



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE SENIOR
SERTIFIKAAT**

GRADE 12/GRAAD 12

MATHEMATICS P1/WISKUNDE V1

NOVEMBER 2023

MARKING GUIDELINES/NASIENRIGLYNE

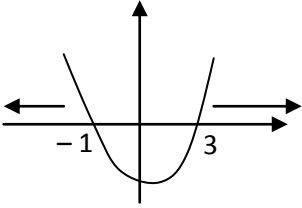
MARKS/PUNTE: 150

**These marking guidelines consist of 17 pages.
*Hierdie nasienriglyne bestaan uit 17 bladsye.***

- NOTE:**
- If a candidate answers a question TWICE, only mark the FIRST attempt.
 - Consistent Accuracy applies in all aspects of the marking memorandum.

- LET WEL:**
- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.
 - Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.

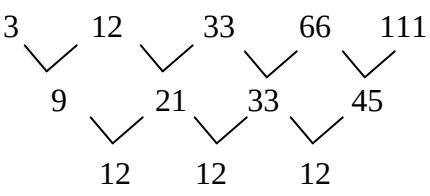
QUESTION 1/VRAAG 1

1.1.1	$x^2 + x - 12 = 0$ $(x - 3)(x + 4) = 0$ $x = 3$ or $x = -4$	✓ factors/formula ✓ answer ✓ answer (3)
1.1.2	$3x^2 - 2x = 6$ $3x^2 - 2x - 6 = 0$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$ $x = 1,79$ or $x = -1,12$	✓ standard form ✓ substitution into correct formula ✓ answer ✓ answer (4)
1.1.3	$\sqrt{2x+1} = x-1$ $2x+1 = (x-1)^2$ $2x+1 = x^2 - 2x + 1$ $x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0$ or $x = 4$ $x \neq 0$ or $x = 4$	✓ squaring both sides ✓ standard form ✓ both x-values ✓ valid answer (4)
1.1.4	$x^2 - 2x > 3$ $x^2 - 2x - 3 > 0$ $(x-3)(x+1) > 0$ CV's: $x = -1$; $x = 3$  $x < -1$ or $x > 3$	✓ standard form ✓ critical values/factors ✓✓ answer (4)

1.2	$\frac{1}{x} + \frac{1}{y} = 1 \quad \dots \quad (1)$ $x + 2 = 2y \quad \dots \quad (2)$ $x = 2y - 2$ $\frac{1}{2y - 2} + \frac{1}{y} = 1$ $y + 2y - 2 = 2y^2 - 2y$ $2y^2 - 5y + 2 = 0$ $(2y - 1)(y - 2) = 0$ $y = \frac{1}{2} \quad \text{or} \quad y = 2$ $x = -1 \quad \text{or} \quad x = 2$ OR/OF $\frac{1}{x} + \frac{1}{y} = 1 \quad \dots \quad (1)$ $x + 2 = 2y \quad \dots \quad (2)$ $y = \frac{x}{2} + 1$ $\frac{1}{x} + \frac{1}{\frac{x}{2} + 1} = 1$ $\frac{1}{x} + \frac{2}{x + 2} = 1$ $x + 2 + 2x = x^2 + 2x$ $x^2 - x - 2 = 0$ $(x + 1)(x - 2) = 0$ $x = -1 \quad \text{or} \quad x = 2$ $y = \frac{1}{2} \quad \text{or} \quad y = 2$	$\checkmark x = 2y - 2$ $\checkmark$ substitution $\checkmark$ standard form $\checkmark$ y-values $\checkmark$ x-values (5) OR/OF $\checkmark y = \frac{x}{2} + 1$ $\checkmark$ substitution $\checkmark$ standard form $\checkmark$ x-values $\checkmark$ y-values (5)
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1.3	$2^{m+1} + 2^m = 3^{n+2} - 3^n$ $2^m(2+1) = 3^n(3^2 - 1)$ $2^m(3) = 3^n(8)$ $2^m(3) = 3^n(2^3)$ $\therefore m = 3 \text{ and } n = 1$ $\therefore m + n = 4$ OR/OF $2^{m+1} + 2^m = 3^{n+2} - 3^n$ $2^m(2+1) = 3^n(3^2 - 1)$ $2^m(3) = 3^n(8)$ $2^m(3) = 3^n(2^3)$ $2^{m-3} = 3^{n-1}$ Only true if $m - 3 = 0$ and $n - 1 = 0$ $\therefore m + n = 4$	✓ factors ✓ $2^m(3) = 3^n(2^3)$ (same bases) ✓ $m = 3$ and $n = 1$ ✓ $m + n = 4$ (4) OR/OF ✓ factors ✓ $2^m(3) = 3^n(2^3)$ (same bases) ✓ $m - 3 = 0$ and $n - 1 = 0$ ✓ $m + n = 4$ (4)
		[24]

