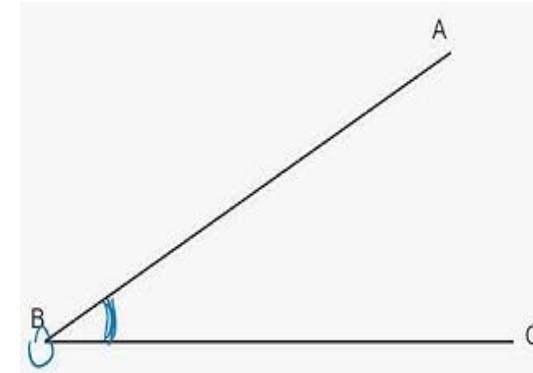
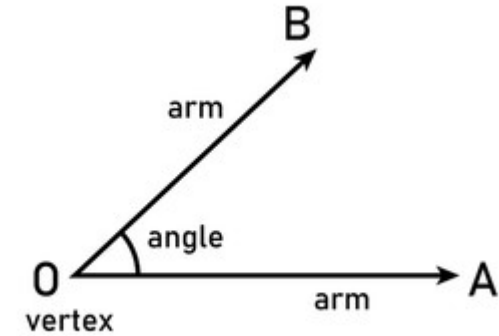


EUCLIDEAN GEOMETRY PART ONE

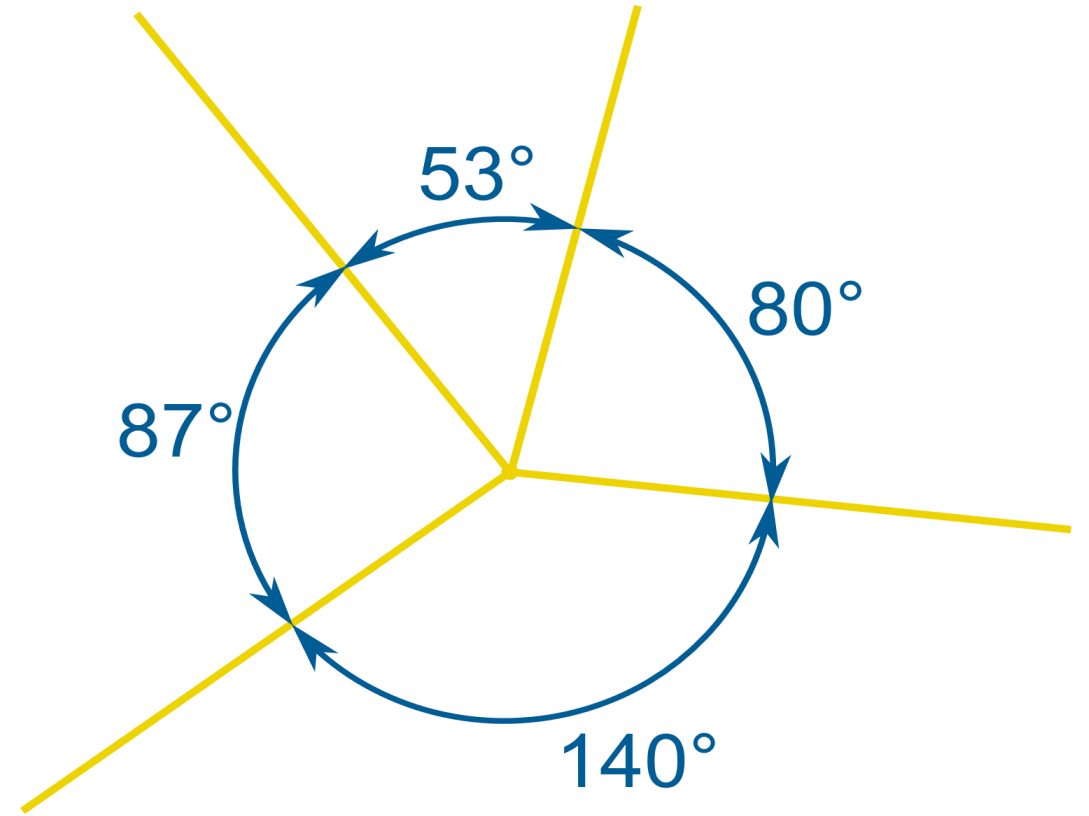
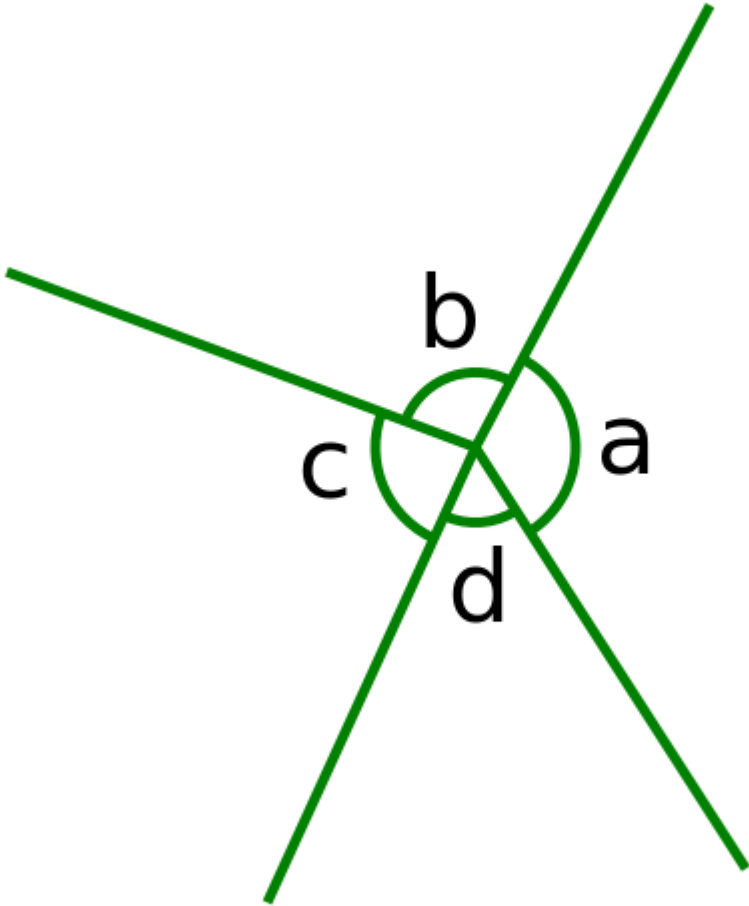
Angles

- An angle is formed when two straight lines meet at a point.
- The point where the angle is formed is called the vertex of an angle.
- Angles are labelled with a caret on a letter, for example, $\hat{B}$
- The $\angle$ symbol is a short method of writing angle in geometry
- Angles can also be labelled according to the line segments that make up the angle.
- For example, in the diagram alongside the angle can be labelled as $\angle BOA$ or $\angle AOB$
- Angles are measured in degrees which is denoted by $^\circ$, a small circle raised above the text, like an exponent



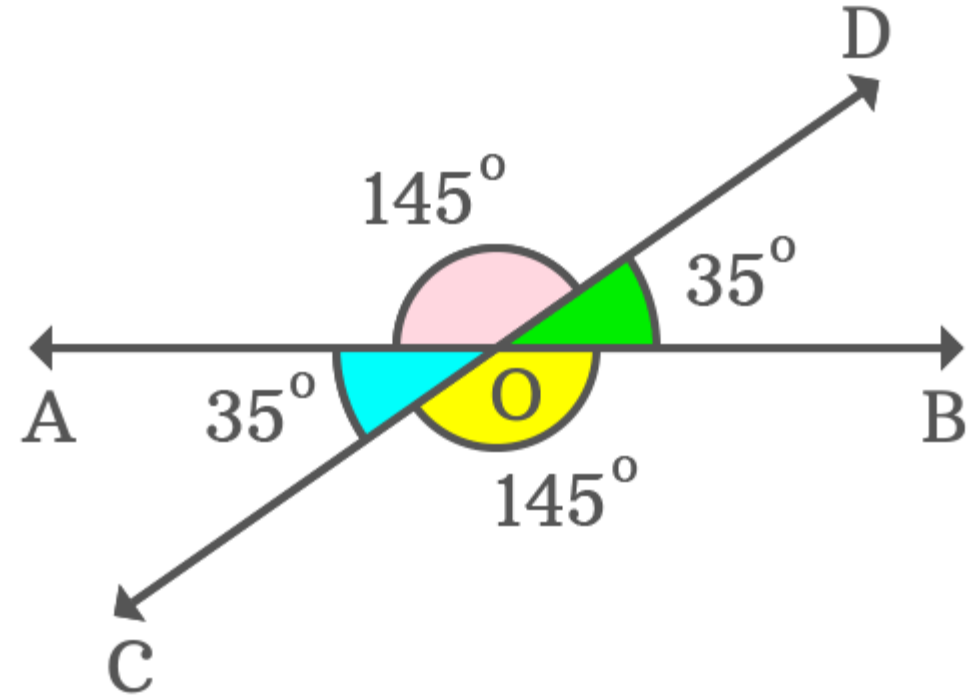
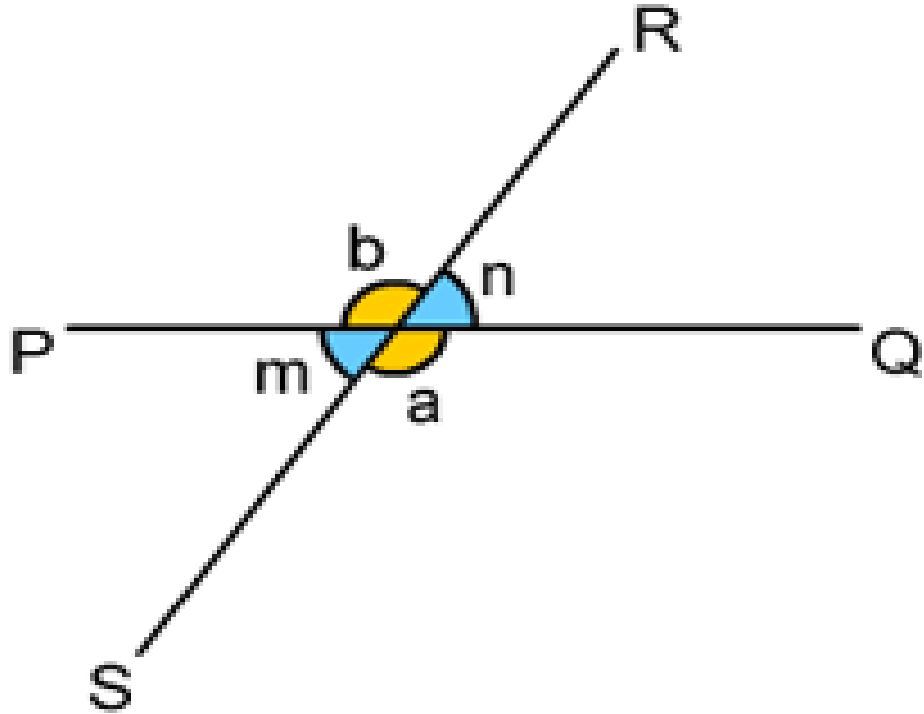
Point **B** is the vertex of the angle between the lines **AB** and **BC**. The angle is referred to as either $\angle ABC$ or $\angle B$. The angle can also be written as **ABC**.

Angles around a point add up to 360°

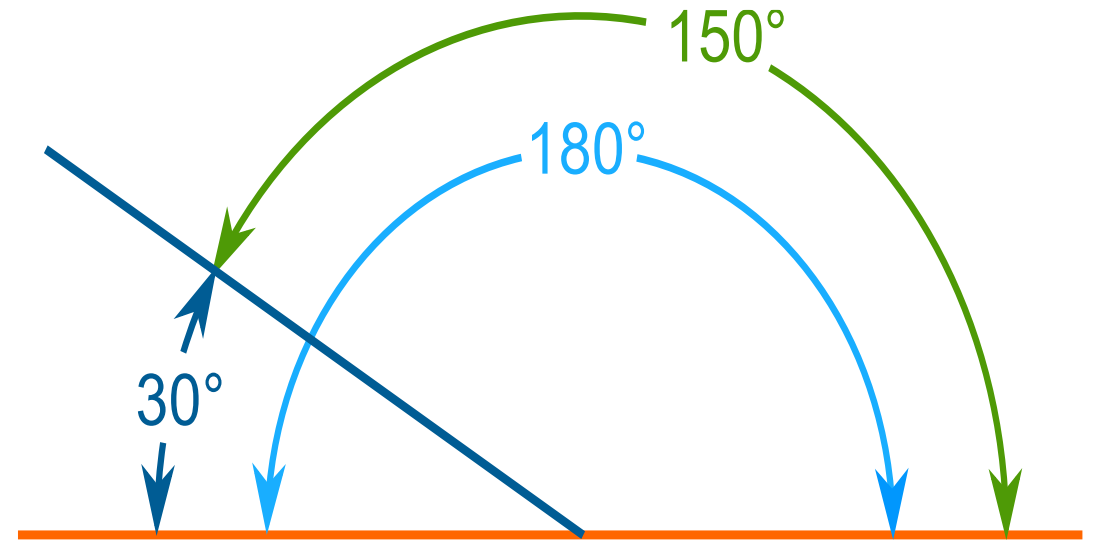
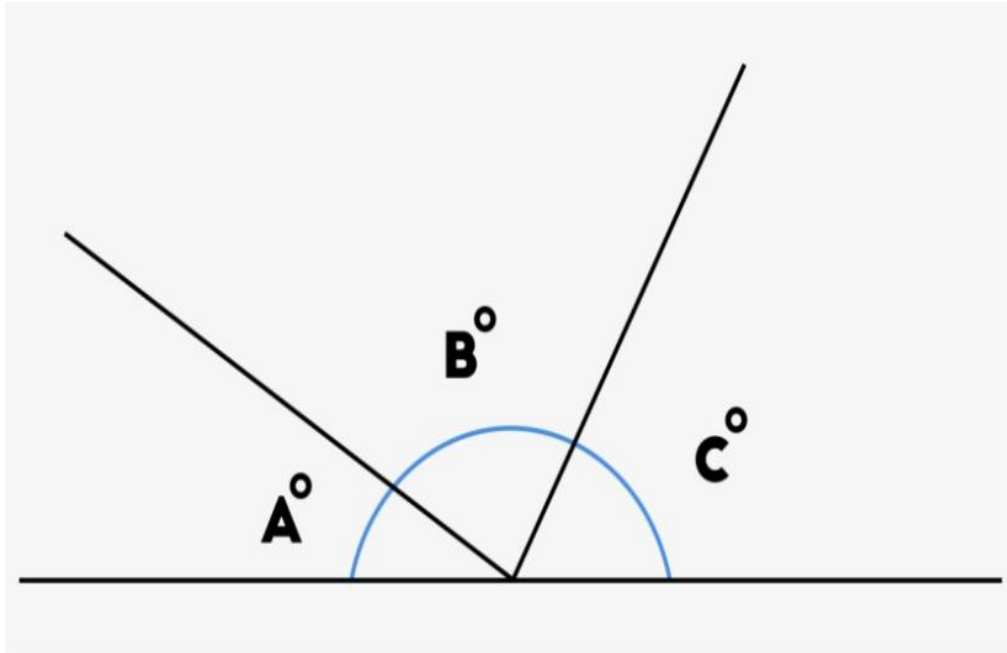


Vertically opposite angles are equal

If two lines PQ and RS cut each other (intersect) ,
then the vertically opposite angles are equal.

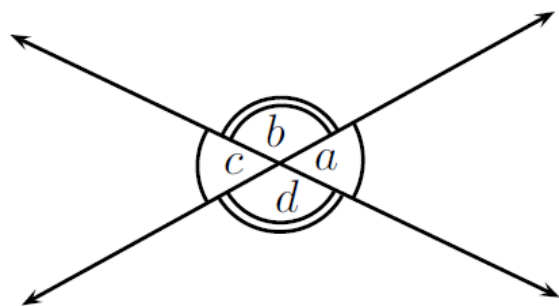


Angles on a straight line add up to 180°



Properties and Notation

- In the diagram below two straight lines intersect at a point, forming the four angles $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$

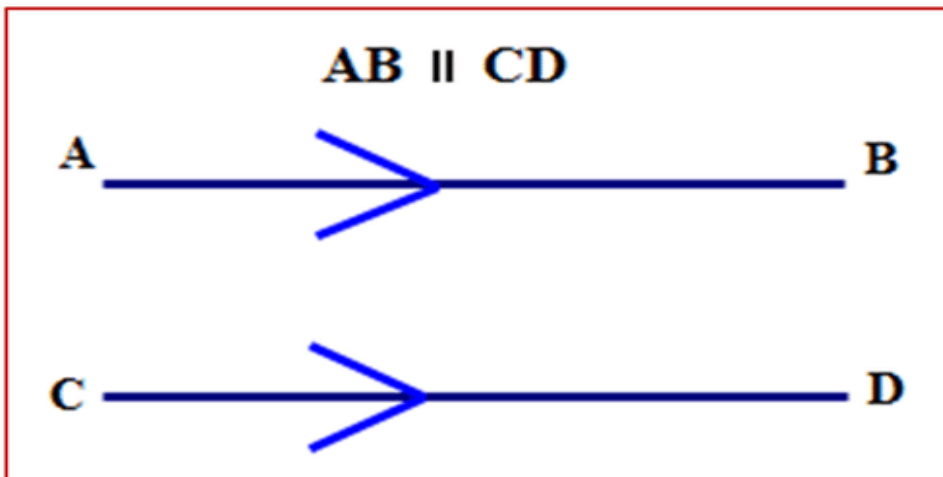
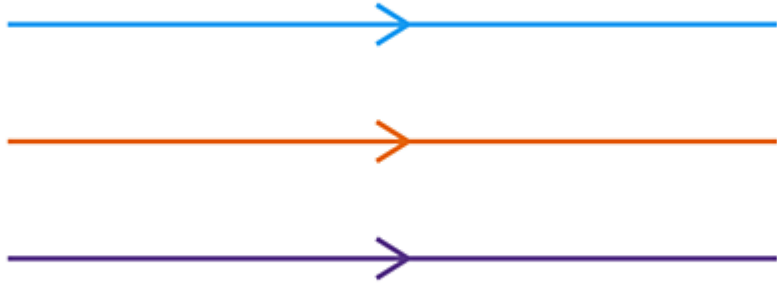


- The following table summarises the different types of angles, with examples from the figure above.

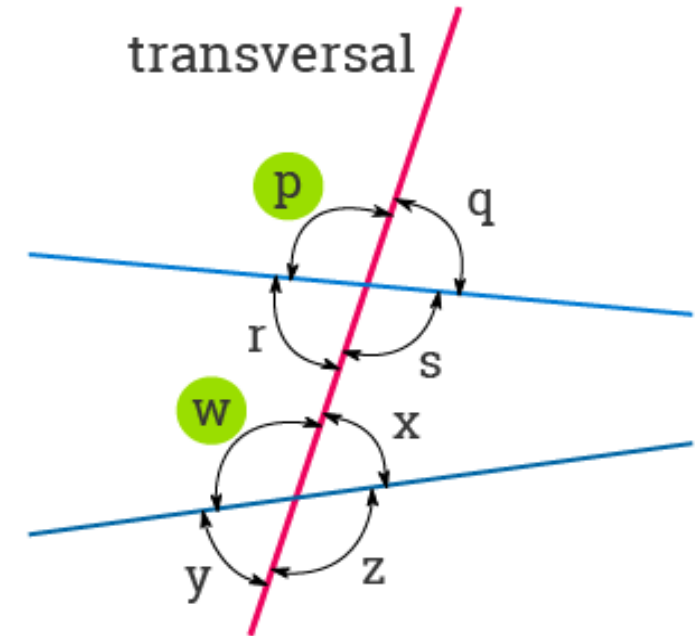
Term	Property	Examples
Acute angle	$0^\circ < \text{angle} < 90^\circ$	$\hat{a}; \hat{c}$
Right angle	Angle = 90°	
Obtuse angle	$90^\circ < \text{angle} < 180^\circ$	$\hat{b}; \hat{d}$
Straight angle	Angle = 180°	$\hat{a} + \hat{b}; \hat{b} + \hat{c}$
Reflex angle	$180^\circ < \text{angle} < 360^\circ$	$\hat{a} + \hat{b} + \hat{c}$
Adjacent angles	Angles that share a vertex and a common side.	$\hat{a}$ and $\hat{d}; \hat{c}$ and $\hat{d}$
Vertically opposite angles	Angles opposite each other when two lines intersect. They share a vertex and are equal.	$\hat{a} = \hat{c}; \hat{b} = \hat{d}$
Supplementary angles	Two angles that add up to 180° .	$\hat{a} + \hat{b} = 180^\circ; \hat{b} + \hat{c} = 180^\circ$
Complementary angles	Two angles that add up to 90° .	
Revolution	The sum of all angles around a point.	$\hat{a} + \hat{b} + \hat{c} + \hat{d} = 360^\circ$

Parallel Lines

- ❖ These are two or more lines that lie in the same plane and never intersect each other are known as **parallel lines**.

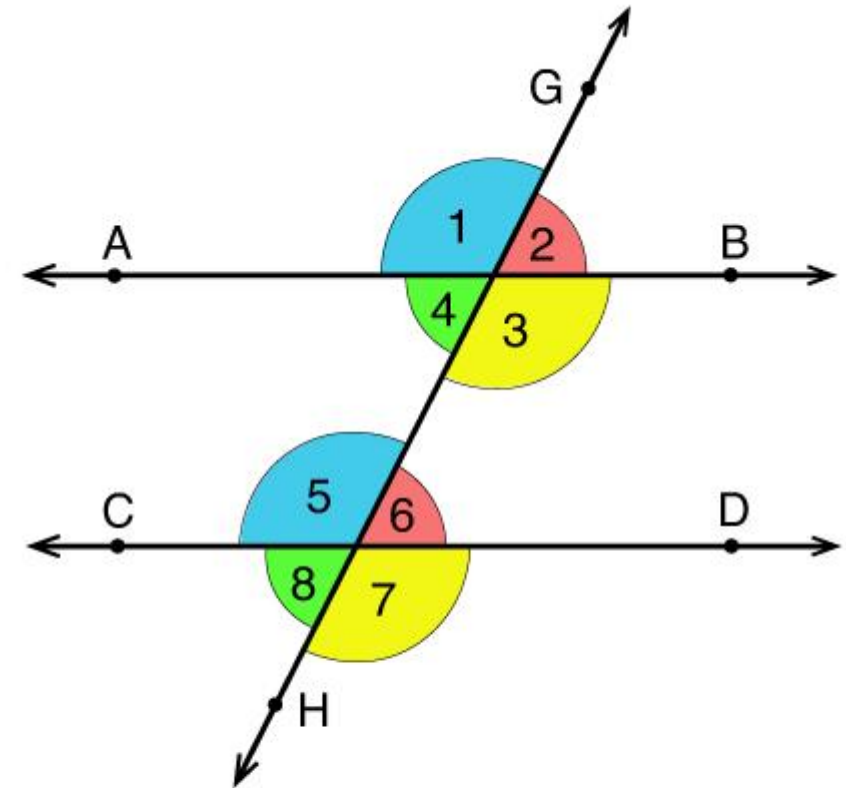
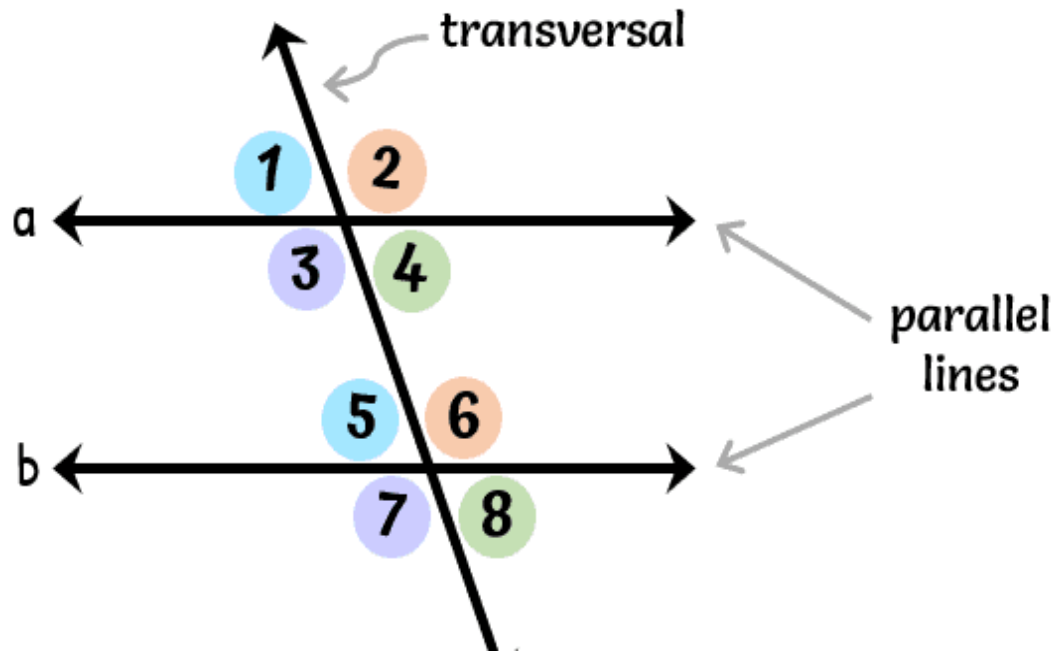


- ❖ When any two parallel lines are intersected by another line called a transversal, many pairs of angles are formed. While some angles are congruent (equal), the others are supplementary



Corresponding Angles

- Corresponding angles are the angles which are formed in matching corners or corresponding corners with the transversal when two parallel lines are intersected by any other line (i.e. the transversal)



