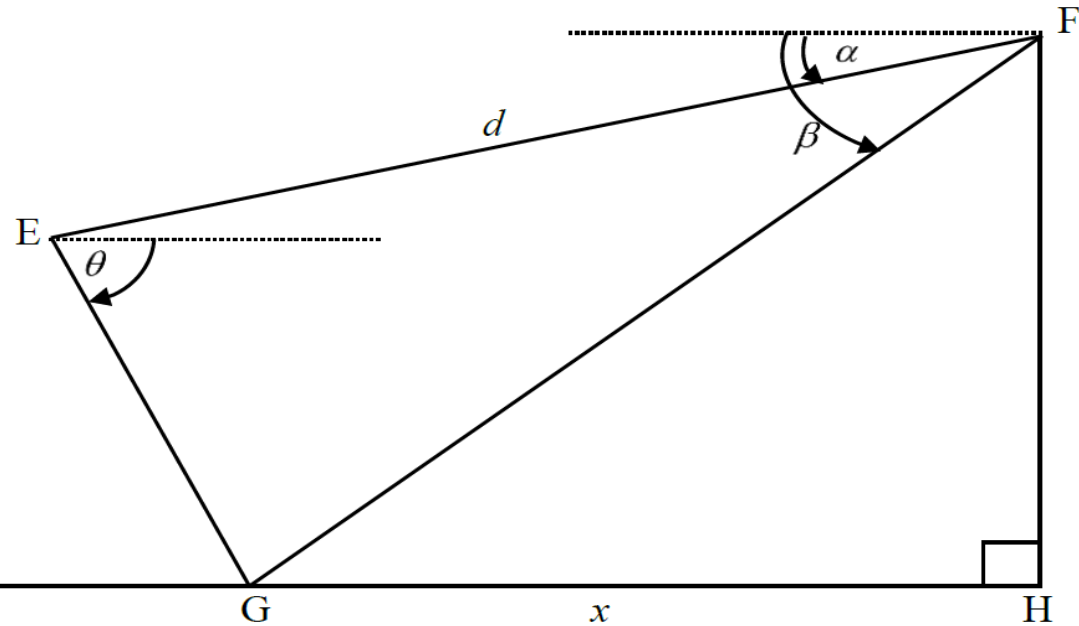


Calculate the:

- 7.1 Height of pole CD. (2)
- 7.2 Length of cable AE. (2)
- 7.3 Length of cable AC. (4)

2024 KwaZulu Natal November Paper 2 Q 6

A cableway connects the top of two mountains across a valley. The distance from E to F is d . From F, the angle of depression to E is α and the angle of depression to G at the bottom of the valley is β . The angle of depression from E to G is θ . H is a point that is directly below F and in the same horizontal plane as G. The distance from G to H is x .



6.1 Write down the size of $\angle FGE$ in terms of θ and β (3)

6.2 Show that:

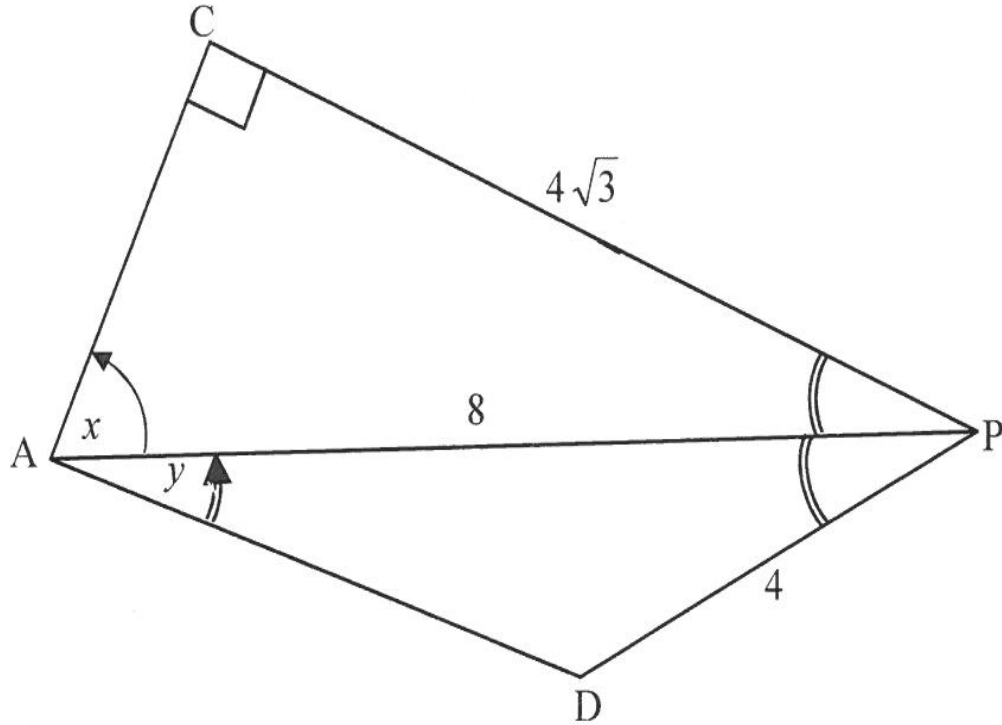
$$FH = \sqrt{\left[\frac{d \sin(\alpha + \theta)}{\sin(\theta + \beta)}\right]^2 - x^2} \quad (5)$$

