



STEM

Explore

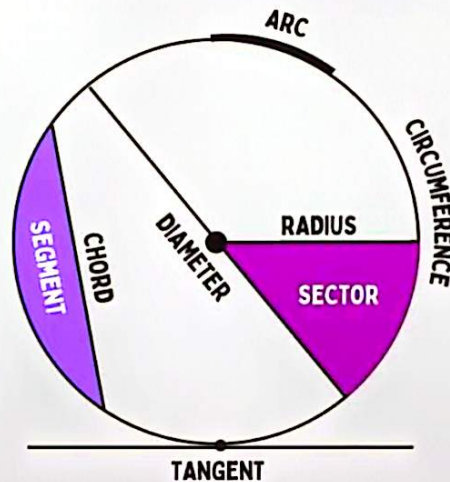
Following

For You



What you need to know first

PARTS OF A CIRCLE



13



Cape Town

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MATH ACADEMY Photo

Grade 11 Mathematics Euclidean Geometry
#grade12 #highschool #grade11

🎵 Contains: Kwa Gae (feat. ...



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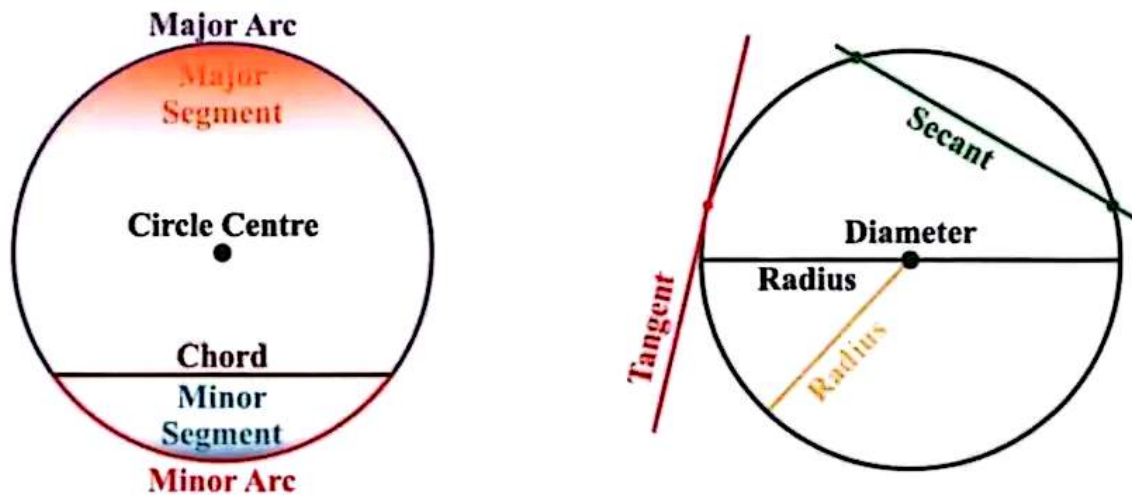


Profile



What you need to know before you attempt Euclidean Geometry

When dealing with circles and circle theorems it is important that you understand the following terminology often used when dealing with circle problems. The diagrams below show the various parts of a circle which you are required to be familiar with.



Term	Explanation
Radius	A line drawn from the centre of the circle to a point on the circumference .
Diameter	A line joining two points on the circumference of the circle passing through the circle centre .
Tangent	A line drawn from a point outside the circle which touches the circle at only one point , called the point of contact. The tangent is always perpendicular to the radius at the point of contact.
Chord	A line joining two points on the circumference of the circle which does not pass through the circle centre .
Secant	A line which cuts through two points on the circumference of a circle.
Theorem	A theorem is a proof of a statement.
Converse	A converse uses the conclusion of a theorem to prove what is given in a theorem.
Corollary	A statement that follows a theorem which is true as a result of the theorem.
Axiom	A general statement which is accepted to be true without a proof.
Rider	The name given to a problem in geometry.

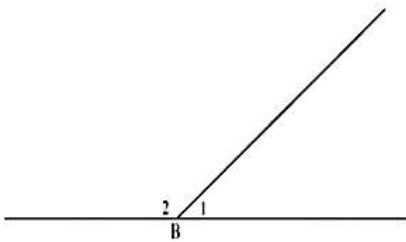
EUCLIDEAN GEOMETRY – GRADE 10

REVISION

LINES AND ANGLES

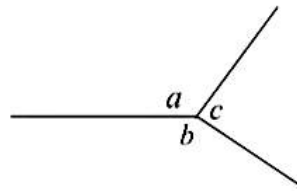
Adjacent supplementary angles

In the diagram, $\hat{B}_1 + \hat{B}_2 = 180^\circ$



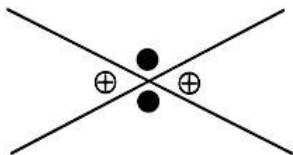
Angles round a point

In the diagram, $a + b + c = 360^\circ$



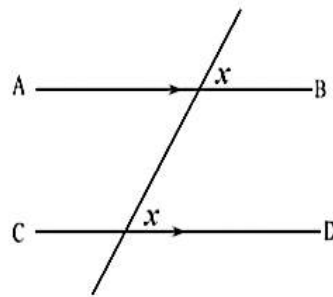
Vertically opposite angles

Vertically opposite angles are equal.



Corresponding angles

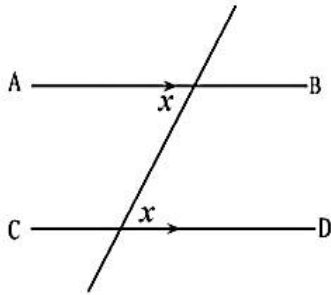
If $AB \parallel CD$, then the corresponding



angles are equal.

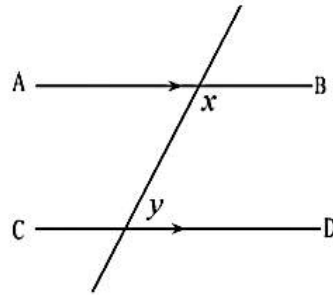
Alternate angles

If $AB \parallel CD$, then the alternate angles are equal.



Co-interior angles

If $AB \parallel CD$, then the co-interior angles add up to 180° , i.e. $x + y = 180^\circ$

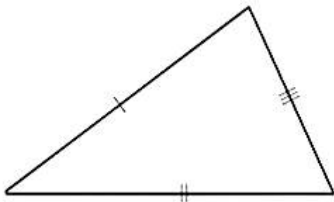


TRIANGLES

There are four kinds of triangles:

Scalene Triangle

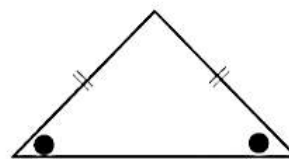
No sides are equal in length



angles are equal

Isosceles Triangle

Two sides are equal

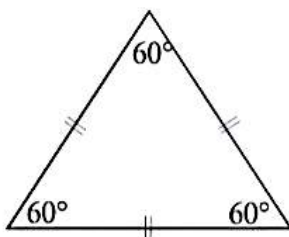


Base

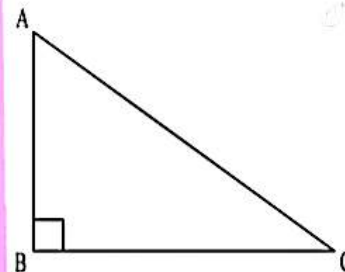
Equilateral

Right-angled triangle

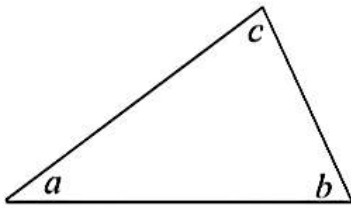
All three sides are equal
All three interior angles are equal



One interior angle is 90°

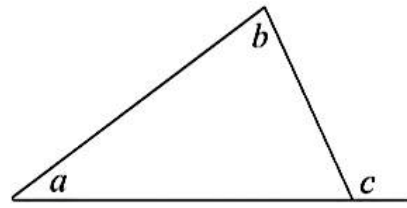


Sum of the angles of a triangle



$$a + b + c = 180^\circ$$

Exterior angle of a triangle



$$c = a + b$$

The Theorem of Pythagoras

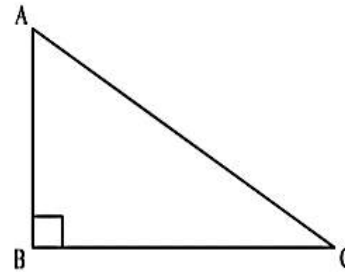
$$AC^2 = AB^2 + BC^2$$

or

$$AB^2 = AC^2 - BC^2$$

or

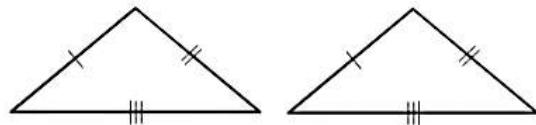
$$BC^2 = AC^2 - AB^2$$



Congruency of triangles (four conditions)

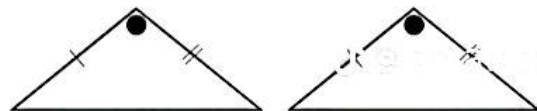
Condition 1

Two triangles are congruent if three sides of one triangle are equal in length to the three sides of the other triangle.



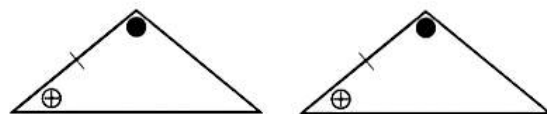
Condition 2

Two triangles are congruent if two sides and the included angle are equal to two sides and the included angle of the other triangle.



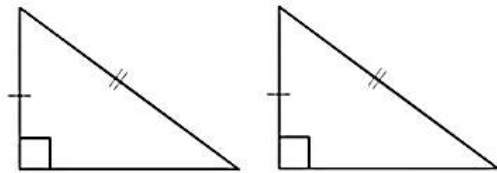
Condition 3

Two triangles are congruent if two angles and one side are equal to two angles and one side of the other triangle.



Condition 4

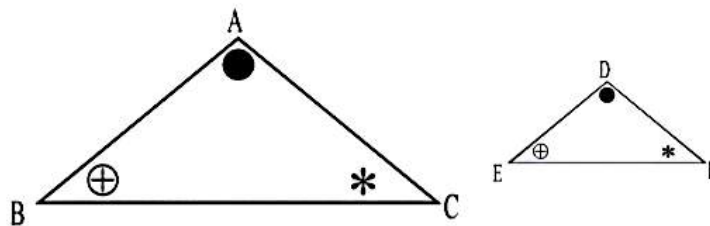
Two right-angled triangles are congruent if the hypotenuse and a side of the one triangle is equal to the hypotenuse and a side of the other triangle.



Similar Triangles

If two triangles are similar (equiangular), then their corresponding sides are in the same proportion.

If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



SOLVING GEOMETRIC RIDERS

STEP 1

- Analyse the diagram given.

STEP 2

- Analyse the diagram by paying attention to **key words** given.
- Look for information in the diagram which can be **helpful and useful**
- Use colours** to mark off equal angles / sides
- Look for **implied information**.

STEP 3

- Brainstorm and develop a **rough proof**.
- Link** the information you have acquired

STEP 4

- Rewrite a **formal proof**

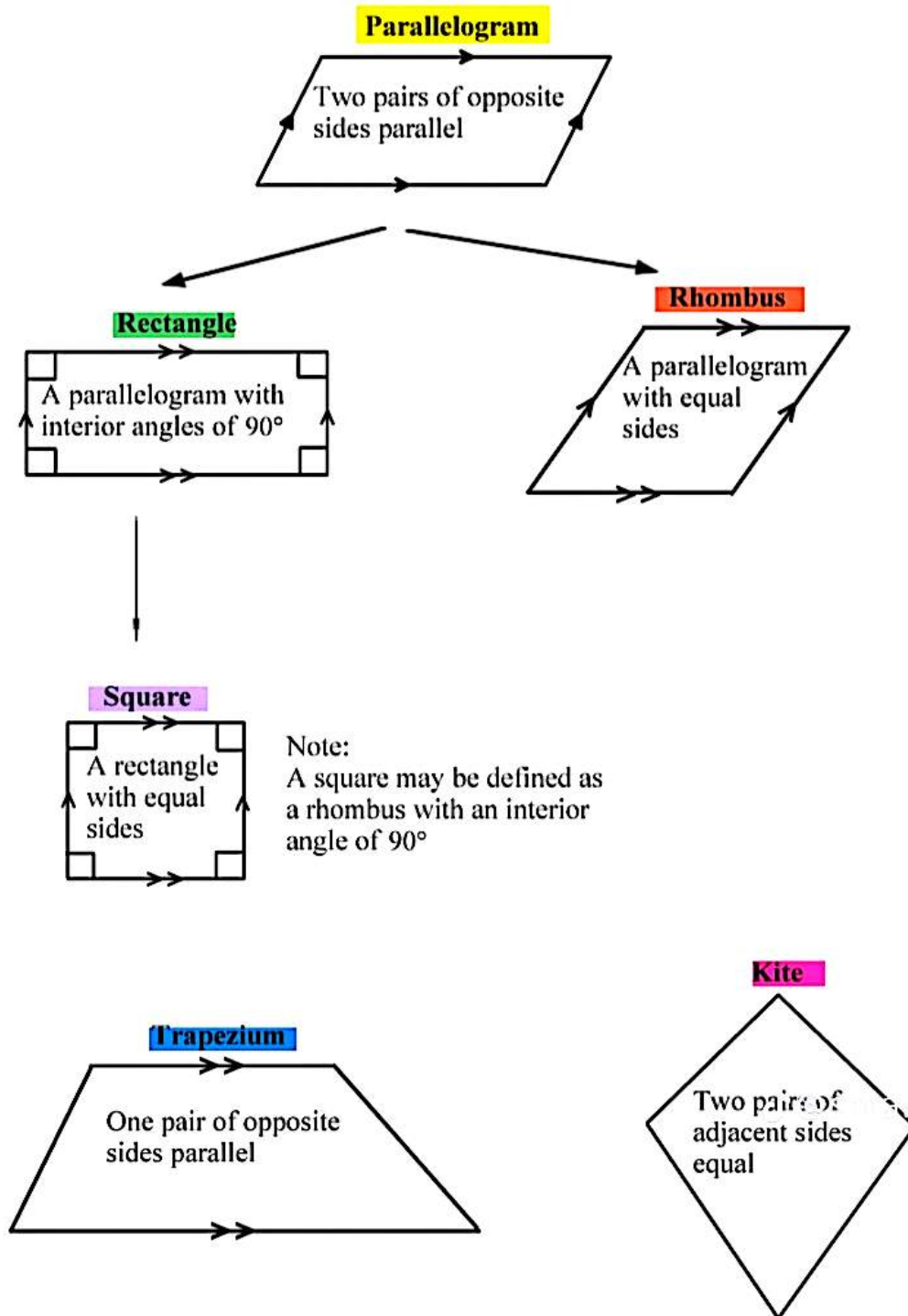
EXAMPLES OF KEY WORDS

- Parallel lines given

QUADRILATERALS

A **polygon** is a two-dimensional figure with three or more straight sides. A **quadrilateral** is a polygon with four straight sides.

