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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA



NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2021



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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

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QUESTION 1

A bakery kept a record of the number of loaves of bread a tuck-shop ordered daily over the last 18 days. The information is shown in the table below.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Calculate the:

1.1.1 Mean number of loaves of bread ordered daily (2)

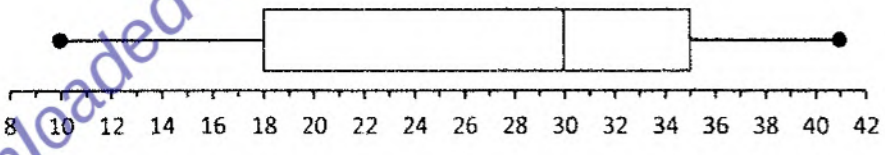
1.1.2 Standard deviation of the data (1)

1.1.3 Number of days on which the number of loaves of bread ordered was more than one standard deviation above the mean (2)

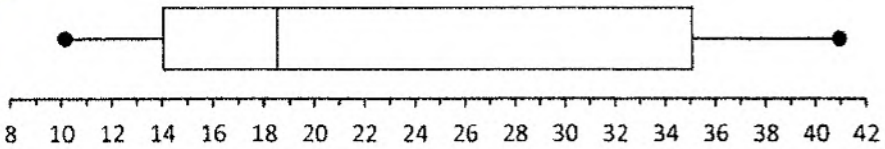
1.2 The tuck-shop owner was not able to sell all the loaves of bread delivered daily. He calculated the mean number of loaves sold over the 18 days to be 20. Calculate the number of loaves of bread which were NOT sold over the 18 days. (2)

1.3 One of the two box and whisker diagrams drawn below represents the data given in the table above.

Graph A:



Graph B:



1.3.1 Which ONE of the two box and whisker diagrams, drawn above, correctly represents the data? Write down a reason for your answer. (2)

1.3.2 Describe the skewness of the data. (1)

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QUESTION 2

A farm stall sells milk in 5-litre containers to the local community. The price varies according to the availability of milk at the farm stall. The price of milk, in rands per 5-litre container, and the number of 5-litre containers of milk sold, are recorded in the table below.

Price of milk in rands per 5-litre container (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)	48	30	26	44	23	32	39	29	42	33

2.1 On the grid provided in the ANSWER BOOK, draw the scatter plot to represent the data. (3)

2.2 Determine the equation of the least squares regression line for the data. (3)

2.3 If the farmer sells a 5-litre container of milk for R38, predict the number of 5-litre containers of milk he will sell. (2)

2.4 Refer to the correlation between the price of 5-litre containers of milk and the number of 5-litre containers of milk sold, and comment on the accuracy of your answer to QUESTION 2.3. (2)

[10]

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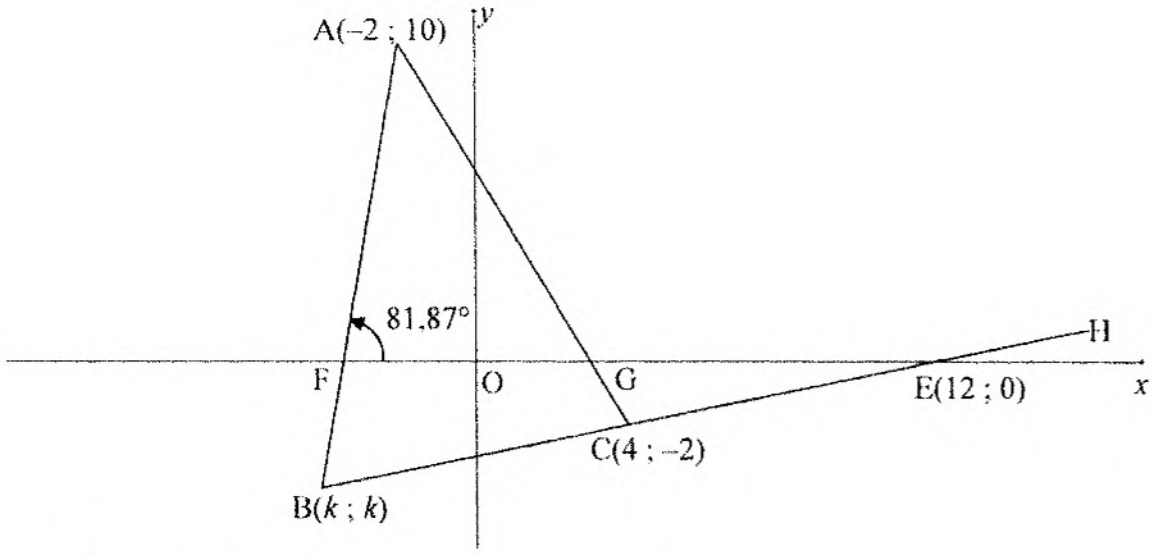