$\angle 1$ corresponds to $\angle 5$

$\angle 2$ corresponds to $\angle 6$

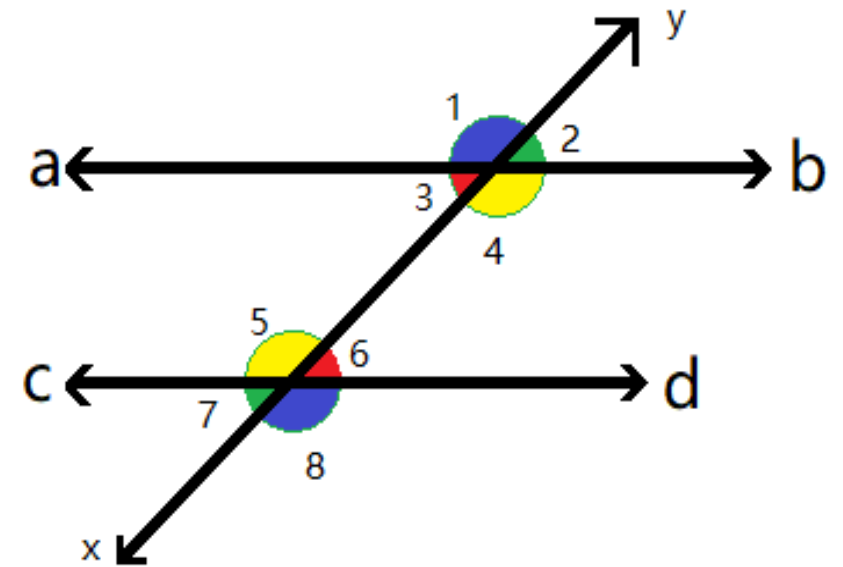
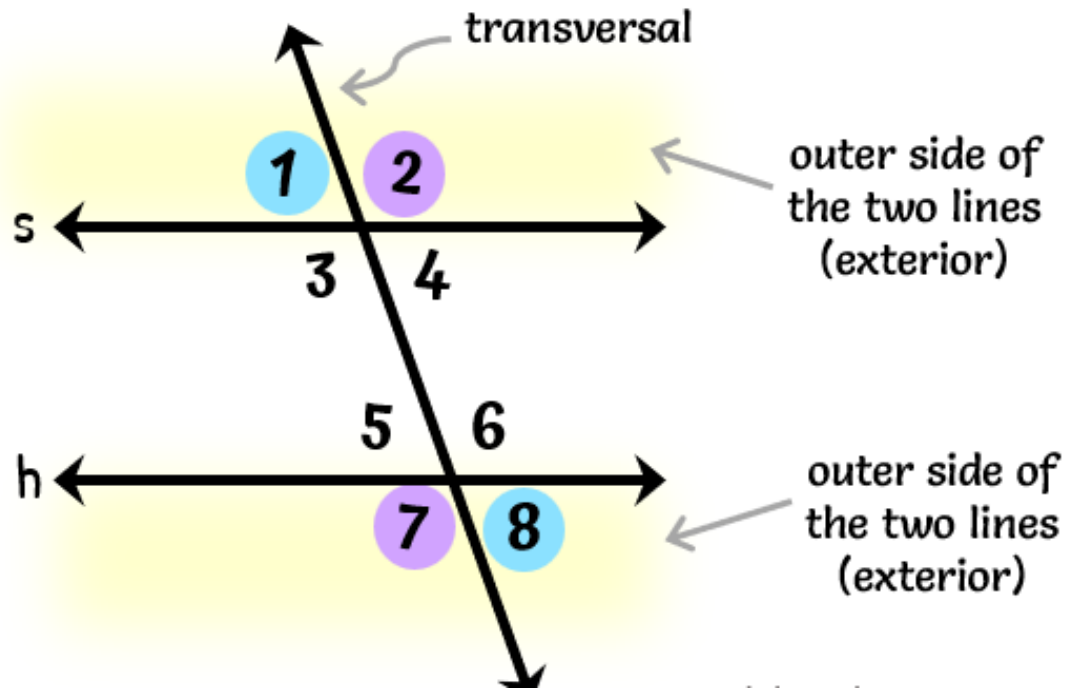
$\angle 3$ corresponds to $\angle 7$

$\angle 4$ corresponds to $\angle 8$

Alternate angles

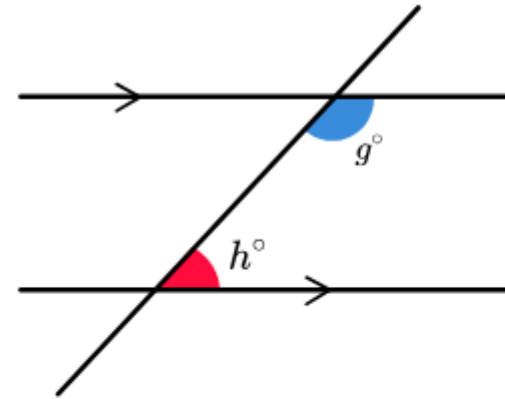
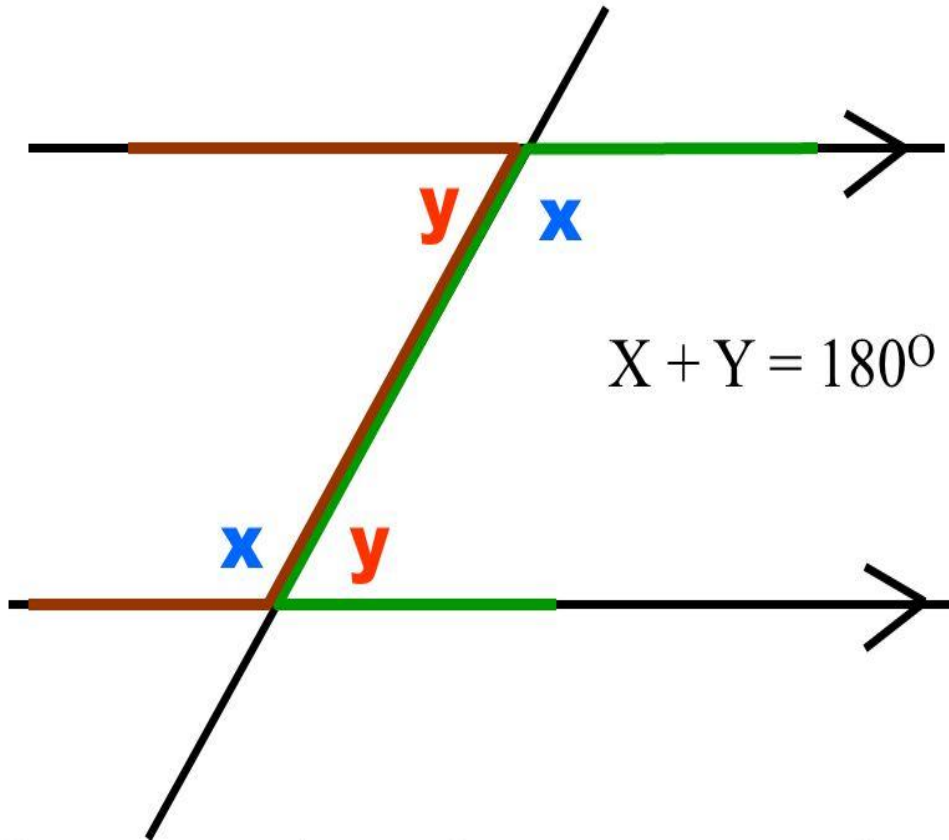
❑ These are angles that occur on opposite sides of the transversal line and have the **same size**.

❑ Alternate angles are equal:

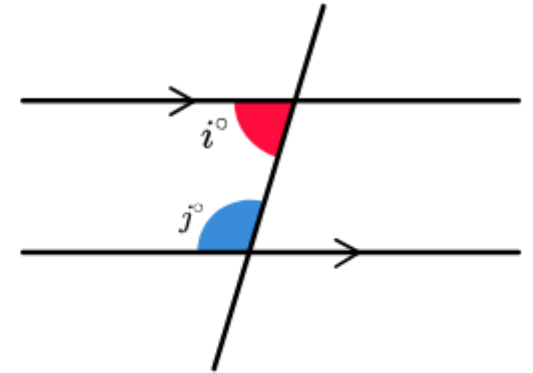


Co-interior angles

- Co-interior angles in parallel lines occur in between two parallel lines when they are intersected by a transversal.
- The two angles that occur on the same side of the transversal always add up to 180°



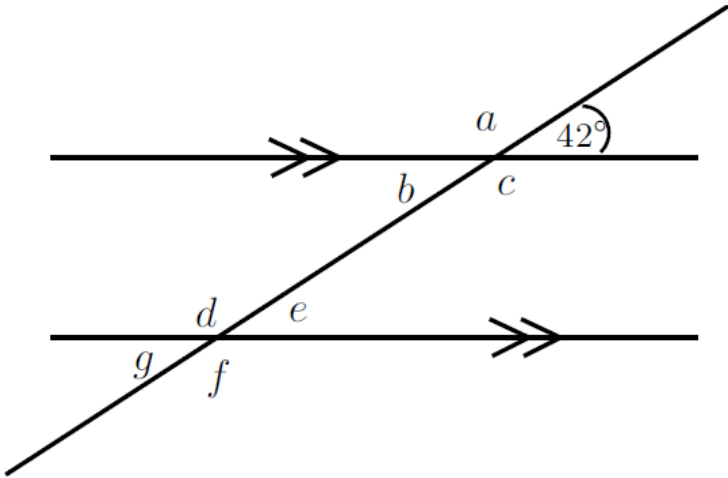
$$g + h = 180^\circ$$



$$i + j = 180^\circ$$

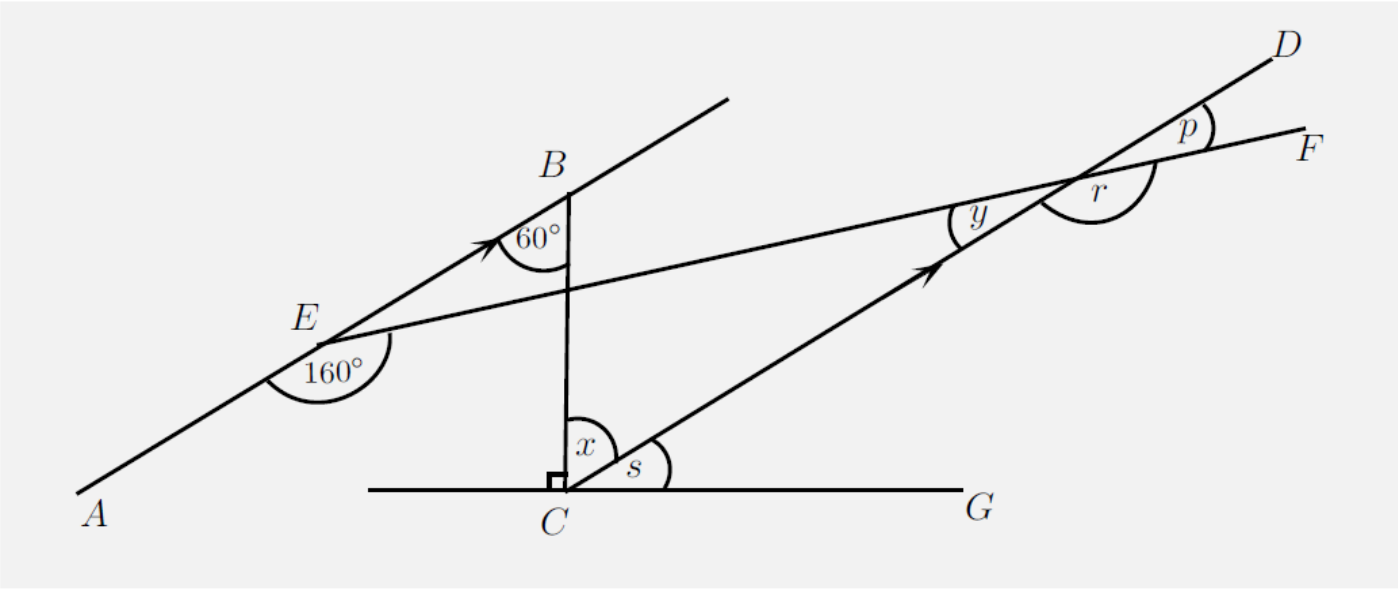
Example 1

Use adjacent, corresponding, co-interior and alternate angles to fill in all the angles labelled with letters in the diagram:



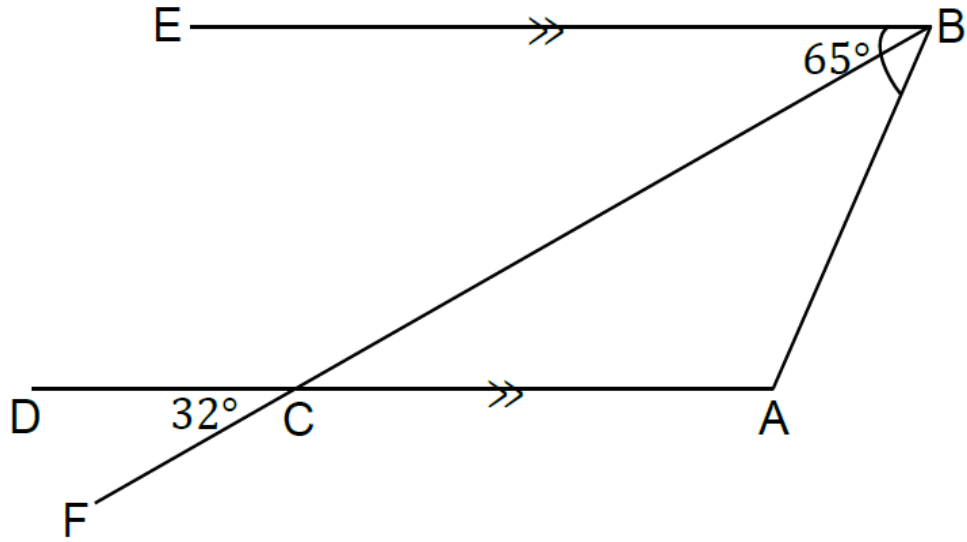
Example 2

Find all the unknown angles.



Example 3

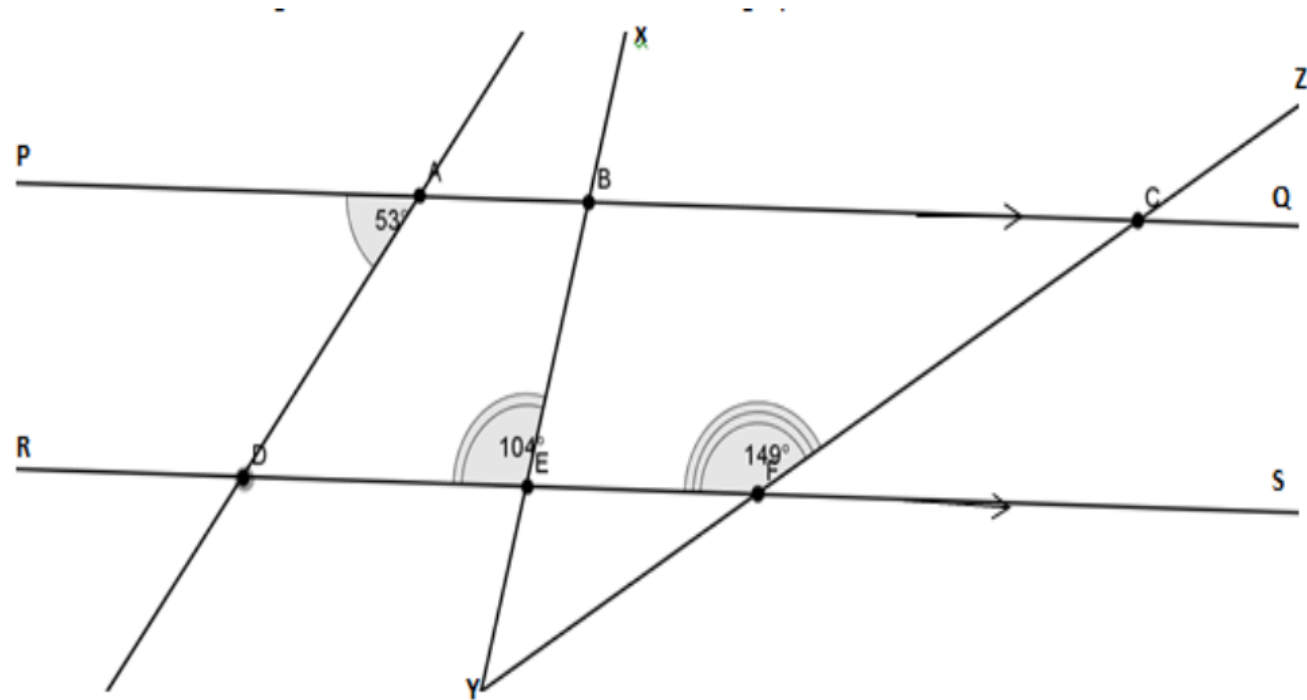
In the diagram below, $\angle ABE = 65^\circ$ and $\angle DCF = 32^\circ$



- a) Calculate the size of $\angle EBC$. Give reasons for your answer.
- b) Calculate the size of $\angle CAB$. Give reasons for your answer.

Example 3

Use the diagram below to answer the following questions

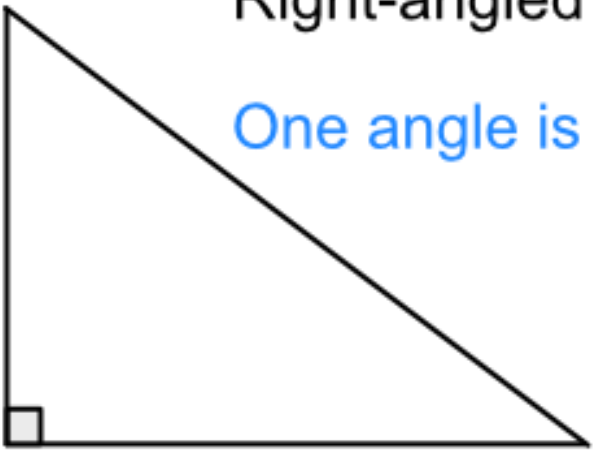


Calculate with reasons

- a) $\angle ADE$
- b) $\angle ABE$
- c) $\angle CBX$
- d) $\angle EFY$
- e) $\angle BCF$
- f) $\angle CBE$
- g) Why is $\angle BCF + \angle CBE \neq 180^\circ$

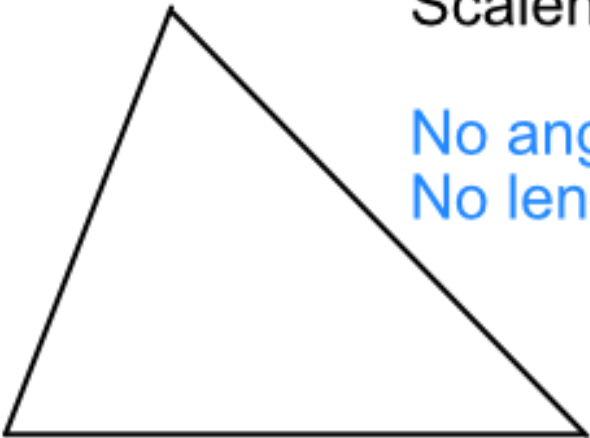
Properties of triangles

Right-angled Triangle



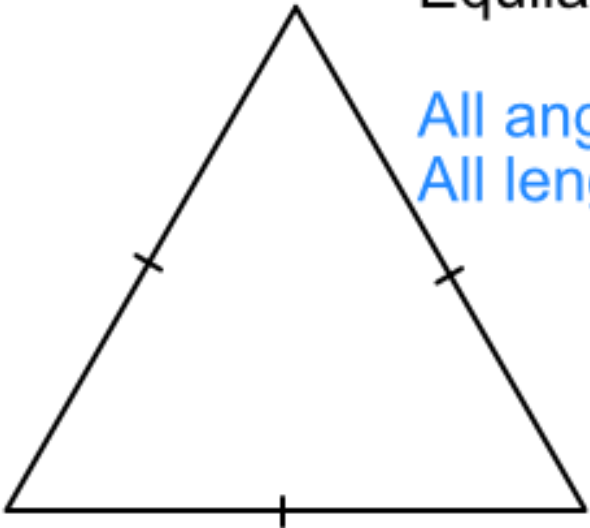
One angle is 90°

Scalene Triangle



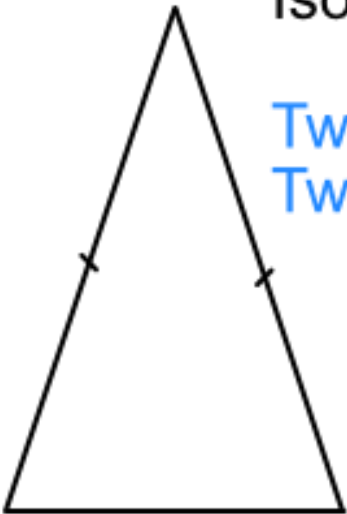
No angles are the same
No lengths are the same

Equilateral Triangle



All angles are the same
All lengths are the same

Isosceles Triangle

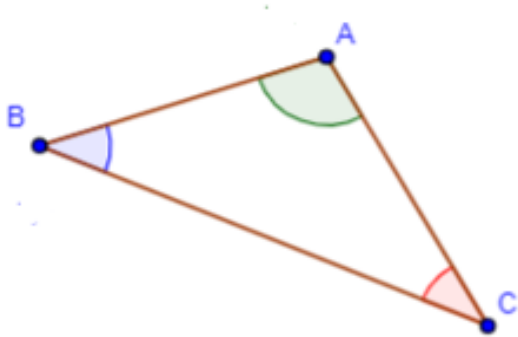


Two angles are the same
Two lengths are the same

Sum of interior angles of a triangle

This fundamental concept in geometry states that the three interior angles of a triangle always add up to 180 degrees

Sum of Angles in a Triangle



$$\angle A + \angle B + \angle C = 180^\circ$$

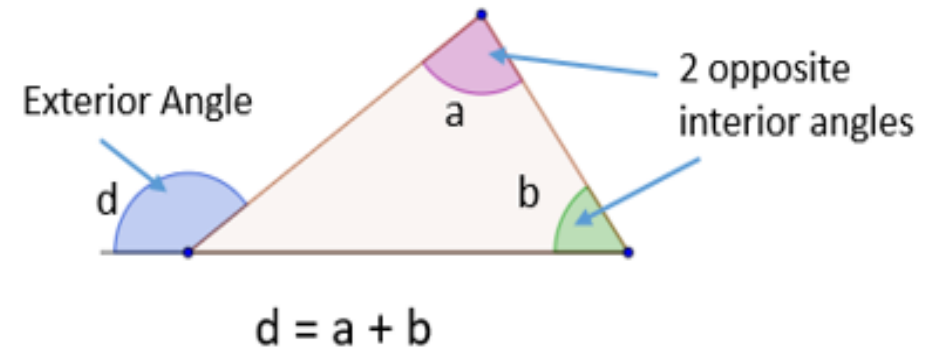
The sum of the angles in a triangle is always 180°

Exterior angle of triangle

An exterior angle of a triangle is formed by extending one of its sides, and its measure equals the sum of the two non-adjacent interior angles (also known as remote interior angles)

Exterior Angle Theorem

The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

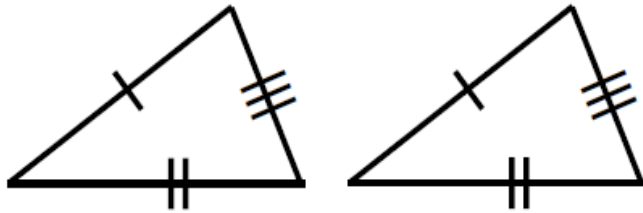


Congruency of triangles

- Congruent triangles are triangles that are the **same size and shape**.
- **There are four conditions to prove congruency in triangles**
- The symbol $\cong$ is used to show congruency

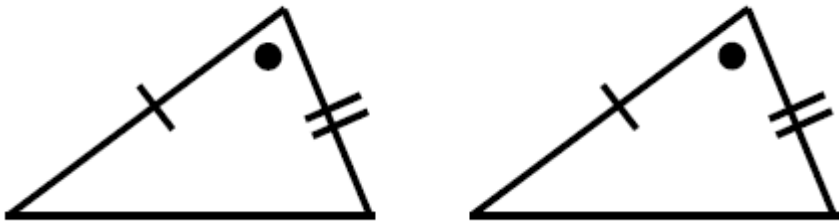
Condition 1

Two triangles are congruent if three sides of one triangle are equal in length to the three sides of the other triangle.



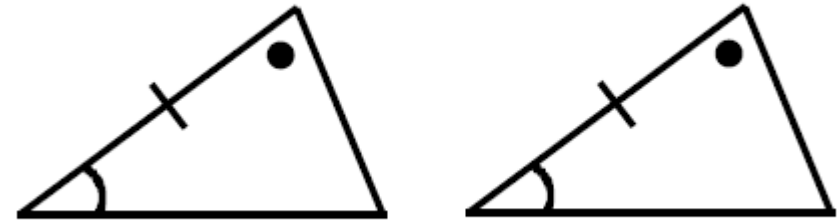
Condition 2

Two triangles are congruent if two sides and the included angle are equal to two sides and the included angle of the other triangle.



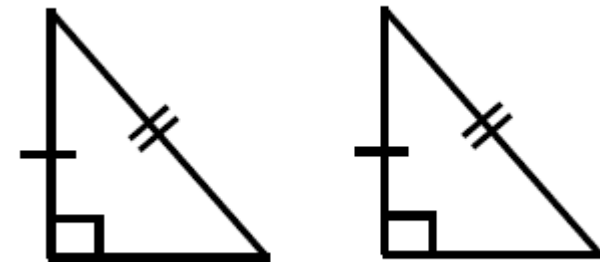
Condition 3

Two triangles are congruent if two angles and one side of a triangle are equal to two angles and a corresponding side of the other triangle.