Definitions of quadrilaterals

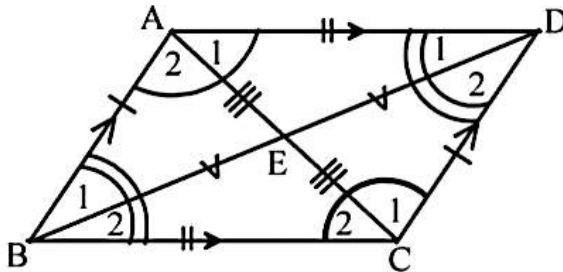


PROPERTIES OF QUADRILATERALS

(It is advisable to first do the investigation and mind map on quadrilaterals before discussing the properties which follow – see Teacher's Guide)

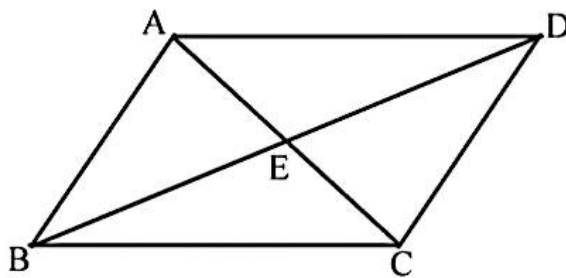
Parallelogram

If ABCD is a parallelogram, you may assume the following properties:



$AD \parallel BC$; $AB \parallel DC$
 $AD = BC$; $AB = DC$
 $AE = EC$; $BE = ED$
 $\hat{D}_1 = \hat{B}_2$; $\hat{D}_2 = \hat{B}_1$; $\hat{C}_1 = \hat{A}_2$; $\hat{C}_2 = \hat{A}_1$
 $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

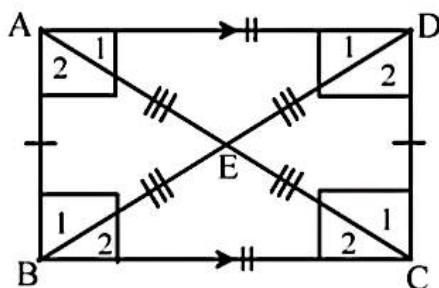
In order to prove that a quadrilateral is a parallelogram, you will need to prove at least one of the following:



$AD \parallel BC$ and $AB \parallel DC$ Opp sides $\parallel$
 $AD = BC$ and $AB = DC$ Opp sides =
 $AE = EC$ and $BE = ED$ Diagonals bisect
 $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$ Opp angles =
 $AB \parallel DC$ and $AB = DC$
 $AD \parallel BC$ and $AD = BC$

Rectangle

If ABCD is a rectangle, you may assume the following properties:



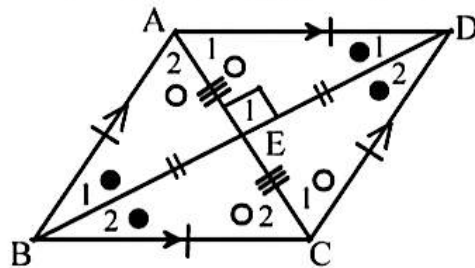
$AD \parallel BC$; $AB \parallel DC$
 $AD = BC$; $AB = DC$
 $AE = EC = BE = ED$
 $\hat{D}_1 = \hat{B}_2$; $\hat{D}_2 = \hat{B}_1$; $\hat{C}_1 = \hat{A}_2$; $\hat{C}_2 = \hat{A}_1$
 $\hat{A} = \hat{C} = \hat{B} = \hat{D} = 90^\circ$

In order to prove that a quadrilateral is a rectangle, you will need to prove one of the following:

- (a) The quadrilateral is a parallelogram with at least one interior angle equal to 90° .
- (b) The diagonals of the quadrilateral are equal in length and bisect each other.

Rhombus

If ABCD is a rhombus, you may assume the following properties:



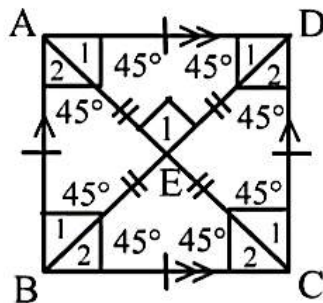
$AD \parallel BC$; $AB \parallel DC$
 $AD = BC = AB = DC$
 $AE = EC$; $BE = ED$
 $\hat{D}_1 = \hat{D}_2 = \hat{B}_1 = \hat{B}_2$
 $\hat{A}_1 = \hat{A}_2 = \hat{C}_1 = \hat{C}_2$; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$
 $\hat{E}_1 = 90^\circ$; $AC \perp BD$

In order to prove that a quadrilateral is a rhombus, you will need to prove one of the following:

- (a) The quadrilateral is a parallelogram with a pair of adjacent sides equal
- (b) The quadrilateral is a parallelogram in which the diagonals bisect at right angles.

Square

If ABCD is a square, you may assume the following properties:



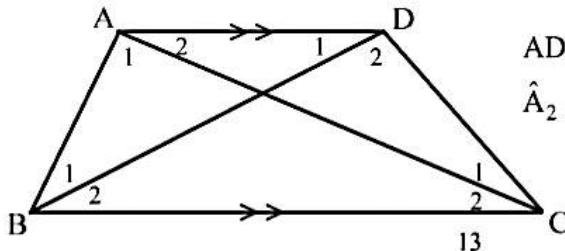
$AD \parallel BC$; $AB \parallel DC$
 $AD = BC = AB = DC$
 $AE = EC = BE = ED$
 $\hat{D}_1 = \hat{D}_2 = \hat{B}_1 = \hat{B}_2 = \hat{A}_1 = \hat{A}_2 = \hat{C}_1 = \hat{C}_2 = 45^\circ$
 $\hat{A} = \hat{C} = \hat{B} = \hat{D} = 90^\circ$
 $\hat{E}_1 = 90^\circ$; $AC \perp BD$

In order to prove that a quadrilateral is a square, you will need to prove one of the following:

- (a) The quadrilateral is a parallelogram with an interior right angle and a pair of adjacent sides equal.
- (b) The quadrilateral is a rhombus with an interior right angle
- (c) The quadrilateral is a rhombus with equal diagonals.

Trapezium

If ABCD is a trapezium, you may assume the following properties:

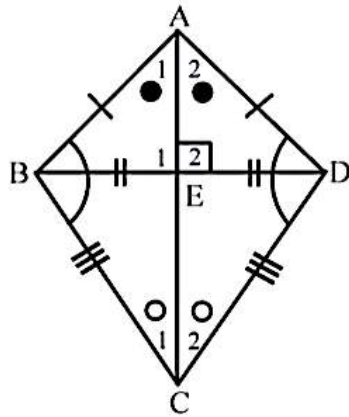


$AD \parallel BC$
 $\hat{A}_2 = \hat{C}_2$; $\hat{D}_1 = \hat{B}_2$

In order to prove that a quadrilateral is a trapezium, you will need to prove that $AD \parallel BC$.

Kite

If ABCD is a kite, you may assume the following properties:



$$AB = AD$$

$$BC = DC$$

$$BE = ED$$

$$\hat{A}_1 = \hat{A}_2$$

$$\hat{C}_1 = \hat{C}_2$$

$$\hat{B} = \hat{D}$$

$$\hat{E}_2 = 90^\circ$$

$$AC \perp BD$$

In order to prove that a quadrilateral is a kite, you will need to prove that the pairs of adjacent sides are equal in length.

GRADE10 ATP

THEOREM 1

The opposite sides and angles of a parallelogram are equal.

Required to prove: $AB = CD$; $AD = BC$; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.

In $\triangle ABC$ and $\triangle CDA$:

(a) $\hat{A}_2 = \hat{C}_1$ alt angles equal

(b) $\hat{A}_1 = \hat{C}_2$ alt angles equal

(c) $AC = AC$ common side

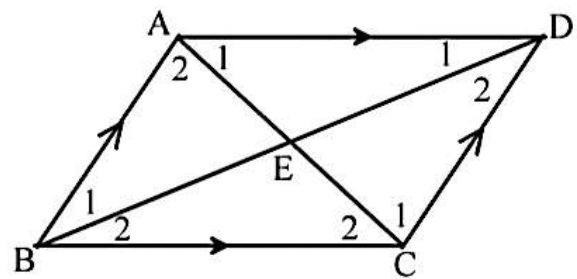
$\therefore \triangle ABC \equiv \triangle CDA$ SAA

$\therefore AB = CD$ and $AD = BC$

Also $\hat{B} = \hat{D}$

Similarly, it can be proved that $\triangle ABD \equiv \triangle CDB$

$\therefore \hat{A} = \hat{C}$



CONVERSE OF THEOREM 1

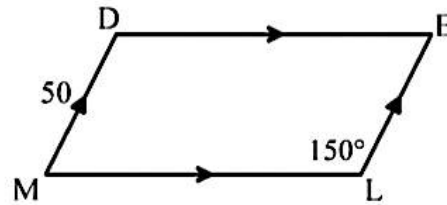
- (a) If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.
- (b) If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.

EXAMPLE 1

DELM is a parallelogram with $DM = 50$ and $\hat{L} = 150^\circ$. Calculate the length of EL and the sizes of $\hat{D}$, $\hat{E}$ and $\hat{M}$.

Solution

$EL = 50$ Opp sides of parm
 $\hat{D} = 150^\circ$ Opp angles of parm
 $\hat{E} = 30^\circ$ Co-interior angles
 $\hat{M} = 30^\circ$ Opp angles of parm

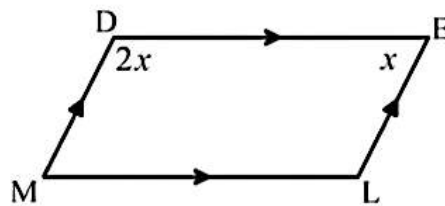


EXAMPLE 2

DELM is a parallelogram. Calculate the value of x and hence the sizes of the interior angles.

Solution

$2x + x = 180^\circ$ Co-interior angles
 $\therefore 3x = 180^\circ$
 $\therefore x = 60^\circ$
 $\therefore \hat{E} = 60^\circ$
 $\therefore \hat{M} = 60^\circ$ Opp angles of parm
 $\hat{D} = 2(60^\circ) = 120^\circ$
 $\hat{L} = 120^\circ$ Opp angles of parm

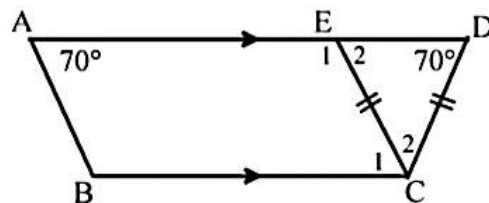


EXAMPLE 3

In trapezium ABCD, $AD \parallel BC$
with $\hat{A} = \hat{D} = 70^\circ$ and $EC = DC$.
Prove that ABCE is a parallelogram.

Solution

$\hat{E}_2 = 70^\circ$ Angles opp equal sides
 $\hat{C}_1 = 70^\circ$ Alt angles equal
 $\therefore \hat{A} = \hat{C}_1$
 $\hat{E}_1 = 110^\circ$ Adjacent suppl angles
 $\hat{B} = 110^\circ$ Co-interior angles
 $\therefore \hat{E}_1 = \hat{B}$

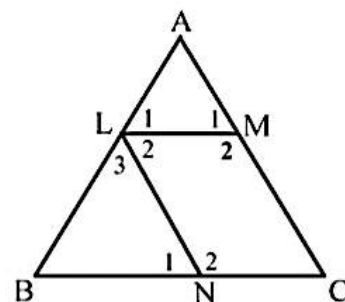
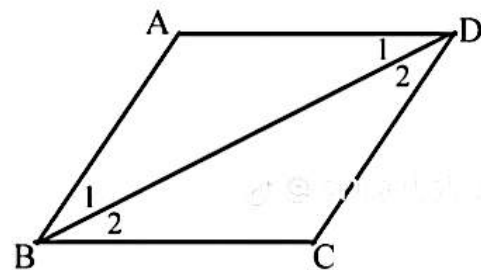
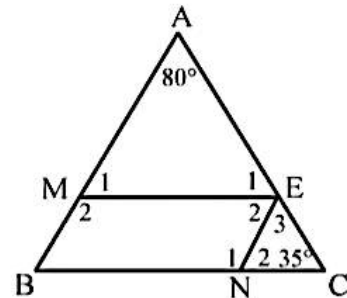
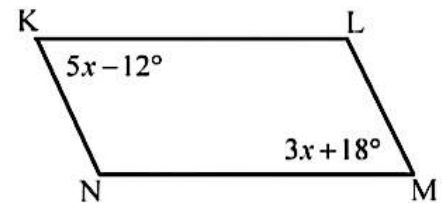
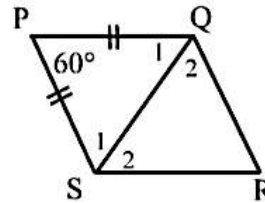
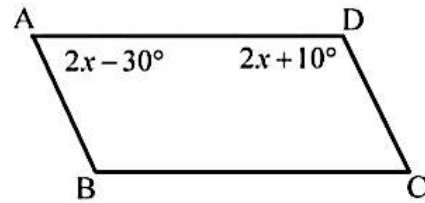


Therefore, ABCE is a parallelogram

Both pair of opposite angles equal

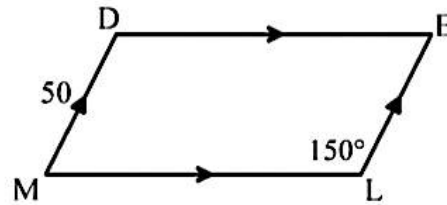
EXERCISE 2

- Determine the sizes of the interior angles of parallelogram ABCD.
- PQRS is a parallelogram with $\hat{P} = 60^\circ$ and $PQ = PS$. Calculate the sizes of $\hat{R}$, $\hat{S}_1$, $\hat{Q}_1$, $\hat{Q}_2$ and $\hat{S}_2$.
- KLMN is a parallelogram. Calculate the size of the interior angles.
- In $\triangle ABC$, $\hat{A} = 80^\circ$ and $\hat{C} = 35^\circ$. Calculate the interior angles of parallelogram MENB.
- In parallelogram ABCD, $AB = AD$ and $\hat{C} = 110^\circ$. Calculate the size of all interior angles.
- $\triangle ABC$ is an equilateral triangle. Determine the interior angles of parallelogram LMCN.



Solution

$EL = 50$ Opp sides of parm
 $\hat{D} = 150^\circ$ Opp angles of parm
 $\hat{E} = 30^\circ$ Co-interior angles
 $\hat{M} = 30^\circ$ Opp angles of parm

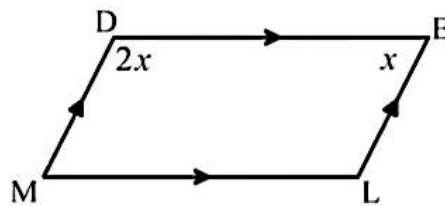


EXAMPLE 2

DELM is a parallelogram. Calculate the value of x and hence the sizes of the interior angles.

Solution

$2x + x = 180^\circ$ Co-interior angles
 $\therefore 3x = 180^\circ$
 $\therefore x = 60^\circ$
 $\therefore \hat{E} = 60^\circ$
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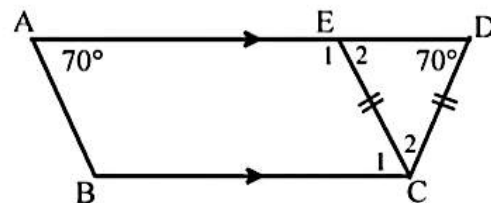


EXAMPLE 3

In trapezium ABCD, $AD \parallel BC$
with $\hat{A} = \hat{D} = 70^\circ$ and $EC = DC$.
Prove that ABCE is a parallelogram.

Solution

$\hat{E}_2 = 70^\circ$ Angles opp equal sides
 $\hat{C}_1 = 70^\circ$ Alt angles equal
 $\therefore \hat{A} = \hat{C}_1$
 $\hat{E}_1 = 110^\circ$ Adjacent suppl angles
 $\hat{B} = 110^\circ$ Co-interior angles
 $\therefore \hat{E}_1 = \hat{B}$



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THEOREM 1

The opposite sides and angles of a parallelogram are equal.

Required to prove: $AB = CD$; $AD = BC$; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.

In $\triangle ABC$ and $\triangle CDA$:

(a) $\hat{A}_2 = \hat{C}_1$ alt angles equal

(b) $\hat{A}_1 = \hat{C}_2$ alt angles equal

(c) $AC = AC$ common side

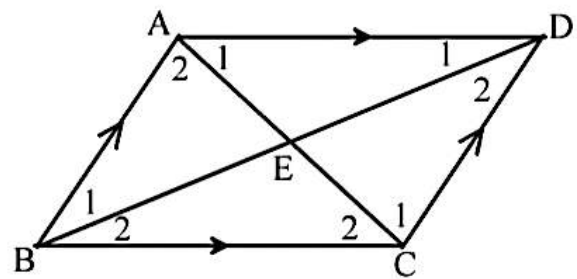
$\therefore \triangle ABC \equiv \triangle CDA$ SAA

$\therefore AB = CD$ and $AD = BC$

Also $\hat{B} = \hat{D}$

Similarly, it can be proved that $\triangle ABD \equiv \triangle CDB$

$\therefore \hat{A} = \hat{C}$



CONVERSE OF THEOREM 1

- (a) If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.
- (b) If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.

EXAMPLE 1

DELM is a parallelogram with $DM = 50$ and $\hat{L} = 150^\circ$. Calculate the length of EL and the sizes of $\hat{D}$, $\hat{E}$ and $\hat{M}$.

THEOREM 2

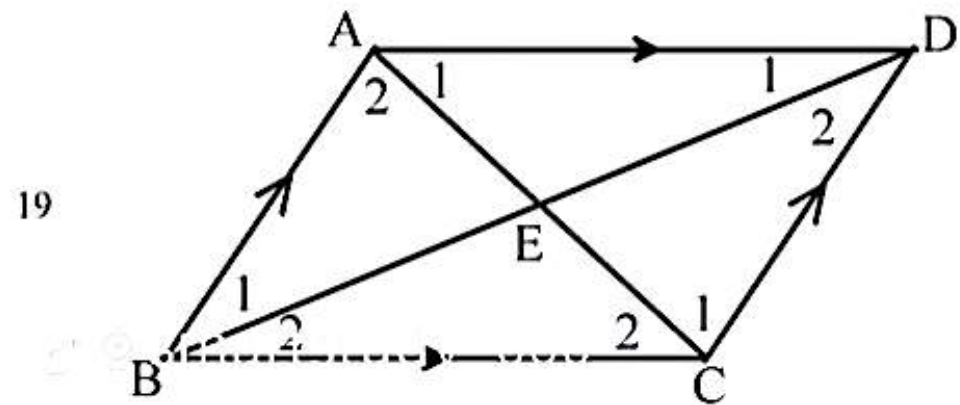
The diagonals of a parallelogram bisect each other.

