

WTS TUTORING



WTS TRIGONOMETRY

GRADE : 12

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WHERE TO START MATHS & SCIENCE IS FOR THE NATION

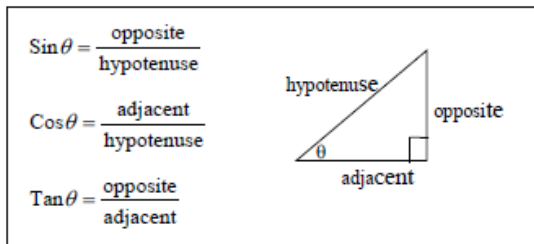
TRIGONOMETRY

±50 MARKS

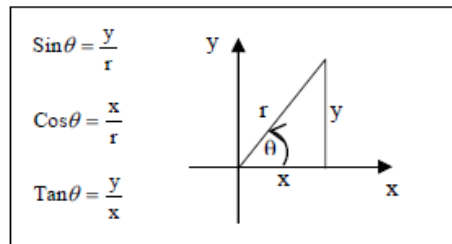
- ✓ It is the study of the relationships between lines and triangles
- ✓ The word trigonometry means ‘measurement of triangles.’

➤ PYTHAGORAS THEOREM

○ In a right-angled Δ



○ On a Cartesian Plane



- ✓ It deals with the length and angles
- ✓ It's given by the equation : $x^2 + y^2 = r^2$
- ✓ The Pythagoras theorem can be used anytime if we have a right angled triangle with two sides and want to find a third side.
- ✓ r is always a longest positive side and opposite to 90°

➤ TRIGONOMETRIC RATIOS

NOTE: SOHCAHTOA

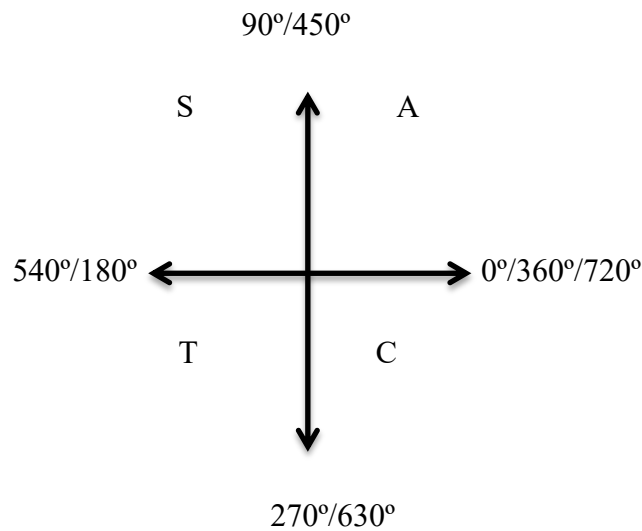
$$\begin{aligned} \diamond \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & [\text{SOH}] \\ &= \frac{y}{r} \end{aligned}$$

$$\begin{aligned} \diamond \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & [\text{CAH}] \\ &= \frac{x}{r} \end{aligned}$$

$$\begin{aligned} \diamond \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & [\text{TOA}] \\ &= \frac{y}{x} \end{aligned}$$

NB: Opposite, Adjacent and Hypotenuse are all sides not an angle.

➤ **ASTC DIAGRAM {All Sbiya's Take Coffee}**

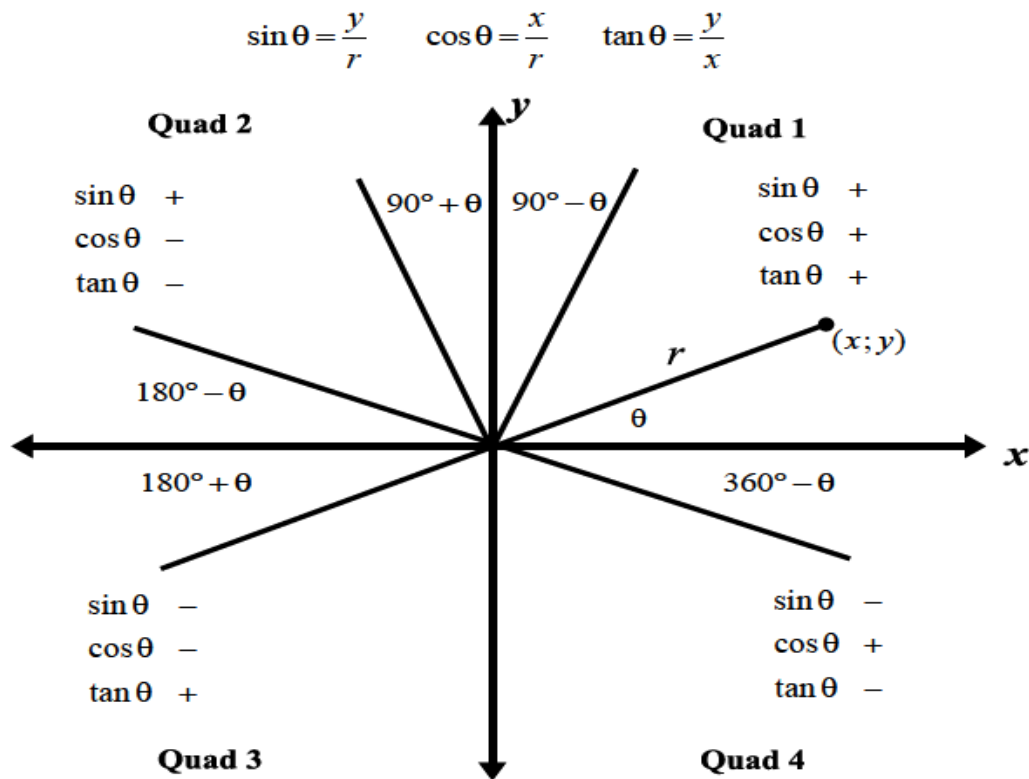


- ❖ It's clearly determines a status of each trigonometric quadrant
- ❖ A [FIRST QUAD AND ALL TRIG RATIOS ARE POSITUVE]
- ❖ S [SECOND QUAD AND ONLY SINE POSITIVE]
- ❖ T [THIRD QUAD AND ONLY TANGENT POSITIVE]
- ❖ C [FOURTH QUAD AND ONLY COSINE POSITIVE]

➤ **CONSIDER A DEMONSTRATION BELOW**

TRIG RATIOS	1 ST	2 ND	3 RD	4 TH
$\sin \theta$	+	+	-	-
$\cos \theta$	+	-	-	+
$\tan \theta$	+	-	+	-

✓ **NOTE!**



➤ **TRIGONOMETRIC IDENTITIES**

- ✓ It is where we are saying left hand side is equal to right hand side.
- ✓ Simplify the complicated side
- ✓ Simplify both if all sides are complicated
- ✓ Always remember to convert tangent to cosine and sine
- ✓ If there is a fraction simple look for LCD
- ✓ Check the common factor
- ✓ Factorise if there's a quadratic express or equation
- ✓ If there nothing to change just simple rationalizes the numerator or denominator.

e.g. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

- ✓ $\beta / \alpha / \theta$: usual represent any angle not a side

➤ **QUOTIENT IDENTITY**

$$✓ \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

➤ **SQUARE IDENTITY**

$$✓ \cos^2 \alpha + \sin^2 \alpha = 1$$

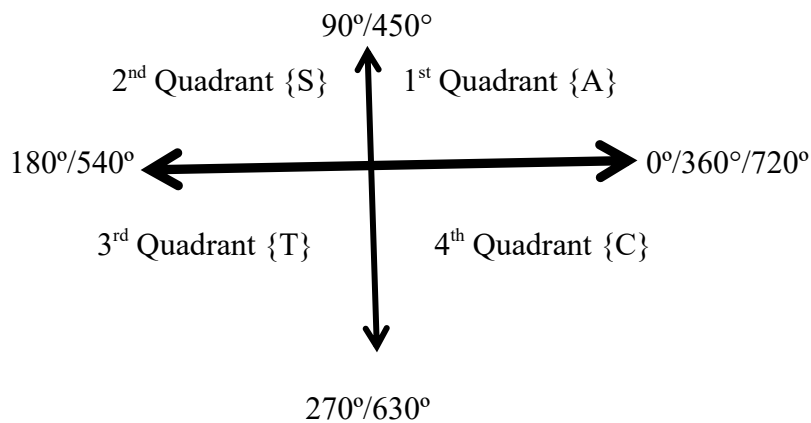
- $\sin^2 \alpha = 1 - \cos^2 \alpha$
- $ = (1 - \cos \alpha) (1 + \cos \alpha)$

- $\sin \alpha = \sqrt{1 - \cos^2 \alpha}$

- $\cos^2 \alpha = 1 - \sin^2 \alpha$
- $ = (1 - \sin \alpha) (1 + \sin \alpha)$
- $\cos \alpha = \sqrt{1 - \sin^2 \alpha}$

• REDUCTION FORMULAR

- ✓ It is a process of reducing at 360° , we are completing the first revolution.
- ✓ Therefore we can complete more than one revolution
- ✓ Here we just considered 0° to 720° , which are two revolutions
- ✓ Take note: negative angle ($-\alpha$), found in the 4th



NOTE

- MOVING FORWARD YOU MUST ADD
- MOVING BACK YOU MUST SUBTRACT

➤ **ANTCLOCKWISE DIRECTION**

- ✓ Reading the angles from right to left
- ✓ Positive angles
- ✓ Give us the angle of inclination

➤ **CLOCKWISE DIRECTION**

- ✓ Reading the angles from left to right
- ✓ Negative angles
- ✓ Angle of inclination will be: $180^\circ - \text{ref angle}$

- **METHOD 1**

- ✓ Simply add 360° up until the angle becomes positive

- **METHOD 2**

- ✓ Take out the negative as a common factor e.g. : $b - a = -(a - b)$
- ✓ And then remember that $(-\alpha)$ is in the 4th quadrant

➤ **CO-FUNCTIONS**

- ✓ Re taking place at vertical axis of ASTC diagram y - axis.
- ✓ Co functions of sine is cosine OR cosine is sine.
- ✓ It includes angles such as 90° , 270° , 450° , 630° and 810° .

➤ **COMPLEMENTARY ANGLES**

- ✓ These are angles that add up to 90° , complementary angles of $\sin\alpha$ and cosine, vice versa

➤ **TRIG RATIOS WITH A SQUARE**

- ✓ Use the closed bracket to include square and simplify inside and then consider the square
- ✓ Conclusion: if there is a square the answer will be always positive.

➤ **ANGLES GREATER THAN 360°**

- ✓ Simplify subtract 360° up until the angle becomes less than 360°

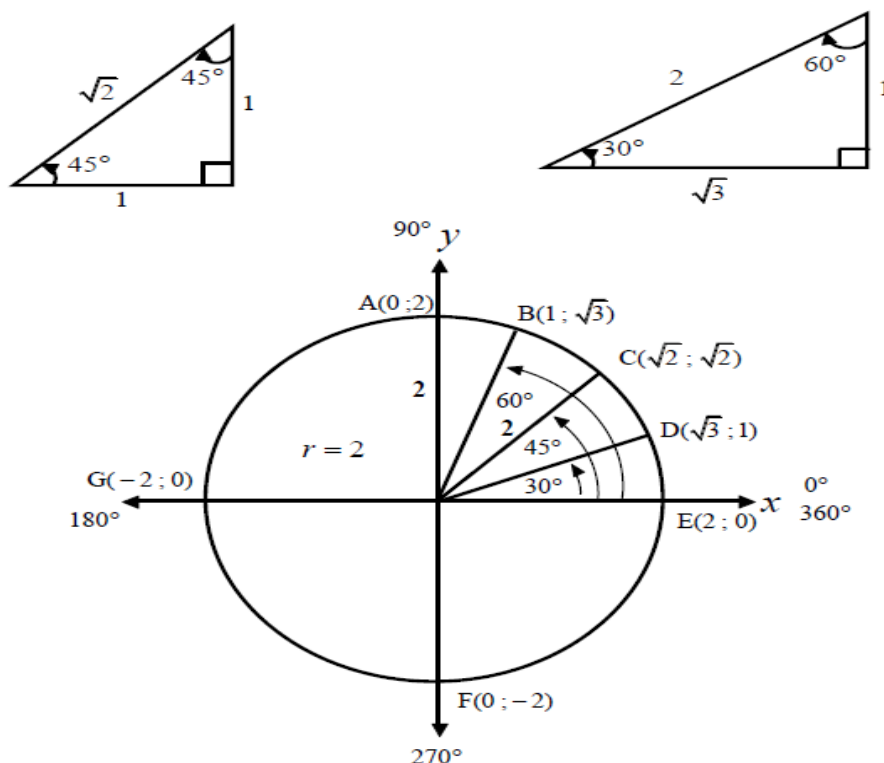
➤ ANGLES

- ✓ It includes all angles on the x and y axis such as $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ, 450^\circ, 630^\circ$ and 720° etc.
- ✓ If given, simply press the calculator

➤ SPECIAL ANGLES

- ✓ Includes $30^\circ; 45^\circ; 60^\circ$
- ✓ If the angle is not an acute angle simply reduce it with special angles.
- ✓ If you have a **Casio calculator** simply use it as it gives the correct rationalized values.
- ✓ If you are using **old sharp calculator** simply use the triangle method

NOTE!



