

WTS TUTORING



WTS NUMBER PATTERNS

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WHERE TO START MATHS & SCIENCE IS FOR THE NATION

- ✓ A number relation shows a relationship between two variables, usually x and y .
- ✓ In other words, we can describe a particular number pattern by means of a mathematical formula.

➤ KEY WORDS:

- ✓ **Sum** : the result of addition
- ✓ **Difference** : the result of subtraction
- ✓ **Product** : the result of multiplication
- ✓ **Natural numbers** : whole numbers greater than or equal to 1
- ✓ **Integer** : a positive or negative whole number or zero.
- ✓ **Even numbers** : any integer that can be divided exactly by 2
- ✓ **Odd numbers** : any integer that cannot be divided exactly by 2
- ✓ **Factor** : a number that divides exactly into a whole number with no remainder
- ✓ **Divisible by** : a number is divisible if, after dividing, there is no remainder.
- ✓ **Prime number** : numbers that have only two factors, the number itself and 1.
- ✓ **Multiple** : the product of two natural numbers
- ✓ **Perfect squares**; are rational numbers which can example end with 1;4;16 ;....but never end in a 3 or an 8 etc
- ✓ **Non-Perfect squares**; are irrational numbers which can example end with 3 or 8 but never end in a 4 or an 16 etc
- ✓ **Which term**: that is a number of terms
- ✓ **nth term**: that is general term
- ✓ **determine the 10 th term**; same as T_{10}
- ✓ **existence of the sequence or series**; the common ratio is between -1 and 1 or
- ✓ **greater than**; the first term to be greater then : $T_n > a$ if a is the given nth term
- ✓ **less than**; the first term to be less then : $T_n < a$ if a is the given nth term

- ✓ **never contain a positive or negative term;** calculate the maximum value of the n th term (y value of turning point)
- ✓ **converging series;** if and only if $-1 < r < 1$

➤ ARITHMETIC SEQUENCE

- ✓ An arithmetic sequence or series is a linear number pattern in which the first difference is constant.
- ✓ The general term formula allows you to determine any specific term of an arithmetic sequence.
- ✓ And the sum of formula determines the sum of a specific number of terms of an arithmetic series.
- ✓ The formulae are as follows:

➤ GENERAL / N-TERM/ LAST TERM (T_n)

- ✓ First common difference : $d = T_n - T_{n-1} = T_2 - T_1 = T_3 - T_2$ (more useful if variables are given / and if the unknown must be calculated)
- ✓ General term: $T_n = a + (n - 1)d$
- ✓ Given the last term: implies that T_n is given and u can calculate the first term or number of terms
- ✓ If other terms are not given use the following: $T_1 = a, T_2 = a + d, T_3 = a + 2d$
NB: hence solving simaltenouesly
- ✓ T_n : the value of the specific term
- ✓ n : number of the term in a sequence and is a counting number (the position of n -th term in the sequence)

➤ SERIES

- ✓ When we add the terms of a sequence together, we form a series.
- ✓ We use the symbol S_n to show the sum of the first n terms of a series.
- ✓ So $S_n = T_1 + T_2 + T_3 + T_4 + \dots + T_n$

PROOF:

The sum of the terms in an arithmetic series

We can also deduce a formula for the sum of the first n terms of an arithmetic series.

Suppose once again that we have an AS with $t_1 = a$, and a common difference of d .

Then:

$$\begin{aligned} S_n &= t_1 + t_2 + \dots + t_{n-1} + t_n \\ &= a + (a + d) + \dots + (a + (n-2)d) + (a + (n-1)d) \end{aligned}$$

Now let's rewrite the equation for the sum of the series, but this time writing the terms in the reverse order. In other words, we start with the last term $(a + (n-1)d)$, and finish with the first term. When we add all the terms in the second equation, then the sum will still be S_n .

Therefore, in the second equation, we have:

$$S_n = (a + (n-1)d) + (a + (n-2)d) + \dots + (a + d) + a$$

Now, let's add the two equations together. The left-hand side is easy, because we just need to add $S_n + S_n$, which equals $2S_n$.

Adding the two sets of values on the right-hand side is a bit more complicated.

However, if we group the terms so that we add the first term of each equation together, then add the second terms together, and so on, then the result is as follows:

$$\begin{array}{ccccccc}
 a & & + a + d & & + \dots & + a + (n-2)d & + a + (n-1)d \\
 a + (n-1)d & + a + (n-2)d & + \dots & + a + d & & & + a \\
 \hline
 2a + (n-1)d & + 2a + (n-1)d & + \dots & + 2a + (n-1)d & + 2a + (n-1)d & &
 \end{array}$$

Therefore, the result of adding the two equations together is:

$$2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \dots + (2a + (n-1)d) + (2a + (n-1)d)$$

On the right-hand side of the resulting equation, we see something remarkable. There are n terms between the plus signs, and each term is equal to $2a + (n-1)d$! Therefore, we can rewrite this result as follows:

$$2S_n = n(2a + (n-1)d)$$

Now, remember that the value that we actually want is the sum of the terms in the series. In other words, we want the value of S_n . We can obtain this by dividing both sides of the equation by 2.

Therefore, the general formula for the sum of the terms in an arithmetic series is:

FORMULA :
$$S_n = \frac{n}{2} (2a + (n-1)d)$$

However, we're still not finished! There is one other form that we can use to express the formula for the sum of an arithmetic series. Remember that the final term, (or *general term*) of the series is given by $a + (n-1)d$.