Required to prove: $AE = EC$ and $BE = ED$

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.

In $\triangle ABE$ and $\triangle CDE$:



- (a) $\hat{A}_2 = \hat{C}_1$ alt angles equal
 (b) $\hat{B}_1 = \hat{D}_2$ alt angles equal
 (c) $AB = DC$ opp sides of parm
 $\therefore \triangle ABE \equiv \triangle CDE$ SAA
 $\therefore BE = ED$ and $AE = EC$

CONVERSE OF THEOREM 2

If the diagonals of quadrilateral bisect each other, then the quadrilateral is a parallelogram.

EXAMPLE 4

Diagonals AC and BD of parallelogram ABCD intersect at M. $AP = QC$ and $AC = 600\text{mm}$, $AB = 500\text{mm}$ and $AP = 100\text{mm}$. Prove that PBQD is a parallelogram.

Solution

$AM = MC$ diagonals of a parm

But $AC = 600\text{mm}$ given

$\therefore AM = MC = 300\text{mm}$

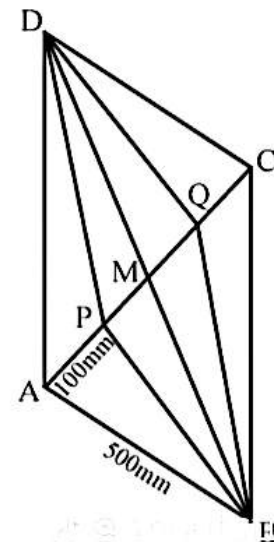
$AP = QC = 100\text{mm}$ given

$\therefore PM = MQ = 200\text{mm}$

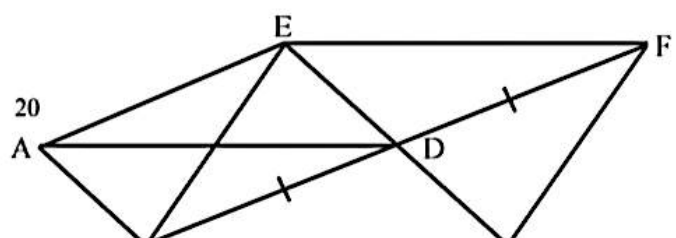
Also $BM = MD$ diagonals of parm

$\therefore PM = MQ$ and $BM = MD$

$\therefore$ PBQD is a parallelogram since diagonals bisect each other.

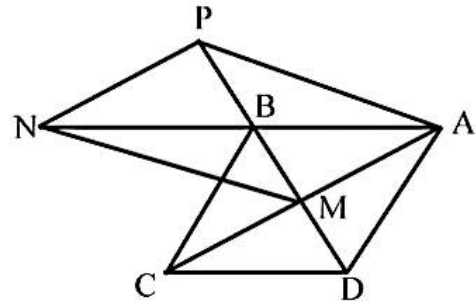


EXERCISE 3

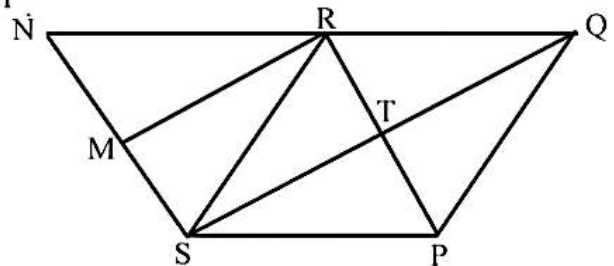


1. Parallelograms ABCD and ABDE are given with $DF = DB$.
Prove that BCFE is a parallelogram.

2. ABCD is a parallelogram. $DM = BP$ and $DC = BN$. Prove:
(a) APNM is a parallelogram.
(b) $PN = MC$



3. PQRS is a parallelogram. PR and QS intersect at T. $QT = RM$ and $SM = PT$.
Prove that:
(a) RTSM is a parallelogram.
(b) $QR = RN$



THEOREM 3

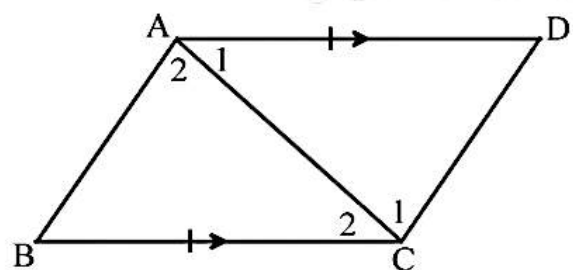
If two opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

Required to prove: ABCD is a parallelogram

Proof:

In $\triangle ABC$ and $\triangle CDA$:

- (a) $\hat{A}_1 = \hat{C}_2$ alt angles equal
(b) $AC = AC$ common side
(c) $AD = BC$ given
 $\therefore \triangle ABC \cong \triangle CDA$ SAS
 $\therefore AB = DC$



and $AD = BC$

$\therefore$ ABCD is a parallelogram since the opposite sides are equal.

EXAMPLE 5

ABCD is a parallelogram with $ED = BF$. Prove that BEFD is a parallelogram.

Solution

$AD \parallel BC$

$\therefore ED \parallel BF$

and $ED = BF$

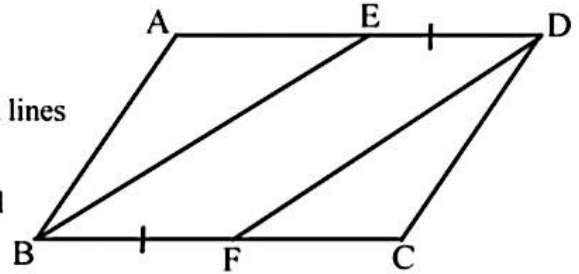
$\therefore$ BEFD is a parm

opp sides parm parallel

AED and BFC are straight lines

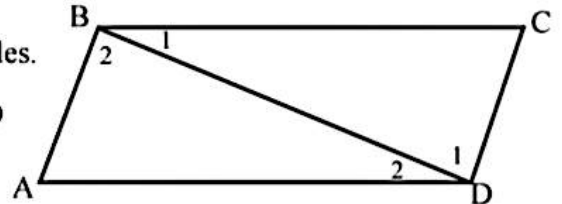
given

one pair of opp sides equal and parallel.

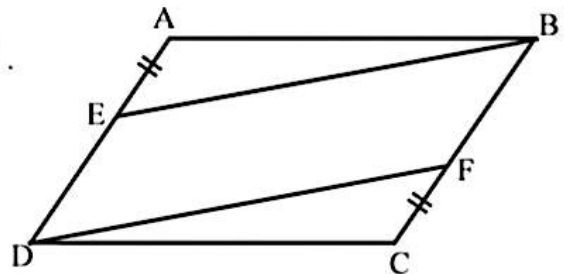


EXERCISE 4

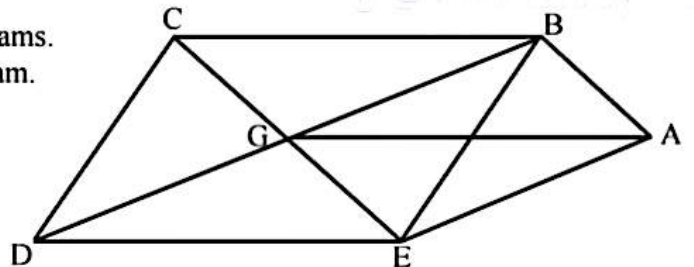
1. $\triangle ABD$ and $\triangle BCD$ are two isosceles triangles.
 $\hat{C} = 75^\circ$ and $\hat{ADB} = 30^\circ$. Prove that ABCD is a parallelogram.



2. ABCD is a parallelogram with $AE = FC$.
Prove that BEDF is a parallelogram.



3. BCDE and ABCG are parallelograms.
Prove that ABGE is a parallelogram.



Grade 11 – Circle Geometry Theorems

1. The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

Given : Any circle O with $OC \perp$ chord AB .

Required : Prove that $AC = BC$.

Construction : Draw OA and OB .

Proof : In $\triangle AOC$ and $\triangle BOC$:

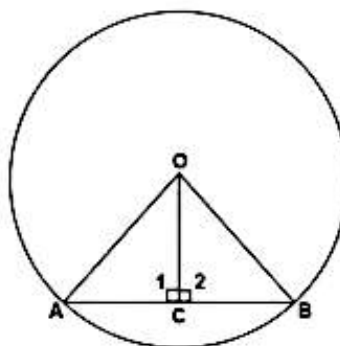
1) $AO = OB$ (radii)

2) OC is common

3) $\hat{C}_1 = \hat{C}_2 = 90^\circ$ (given)

$\therefore \triangle AOC \equiv \triangle BOC$ (RHS)

$\therefore AC = BC$



2. The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord (*Theorem 1 converse*).

Given : Any circle O and C the midpoint of chord AB.

Required : Prove that $OC \perp AB$.

Construction : Draw OC , OA and OB .

Proof : In $\triangle AOC$ and $\triangle BOC$:

1) $AO = OB$ (radii)

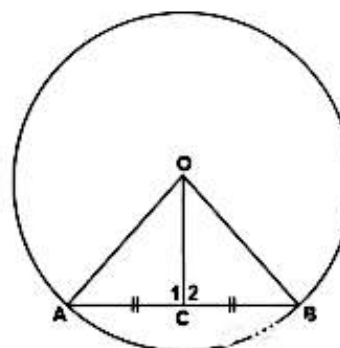
2) OC is common

3) $AC = BC$ (given)

$\therefore \triangle AOC \equiv \triangle BOC$ (S S S)

$\therefore \hat{C}_1 = \hat{C}_2 = 90^\circ$ (straight line)

$\therefore OC \perp AB$



3. The angle subtended by an arc (or chord) of a circle at the centre is double the angle it subtends at any point on the circle. (The central angle is double the inscribed angle subtended by the same arc.)

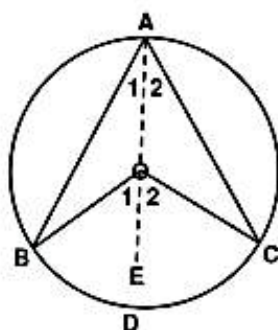


Figure 1

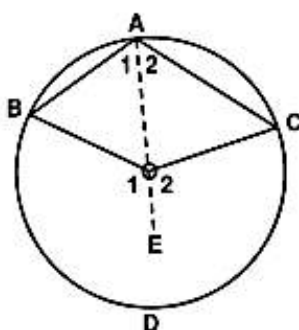


Figure 2

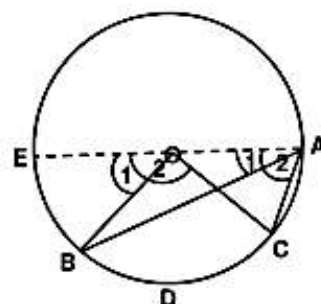


Figure 3

Given : Any circle O with central angle BOC and inscribed angle BAC , both subtended by an arc BDC.

Required : Prove that $\hat{BOC} = 2\hat{BAC}$

Construction : Join AO and produce to E.

Proof :

$$\hat{A}_1 = \hat{B} \quad (\text{OA} = \text{OB} = \text{radius})$$

$$\hat{O}_1 = \hat{B} + \hat{A}_1 \quad (\text{ext. } \angle \text{ of } \Delta = \text{sum of opp.})$$

$$\hat{O}_1 = 2\hat{A}_1$$

Similarly $\hat{O}_2 = 2\hat{A}_2$

$$\therefore \hat{BOC} = 2\hat{BAC}$$

In Figure 1 and 2

$$\hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$$

$$\hat{BOC} = 2(\hat{A}_1 + \hat{A}_2)$$

$$\hat{BOC} = 2\hat{BAC}$$

In Figure 3

$$\hat{O}_2 - \hat{O}_1 = 2\hat{A}_2 - 2\hat{A}_1$$

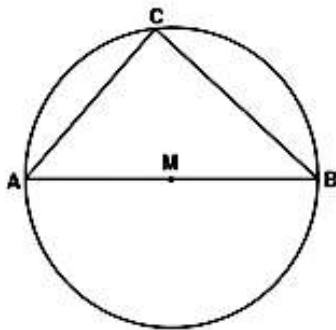
$$\hat{O}_2 - \hat{O}_1 = 2(\hat{A}_2 - \hat{A}_1)$$

$$\hat{O}_2 - \hat{O}_1 = 2\hat{BAC}$$

$$\hat{BOC} = 2\hat{BAC}$$

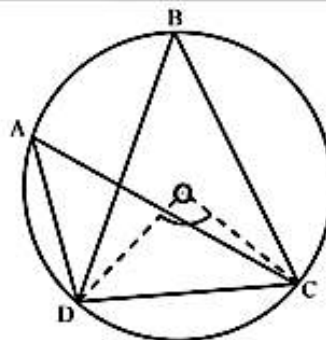
4. The inscribed angle subtended by the diameter of a circle is a right angle.
($\angle$ in semi circle)

Given AB, the diameter of the circle passing through centre M.



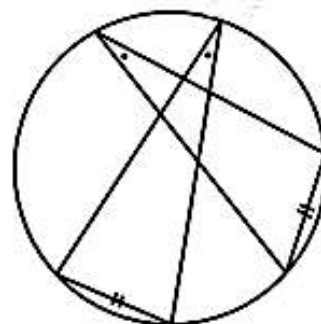
In the sketch AB is the diameter of the circle.
Therefore $\hat{ACB} = 90^\circ$ (angle in semi circle)

5. If the angles subtended by a chord (or arc) of the circle are on the same side of the chord (or arc), then the angles are equal. (Reason: $\angle$ s in same segment)

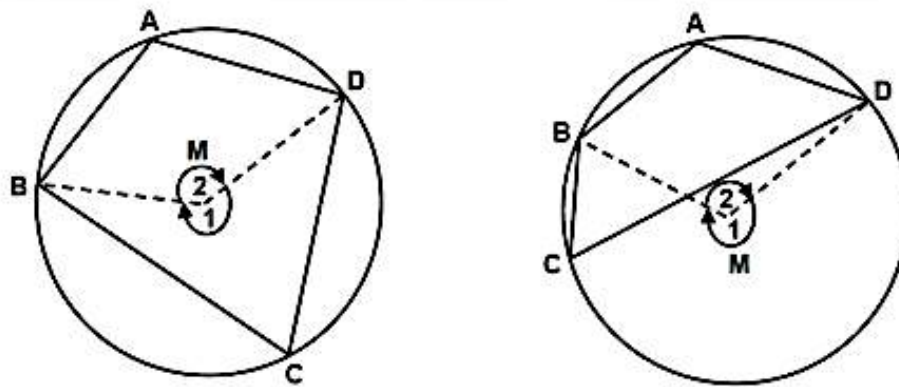


Given : Circle O with inscribed angles A and B subtended by chord DC.
Required : Prove that $\hat{A} = \hat{B}$
Construction : Draw central angle $\hat{DOC}$.
Proof : $\hat{A} = \frac{1}{2} \hat{DOC}$ (angle at centre = $2 \times$ angle circumference)
 $\hat{B} = \frac{1}{2} \hat{DOC}$ (angle at centre = $2 \times$ angle circumference)
 $\therefore \hat{A} = \hat{B}$

We can deduce from this theorem that if angles at the circumference of a circle are subtended by arcs (or chords) of equal length, then the angles are equal.



6. The opposite angles of a cyclic quadrilateral are supplementary (together 180°).



Given : Circle M with cyclic quadrilateral ABCD .

Prove : (a) $\hat{A} + \hat{C} = 180^\circ$
(b) $\hat{ABC} + \hat{ADC} = 180^\circ$

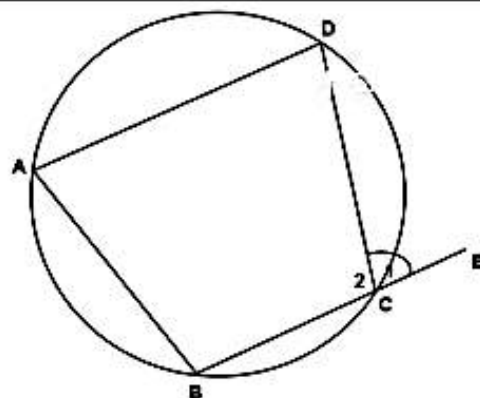
Construction : Draw BM and DM .

Proof : $\hat{M}_1 = 2\hat{A}$ (angle at central = $2 \times$ angle circumference)
 $\hat{M}_2 = 2\hat{C}$ (angle at central = $2 \times$ angle circumference)
 $\hat{M}_1 + \hat{M}_2 = 2(\hat{A} + \hat{C})$
 But $\hat{M}_1 + \hat{M}_2 = 360^\circ$
 Therefore $\hat{A} + \hat{C} = 180^\circ$
 But $\hat{ABC} + \hat{ADC} + \hat{A} + \hat{C} = 360^\circ$ (angles of quadrilateral)
 Therefore $\hat{ABC} + \hat{ADC} = 180^\circ$

7. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Cyclic quad ABCD with BC produced to E.