➤ RESTRICTIONS VS INTERVALS

- ✓ The closed brackets [] are used with $\leq$ or $\geq$ {the value is included}
- ✓ The open brackets () are used with $<$ or $>$ {the value is excluded}
- ✓ This can be more useful in general solution and in questions requiring the use of the diagram

➤ **COMPOUND ANGLES**

- ✓ It deals with angles which are not the same e.g. $(\infty + \alpha)$
- ✓ It deals with a relationship between sine and cosine, will be given in the formula sheet
- ✓ **In sine compound:** it keeps the original sign between the terms.
- ✓ **In cosine compound:** it changes the sign between the terms..

➤ **THESE ARE EXAMINABLE, LEARN THEM WELL**

PROOF:

$$\begin{aligned}\cos(\alpha + \beta) &= \cos[\alpha - (-\beta)] = \cos\alpha.\cos(-\beta) + \sin\alpha.\sin(-\beta) \\ &= \cos\alpha.\cos\beta + \sin\alpha.(-\sin\beta) \\ &= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta\end{aligned}$$

PROOF:

$$\begin{aligned}\sin(\alpha + \beta) &= \cos[90^\circ - (\alpha + \beta)] = \cos[90^\circ - \alpha - \beta] = \cos[(90^\circ - \alpha) - \beta] \\ &= \cos(90^\circ - \alpha).\cos(\beta) + \sin(90^\circ - \alpha).\sin(\beta) \\ &= \sin\alpha.\cos\beta + \cos\alpha.\sin\beta\end{aligned}$$

PROOF:

$$\begin{aligned}\sin(\alpha - \beta) &= \cos[90^\circ - (\alpha - \beta)] = \cos[90^\circ - \alpha + \beta] = \cos[(90^\circ + \beta) - \alpha] \\ &= \cos(90^\circ + \beta).\cos\alpha + \sin(90^\circ + \beta).\sin\alpha \\ &= -\sin\beta.\cos\alpha + \cos\beta.\sin\alpha \\ &= \sin\alpha.\cos\beta - \cos\alpha.\sin\beta\end{aligned}$$

NOTE!!

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \dots\dots \text{think "cos cos sin sin"} \text{ ☺}$$

Note the opposite signs

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \dots\dots \text{think "sin cos cos sin"} \text{ ☺}$$

Note the same signs

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

➤ **DOUBLE ANGLES**

- ✓ It deals with angles which are the same e.g. $\theta = \beta$
- ✓ It deals with the relationship between sine and cosine.
- ✓ It will be given in the formula sheet.

NOTE!

$$\sin 2\alpha = \sin(\alpha + \alpha)$$

$$\sin 2\alpha = \sin \alpha \cos \alpha + \cos \alpha \sin \alpha$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$= 2 (\sin \alpha)(\cos \alpha) \dots \text{(note that } \sin \alpha \text{ and } \cos \alpha \text{ are numbers, or factors in this case)}$$

$$\cos 2\alpha = \cos(\alpha + \alpha)$$

$$\cos 2\alpha = \cos \alpha \cos \alpha - \sin \alpha \sin \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = (1 - \sin^2 \alpha) - \sin^2 \alpha \quad \text{OR} \quad \cos 2\alpha = \cos^2 \alpha - (1 - \cos^2 \alpha)$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

➤ **WITH THE AID OF A DIAGRAM**

- ✓ Firstly make the trig ratios be the subject of the formula
- ✓ Check the quadrant
- ✓ Draw the right angled triangle and always the line must lie on the x-axis
- ✓ Use the Pythagoras to calculate the unknown side
- ✓ And then simplify any question, take note of double and compound angles
- ✓ Remember to convert the given angle with special angles
- ✓ And then simplify any question, take note of double and compound angles

➤ **EXPRESSING IN TERMS VARIABLES**

- ✓ Always draw the diagram on the first quadrant
- ✓ Use the Pythagoras to calculate the unknown side
- ✓ And then simplify any question ,take note of double and compound angles
- ✓ Remember to convert the given angle with special angles

➤ **REDUCTIONS AND IDENTITIES**

- ✓ Use ASTC diagram and identities
- ✓ In any calculation relate to special angles
- ✓ Always try to convert all terms into sine and cosine e.g. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- ✓ Always try to convert double to single angles
- ✓ Think about double and compound angles.
- ✓ Addition and subtraction of fractions: find LCD then simplify
- ✓ Factories: take out the HCF, difference of two squares, trinomials, grouping, sum or difference of two cubes.
- ✓ If nothing to factorize: simply rationalize
- ✓ Do not cancel over a + and -.
- ✓ Remember a common denominator.
- ✓ Keep looking at the RHS

➤ **ALGEBRAICALLY TRIGONOMETRY**

- ✓ Try to manipulate, the part or the given part into a required one.
- ✓ The skills of squaring both sides and then simplifying are needed.
- ✓ The skill of working from double angles into single angles is required.

➤ **INVALID / UNDEFINED VALUES**

- ✓ Let denominator equal to zero
- ✓ If tangent is give simple consider the asymptotes.
- ✓ Solve as like general solution
- ✓ Substitute with integers, and then consider the domain

➤ **MAXIMUM AND MINIMUM VALUES**

- ✓ They are indicated by the amplitude of the trig function.
- ✓ Maximum is the turning point above the horizontal
- ✓ Minimum is the turning point below the horizontal or x axis.
- ✓ Simplify up to the single trig ratio.

➤ **TRIGONOMETRIC EQUATIONS AND GENERAL SOLUTIONS**

- ✓ Indicate all angles including negative ones
- ✓ Take note of the reference angle :acute angle
- ✓ Use $k \cdot 360^\circ$ for sine and cosine due to their period
- ✓ K represents elements of integers $k \in (-1,0,1)$
- ✓ Always make a trig ratio a subject of the formula e.g. $a \sin b = c$
- ✓ Always remember to simplify before considering the quadrant
- ✓ Consider the quadrant / Cartesian plan in the form ASTC (ALL SBIYA'S TAKE COFFIE)

➤ **QUADRANTS**

- ✓ $1^{\text{st}} = \text{reference angle.}$
- ✓ $2^{\text{nd}} = 180^\circ - \text{reference angle.}$
- ✓ $3^{\text{rd}} = 180^\circ + \text{reference angle.}$
- ✓ $4^{\text{th}} = 360^\circ - \text{reference angle.}$
- ✓ Remember to apply co-functions in case of sine and cosine given.
- ✓ Remember again to convert sine and cosine to tangent

➤ **TRIGONOMETRY SUMMARY**

• **CALCULATE THE VALUE OF A TRIG EXPRESSION WITHOUT USING A CALCULATOR**

- ✓ Establish whether you need a rough sketch or special triangles or ASTC rules.

• **EXPRESS TRIG RATIO IN TERMS OF THE GIVEN VARIABLE**

- ✓ Draw a rough sketch with given angle and label 2 of the sides.
- ✓ The 3rd side can then be determined using Pythagoras.
- ✓ Express each of the angles in question in terms of the angle in the rough sketch.

• **SIMPLIFY A TRIGONOMETRICALLY EXPRESSION**

- ✓ Use the ASTC rule to simplify the given expression if possible.
- ✓ See if any of the identities can be used to simplify it, if not see if it can be factorized.
- ✓ Check again if any identity can be used.
- ✓ This includes using the compound and double angle identities

- **PROVE A GIVEN IDENTITY**

- ✓ Simplify the one side of the equation using reduction formulae and identities until it cannot simplify any further

- **SOLVE A TRIG EQUATION**

- ✓ Find the reference angle by ignoring the “-“sign and write down the two solutions
- ✓ Then write down the general solution for the given eq.
- ✓ From the general solution you can determine the solution for any specified interval by using various values of k

➤ TRIGONOMETRY GRAPHS

- **Consider:** $y = asinb(x - p) + q$ or $y = acosb(x - p) + q$

✓ a: amplitude = $\frac{1}{2}$ (distance between the y – values of T. P)

✓ b: co – efficient of an angle used to calculate the period

- **p affects the horizontal shift of the graph**

✓ If $p > 0$ (positive), the graph moves p degrees to the left.

✓ If $p < 0$ (negative), the graph moves p degrees to the right.

- **q affects the horizontal shift of the graph**

✓ If $q > 0$ (positive), the graph moves upwards

✓ If $q < 0$ (negative), the graphs moves downwards.

- **Consider:** $y = acosbx$ and $y = atanbx$

✓ Period(P) = $\frac{360^\circ}{b}$ for sine and cosine

✓ Period(P) = $\frac{180^\circ}{b}$ for tangent

- **Scale : with no transformation:** $y = acosbx$

✓ $scale = \frac{Period}{4}$

- **Scale: with transformation:** $y = asinb(x - p) + q$ / $y = acosb(x - p) + q$

✓ $scale = (90^\circ - p)$ with $p > 0$

- **Scale: with transformation:** $y = atanb(x - p) + q$

✓ $scale = (45^\circ - p)$ with $p > 0$

- **RANGE**

✓ Y values of the turning point.

✓ Use the closed brackets

✓ Start with the minimum to maximum value inside the brackets

➤ **DOMAIN**

- ✓ Always given.
- ✓ And is not the same as period

➤ **CALCULATING THE UNKNOWN**

- ✓ For one variable: use one point along the line
- ✓ For two variables: use two points along the line and solve the two equations simultaneously

➤ **POINT OF INTERSECTION**

- ✓ Means the point where two graphs are equal : $f(x) = g(x)$
- ✓ Equate the two graphs.
- ✓ Solve simultaneous.
- ✓ And the substitute with integers for the size of an angle.
- ✓ Co-functions can be used in the case of sine and cosine

➤ **FOR WHICH VALUE(S) OF X:**

- a) $f'(x) > 0$ {where the graph is increasing}
- b) $f'(x) < 0$ {where the graph is decreasing}
- c) $f(x).g(x) > 0$ {where both graphs are above or below}
- d) $f(x).g(x) < 0$ {One graph above and other one below the horizontal}
- e) $f(x) > 0$ {above the horizontal}
- f) $f(x) < 0$ {below the horizontal}
- g) $g'(x) = 0$ {x-value of a turning point}
- h) $g'(x) > 0$ and $f'(x) > 0$ {both graphs increasing}
- i) $g'(x) < 0$ and $f'(x) < 0$ {both graphs decreasing}
- j) $g(x) - f(x) = 1$ {graph $g(x)$ is above $f(x)$ and the distance is 1 unit}
- k) $x.f(x) > 0$ {+ve x-value and +ve y-value or -ve x-value and -ve y-value}
- l) $f(x).f'(x) < 0$ {+ve y-value and the decreasing curve}

Key note:

- ✓ Remember to manipulate the new given equation into the original one
- ✓ Maximum and minimum values means the y values of the turning point

➤ TRIANGLE SOLUTIONS

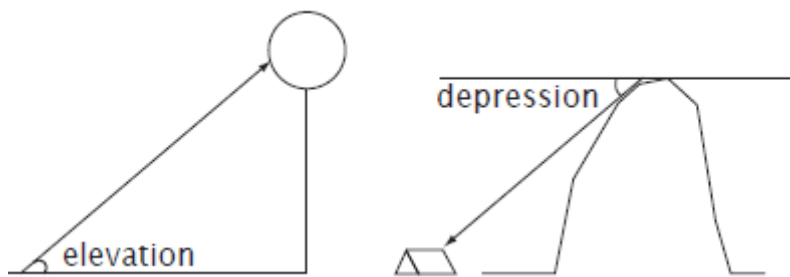
- ✓ Can only be used if the triangle is not a right angled triangle
- ✓ If a triangle is a right angled triangle simply use trig ratios
- ✓ First calculate the unknowns using the information given

Take note:

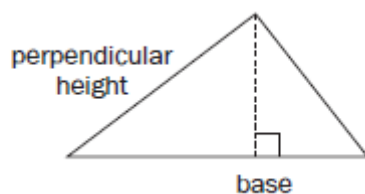
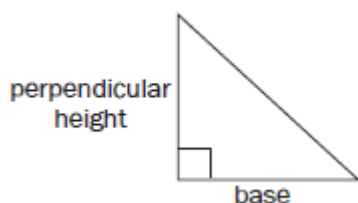
- ✓ Exterior angles of a triangle
- ✓ Sum of angles of a triangle
- ✓ Conditions of parallel lines
- ✓ Theorem from grade 11 work