QUESTION 2/VRAAG 2

2.1.1	$7 + 12 + 17 + \dots$ $T_n = a + (n-1)d$ $T_{91} = 7 + (91-1)(5)$ $T_{91} = 457$ OR/OF $d = 5$ $T_n = 5n + 2$ $T_{91} = 5(91) + 2$ $T_{91} = 457$	$\checkmark d = 5$ $\checkmark$ substitution into correct formula $\checkmark$ answer (3) OR/OF $\checkmark d = 5$ $\checkmark$ substitution $n = 91$ $\checkmark$ answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$ $S_9 = 21\ 112$ OR/OF $S_n = \frac{n}{2}(a + l)$ $S_{91} = \frac{91}{2}(7 + 457)$ $S_{91} = 21\ 112$	$\checkmark$ substitution into correct formula $\checkmark$ answer (2) OR/OF $\checkmark$ substitution into correct formula $\checkmark$ answer (2)
2.1.3	$T_n = 7 + (n-1)(5)$ $5n + 2 = 517$ $5n = 515$ $n = 103$	$\checkmark$ substitution into correct formula $\checkmark$ equate $\checkmark$ answer (3)
2.2.1	$T_1 = 3; T_2 - T_1 = 9 \text{ and } T_3 - T_2 = 21$  $\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$ OR/OF $2a = 12$ $a = 6$ $3(6) + b = 9$ $b = -9$ $6 - 9 + c = 3$ $T_5 = 6(5)^2 - 9(5) + 6 = 111$	$\checkmark$ constant second diff = 12 $\checkmark$ first differences : 33 and 45 (2) OR/OF $\checkmark$ constant second diff = 12 $\checkmark$ substitute 5 (2)

2.2.2	$2a = 12$ $a = 6$ $3(6) + b = 9$ or $5 \times 6 + b = 21$ $b = -9$ $6 - 9 + c = 3$ $c = 6$ $T_n = 6n^2 - 9n + 6$	$\checkmark 2a = 12$ $\checkmark 3(6) + b = 9 / 5 \times 6 + b = 21$ $\checkmark 6 - 9 + c = 3$ (3)
2.2.3	$T_n' = 12n - 9 > 0$ $n > \frac{3}{4}$ $\therefore T_n$ is increasing for $n \in N$ OR/OF $n = -\frac{b}{2a} = -\frac{-9}{2(6)}$ $n = \frac{3}{4}$ $\therefore$ min at $n = 1$ for $n \in N$ $\therefore T_n$ is increasing for $n \in N$	$\checkmark T_n' = 12n - 9$ $\checkmark n > \frac{3}{4}$ $\checkmark$ increasing for $n \in N$ (3) OR/OF $\checkmark n = -\frac{b}{2a} = \frac{9}{2(6)}$ $\checkmark n = \frac{3}{4}$ $\checkmark$ increasing for $n \in N$ (3)
		[16]