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QUESTION 3

In the diagram, A(–2 ; 10), B(k ; k) and C(4 ; –2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersects the x-axis at F and G respectively. The angle of inclination of line AB is 81,87°.



3.1 Calculate the gradient of:

3.1.1 BE (2)

3.1.2 AB (2)

3.2 Determine the equation of BE in the form $y = mx + c$ (2)

3.3 Calculate the:

3.3.1 Coordinates of B, where $k < 0$ (2)



3.3.2 Size of $\hat{A}$ (4)

3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram ACES, where S is a point in the first quadrant (2)

3.4 Another point T(p ; p), where $p > 0$, is plotted such that ET = BE = $4\sqrt{17}$ units.

3.4.1 Calculate the coordinates of T. (5)

3.4.2 Determine the equation of the:

(a) Circle with centre at E and passing through B and T in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

(b) Tangent to the circle at point B(k ; k) (3)

[24]

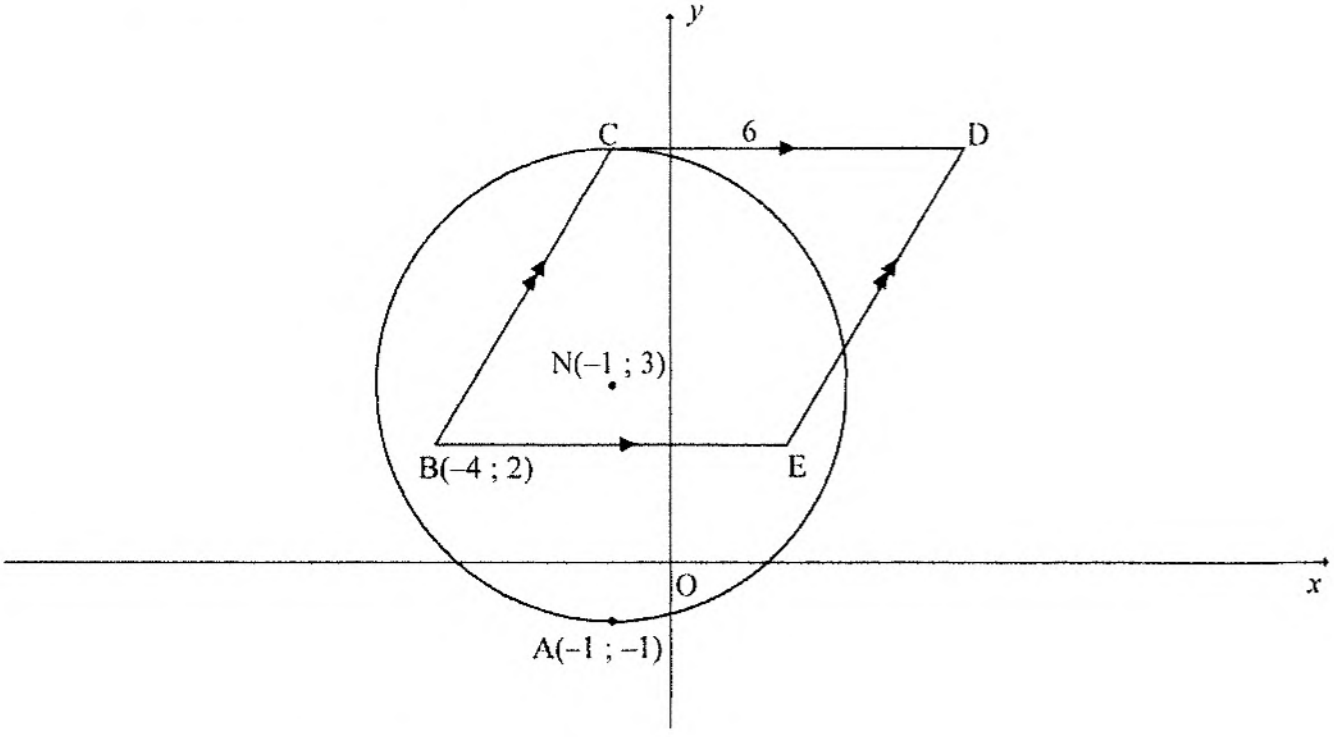
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QUESTION 4