ANGLES OF ELEVATION AND DEPRESSION

- ✓ angle of elevation measures up
- ✓ angle of depression measures down



➤ **FOR AREA RULE**



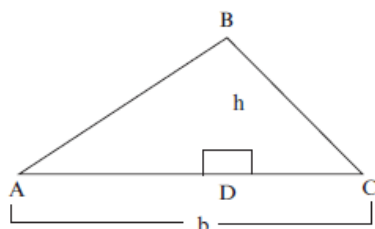
Area of a right-angled triangle

$$\text{Area } \Delta = \frac{1}{2} \text{ base} \times \text{perpendicular height}$$

$$\text{Area } \Delta = \frac{1}{2} bh$$

PROOF:

If $\hat{A}$ is acute



$$\text{Area of } \Delta ABC = \frac{1}{2} bh \dots \dots \dots (1)$$

$$\text{But } \sin A = \frac{h}{c} \therefore h = c \sin A$$

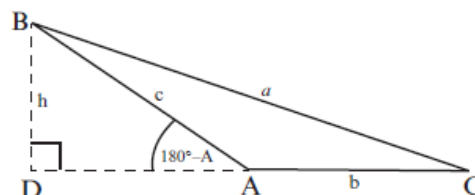
Substituting into (1)

$$\text{Area of } \Delta ABC = \frac{1}{2} bc \sin A$$

Similarly it can be shown that

$$\begin{aligned} \text{Area of } \Delta ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \end{aligned}$$

If $\hat{A}$ is obtuse



$$\text{Area of } \Delta ABC = \frac{1}{2} bh \dots \dots \dots (1)$$

$$\text{But } \sin (180^\circ - A) = \frac{h}{c} \therefore h = c \sin A$$

Substituting into (1)

$$\text{Area of } \Delta ABC = \frac{1}{2} bc \sin A$$

Similarly it can be shown that

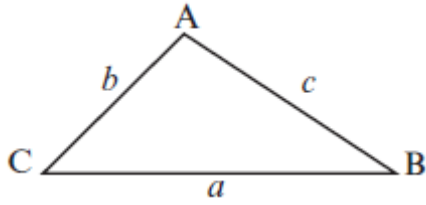
$$\begin{aligned} \text{Area of } \Delta ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \end{aligned}$$

➤ **CONDITIONS**

✓ SAS

➤ **FORMULA:**

The area of any $\triangle ABC$ is half the product of two sides and sine of the included angle.



So if you choose to use angle A, then

$$\text{Area } \triangle ABC = \frac{1}{2} bc \sin A$$

If you choose to use angle B, then

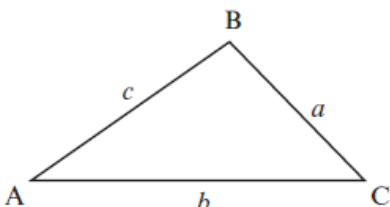
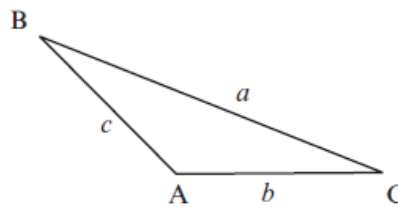
$$\text{Area } \triangle ABC = \frac{1}{2} ac \sin B$$

If you choose to use angle C, then

$$\text{Area } \triangle ABC = \frac{1}{2} ab \sin C$$

➤ **FOR SINE RULE**

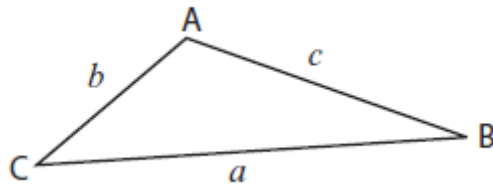
PROOF:

If $\hat{A}$ is acute 	If $\hat{A}$ is obtuse 
Using Area rule for $\triangle ABC$: $\frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$ Dividing each by $\frac{1}{2} abc$ results: $\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$	

➤ **CONDITIONS**

- ✓ SSA
- ✓ AAS

➤ **FORMULA:**



In any triangle ABC:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

OR

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

➤ **FOR COSINE RULE**

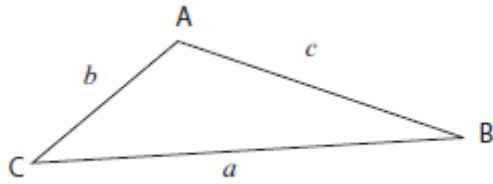
PROOF:

If $\hat{A}$ is acute In $\triangle BDC$: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem) $= BD^2 + (b - AD)^2$ $= BD^2 + b^2 - 2bAD + AD^2$ But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem) Thus $a^2 = b^2 + c^2 - 2bAD$(1) In $\triangle ABD$: $\cos A = \frac{AD}{c} \therefore AD = c \cos A$(2) Substituting (2) into (1) $\therefore a^2 = b^2 + c^2 - 2bc \cos A$ Similarly it can be shown that: $b^2 = a^2 + c^2 - 2ac \cos B$ and $c^2 = a^2 + b^2 - 2ab \cos C$	If $\hat{A}$ is obtuse In $\triangle BDC$: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem) $= BD^2 + (b + AD)^2$ $= BD^2 + b^2 + 2bAD + AD^2$ But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem) Thus $a^2 = b^2 + c^2 + 2bAD$(1) In $\triangle ABD$: $\cos(180^\circ - A) = \frac{AD}{c} \therefore AD = -c \cos A$(2) Substituting (2) into (1) $\therefore a^2 = b^2 + c^2 - 2bc \cos A$ Similarly it can be shown that: $b^2 = a^2 + c^2 - 2ac \cos B$ and $c^2 = a^2 + b^2 - 2ab \cos C$
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➤ **CONDITIONS**

- ✓ SAS
- ✓ SSS

➤ **FORMULA:**

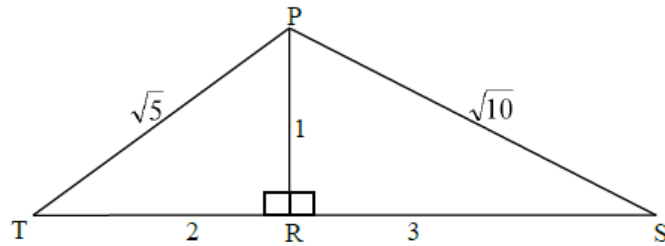


In any triangle ABC:

- ✓ If you choose to use angle A, then → $a^2 = b^2 + c^2 - 2bc \cos A$
- ✓ If you choose to use angle B, then → $b^2 = a^2 + c^2 - 2ac \cos B$
- ✓ If you choose to use angle C, then → $c^2 = a^2 + b^2 - 2ab \cos C$

KWV: 01**QUESTION 5**

- 5.1 In the diagram $PR \perp TS$ in obtuse triangle PTS .
 $PT = \sqrt{5}$; $TR = 2$; $PR = 1$; $PS = \sqrt{10}$ and $RS = 3$



- 5.1.1 Write down the value of:

(a) $\sin \hat{T}$ (1)

(b) $\cos \hat{S}$ (1)

- 5.1.2 Calculate, WITHOUT using a calculator, the value of $\cos(\hat{T} + \hat{S})$ (5)

- 5.2 Determine the value of:

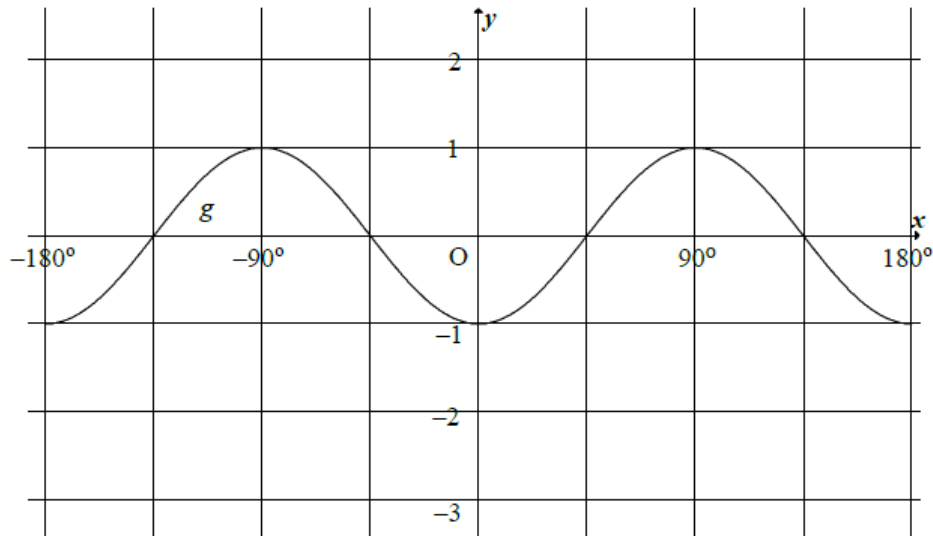
$$\frac{1}{\cos(360^\circ - \theta) \cdot \sin(90^\circ - \theta)} - \tan^2(180^\circ + \theta) \quad (6)$$

- 5.3 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)
[18]

QUESTION 6

6.1 Determine the general solution of $4\sin x + 2\cos 2x = 2$ (6)

6.2 The graph of $g(x) = -\cos 2x$ for $x \in [-180^\circ; 180^\circ]$ is drawn below.



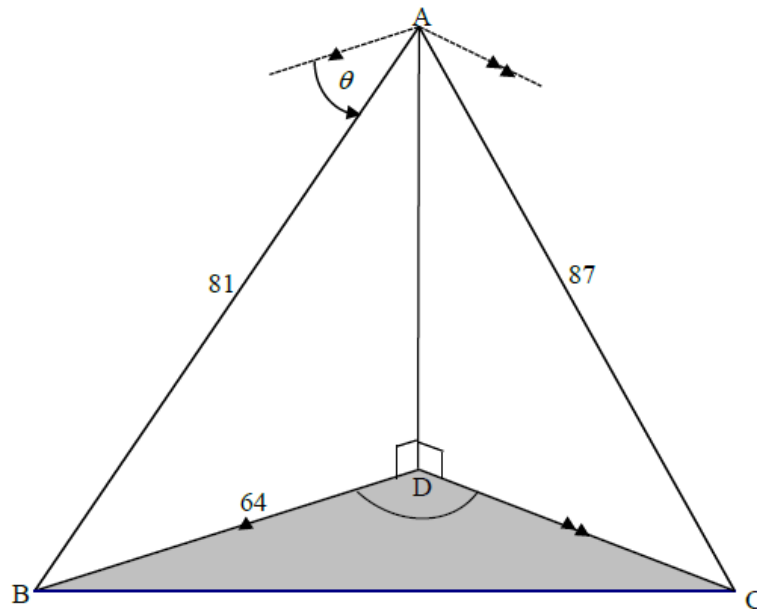
6.2.1 Draw the graph of $f(x) = 2\sin x - 1$ for $x \in [-180^\circ; 180^\circ]$ on the set of axes provided in the ANSWER BOOK. (3)

6.2.2 Write down the values of x for which g is strictly decreasing in the interval $x \in [-180^\circ; 0^\circ]$ (2)

6.2.3 Write down the value(s) of x for which $f(x + 30^\circ) - g(x + 30^\circ) = 0$ for $x \in [-180^\circ; 180^\circ]$ (2)
[13]

QUESTION 7

From point A an observer spots two boats, B and C , at anchor. The angle of depression of boat B from A is θ . D is a point directly below A and is on the same horizontal plane as B and C . $BD = 64$ m, $AB = 81$ m and $AC = 87$ m.