QUESTION 3/VRAAG 3

3.1.1	$T_n = ar^{n-1}$ $T_n = 3(2)^{n-1}$	$\checkmark T_n = 3(2)^{n-1}$ (1)
3.1.2	$\sum_{p=1}^k \frac{3}{2} \cdot 2^p = 98\,301$ $\sum_{p=1}^k \frac{3}{2} \cdot 2^p = 3 + 6 + 12 + \dots$ $n = k$ $\frac{3[(2)^k - 1]}{2 - 1} = 98\,301$ $(2)^k = 32\,768$ $2^k = 2^{15} \quad \text{OR/OF} \quad k = \log_2 32\,768$ $\therefore k = 15$	$\checkmark$ expansion $\checkmark n = k$ $\checkmark$ substitution into correct formula $\checkmark k = 15$ (4)
3.2	$S_{22} = \frac{22}{2} [2a + 21(3)]$ $S_{22} = 22a + 693$ $S_{\infty} = \frac{a}{1 - \frac{1}{3}}$ $= \frac{3a}{2}$ $\therefore 22a + 693 = \frac{3a}{2} + 734$ $44a + 1386 = 3a + 1468$ $41a = 82$ $a = 2$	$\checkmark$ substitution into S_n $\checkmark S_{22} = 22a + 693$ $\checkmark$ substitution into S_{∞} $\checkmark S_{22} = S_{\infty} + 734$ $\checkmark$ answer (5)
		[10]

QUESTION 4/VRAAG 4

4.1	$y = -4$	✓ $y = -4$ (1)
4.2	x – intercept: $0 = 2^x - 4$ $4 = 2^x$ $x = 2$ $\therefore B(2;0)$	✓ $y = 0$ ✓ $x = 2$ (2)
4.3	$y = 2^0 - 4 = -3$ $\therefore A(0; -3)$ $y = mx + c$ $m = \frac{3}{2}$ $k(x) = \frac{3}{2}x - 3$	✓ $y = -3$ ✓ gradient ✓ equation (3)
4.4	$k(1) = \frac{3}{2}(1) - 3 = \frac{-3}{2}$ $f(1) = 2^1 - 4 = -2$ Vertical distance $= -\frac{3}{2} - (-2) = \frac{1}{2}$ units	✓ $k(1)$ ✓ $f(1) = -2$ ✓ answer (3)
4.5	$g(x) = f(x) + 4$ $g(x) = 2^x ; x \in [-2; 4)$	✓ $g(x) = 2^x$ (1)
4.6	Range of $g : y \in \left[\frac{1}{4}; 16\right)$ Domain of $g^{-1} : x \in \left[\frac{1}{4}; 16\right)$ or/of $\frac{1}{4} \leq x < 16$	✓ ✓ $x \in \left[\frac{1}{4}; 16\right)$ (2)
4.7	$g : y = 2^x$ $g^{-1} : x = 2^y$ $g^{-1}(x) = \log_2 x, x \in \left[\frac{1}{4}; 16\right)$	✓ swop x and y ✓ equation (2)
		[14]

QUESTION 5/VRAAG 5

5.1	(1 ; 8)	✓ $x = 1$ ✓ $y = 8$ (2)
5.2	$y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$ $C\left(0; \frac{15}{2}\right)$	✓ $x = 0$ ✓ answer (2)
5.3	$8 = \frac{d}{1}$ $\therefore d = 8$	✓ substitution (1 ; 8) (1)
5.4	$y \in R ; y \neq 0$	✓ $y \neq 0$ (1)
5.5	$-3 \leq x < 0$ or $x \geq 5$ OR/OF $x \in [-3 ; 0) \cup [5 ; \infty)$	✓✓ $-3 \leq x < 0$ ✓ $x \geq 5$ (3)
5.6	$-2x + k = \frac{8}{x}$ $-2x^2 + kx - 8 = 0$ $\Delta = (k)^2 - 4(-2)(-8)$ $k^2 - 64 < 0$ $CV : k = 8 ; k = -8$ $\therefore -8 < k < 8 \quad \text{or/of} \quad k \in (-8 ; 8)$ OR/OF $g'(x) = h'(x)$ $-\frac{8}{x^2} = -2$ $-8 = -2x^2$ $x = \pm 2$ $y = \pm 4 \quad \therefore B(2 ; 4) \text{ and } A(-2 ; -4)$ For tangents: $h(x) = -2x + k \quad \text{or} \quad h(x) = -2x + k$ $4 = -2(2) + k \quad -4 = -2(-2) + k$ $k = 8 \quad k = -8$ $\therefore -8 < k < 8 \quad \text{or/of} \quad k \in (-8 ; 8)$	✓ $-2x + k = \frac{8}{x}$ ✓ standard form ✓ substitution into Δ ✓ $\Delta < 0$ or $\Delta = 0$ ✓ inequality (5) OR/OF ✓ $-\frac{8}{x^2}$ ✓ $= -2$ ✓ x-values ✓ y-values ✓ inequality (5)