In the diagram, the circle centred at N(–1 ; 3) passes through A(–1 ; –1) and C. B(–4 ; 2), C, D and E are joined to form a parallelogram such that BE is parallel to the x-axis. CD is a tangent to the circle at C and CD = 6 units.



4.1 Write down the length of the radius of the circle. (1)

4.2 Calculate the:

4.2.1 Coordinates of C (2)

4.2.2 Coordinates of D (2)

4.2.3 Area of ΔBCD (3)

4.3 The circle, centred at N, is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F.

Calculate the:

4.3.1 Length of NM (3)

4.3.2 Midpoint of AF (4)

[15]

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QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$$ (6)

5.2 Prove the identity: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)} = 2\cos x - 1$ (4)

5.3 Given: $\sin 36° = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p:

5.3.1 $\tan 36°$ (3)

5.3.2 $\cos 108°$ (4)

[17]

QUESTION 6

6.1 Given: $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

6.1.2 Simplify completely: $2\cos 6x\cos 4x - \cos 10x + 2\sin^2 x$ (5)

6.2 Determine the general solution of $\tan x = 2\sin 2x$ where $\cos x < 0$. (7)

[15]

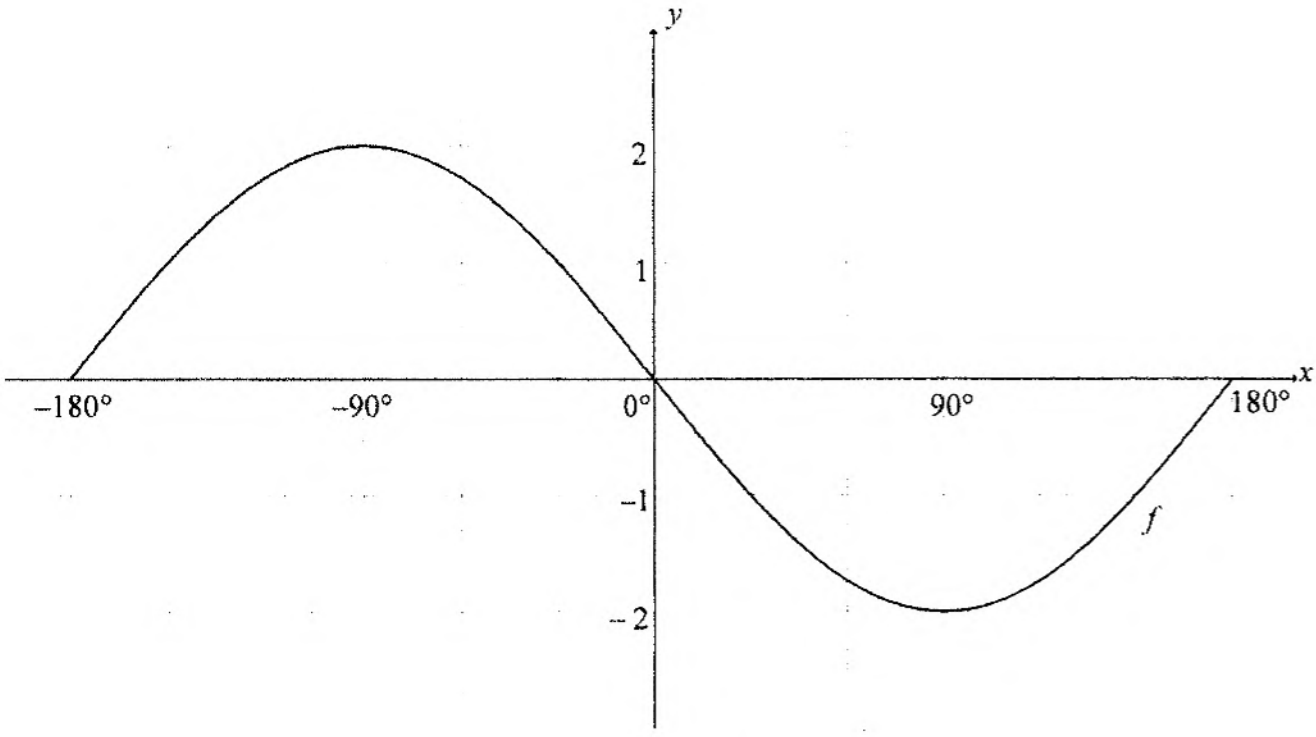
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QUESTION 7