Condition 4

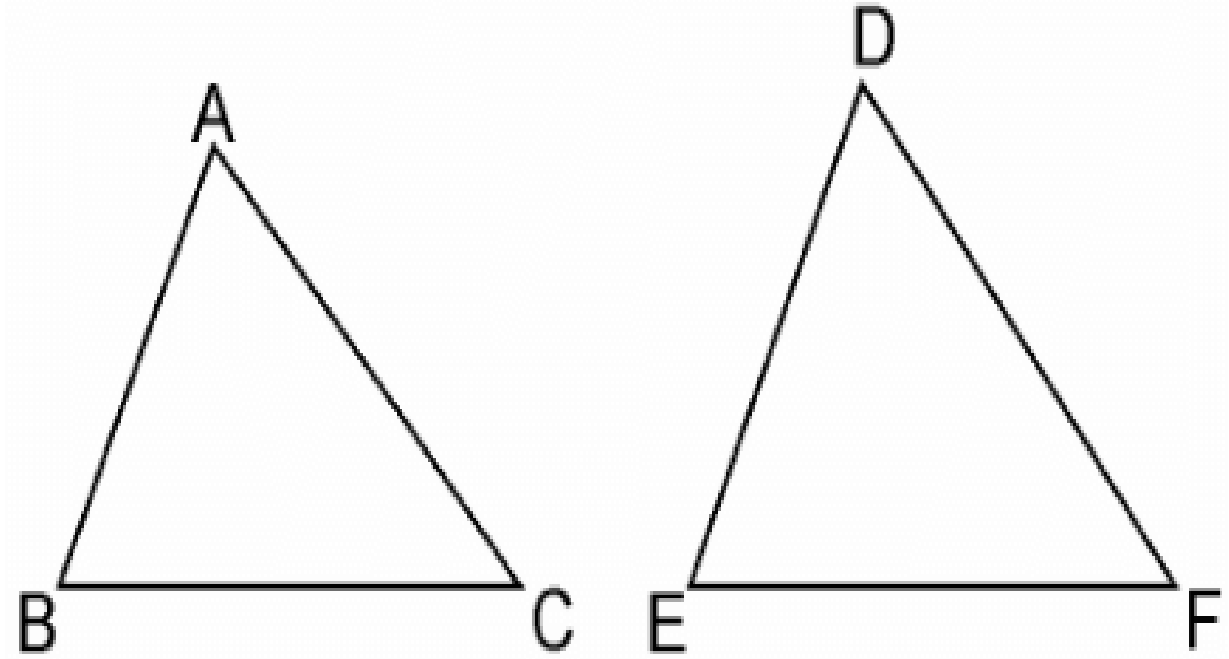
Two right-angled triangles are congruent if the hypotenuse and a side of the one triangle is equal to the hypotenuse and a side of the other triangle.



Similar Triangles

- ☐ Two polygons are similar if they have the same shape but not necessarily the same size.
- ☐ Two polygons (which have the same number of sides) are similar if they meet **both** of the following conditions
- ☐ Two conditions must **both** be satisfied for two polygons to be similar:
 - (a) The corresponding angles must be equal.
 - (b) The ratio of the corresponding sides must be in the same proportion.
- ☐ The symbol $|||$ is used to show similarity

Consider the following triangles:



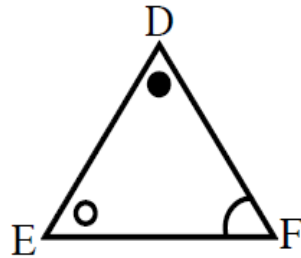
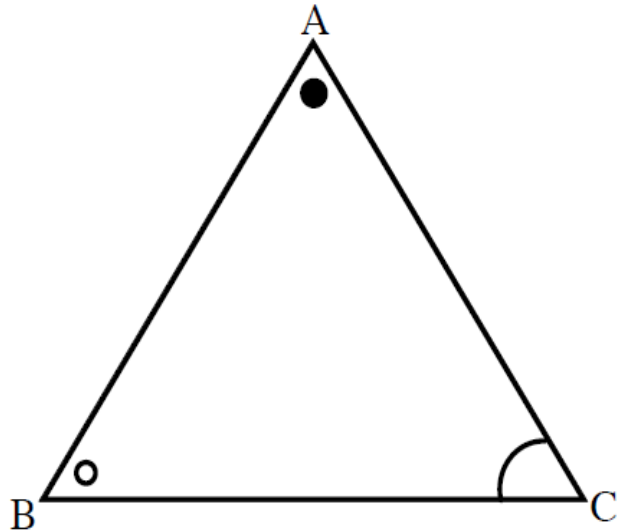
If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$
 $\therefore \triangle ABC ||| \triangle DEF$

(equiangular)
(similar triangles)

- ❖ If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).

Given that :

$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F \quad (\text{equiangular})$$



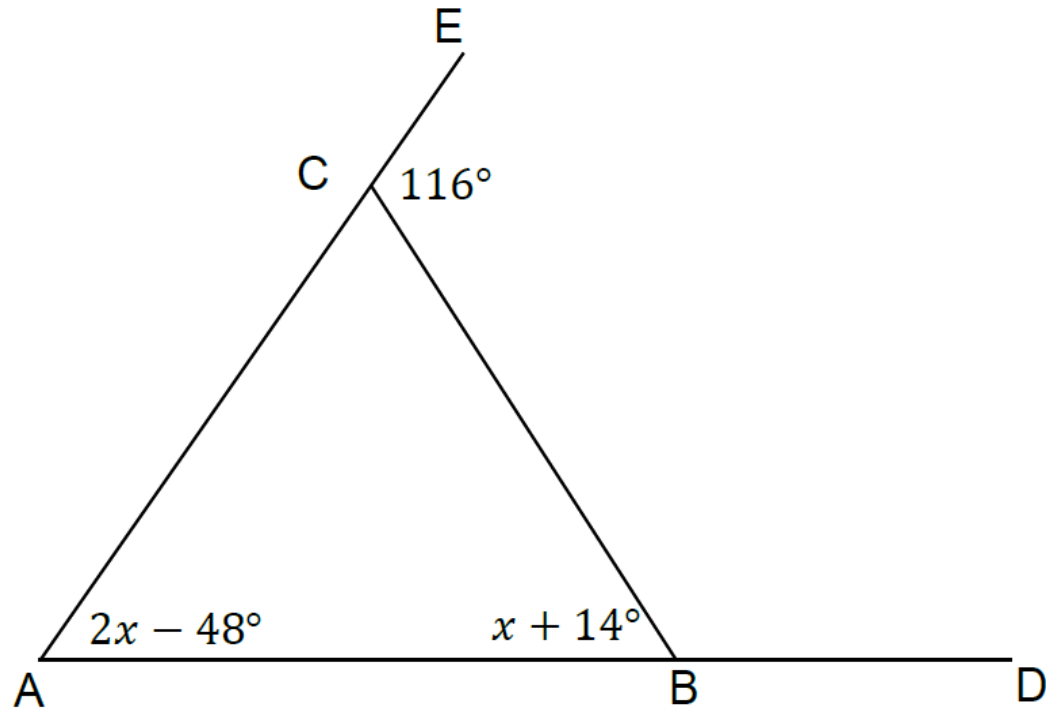
Then:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad (\text{similar } \Delta\text{s in proportion})$$

$$\Delta ABC \parallel \parallel \Delta DEF$$

Example 5

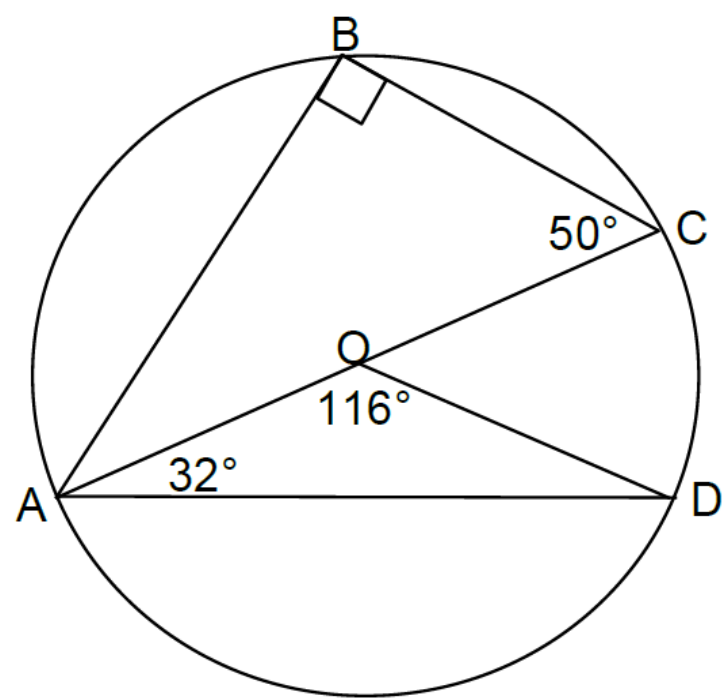
In the diagram below, $\angle CAB = 2x - 48^\circ$, $\angle ABC = x + 14^\circ$ and $\angle BCE = 116^\circ$



- Calculate the value of x . Give reasons for your answer.
- Calculate the actual size of $\angle CAB$.
- What type of Δ is ΔABC ? Give reasons for your answer.

Example 6

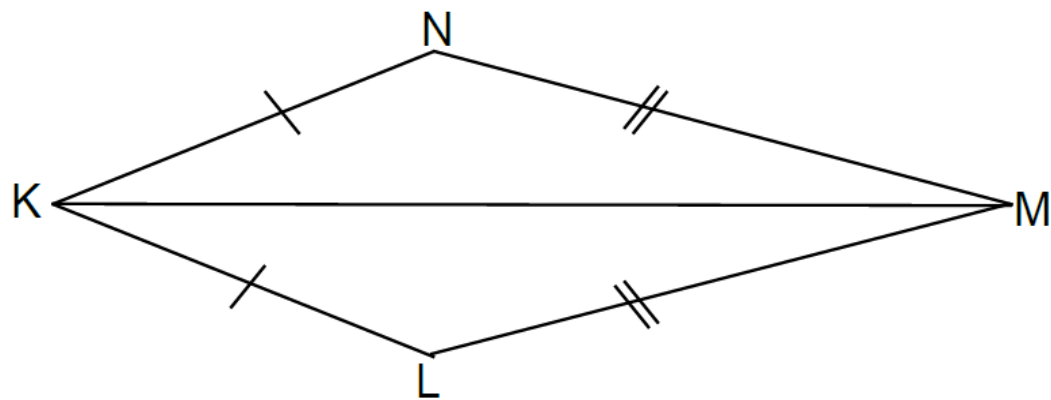
In the figure below, O is the center of the circle



- a) Calculate the size of $\angle CAB$. Give a reason for your answer.
- b) Calculate the size of $\angle ADO$. Give a reason for your answer.

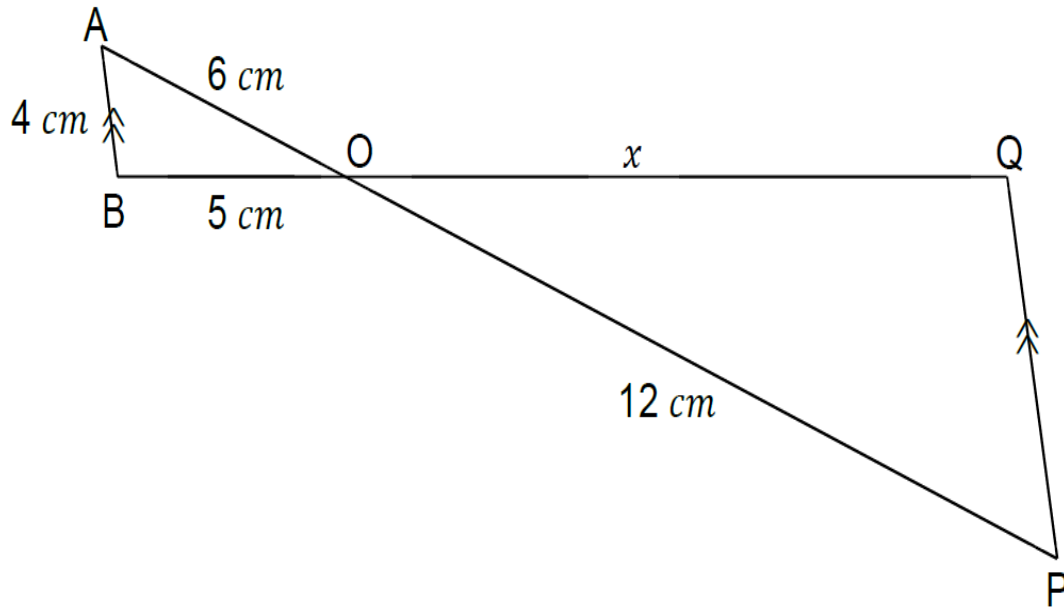
Example 7

In the diagram below, prove that $\triangle KLM \equiv \triangle KNM$.



Example 8

In the diagram below, $AB \parallel PQ$.



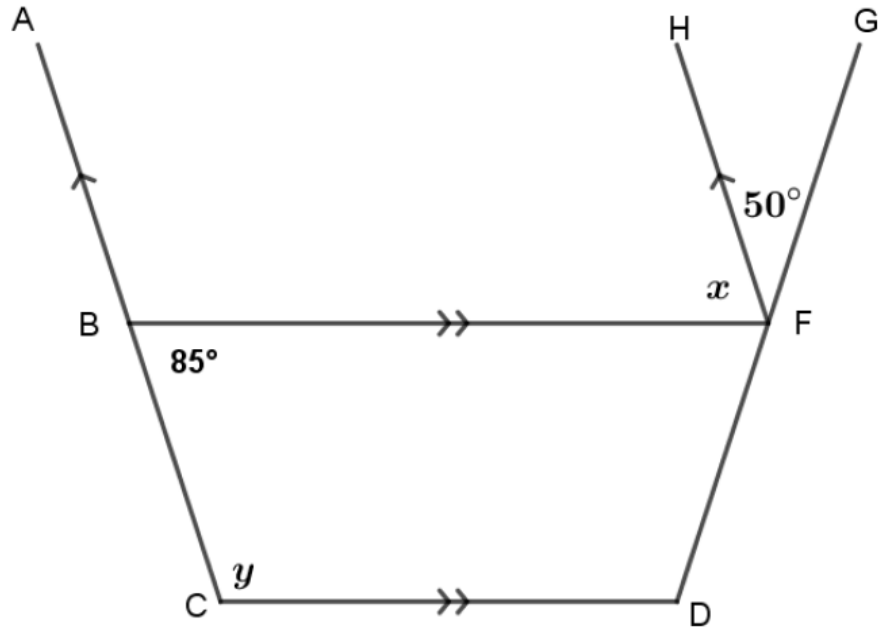
- Prove that $\triangle ABO \parallel \triangle PQO$.
- Calculate the value of x .



PAST EXAM PAPERS

2018 Eastern Cape November Q 5.1-5.2

In the diagram below $AC \parallel HF$ and $BF \parallel CD$. $\angle CAB = 85^\circ$
and $\angle GFH = 50^\circ$ Find with reasons, the size of:



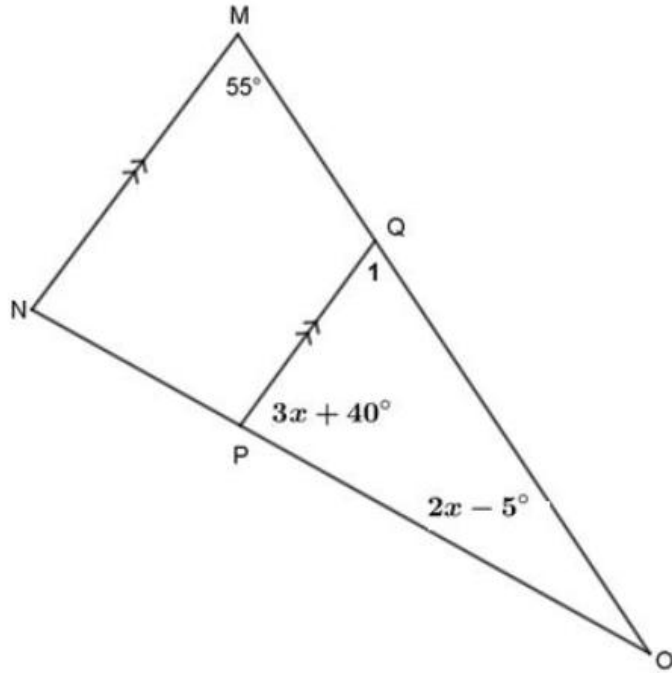
5.1.1 x (2)

5.1.2 y (2)

CONTINUED.....

In the diagram below is $\triangle MNO$ with $MN \parallel QP$.

$\angle OMN = 55^\circ$, $\angle QPO = 3x + 40^\circ$ and $\angle MON = 2x - 5^\circ$



5.2.1 Calculate the value of x . Give reasons for your answer.

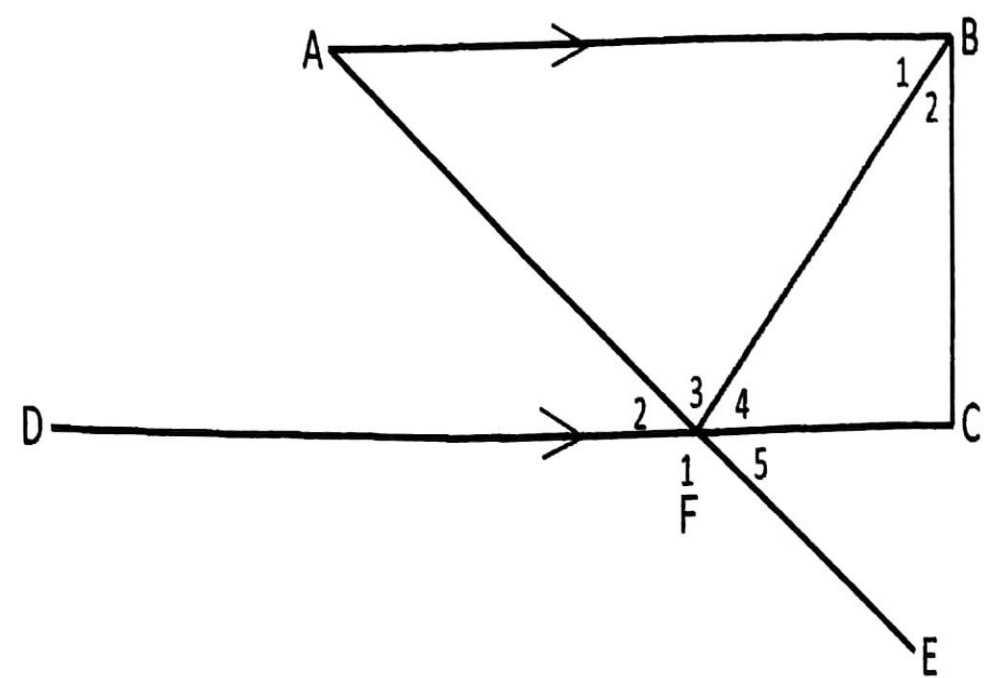
(4)

5.2.2 Calculate the size of $\angle MON$

(2)

2017 Gauteng November Q 7.1;7.3-7.4

Given that $AB \parallel CD$

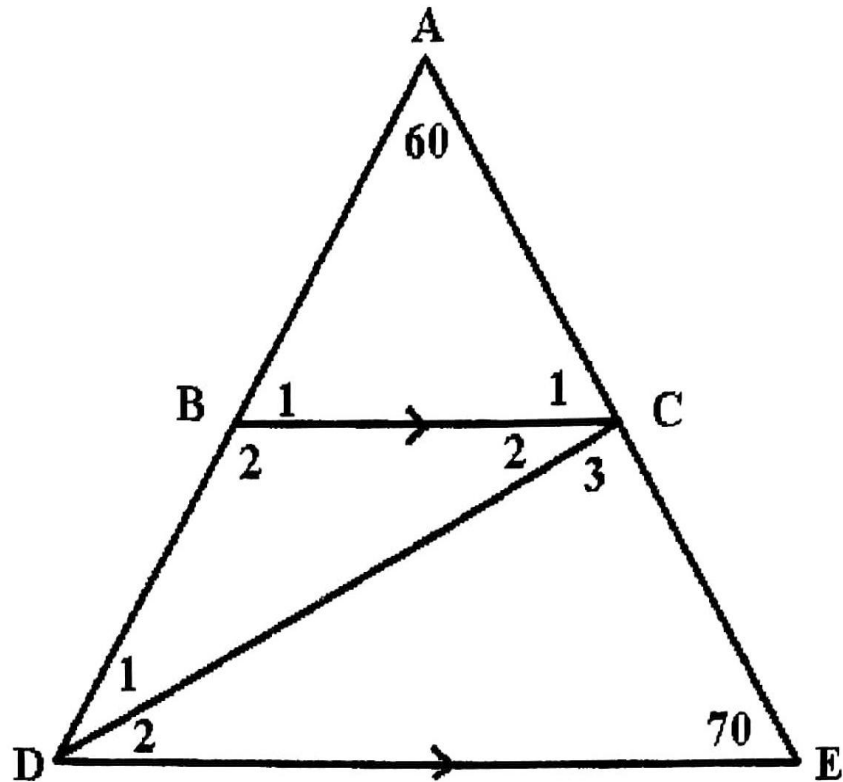


Complete the following table

STATEMENT	REASON
$\angle F_2 = \angle F_5$	
$\angle B_1 =$	
$\angle C + \angle ABC$	
If $\angle B_2 = \angle F_4$ then $BC =$	

CONTINUED.....

Study the following diagram and then answer the questions that follow. $BC \parallel DE$ and $\angle D_1 = \angle D_2$. Determine the size of the following angles with reasons.



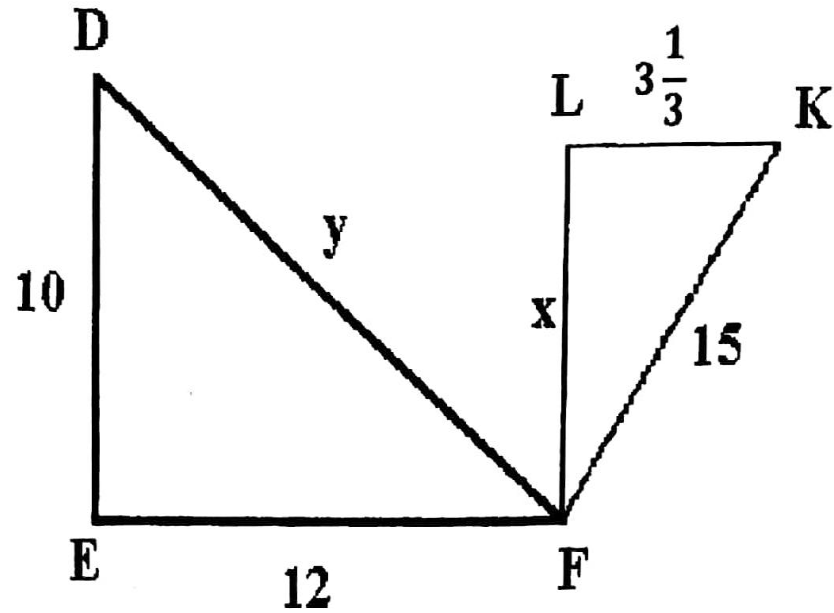
7.3.1 $\angle D_2$ (2)

7.3.2 $\angle C_1$, $\angle C_2$ and $\angle C_3$ (6)

7.3.3 $\angle B_1$ and $\angle B_2$ (4)

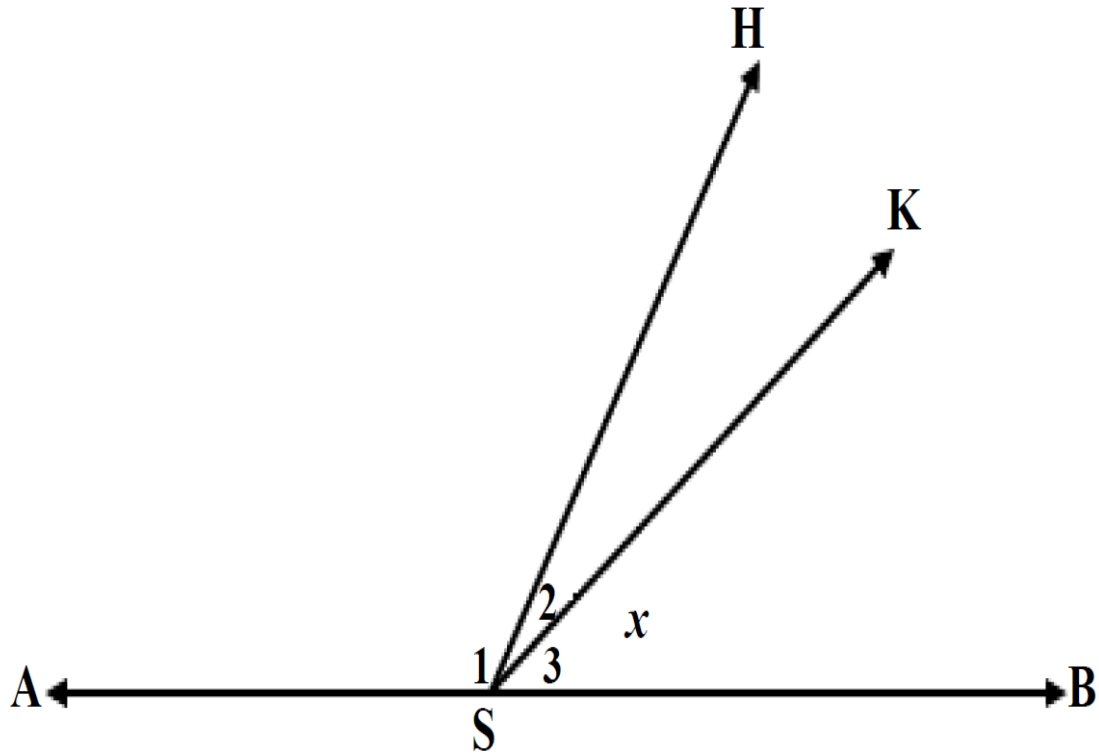
CONTINUED.....