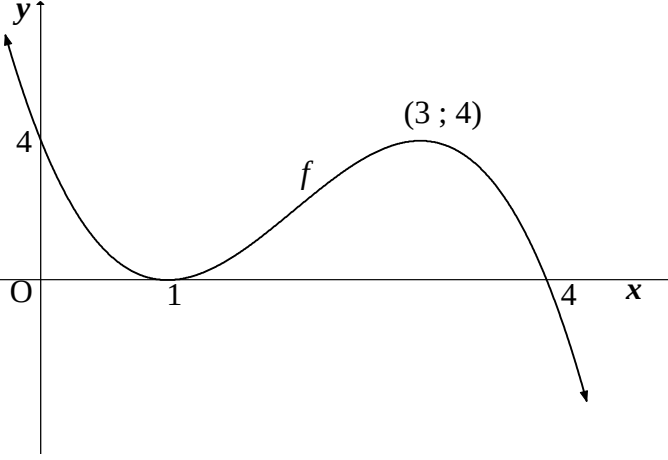
QUESTION 6/VRAAG 6

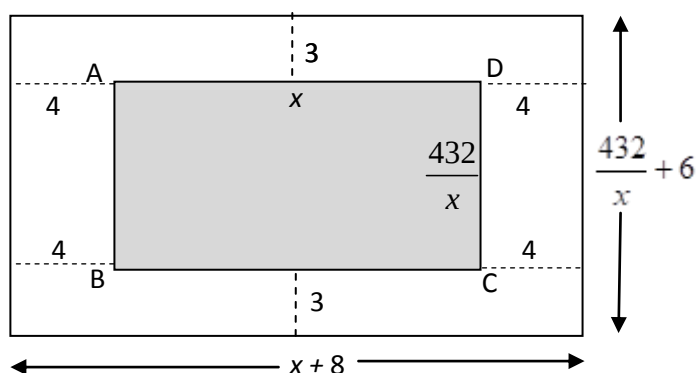
6.1.1	$A = P(1+i)^n$ $19\,319,48 = 18\,500\left(1 + \frac{r}{1200}\right)^6$ $\left(1 + \frac{r}{1200}\right) = \sqrt[6]{1,04429\dots}$ $\frac{r}{1200} = 0,00725\dots$ $r = 8,7\%$	✓ $n = 6$ ✓ substitution into correct formula ✓ answer (3)
6.1.2	$1 + \frac{i}{100} = \left(1 + \frac{8,7}{1200}\right)^{12}$ $r = 9,06\%$	✓ substitution into correct formula ✓ answer (2)
6.2.1	$A = P(1-in)$ $0 = 10\,000(1 - 0,2n)$ $n = 5$	✓ substitution into correct formula ✓ answer (2)
6.2.2	$F = \frac{x[(1+i)^n - 1]}{i}$ $20\,000 = \frac{x\left[\left(1 + \frac{8,7}{1200}\right)^{60} - 1\right]}{\frac{8,7}{1200}}$ $x = R267,26$	✓ i ✓ n ✓ substitution into correct formula ✓ answer (4)
6.3	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $1\,600\,000 = \frac{20\,000\left[1 - \left(1 + \frac{0,112}{12}\right)^{-n}\right]}{\frac{0,112}{12}}$ $\frac{56}{75} = 1 - \left(1 + \frac{0,112}{12}\right)^{-n}$ $\left(1 + \frac{0,112}{12}\right)^{-n} = \frac{19}{75}$ $-n = \log_{\left(1 + \frac{0,112}{12}\right)}\left(\frac{19}{75}\right)$ $-n = -147,80$ Tino will make 147 withdrawals of R20 000	✓ i ✓ substitution into correct formula ✓ correct use of logs ✓ $-n = -147,80$ ✓ $n = 147$ (5)
		[16]