In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for the interval $x \in [-180° ; 180°]$.



7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60°)$ for $x \in [-180° ; 180°]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)

7.2 Write down the period of $f(3x)$. (2)

7.3 **Use the graphs** to determine the value of x in the interval $x \in [-180° ; 180°]$ for which $f(x) - g(x) = 1$. (1)

7.4 Write down the range of k, if $k(x) = \frac{1}{2}g(x) + 1$. (2)

[8]

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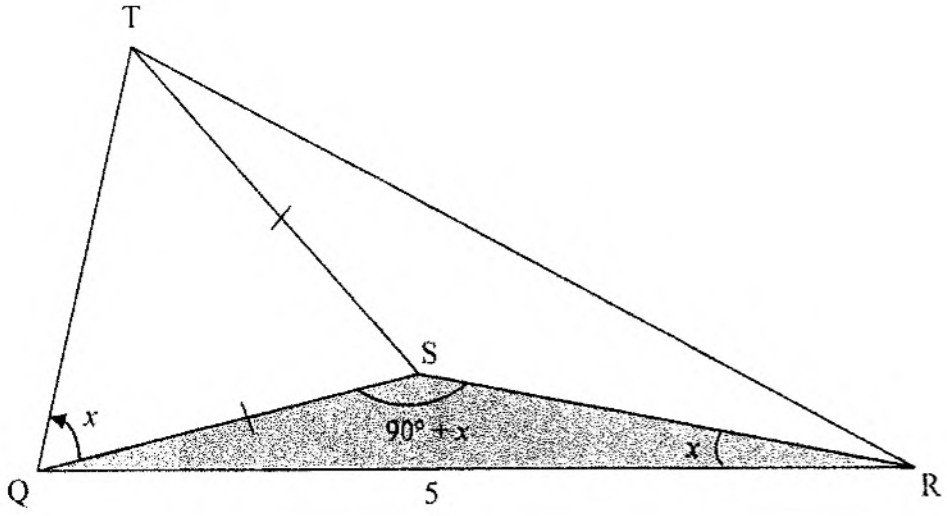
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QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x.

$\hat{QSR} = 90° + x$, $\hat{QRS} = x$, QR = 5 units and TS = SQ.



8.1 Prove that $QS = 5 \tan x$ (3)

8.2 Prove that the length of $QT = 10 \sin x$ (5)

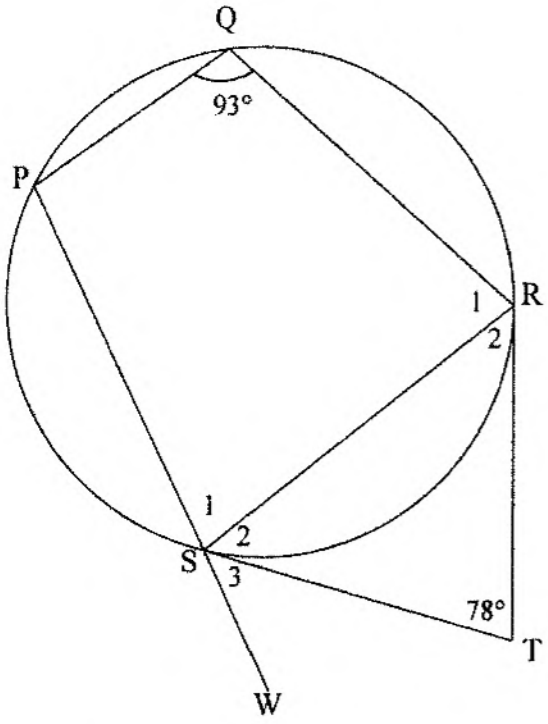
8.3 Calculate the area of ΔTQR if $\hat{TQR} = 70°$ and $x = 25°$. (2)
[10]

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QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T} = 78°$ and $\hat{Q} = 93°$.