7.4 In the diagram below, $\triangle DEF \parallel \triangle KLF$. Determine the values of x and y . Round off your answer to the nearest whole numbers (3)



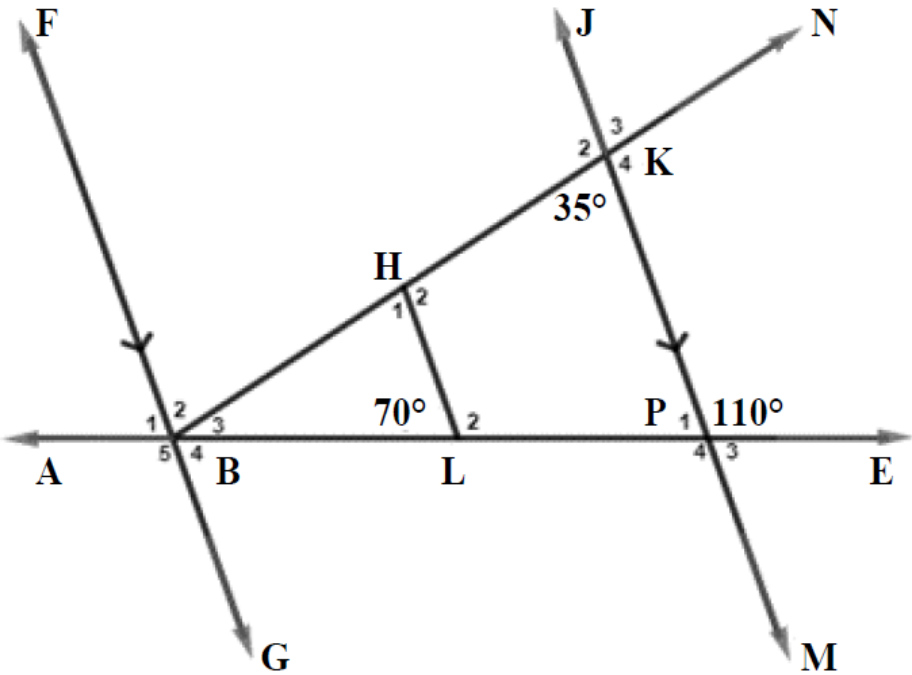
2022 Gauteng November Paper 2 Q 2.2-2.3

In the diagram below, it is given that $\angle S_3 = 2\angle S_2 = x$ and $\angle S_1 = 3\angle S_2$. Calculate the value of x



CONTINUED.....

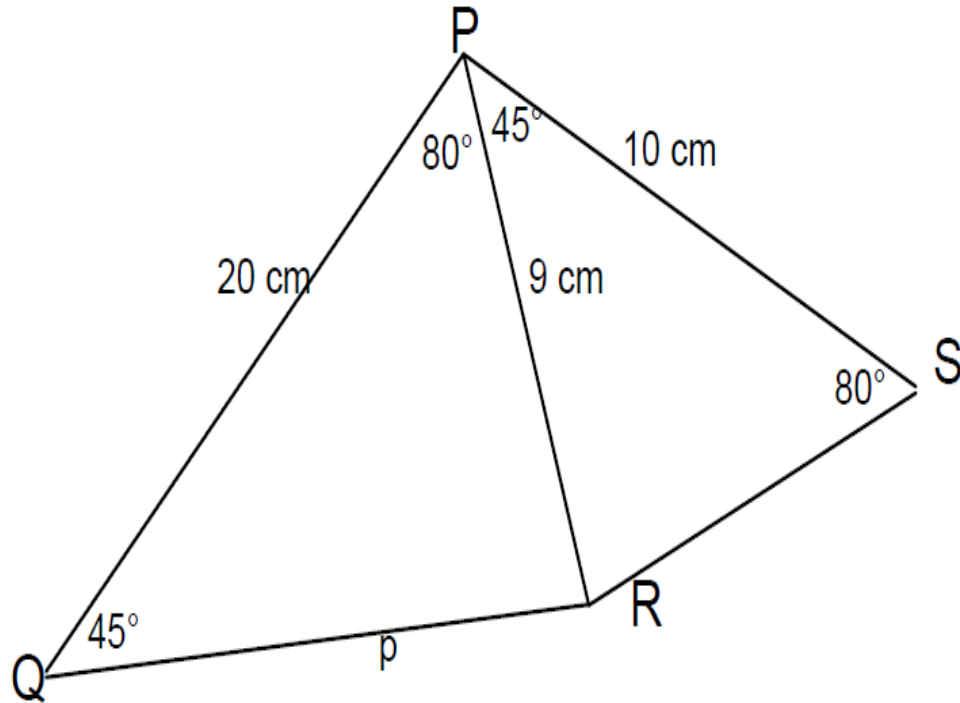
In the diagram below, calculate with reasons the following angles



- 2.3.1 $\angle B_3$ (2)
- 2.3.2 $\angle B_5$ (2)
- 2.3.3 Prove with reasons that $HL \parallel KM$ (3)

2011 November Paper Q 4.1-4.2

In the figure below; $\triangle PQR \parallel \triangle SPR$.

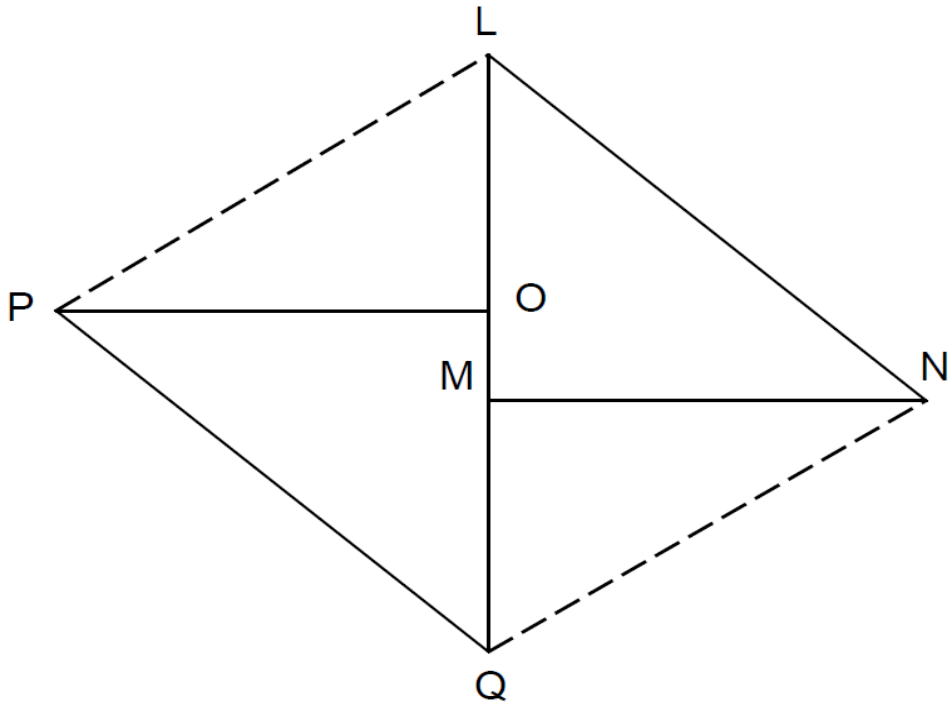


4.1.1 Calculate the length of p. (2)

4.1.2 Using the answer from QUESTION 4.1.1 and give reasons for $\triangle PQR \parallel \triangle SPR$ (1)

CONTINUED.....

In the diagram below $LO = MQ = 6\text{ cm}$; $OM = 3\text{ cm}$;
 $\angle OPQ = 30^\circ$. $PQ = LN$; $PO \perp LQ$ and $MN \perp LQ$.

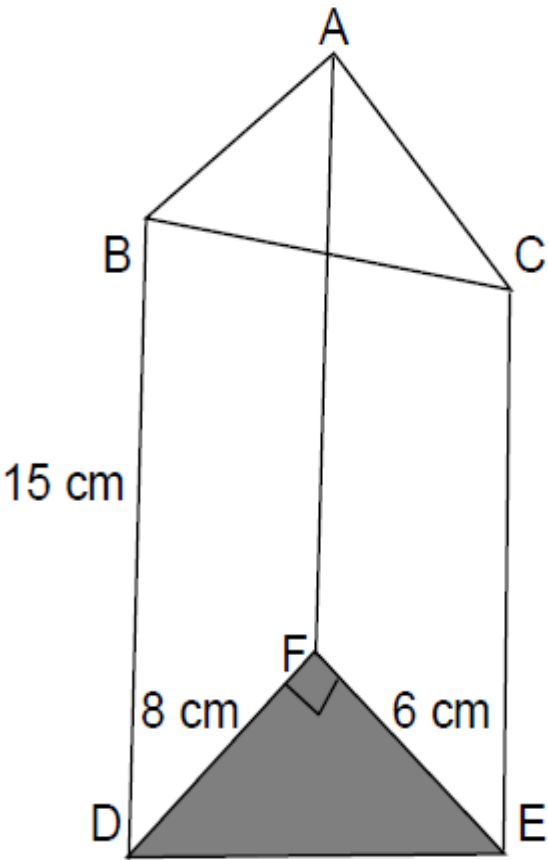


4.2.1 Show with reasons that $LM = QO$. (2)

4.2.2 Prove that $\triangle POQ \cong \triangle NML$. (3)

2013 Eastern Cape November Paper Q 6.3

A triangular prism is drawn. The base is a right-angled triangle with $DF = 8\text{ cm}$, $FE = 6\text{ cm}$ and its height is 15 cm . B is joined to F.

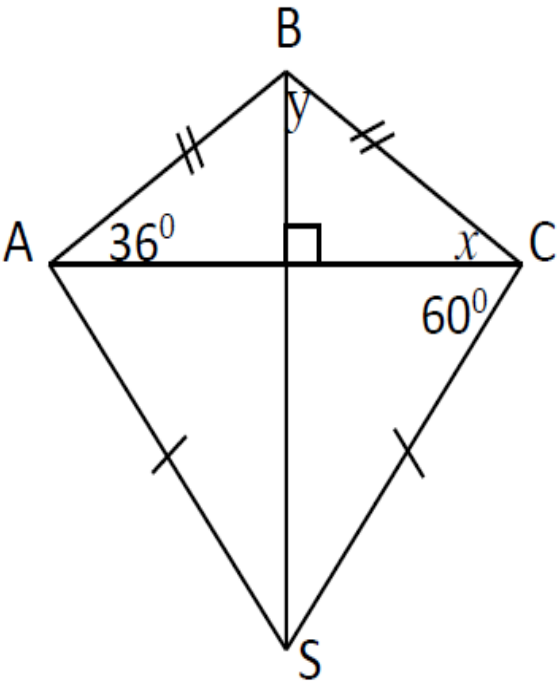


6.3.1 Calculate the length of BF. (2)

6.3.2 Show that $\triangle FDE$ and $\triangle ABC$ are congruent. (4)

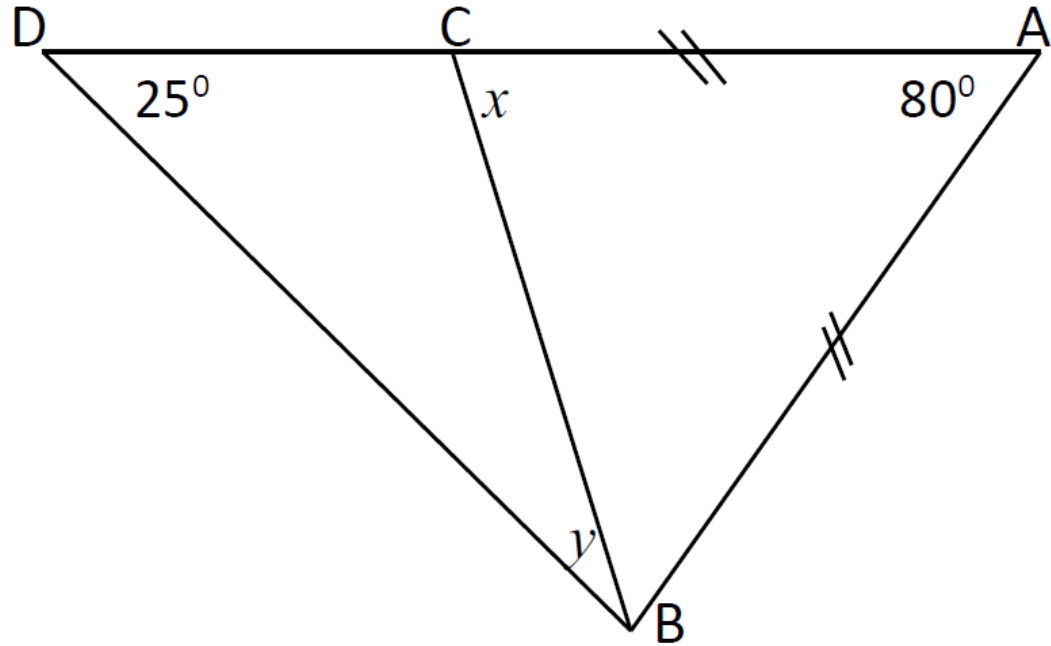
2015 Gauteng November Q 6.1-6.4

6.1.1 Determine with reasons, the sizes of angles x and y in the diagram below (4)



CONTINUED.....

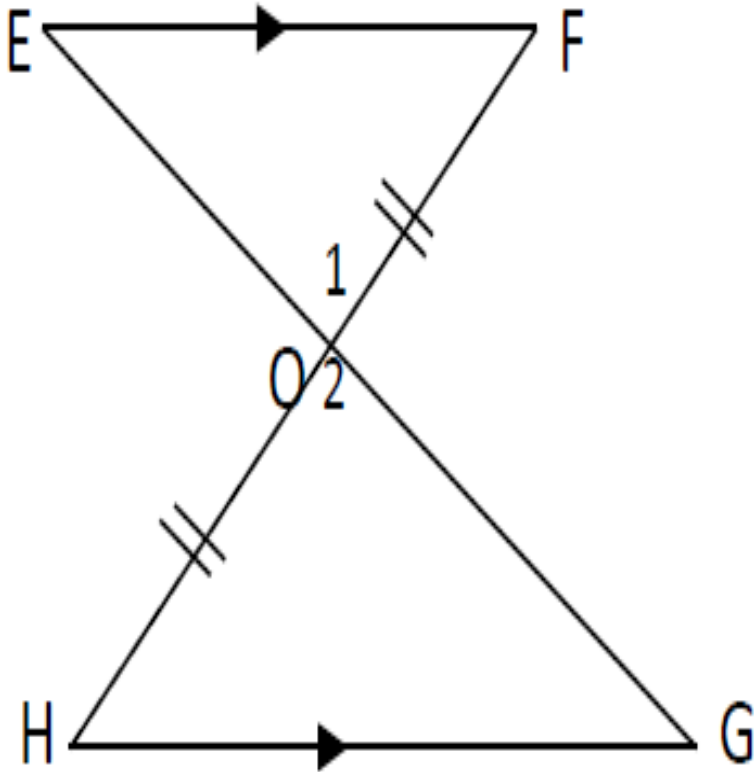
6.1.2 Determine with reasons, the sizes of angles x and y in the diagram below (4)



CONTINUED...

6.2 Study the sketch below and prove that

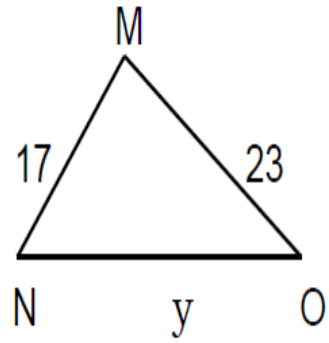
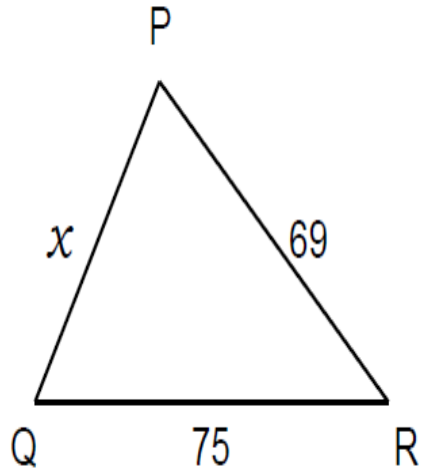
$$\triangle EFO \equiv \triangle GHO \quad (4)$$



CONTINUED.....

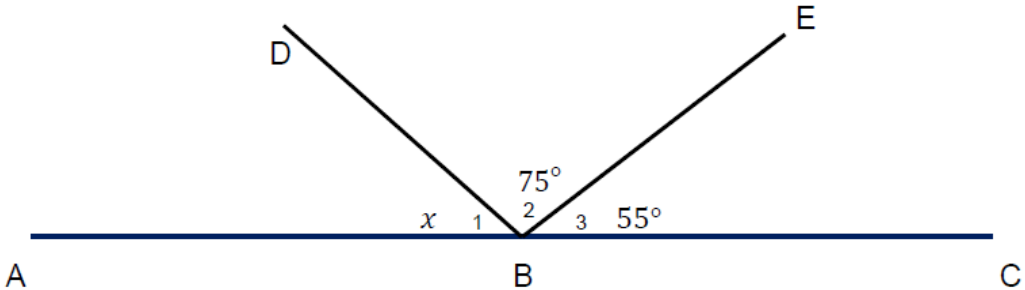
6.3 Given that $\triangle PQR \parallel \triangle MNO$

Find the lengths of unknown sides, x and y (5)

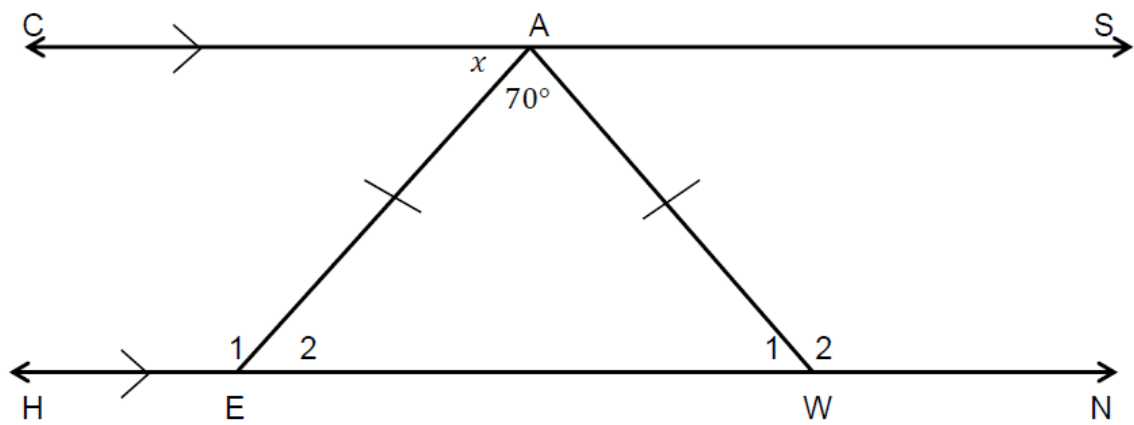


- 6.2.1 Give a reason why $\angle E_2 = x$
(1)
- 6.2.2 Hence, determine the value of x
(3)

6.1 In the figure below, ABC is a straight line $\angle B_2 = 75^\circ$ and $\angle B_3 = 55^\circ$.
Determine, with reasons the size of x . (2)

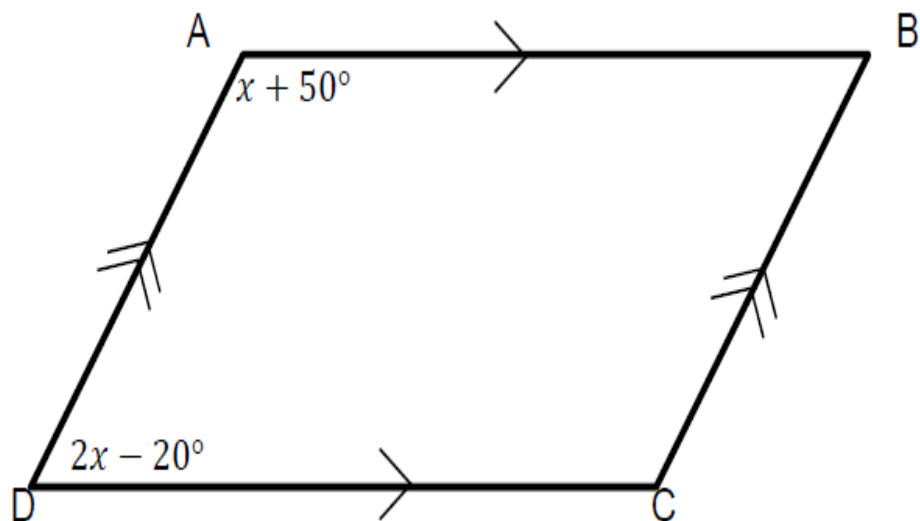


6.2 In the figure below, $CS \parallel HN$. $\angle EAW = 70^\circ$,
 $AE = AW$ and $\angle CAE = x$



CONTINUED.....

6.3 In the figure below, ABCD is a parallelogram.
Calculate the size of x . (4)

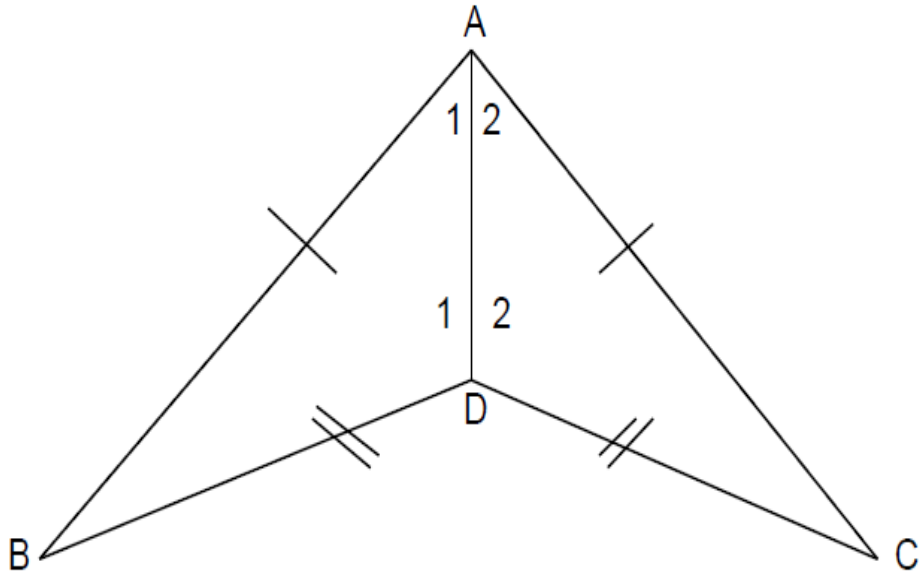


CONTINUED.....

6.4 In the figure below, $AB = AC$ and $BD = CD$.

6.4.1 Prove that $\triangle ABD \equiv \triangle ACD$ (4)

6.4.2 Hence, prove that DA bisects $\angle BAC$ (1)

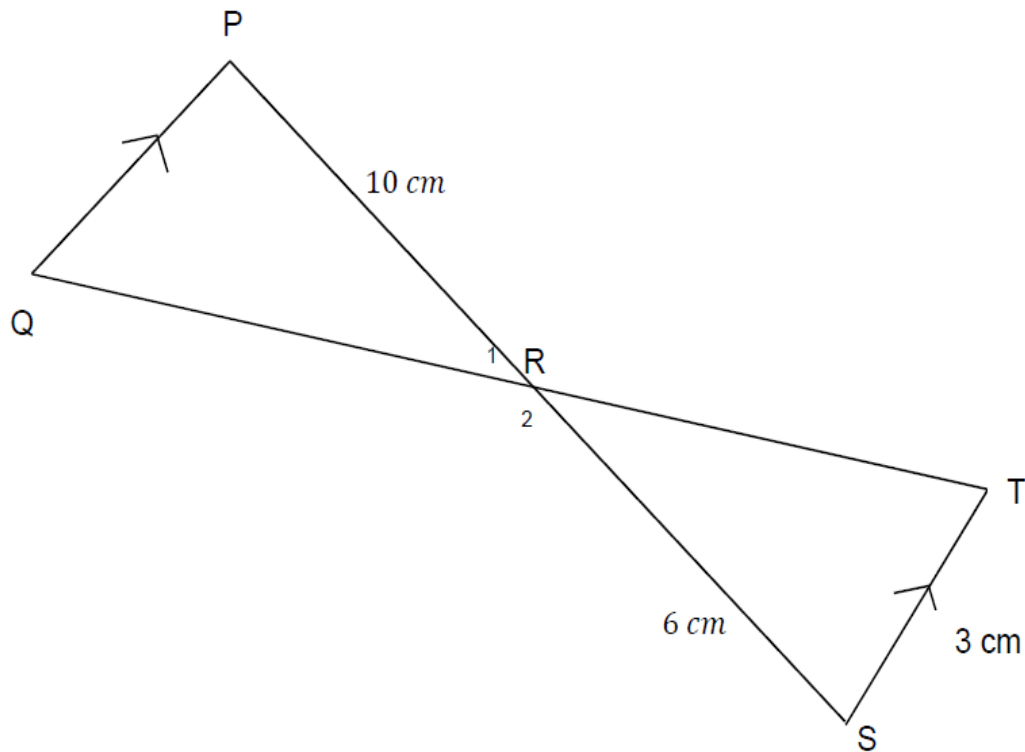


CONTINUED.....