QUESTION 7/VRAAG 7

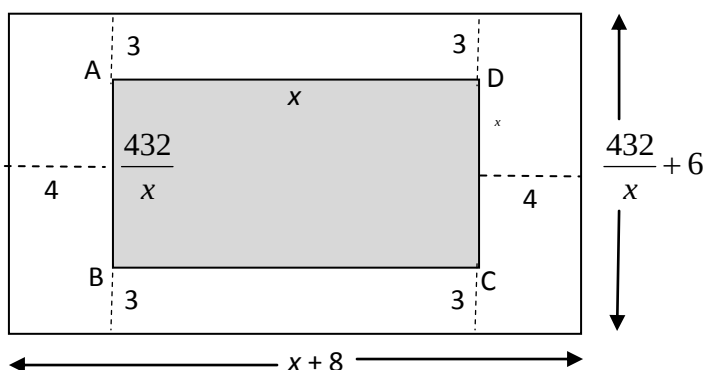
7.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4(x+h)^2 - (-4x^2)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4x^2 - 8xh - 4h^2 + 4x^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $f'(x) = \lim_{h \rightarrow 0} (-8x - 4h)$ $f'(x) = -8x$ OR/OF $f(x+h) = -4(x+h)^2 = -4x^2 - 8xh - 4h^2$ $f(x+h) - f(x) = -4x^2 - 8xh - 4h^2 - (-4x^2)$ $= -8xh - 4h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $f'(x) = \lim_{h \rightarrow 0} (-8x - 4h)$ $f'(x) = -8x$	✓ substitution into correct formula ✓ $f(x+h) = -4x^2 - 8xh - 4h^2$ ✓ simplification ✓ common factor ✓ answer (5) OR/OF ✓ $f(x+h) = -4x^2 - 8xh - 4h^2$ ✓ simplification ✓ substitution into correct formula ✓ common factor ✓ answer (5)
7.2.1	$f(x) = 2x^3 - 3x$ $f'(x) = 6x^2 - 3$	✓ $6x^2$ ✓ -3 (2)
7.2.2	$D_x \left[7\sqrt[3]{x^2} + 2x^{-5} \right]$ $D_x \left[7x^{\frac{2}{3}} + 2x^{-5} \right]$ $= \frac{14}{3} x^{-\frac{1}{3}} - 10x^{-6}$	✓ $x^{\frac{2}{3}}$ ✓ derivative with rational exp ✓ $-10x^{-6}$ (3)
7.3	$-6x^2 + 8 > 0$ $x^2 < \frac{8}{6}$ CV's: $x = -\frac{2}{\sqrt{3}}$ or $x = \frac{2}{\sqrt{3}}$ Positive for: $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$	✓ CV's: $x = \pm \frac{2}{\sqrt{3}}$ ✓ ✓ answer (3)
[13]		

QUESTION 8/VRAAG 8

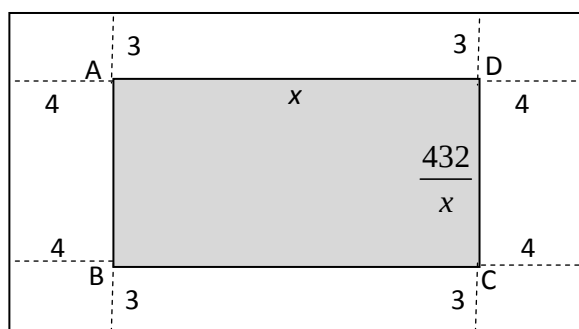
8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3 \text{ or } x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)	✓ $f'(x) = -3x^2 + 12x - 9$ ✓ $f'(x) = 0$ ✓ both x-values ✓ both y-values (4)
8.2		✓ y-intercept ✓ both x-intercepts ✓ both turning points ✓ shape (4)
8.3	$0 < k < 4$ or/of $k \in (0 ; 4)$	✓✓ k between y-values of turning points (2)
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x - 2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$ OR/OF Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x - 2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$	✓ $f''(x) = -6x + 12$ ✓ $f''(x) = 0$ ✓ x-value ✓ y-value ✓ gradient at x-value ✓ equation of tangent (6) OR/OF ✓✓ $\frac{3+1}{2}$ ✓ x-value ✓ y-value ✓ gradient at x-value ✓ equation of tangent (6)
8.5	$\tan \theta = 3$ $\therefore \theta = 71,57^\circ$	✓ gradient of g ✓ answer (2)
		[18]



