

NAME: _____ TEACHER: VvR / AG / JO



Redhill School

REDHILL SENIOR SCHOOL

GRADE 12 IEB

PRELIMINARY EXAMINATION

MATHEMATICS: PAPER II

%

EXAMINER: A GOMES

MODERATOR: V VAN ROOY

TIME: 3 HOURS

MARKS: 150

SECTION A								
QUESTION	1	2	3	4	5	6	7	SUBTOTAL
MARKS	13	11	6	5	13	21	7	76
MARK								
SECTION B								
QUESTION	8	9	10	11	12	13	14	SUBTOTAL
MARKS	9	10	7	9	5	18	16	74
MARK								
						TOTAL		/150

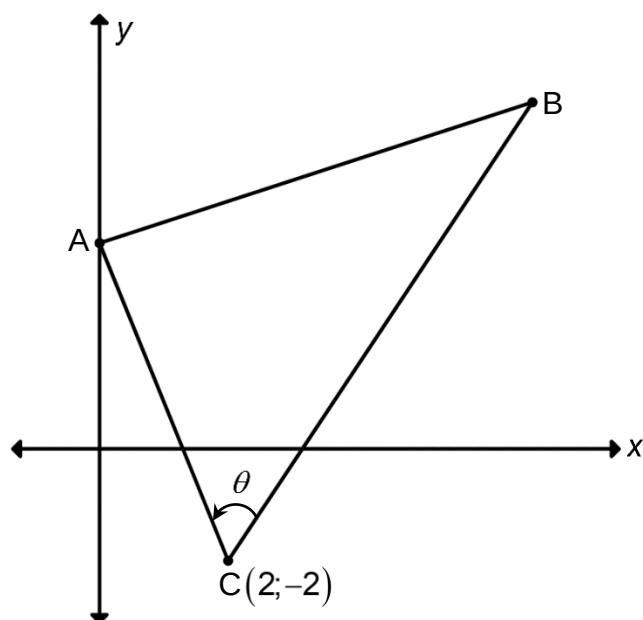
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of **14 QUESTIONS** and **32 PAGES**. An information sheet is attached separately.
2. Read the questions carefully.
3. Answer all the questions **ON THIS QUESTION PAPER**.
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers. Answers only will NOT necessarily be awarded full marks.
6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to TWO decimal places, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly **in pen**.

SECTION A

QUESTION 1

- a) In the diagram below $\triangle ABC$ is shown. Point C has coordinates $(2; -2)$, line AB has equation $2y - x - 4 = 0$ and BC has equation $\frac{1}{3}y + \frac{8}{3} = x$.
The angle $\hat{BCA} = \theta$.



- (1) Determine the coordinates of A. (2)

- (2) Calculate the value of θ . (4)

- (3) Is BC a diameter to the circle passing through A, B and C? Support your answer by means of calculations. (3)

b) Consider the following:

Circle: $x^2 + y^2 + 2x - 8y = 2008$

Line: $y = -3x + k$

Determine the value of k if the line passes through the centre of the circle. (4)

[13]

QUESTION 2

- a) Determine the numerical value of the expression below **without the use of a calculator**:

$$\frac{4\cos^2 198^\circ - 2}{7\sin 27^\circ \cos 333^\circ} \quad (6)$$

b) Consider the following: $\frac{\cos 2x - \sin 4x}{\cos x - \sin x} = \cos 3x - \sin 3x$.

(1) Using compound angles show that $\cos 2x - \sin 4x$ can be re-written as:

$$\cos 3x \cos x - \cos 3x \sin x + \sin 3x \sin x - \sin 3x \cos x \quad (2)$$

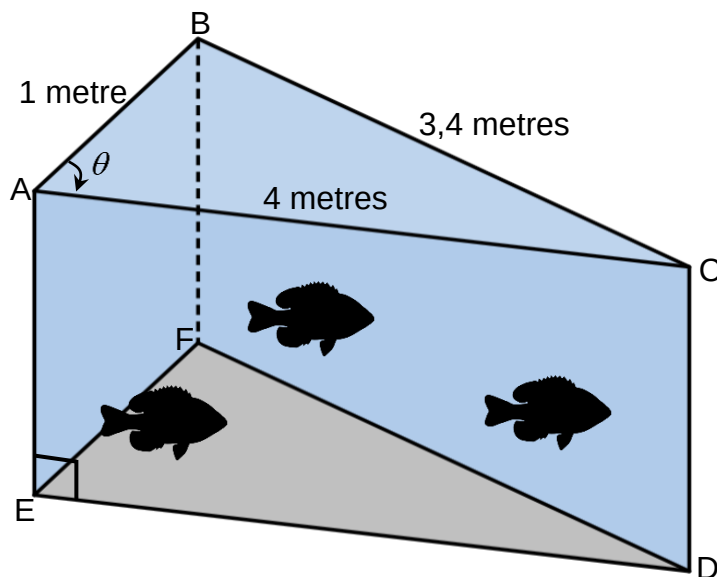
(2) Hence, prove the identity. (3)

[11]

QUESTION 3

A triangular fish tank, being designed for an aquarium, is shown in the diagram below.

- $\triangle EFD$ forms the base of the fish tank.
- The length of AB is 1 metre, the length of BC is 3,4 metres and AC is 4 metres.
- $\hat{BAC} = \theta$.



a) Determine the size of θ . (3)

b) Calculate the volume of water the fish tank can hold if AE is 2 metres long. (3)

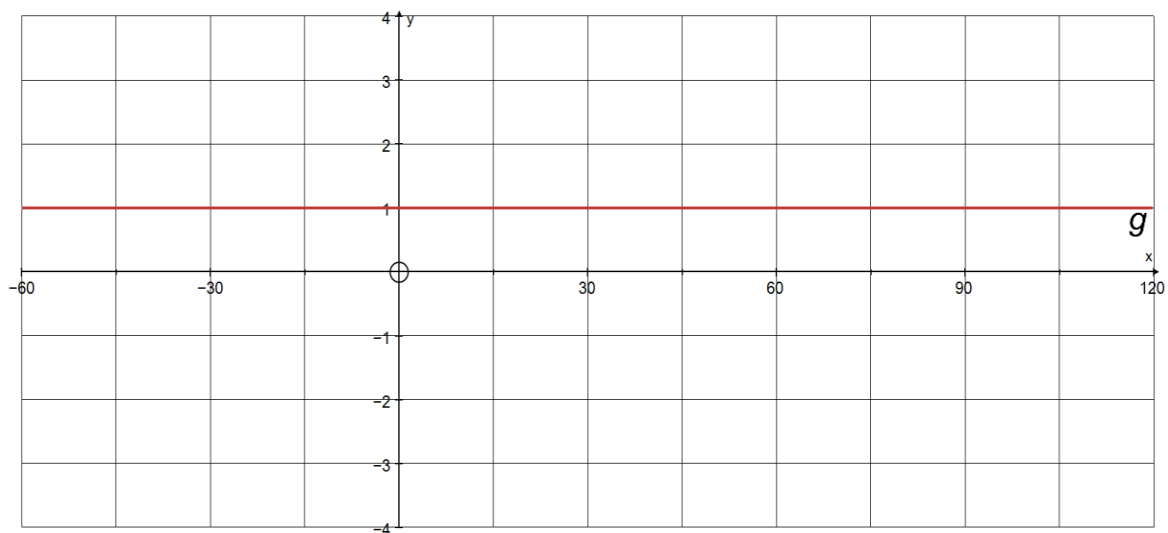
[6]

QUESTION 4

Consider the functions: $f(x) = \tan(x - 30^\circ)$

$$g(x) = 1$$

- a) The graph of g is shown below, sketch the graph of f on the axes below (3)
for $x \in [-60^\circ; 120^\circ]$, indicating all intercepts with the axes and asymptotes
where applicable.