Therefore, if we represent this final term as l , then we can rewrite the formula as:

FORMULA:
$$S_n = \frac{n}{2} (a + l)$$

where l is the final term in the series.

➤ GEOMETRIC SEQUENCE

- ✓ A geometric sequence or series is an exponential number pattern in which the ratio is constant.
- ✓ The general term formula allows you to determine any specific term of a geometric sequence.
- ✓ You have also learnt formulae to determine the sum of a specific number of terms of a geometric series.
- ✓ The formulae are as follows:

➤ GENERAL / N-TERM/ LAST TERM (T_n)

- ✓ Common ratio: (more useful if variables are given / and if the unknown must be calculated, hence applying **orlando pirates sign : kwv rule**)

To find r , we can use the following formula:

$$r = \frac{T_n}{T_{n-1}} = \frac{T_{n+1}}{T_n}$$

For example:

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$$

- ✓ General term: $T_n = ar^{n-1}$
- ✓ Given the last term :implies that T_n is given and u can calculate the first term or number of terms
- ✓ If few terms are given: $T_1 = a$, $T_2 = ar$, $T_3 = ar^2$ NB: solving simaltenouesly
- ✓ Take notes of exponential and logarithms laws

➤ SERIES

- ✓ When we add the terms of a sequence together, we form a series.
- ✓ We use the symbol S_n to show the sum of the first n terms of a series.
- ✓ So $S_n = T_1 + T_2 + T_3 + T_4 + \dots + T_n$

PROOF:

The sum of the terms in a geometric series

The next step is to deduce a formula for the sum S_n of the first n terms of a GS. Once again, suppose that we have a GS with a first term (t_1) equal to a , and a common ratio of r .

We express the sum of the GS as follows:

$$\begin{aligned} S_n &= t_1 + t_2 + \dots + t_n \\ &= a + ar + ar^2 + \dots + ar^{n-1} \end{aligned}$$

Now, if we multiply S_n by r , we get:

$$r \times S_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$

Finally, if we subtract $r \times S_n$ from S_n , then we have:

$$\begin{array}{r} a + ar + ar^2 + \dots + ar^{n-1} \\ - \quad ar + ar^2 + \dots + ar^{n-1} + ar^n \\ \hline a + 0 + 0 + \dots + 0 - ar^n \\ \hline \hline \end{array}$$

Therefore:

$$S_n - r \times S_n = a - ar^n$$

$$S_n(1 - r) = a - ar^n$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Therefore, the general formula for the sum of n terms in a GS is:

$$S_n = \frac{a(1 - r^n)}{1 - r} \text{ or } S_n = \frac{a(r^n - 1)}{r - 1}, \quad \text{since } \frac{a(1 - r^n)}{1 - r} = \frac{-a(r^n - 1)}{-(r - 1)}$$

- Take note there is no difference between two formulae

➤ CONVERGES SERIES

- ✓ The sum of a convergent geometric series tends towards a particular value as the number of terms in the series increases.
- ✓ However, sum will never actually reach that value. We express this mathematically by using the concept of a limit.
- ✓ This is a very useful concept, because it enables us to define a second concept, namely the sum to infinity of the series, which we define as:
- ✓ Is for geometric pattern
- ✓ Note: $-1 < r < 1$ { r is a common ratio $r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$ }
- ✓ Always work with inequalities
- ✓ Sum to infinity: $S_\infty = \frac{a}{1 - r}$

➤ **WHAT ABOUT GEOMETRIC SEQUENCES WITH $R < -1$ OR $R > 1$?**

- ✓ In sequences in which $r < -1$ or $r > 1$
- ✓ We find that $n S$ does not approach a definite limit as n increases.
- ✓ In such a case, the sequence diverges (or the sequence is divergent).

KUTHI HUUUUUU!

If you are asked to find the general term (T_n) of a sequence, you must first determine whether it is:

- linear (arithmetic) – the *first difference* is a constant;
- quadratic – the *second difference* is a constant; or
- geometric – there is a common ratio.

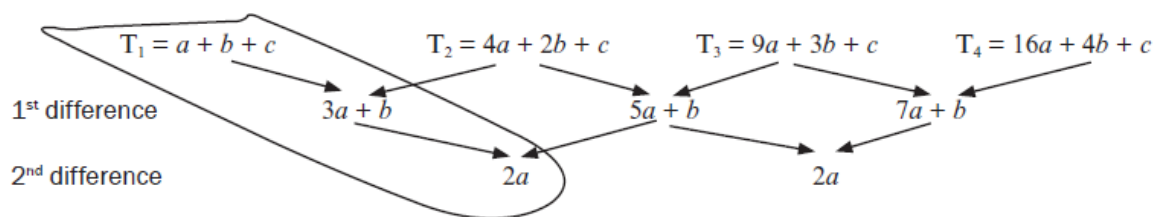
➤ QUADRATIC PATTERNS

- ✓ It is a sequence whereby there is a constant second difference (d).