6.5 In $\triangle PQR$ and $\triangle STR$, $PQ \parallel ST$, $PR = 10\text{cm}$, $ST = 3\text{cm}$ and $SR = 6\text{cm}$

6.5.1 Show that $\triangle PQR \sim \triangle STR$ (4)

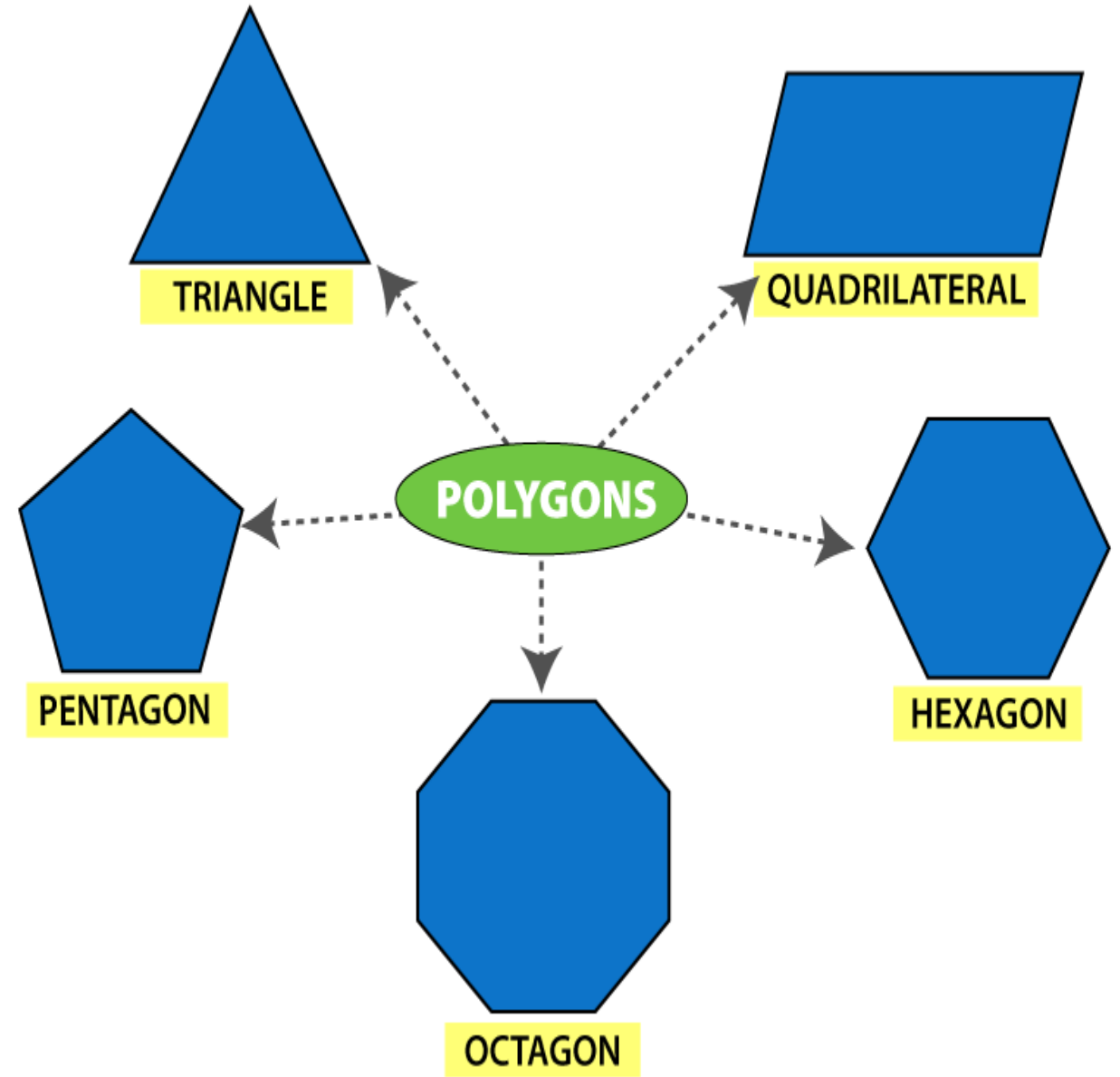
6.5.2 Calculate the length of PQ (3)



EUCLIDEAN GEOMETRY PART TWO

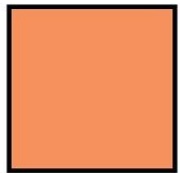
Polygons

- A polygon is a closed, two-dimensional (flat) shape formed by three or more straight, connecting line segments.
- Polygons cannot have curved sides or openings, and they must lie on a flat plane.
- Common examples include triangles, squares, pentagons, and hexagons



Quadrilaterals

- A quadrilateral is a polygon with four straight sides, four vertices (corners), and four interior angles that sum to 360°
- The name derives from "quad" (four) and "lateral" (side), defining any four-sided shape.
- Common examples include squares, rectangles, parallelograms, rhombuses, trapezoids, and kites



Square



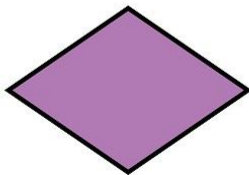
Rectangle



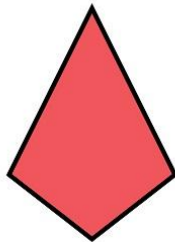
Parallelogram



Trapezoid

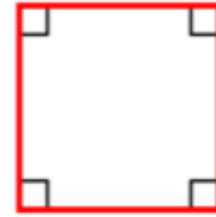


Rhombus



Kite

Quadrilaterals



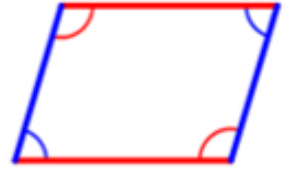
square

All sides equal
All angles 90°



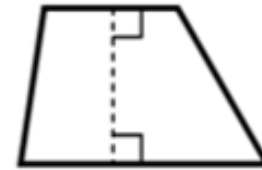
rectangle

Opposite sides equal
All angles 90°



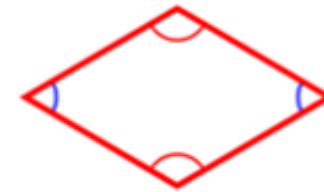
parallelogram

Opposite sides equal
and parallel



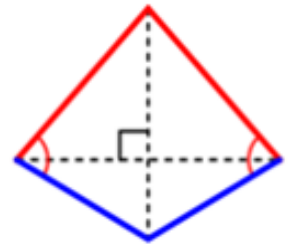
trapezoid (US)
trapezium (UK)

At least 1 pair of
parallel sides



rhombus

All sides equal
Opposite sides parallel

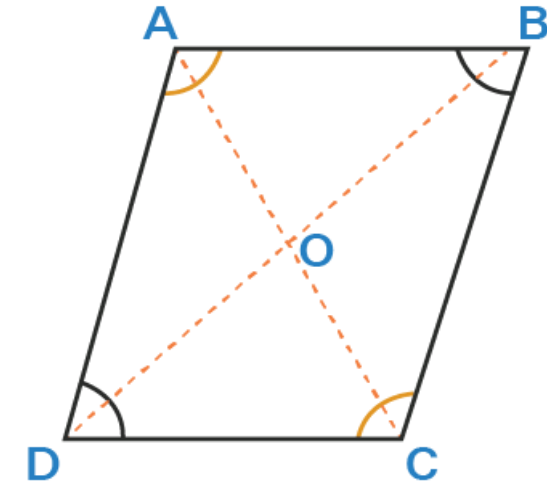
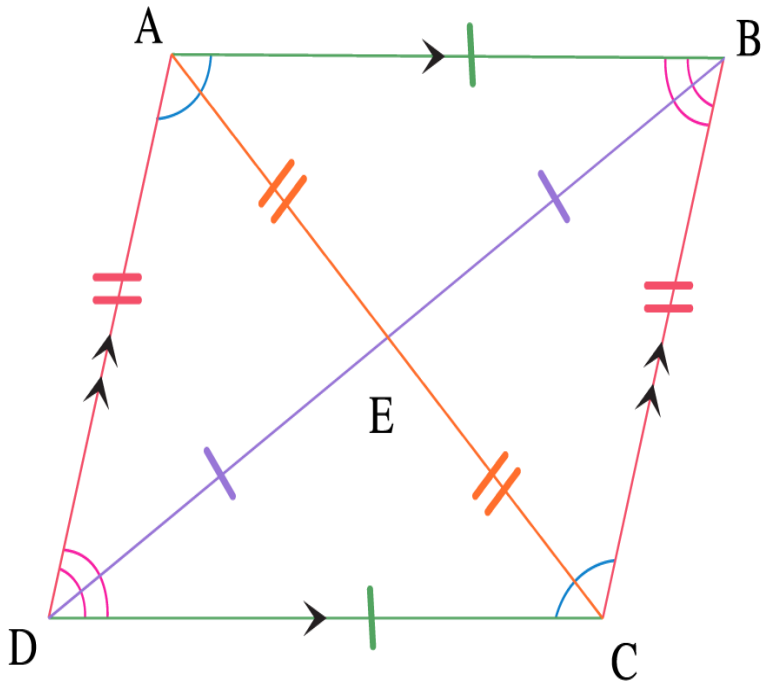


kite

Adjacent pairs of
sides equal

Properties of a parallelogram

- ❖ The diagonals of a parallelogram bisect each other.
- ❖ The opposite angles of a parallelogram are equal.
- ❖ The interior angles add up to 360°
- ❖ The opposite sides of a parallelogram are parallel and equal.



Sides

$$AB = CD$$

$$BC = AD$$

Diagonals

$$AO = OC$$

$$DO = OB$$

Angles

$$\angle A = \angle C$$

$$\angle B = \angle D$$

$$\angle A + \angle D = 180^\circ$$

$$\angle A + \angle B = 180^\circ$$

$$\angle B + \angle C = 180^\circ$$

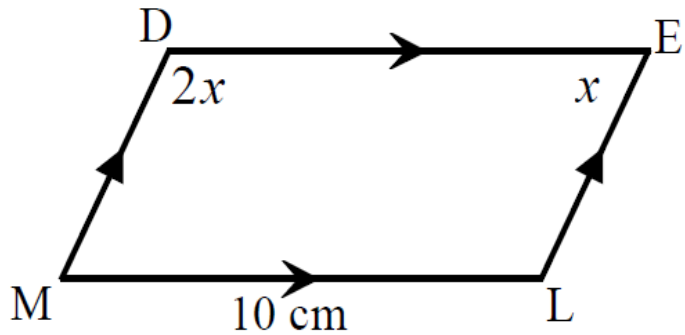
$$\angle C + \angle D = 180^\circ$$

Example 9

DELM is a parallelogram.

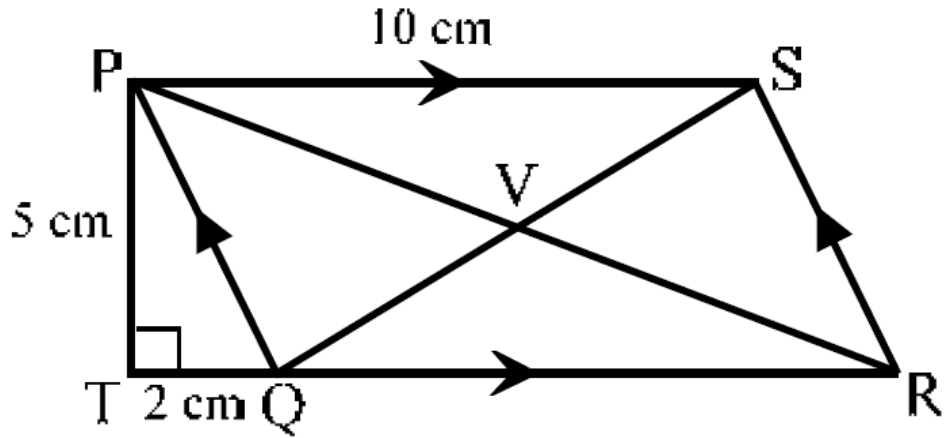
(a) Calculate the value of x and hence the sizes of the interior angles.

(b) If $DE = 2DM$ and $ML = 10\text{ cm}$, determine the length of the other sides of DELM.



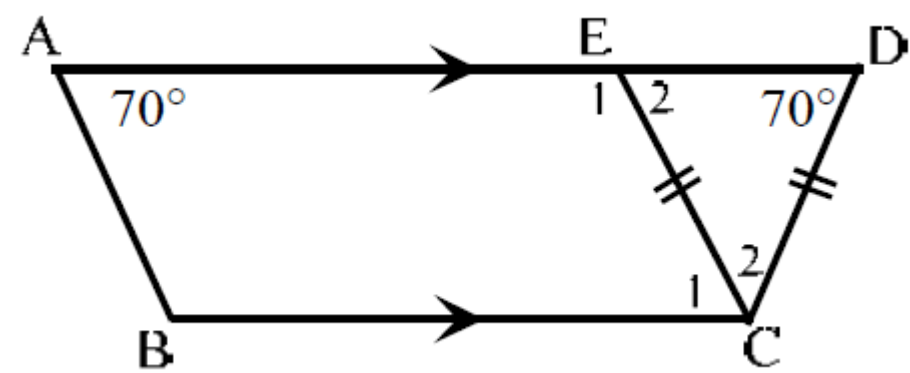
Example 10

In the diagram, PQRS is a parallelogram. $PT \perp TQR$,
 $PT = 5\text{ cm}$, $TQ = 2\text{ cm}$ and $PS = 10\text{ cm}$.
Calculate the length of VR.

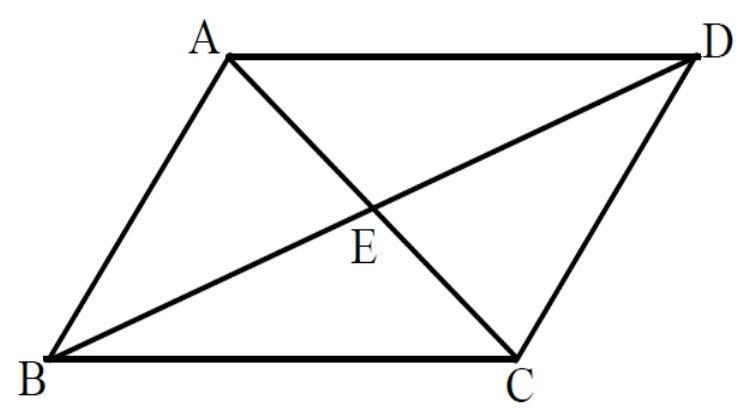


Example 11

In trapezium ABCD, $AD \parallel BC$ with $\angle A = \angle D = 70^\circ$ and $EC = DC$.
Prove that ABCE is a parallelogram.



Note: In order to prove that a quadrilateral is a parallelogram, you will need to prove at least one of the following:



- $AD \parallel BC$ and $AB \parallel DC$
 - $AD = BC$ and $AB = DC$
 - $AE = EC$ and $BE = ED$
 - $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$
 - $AB \parallel DC$ and $AB = DC$
 - $AD \parallel BC$ and $AD = BC$
- Opp sides $\parallel$
Opp sides =
Diagonals bisect
Opp angles =
One pair opp sides = and $\parallel$
One pair opp sides = and $\parallel$

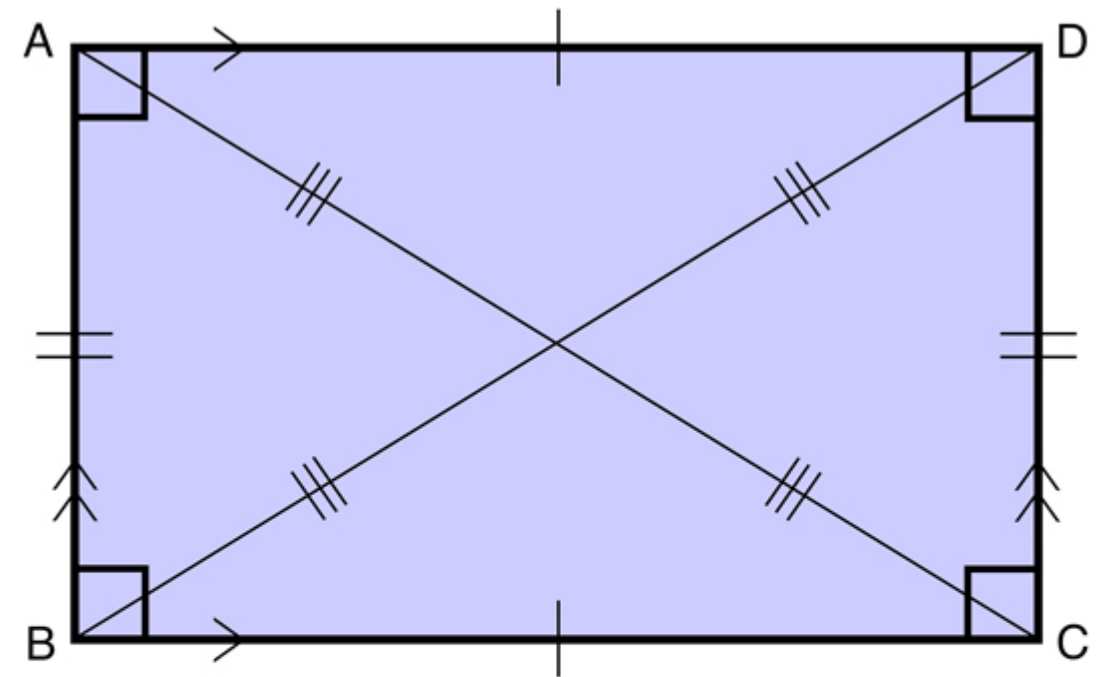
Properties of a rectangle

- The diagonals of a rectangle bisect each other and are equal in length.
- The interior angles of a rectangle are equal to 90° .
- The interior angles add up to 360°
- The opposite sides of a rectangle are parallel and equal.

Note:

In order to prove that a quadrilateral is a **rectangle**, you will need to prove one of the following:

- (a) The quadrilateral is a parallelogram with at least one interior angle equal to 90° .
- (b) The diagonals of the quadrilateral are equal in length and bisect each other.



① Has four sides and four vertices

④ All four angles are right angles

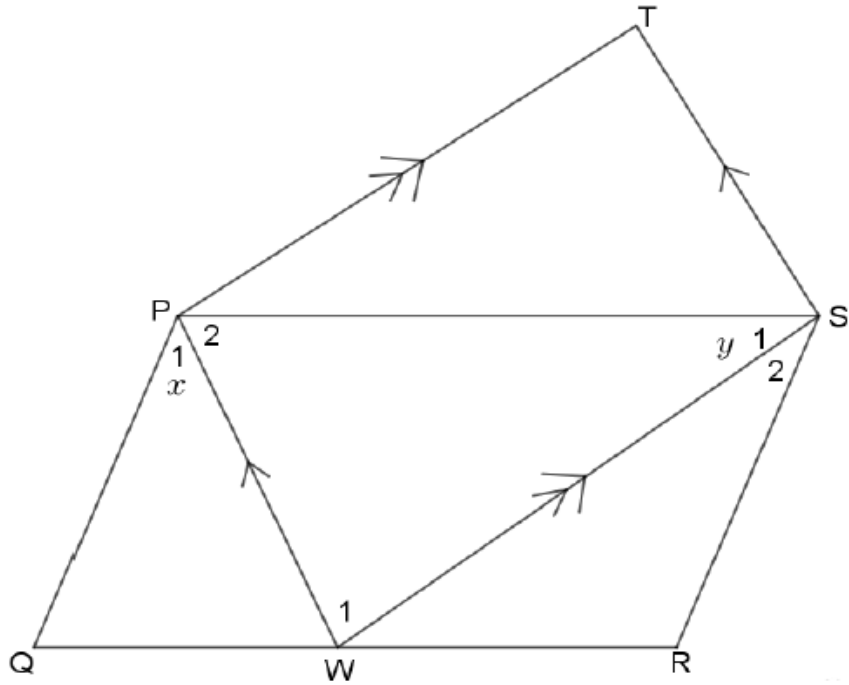
② Opposite sides are equal

⑤ Diagonals are equal and bisect each other

③ Opposite sides are parallel

2022 Gauteng November Paper 2 Q 8

In parallelogram PQRS, PW bisects $\angle QPS$ and SW bisects $\angle PSR$. PW $\parallel$ ST and PT $\parallel$ WS.



8.1 Prove that $x + y = 90^\circ$ (4)

8.2 Prove that PWST is a rectangle. (4)

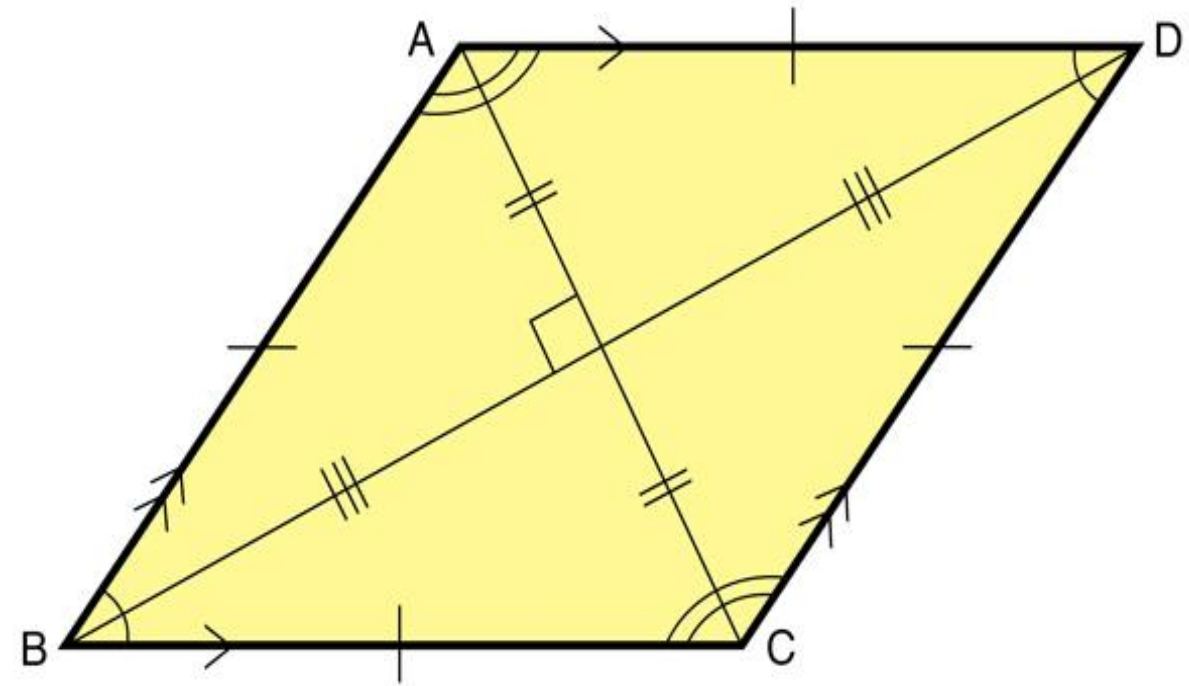
Properties of a rhombus

- The diagonals of a rhombus bisect each other at right angles.
- The diagonals bisect the vertex angles.
- The opposite angles of a rhombus are equal.
- The interior angles add up to 360° .
- The opposite sides of a rhombus are parallel and all sides are equal.

Note:

In order to prove that a quadrilateral is a **rhombus**, you will need to prove one of the following:

- (a) The quadrilateral is a parallelogram with a pair of adjacent sides equal.
- (b) The quadrilateral is a parallelogram in which the diagonals bisect at right angles.



① All four sides are equal

② Opposite sides are parallel

③ Opposite angles are equal

④ Diagonals are perpendicular and bisect each other

⑤ Adjacent angles add up to 180°

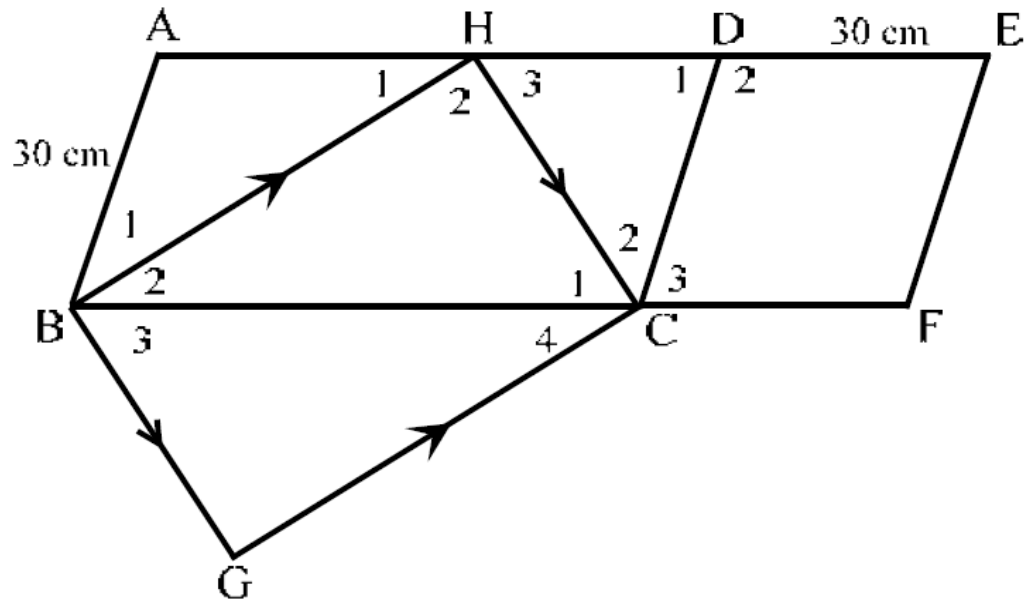
Example 12

ABCD is a parallelogram. BH bisects $\angle ABC$ and HC bisects $\angle BCD$. $\angle ABC = 60^\circ$, $\angle F = 120^\circ$. $BH \parallel GC$ and $BG \parallel HC$. AD is produced to E such that $AB = DE = 30 \text{ cm}$. BC is produced to F.