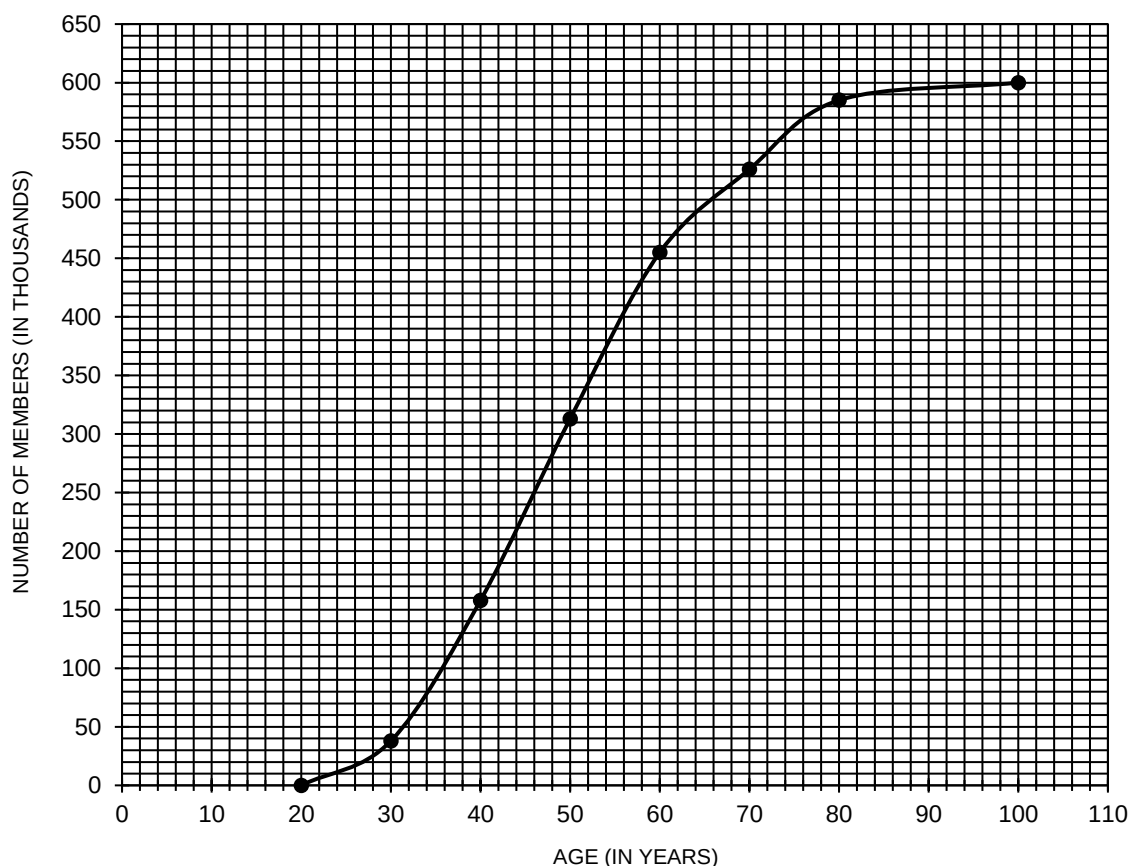


- b) Determine the value(s) of x for which $f(x) \geq g(x)$ for $x \in [-60^\circ; 120^\circ]$. (2)

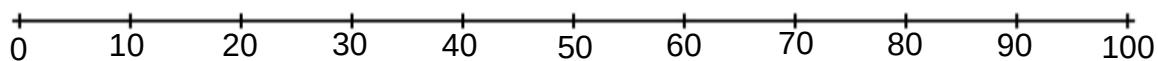
[5]

QUESTION 5

- a) A medical aid scheme in South Africa does a survey on the ages of their 600 000 members.
The data collected is collated and the ogive curve below is produced.



- (1) Complete the box and whisker plot below using the scale provided.
(Indicate on the ogive where the values have been found and show the exact values on the box and whisker plot.) (4)



- (2) How many members does this medical aid have that are between the ages of 30 and 50? (2)

- b) A smaller sample of members, each of who were diagnosed with high blood pressure was then taken.