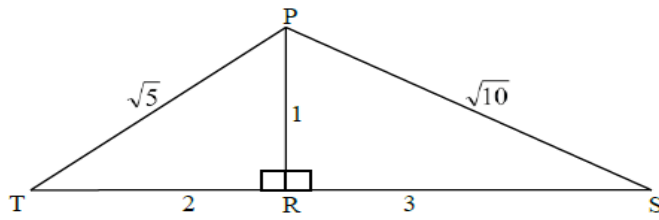


- 7.1 Calculate the size of θ to the nearest degree. (3)
- 7.2 If it is given that $\hat{BAC} = 82,6^\circ$, calculate BC , the distance between the boats. (3)
- 7.3 If $\hat{BDC} = 110^\circ$, calculate the size of $\hat{DCB}$. (3)
- [9]

MEMO: 01

QUESTION/VRAG 5

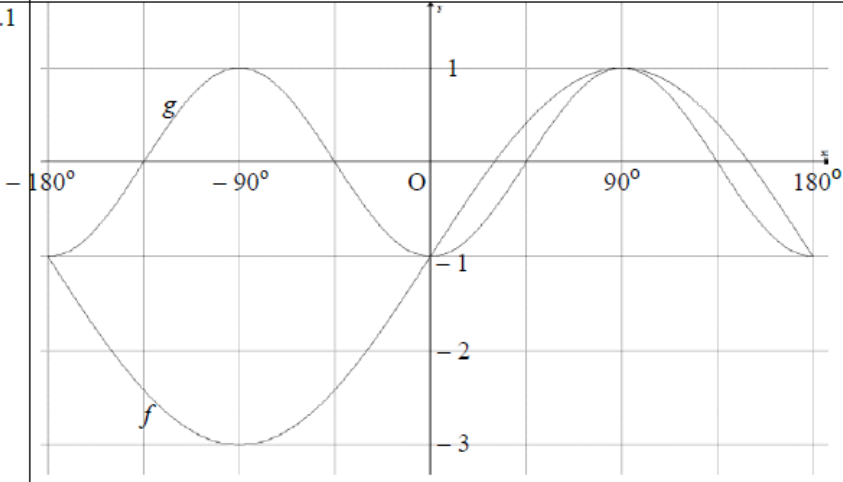
5.1



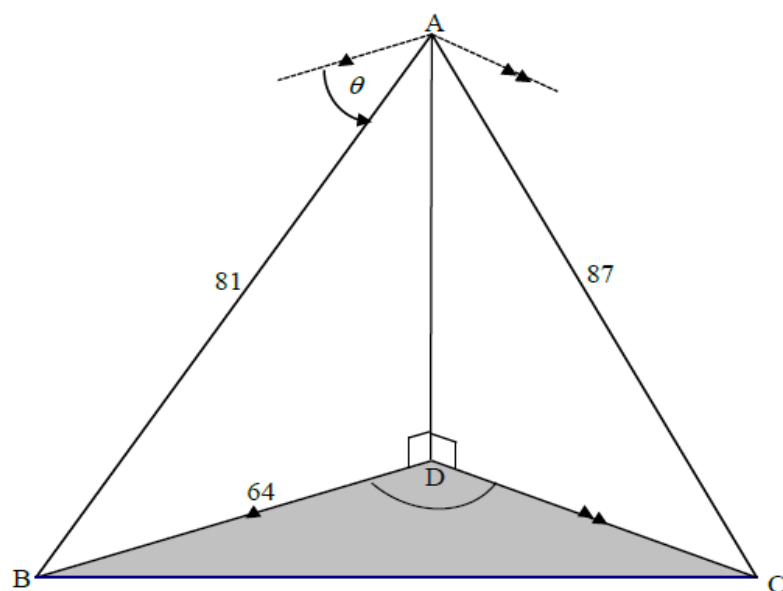
5.1.1(a)	$\sin T = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0,45$	✓ value (1)
5.1.1(b)	$\cos S = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} = 0,95$	✓ value (1)
5.1.2	$\cos(T + S) = \cos T \cos S - \sin T \sin S$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$ $= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$ $= \frac{5}{\sqrt{50}} \text{ or } \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$	✓ expansion ✓ $\frac{2}{\sqrt{5}}$ ✓ $\frac{1}{\sqrt{10}}$ ✓ simplification ✓ answer (5)
5.2	$\frac{1}{\cos(360^\circ - \theta) \sin(90^\circ - \theta)} - \tan^2(180^\circ + \theta)$ $= \frac{1}{(\cos \theta)(\cos \theta)} - \tan^2 \theta$ $= \frac{1}{\cos^2 \theta} - \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right)$ $= \frac{1 - \sin^2 \theta}{\cos^2 \theta}$ $= \frac{\cos^2 \theta}{\cos^2 \theta} \text{ OR } \frac{1 - \sin^2 \theta}{1 - \sin^2 \theta}$ $= 1$	✓ $\cos \theta$ ✓ $\cos \theta$ ✓ $\tan^2 \theta$ ✓ $\frac{\sin^2 \theta}{\cos^2 \theta}$ ✓ identity ✓ answer (6)

5.3	$(\sin x - \cos x)^2 = \left(\frac{3}{4}\right)^2$ $\sin^2 x - 2 \sin x \cos x + \cos^2 x = \frac{9}{16}$ $1 - 2 \sin x \cos x = \frac{9}{16}$ $2 \sin x \cos x = \frac{7}{16}$ $\therefore \sin 2x = \frac{7}{16}$	✓ squaring both sides ✓ expanding LHS ✓ using identity ✓ simplifying ✓ answer (5) [18]
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QUESTION/VRAAG 6

6.1	$4 \sin x + 2 \cos 2x = 2$ $2 \sin x + \cos 2x - 1 = 0$ $2 \sin x + (1 - 2 \sin^2 x) - 1 = 0$ $2 \sin^2 x - 2 \sin x = 0$ $2 \sin x (\sin x - 1) = 0$ $2 \sin x = 0$ or $\sin x - 1 = 0$ $\sin x = 0$ or $\sin x = 1$ $x = k.180^\circ$ or $x = 90^\circ + k.360, k \in \mathbb{Z}$	$\checkmark$ using identity $\checkmark$ standard form $\checkmark$ factors $\checkmark \sin x = 0$ or $\sin x = 1$ $\checkmark k.180^\circ$ $\checkmark 90^\circ + k.360, k \in \mathbb{Z}$ (6)
6.2.1		$\checkmark$ turning point $(-90^\circ; -3)$ $\checkmark$ turning point $(90^\circ; 1)$ $\checkmark (-180^\circ; -1)$ & $(0^\circ; -1)$ (3)
6.2.2	$(-90^\circ; 0^\circ)$ OR/OF $-90^\circ < x < 0^\circ$	$\checkmark \checkmark$ answer (2) $\checkmark \checkmark$ answer (2)
6.2.3	$f(x) = g(x)$ $\therefore -180^\circ; 0^\circ; 90^\circ; 180^\circ$ $f(x + 30^\circ) = g(x + 30^\circ)$ $\therefore x = -30^\circ; 60^\circ; 150^\circ$	$\checkmark$ any ONE correct $\checkmark$ other 2 correct (2) [13]

QUESTION/VRAAG 7



7.1	$\hat{A}BD = \theta$ [alternate $\angle$ s; $\parallel$ lines] $\cos \theta = \frac{BD}{AB} = \frac{64}{81}$ $\theta = 38^\circ$ OR/OF $\sin \hat{B}AD = \frac{64}{81}$ $\hat{B}AD = 52,18^\circ$ $\theta = 38^\circ$	✓ correct trig ratio ✓ substitution into correct ratio ✓ answer (to the nearest degree) (3) ✓ correct trig ratio ✓ substitution into correct ratio ✓ answer (to the nearest degree) (3)
7.2	$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos \hat{B}AC$ $= 81^2 + 87^2 - 2(81)(87) \cos 82,6^\circ$ $= 12314,754...$ $BC = 110,97 \text{ m}$	✓ use cosine rule ✓ correct substitution into cosine rule ✓ answer (3)

7.3	$\frac{\sin \hat{D}CB}{BD} = \frac{\sin \hat{B}DC}{BC}$ $\sin \hat{D}CB = \frac{BD \cdot \sin \hat{B}DC}{BC}$ $\sin \hat{D}CB = \frac{64 \cdot \sin 110^\circ}{110,97}$ $\therefore \hat{D}CB = 32,82^\circ$	✓ use sine rule ✓ substitution ✓ answer (3) [9]
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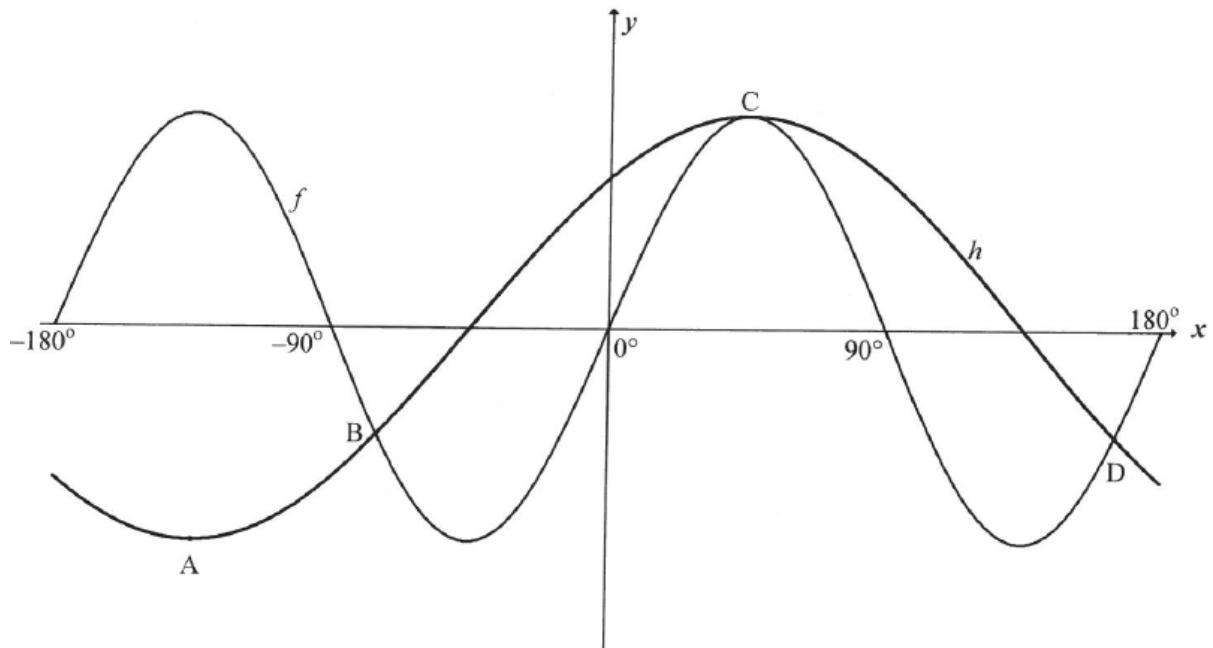
KWV: 02

QUESTION 5

- 5.1 Given: $\sin A = 2p$ and $\cos A = p$
- 5.1.1 Determine the value of $\tan A$. (2)
- 5.1.2 **Without using a calculator**, determine the value of p , if $A \in [180^\circ ; 270^\circ]$. (3)
- 5.2 Determine the general solution of $2\sin^2 x - 5\sin x + 2 = 0$ (6)
- 5.3 5.3.1 Expand $\sin(x + 300^\circ)$ using an appropriate compound angle formula. (1)
- 5.3.2 **Without using a calculator**, determine the value of $\sin(x + 300^\circ) - \cos(x - 150^\circ)$. (5)
- 5.4 Prove the identity: $\frac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x$. (5)
- 5.5 Consider: $\sin x + \cos x = \sqrt{1 + k}$
- 5.5.1 Determine k as a single trigonometric ratio. (3)
- 5.5.2 Hence, determine the maximum value of $\sin x + \cos x$. (2)
- [27]

QUESTION 6

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45^\circ)$ for the interval $x \in [-180^\circ; 180^\circ]$. $A(-135^\circ; -1)$ is a minimum point on graph h and $C(45^\circ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and $D\left(165^\circ; -\frac{1}{2}\right)$.



- 6.1 Write down the period of f . (1)
- 6.2 Determine the x -coordinate of B . (1)
- 6.3 Use the graphs to solve $2\sin x \cdot \cos x \leq \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180^\circ; 180^\circ]$. Show ALL working. (4)
- [6]**

QUESTION 7

A rectangular box with lid $ABCD$ is given in FIGURE (i) below. The lid is opened through 60° to position $HKCD$, as shown in the FIGURE (ii) below. $EF = 12\text{ cm}$, $FG = 6\text{ cm}$ and $BG = 8\text{ cm}$.