9.1 Give a reason why ST = TR. (1)

9.2 Calculate, giving reasons, the size of:

9.2.1 $\hat{S}_2$ (2)

9.2.2 $\hat{S}_3$ (2)

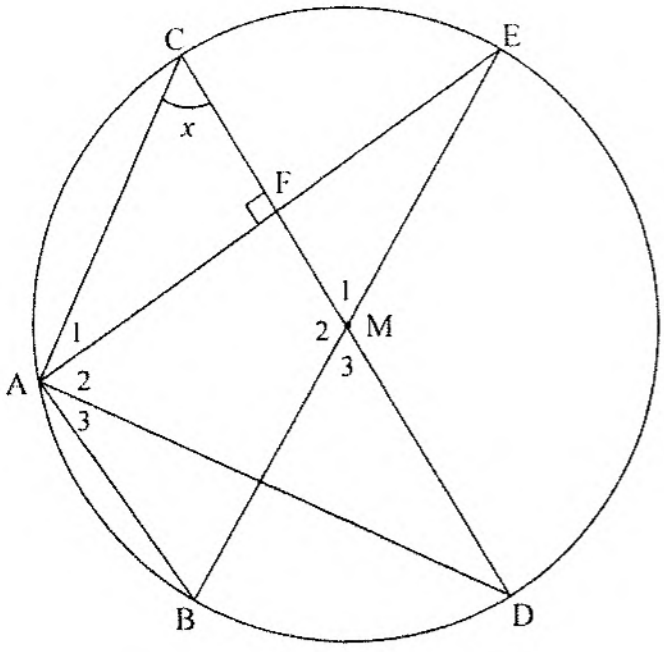
[5]

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QUESTION 10

In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. AE ⊥ CD. Let $\hat{C} = x$.



10.1 Give a reason why AF = FE. (1)

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)

10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)

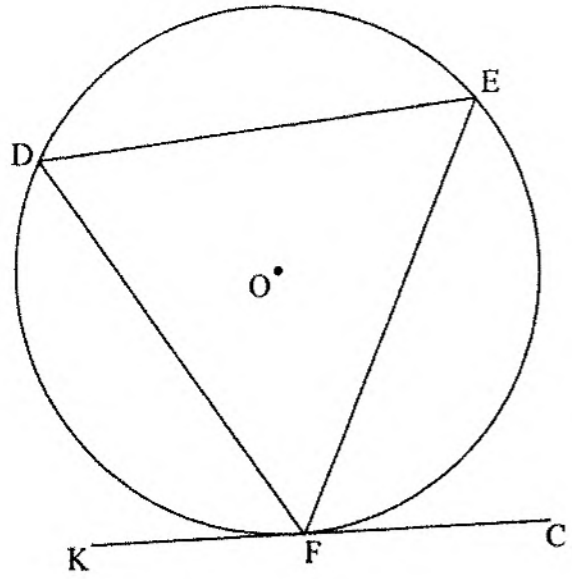
10.4 Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE. (5)
[13]