Each of these members were asked what dosage they took of a specific medication they were prescribed to treat their high blood pressure as well as their systolic blood pressure.

x: Dosage (mg)	5	5,5	6,5	4	8	12	7	9,5
y: Systolic Blood pressure (mmHg)	126	130	134	158	116	107	129	128

- (1) Using the data above find the equation of the least squares regression line that represents the data. (2)

- (2) Write down AND comment on the correlation coefficient of this data. (2)

- (3) An additional member was surveyed and was found to be taking a dosage of 15mg with a systolic blood pressure of 177mmHg.

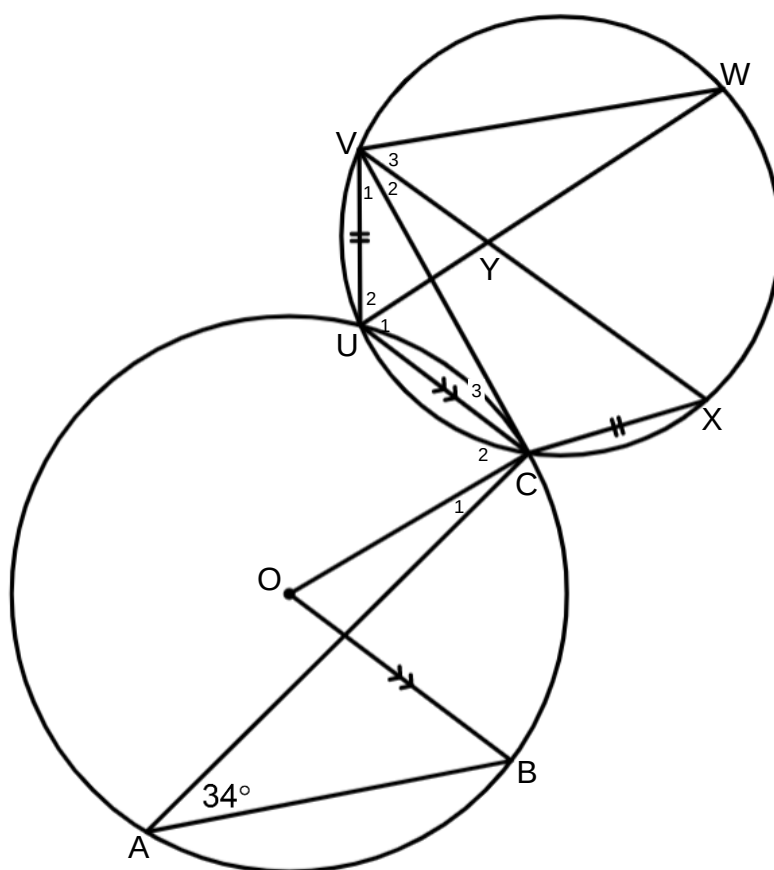
How would this members results affect the gradient of the regression line and the correlation coefficient? Explain. (3)

[13]

QUESTION 6

a) In the diagram below:

- O is the centre of the larger circle.
- A, B, C and U are points on the circumference of the larger circle.
- C, U, V, W and X are points on the circumference of the smaller circle.
- Chords $UV = CX$.
- VC is a tangent to the larger circle at C.
- $OB \parallel UC$.
- $\hat{CAB} = 34^\circ$.



- (1) A student is trying to calculate the size of $\hat{V}_2$, one of the statements and reasons they give whilst answering the question is shown in the block below:

$\hat{C}_2 = \hat{V}_1 + \hat{V}_2$	Ext. $\angle$ cyclic quad
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Explain why this is incorrect.

(1)

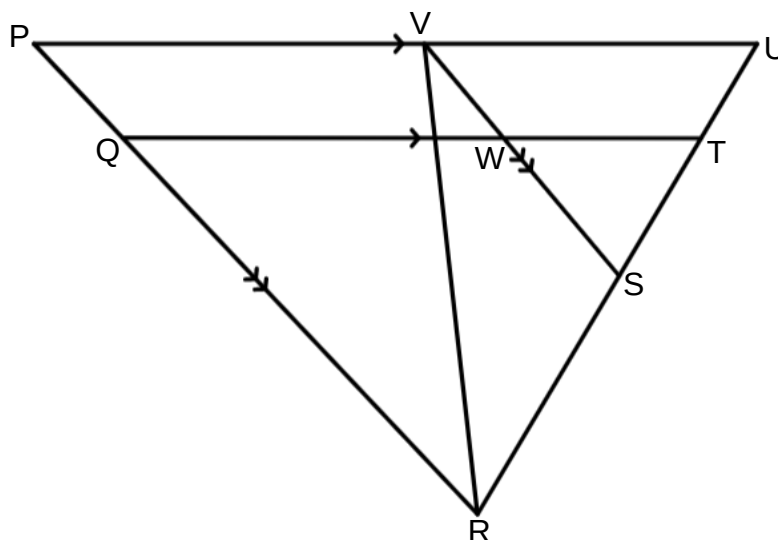
(2) Determine, with reasons, the size of:

(i) $\hat{C}_2$ (4)

(ii) $\hat{W}$ (3)

(3) Prove that $UC \parallel VX$. (3)

- b) In the sketch below $PU \parallel QT$ and $PR \parallel VS$. $PR = 6PQ$ and $PV = \frac{5}{6}VU$.



Determine:

(1) $\frac{TU}{TR}$ (2)

(2) $\frac{TU}{SR}$ (3)

$$(3) \quad \frac{\text{Area of } \triangle VUS}{\text{Area of } \triangle PUR}$$

(3)

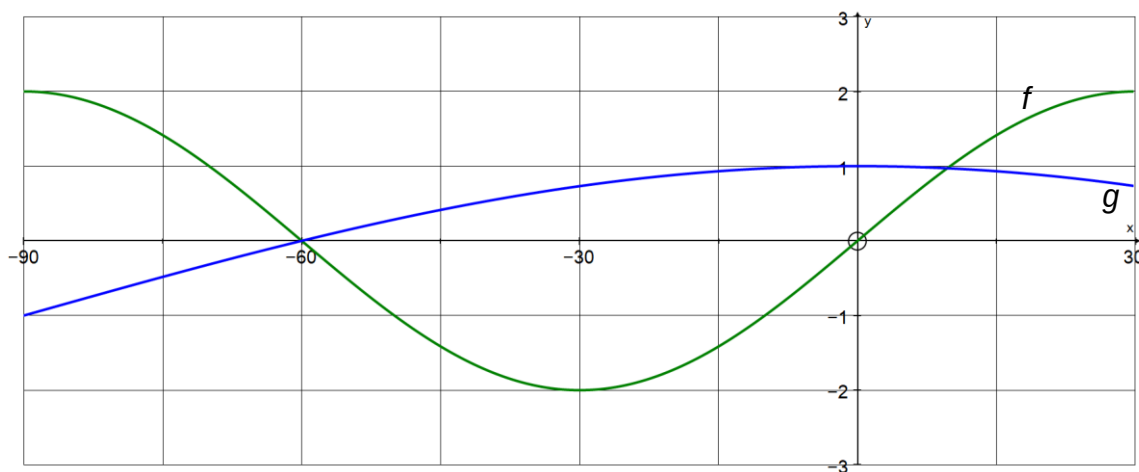
$$(4) \quad \frac{\text{Area of } \triangle PUR}{\text{Area of } \triangle VRU}$$

(2)

[21]

QUESTION 7

In the diagram below the functions $f(x) = 2\sin(k \cdot x)$ and $g(x) = 2\cos x - p$ are sketched for $-90^\circ \leq x \leq 30^\circ$.



- What is the period of f ? (1)
- Write down the values of k and p . (2)
- Determine the range of $y = 2g(x) + 3$. (2)
- For which values of m will $g(x) = m - 1$ have real roots? (2)

[7]