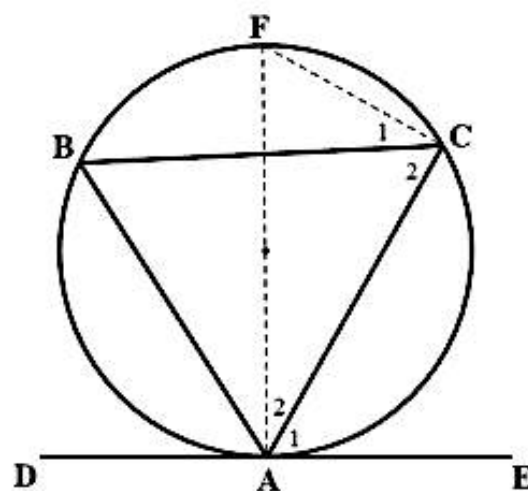
$$\begin{aligned}\hat{C}_1 + \hat{C}_2 &= 180^\circ && \text{(sum angles st. line)} \\ \hat{A} + \hat{C}_2 &= 180^\circ && \text{(proved)} \\ \hat{A} &= \hat{C}_1\end{aligned}$$



When asked to prove that a quadrilateral is a cyclic quadrilateral:

- * the sum of a pair of opposite interior angles is 180°
OR
- * an exterior angle of the quadrilateral is equal to the opposite interior angle
OR
- * a side of the quadrilateral subtends equal angles at the opposite vertices (angles in the same segment)

8. The angle between a tangent and a chord is equal to the angle in the alternate segment.
(Tangent chord theorem)



Given : DE is a tangent to circle centre O at A. B and C are points on the circle.

Prove : $\hat{A}_1 = \hat{B}$

Construction : Diameter AOF, connect F and C.

Proof : $\hat{A}_1 + \hat{A}_2 = 90^\circ$ (Radius $\perp$ Tangent)

$\hat{FCA} = 90^\circ$ ($\angle$ in semi $\odot$)

$\hat{F} = 90^\circ - \hat{A}_2$ (sum $\angle$ s triangle)

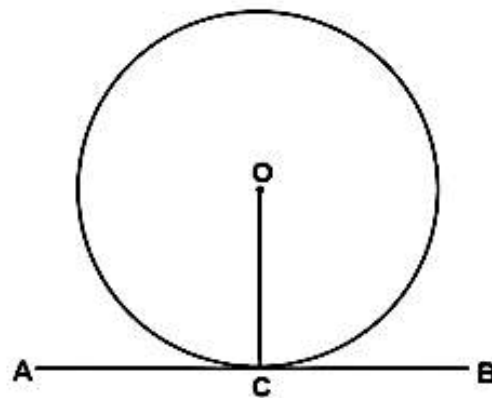
and $\hat{A}_1 = 90^\circ - \hat{A}_2$

$\therefore \hat{A}_1 = \hat{F}$

but $\hat{B} = \hat{F}$ ($\angle$ s in same segment)

$\therefore \hat{A}_1 = \hat{B}$

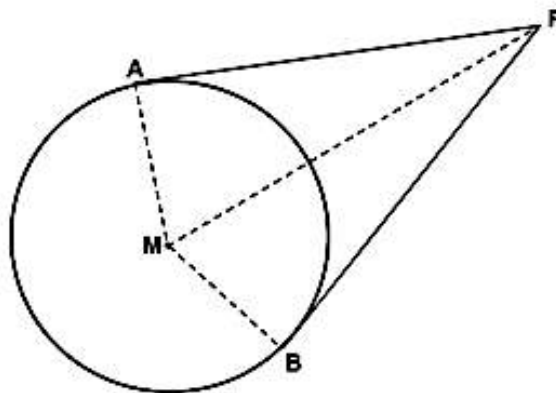
9. A tangent to a circle is perpendicular to the radius at the point of contact.



E.g. if OC is a radius and tangent AB touches the circle at C, then $AB \perp OC$.

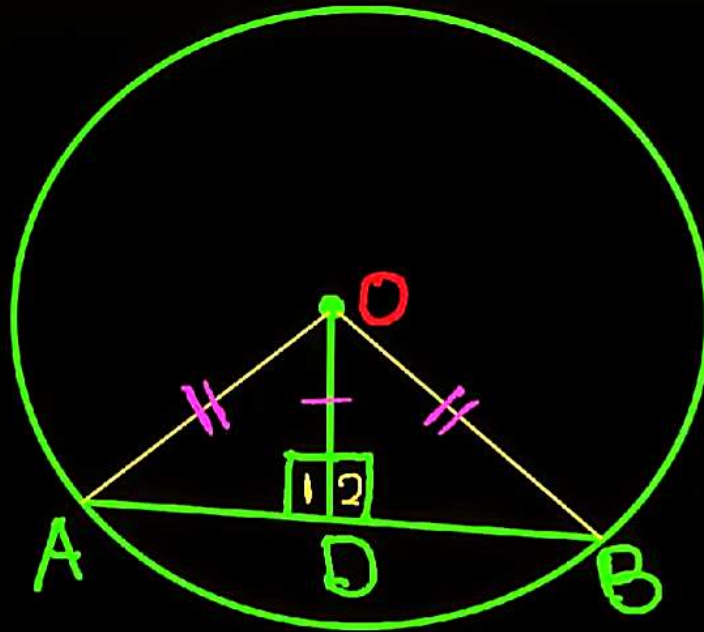
** There is no need to learn the proof for this theorem.

10. Two tangents drawn to a circle from the same point outside the circle, are equal in length.



** There is no need to learn the proof for this theorem.

Theorem 1. Know the statement as well



Prove that $AD = DB$.
Congruency $\rightarrow$ RHS

C. CIRCLE THEOREMS

THEOREM 1: LINE SEGMENT THROUGH CENTRE AND MIDPOINT

The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

Construction: Create Two triangle
Join AO and OB

In a $\triangle OAD$ and OBD

$\hat{O}_1 = \hat{O}_2 = 90^\circ$, given.

$OA = OB$, Radii

$OD = OD$, Common

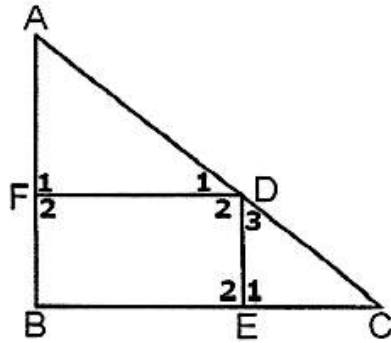
$\triangle OAD \equiv \triangle OBD$, RHS

$\therefore AD = BD$, Congruent triangles.

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Worksheet 9 C

2.



BEDF is a rectangle.

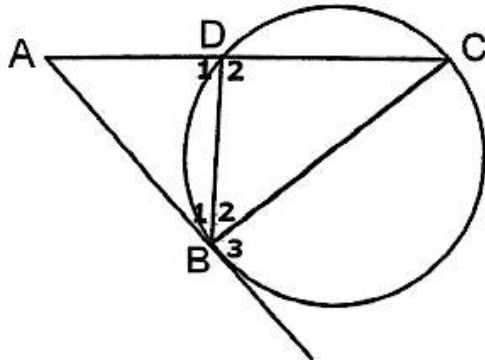
2.1 Prove that $\triangle ABC \sim \triangle AFD$.

2.2 If $BC = 40$ units, $AB = 20$ units and $AF = k$ units, show that $FD = 2k$ units.

2.3 Write FB i.t.o. k and show that the area of the rectangle $= -2k^2 + 40k$.

2.4 Determine the value of k for which the area of the rectangle will be a maximum.

3.



AB is a tangent.

3.1 Prove that $\triangle ABD \sim \triangle ACB$.

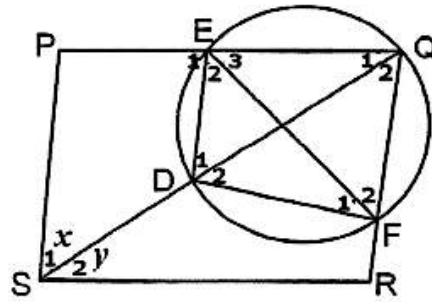
3.2 If $DC = 5$ and $AD = 4$, calculate the length of AB .

3.3 If BC is the diameter of the circle, Show that the radius,

$$r = \frac{3}{2}\sqrt{5}$$

Worksheet 9 C

4.



PQRS is a parallelogram.

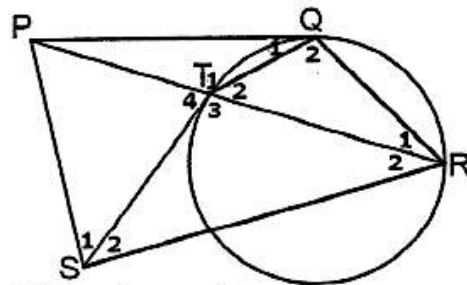
4.1 Name 2 angles equal to x .

4.2 Name 2 angles equal to y .

4.3 Determine a triangle similar to $\triangle FED$.

4.4 Prove that $ED \cdot PQ = QR \cdot DF$.

5.



PQ is a tangent.

5.1 Prove by using similarity that $PQ^2 = PT \cdot PR$.

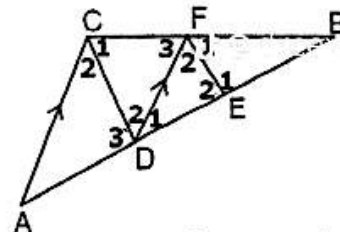
5.2 If given $\angle PSR = \angle T_4$,

Prove that $\triangle PRS \sim \triangle PTS$.

5.3 Complete: $\frac{PR}{PS} = \frac{PT}{PT}$

5.4 Hence prove that $PQ = PS$.

6.

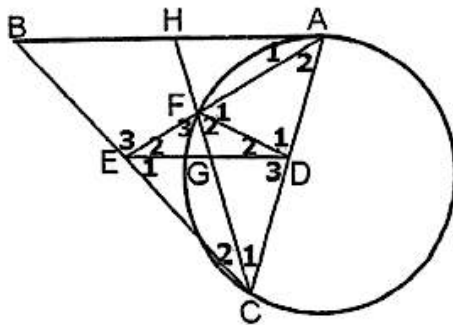


FD bisects $\angle CDB$ and $\angle F_1 = \angle D_1$.

6.1 Name 4 angles equal to $\angle D_1$.

Worksheet 9 E

3.



AB is a tangent. $DE \parallel AB$.

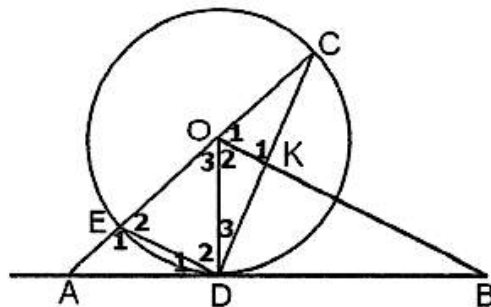
3.1 Prove that:

3.1.1 CDFE a cyclic quadrilateral

3.1.2 $\triangle CDF \parallel \triangle AEB$

3.1.3 $\frac{DA \cdot CG}{GH} = \frac{AE \cdot DF}{EB}$

4.



O is the centre of the circle.

$OB \parallel ED$. AB is a tangent. $\hat{D}_3 = x$.

4.1 Name 3 angles equal to x .

4.2 Determine the size of:

4.2.1 $\hat{K}_1$

4.2.2 $\hat{ADC}$ i.t.o. x

4.3 Prove that:

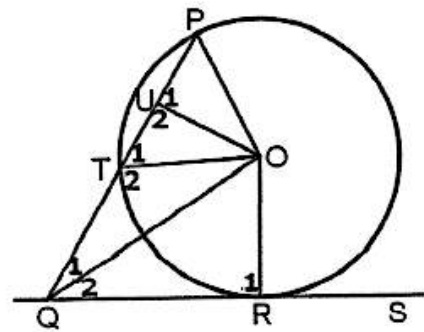
4.3.1 $DK = KC$

4.3.2 $\triangle ACD \parallel \triangle ADE$

4.3.3 $CK \cdot AD = \frac{AC \cdot ED}{2}$

Worksheet 9 E

5.



O is the centre of the circle.

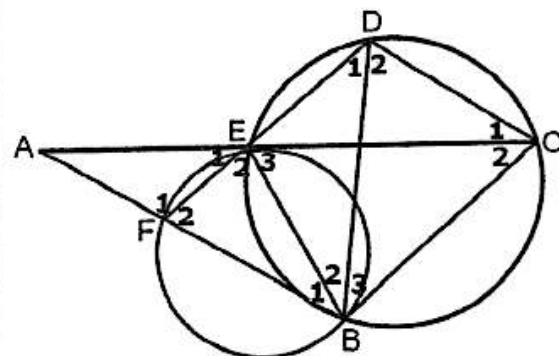
QRS is a tangent and $TU = UP$.

Prove that:

5.1 QROU a cyclic quadrilateral

5.2 $OQ^2 + UT^2 = QU^2 + OT^2$

6.



AB is a tangent to the bigger circle. $DF \parallel BC$.

6.1 Prove that:

6.1.1 $\triangle BCD \parallel \triangle BEF$

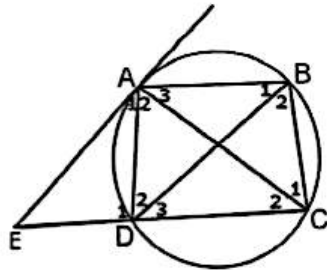
6.1.2 AE is a tangent to circle FEB

6.1.3 $\triangle ABE \parallel \triangle AEF$

6.2 If $4FB = 3AB$, show that
 $AB^2 = 4AE^2$.

Worksheet 9 B

3.



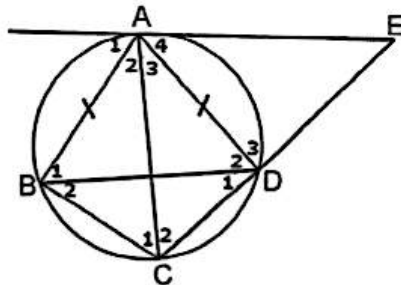
EA is a tangent, $AB \parallel DC$, $\hat{A}_1 = x$.

3.1 Name 4 angles equal to x .3.2 Prove that $\triangle ABC \parallel \triangle ADE$.

3.3 Hence show that

$$BA = \frac{AC \cdot DA}{AE}.$$

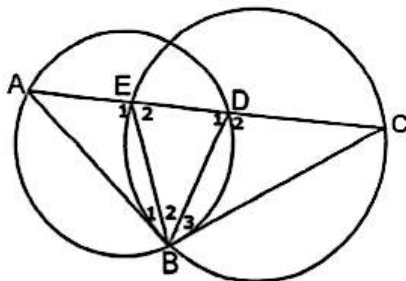
4.



AE is a tangent. Prove:

4.1 $\triangle ADE \parallel \triangle CBA$ 4.2 $AB^2 = DE \cdot BC$

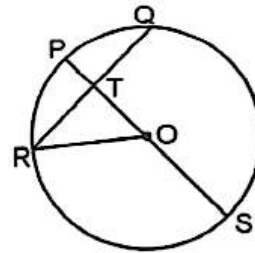
5.



AB is a tangent to the bigger circle. BC is a tangent to the smaller circle. Prove that:

5.1 $BD = BE$ 5.2 $\triangle BCD \parallel \triangle ABE$ 5.3 $BC \cdot AE = AB \cdot BE$ **Worksheet 9 B**

6.



O is the centre of the circle.

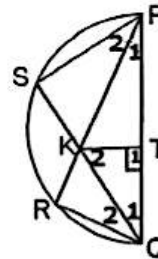
$PO \perp RQ$ and $\frac{PS}{PT} = \frac{9}{4}$.

$QT = 12$ cm.

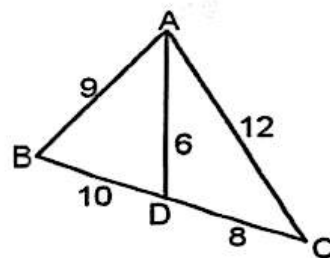
6.1 If $PT = 4a$ units, determine TO l.t.o. a .

6.2 Hence calculate the length of the radius.

7.

7.1 Prove that $\triangle QSP \parallel \triangle QTK$.7.2 Prove that $PS^2 = \frac{SP^2 QK^2}{TK^2} - SQ^2$ **Worksheet 9 C**

1.

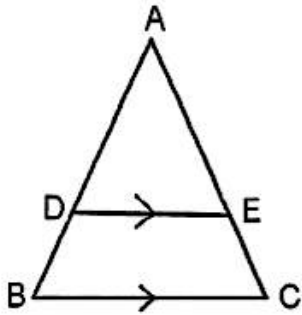


Prove that $\triangle CAD \parallel \triangle CBA$.