➤ GENERAL TERM

- ✓ Second common difference : $2a = d$
- ✓ If the unknown given workout the second difference using given variable and hence equate the second difference in order to solve the unknown.
- ✓ General term: $T_n = an^2 + bn + c$ (NB: substitute the values at the end)

KEY:



If the sequence is quadratic, $T_n = an^2 + bn + c$, we then use the formula:

$a + b + c$ = the first term of the sequence

$3a + b$ = the first value of the first difference

$2a$ = the second difference

- ✓ $a + b + c = T_1$
 - ✓ $3a + b = T_2 - T_1$
 - ✓ $2a = d$
- Given the T_n implies that you can calculate the number of terms
 - If few terms are given substitute into: $T_n = an^2 + bn + c$ and
 - hence solve simultaneously

**➤ DETERMINE BETWEEN TWO CONSECUTIVE
TERMS**

- ✓ **To calculate the first difference between given terms**

❖ Using linear nth term

- ✓ The term will always lie in the lowest number
- ✓ Firstly calculate the n term and substitute with : T_{n-1}

❖ Using quadratic nth term

- ✓ Simple substitute into the nth term by calculating the difference starting with lower number.

❖ To calculate two terms between

- ✓ Firstly calculate the number of terms and substitute into nth term and take note to increase the number for the difference.

NB: $d_n = T_n - T_{n+1}$

➤ COMBINATION OF AP & GP

- ✓ Separate the sequence into two
- ✓ Divide the number of terms in a counting number form and consider the odd and even position.
- ✓ The constant sequence will be given by : $S_n = n \cdot c$
- ✓ The number of terms : $T_n = T_n - T_{n-1}$

➤ GIVEN SUM FORMULA

- ✓ Take note of the formula of calculating the term : $T_n = S_n - S_{n-1}$
- ✓ Be in position to calculate the first three terms from the given sum formula, hence general term

➤ INTERPRETATIONS

KEY:

- ✓ Given the last term: implies that T_n is given and you can calculate the first term or number of terms
- ✓ If other terms are omitted use the following key : $T_1 = a, T_2 = a + d, T_3 = a + 2d$
and or : $T_1 = a, T_2 = ar, T_3 = ar^2$

NB: hence solving simaltenouesly

➤ SIGMA NOTATION

- ✓ Sigma notation offers a useful shorthand method to indicate the sum of a number of terms in a series.
- ✓ The symbol $\sum$ is the Greek capital letter S, and indicates sum.
- ✓ We use the symbol as follows

$$\sum_i^n T_n$$

- ✓ Means the sum of n-th terms
- ✓ n : stand for last value after substituting in the n-th terms
- ✓ i : stand for the first value after substituting in the n-th terms
- ✓ T_n : stand for the general term
- ✓ NB: always determine the first 3 terms in order to know the type of the pattern e.g. d-arithmetic or r-geometric

- ✓ Number of terms : $n = top - bottom + 1$
- ✓ Given the sum: you can calculate the number of terms
- ✓ If the question says evaluate or calculate it means determine the sum of n-th terms
- ✓ For the sum: First three terms-hence the pattern-no. of terms –the formula
- ✓ Writing in sigma notation: First three terms-hence the pattern-no. of terms –and lastly
 $i = 1$ for the first value

KWV: 01

QUESTION 2

- 2.1 Given the arithmetic series: $a + 13 + b + 27 + \dots$
- 2.1.1 Show that $a = 6$ and $b = 20$ (2)
- 2.1.2 Calculate the sum of the first 20 terms of the series. (3)
- 2.1.3 Write the series in QUESTION 2.1.2 in sigma notation. (2)
- 2.2 Given the geometric series: $(x - 2) + (x^2 - 4) + (x^3 + 2x^2 - 4x - 8) + \dots$
- 2.2.1 Determine the values of x for which the series converges. (4)
- 2.2.2 If $x = -\frac{3}{2}$, calculate the sum to infinity of the given series. (3)
- [14]**

QUESTION 3

The first four terms of a quadratic number pattern are $-1 ; 2 ; 9 ; 20$.

- 3.1 Determine the general term of the quadratic number pattern. (4)
- 3.2 Calculate the value of the 48th term of the quadratic number pattern. (2)
- 3.3 Show that the sum of the first differences of this quadratic number pattern can be given by $S_n = 2n^2 + n$ (3)
- 3.4 If the sum of the first 69 first differences in QUESTION 3.3 equals 9 591 (that is, $S_{69} = 9\,591$), which term of the quadratic number pattern has a value of 9 590? (2)
- [11]**

MEMO: 01

QUESTION/VRAAG 2

2.1.1	$27 - b = b - 13$ $b = \frac{27 + 13}{2}$ $b = 20$ $27 - 20 = 13 - a$ $a = 6$ OR/OF $27 - 13 = 2d$ $d = 7$ $b = 13 + 7 = 20$ $a = 13 - 7 = 6$	$\checkmark 27 - b = b - 13$ $\checkmark 27 - 20 = 13 - a$ (2) $\checkmark d = 7$ or $27 - 13 = 2d$ $\checkmark b = 13 + 7$ $a = 13 - 7$ (2)
2.1.2	$a = 6 \quad d = 7$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_{20} = \frac{20}{2}[2(6) + (20 - 1)(7)]$ $= 1450$ OR/OF $T_{20} = a + 19(d)$ $= 6 + 19(7)$ $= 139$ $S_n = \frac{n}{2}[a + T_n]$ $S_{20} = \frac{20}{2}[6 + 139]$ $= 1450$	$\checkmark d = 7$ $\checkmark$ correct substitution into correct formula $\checkmark$ answer (3) $\checkmark d = 7$ $\checkmark T_{20} = 139$ $\checkmark$ answer (3)