76 marks

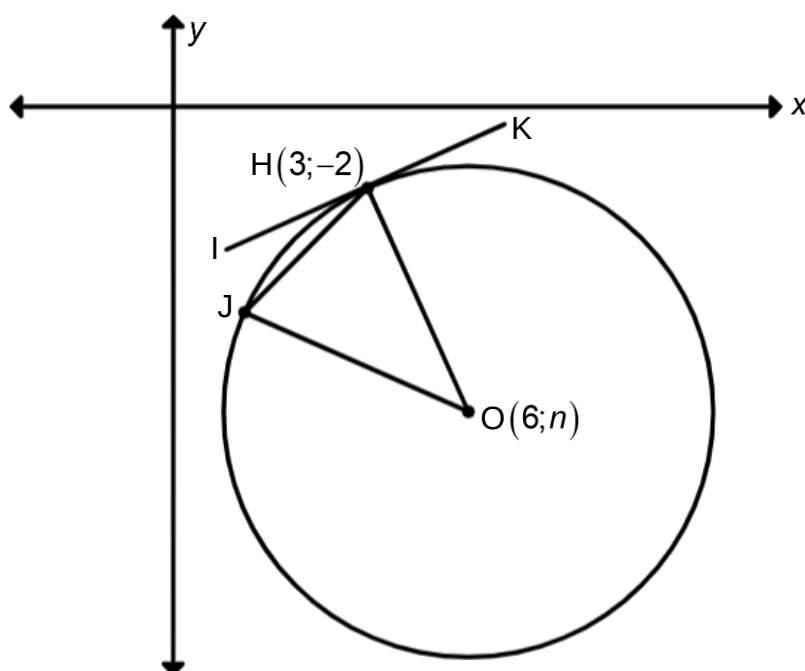
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SECTION B

QUESTION 8

In the diagram below the circle with centre O is sketched:

- The circle has equation $(x - 6)^2 + (y - n)^2 = r^2$
- The points H(3; -2) and J are on the circumference of the circle.
- The line IK is a tangent to the circle through H.
- H and J are joined by a chord with equation $y = x - 5$.



- a) If the gradient of the tangent IK is $\frac{3}{5}$, show that $n = -7$. (2)

b) Determine the length of the radius of the circle. (2)

c) Find the coordinates of J. (4)

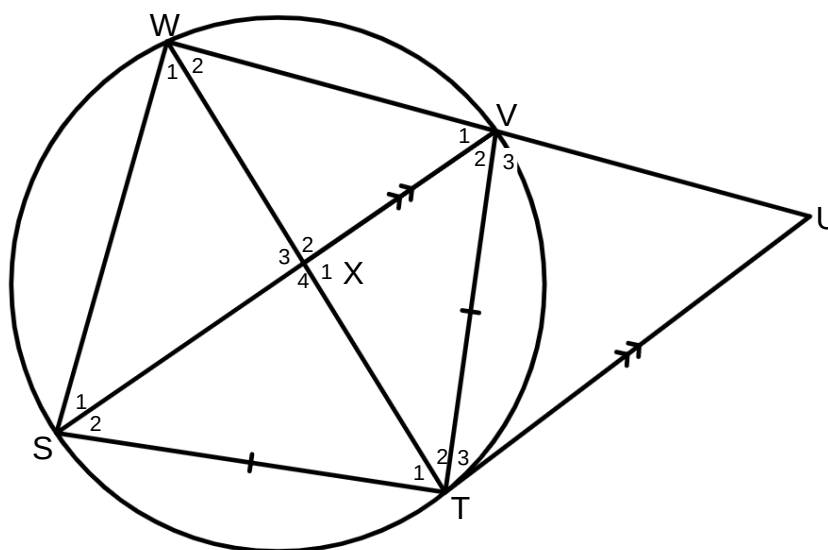
d) What is the minimum distance between the circumference of the circle and the x -axis? (1)

[9]

QUESTION 9

In the diagram below:

- $TV = ST$.
- Chords WT and VS intersect at X .
- $SV \parallel TU$.



a) Prove that UT is a tangent to the circle at T .