Study the proves of the following theorems:

Theorem 1 (a)

A line parallel to one side of a Δ divides the other 2 sides in the same proportion.



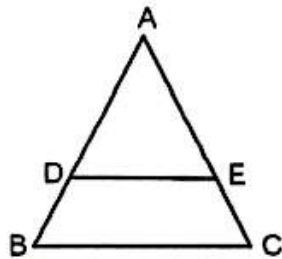
$$\frac{AD}{DB} = \frac{AE}{EC} \quad [DE \parallel BC]$$

$$\text{or } \frac{AD}{AB} = \frac{AE}{AC} \quad [DE \parallel BC]$$

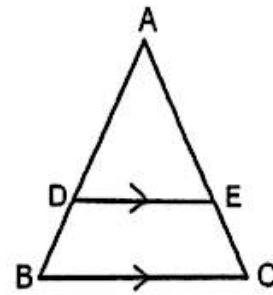
$$\text{or } \frac{AB}{DB} = \frac{AC}{EC} \quad [DE \parallel BC]$$

Theorem 1 (b)

If a line divides 2 sides of a Δ in the same proportion, then the line is parallel to the 3rd side.

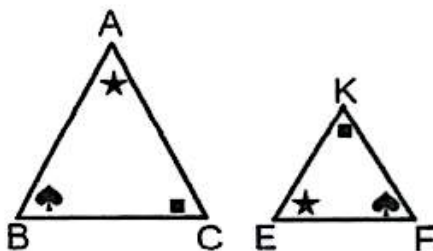


$$\begin{aligned} \text{If } \frac{AD}{DB} &= \frac{AE}{EC} \\ \text{then} \\ DE &\parallel BC \end{aligned}$$



Theorem 2 (a)

If the corresponding $\angle^s$ of 2 Δ^s are equal, then the corresponding sides are in proportion, and thus the Δ^s are similar.

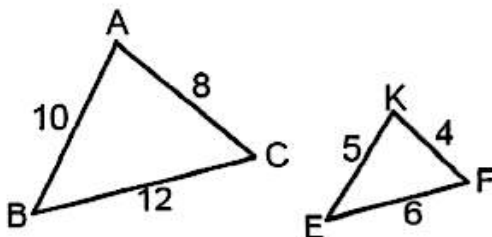


$$\Delta ABC \parallel \Delta EFK (\angle\angle\angle)$$

$$\therefore \frac{AB}{EF} = \frac{BC}{FK} = \frac{AC}{EK}$$

Theorem 2 (b)

If the corresponding sides of 2 Δ^s are proportional, then the $\angle^s$ are equiangular, and thus the Δ^s are similar.



$$\frac{AB}{KE} = \frac{BC}{EF} = \frac{AC}{KF} = \frac{1}{2}$$

$$\therefore \hat{A} = \hat{K}, \hat{B} = \hat{E} \text{ and } \hat{C} = \hat{F}$$

$$\therefore ABC \parallel \Delta KEF$$

Worksheet 9 C

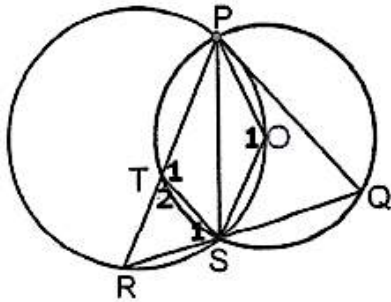
6.2 Prove that:

6.2.1 $\frac{FB}{FE} = \frac{DB}{DF}$

6.2.2 $\frac{CF}{FB} = \frac{CD}{BD}$

6.3 If $\hat{BDC} = 90^\circ$ and $DC = 5$, $BC = 13$, calculate the length of DB and FB .

7.



O is the centre of the smaller circle. $\hat{O}_1 = 2x$.

7.1 Prove that:

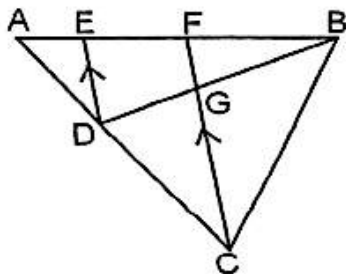
7.1.1 $TS \parallel PQ$

7.1.2 $\triangle STR \parallel \triangle QPR$

7.2 Determine $\frac{ST}{PQ}$ if $RT:TP = 4:7$.

Worksheet 9 D

1.



$BF = 8$ cm, $AF = 12$ cm, $ED = 4$ cm.
 $CD:DA = 2:1$

1.1 Determine: $\frac{AE}{EF}$.

1.2 Prove that $2FE = EB$.

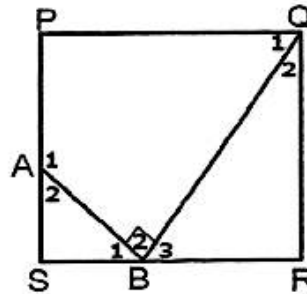
1.3 Calculate the:

1.3.1 length van FG .

Worksheet 9 D

1.3.2 numerical value of:
 $\frac{\text{area van } \triangle AFC}{\text{area van } \triangle CBF}$

2.

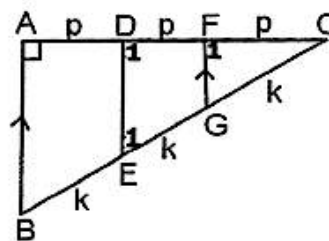


PQRS is a square with length of the sides = k units. $\hat{B}_1 = x$
Prove that:

2.1 $\triangle ASB \parallel \triangle BRQ$

2.2 Determine the area of $\triangle ASB$ i.t.o. k and p if $BR = p$ units.

3.

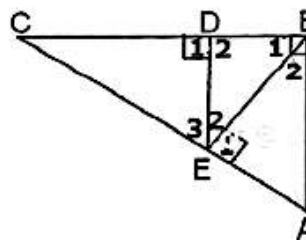


$FG \parallel AB$, $FG = y$, $AD = DF = FC = p$,
 $BE = EG = GC = k$, $\hat{A} = 90^\circ$.

3.1 Determine AB i.t.o. y .

3.2 Determine:
area $DFGE$: area $\triangle ABC$.

4.



$EC:EA = 5:4$, $AE = 8$ cm, $\hat{C} = x$.

4.1 Prove that $\triangle ABE \parallel \triangle BCE$.

4.2 Calculate the lengths of:

4.2.1 BE

4.2.2 DC (answer in surd form)

MODULE 9: Euclidean Geometry

Similarity

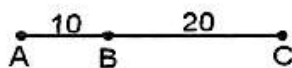
Revise the theory of module 12 in the grade 11 X-Factor.

Ratio and Proportion

1. Ratio



$$\frac{AB}{BC} = \frac{10}{20} = \frac{1}{2} \quad \text{or} \quad AB:BC = 1:2$$



$$\frac{AB}{AC} = \frac{10}{30} = \frac{1}{3} \quad \text{or} \quad AB:AC = 1:3$$

$$\therefore 3AB = AC$$

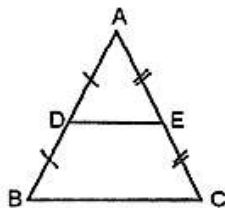
$$\therefore AB = \frac{1}{3}AC$$

2. Proportion

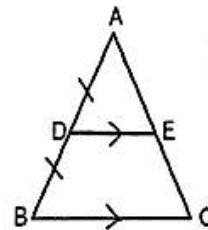
When 2 or more ratios are equal. $\frac{AB}{AC} = \frac{DE}{EF}$

1. Midpoint theorem

1.1



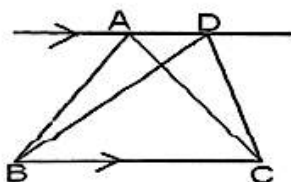
If $AD = DB$
and $AE = EC$
then $DE \parallel BC$ and
 $DE = \frac{1}{2}BC$



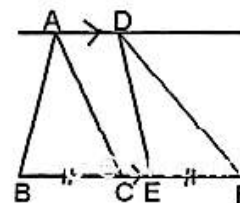
1.2 If $AD = DB$
and $DE \parallel BC$
then $AE = EC$

2. Area of $\Delta = \frac{1}{2}.b.h$

2.1 Δ^s on the same base or equal bases and between the same parallel lines are equal in area.

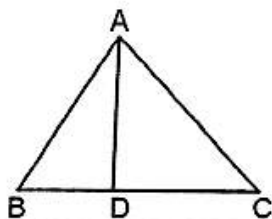


area $\Delta ABC = \text{area } \Delta DBC$



area $\Delta ABC = \text{area } \Delta DEF$

2.2



$$\frac{\text{area } \Delta ABD}{\text{area } \Delta ADC} = \frac{\frac{1}{2}.BD.h}{\frac{1}{2}.DC.h} = \frac{BD}{DC}$$

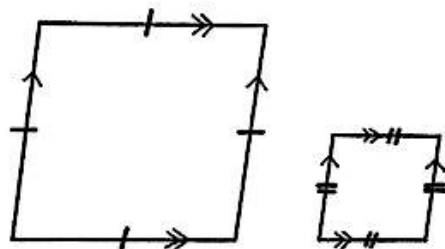
ΔABD and ΔADC have the same height

Sketches not to scale. Round answers off to one decimal place. Give reasons with all answers.

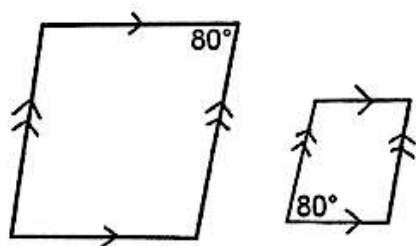
Worksheet 9 A

1. Which of the following pairs of polygons are **not** similar?

(a)

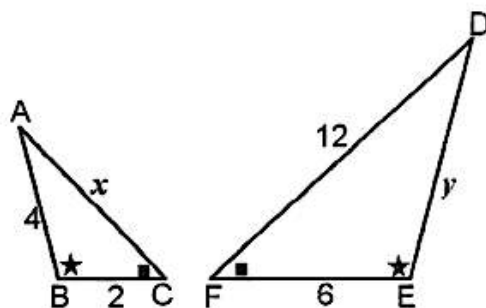


(b)

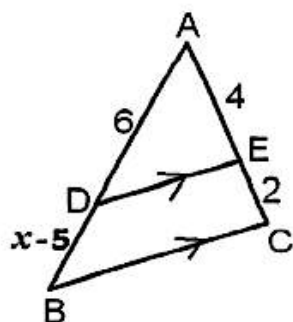


2. Calculate the unknown in the following figures:

2.1

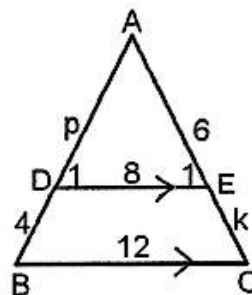


2.2

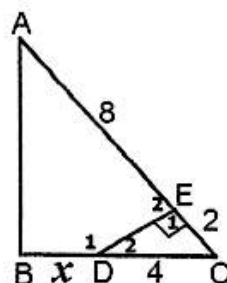


Worksheet 9 A

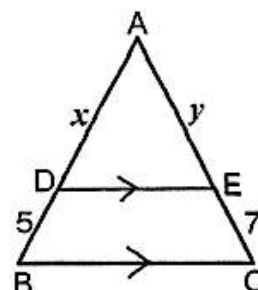
2.3



2.4

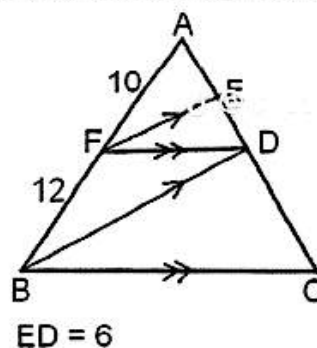


3.

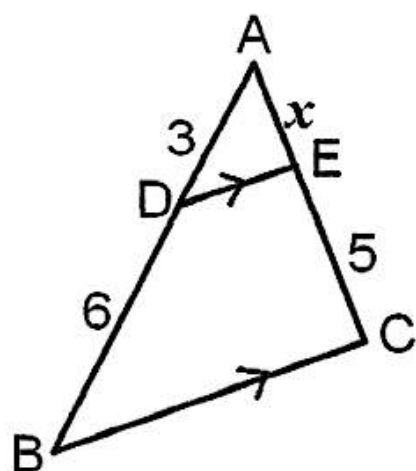


Determine: $\frac{x}{y}$

4. Calculate the length of AC.



Examples



1.1 Prove $AD \cdot EC = DB \cdot AE$

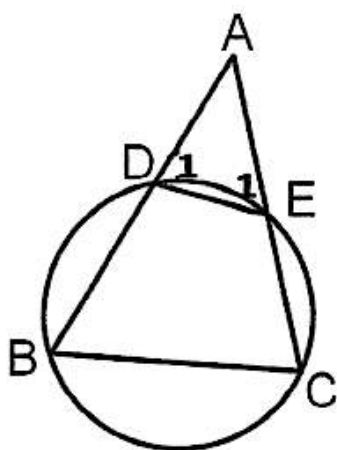
1.2 Calculate x .

Answer

$$1.1 \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \left| \begin{array}{l} DE \parallel BC \end{array} \right.$$

$$\therefore AD \cdot EC = AE \cdot DB$$

$$1.2 \quad \begin{array}{l} 3 \cdot 5 = 6 \cdot x \\ \therefore 6x = 15 \\ \therefore x = 2,5 \end{array}$$



2. Prove that:

2.1 $\triangle ABC \parallel \triangle AED$

2.2 $AB \cdot ED = AE \cdot BC$

Answer

2.1 In $\triangle ABC$ en $\triangle AED$

$$\hat{A} = \hat{A}$$

$$\hat{B} = \hat{E}_1$$

$$\hat{C} = \hat{D}_1$$

$$\therefore \triangle ABC \parallel \triangle AED$$

common
ext. $\angle$ cyclic quad.
ext. $\angle$ cyclic quad.
 $\angle \angle \angle$

$$2.2 \quad \therefore \frac{AB}{AE} = \frac{BC}{ED} = \left(\frac{AC}{AD} \right) \quad \left| \begin{array}{l} \triangle ABC \parallel \triangle AED \end{array} \right.$$

$$\therefore AB \cdot ED = AE \cdot BC$$

NB order!

Remember:

If $\frac{AB}{KE} = \frac{BC}{EF}$

$$\frac{a}{b} = \frac{c}{d}$$

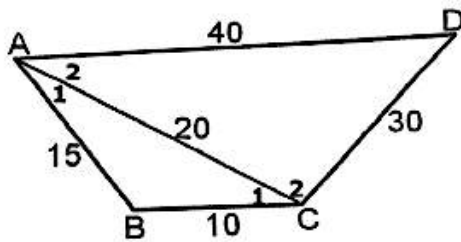
then $\frac{AB}{BC} = \frac{KE}{EF}$

$$\therefore \frac{a}{c} = \frac{b}{d}$$

Worksheets are from easy to more difficult. Use as examples.

Worksheet 9 A

5.

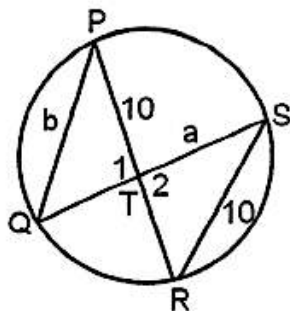


Prove that:

5.1 $\triangle BCA \parallel \triangle CAD$

5.2 $AD \parallel BC$

6.

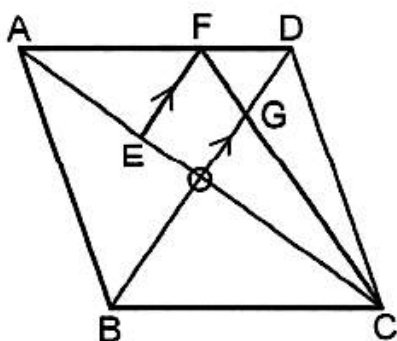


$QT:RT = 8:5$. Calculate a and b

7. ABCD is a parallelogram.

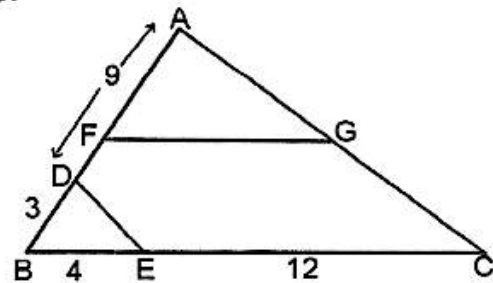
$$\frac{DF}{FA} = \frac{3}{4}$$

Determine: $\frac{CO}{OE}$.



Worksheet 9 A

8.