2.1.3	$T_n = 6 + (n-1)(7)$ $= 7n - 1$ $\sum_{n=1}^{20} (6 + 7(n-1))$ $= \sum_{n=1}^{20} (7n - 1)$	$\checkmark T_n = 6 + (n-1)(7) \text{ or } 7n-1$ $\checkmark \sum_{n=1}^{20}$ (2)
2.2.1	$r = \frac{(x-2)(x+2)}{x-2} \quad \text{or} \quad r = \frac{(x^2-4)(x+2)}{x^2-4}$ $= x+2$ For convergence/Om te konvergeer: $-1 < r < 1$ $-1 < x+2 < 1$ $-3 < x < -1$	$\checkmark \frac{(x^2-4)}{x-2} \text{ or } \frac{(x-2)(x+2)}{x-2} \text{ or } \frac{(x^2-4)(x+2)}{x^2-4}$ $\checkmark r = x+2$ $\checkmark -1 < r < 1$ $\checkmark \text{ answer}$ (4)
2.2.2	$\left(-\frac{7}{2}\right) + \left(-\frac{7}{4}\right) + \left(-\frac{7}{8}\right) + \dots$ $S_{\infty} = \frac{a}{1-r}$ $= \frac{-\frac{7}{2}}{1-\frac{1}{2}}$ $= -7$ OR/OF $S_{\infty} = \frac{a}{1-r}$ $= \frac{(x-2)}{1-(x+2)}$ $= \frac{x-2}{-x-1}$ $= \frac{-\frac{3}{2}-2}{\frac{3}{2}-1}$ $= \frac{-\frac{7}{2}}{\frac{1}{2}}$ $= -7$	$\checkmark a = -\frac{7}{2}$ $\checkmark \text{ substitution into correct formula}$ $\checkmark \text{ answer}$ (3) $\checkmark \text{ substitution into correct formula}$ $\checkmark \text{ substitution of } x = -\frac{3}{2}$ $\checkmark \text{ answer}$ (3) [14]

QUESTION/VRAAG 3

3.1	$2a = 4$ $a = 2$ $3a + b = 3$ $b = -3$ $a + b + c = -1$ $c = 0$ $T_n = 2n^2 - 3n$ OR/OF $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= (-1) + (n-1)(3) + \frac{(n-1)(n-2)}{2}(4)$ $= -1 + 3n - 3 + 2n^2 - 6n + 4$ $= 2n^2 - 3n$	✓ 2nd difference = 4 ✓ $a = 2$ ✓ $b = -3$ ✓ $T_n = 2n^2 - 3n$ (4) ✓ formula ✓ 2nd difference = 4 ✓ simplifying ✓ $T_n = 2n^2 - 3n$ (4)
3.2	$T_n = 2n^2 - 3n$ $T_{48} = 2(48)^2 - 3(48)$ $= 4464$	✓ substitution ✓ answer (2)
3.3	$3 + 7 + 11 + \dots$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(3) + (n-1)4]$ $= \frac{n}{2}[6 + 4n - 4]$ $= 2n^2 + n$	✓ $a = 3$ ✓ $d = 4$ ✓ substitution into correct formula (3)
3.4	$S_{69} = 9591$ and $T_1 = -1$ (of the original sequence/van die oorspronklike ry) $9591 + (-1) = 9590$ $S_{69} + T_1 = 9590$ The 70 th term of the original sequence will have a value of 9590 OR/OF $2n^2 - 3n = 9590$ $2n^2 - 3n - 9590 = 0$ $(n-70)(2n+137) = 0$ $n = 70$ $T_{70} = 9590$	✓ $(9591) + (-1)$ ✓ 70 (2) ✓ $2n^2 - 3n - 9590 = 0$ ✓ 70 (2) [11]

KWV: 02

QUESTION 2

2.1 Given the geometric sequence: $3; 2; k; \dots$

2.1.1 Write down the value of the common ratio. (1)

2.1.2 Calculate the value of k . (2)

2.1.3 Calculate the value of n if $T_n = \frac{128}{729}$. (4)

2.2 In a Mathematics competition, the total prize money for the finalists is R30 500. Each finalist will receive a part of the prize money according to his/her position at the end of the competition. The table below shows the position of the finalists at the end of the competition and the prize money received.

POSITION OF THE FINALIST AT THE END OF THE COMPETITION	PRIZE MONEY
Last	R100
Second from last	R250
Third from last	R400
Fourth from last	R550
.	.
.	.
.	.
.	.
First	Rx