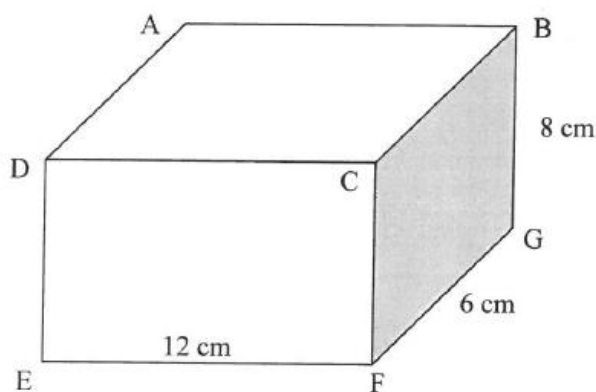


FIGURE (I)

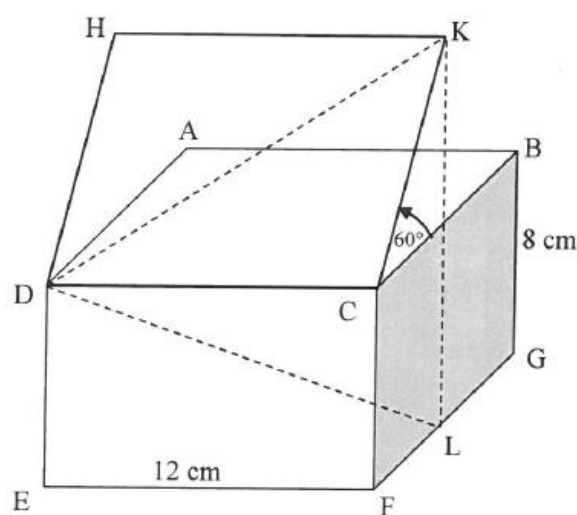


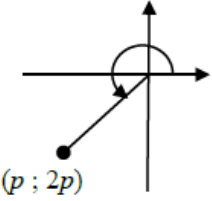
FIGURE (II)

- 7.1 Write down the length of KC . (1)
- 7.2 Determine KL , the perpendicular height of K , above the base of the box. (3)
- 7.3 Hence, determine the value of $\frac{\sin \hat{KDL}}{\sin \hat{DLK}}$. (4)

[8]

MEMO: 02

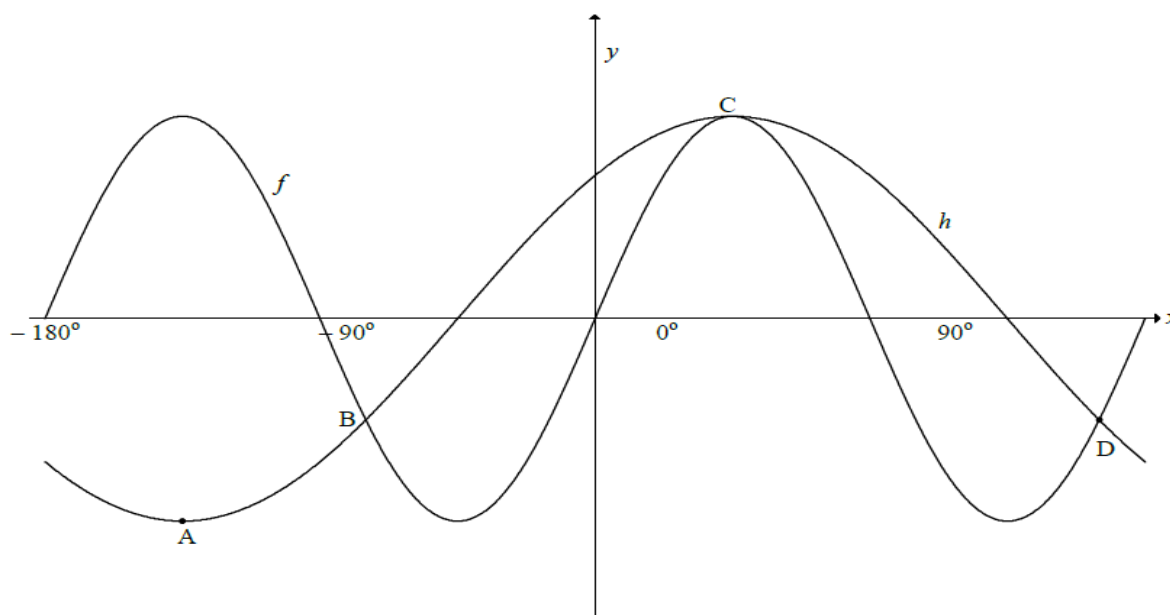
QUESTION/VRAAG 5

5.1.1	$\tan A = \frac{\sin A}{\cos A}$ $= \frac{2p}{p}$ $= 2$ OR/OF $\tan A = \frac{2p}{p}$ $= 2$	✓ identity ✓ value of tan A (2)  ✓ $\frac{y}{x}$ ✓ value of tan A (2)
5.1.2	$\sin^2 A + \cos^2 A = 1$ $(2p)^2 + p^2 = 1$ $4p^2 + p^2 = 1$ $5p^2 = 1$ $p^2 = \frac{1}{5}$ $\therefore p = -\frac{1}{\sqrt{5}}$	✓ $(2p)^2 + p^2 = 1$ ✓ simplification of LHS ✓ answer (3)
5.2	$2 \sin^2 x - 5 \sin x + 2 = 0$ $(2 \sin x - 1)(\sin x - 2) = 0$ $\sin x = \frac{1}{2} \text{ or } \sin x = 2 (\text{no solution})$ ref $\angle = 30^\circ$ $\therefore x = 30^\circ + k.360^\circ \text{ or } x = 150^\circ + k.360^\circ; k \in \mathbb{Z}$	✓ factors or formula ✓ both equations ✓ no solution/geen opl ✓ $30^\circ + k.360^\circ$ ✓ $150^\circ + k.360^\circ$; ✓ $k \in \mathbb{Z}$ (6)
5.3.1	$\sin(x + 300^\circ) = \sin x \cos 300^\circ + \cos x \sin 300^\circ$	✓ expansion/uitbreiding (1)
5.3.2	$\sin(x + 300^\circ) - \cos(x - 150^\circ)$ $= \sin x \cos 300^\circ + \cos x \sin 300^\circ - (\cos x \cos 150^\circ + \sin x \sin 150^\circ)$ $= \sin x \cos 60^\circ - \cos x \sin 60^\circ - (-\cos x \cos 30^\circ + \sin x \sin 30^\circ)$ $= \sin x \cos 60^\circ - \cos x \sin 60^\circ + \cos x \cos 30^\circ - \sin x \sin 30^\circ$ $= \frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x + \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$ $= 0$ OR/OF	✓ 2nd expansion/ 2de uitbreiding ✓✓ reduction/reduksie ✓ special angle values/ spesiale hoekwaardes ✓ answer (5)

	$\sin(x + 300^\circ) - \cos(x - 150^\circ)$ $= \sin x \cos 300^\circ + \cos x \sin 300^\circ - (\cos x \cos 150^\circ + \sin x \sin 150^\circ)$ $= \sin x \cos 60^\circ - \cos x \sin 60^\circ - (-\cos x \cos 30^\circ + \sin x \sin 30^\circ)$ $= \sin x \cos 60^\circ - \cos x \sin 60^\circ + \cos x \cos 30^\circ - \sin x \sin 30^\circ$ $= \sin x \sin 30^\circ - \cos x \sin 60^\circ + \cos x \sin 60^\circ - \sin x \sin 30^\circ$ $= 0$	✓ 2 nd expansion/ 2de uitbreiding ✓ ✓ reduction/reduksie ✓ co-ratios / ko-verh ✓ answer (5)
5.4	Consider: $\frac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x$ $\text{LHS} = \frac{\left(\frac{\sin x}{\cos x} + 1\right)}{\left(\sin x \cdot \frac{\sin x}{\cos x} + \cos x\right)} = \frac{\left(\frac{\sin x + \cos x}{\cos x}\right)}{\left(\frac{\sin^2 x + \cos^2 x}{\cos x}\right)}$ $= \frac{\sin x + \cos x}{\frac{\cos x}{1}}$ $= \frac{\sin x + \cos x}{\cos x} \times \frac{\cos x}{1}$ $= \sin x + \cos x$ $= \text{RHS}$ OR/OF $\text{LHS} = \frac{\left(\frac{\sin x}{\cos x} + 1\right)}{\left(\sin x \cdot \frac{\sin x}{\cos x} + \cos x\right)} = \frac{\left(\frac{\sin x}{\cos x} + 1\right)}{\left(\frac{\sin^2 x + \cos^2 x}{\cos x}\right)}$ $= \frac{\left(\frac{\sin x}{\cos x} + 1\right)}{\frac{1}{\cos x}}$ $= \left(\frac{\sin x}{\cos x} + 1\right) \times \frac{\cos x}{1}$ $= \sin x + \cos x$ $= \text{RHS}$	✓ identity of tan x ✓ $\frac{\sin x + \cos x}{\cos x}$ ✓ $\frac{\sin^2 x + \cos^2 x}{\cos x}$ ✓ $\sin^2 x + \cos^2 x = 1$ ✓ simplify (5)
5.5.1	$(\sqrt{1+k})^2 = (\sin x + \cos x)^2$ $1+k = \sin^2 x + 2 \sin x \cos x + \cos^2 x$ $1+k = 1 + \sin 2x$ $k = \sin 2x$	✓ square both sides ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\sin 2x$ (3)

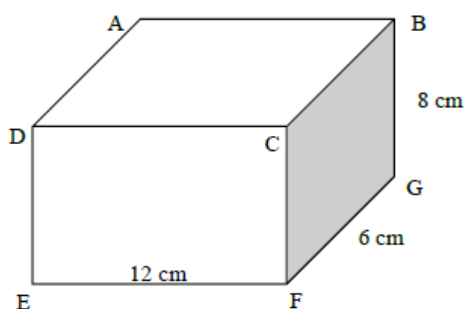
5.5.2	From 5.5.1 $\sin x + \cos x = \sqrt{1 + \sin 2x}$ $\therefore \text{max value: } \sin x + \cos x = \sqrt{1+1}$ $= \sqrt{2}$ OR/OF Maximum value of $1 + \sin 2x = 1 + 1$ $= 2$ $\therefore \text{maximum value of } \sin x + \cos x = \sqrt{2}$ OR/OF $(\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$ $= 1 + \sin 2x$ $\therefore \text{max value } (\sin x + \cos x)^2 = 1 + 1 = 2$ $\therefore \text{max value } \sin x + \cos x = \sqrt{2}$	✓ max of $\sin 2x = 1$ ✓ answer (2) ✓ max of $\sin 2x = 1$ ✓ answer (2) ✓ max of $\sin 2x = 1$ ✓ answer (2)
		[27]

QUESTION/VRAAG 6



6.1	Period = 180°	✓ answer (1)
6.2	-75°	✓ answer (1)
6.3	$\sin 2x \leq \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$ $\sin 2x \leq \cos 45^\circ \cdot \cos x + \sin 45^\circ \cdot \sin x$ $\sin 2x \leq \cos(x - 45^\circ)$ $x \in [-75^\circ ; 165^\circ]$	✓ $\cos 45^\circ \cdot \cos x + \sin 45^\circ \cdot \sin x$ ✓ $\cos(x - 45^\circ)$ ✓ ✓ answer (4)
		[6]

QUESTION/VRAAG 7



Figure/Figuur (i)

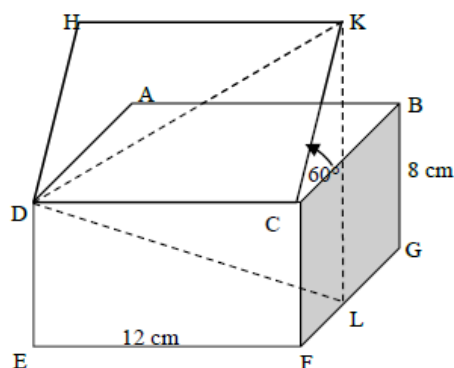
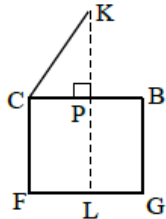


Figure / Figuur (ii)

7.1	KC = 6 cm	✓ answer (1)
7.2	Let P be the point of intersection of KL and CB $\frac{KP}{KC} = \sin 60^\circ$ $KP = 6 \sin 60^\circ$ $KP = 3\sqrt{3} \text{ or } 5,20$ $\therefore KL = 8 + 3\sqrt{3} \text{ or } 13,20 \text{ cm}$ 	✓ trig ratio ✓ length of KP ✓ answer (3)
7.3	$DK^2 = 6^2 + 12^2$ $DK = \sqrt{180} \text{ or } 6\sqrt{5} \text{ or } 13,42 \text{ cm}$ $\frac{\sin \hat{KDL}}{KL} = \frac{\sin \hat{DLK}}{DK}$ $\frac{\sin \hat{KDL}}{\sin \hat{DLK}} = \frac{KL}{DK}$ $= \frac{8 + 3\sqrt{3}}{6\sqrt{5}} \text{ or } \frac{13,20}{13,42} \text{ or } 0,98$	✓ $DK = 6\sqrt{5}$ ✓ use of sine rule $\frac{\sin \hat{KDL}}{\sin \hat{DLK}} = \frac{KL}{DK}$ ✓ answer (4) [8]

KWV: 03**QUESTION 5**

- 5.1 In $\triangle MNP$, $\hat{N} = 90^\circ$ and $\sin M = \frac{15}{17}$.