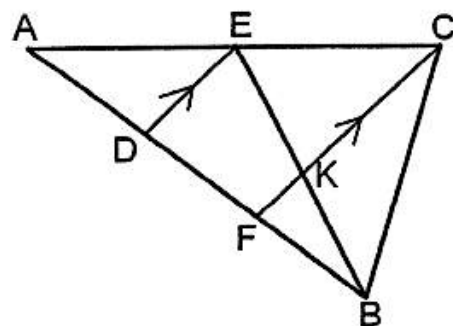
$AD = 9$, $DF = k$.

8.1 Prove that: $DE \parallel AC$.

8.2 If $FG \parallel BC$ and $\frac{AG}{GC} = \frac{3}{4}$,
calculate the length of DF.

Worksheet 9 B

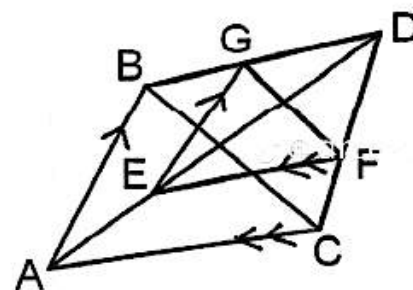
1.



$EC:EA = 3:2$. $AF:FB = 4:3$.

Determine: $BK:KE$.

2. Prove that: $GF \parallel BC$.



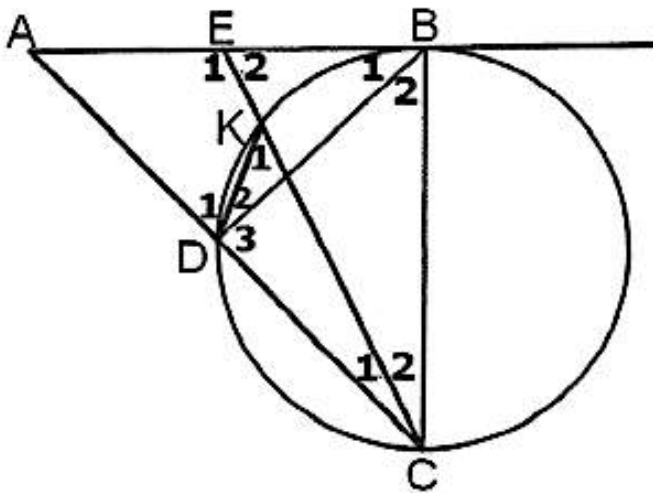
 GRADE 12 

 EUCLIDEAN
GEOMETRY 

 @drcextraclass

Worksheet 9 E

7.



$CK = 8$, $EK = 4$, $DC = 6$,
 $DK = 3$. BC is the diameter of the
circle. AB is a tangent to the
circle.

7.1 Prove that:

7.1.1 $\hat{B}_2 = \hat{A}$

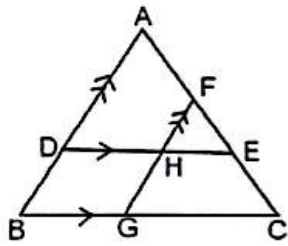
7.1.2 $AEKD$ a cyclic quadrilateral

7.1.3 $CK = \frac{CA \cdot DK}{AE}$

7.2 Calculate the length of AE and
 AD .

Worksheet 9 D

5.



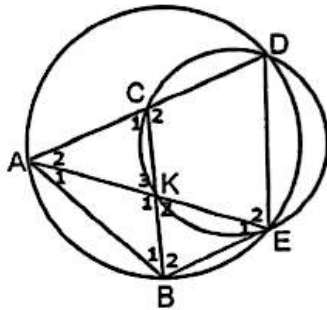
$CG:GB = 4:3$, $\frac{AD}{DB} = \frac{2}{1}$ and
 $AC = 21$ cm

5.1 Calculate the lengths of:

5.1.1 FC 5.1.2 CE

5.2 Determine: $\frac{EH}{HD}$.

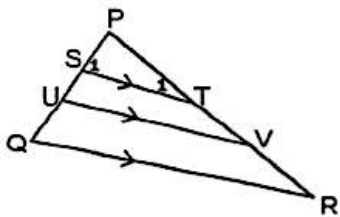
6.



Prove that:

6.1 $\angle ABE = \angle AKB$ 6.2 $\triangle BEA \sim \triangle KBA$ 6.3 $AB = \sqrt{DA \cdot AC}$

7.



$PS = 3$ cm, $PT = 4$ cm, $VR = 3$ cm,
 $SU = 2$ cm, $ST = 5$ cm

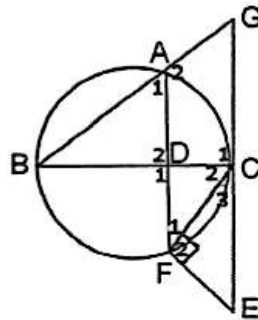
7.1 Determine whether $\triangle PQR$ is equiangular.

7.2 Calculate:

7.2.1 TV 7.2.2 QR

Worksheet 9 E

1.



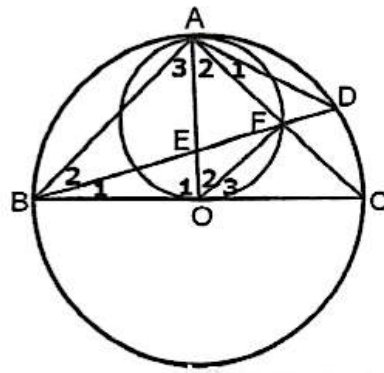
GCE is a tangent, $FE \perp FC$,
 BC is a diameter and $AD = DF$

1.1 Prove that:

1.1.1 $AF \parallel GE$ 1.1.2 $\triangle FEC \sim \triangle DAB$ 1.1.3 $\frac{FE}{FC} = \frac{FD}{DB}$

1.2 Calculate the length of AB if
 $FC = 12$ units, $BD = 10$ units and
 $FE = 6$ units (Answer in simplest
surd form)

2.



O is the centre of the bigger circle.
 $OF \parallel AB$.

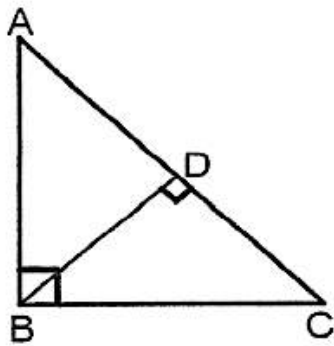
2.1 Determine: $\frac{FC}{FA}$.

2.2 Prove that:

2.2.1 $\triangle BFC \sim \triangle AFD$ 2.2.2 $AF^2 = FD \cdot BF$

Theorem 3 Pythagoras

In any right-angled Δ , the square on the hypotenuse is equal to the sum of the squares on the other 2 sides.



$$\boxed{AC^2 = AB^2 + BC^2}$$

(Prove the theorem with similarity)

Deductions: 1. $\Delta ABC \parallel \Delta ADB$ $\therefore AB^2 = AC \cdot AD$	$AC \perp BD$ and $AB \perp BC$
2. $\Delta ABC \parallel \Delta BDC$ $\therefore BC^2 = AC \cdot DC$	$AC \perp BD$ and $AB \perp BC$
3. $\Delta ADB \parallel \Delta BDC$ $\therefore BD^2 = CD \cdot DA$	$AC \perp BD$ and $AB \perp BC$

Summary

1. If you have to proof that **2 Δ^s are similar** then prove that:
 - 1.1 **2 pairs of angles are equal** or
 - 1.2 **all pairs of corresponding sides are in the same proportion.**
2. If you have to proof that **polygons are similar** then prove that:
 - 2.1 **all pairs of corresponding $\angle^s$ are equal AND**
 - 2.2 **all pairs of corresponding sides are in the same proportion.**

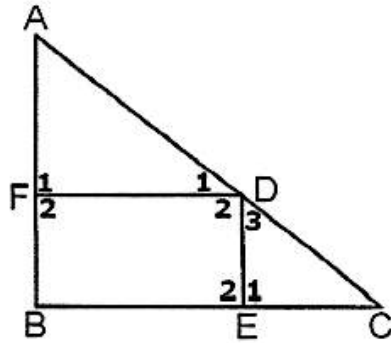
NB: ORDER - of the similar Δ^s is very important!

If $\Delta ABC \parallel \Delta KEF$

$$\therefore \frac{AB}{KE} = \frac{BC}{EF} = \frac{AC}{KF}$$

Worksheet 9 C

2.



BEDF is a rectangle.

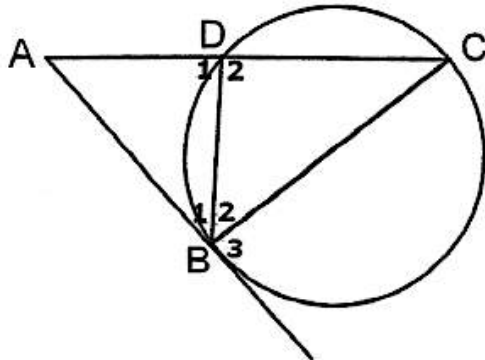
2.1 Prove that $\triangle ABC \sim \triangle AFD$.

2.2 If $BC = 40$ units, $AB = 20$ units and $AF = k$ units, show that $FD = 2k$ units.

2.3 Write FB i.t.o. k and show that the area of the rectangle $= -2k^2 + 40k$.

2.4 Determine the value of k for which the area of the rectangle will be a maximum.

3.



AB is a tangent.

3.1 Prove that $\triangle ABD \sim \triangle ACB$.

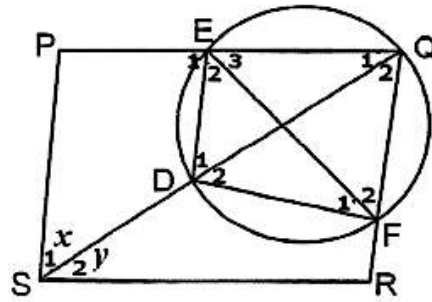
3.2 If $DC = 5$ and $AD = 4$, calculate the length of AB .

3.3 If BC is the diameter of the circle, Show that the radius,

$$r = \frac{3}{2}\sqrt{5}$$

Worksheet 9 C

4.



PQRS is a parallelogram.

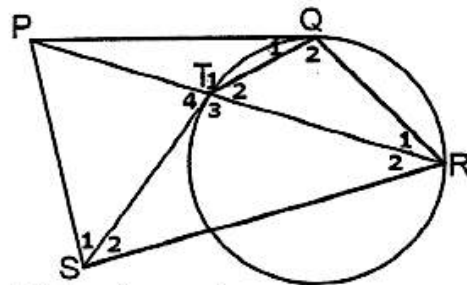
4.1 Name 2 angles equal to x .

4.2 Name 2 angles equal to y .

4.3 Determine a triangle similar to $\triangle FED$.

4.4 Prove that $ED \cdot PQ = QR \cdot DF$.

5.



PQ is a tangent.

5.1 Prove by using similarity that $PQ^2 = PT \cdot PR$.

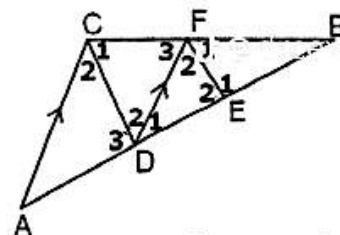
5.2 If given $\angle PSR = \angle T_4$,

Prove that $\triangle PRS \sim \triangle PTS$.

5.3 Complete: $\frac{PR}{PS} = \frac{PT}{PT}$

5.4 Hence prove that $PQ = PS$.

6.

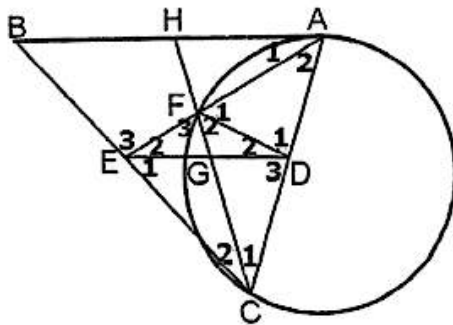


FD bisects $\angle CDB$ and $\angle F_1 = \angle D_1$.

6.1 Name 4 angles equal to $\angle D_1$.

Worksheet 9 E

3.



AB is a tangent. $DE \parallel AB$.

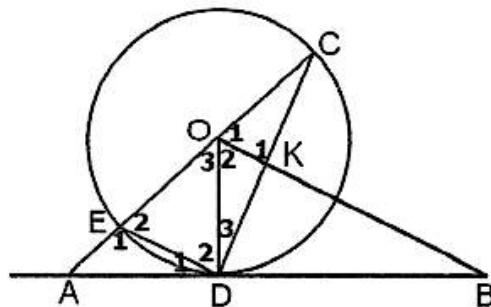
3.1 Prove that:

3.1.1 CDFE a cyclic quadrilateral

3.1.2 $\triangle CDF \parallel \triangle AEB$

3.1.3 $\frac{DA \cdot CG}{GH} = \frac{AE \cdot DF}{EB}$

4.



O is the centre of the circle.
OB $\parallel$ ED. AB is a tangent. $\hat{D}_3 = x$.

4.1 Name 3 angles equal to x .

4.2 Determine the size of:

4.2.1 $\hat{K}_1$

4.2.2 $\hat{ADC}$ i.t.o. x

4.3 Prove that:

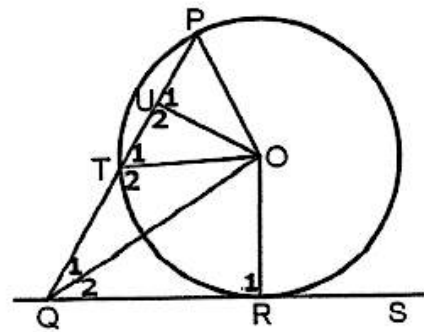
4.3.1 $DK = KC$

4.3.2 $\triangle ACD \parallel \triangle ADE$

4.3.3 $CK \cdot AD = \frac{AC \cdot ED}{2}$

Worksheet 9 E

5.



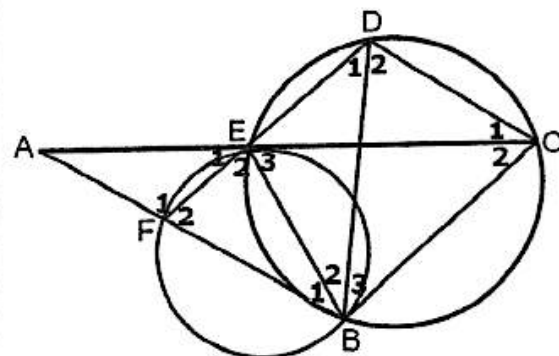
O is the centre of the circle.
QRS is a tangent and $TU = UP$.

Prove that:

5.1 QROU a cyclic quadrilateral

5.2 $OQ^2 + UT^2 = QU^2 + OT^2$

6.



AB is a tangent to the bigger circle. $DF \parallel BC$.

6.1 Prove that:

6.1.1 $\triangle BCD \parallel \triangle BEF$

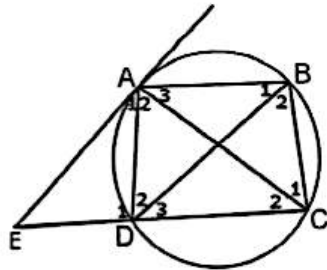
6.1.2 AE is a tangent to circle FEB

6.1.3 $\triangle ABE \parallel \triangle AEF$

6.2 If $4FB = 3AB$, show that
 $AB^2 = 4AE^2$.

Worksheet 9 B

3.



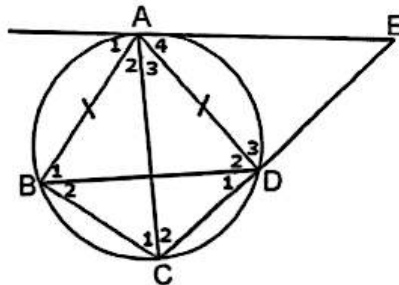
EA is a tangent, $AB \parallel DC$, $\hat{A}_1 = x$.

3.1 Name 4 angles equal to x .3.2 Prove that $\triangle ABC \parallel \triangle ADE$.

3.3 Hence show that

$$BA = \frac{AC \cdot DA}{AE}.$$

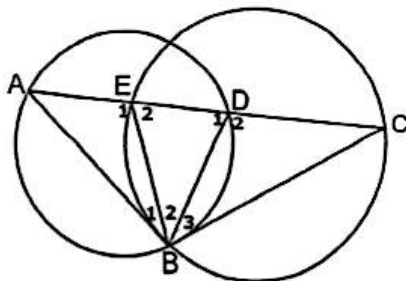
4.



AE is a tangent. Prove:

4.1 $\triangle ADE \parallel \triangle CBA$ 4.2 $AB^2 = DE \cdot BC$

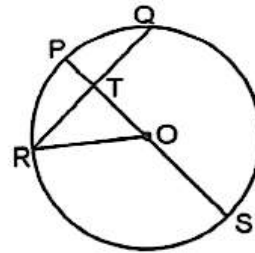
5.



AB is a tangent to the bigger circle. BC is a tangent to the smaller circle. Prove that:

5.1 $BD = BE$ 5.2 $\triangle BCD \parallel \triangle ABE$ 5.3 $BC \cdot AE = AB \cdot BE$ **Worksheet 9 B**

6.