2.2.1 Calculate the prize money of the finalist finishing 18th from last. (2)

2.2.2 Calculate x . (6)
[15]

QUESTION 3

Given the quadratic sequence: $0; 17; 32; \dots$

3.1 Determine an expression for the general term, T_n , of the quadratic sequence. (4)

3.2 Which terms in the quadratic sequence have a value of 56? (3)

3.3 Hence, or otherwise, calculate the value of $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$. (4)
[11]

MEMO: 02

QUESTION/VRAAG 2

2.1.1	$3; 2; k; \dots$ $r = \frac{2}{3}$	$\checkmark r = \frac{2}{3} / 0,67$ (1)
2.1.2	$r = \frac{T_3}{T_2}$ $T_3 = r \times T_2$ $= \frac{2}{3} \times 2$ $= \frac{4}{3}$ Thus $k = \frac{4}{3}$	$\checkmark \frac{2}{3} \times 2$ $\checkmark \frac{4}{3} / 1,34$ (2)
2.1.3	$T_n = ar^{n-1}$ $\frac{128}{729} = 3 \times \left(\frac{2}{3}\right)^{n-1}$ $\left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187}$ $\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^7$ $n-1 = 7$ $n = 8$ OR / OF $T_n = ar^{n-1}$ $\frac{128}{729} = 3 \times \left(\frac{2}{3}\right)^{n-1}$ $\left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187}$ $n-1 = \log_{\frac{2}{3}} \frac{128}{2189}$ $n-1 = 7$ $n = 8$	$\checkmark \frac{128}{729} = 3 \times \left(\frac{2}{3}\right)^{n-1}$ $\checkmark \left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187}$ $\checkmark \left(\frac{2}{3}\right)^7$ $\checkmark$ answer OR / OF $\checkmark \frac{128}{729} = 3 \times \left(\frac{2}{3}\right)^{n-1}$ $\checkmark \left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187}$ $\checkmark n-1 = \log_{\frac{2}{3}} \frac{128}{2189}$ $\checkmark$ answer (4)
2.2.1	$T_n = a + (n-1)d$ $T_{18} = 100 + (18-1)(150)$ $= R\ 2\ 650$	$\checkmark$ substitution of n , a and d into AS $\checkmark\ 2\ 650$ (2)

2.2.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $30\,500 = \frac{n}{2}[2(100) + (n-1)(150)]$ $61\,000 = n(150n + 50)$ $61\,000 = 150n^2 + 50n$ $3n^2 + n - 1\,220 = 0$ $(3n + 61)(n - 20) = 0$ $n = -\frac{61}{3} \text{ or } n = 20$ N/A $x = 100 + (20-1)(150)$ $= R\,2\,950$	✓ substitute 30 500, a and d into sum formula for AS ✓ simplification ✓ factors or quad formula ✓ $n = 20$ ✓ substitution T_n of AS ✓ 2 950 (6) [15]
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QUESTION/VRAAG 3

3.1	First differences: 17; 15 Second difference: -2 $T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2} = \frac{-2}{2} = -1$ $3a + b = 17$ $3(-1) + b = 17$ $b = 20$ $a + b + c = 0$ $-1 + 20 + c = 0$ $c = -19$ $T_n = -n^2 + 20n - 19$ OR / OF First differences: 17; 15 $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= (0) + (n-1)(17) + \frac{(n-1)(n-2)}{2}(-2)$ $= 17n - 17 - n^2 + 3n - 2$ $= -n^2 + 20n - 19$	✓ 17; 15 ✓ value of a ✓ value of b ✓ value of c ✓ 17; 15 ✓ value of a ✓ value of b ✓ value of c (4)
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3.2	$56 = -n^2 + 20n - 19$ $n^2 - 20n + 75 = 0$ $(n - 15)(n - 5) = 0$ $n = 5 \text{ or } n = 15$	✓ $T_n = 56$ ✓ factors ✓ both answers (3)
3.3	$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$ $= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T_{15}$ $= (T_5 - T_{15}) + (T_6 - T_{14}) + \dots + (T_9 - T_{13}) + T_{10}$ $= T_{10}$ because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ $T_{10} = -(10)^2 + 20(10) - 19$ $= 81$ OR/OF $ \begin{array}{ccccccc} & & T_5 & & T_{10} & & T_{15} \\ 0; & 17; & 32; & 45; & 56; & 65; & 72; & 77; & 80; & 81; & 80; & 77; & 72; & 65; & 56 \end{array} $ Hence, $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$ $= (56 + 65 + 72 + 77 + 80 + 81) - (80 + 77 + 72 + 65 + 56)$ $= 81$	✓ ✓ symmetry of terms ✓ T_{10} ✓ 81 (4) ✓ writing out the symmetry of terms ✓ $56 + 65 + 72 + 77 + 80 + 81$ ✓ $80 + 77 + 72 + 65 + 56$ ✓ 81 (4) [11]

KWV: 03

QUESTION 2

- 2.1 Given the quadratic pattern: 5 ; 10 ; 17 ; 26 ; ...
- 2.1.1 Write down the next TWO terms of the pattern. (2)
- 2.1.2 Determine the formula for the n^{th} term of the pattern. (4)
- 2.1.3 Which term of the pattern will have a value of 1 765? (4)
- 2.2 The first 24 terms of an arithmetic series are: $35 + 42 + 49 + \dots + 196$.
- Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7. (5)
- [15]**

QUESTION 3

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

- 3.1 What distance will he cover on day 3 of the trip? (2)
- 3.2 On what day of the trip will Themba pass the halfway point? (4)
- 3.3 Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage. (3)
- [9]**