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QUESTION 11

11.1 In the diagram, chords DE, EF and DF are drawn in the circle with centre O. KFC is a tangent to the circle at F.

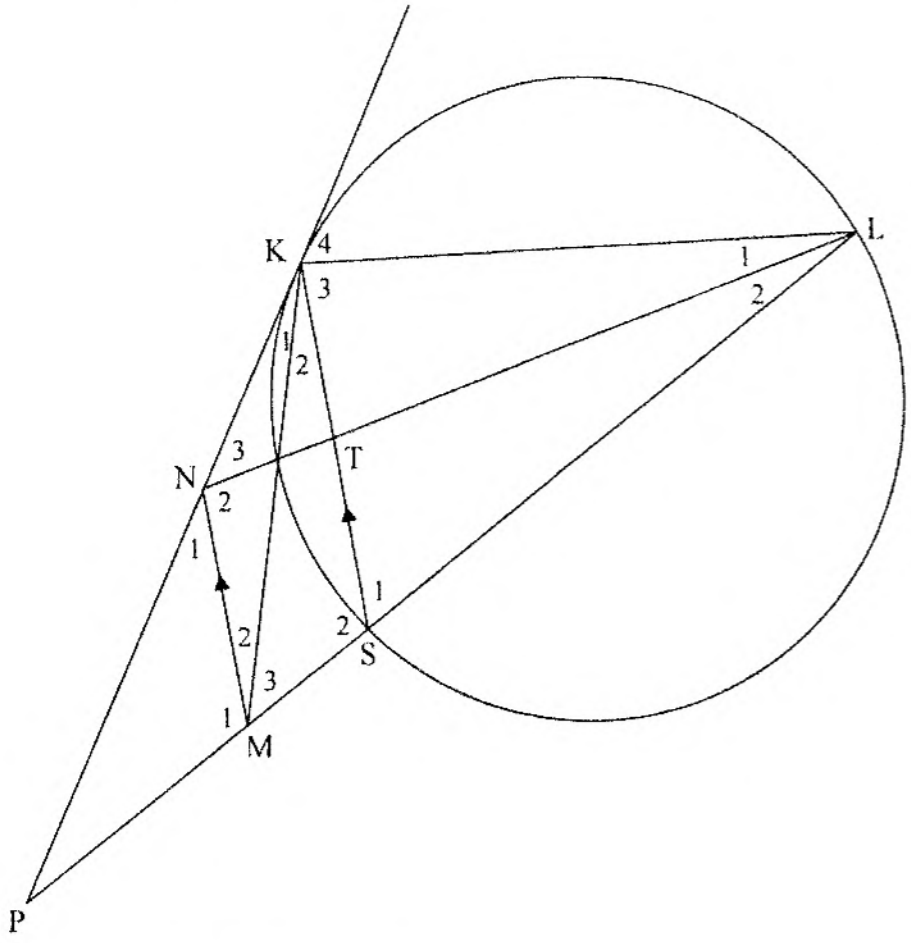


Prove the theorem which states that $D\hat{F}K = \hat{E}$. (5)

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11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN ∥ SK. Chord KS and LN intersect at T.



11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) KLMN is a cyclic quadrilateral (1)

11.2.2 Prove, giving reasons, that $\Delta LKN \,|||\, \Delta KSM$. (5)

11.2.3 If LK = 12 units and 3KN = 4SM, determine the length of KS. (4)

11.2.4 If it is further given that NL = 16 units, LS = 13 units and KN = 8 units, determine, with reasons, the length of LT. (4)

[23]

TOTAL: 150

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INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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DATE / *DATUM*	
BOOK NUMBER / *BOEKNOMMER*	OF / *VAN* BOOKS / *BOEKE*
SUBJECT CODE / *VAKKODE*	
PAPER NUMBER / *VRAESTELNOMMER*	2
SUBJECT NAME / *VAKNAAM*	**MATHEMATICS/*WISKUNDE***

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Question / *Vraag*	Marks / *Punte*		Marker's Code & Initials / *Nasiener se kode & Voorletters*	Marks / *Punte*		SM	Marks / *Punte*		DCM / *AHN*	Marks / *Punte*		CM / *JIN*	Marks / *Punte*		IM	Marks / *Punte*		EM
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READ INSTRUCTIONS ON THE NEXT PAGE
LEES INSTRUKSIES OP VOLGENDE BLADSY

This answer book consists of 25 pages./*Hierdie antwoordeboek bestaan uit 25 bladsye.*

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PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	*VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG*
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	*1. Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes soos verskaf en plak jou stafieskodeplakker in die ruimte soos verskaf.*
2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.	*2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.*
3. Answer ALL questions in the spaces provided.	*3. Beantwoord ALLE vrae in die ruimtes wat voorsien is.*
4. NO pages may be torn from this answer book.	*4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.*
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	*5. Lees die instruksies wat op jou eksamenrooster gedruk is sorgvuldig deur, asook enige ander instruksies wat op elke eksamenvraestel gegee word.*
6. Candidates may NOT retain an answer book or remove it from the examination room.	*6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.*
7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.	*7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.*
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	*8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.*
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	*9. In geval jy bykomende ruimte benodig vir jou antwoorde:* *9.1 Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word.* *9.2 As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.* *9.3 Trek 'n lyn na elke antwoord.*
10. Draw a neat line through any work/rough work that must NOT be marked.	*10. Trek 'n netjiese lyn deur enige werk/rofwerk wat NIE nagesien moet word NIE.*