(3)

b) Show that $\triangle UWT \parallel \triangle UTV$ (3)

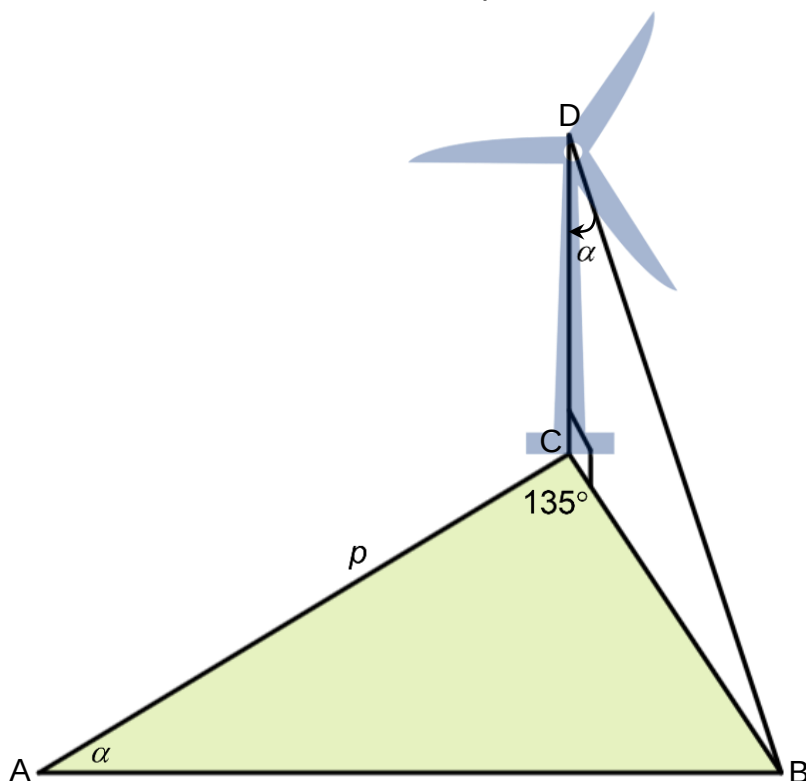
c) Prove that $TU^2 \cdot TX = WT \cdot UV^2$ (4)

[10]

QUESTION 10

With the increase in global warming companies around the world have invested heavily in renewable sources of energy, such as wind energy, in the form of wind farms.

The diagram below shows a vertical wind turbine on the edge of a triangular shaped field. $\hat{BAC} = \hat{CDB} = \alpha$, $\hat{ACB} = 135^\circ$ and $AC = p$ units.



- a) Show that the length of BC can be expressed as $\frac{p \cdot \sin \alpha}{\sin(45^\circ - \alpha)}$ (2)

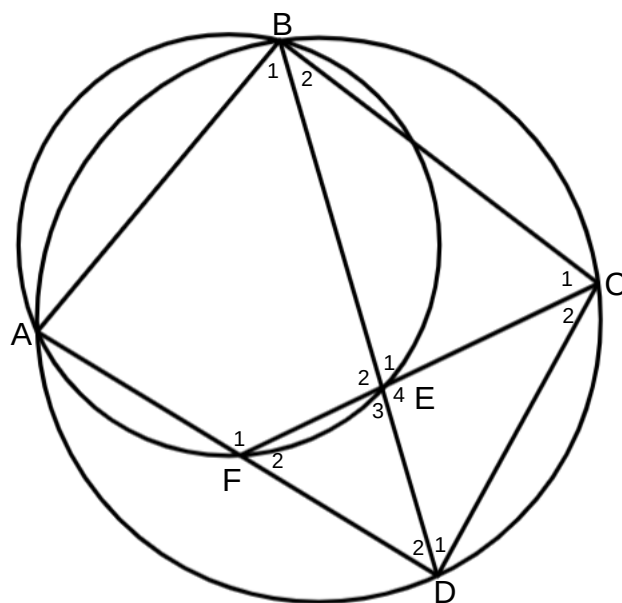
b) Hence, show that height of the wind turbine $DC = \frac{\sqrt{2}p \cos \alpha}{\cos \alpha - \sin \alpha}$ (5)

[7]

QUESTION 11

In the diagram below:

- Circle ABCD and circle ABEF intersect at A and B.
- BED, AFD and FEC are straight lines.
- BD, CD and BC are chords in the larger circle.



Prove that:

a) $CD^2 = ED \cdot BD$

(6)

Additional space for (a)...

b) $CD^2 - ED^2 = BE \cdot ED$ (3)

[9]

QUESTION 12

After the 2022 preliminary exams, 10 students are selected at random, and the results of their maths paper 2 exam are analysed.