MEMO: 03

QUESTION/VRAAG 2

2.1.1	37; 50	✓ 37 ✓ 50 (2)
2.1.2	$a = \frac{\text{second difference}}{2} = \frac{2}{2} = 1$ $3a + b = 5$ $3 + b = 5$ $b = 2$ $a + b + c = 5$ $1 + 2 + c = 5$ $c = 2$ $T_n = an^2 + bn + c$ $= n^2 + 2n + 2$	✓ second difference of 2 ✓ $a = 1$ ✓ $b = 2$ ✓ $c = 2$ (4)
2.1.3	$n^2 + 2n + 2 = 1765$ $n^2 + 2n - 1763 = 0$ $(n + 43)(n - 41) = 0$ $n = -43 \text{ or } n = 41$ N/A OR/OF $n^2 + 2n + 2 = 1765$ $n^2 + 2n - 1763 = 0$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-2 \pm \sqrt{2^2 - 4(1)(-1763)}}{2(1)}$ $= \frac{-2 \pm \sqrt{7056}}{2}$ $n = -43 \text{ or } n = 41$ N/A	✓ equating T_n to 1765 ✓ standard form ✓ factors ✓ answer with rejection (4) OR/OF ✓ equating T_n to 1765 ✓ standard form ✓ subt in correct formula ✓ answer with rejection (4)

QUESTION/VRAAG 3

3.1	$r = 0,94; \quad a = 100$ $T_3 = ar^2$ $= 100(0,94)^2$ $= 88,36 \text{ km}$	$\checkmark r = 0,94$ $\checkmark \text{ answer} \quad (2)$
3.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $750 = \frac{100(0,94^n - 1)}{0,94 - 1}$ $\frac{750(-0,06)}{100} = 0,94^n - 1$ $0,94^n = 1 - \frac{9}{20} \quad \text{or} \quad \left(\frac{47}{50}\right)^n = \frac{11}{20}$ $0,94^n = 0,55$ $n = \frac{\log 0,55}{\log 0,94}$ $= 9,66$ He will pass the halfway point on the 10th day <i>Hy sal die halfpadmerk verbystee op die 10^{de} dag</i>	$\checkmark \text{ substitution into correct formula}$ $\checkmark 0,94^n = 0,55$ $\checkmark \text{ use of logarithms}$ $\checkmark \text{ answer} \quad (4)$
3.3	$S_\infty = \frac{a}{1 - r}$ $1500 < \frac{100}{1 - r}$ $1 - r < \frac{100}{1500}$ $r > \frac{14}{15} \quad \text{or} \quad 93,33\%$	$\checkmark \text{ use of } S_\infty \text{ formula}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer} \quad (3)$

[9]

KWV: 04

QUESTION 2

- 2.1 The first FOUR terms of a quadratic pattern are: 15 ; 29 ; 41 ; 51
- 2.1.1 Write down the value of the 5th term. (1)
- 2.1.2 Determine an expression for the n^{th} term of the pattern in the form $T_n = an^2 + bn + c$. (4)
- 2.1.3 Determine the value of T_{27} (2)
- 2.2 Given a geometric sequence: 36 ; -18 ; 9 ; ...
- 2.2.1 Determine the value of r , the common ratio. (1)
- 2.2.2 Calculate n if $T_n = \frac{9}{4\,096}$ (3)
- 2.2.3 Calculate S_∞ (2)
- 2.2.4 Calculate the value of $\frac{T_1 + T_3 + T_5 + T_7 + \dots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \dots + T_{500}}$ (4)
- [17]

QUESTION 3

- 3.1 The first three terms of an arithmetic sequence are: $2p + 3$; $p + 6$ and $p - 2$.
- 3.1.1 Show that $p = 11$. (2)
- 3.1.2 Calculate the smallest value of n for which $T_n < -55$. (3)
- 3.2 Given that $\sum_{k=1}^6 (x - 3k) = \sum_{k=1}^9 (x - 3k)$, prove that $\sum_{k=1}^{15} (x - 3k) = 0$. (5)
- [10]

MEMO: 03

QUESTION/VRAAG 2

2.1.1	59	✓ answer (1)
2.1.2	$ \begin{array}{ccccccc} 15 & & 29 & & 41 & & 51 \\ & \backslash & / & \backslash & / & \backslash & / \\ & 14 & & 12 & & 10 & \\ & & \backslash & / & \backslash & / & \\ & & -2 & & -2 & & \end{array} $ $ \begin{aligned} 2a &= -2 \\ a &= -1 \\ 3(-1) + b &= 14 \\ b &= 17 \\ (-1) + (17) + c &= 15 \\ c &= -1 \\ T_n &= -n^2 + 17n - 1 \end{aligned} $	✓ second difference of -2 ✓ a ✓ b ✓ c (4)
2.1.3	$ \begin{aligned} T_{27} &= -(27)^2 + 17(27) - 1 \\ &= -271 \end{aligned} $	✓ substitution ✓ answer (2)
2.2.1	$r = \frac{-18}{36} = -\frac{1}{2}$	✓ answer (1)
2.2.2	$ \begin{aligned} T_n &= 36 \left(-\frac{1}{2} \right)^{n-1} \\ \frac{9}{4096} &= 36 \left(-\frac{1}{2} \right)^{n-1} \\ \frac{1}{16384} &= \left(-\frac{1}{2} \right)^{n-1} \\ \left(-\frac{1}{2} \right)^{14} &= \left(-\frac{1}{2} \right)^{n-1} \\ 14 &= n - 1 \\ n &= 15 \\ \text{OR/OF} \\ 36; -18; 9; \frac{-9}{2}; \frac{9}{4}; \frac{-9}{8}; \dots; \frac{9}{4096} \\ \text{If you look only at the denominator: } 2; 4; 8; \dots; 4096 \\ 2^k &= 4096 \\ 2^k &= 2^{12} \\ k &= 12 \\ \therefore n &= 15 \text{ terms} \end{aligned} $	✓ $T_n = 36 \left(-\frac{1}{2} \right)^{n-1}$ ✓ $\frac{1}{16384} = \left(-\frac{1}{2} \right)^{n-1}$ ✓ answer (3) OR/OF ✓ $2^k = 4096$ ✓ $k = 12$ ✓ answer (3)