2014 November Paper 2 Q 5

In the figure below, $\triangle ACP$ and $\triangle ADP$ are triangles with $\angle C = 90^\circ$,

$CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $\angle DPC$.

Let $\angle CAP = x$ and $\angle DAP = y$.



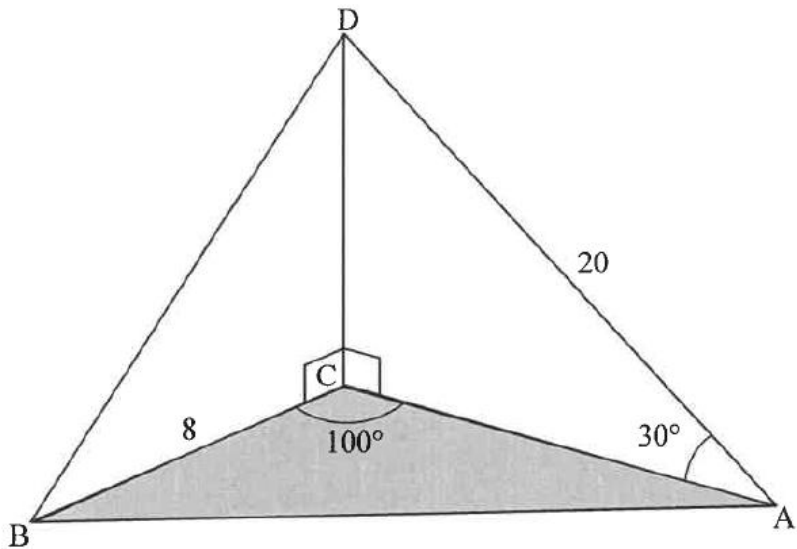
5.1 Show by calculation, that $x = 60^\circ$. (2)

5.2 Calculate the length of AD . (4)

5.3 Determine y . (3)

2023 Senior Certificate Paper 2 Q 7

In the diagram A, B and C are points in the Same horizontal plane. D is a point directly above C , that is $DC \perp BC$. It is given that $\angle ACB = 100^\circ$, $\angle CAD = 30^\circ$ $AD = 20$ units and $BC = 8$ units



7.2 It is further given that $\angle ABD = 73,4^\circ$, calculate the size of $\angle ADB$ (3)

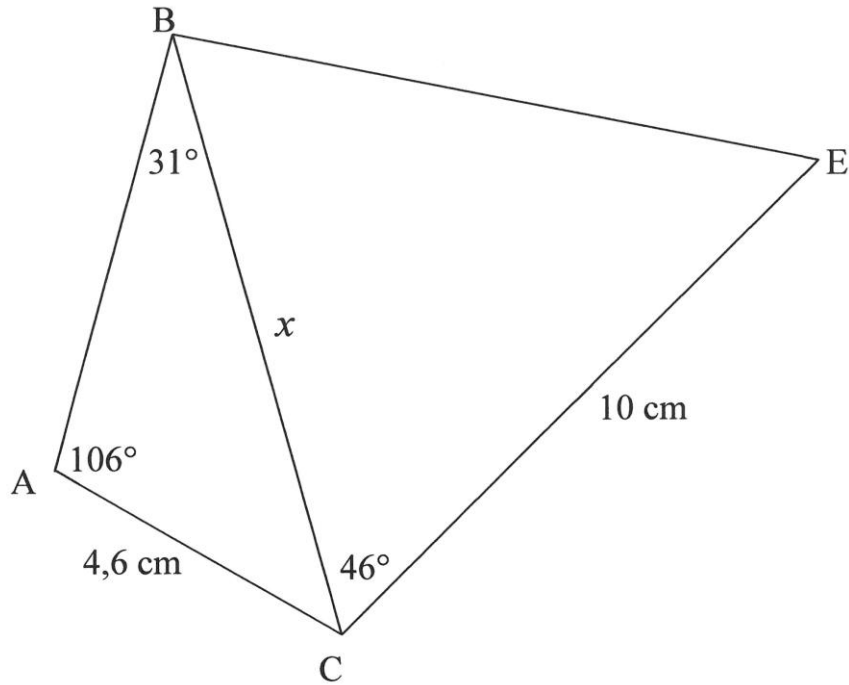
7.1 Calculate the length of:

7.1.1 AC (2)

7.1.2 AB (3)

2019 November Paper 2 Q 7

In the diagram A, C, E and B are the vertices of a quadrilateral. $\angle ABC = 31^\circ$, $\angle BAC = 106^\circ$, $AC = 4,6\text{m}$, $CE = 10\text{cm}$, and $BC = x$.

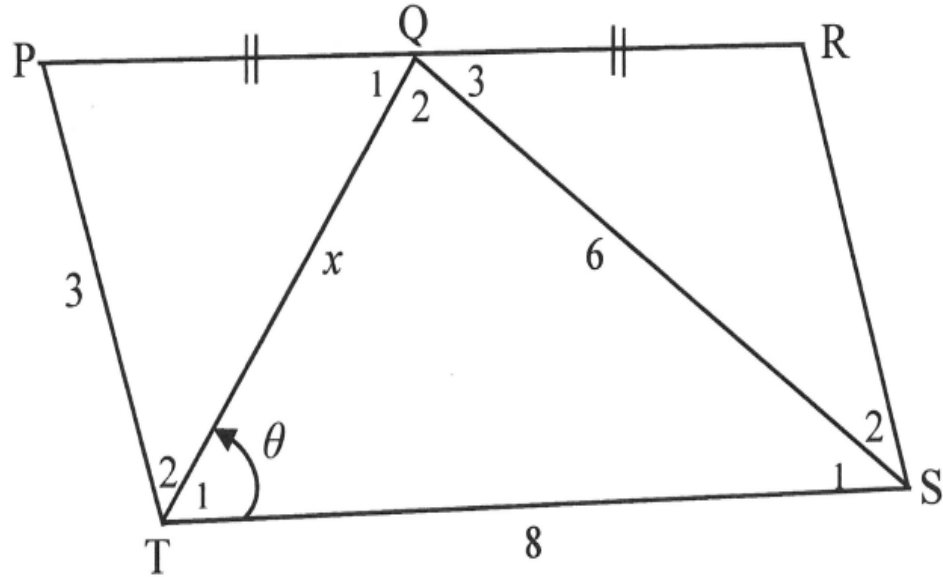


7.1.1 Calculate the length of BC (3)

7.1.2 Calculate the area of the quadrilateral ACEB (4)

CONTINUED...

7.2 In the diagram below, PTRS is a parallelogram. Q is the midpoint of PR. $QS = 6$ units, $PT = 3$ units, $TS = 8$ units. $QT = x$ units and $\angle T_1 = \theta$



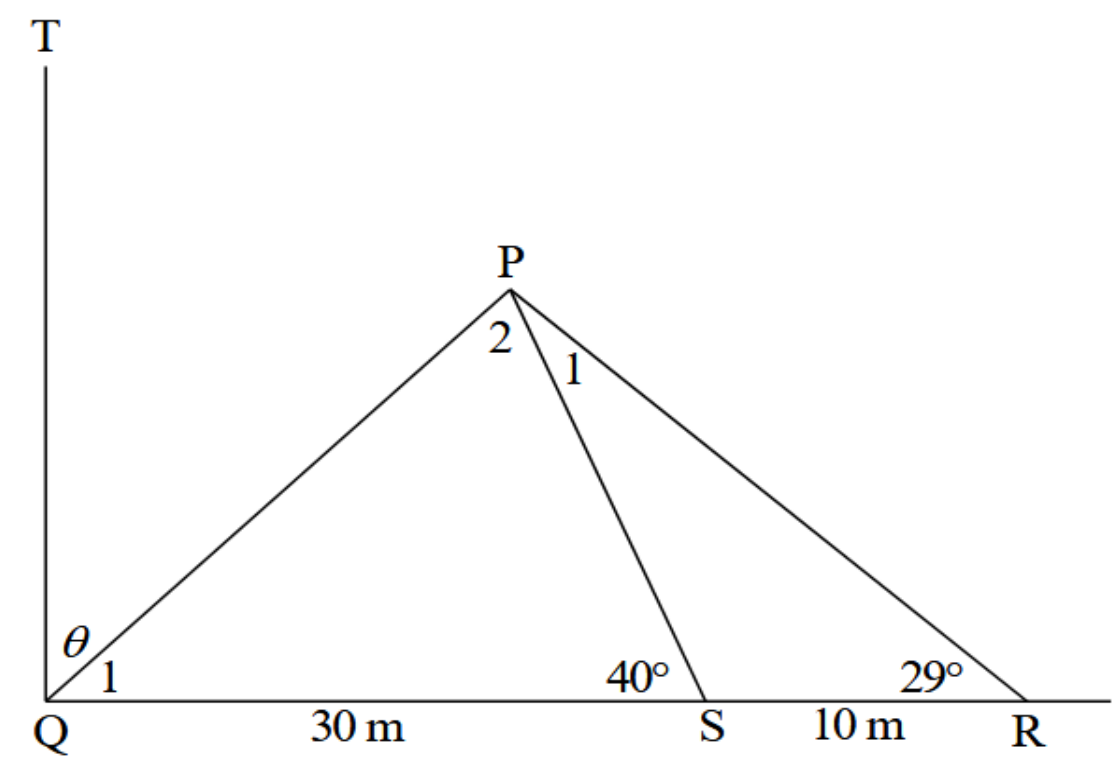
7.2.1 Show that:

$$\cos \theta = \frac{x^2 + 28}{16x} \quad (3)$$

7.2.2 Hence, determine the length of QT (6)

2022 Eastern Cape Paper 2 Q 7

Study the diagram below and then answer the questions that follow. $\angle TQR = 90^\circ$, $\angle QSP = 40^\circ$, $\angle SRP = 29^\circ$ and $QS = 30\text{m}$ and $SR = 10\text{m}$. It is also given that $\angle TQP = \theta$.



- 7.1 Give a reason why $\angle P_1 = 11^\circ$. (1)
- 7.2 Calculate the length of PS. (3)
- 7.3 Determine the value of θ , correct to the nearest degree. (5)

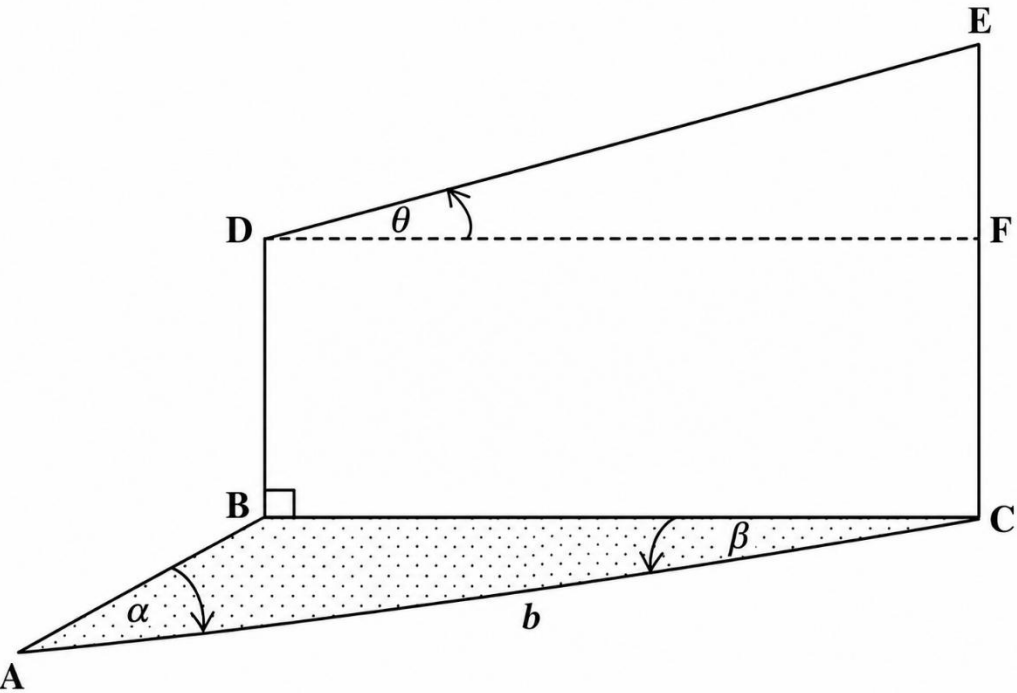
2010 March Paper 2 Q 12.1

12.1 Prove that

$$DE = \frac{bsin\alpha}{sin(\alpha + \beta)cos\theta}$$

(6)

In the diagram below A, B and C are three points in the same horizontal plane. D is vertically above B and E is vertically above C. The angle of elevation of E from D is θ . F is a point on EC such that $DF \parallel BC$. $\angle BAC = \alpha$ and $\angle ACB = \beta$ and $AC = b$ metres.



SINE, COSINE AND AREA RULES PART TWO