The one standard deviation interval about the mean is $[49,28;80,72]$.

Determine the interval that shows two standard deviations about the mean.

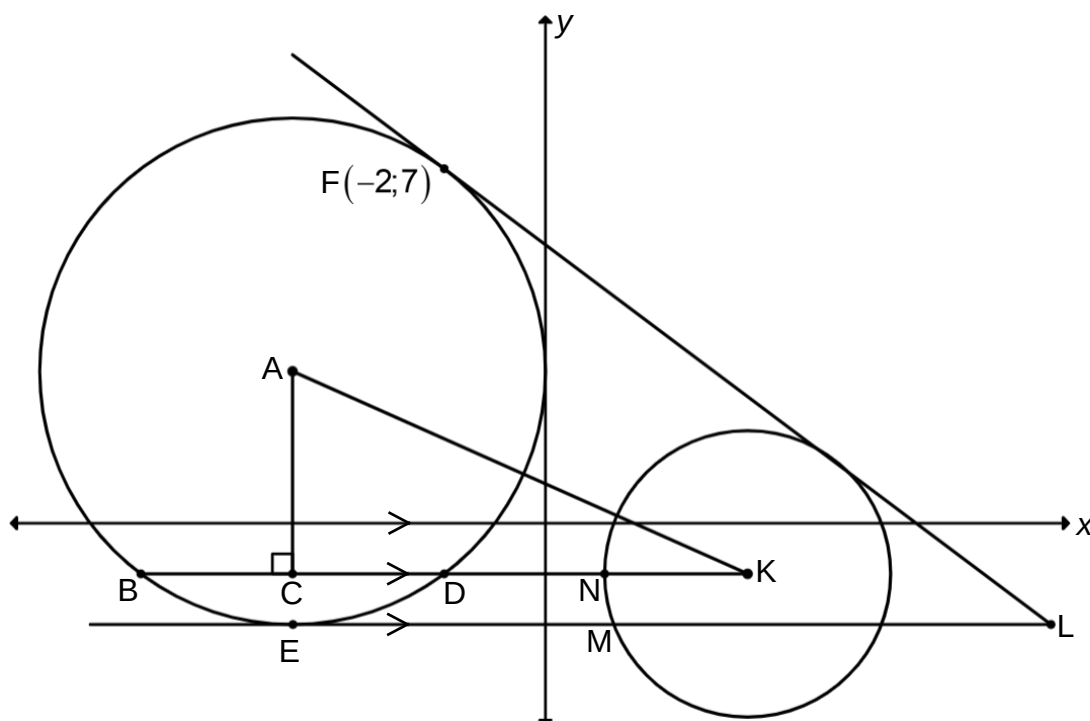
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QUESTION 13

In the diagram below A is the centre of the circle $(x + 5)^2 + (y - 3)^2 = 25$ and K is the centre of the circle $(x - a)^2 + (y - b)^2 = 8$.

- FL is a tangent to the circle at $(-2; 7)$ and EL is a tangent at E.
- B, E and D lie on the circumference of the circle centred at A.
- M and N lie on the circumference of the circle centred at K.
- $AC \perp BD$, $CK \parallel EL$ and $CK \parallel x$ -axis.
- $BD = 6$ units.



a) Write down the length of the radius of the circle centred at A. (1)

b) Determine the coordinates of B and D. (4)

c) (1) Write down the equation of EL (1)

(2) Hence, find the coordinates of L. (5)

d) If $DN = 6 - 2\sqrt{2}$, determine the length of AK. (3)

- e) Another circle $(x + 7)^2 + (y - 2)^2 = p^2$ touches the circle centred at A once.
Determine the possible values of p . (Leave your answer in surd form.) (4)

[18]

QUESTION 14

- a) If $\cos x > 0$, determine the general solution of $2\sin 2x = \tan x$. (7)

- b) If it is given that $\cos 23^\circ + \sin 23^\circ = k$, then, without the use of a calculator and in terms of k , determine the value of:

(1) $\cos 46^\circ$

(6)

(2) $\cos 23^\circ$

(3)

[16]

74 marks

ADDITIONAL SPACE FOR WORKING