O is the centre of the circle.

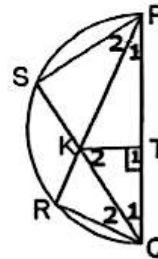
$PO \perp RQ$ and $\frac{PS}{PT} = \frac{9}{4}$.

$QT = 12$ cm.

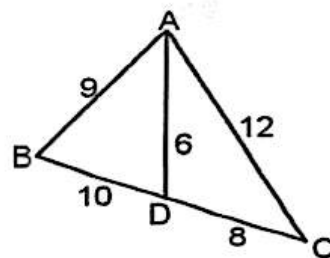
6.1 If $PT = 4a$ units, determine TO l.t.o. a .

6.2 Hence calculate the length of the radius.

7.

7.1 Prove that $\triangle QSP \parallel \triangle QTK$.7.2 Prove that $PS^2 = \frac{SP^2 QK^2}{TK^2} - SQ^2$ **Worksheet 9 C**

1.

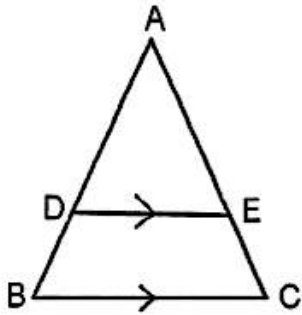


Prove that $\triangle CAD \parallel \triangle CBA$.

Study the proves of the following theorems:

Theorem 1 (a)

A line parallel to one side of a Δ divides the other 2 sides in the same proportion.



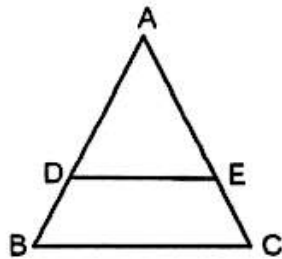
$$\frac{AD}{DB} = \frac{AE}{EC} \quad [DE \parallel BC]$$

$$\text{or } \frac{AD}{AB} = \frac{AE}{AC} \quad [DE \parallel BC]$$

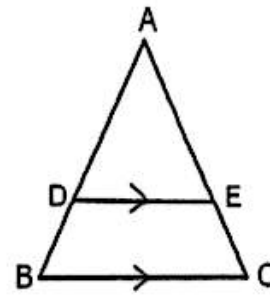
$$\text{or } \frac{AB}{DB} = \frac{AC}{EC} \quad [DE \parallel BC]$$

Theorem 1 (b)

If a line divides 2 sides of a Δ in the same proportion, then the line is parallel to the 3rd side.

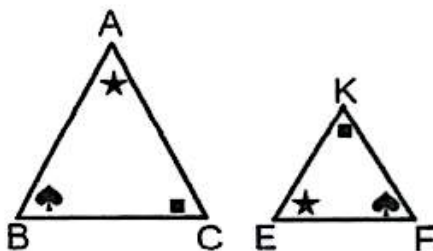


$$\begin{aligned} \text{If } \frac{AD}{DB} &= \frac{AE}{EC} \\ \text{then} \\ DE &\parallel BC \end{aligned}$$



Theorem 2 (a)

If the corresponding $\angle^s$ of 2 Δ^s are equal, then the corresponding sides are in proportion, and thus the Δ^s are similar.

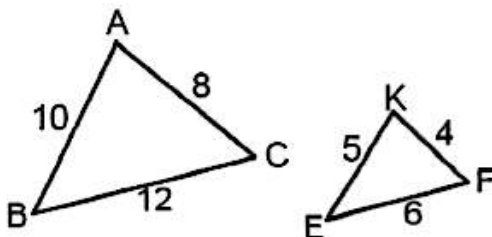


$$\Delta ABC \parallel \Delta EFK (\angle\angle\angle)$$

$$\therefore \frac{AB}{EF} = \frac{BC}{FK} = \frac{AC}{EK}$$

Theorem 2 (b)

If the corresponding sides of 2 Δ^s are proportional, then the $\angle^s$ are equiangular, and thus the Δ^s are similar.



$$\frac{AB}{KE} = \frac{BC}{EF} = \frac{AC}{KF} = \frac{1}{2}$$

$$\therefore \hat{A} = \hat{K}, \hat{B} = \hat{E} \text{ and } \hat{C} = \hat{F}$$

$$\therefore ABC \parallel \Delta KEF$$

Worksheet 9 C

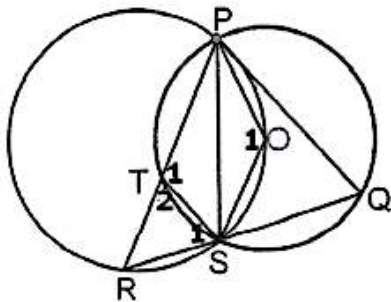
6.2 Prove that:

6.2.1 $\frac{FB}{FE} = \frac{DB}{DF}$

6.2.2 $\frac{CF}{FB} = \frac{CD}{BD}$

6.3 If $\hat{BDC} = 90^\circ$ and $DC = 5$, $BC = 13$, calculate the length of DB and FB .

7.



O is the centre of the smaller circle. $\hat{O}_1 = 2x$.

7.1 Prove that:

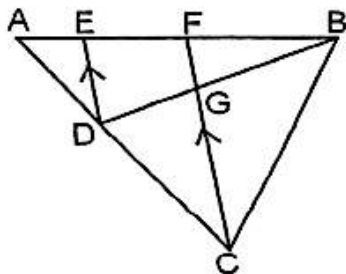
7.1.1 $TS \parallel PQ$

7.1.2 $\triangle STR \parallel \triangle QPR$

7.2 Determine $\frac{ST}{PQ}$ if $RT:TP = 4:7$.

Worksheet 9 D

1.



$BF = 8$ cm, $AF = 12$ cm, $ED = 4$ cm.
 $CD:DA = 2:1$

1.1 Determine: $\frac{AE}{EF}$.

1.2 Prove that $2FE = EB$.

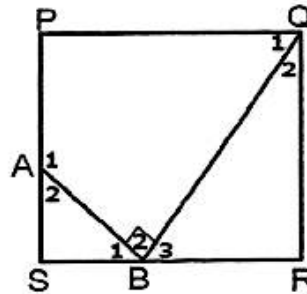
1.3 Calculate the:

1.3.1 length van FG .

Worksheet 9 D

1.3.2 numerical value of:
 $\frac{\text{area van } \triangle AFC}{\text{area van } \triangle CBF}$

2.

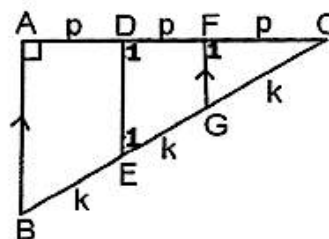


PQRS is a square with length of the sides = k units. $\hat{B}_1 = x$
Prove that:

2.1 $\triangle ASB \parallel \triangle BRQ$

2.2 Determine the area of $\triangle ASB$ i.t.o. k and p if $BR = p$ units.

3.

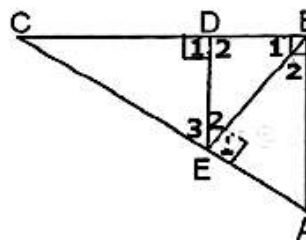


$FG \parallel AB$, $FG = y$, $AD = DF = FC = p$,
 $BE = EG = GC = k$, $\hat{A} = 90^\circ$.

3.1 Determine AB i.t.o. y .

3.2 Determine:
area $DFGE$: area $\triangle ABC$.

4.



$EC:EA = 5:4$, $AE = 8$ cm, $\hat{C} = x$.

4.1 Prove that $\triangle ABE \parallel \triangle BCE$.

4.2 Calculate the lengths of:

4.2.1 BE

4.2.2 DC (answer in surd form)

MODULE 9: Euclidean Geometry

Similarity

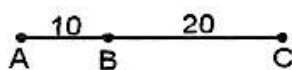
Revise the theory of module 12 in the grade 11 X-Factor.

Ratio and Proportion

1. Ratio



$$\frac{AB}{BC} = \frac{10}{20} = \frac{1}{2} \quad \text{or} \quad AB:BC = 1:2$$



$$\frac{AB}{AC} = \frac{10}{30} = \frac{1}{3} \quad \text{or} \quad AB:AC = 1:3$$

$$\therefore 3AB = AC$$

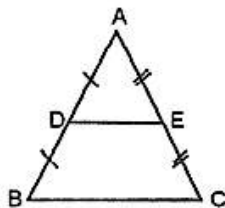
$$\therefore AB = \frac{1}{3}AC$$

2. Proportion

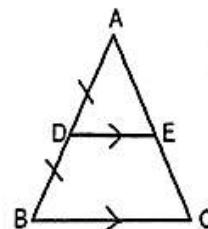
When 2 or more ratios are equal. $\frac{AB}{AC} = \frac{DE}{EF}$

1. Midpoint theorem

1.1



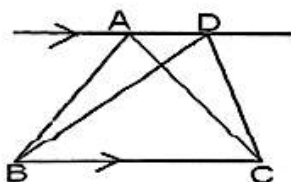
If **AD = DB**
and **AE = EC**
then **DE || BC** and
DE = 1/2 BC



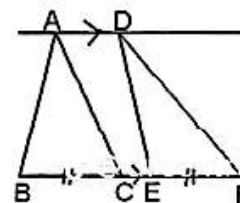
1.2 If **AD = DB**
and **DE || BC**
then **AE = EC**

2. Area of $\Delta = \frac{1}{2} \cdot b \cdot h$

2.1 Δ^s on the same base or equal bases and between the same parallel lines are equal in area.

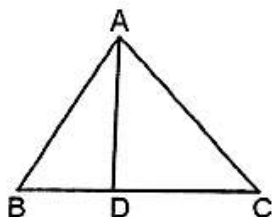


area $\Delta ABC = \text{area } \Delta DBC$



area $\Delta ABC = \text{area } \Delta DEF$

2.2



$$\frac{\text{area } \Delta ABD}{\text{area } \Delta ADC} = \frac{\frac{1}{2} \cdot BD \cdot h}{\frac{1}{2} \cdot DC \cdot h} = \frac{BD}{DC}$$

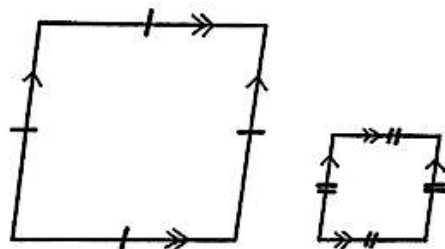
ΔABD and ΔADC have the same height

Sketches not to scale. Round answers off to one decimal place. Give reasons with all answers.

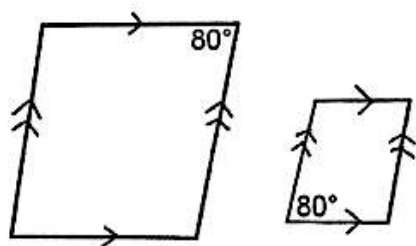
Worksheet 9 A

1. Which of the following pairs of polygons are **not** similar?

(a)

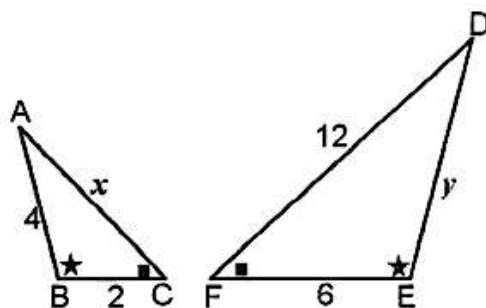


(b)

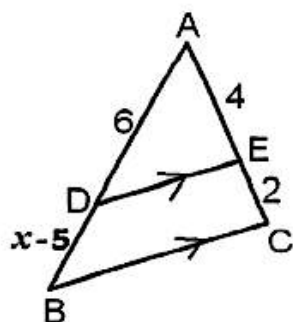


2. Calculate the unknown in the following figures:

2.1

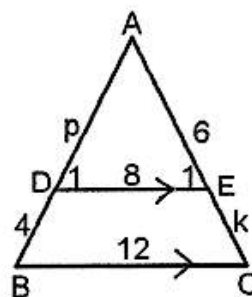


2.2

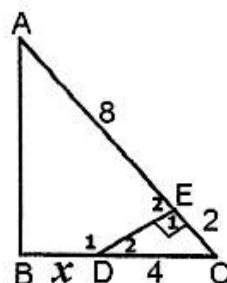


Worksheet 9 A

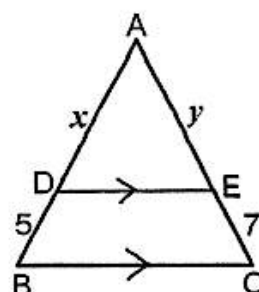
2.3



2.4

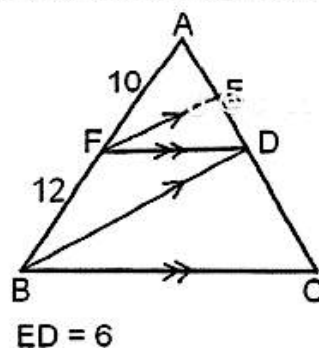


3.

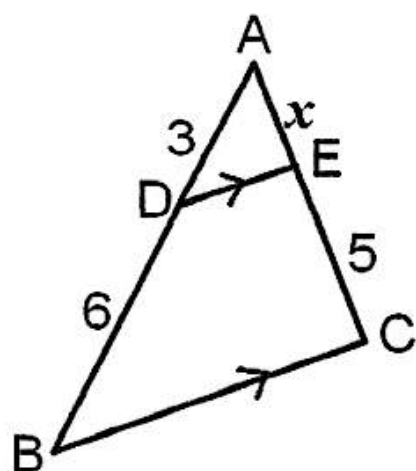


Determine: $\frac{x}{y}$

4. Calculate the length of AC.



Examples



1.1 Prove $AD \cdot EC = DB \cdot AE$

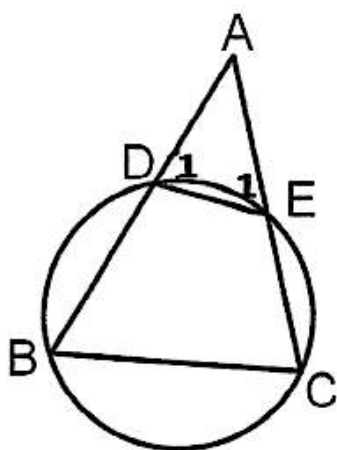
1.2 Calculate x .

Answer

$$1.1 \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \left| \begin{array}{l} DE \parallel BC \end{array} \right.$$

$$\therefore AD \cdot EC = AE \cdot DB$$

$$1.2 \quad \begin{array}{l} 3 \cdot 5 = 6 \cdot x \\ \therefore 6x = 15 \\ \therefore x = 2,5 \end{array}$$



2. Prove that:

2.1 $\triangle ABC \parallel \triangle AED$

2.2 $AB \cdot ED = AE \cdot BC$

Answer

2.1 In $\triangle ABC$ en $\triangle AED$

$$\hat{A} = \hat{A}$$

$$\hat{B} = \hat{E}_1$$

$$\hat{C} = \hat{D}_1$$

$$\therefore \triangle ABC \parallel \triangle AED$$

common
ext. $\angle$ cyclic quad.
ext. $\angle$ cyclic quad.
 $\angle \angle \angle$

$$2.2 \quad \therefore \frac{AB}{AE} = \frac{BC}{ED} = \left(\frac{AC}{AD} \right) \quad \left| \begin{array}{l} \triangle ABC \parallel \triangle AED \end{array} \right.$$

$$\therefore AB \cdot ED = AE \cdot BC$$

NB order!

Remember:

If $\frac{AB}{KE} = \frac{BC}{EF}$

$$\frac{a}{b} = \frac{c}{d}$$

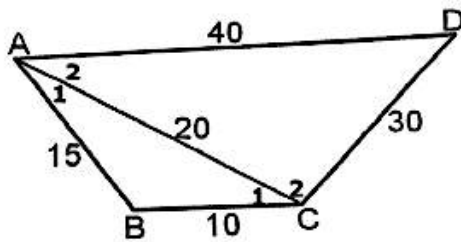
then $\frac{AB}{BC} = \frac{KE}{EF}$

$$\therefore \frac{a}{c} = \frac{b}{d}$$

Worksheets are from easy to more difficult. Use as examples.

Worksheet 9 A

5.

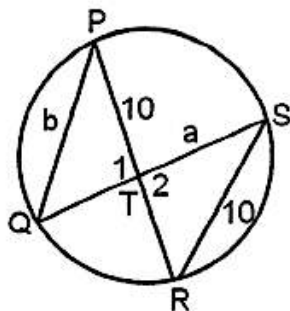


Prove that:

5.1 $\triangle BCA \parallel \triangle CAD$

5.2 $AD \parallel BC$

6.