Prove that:

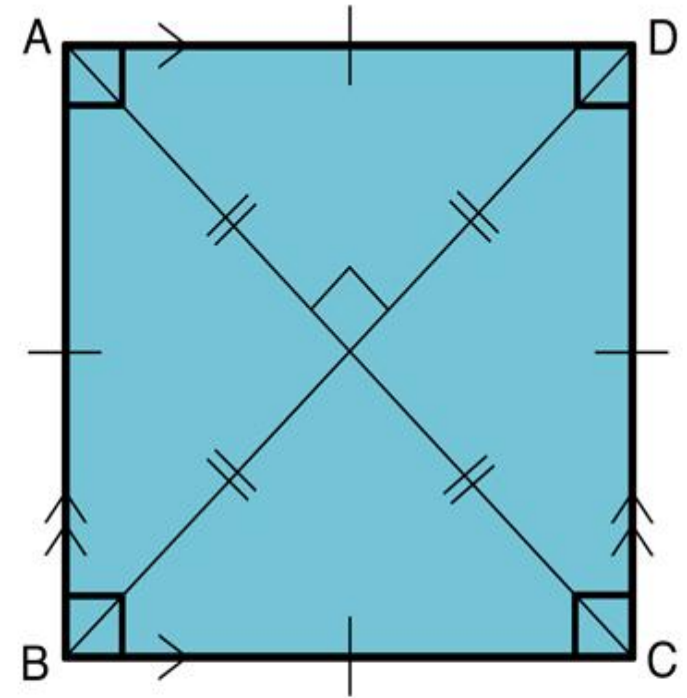
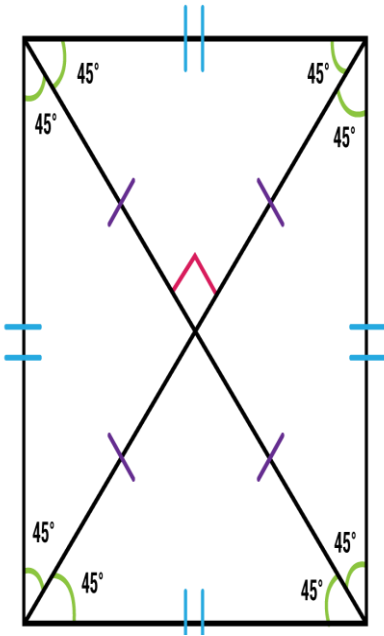
(a) BGCH is a rectangle.

(b) DCFE is a rhombus.



Properties of a square

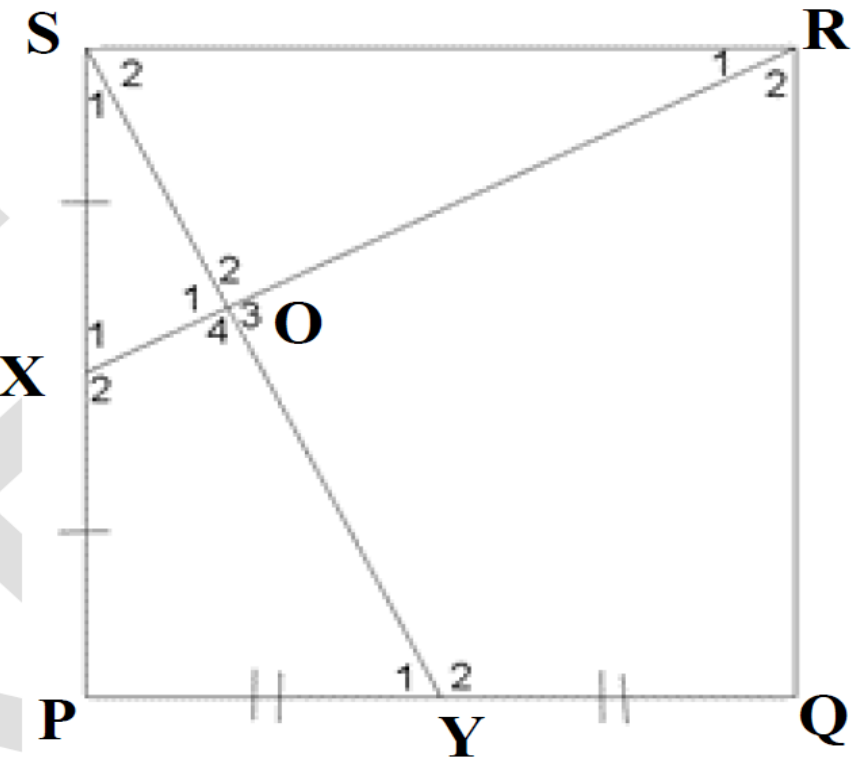
- ❑ The diagonals of a square bisect each other at right angles and are equal in length.
- ❑ The diagonals bisect the vertex angles.
- ❑ The interior angles of a square are equal to 90° .
- ❑ The interior angles add up to 360°
- ❑ The opposite sides of a square are parallel and all sides are equal.



- ① Has four sides and four vertices
- ② All four sides are equal
- ③ All four angles are right angles
- ④ Opposite sides are parallel
- ⑤ Diagonals are perpendicular (bisect at 90°)

2023 Gauteng November Paper 2 Q 9.2

PQRS is a square. X is a midpoint of SP, and Y is the midpoint of PQ.



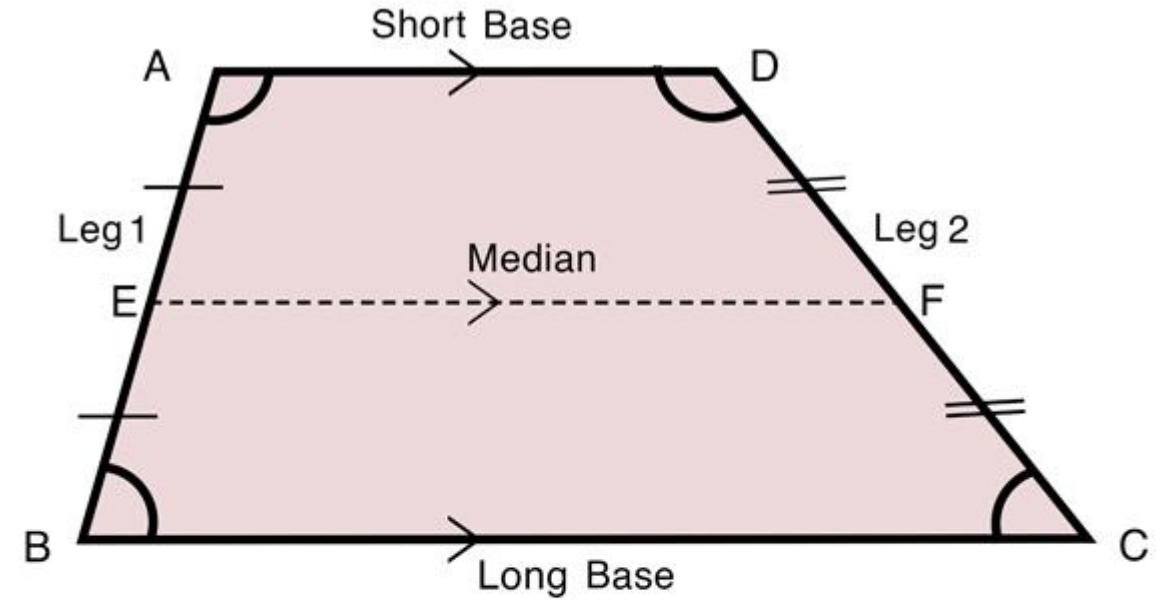
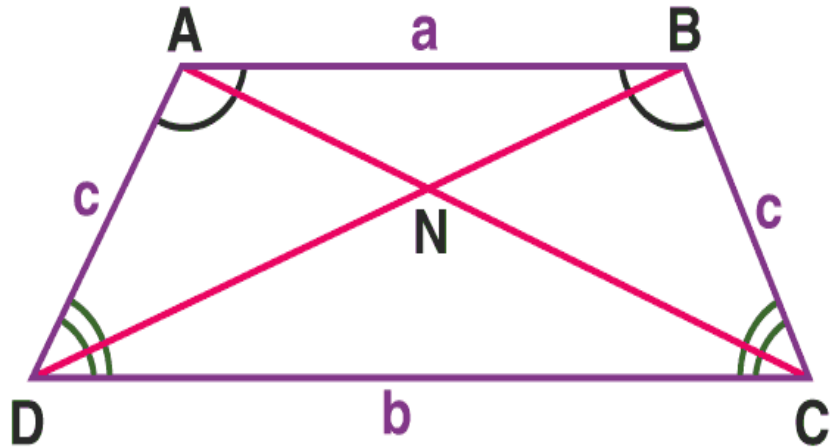
9.2.1 Prove that $\triangle SPYP \equiv \triangle RSX$ (6)

9.2.2 Given that $\angle R_1 = x$. Identify, with reasons, another angle equal to x (2)

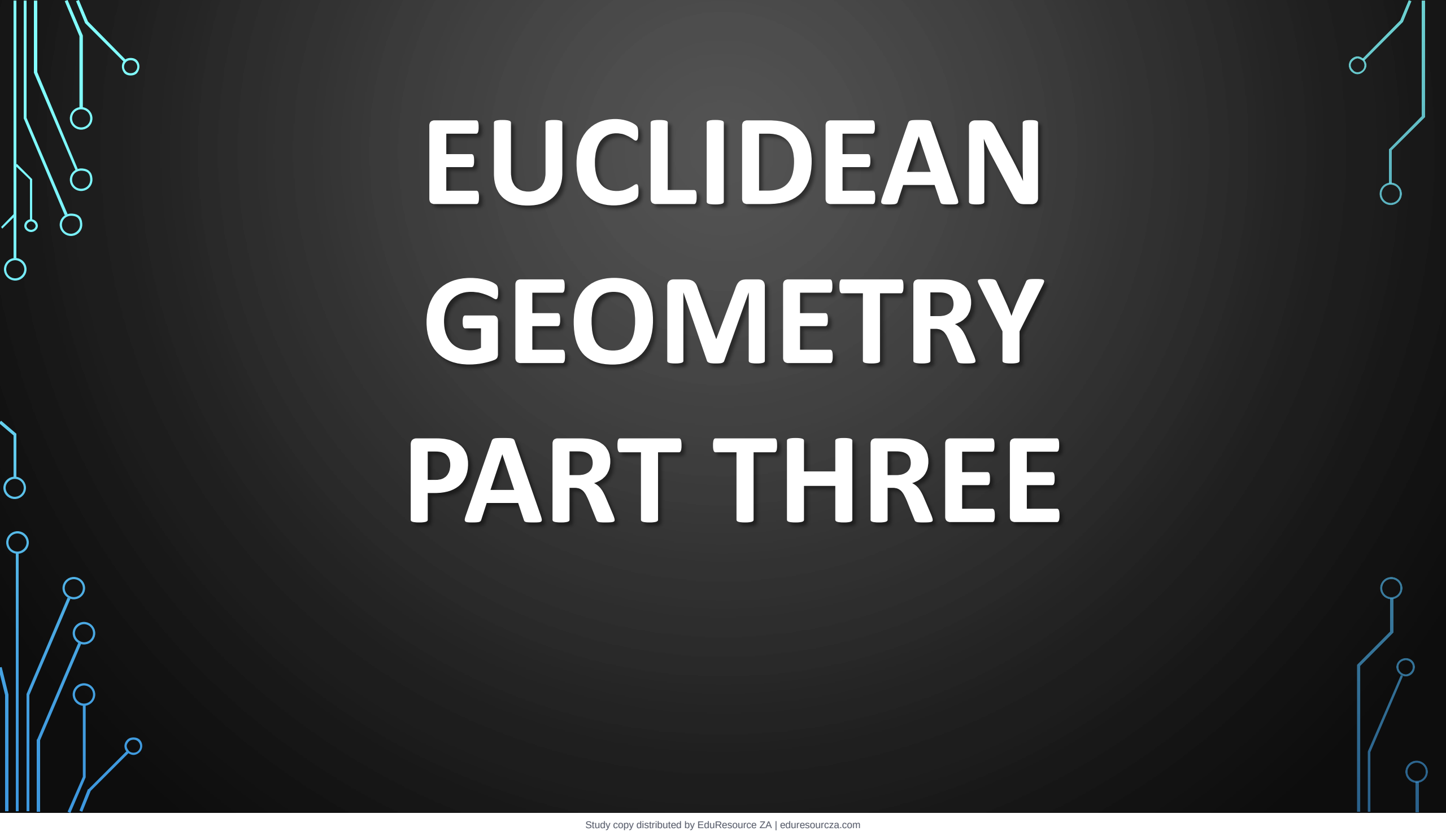
9.2.3 Prove that $SY \perp RX$. (5)

Properties of a trapezium

- The diagonals of a trapezium intersect but **do not bisect each other**.
- They lie between parallel lines and therefore the alternate angles are equal.
- The interior angles add up to 360°
- One pair of opposite sides are parallel.



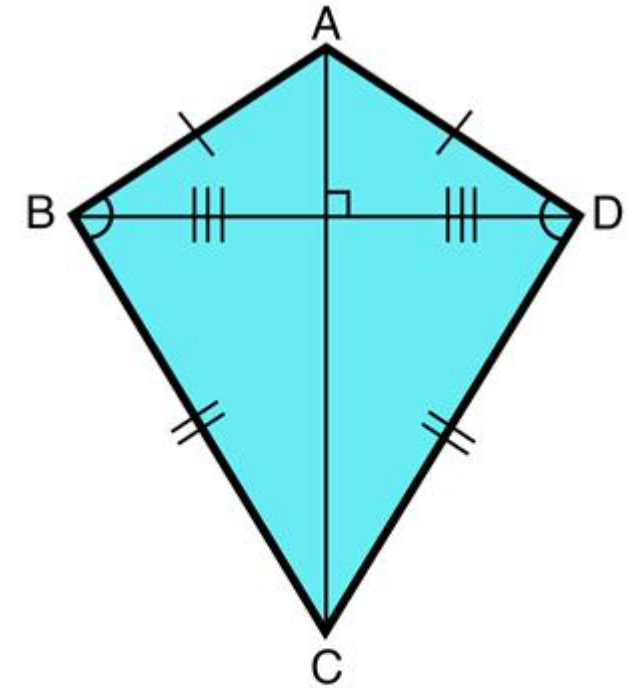
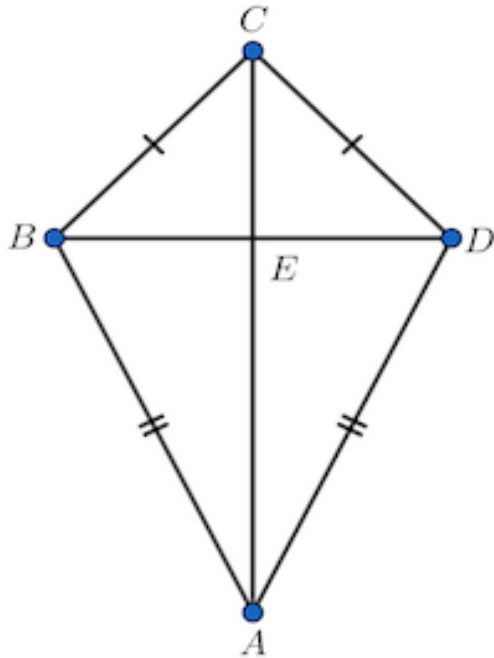
- ① Has four sides and four angles
- ② One pair of opposite sides are parallel (short & long base)
- ③ Adjacent angles add up to 180°
- ④ The median is parallel to the bases, & divides the non-parallel sides (the legs) equally

The background is a dark grey gradient. In the four corners, there are decorative elements consisting of light blue lines and small circles, resembling a circuit board or a stylized network diagram. The lines are of varying lengths and angles, connecting the circles in a non-uniform pattern.

EUCLIDEAN GEOMETRY PART THREE

Properties of a kite

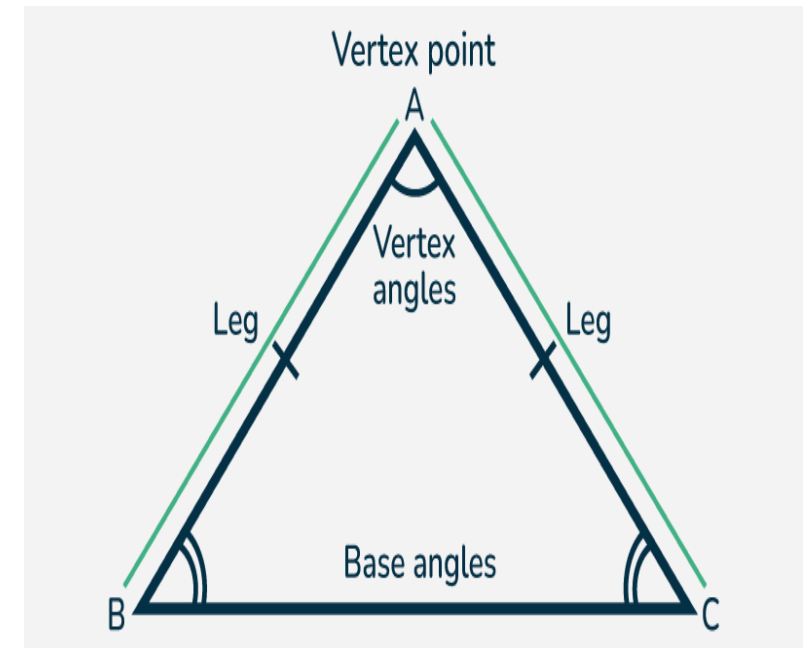
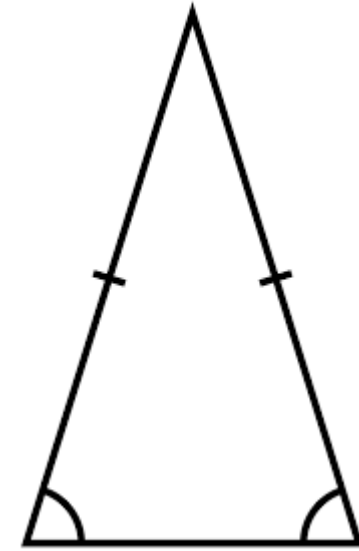
- The diagonals are perpendicular and one diagonal bisects the other.
- One of the diagonals bisects the vertex angles.
- One pair of opposite angles are equal.
- The interior angles add up to 360°
- Two pairs of adjacent sides are equal.



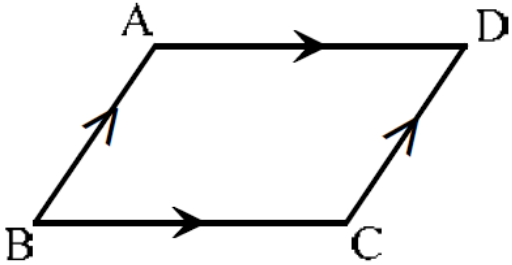
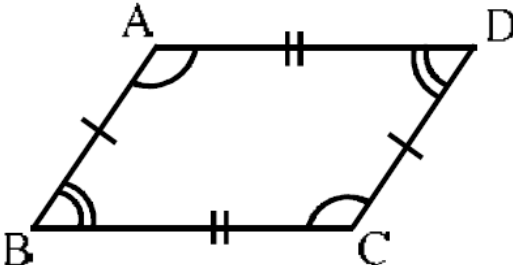
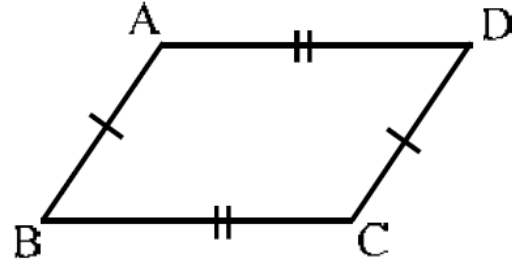
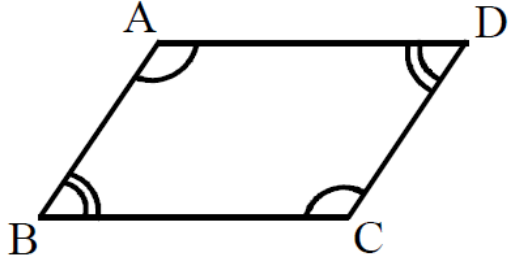
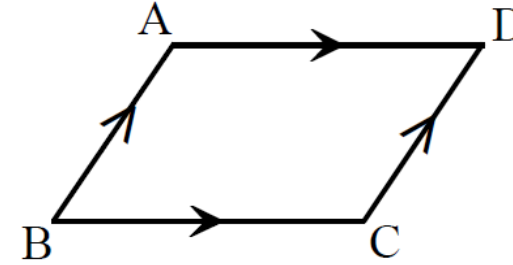
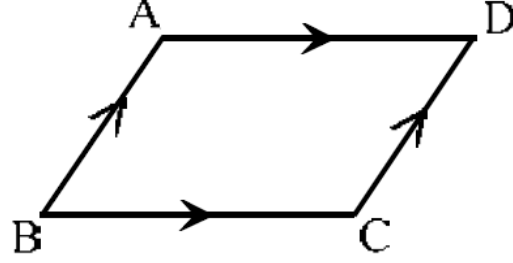
- ① Has two pairs of adjacent equal sides
- ② One pair of opposite angle is equal
- ③ Diagonals are perpendicular to each other with the longer diagonal bisecting the shorter one

The concept of a theorem and its converse

- **A theorem is a statement that can be demonstrated to be true by accepted mathematical operations and arguments.**
- In general, a theorem is an embodiment of some general principle that makes it part of a larger theory.
- The process of showing a theorem to be correct is called a **proof**.
- In Grade 10, we introduce the concept of a theorem and its proof.
- **A converse of a theorem is a statement formed by interchanging what is given in a theorem and what is to be proved.**
- For example, the isosceles triangle theorem states that if two sides of a triangle are equal then the two base angles are equal.
- In the converse, the given (that two sides are equal) and what is to be proved (that two angles are equal) are swapped
- The converse is the statement that if two angles of a triangle are equal then two sides are equal.



- In Grade 10, we will discuss and then prove three theorems relating to parallelograms

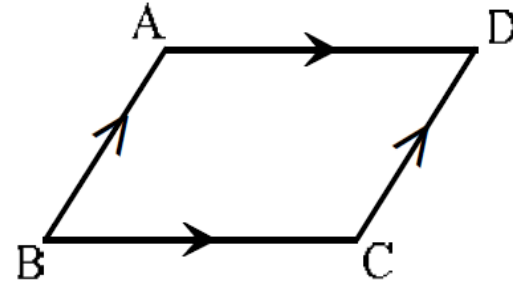
Statement of theorem	Diagrams	
<u>Theorem 1</u> The opposite sides and angles of a parallelogram are equal. (opp sides of parm equal) (opp $\angle$s of parm equal)	If ABCD is a parallelogram (AD$\parallel$BC and AB$\parallel$DC) 	then AD = BC , AB = DC , $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$ 
<u>Theorem 1 (Converse)</u> (a) If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram. (opp sides of quad equal) (b) If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram. (opp $\angle$s of quad equal)	If AD = BC and AB = DC  If $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$ 	then ABCD is a parm  then ABCD is a parm 

Theorem 2

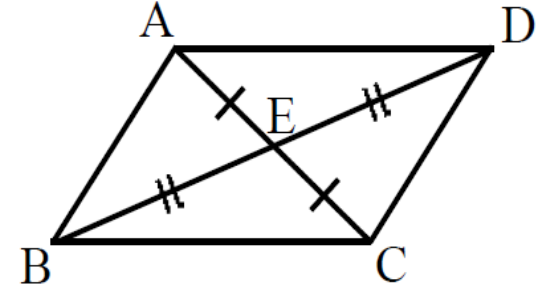
The diagonals of a parallelogram bisect each other.