2.2.3	$S_{\infty} = \frac{a}{1-r}$ $= \frac{36}{1 - \left(-\frac{1}{2}\right)}$ $= 24$	✓ correct subst into correct formula with $-1 < r < 1$ ✓ answer if $-1 < r < 1$ (2)
2.2.4	$S_{250 \text{ even}} = \frac{-18 \left(\left(\frac{1}{4} \right)^{250} - 1 \right)}{\frac{1}{4} - 1}$ $= -24$ $S_{250 \text{ odd}} = \frac{36 \left(\left(\frac{1}{4} \right)^{250} - 1 \right)}{\frac{1}{4} - 1}$ $= 48$ $\frac{S_{\text{odd}}}{S_{\text{even}}} = \frac{48}{-24}$ $= -2$ OR/OF $\frac{T_1 + T_3 + T_5 + T_7 + \dots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \dots + T_{500}}$ $= \frac{a + ar^2 + ar^4 + \dots + ar^{498}}{ar + ar^3 + ar^5 + \dots + ar^{499}}$ $= \frac{a + ar^2 + ar^4 + \dots + ar^{498}}{r(a + ar^2 + ar^4 + \dots + ar^{498})}$ $= \frac{1}{r}$ $= -2$	✓ $r = \frac{1}{4}$ and $n = 250$ ✓ $S_{250 \text{ even}} = -24$ ✓ $S_{250 \text{ odd}} = 48$ ✓ answer (4) OR/OF ✓ $a + ar^2 + ar^4 + \dots + ar^{498}$ ✓ $ar + ar^3 + ar^5 + \dots + ar^{499}$ ✓ $r(a + ar^2 + ar^4 + \dots + ar^{498})$ ✓ answer (4) [17]

QUESTION/VRAAG 3

3.1.1	$p + 6 - (2p + 3) = p - 2 - (p + 6)$ $-p + 3 = -8$ $p = 11$	✓ equating i.t.o p ✓ simplifying (2)
3.1.2	$T_n = 25 + (n-1)(-8) = 33 - 8n$ $33 - 8n < -55$ $-8n < -88$ $n > 11$ ∴ Term 12 will be the first term smaller than -55 ∴ Term 12 sal die eerste term kleiner as -55 wees.	✓ subst into T_n formula ✓ $n > 11$ ✓ $n = 12$ (3)
3.2	$S_6 = \frac{n}{2}[a + l] = \frac{6}{2}[(x-3) + (x-18)]$ $= 6x - 63$ $S_9 = \frac{n}{2}[a + l] = \frac{9}{2}[(x-3) + (x-27)]$ $= 9x - 135$ $6x - 63 = 9x - 135$ $3x = 72$ $x = 24$ $\therefore S_{15} = \frac{n}{2}[a + l] = \frac{15}{2}[(x-3) + (x-45)]$ $= \frac{15}{2}[2x - 48]$ $= \frac{15}{2}[2(24) - 48] = 0 = \text{RHS}$ OR/OF $\sum_{k=7}^9 (x - 3k) = 0$ $(x - 21) + (x - 24) + (x - 27) = 0$ $\therefore 3x - 72 = 0$ $3x = 72$ $x = 24$ $\sum_{k=1}^{15} (24 - 3k)$ $= 21 + 18 + 15 + \dots + -21.$ $S_n = \frac{n}{2}[a + l]$ $= \frac{15}{2}[21 - 21]$ $= 0 = \text{RHS}$ OR/OF	✓ $6x - 63$ ✓ $9x - 135$ ✓ 24 ✓ $\frac{15}{2}[(x-3) + (x-45)]$ ✓ substitution of x (5) OR/OF ✓ expansion ✓ $3x - 72 = 0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) OR/OF

$ \begin{aligned} &(x-3) + (x-6) + (x-9) + (x-12) + (x-15) + (x-18) \\ &= (x-3) + (x-6) + (x-9) + (x-12) + (x-15) + (x-18) \\ &\quad + (x-21) + (x-24) + (x-27) \\ &\therefore 3x - 72 = 0 \\ &\quad 3x = 72 \\ &\quad x = 24 \end{aligned} $ $ \begin{aligned} &\sum_{k=1}^{15} (24 - 3k) \\ &= 21 + 18 + 15 + \dots + -21. \end{aligned} $ $ \begin{aligned} S_n &= \frac{n}{2} [a + l] \\ &= \frac{15}{2} [21 - 21] \\ &= 0 = \text{RHS} \end{aligned} $	✓ expansion ✓ $3x - 72 = 0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) [10]
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KWV: 05