Determine, **without using a calculator**:

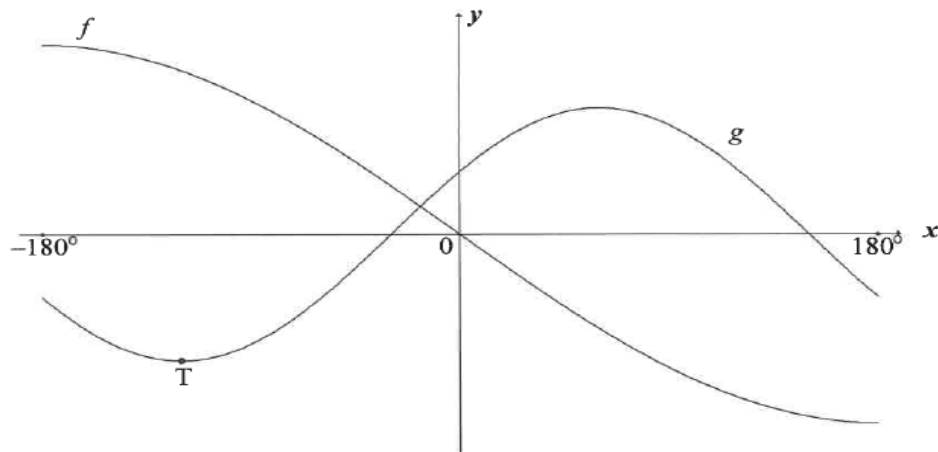
- 5.1.1 $\tan M$ (3)
- 5.1.2 The length of NP if MP = 51 (2)
- 5.2 Simplify to a single term: $\cos(x - 360^\circ) \cdot \sin(90^\circ + x) + \cos^2(-x) - 1$ (4)
- 5.3 Consider: $\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ)$
- 5.3.1 Write as a single trigonometric term in its simplest form. (2)
- 5.3.2 Determine the general solution of the following equation:

$$\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ) = \cos(2x - 20^\circ) \quad (7)$$

[18]

QUESTION 6

In the diagram, the graphs of $f(x) = -3 \sin \frac{x}{2}$ and $g(x) = 2 \cos(x - 60^\circ)$ are drawn in the interval $x \in [-180^\circ; 180^\circ]$. $T(p; q)$ is a turning point of g with $p < 0$.

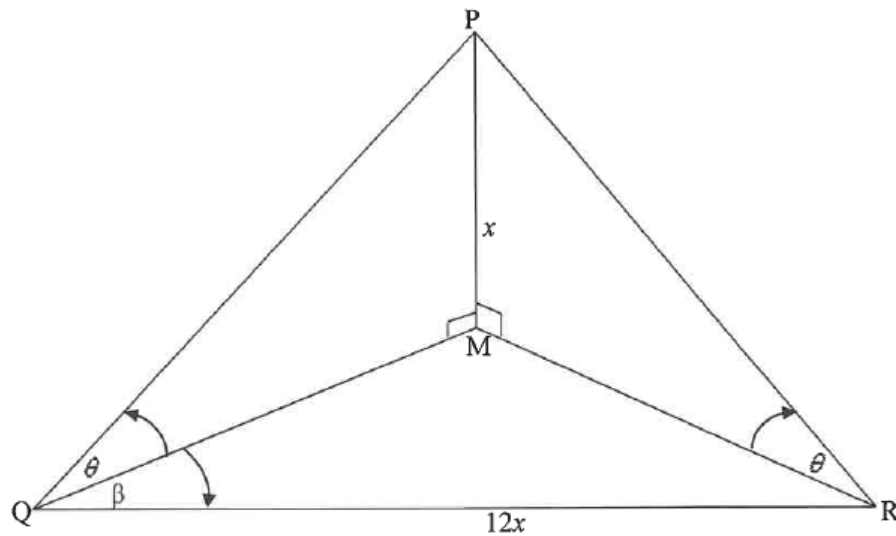


- 6.1 Write down the period of f . (1)
- 6.2 Write down the range of g . (2)
- 6.3 Calculate $f(p) - g(p)$. (3)
- 6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180^\circ; 180^\circ]$ for which:
- 6.4.1 $g(x) > 0$ (3)
- 6.4.2 $g(x) \cdot g'(x) > 0$ (4)

[13]

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ . Q, M and R lie in the same horizontal plane.



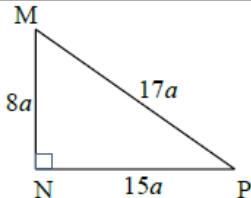
7.1 Write QM in terms of x and θ . (2)

7.2 Prove that $\tan \theta = \frac{\cos \beta}{6}$. (4)

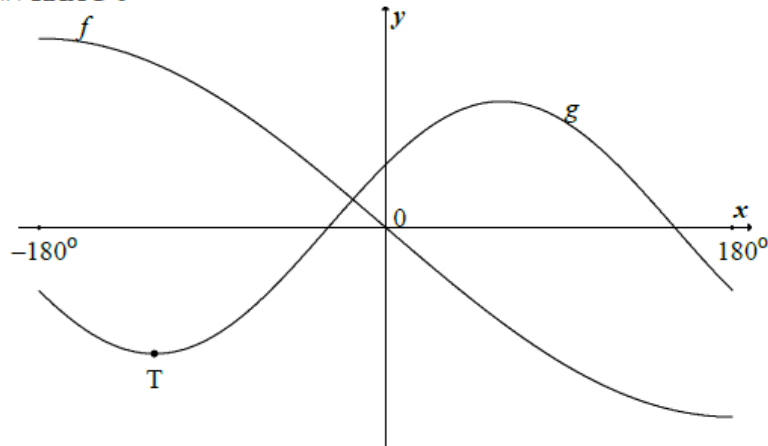
7.3 If $\beta = 40^\circ$ and $QM = 60$ metres, calculate the height of the lighthouse to the nearest metre. (3)
[9]

MEMO: 03

QUESTION/VRAAG 5

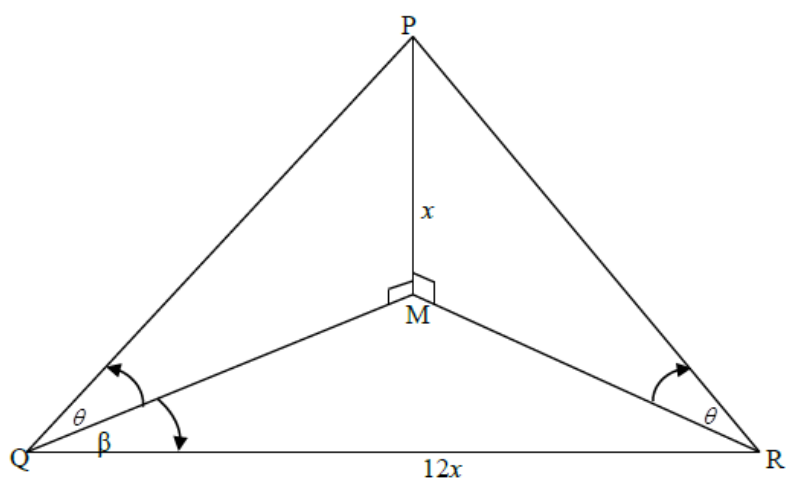
5.1.1	Given : $\sin M = \frac{15}{17}$ $MN^2 = 17^2 - 15^2$ $= 64$ $MN = 8$ OR $\therefore \tan M = \frac{15}{8}$	 ✓ sketch or Pyth ✓ $MN = 8$ ✓ answer (3)
5.1.2	$\sin M = \frac{NP}{MP}$ $\frac{NP}{51} = \frac{15a}{17a}$ $\therefore NP = 45$	✓ equating trig ratios ✓ answer (2)
5.2	$\cos(x - 360^\circ) \cdot \sin(90^\circ + x) + \cos^2(-x) - 1$ $= \cos x \cdot \cos x + \cos^2 x - 1$ $= \cos^2 x + \cos^2 x - 1$ $= 2\cos^2 x - 1$ $= \cos 2x$	✓ $\cos x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ identity (4)
5.3.1	$\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ)$ $= \sin[(2x + 40^\circ) - (x + 30^\circ)]$ $= \sin(x + 10^\circ)$	✓ reduction ✓ answer (2)
5.3.2	$\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ) = \cos(2x - 20^\circ)$ $\therefore \cos(2x - 20^\circ) = \sin(x + 10^\circ)$ $\cos(2x - 20^\circ) = \cos[90^\circ - (x + 10^\circ)]$ $2x - 20^\circ = 80^\circ - x + k \cdot 360^\circ$ or $2x - 20^\circ = 360^\circ - (80^\circ - x) + k \cdot 360^\circ$ $3x = 100^\circ + k \cdot 360^\circ$ or $2x - 20^\circ = 280^\circ + x + k \cdot 360^\circ$ $x = 33,33^\circ + k \cdot 120^\circ$ or $x = 300^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ OR/OF $\therefore \cos(2x - 20^\circ) = \sin(x + 10^\circ)$ $\sin[90^\circ - (2x - 20^\circ)] = \sin(x + 10^\circ)$ $110^\circ - 2x = x + 10^\circ + k \cdot 360^\circ$ or $110^\circ - 2x = 180^\circ - (x + 10^\circ) + k \cdot 360^\circ$ $3x = 100^\circ - k \cdot 360^\circ$ or $110^\circ - 2x = 170^\circ - x + k \cdot 360^\circ$ $x = 33,33^\circ - k \cdot 120^\circ$ or $x = -60^\circ - k \cdot 360^\circ ; k \in \mathbb{Z}$	✓ equating ✓ co ratio ✓ $80^\circ - x$ ✓ $280^\circ + x$ ✓ simplification/vereenv ✓ $x = 33,33^\circ + k \cdot 120^\circ$ ✓ $x = 300^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ (7) ✓ equating ✓ co ratio ✓ $x + 10^\circ$ ✓ $170^\circ - x$ ✓ simplification/vereenv ✓ $x = 33,33^\circ - k \cdot 120^\circ$ ✓ $x = -60^\circ - k \cdot 360^\circ ; k \in \mathbb{Z}$ (7)
		[18]

QUESTION/VRAAG 6



6.1	Period = 720°	✓ answer (1)
6.2	$y \in [-2 ; 2]$ OR/OF $-2 \leq y \leq 2$	✓✓ answer (2) ✓✓ answer (2)
6.3	$f(-120^\circ) - g(-120^\circ)$ $= -3 \sin\left(-\frac{120^\circ}{2}\right) - 2 \cos(-120^\circ - 60^\circ)$ $= \frac{4 + 3\sqrt{3}}{2}$ or 4,60 (4,5980...)	✓ $x = -120^\circ$ ✓ substitution ✓ answer (3)
6.4.1	x-intercepts of g at $-90^\circ + 60^\circ = -30^\circ$ and $90^\circ + 60^\circ = 150^\circ$ $\therefore x \in (-30^\circ ; 150^\circ)$ OR/OF x-intercepts of g at $-90^\circ + 60^\circ = -30^\circ$ and $90^\circ + 60^\circ = 150^\circ$ $-30^\circ < x < 150^\circ$	✓ value ✓ value ✓ answer (3) ✓ value ✓ value ✓ answer (3)
6.4.2	$x \in [-180^\circ ; -120^\circ) \cup (-30^\circ ; 60^\circ) \cup (150^\circ ; 180^\circ]$ OR/OF $-180^\circ \leq x < -120^\circ$ or $-30^\circ < x < 60^\circ$ or $150^\circ < x \leq 180^\circ$	✓ $[-180^\circ ; -120^\circ)$ ✓ $(-30^\circ ; 60^\circ)$ ✓ $(150^\circ ; 180^\circ]$ ✓ notation for inclusive in the first/last interval (4) ✓ $-180^\circ \leq x < -120^\circ$ ✓ $-30^\circ < x < 60^\circ$ ✓ $150^\circ < x \leq 180^\circ$ 1 mark: each interval ✓ notation for inclusive in the first/last interval (4)
		[13]