(diags of parm bisect)

If ABCD is a parallelogram



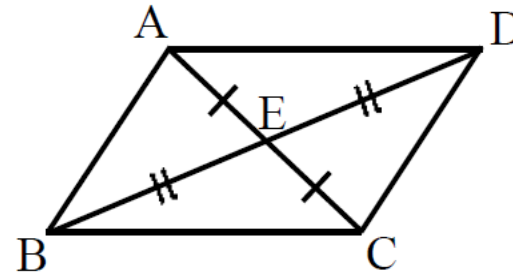
then $AE = EC$ and $BE = ED$

**Theorem 2 (Converse)**

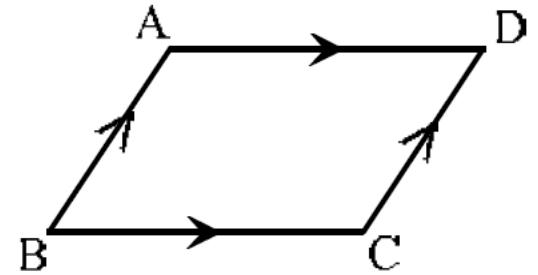
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

(diags of quad bisect)

If $AE = EC$ and $BE = ED$



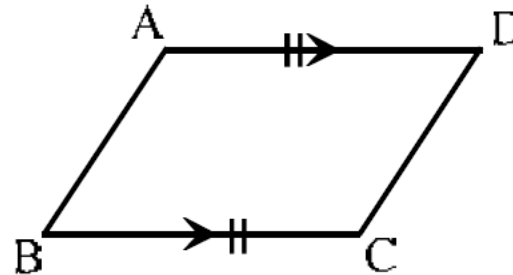
then ABCD is a parm

**Theorem 3**

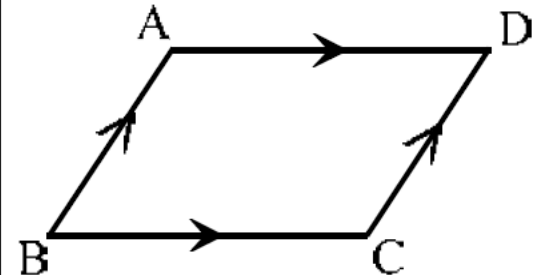
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

(one pair opp sides equal and $\parallel$)

If $AD \parallel BC$ and $AD = BC$



then ABCD is a parm



THEOREM 1

The opposite sides and angles of a parallelogram are equal.

Required to prove: $AB = CD$; $AD = BC$; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.

In $\triangle ABC$ and $\triangle CDA$:

- (a) $\hat{A}_2 = \hat{C}_1$ alt $\angle$ s ; $AB \parallel DC$
- (b) $\hat{A}_1 = \hat{C}_2$ alt $\angle$ s ; $AD \parallel BC$
- (c) $AC = AC$ common side

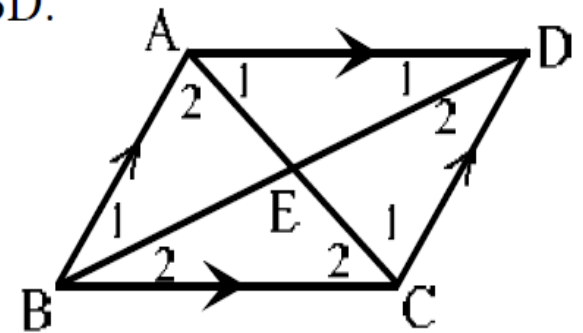
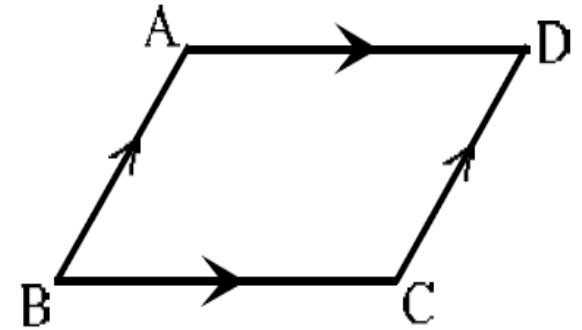
$\therefore \triangle ABC \equiv \triangle CDA$ SAA

$\therefore AB = CD$ and $AD = BC$

Also $\hat{B} = \hat{D}$

Similarly, it can be proved that $\triangle ABD \equiv \triangle CDB$

$\therefore \hat{A} = \hat{C}$



THEOREM 2

The diagonals of a parallelogram bisect each other.

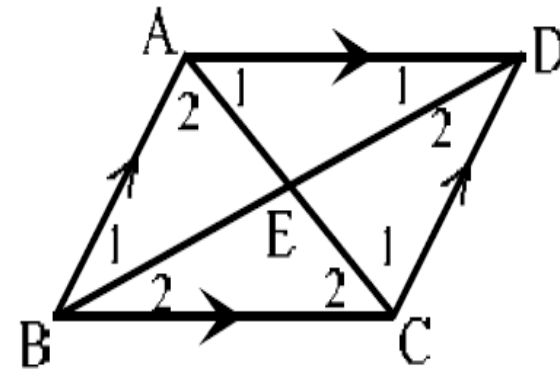
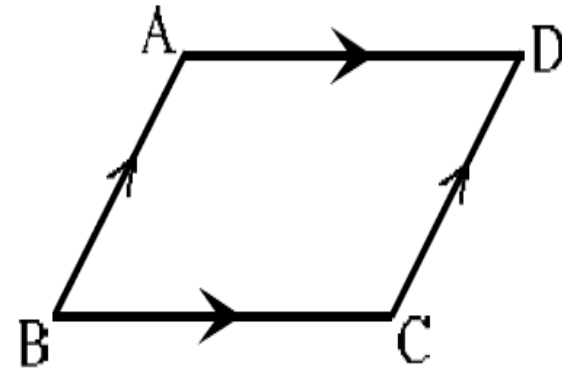
Required to prove: $AE = EC$ and $BE = ED$

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.

In $\triangle ABE$ and $\triangle CDE$:

- (a) $\hat{A}_2 = \hat{C}_1$ alt $\angle$ s ; $AB \parallel DC$
 - (b) $\hat{B}_1 = \hat{D}_2$ alt $\angle$ s ; $AB \parallel DC$
 - (c) $AB = DC$ opp sides of a parm
- $\therefore \triangle ABE \equiv \triangle CDE$ SAA
- $\therefore BE = ED$ and $AE = EC$



THEOREM 3

If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

Required to prove: ABCD is a parallelogram

Proof:

In $\triangle ABC$ and $\triangle CDA$:

(a) $\hat{A}_1 = \hat{C}_2$ alt $\angle$ s; $AD \parallel BC$

(b) $AC = AC$ common side

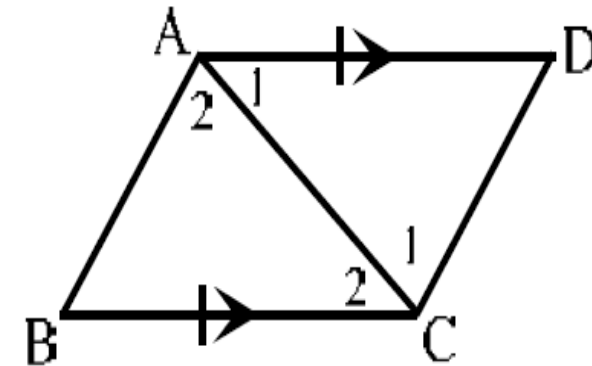
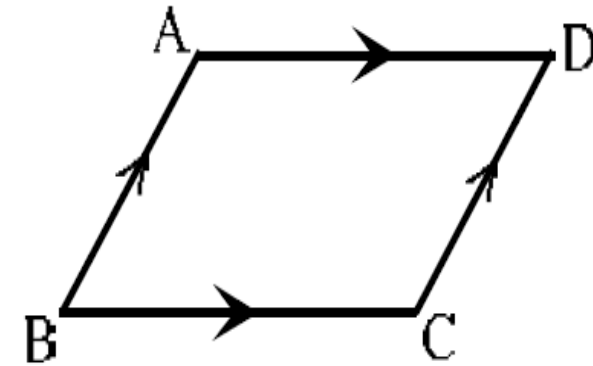
(c) $AD = BC$ given

$\therefore \triangle ABC \equiv \triangle CDA$ SAS

$\therefore \hat{A}_2 = \hat{C}_1$

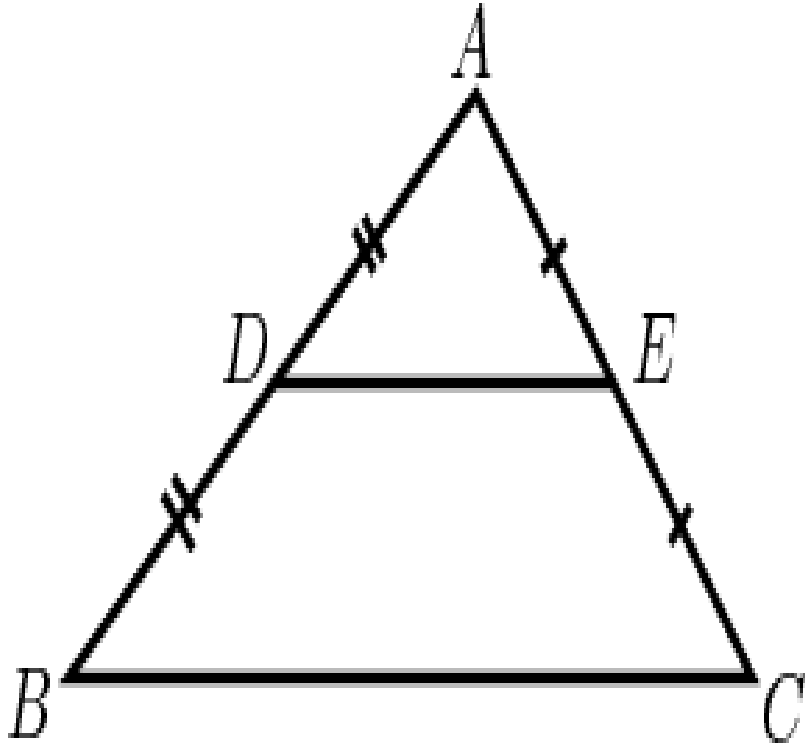
$\therefore AB \parallel DC$ alt $\angle$ s =

$\therefore ABCD$ is a parallelogram since the opposite sides are parallel



Mid-point Theorem in a Triangle

- The line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half the length of the third side.

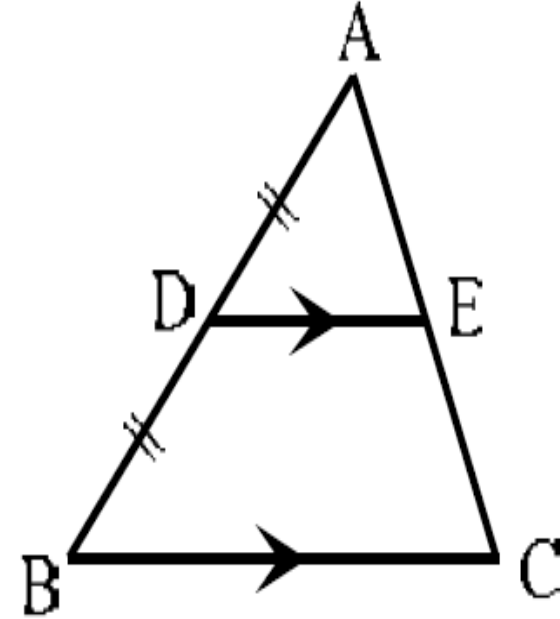


Given that $DE \parallel BC$, D as the midpoint of AB and E as the midpoint of AC then:

$$DE = \frac{1}{2}BC$$

Converse:

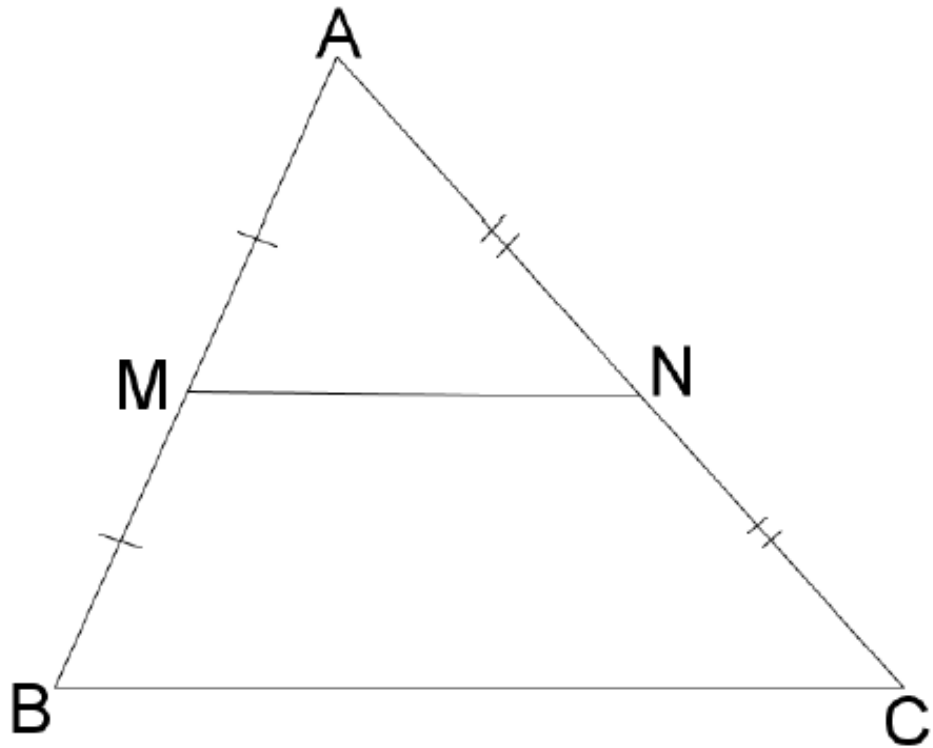
- The line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side



If $AD = DB$ and $DE \parallel BC$, then $AE = EC$ and $DE = \frac{1}{2}BC$

2021 Gauteng November Paper 2 Q 8.1

8.1 In $\triangle ABC$, **M** and **N** are the midpoints of **AB** and **AC**, respectively.

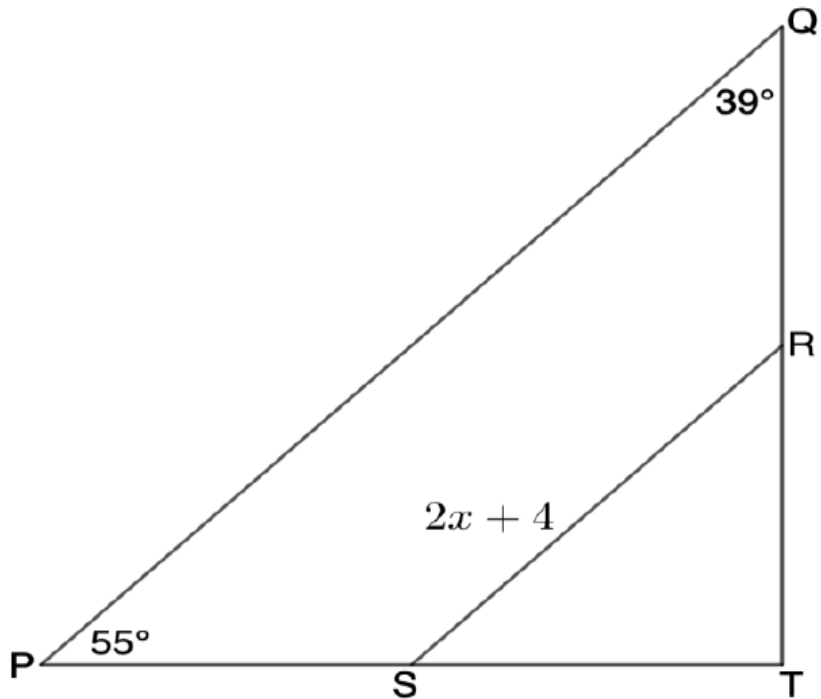


Prove the theorem that states that $MN \parallel BC$. (6)

PAST EXAM PAPERS

2024 North West November Paper 2 Q 9

In the diagram below, R is the midpoint of QT and S is the midpoint of PT. The length of SR is $2x + 4$. and $\angle Q = 39^\circ$ and $\angle P = 55^\circ$

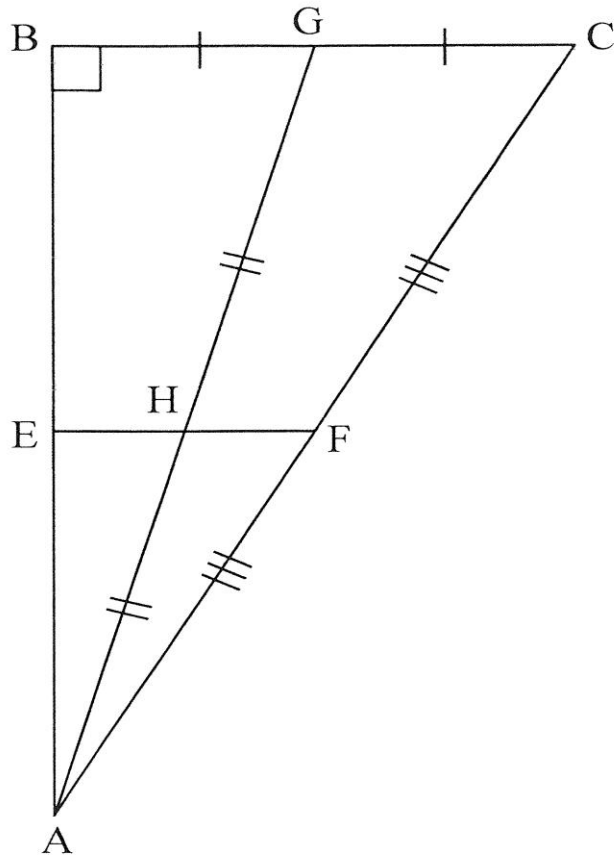


- 9.1 Determine the length of PQ in terms of x . (2)
- 9.2 If the length of PQ is 18, determine the value of x . (3)
- 9.3 Prove that $\triangle TRS \parallel \triangle TQP$ (5)

2017 November Paper 2 Q 8.39.3 Hence, calculate the area of $\triangle ABC$

(3)

$\triangle ABC$ is right angled at B. F and G are the midpoints of AC and BC respectively. H is the midpoint of AG. E lies on AB such that FHE is a straight line.

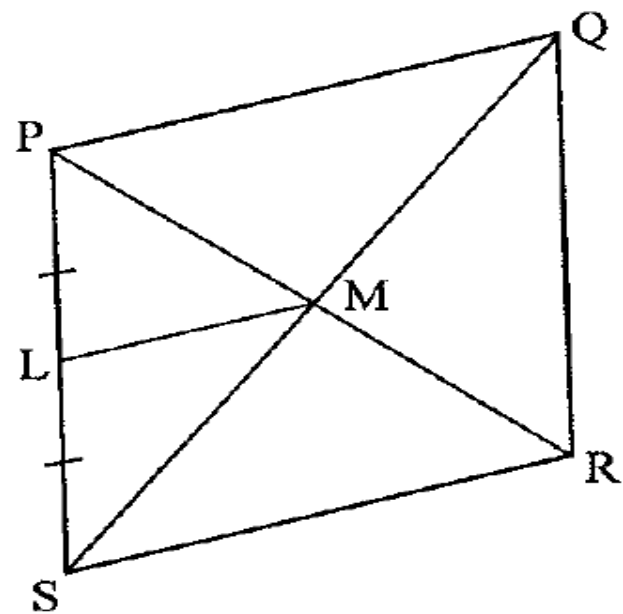


9.1 Prove that E is the midpoint of AB (3)

9.2 If $EH = 3,5$ cm and the area of $\triangle AEH = 9,5\text{cm}^2$, calculate the length of AB (3)

2019 November Paper 2 Q 7.2

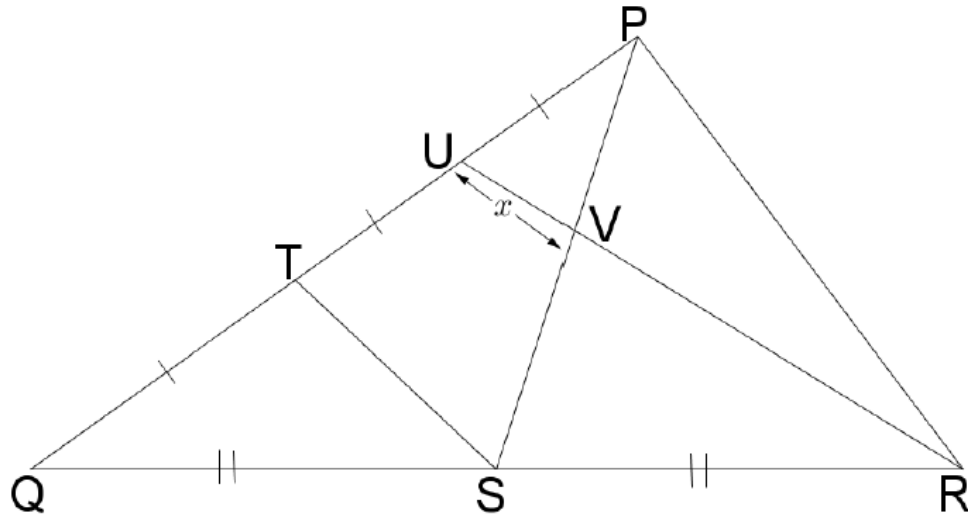
PQRS is a rhombus with diagonals PR and SQ intersecting at M.
The perimeter of the rhombus is 12 cm.
L is the midpoint of PS.



Calculate the length of LM. (4)

2021 Gauteng November Paper 2 Q 8.2

In $\triangle PQR$, $QT = TU = UP$. $QS = SR$ and $UV = x \text{ cm}$.



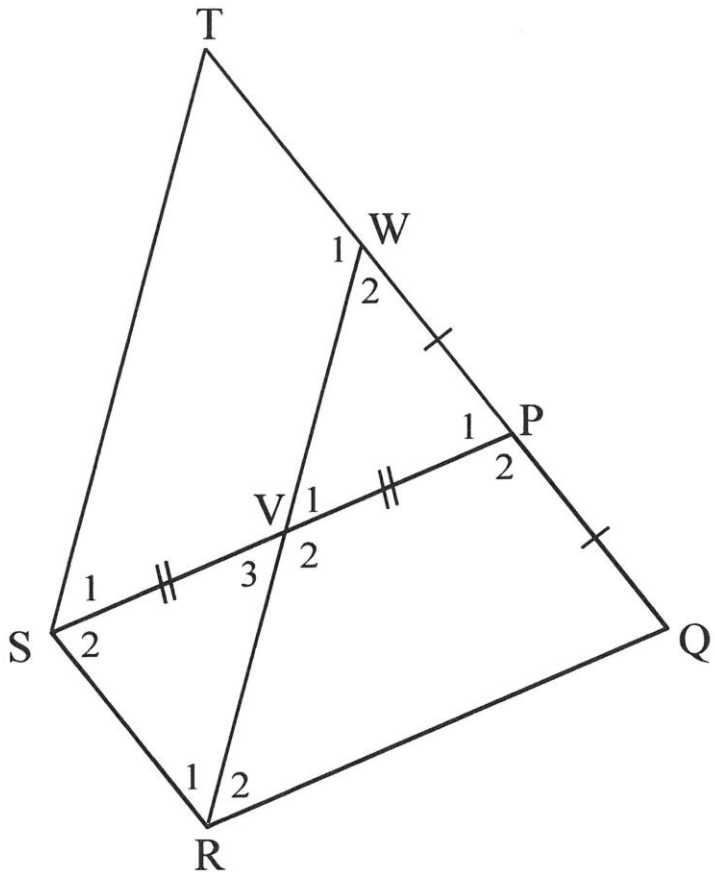
Determine the length of VR in terms of x . (6)

2019 November Paper 2 Q 8.1 and 8.2.2-8.2.3

8.1 Complete the statement so that it is TRUE:

The diagonals of a parallelogram each other (1)

8.2 In the diagram below, P is the midpoint of side WQ of $\triangle WQR$. V is on WR such that $VP \parallel RQ$. PV is produced by its own length to S. PW is produced to T and ST is drawn.