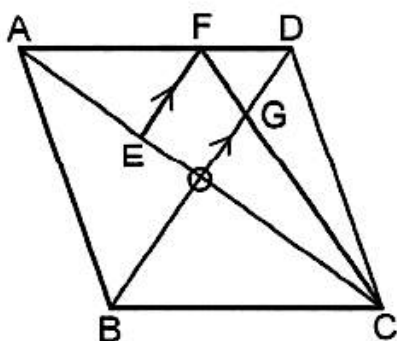


$QT:RT = 8:5$. Calculate a and b

7. ABCD is a parallelogram.

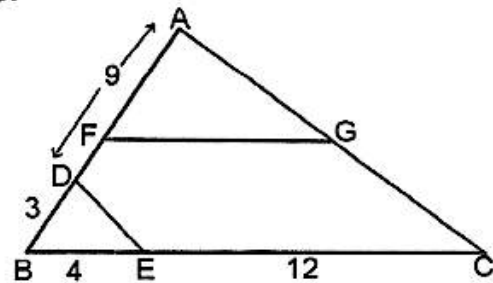
$$\frac{DF}{FA} = \frac{3}{4}$$

Determine: $\frac{CO}{OE}$



Worksheet 9 A

8.



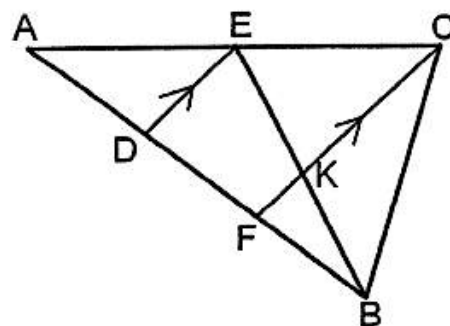
$AD = 9$, $DF = k$.

8.1 Prove that: $DE \parallel AC$.

8.2 If $FG \parallel BC$ and $\frac{AG}{GC} = \frac{3}{4}$,
calculate the length of DF.

Worksheet 9 B

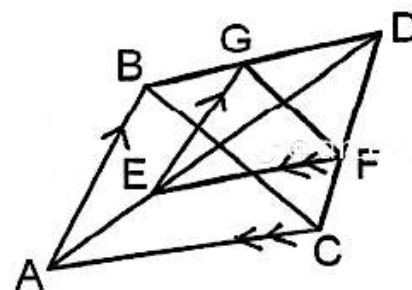
1.



$EC:EA = 3:2$. $AF:FB = 4:3$.

Determine: $BK:KE$.

2. Prove that: $GF \parallel BC$.



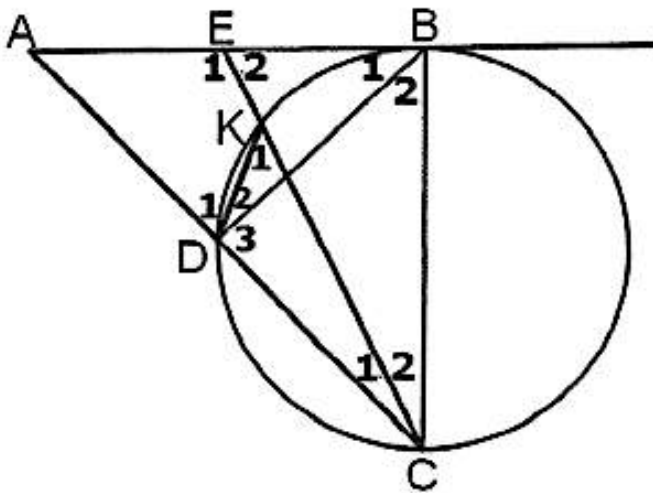
 GRADE 12 

 EUCLIDEAN
GEOMETRY 

 @drcextraclasses

Worksheet 9 E

7.



CK = 8, EK = 4, DC = 6,
DK = 3. BC is the diameter of the
circle. AB is a tangent to the
circle.

7.1 Prove that:

7.1.1 $\hat{B}_2 = \hat{A}$

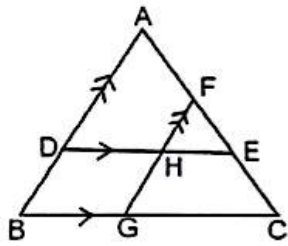
7.1.2 AEKD a cyclic quadrilateral

7.1.3 $CK = \frac{CA \cdot DK}{AE}$

7.2 Calculate the length of AE and
AD.

Worksheet 9 D

5.



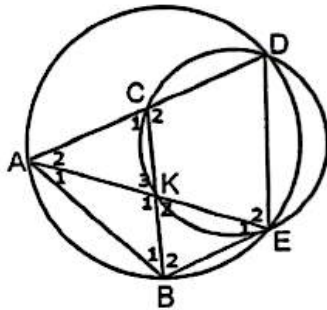
$CG:GB = 4:3$, $\frac{AD}{DB} = \frac{2}{1}$ and
 $AC = 21$ cm

5.1 Calculate the lengths of:

5.1.1 FC 5.1.2 CE

5.2 Determine: $\frac{EH}{HD}$.

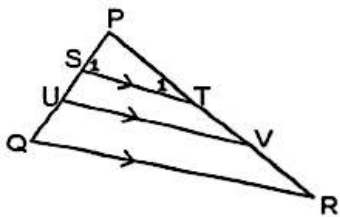
6.



Prove that:

6.1 $\angle ABE = \angle AKB$ 6.2 $\triangle BEA \sim \triangle KBA$ 6.3 $AB = \sqrt{DA \cdot AC}$

7.



$PS = 3$ cm, $PT = 4$ cm, $VR = 3$ cm,
 $SU = 2$ cm, $ST = 5$ cm

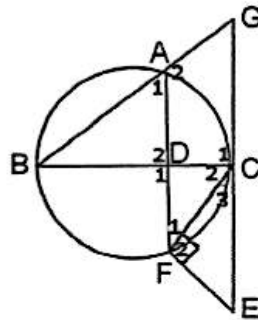
7.1 Determine whether $\triangle PQR$ is equiangular.

7.2 Calculate:

7.2.1 TV 7.2.2 QR

Worksheet 9 E

1.



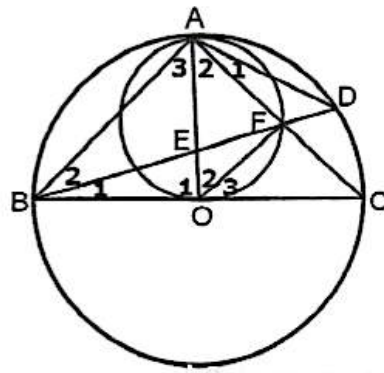
GCE is a tangent, $FE \perp FC$,
 BC is a diameter and $AD = DF$

1.1 Prove that:

1.1.1 $AF \parallel GE$ 1.1.2 $\triangle FEC \sim \triangle DAB$ 1.1.3 $\frac{FE}{FC} = \frac{FD}{DB}$

1.2 Calculate the length of AB if
 $FC = 12$ units, $BD = 10$ units and
 $FE = 6$ units (Answer in simplest
surd form)

2.



O is the centre of the bigger circle.
 $OF \parallel AB$.

2.1 Determine: $\frac{FC}{FA}$.

2.2 Prove that:

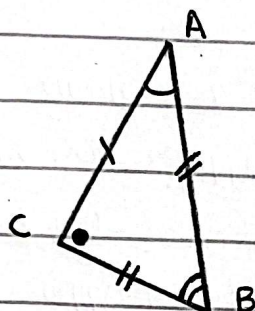
2.2.1 $\triangle BFC \sim \triangle AFD$ 2.2.2 $AF^2 = FD \cdot BF$

Triangles

There are four kinds of triangles:

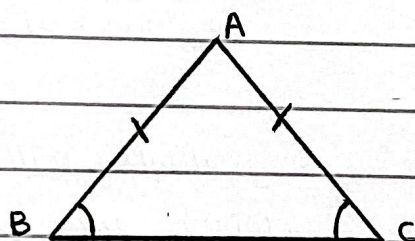
- Scalene

All sides and angles are different



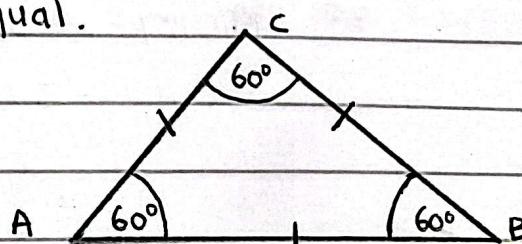
- Isosceles

Two sides are equal in length. The angles opposite the equal sides are also equal



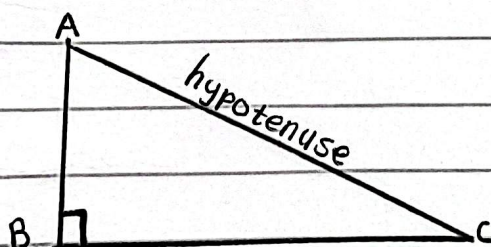
- Equilateral

All three sides are equal in length and all three angles are equal.



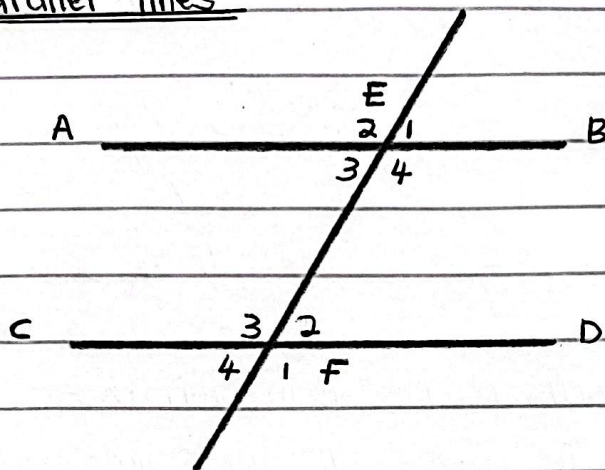
- Right-Angled

A right angle triangle has an angle that is equal to 90°



Worked Examples

1. Parallel lines



Alternating angles

$$\hat{E}_3 = \hat{F}_2 \quad (\text{Alt. } \angle\text{'s } AB \parallel CD)$$

$$\hat{E}_4 = \hat{F}_3 \quad (\text{Alt. } \angle\text{'s } AB \parallel CD)$$

Co-interior Angles

$$\hat{E}_4 + \hat{F}_2 = 180^\circ$$

$$\hat{F}_3 + \hat{E}_2 = 180^\circ$$

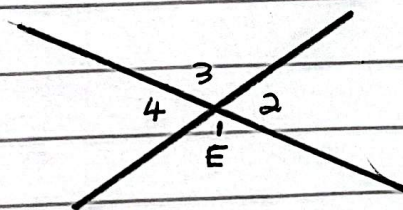
Corresponding angles

$$\hat{E}_1 = \hat{F}_2 \quad (\text{Corresp. } \angle\text{'s } AB \parallel CD)$$

$$\hat{F}_4 = \hat{E}_3 \quad (\text{Corresp. } \angle\text{'s } AB \parallel CD)$$

$$\hat{E}_4 = \hat{F}_1 \quad (\text{Corresp. } \angle\text{'s } AB \parallel CD)$$

2. Crossing Lines



$$\hat{E}_1 = \hat{E}_3 \quad (\text{Vert opp } \angle\text{'s})$$

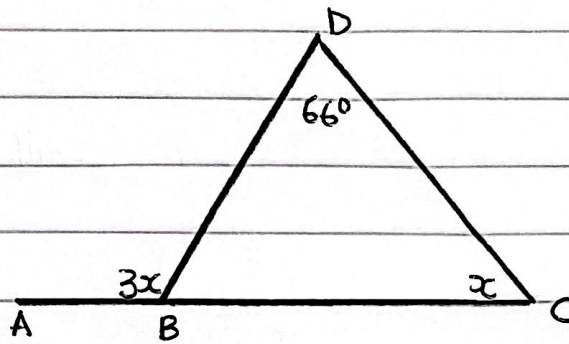
$$\hat{E}_2 = \hat{E}_4 \quad (\text{Vert opp } \angle\text{'s})$$

$$\hat{E}_1 + \hat{E}_2 + \hat{E}_3 + \hat{E}_4 = 360^\circ$$

($\angle\text{'s around pt}$)

3. Properties of Triangles

i)



Calculate the value of x

$$\hat{D} + \hat{C} = \hat{B} \quad (\text{Sum of opp. interior } \angle\text{'s} = \text{exterior } \angle\text{'s})$$

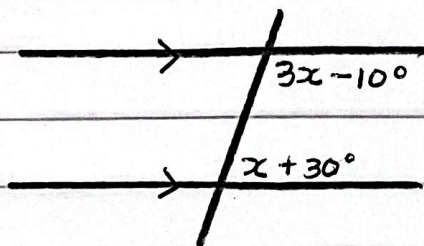
$$66^\circ + x = 3x$$

$$3x - x = 66^\circ$$

$$\frac{2x}{2} = \frac{66^\circ}{2}$$

$$x = 33^\circ$$

ii) find the value of x



$$(3x - 10^\circ) + (x + 30^\circ) = 180^\circ \quad (\text{Co-interior } \angle\text{'s AB||CD})$$

$$3x + x = 180^\circ - 30^\circ + 10^\circ$$

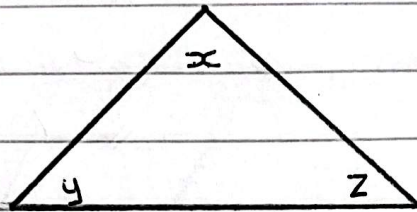
$$\frac{4x}{4} = \frac{160^\circ}{4}$$

$$x = 40^\circ$$

🎵 @mathscience.life

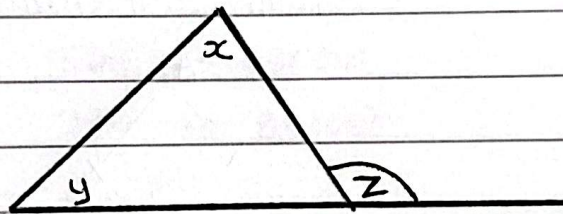
Rules and Theorems of triangles

- Sum of the triangle angles inside a triangle adds up to 180°



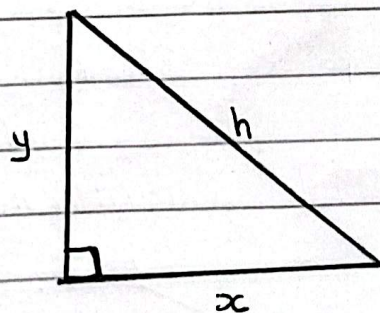
$$x + y + z = 180^\circ$$

- The exterior angle of a triangle is equal to the sum of the opposite interior angles.



$$z = x + y$$

- The Theorem of Pythagoras states that in a right-angle triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides



$$h^2 = x^2 + y^2$$

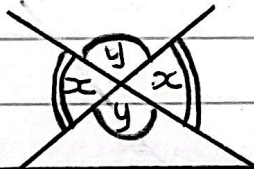
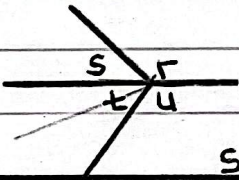
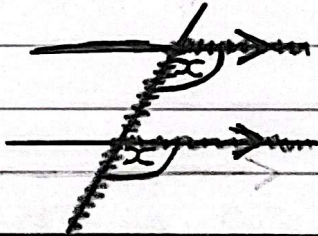
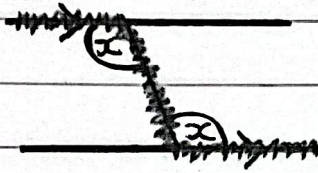
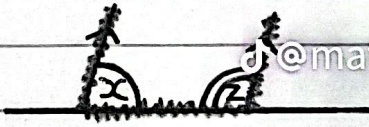
or

$$x^2 = h^2 - y^2$$

or

$$y^2 = h^2 - x^2$$

Revision of lines, Angles, and Triangles

Definations	Examples
Adjacent angles on a straight line are supplementary (add up to 180°)	 $x + y = 180^\circ$
Vertically opposite angles are equal	
Angles around a point add up to 360°	 $s + r + t + u = 360^\circ$
Corresponding angles of parallel lines are equal. (f-shape)	
Alternate angle of parallel lines are equal (Z-shape)	
Co-interior angles of parallel lines are supplementary (u-shape)	

SOLVING GEOMETRIC RIDERS

STEP 1

- Analyze the diagram given.

STEP 2

- Analyze the diagram by paying attention to key words given.
- Look for information in the diagram which can be helpful and useful
- Use colors to mark off equal angles / sides
- Look for implied information.

STEP 3

- Brainstorm and develop a rough proof.
- Link the information you have acquired

STEP 4

- Rewrite a formal proof

EXAMPLES OF KEY WORDS

- Parallel lines given
- Triangle information
- How to prove that lines are parallel?
- Centre of circle given
- Diameter given
- Angles on the same segment
- Chords
- Cyclic quads
- Tangents etc

drawn by EduResource ZA