QUESTION 2

- 2.1 Given the arithmetic series: $18 + 24 + 30 + \dots + 300$
- 2.1.1 Determine the number of terms in this series. (3)
- 2.1.2 Calculate the sum of this series. (2)
- 2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)
- 2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.
- 2.2.1 Determine the n^{th} term of the sequence. (2)
- 2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)
- 2.2.3 Calculate the sum to infinity of this sequence. (2)
- [16]

QUESTION 3

- 3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$.
- 3.1.1 Determine the value of a . (2)
- 3.1.2 Determine the formula for T_n . (4)
- 3.2 Given the series: $(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \dots + (81 \times 82)$
- Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)
- [10]

MEMO: 05

QUESTION/VRAAG 2

2.1.1	$T_n = a + (n-1)d$ $300 = 18 + (n-1)6$ $300 = 18 + 6n - 6$ $6n = 288$ $n = 48$	$\checkmark a = 18$ and $d = 6$ $\checkmark T_n = 300$ $\checkmark$ answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{48}{2}[2(18) + 47(6)]$ $= 7632$	$\checkmark$ substitution in formula $\checkmark$ answer (2)
2.1.3	Sum of all numbers from 1 to 300 / <i>Som van alle getalle van 1 tot 300</i> $= \frac{300}{2}[2(1) + 299(1)]$ $= \frac{300(301)}{2}$ $= 45150$ Sum of numbers not divisible by 6 / <i>Som van getalle wat nie deelbaar deur 6 is nie</i> $= 45150 - (7632 + 6 + 12)$ $= 37500$	$\checkmark$ substitution $\checkmark$ answer $\checkmark (7632 + 6 + 12)$ $\checkmark$ answer (4)
2.2.1	16, 8; 4; $r = \frac{1}{2}$ $T_n = ar^{n-1}$ $= 16\left(\frac{1}{2}\right)^{n-1}$ $= 2^4(2^{-n+1})$ $= 2^{5-n}$	$\checkmark r = \frac{1}{2}$ $\checkmark$ answer (in any format) (2)
2.2.2	$16 + 8 + 4 + 2 + 1 + \frac{1}{2} = 31$ $S_5 = 31$ $n > 5$ or $n \geq 6$	$\checkmark 16 + 8 + 4 + 2 + 1 + \frac{1}{2}$ $\checkmark S_5 = 31$ $\checkmark n > 5 / n \geq 6$ (3)

	OR $S_n = \frac{a(1-r^n)}{1-r}$ $31 < \frac{16\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ $31 < 32(1-2^{-n})$ $\frac{31}{32} - 1 < -2^{-n}$ $\frac{1}{32} > 2^{-n}$ $2^{-5} > 2^{-n}$ $n > 5$ or $n \geq 6$	✓ $S_n > 31$ ✓ simplification ✓ $n > 5 / n \geq 6$ (3)
2.2.3	$S_\infty = \frac{a}{1-r}$ $= \frac{16}{1-\frac{1}{2}}$ $= 32$ OR $16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} \dots\dots$ Answer gets <u>closer and closer to 32</u> the more terms gets added together <i>Antwoord beweeg nader en nader aan 32 hoe meer terme bymekaar getel word</i>	✓ substitution of a and r ✓ answer (2) ✓ expanding the series ✓ answer (2) [16]

QUESTION/VRAAG 3

3.1.1	1; x; y; z..... $T_n = 4n + 6$ 10; 14; 18..... 2nd difference = 4 ✓ $2a = 4$ ✓ $a = 2$ (2) OR $T_n = 4n + 6$ $d = 4$ $2a = 4$ $a = 2$ (2)	
3.1.2	3rd difference = 10 ✓ 1st differences 10; 14; 18..... ✓ $3a + b = 10$ $a + b + c = 1$ $2 + 4 + c = 1$ $c = -5$ ✓ $a + b + c = 1$ ✓ $T_n = 2n^2 + 4n - 5$ (4)	

3.2	Consider the sequence made up by the first factors of each term: <i>Beskou die ry wat deur die eerste faktore van elke term gevorm word:</i> 1; 5; 9; 13; ... 81 An arithmetic sequence / <i>rekenkundige ry</i>: $T_n = a + (n-1)d$ $= 1 + (n-1)4$ $= 4n - 3$ To find the no. of terms: $81 = 4n - 3$ <i>Aantal terme:</i> $4n = 84$ $\therefore n = 21$ The second factor is 1 more than the first factor / <i>Tweede faktor is 1 meer as die eerste faktor:</i> $T_n = 4n - 3 + 1$ $= 4n - 2$ OR Consider the sequence made up by the second factors of each term: <i>Beskou die ry wat deur die tweede faktore van elke term gevorm word:</i> 2; 6; 10; 14; ... 82 Also an arithmetic sequence / <i>rekenkundige ry</i>: $T_n = a + (n-1)d$ $= 2 + (n-1)4$ $= 4n - 2$ In sigma notation: $\sum_{n=1}^{21} (4n-3)(4n-2) \quad \text{or} \quad \sum_{n=1}^{21} 2(4n-3)(2n-1) \quad \text{or} \quad \sum_{n=1}^{21} (16n^2 - 20n + 6)$	✓ $T_n = 4n - 3$ ✓ no. of terms ✓ $T_n = 4n - 2$ ✓ $T_n = 4n - 2$ ✓ answer in sigma notation (4) [10]
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