8.2.1 Give a reason why $WV = VR$ (1)

8.2.2 Prove that: (3)

a) $\triangle VWP \equiv \triangle VRS$ (3)

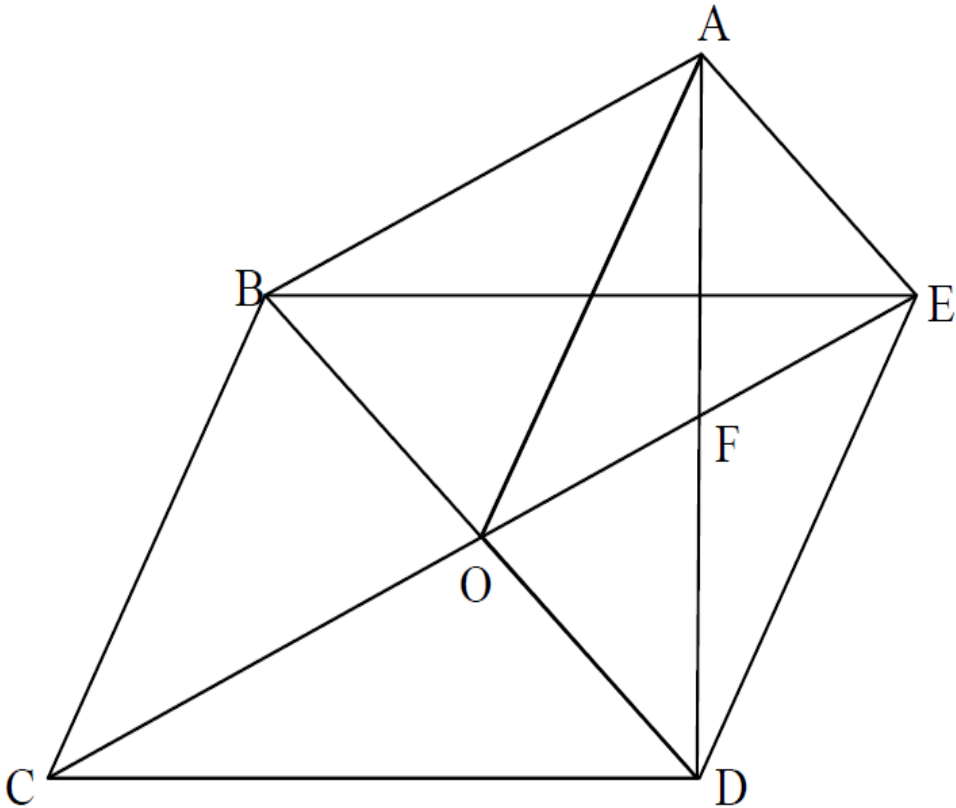
b) SWPR is a parallelogram (2)

c) PQRS is a parallelogram (3)

8.2.3 If it is further given that RSTW is a parallelogram, show that $TQ = 3SR$ (2)

2012 Exemplar Paper 2 Q 9

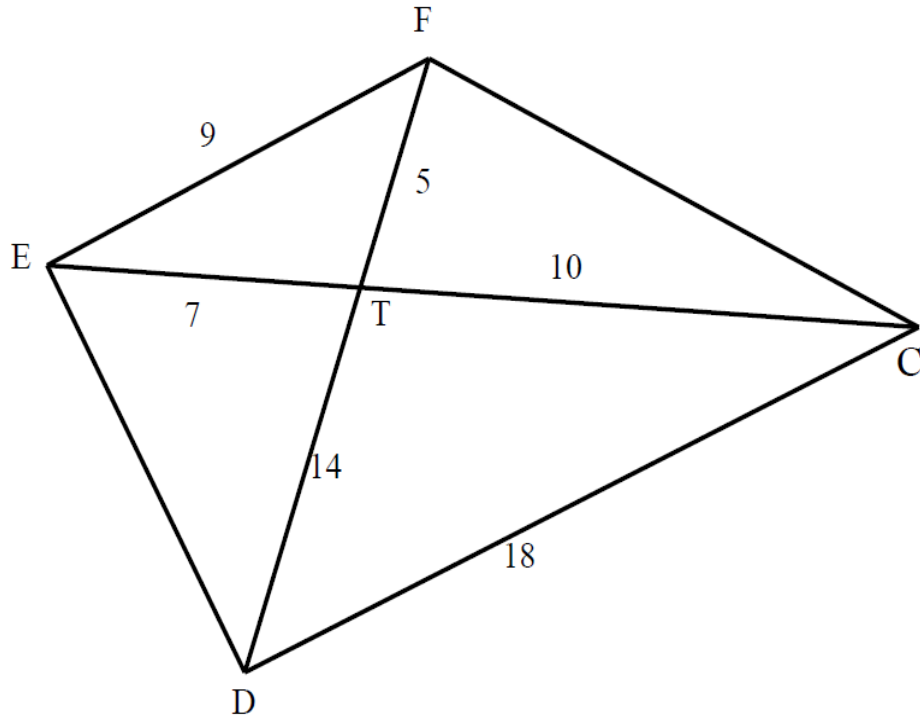
In the diagram, BCDE and AODE are parallelograms.



- 9.1 Prove that $OF \parallel AB$. (4)
- 9.2 Prove that ABOE is a parallelogram. (4)
- 9.3 Prove that $\triangle ABO \cong \triangle EOD$. (5)

2020 Eastern Cape November Paper 2 Q 5.4

5.4 In the diagram below, $FT = 5$ cm, $ET = 7$ cm, $EF = 9$ cm
 $CT = 10$ cm, $DT = 14$ cm and $CD = 18$ cm

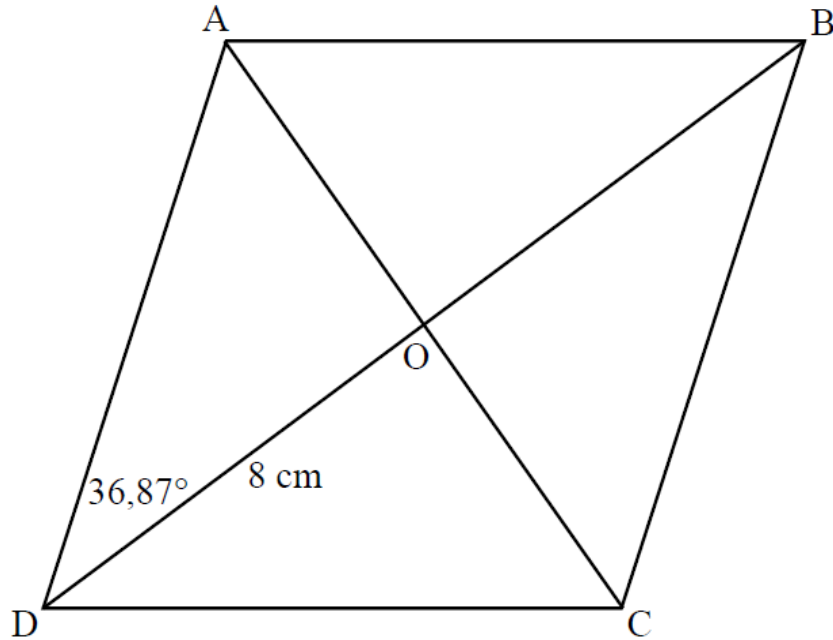


5.4.1 Prove that $\triangle EFT \parallel \triangle DCT$. (3)

5.4.2 If it is further given that $\angle DFC = \angle TDC$,
prove that $\angle FEC = \angle TFC$. (3)

2015 November Paper 2 Q 8

In the diagram, ABCD is a rhombus having diagonals AC and BD intersecting in O. $\angle AOD = 36,87^\circ$ and $DO = 8\text{cm}$



8.1 Write down the sizes of the following angles:

8.1.1 $\angle CDO$ (1)

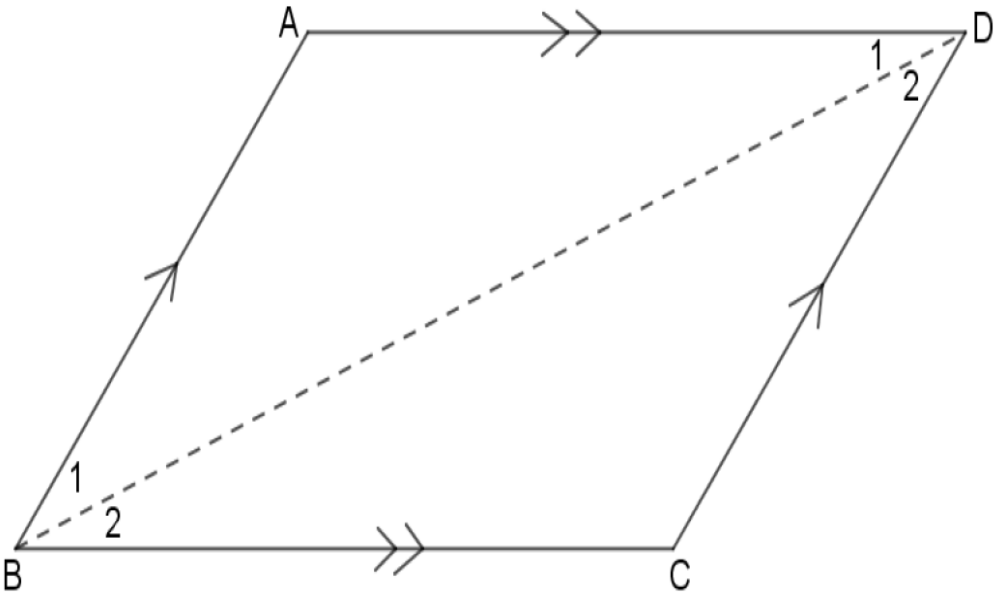
8.1.2 $\angle AOD$ (1)

8.2 Calculate the length of AO (2)

8.3 If E is a point on AB such that $OE \parallel DA$, calculate the length of OE. (4)

2022 Gauteng November Paper 2 Q 9

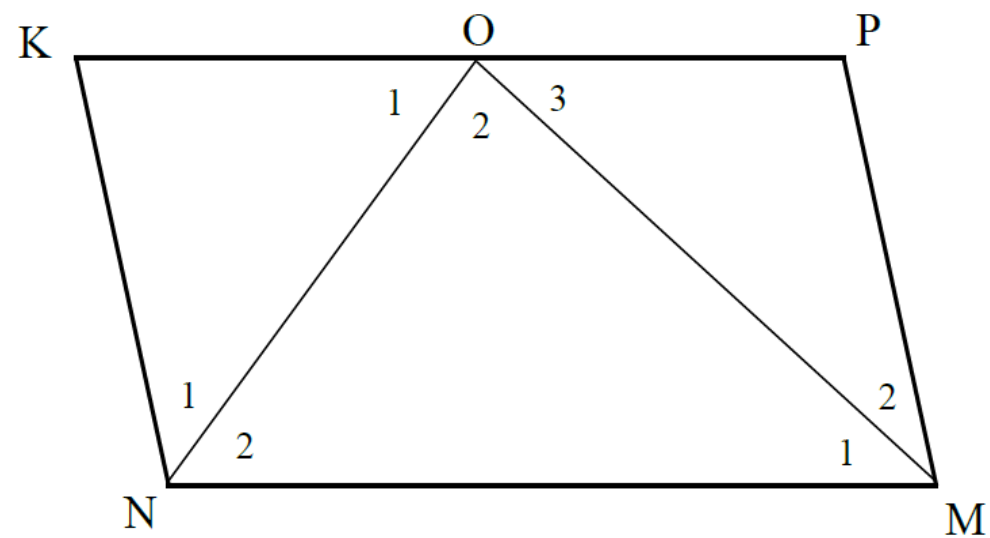
ABCD is a parallelogram with $AD \parallel BC$ and $BA \parallel CD$.



Using the diagram above, prove the theorem that states that the opposite sides of a parallelogram are equal. (5)

2016 November Paper 2 Q 8.3

8.3 In the sketch below, KPMN is a parallelogram. ON bisects $\angle KNM$ and OM bisects $\angle NMP$.



8.3.1 Show that $\angle NOM = 90^\circ$ (3)

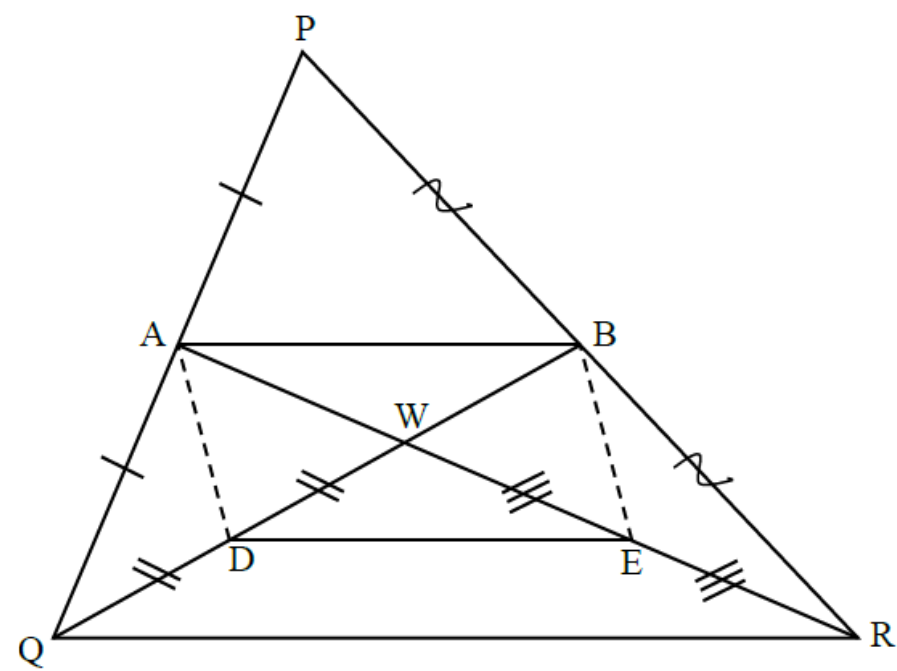
8.3.2 Prove that O is the midpoint of KP. (6)

2016 November Paper 2 Q 9

9.1 Complete the following statement:

The line through the midpoint of two sides in a triangle is parallel to and ... the third side. (1)

9.2 In $\triangle PQR$, A and B are the midpoints of sides PQ and PR respectively. AR and BQ intersect at W. D and E are points on WQ and WR respectively such that $WD = DQ$ and $WE = ER$.



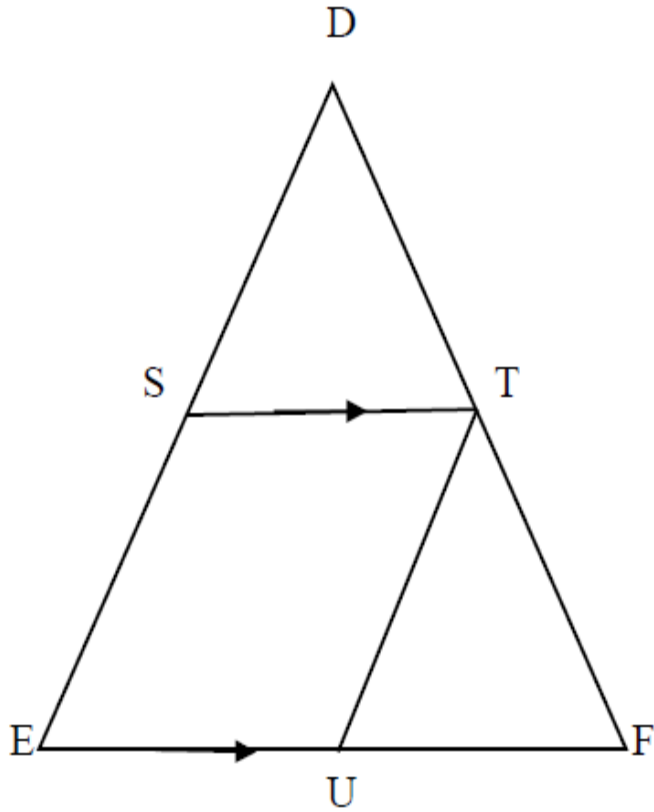
Prove that ADEB is a parallelogram. (5)

2020 Eastern Cape November Paper 2 Q 5.5

5.5.1 Complete the following statement for $\triangle ABC$:

If D is a point on AB and E is a point on AC such that $AD = DB$
and $DE \parallel BC$, then ... (1)

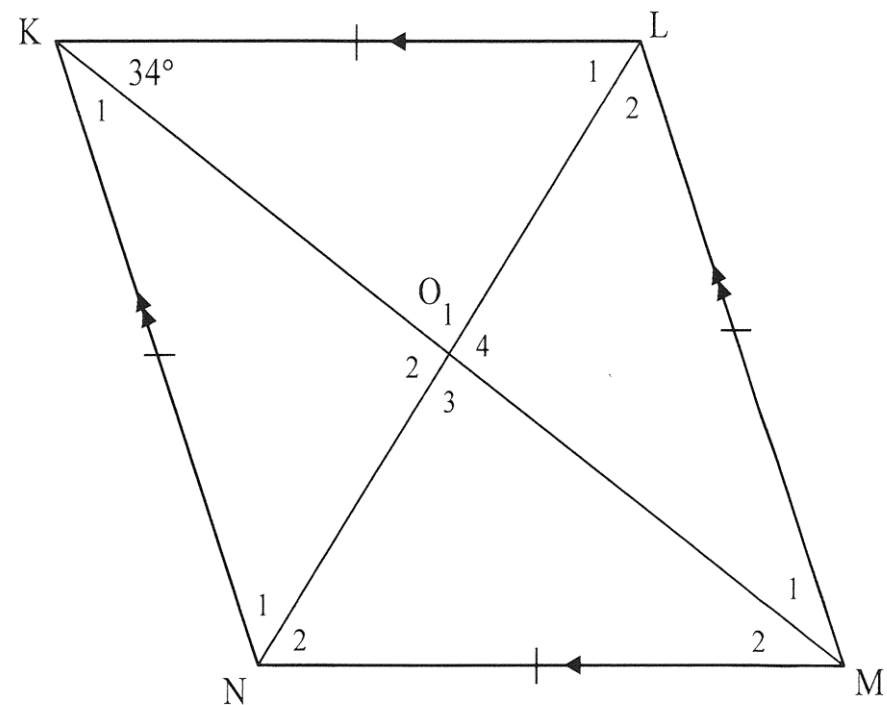
5.5.2 In $\triangle DEF$, $DS = SE$, $EU = UF$ and $ST \parallel EF$.



Prove that SEUT is a parallelogram. (4)

2017 November Paper 2 Q 8.1

KLMN is rhombus with diagonals intersecting at O.
 $\angle LKM = 34^\circ$

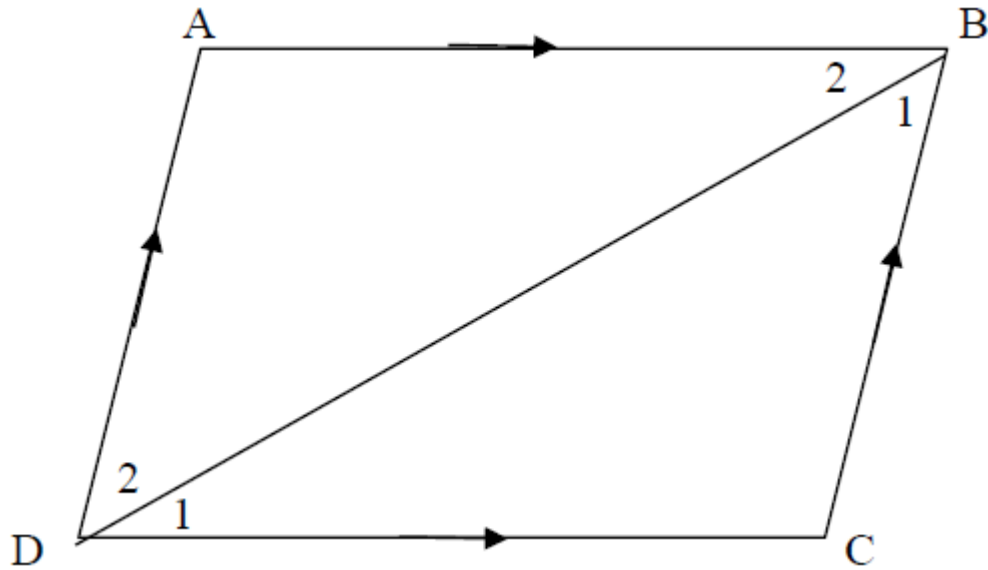


- 8.1 Write down the size of rhombus $\angle O_1$ (1)
- 8.2 Calculate the size of $\angle L_1$ (2)
- 8.1.3 Calculate the size of $\angle KNM$ (2)

2020 Eastern Cape November Paper 2 Q 5.1

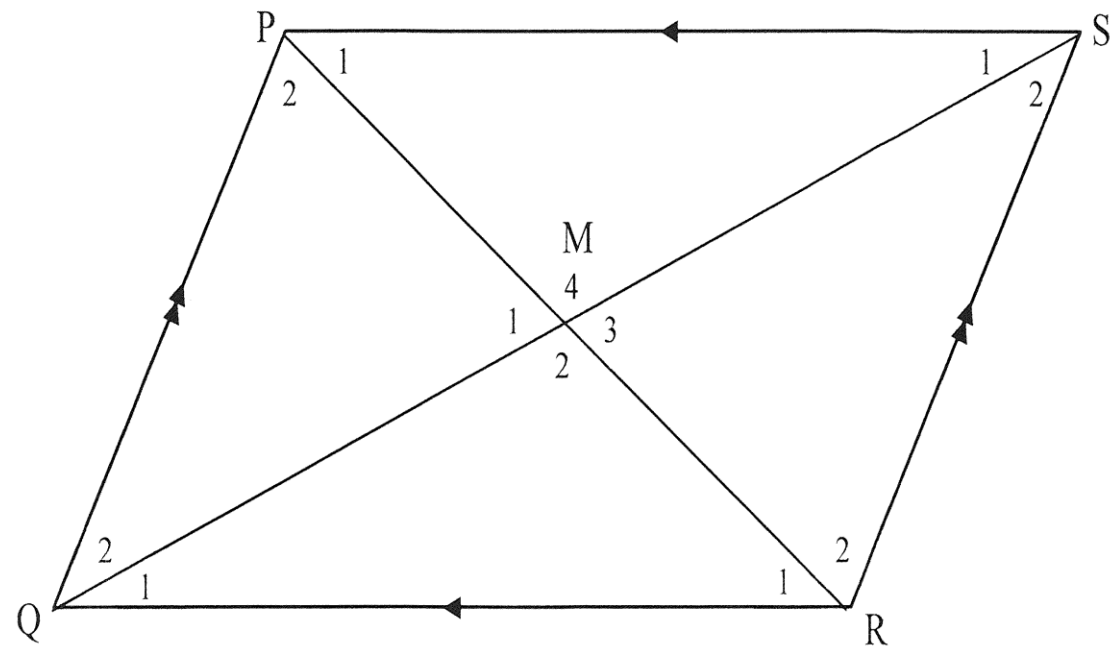
Use the diagram below to prove that the opposite sides of a parallelogram are equal, i.e., $AB = CD$ and $AD = BC$. (4)

Hint: Prove that $\triangle ABD \equiv \triangle CDB$



2017 November Paper 2 Q 8.2

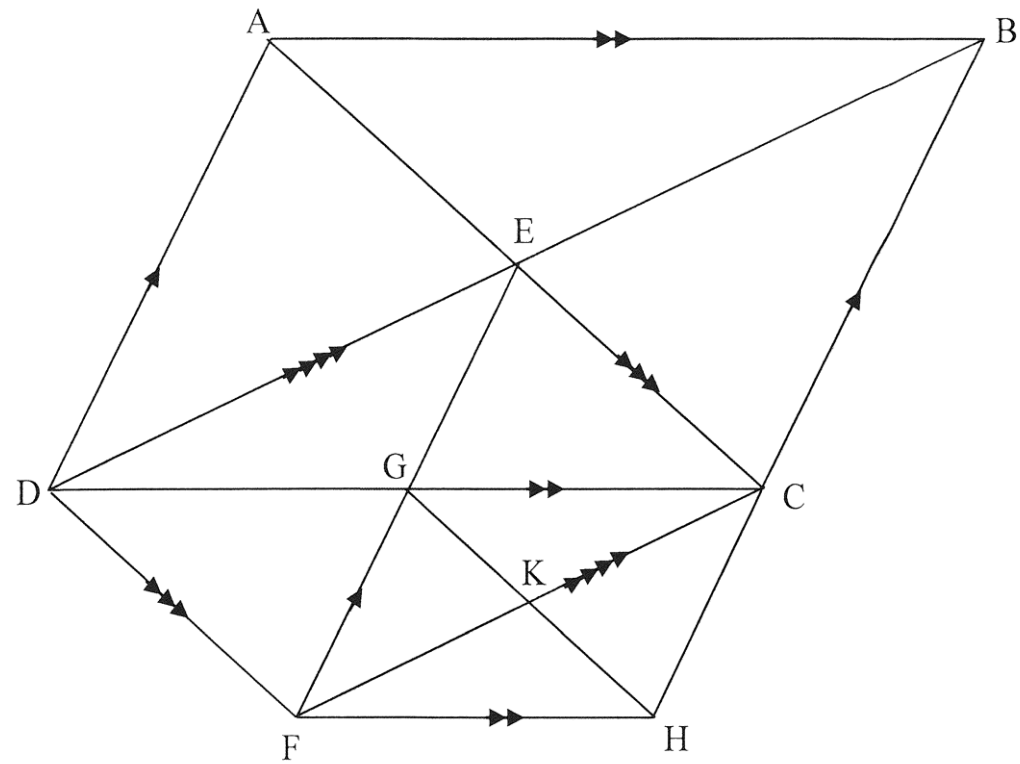
Given parallelogram PQRS with diagonals PR and QS intersecting at M.



Prove that the diagonals bisect each other (4)

2017 November Paper 2 Q 8.3

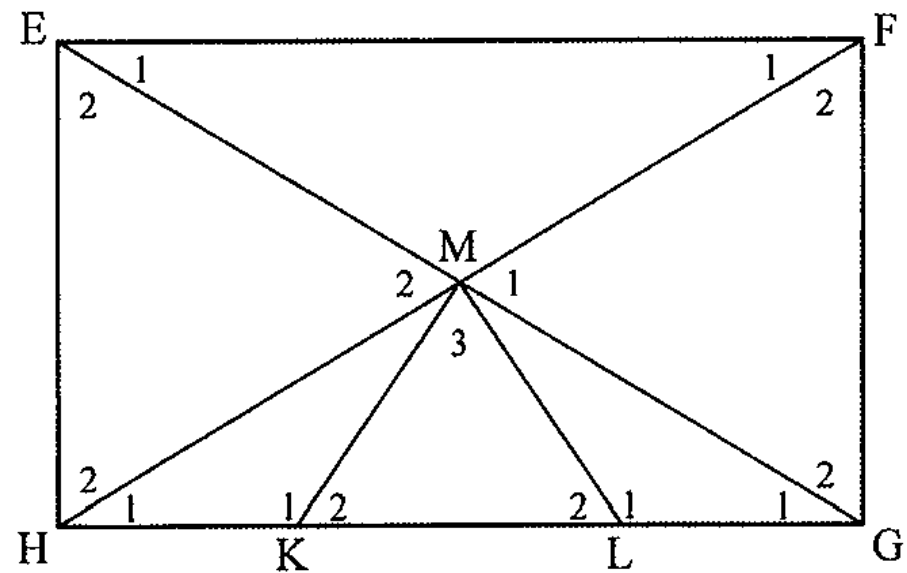
In the diagram, ABCD is a parallelogram with diagonals intersecting at E. The diagonals of parallelogram DECF at G. The diagonals of the parallelogram FGCH intersect at K



Prove that $DB = 4KC$ (4)

2019 November Paper 2 Q 7.1

In the diagram below, EFGH is a rectangle having diagonals intersecting at M. $\angle M_2 = 60^\circ$ and $\angle L_2 = 40^\circ$



Calculate the size of:

7.1.1 $\angle F_1$ (2)

7.1.2 $\angle GML$ (3)