QUESTION/VRAAG 7



7.1	In $\triangle PMQ$: $\tan \theta = \frac{x}{QM}$ $\therefore QM = \frac{x}{\tan \theta}$ OR/OF $\frac{x}{\sin \theta} = \frac{MQ}{\sin P}$ $MQ = \frac{x \sin P}{\sin \theta}$ $= \frac{x \cos \theta}{\sin \theta}$ $= \frac{x}{\tan \theta}$	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	In $\triangle PMR$: $\tan \theta = \frac{x}{MR}$ OR $\triangle PMQ \cong \triangle PMR$ [AAS/HHS] $\therefore MR = \frac{x}{\tan \theta} = QM$ $\hat{QMR} = 180^\circ - 2\beta$ $\frac{\sin \beta}{MR} = \frac{\sin \hat{QMR}}{12x}$ $\sin \beta \times \frac{\tan \theta}{x} = \frac{\sin(180^\circ - 2\beta)}{12x}$ $\tan \theta = \frac{\sin 2\beta}{12x} \times \frac{x}{\sin \beta}$ $\tan \theta = \frac{2 \sin \beta \cos \beta}{12x} \times \frac{x}{\sin \beta}$ $\tan \theta = \frac{\cos \beta}{6}$ OR	✓ $MR = QM$ ✓ correct substitution into the sine rule in $\triangle QMR$ ✓ reduction ✓ double angle (4)

	In $\triangle PMR$: $\tan \theta = \frac{x}{MR}$ OR $\triangle PMQ \cong \triangle PMR$ [AAS/HHS] $MR^2 = QM^2 + QR^2 - 2QM \cdot QR \cos \beta$ $MR^2 = \left(\frac{x}{\tan \theta}\right)^2 + (12x)^2 - 2\left(\frac{x}{\tan \theta}\right)(12x)(\cos \beta)$ $\frac{x^2}{\tan^2 \theta} = \frac{x^2}{\tan^2 \theta} + 144x^2 - 24\left(\frac{x^2}{\tan \theta}\right)(\cos \beta)$ $24\left(\frac{x^2}{\tan \theta}\right)(\cos \beta) = 144x^2$ $\cos \beta = 6 \tan \theta$ $\tan \theta = \frac{\cos \beta}{6}$	✓ correct substitution into the cosine rule in $\triangle QMR$ ✓ substitution ✓ $MR = QM$ ✓ simplification (4)
7.3	$\frac{x}{QM} = \frac{\cos \beta}{6}$ [both equal $\tan \theta$] $x = \frac{60 \cos 40}{6}$ $x = 7,66$ The height of the lighthouse is 8 metres	✓ equating ✓ subst. $QM = 60$ and $\beta = 40^\circ$ ✓ answer (3)
		[9]

QUESTION 5

5.1 Without using a calculator, write the following expressions in terms of $\sin 11^\circ$:

5.1.1 $\sin 191^\circ$ (1)

5.1.2 $\cos 22^\circ$ (1)

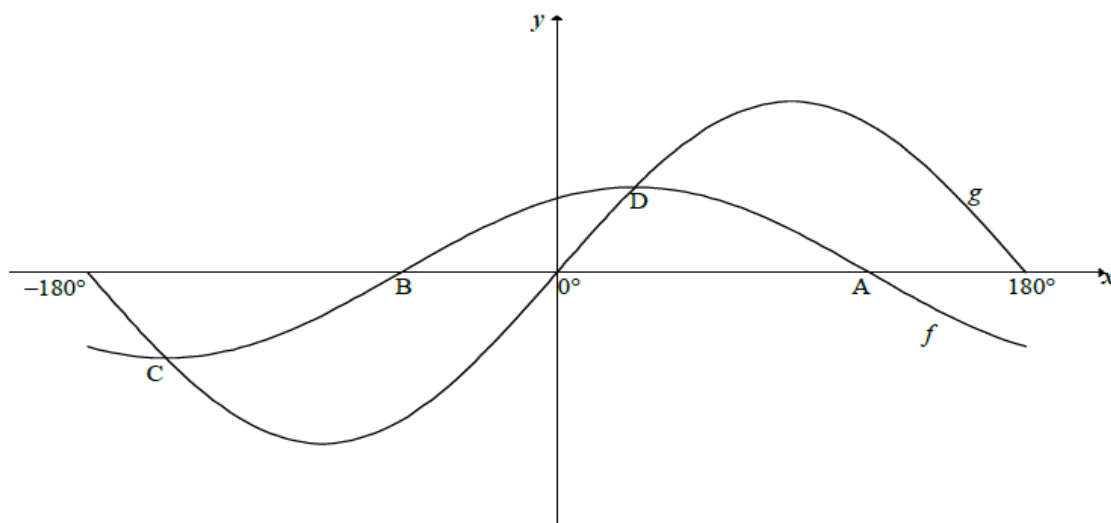
5.2 Simplify $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90^\circ$
Without using a calculator, determine the value of $\sin 2P$. (5)
[12]

QUESTION 6

6.1 Determine the general solution of $\cos(x - 30^\circ) = 2 \sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x - 30^\circ)$ and $g(x) = 2 \sin x$ are drawn for the interval $x \in [-180^\circ; 180^\circ]$. A and B are the x -intercepts of f . The two graphs intersect at C and D, the minimum and maximum turning points respectively of f .



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180^\circ; 180^\circ]$, for which:

(a) Both graphs are increasing (2)

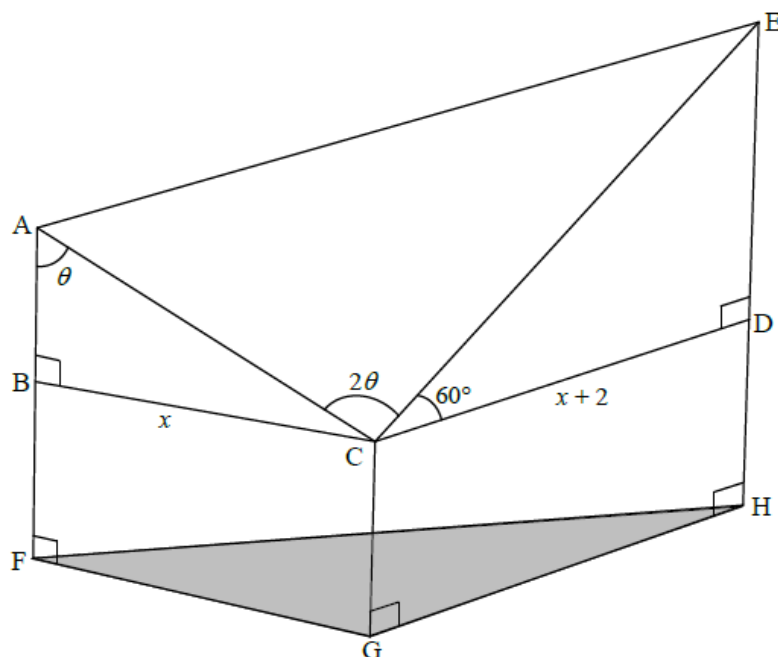
(b) $f(x + 10^\circ) > g(x + 10^\circ)$ (2)

6.2.3 Determine the range of $y = 2^{\sin x + 3}$ (5)
[18]

QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. $\triangle ACE$ forms the roof of an entertainment centre.

$BC = x$, $CD = x + 2$, $\hat{BAC} = \theta$, $\hat{ACE} = 2\theta$ and $\hat{ECD} = 60^\circ$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

7.1.2 CE in terms of x (2)

7.2 Show that the area of the roof $\triangle ACE$ is given by $2x(x+2)\cos\theta$. (3)

7.3 If $\theta = 55^\circ$ and $BC = 12$ metres, calculate the length of AE. (4)
[11]

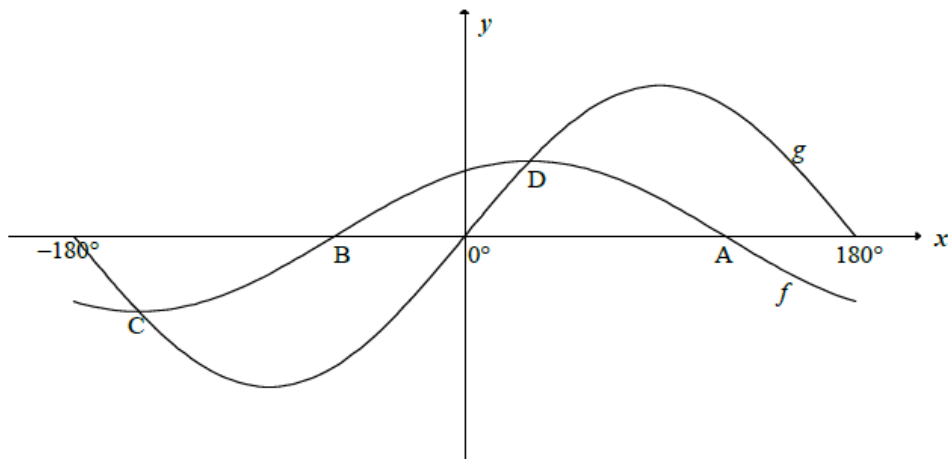
MEMO: 04

QUESTION/VRAG 5

5.1.1	$\sin 191^\circ$ $= -\sin 11^\circ$	$\checkmark -\sin 11^\circ$ (1)
5.1.2	$\cos 22^\circ$ $= \cos(2 \times 11^\circ)$ $= 1 - 2\sin^2 11^\circ$	$\checkmark$ answer (1)
5.2	$\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{1}{\sqrt{2}}\right) + \cos x \left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ OR $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{\sqrt{2}}{2}\right) + \cos x \left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	$\checkmark -\cos x$ $\checkmark$ expansion $\checkmark$ special angle ratios $\checkmark$ simplification of last 2 terms $\checkmark$ answer (5)
5.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$ $\sin^2 P + 2 \sin P \cos P + \cos^2 P = \frac{49}{25}$ $2 \sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$ $= \frac{24}{25}$	$\checkmark \sin Q = \cos P$ $\checkmark$ squaring $\checkmark$ expansion $\checkmark \sin^2 P + \cos^2 P = 1$ $\checkmark$ answer (5)
		[12]

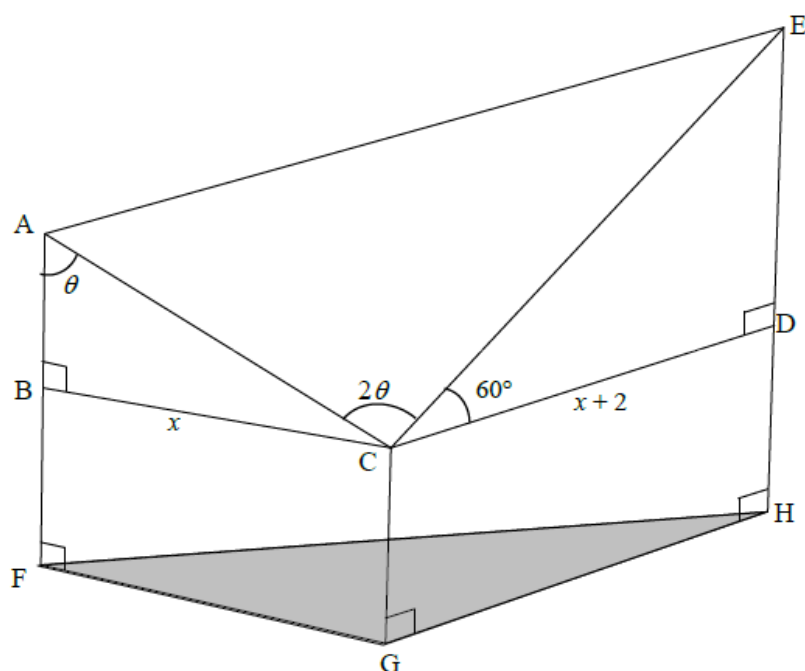
QUESTION/VRAAG 6

6.1	$\cos(x - 30^\circ) = 2 \sin x$ $\cos x \cos 30^\circ + \sin x \sin 30^\circ = 2 \sin x$ $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = 2 \sin x$ $\frac{\sqrt{3}}{2} \cos x = \frac{3}{2} \sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ + k \cdot 180^\circ; \quad k \in \mathbb{Z}$ OR $x = 30^\circ + k \cdot 360^\circ$ or $x = 210^\circ + k \cdot 360^\circ; \quad k \in \mathbb{Z}$	✓ expansion ✓ special $\angle$ s ✓ simplification ✓ equation in tan ✓ 30° ✓ $k \cdot 180^\circ; k \in \mathbb{Z}$ OR ✓ 30° and 210° ✓ $k \cdot 360^\circ; k \in \mathbb{Z}$ (6)
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6.2.1(a)	A(120° ; 0)	✓ answer (1)
6.2.1(b)	C(-150° ; -1)	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90^\circ ; 30^\circ)$ OR $-90^\circ < x < 30^\circ$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160^\circ ; 20^\circ)$ OR $-160^\circ < x < 20^\circ$	✓ endpoints ✓ correct interval (2)
6.2.3	$y = 2^{2 \sin x + 3}$ Range of $y = 2 \sin x$: $y \in [-2 ; 2]$ OR $-2 \leq y \leq 2$ Range of $y = 2 \sin x + 3$: $y \in [1 ; 5]$ OR $1 \leq y \leq 5$ Range: $y = 2^{2 \sin x + 3}$: $y \in [2 ; 32]$ OR $2 \leq y \leq 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
		[18]

QUESTION/VRAAG 7



7.1.1	$\sin \theta = \frac{x}{AC}$ $AC = \frac{x}{\sin \theta}$ OR $\frac{\sin \theta}{x} = \frac{\sin 90^\circ}{AC}$ $AC = \frac{x}{\sin \theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60^\circ = \frac{x+2}{CE}$ $CE = \frac{x+2}{\cos 60^\circ}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$ OR $\frac{\sin 30^\circ}{x+2} = \frac{\sin 90^\circ}{CE}$ $CE = \frac{x+2}{\sin 30^\circ}$ $= 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	$\text{Area } \triangle ACE = \frac{1}{2} AC \cdot EC \cdot \sin \hat{ACE}$ $= \frac{1}{2} \left(\frac{x}{\sin \theta} \right) (2(x+2)) \sin 2\theta$ $= \frac{x(x+2) \times 2 \sin \theta \cos \theta}{\sin \theta}$ $= 2x(x+2) \cos \theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin \theta} (2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)
7.3	$EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC) \cos \hat{ACE}$ $= \left(\frac{12}{\sin 55^\circ} \right)^2 + 28^2 - 2 \left(\frac{12}{\sin 55^\circ} \right) (28) \cos 110^\circ$ $AE = 35,77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]

KWV: 05

QUESTION 5

- 5.1 Given that $\sin \alpha = -\frac{4}{5}$ and $90^\circ < \alpha < 270^\circ$.