$$\text{total area} = 2(x+8)(3) + 2\left(\frac{432}{x}\right)(4) + \left(\frac{432}{x}\right)(x)$$

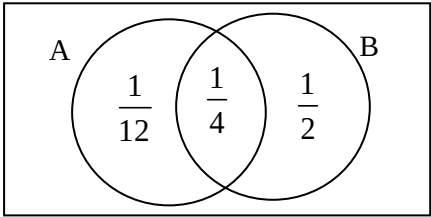
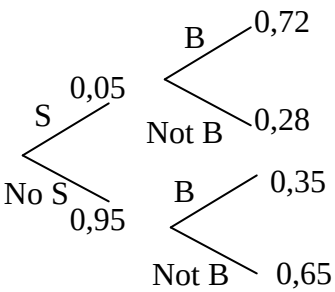


$$\text{total area} = 2(4)\left(\frac{432}{x} + 6\right) + (x)\left(\frac{432}{x} + 6\right)$$



$$\text{total area} = 4(4)(3) + 2(x)(3) + \left(\frac{432}{x}\right)(x) + 2\left(\frac{432}{x}\right)(4)$$

QUESTION 10/VRAAG 10

10.1.1	$P(A \text{ and } B) = P(A) \times P(B)$ $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$	$\checkmark \frac{1}{3} \times \frac{3}{4}$ $\checkmark \frac{1}{4}$ (2)
10.1.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ OR/OF  $P(A \text{ or } B) = \frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$	$\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (2) OR/OF $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (2)
10.2.1		$\checkmark \text{ branch 1 with probabilities}$ $\checkmark \text{ branch 2 with probabilities}$ $\checkmark \text{ branch 3 with probabilities}$ (3)
10.2.2	$P(\text{NOT below } 0^\circ)$ $= P(S; \text{NOT below } 0^\circ) + P(NS; \text{NOT below } 0^\circ)$ $= 0,05 \times 0,28 + 0,95 \times 0,65$ $= 0,6315$	$\checkmark \text{ value of } P(S; \text{NOT below } 0^\circ)$ $\checkmark \text{ value of } P(NS; \text{NOT below } 0^\circ)$ $\checkmark \text{ answer}$ (3)
10.3.1	$n(S) = 10!$	$\checkmark 10!$ (1)

10.3.2	4 Options; $2 \times 8 \times 7 \times 6 \times 5 \times 4 \times 1 \times 3 \times 2 \times 1 = 80\,640$ $8 \times 2 \times 7 \times 6 \times 5 \times 4 \times 3 \times 1 \times 2 \times 1 = 80\,640$ $8 \times 7 \times 2 \times 6 \times 5 \times 4 \times 3 \times 1 \times 1 \times 1 = 80\,640$ $8 \times 7 \times 6 \times 2 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 80\,640$ Total number of possibilities = 322 560 $P(5 \text{ learners in between}) = \frac{322\,560}{10!} = \frac{4}{45}$ OR/OF $2 \times 8 \times 7 \times 6 \times 5 \times 4 \times 1 \times 3 \times 2 \times 1$ 4 possible starting positions $\therefore 4(2 \times 8! \times 1) = 322\,560$ $8(8!) = 322\,560$ $P(5 \text{ learners in between}) = \frac{322\,560}{10!} = \frac{4}{45}$	✓ $(2 \times 8!)$ ✓✓ $4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4) OR/OF ✓ $(2 \times 8!)$ ✓✓ $4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4)
		[15]

TOTAL/TOTAAL: 150