WITHOUT using a calculator, determine the value of each of the following in its simplest form:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos \alpha$ (2)

5.1.3 $\sin(\alpha - 45^\circ)$ (3)

- 5.2 Consider the identity: $\frac{8 \sin(180^\circ - x) \cos(x - 360^\circ)}{\sin^2 x - \sin^2(90^\circ + x)} = -4 \tan 2x$

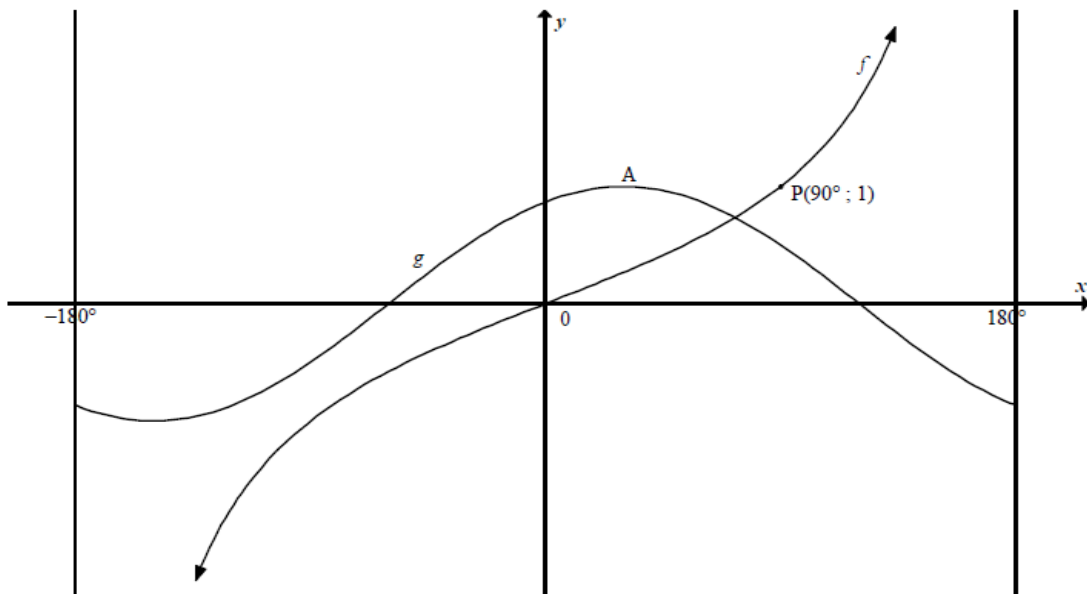
5.2.1 Prove the identity. (6)

5.2.2 For which value(s) of x in the interval $0^\circ < x < 180^\circ$ will the identity be undefined? (2)

- 5.3 Determine the general solution of $\cos 2\theta + 4 \sin^2 \theta - 5 \sin \theta - 4 = 0$. (7)
[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30^\circ)$ are drawn on the same system of axes for $-180^\circ \leq x \leq 180^\circ$. The point $P(90^\circ; 1)$ lies on f . Use the diagram to answer the following questions.

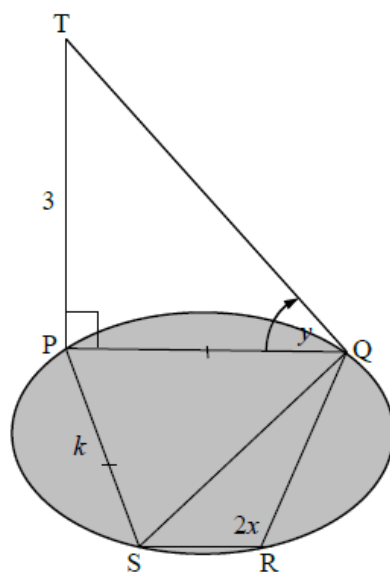


- 6.1 Determine the value of b . (1)
- 6.2 Write down the coordinates of A, a turning point of g . (2)
- 6.3 Write down the equation of the asymptote(s) of $y = \tan b(x + 20^\circ)$ for $x \in [-180^\circ; 180^\circ]$. (1)
- 6.4 Determine the range of h if $h(x) = 2g(x) + 1$. (2)
[6]

QUESTION 7

7.1 Prove that in any acute-angled $\triangle ABC$, $\frac{\sin A}{a} = \frac{\sin B}{b}$. (5)

7.2 The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is y° . $PQ = PS = k$ units, $TP = 3$ units and $\angle SRQ = 2x^\circ$.



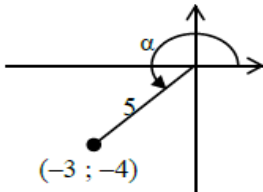
7.2.1 Show, giving reasons, that $\angle PSQ = x$. (2)

7.2.2 Prove that $SQ = 2k \cos x$. (4)

7.2.3 Hence, prove that $SQ = \frac{6 \cos x}{\tan y}$. (2)
[13]

MEMO: 05

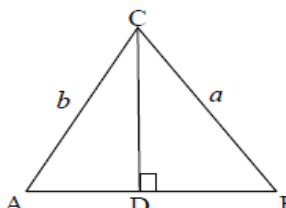
QUESTION 5

5.1.1	$-\sin \alpha$ $= -\left(-\frac{4}{5}\right) = \frac{4}{5}$	✓ reduction ✓ answer (2)
5.1.2	$(-4)^2 + b^2 = 5^2$ $b^2 = 25 - 16 = 9$ $b = -3$ $\cos \alpha = \frac{-3}{5}$ 	✓ $b = -3$ ✓ answer (2)
5.1.3	$\sin(\alpha - 45^\circ)$ $= \sin \alpha \cos 45^\circ - \cos \alpha \sin 45^\circ$ $= -\frac{4}{5} \cdot \frac{1}{\sqrt{2}} - \left(-\frac{3}{5}\right) \cdot \frac{1}{\sqrt{2}}$ $= -\frac{1}{5\sqrt{2}}$ OR $\sin(\alpha - 45^\circ)$ $= \sin \alpha \cos 45^\circ - \cos \alpha \sin 45^\circ$ $= -\frac{4}{5} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{3}{5}\right) \cdot \frac{\sqrt{2}}{2}$ $= -\frac{\sqrt{2}}{10}$	✓ expansion ✓ $\frac{1}{\sqrt{2}}$ ✓ answer in simplest form (3)
5.2.1	$LHS = \frac{8 \sin x \cdot \cos x}{\sin^2 x - \cos^2 x}$ $= \frac{4(2 \sin x \cdot \cos x)}{\sin^2 x - \cos^2 x}$ $= \frac{4 \sin 2x}{-(\cos^2 x - \sin^2 x)}$ $= \frac{4 \sin 2x}{-\cos 2x}$ $= -4 \tan 2x$	✓ $\sin x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ $4 \sin 2x$ ✓ factorise ✓ $-\cos 2x$ (6)
5.2.2	Undefined when $\cos 2x = 0$ or $\tan 2x = \infty$: $x = 45^\circ$ and $x = 135^\circ$	✓ 45° ✓ 135° (2)
5.3	$1 - 2 \sin^2 \theta + 4 \sin^2 \theta - 5 \sin \theta - 4 = 0$ $2 \sin^2 \theta - 5 \sin \theta - 3 = 0$ $(2 \sin \theta + 1)(\sin \theta - 3) = 0$ $\therefore \sin \theta = -\frac{1}{2} \quad \text{or} \quad \sin \theta = 3 \text{ (no solution)}$ $\therefore \theta = 210^\circ + 360^\circ k \quad \text{or} \quad \theta = 330^\circ + 360^\circ k \quad ; k \in \mathbb{Z}$ OR $\therefore \theta = 210^\circ + 360^\circ k \quad \text{of} \quad \theta = 30^\circ + 360^\circ k \quad ; k \in \mathbb{Z}$	✓ $1 - 2 \sin^2 \theta$ ✓ standard form ✓ factors ✓ no solution ✓ 210° ✓ 330° ✓ $+ 360^\circ k \quad ; k \in \mathbb{Z}$ (7) [22]

QUESTION 6

6.1	$b = \frac{1}{2}$	✓ value of b (1)
6.2	A(30° ; 1)	✓ 30° ✓ 1 (2)
6.3	$x = 160^\circ$	✓ $x = 160^\circ$ (1)
6.4	$h(x) = 2\cos(x - 30^\circ) + 1$ $y \in [-1 ; 3]$ OR $-1 \leq y \leq 3$	✓ critical values ✓ notation (2) [6]

QUESTION 7

7.1	Draw $CD \perp AB$ In $\triangle ACD$: $\sin A = \frac{CD}{b} \therefore CD = b \cdot \sin A$ In $\triangle CBD$: $\sin B = \frac{CD}{a} \therefore CD = a \cdot \sin B$ $\therefore b \cdot \sin A = a \cdot \sin B$ $\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$	 ✓ construction ✓ $\sin A$ ✓ making CD the subject ✓ $\sin B$ ✓ $b \cdot \sin A = a \cdot \sin B$ (5)
7.2.1	$\hat{SPQ} = 180^\circ - 2x$ (opp $\angle$s of cyclic quad) $\hat{PSQ} + \hat{PQS} = 2x$ (sum of $\angle$s in $\triangle$) $\hat{PSQ} = \hat{PQS} = x$ ($\angle$s opp equal sides)	✓ $\hat{SPQ} = 180^\circ - 2x$ (S/R) ✓ reason (2)
7.2.2	$\frac{\sin \hat{SPQ}}{\sin(180^\circ - 2x)} = \frac{\sin \hat{PSQ}}{\sin x}$ $\frac{SQ}{\sin(180^\circ - 2x)} = \frac{PQ}{\sin x}$ $SQ = \frac{k \sin 2x}{\sin x}$ $SQ = \frac{k(2 \sin x \cdot \cos x)}{\sin x} = 2k \cos x$ OR $SQ^2 = PQ^2 + PS^2 - 2PQ \cdot PS \cdot \cos \hat{SPQ}$ $= k^2 + k^2 - 2 \cdot k \cdot k \cdot \cos(180^\circ - 2x)$ $= 2k^2 + 2k^2 \cos 2x$ $= 2k^2 + 2k^2(2\cos^2 x - 1)$ $= 4k^2 \cos^2 x$ $SQ = 2k \cos x$	✓ substitution into correct formula ✓ $\sin 2x$ ✓ SQ subject ✓ $2 \sin x \cdot \cos x$ (4) ✓ substitution into correct formula ✓ $-\cos 2x$ ✓ $2\cos^2 x - 1$ ✓ simplification (4)
7.2.3	$\tan y = \frac{3}{k}$ $k = \frac{3}{\tan y}$ $SQ = 2 \cos x \left(\frac{3}{\tan y} \right)$ $\therefore$ $= \frac{6 \cos x}{\tan y}$	✓ tan ratio ✓ k subject and substitution (2) [13]

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