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QUESTION/*VRAAG* 1

1.1

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19	19	20	21	35	35	37	40	41

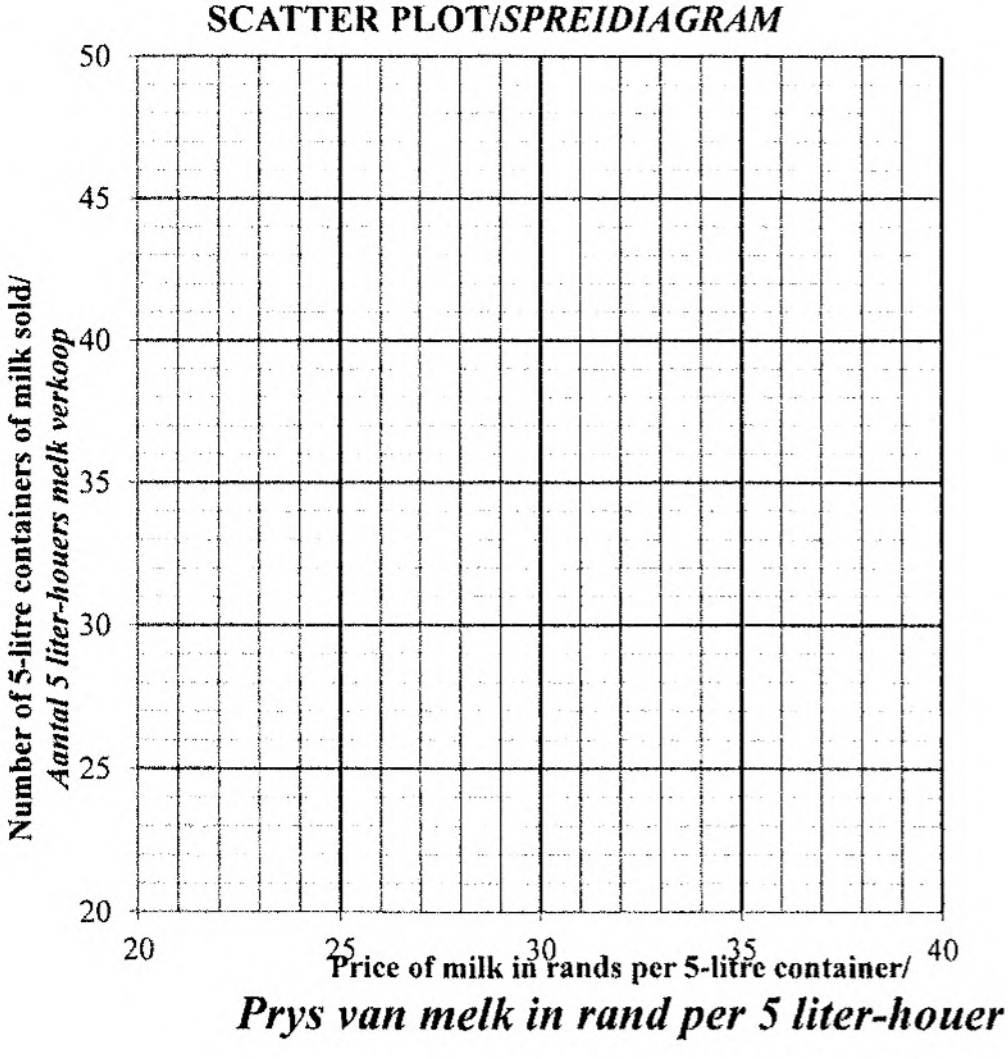
	Solution/*Oplossing*	**Marks** ***Punte***
1.1.1		(2)
1.1.2		(1)
1.1.3		(2)
1.2		(2)
1.3.1		(2)
1.3.2		(1)
		[10]

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QUESTION/*VRAAG* 2

Price of milk in rands per 5-litre container (*x*)/ ***Prys van melk, in rand, per 5 liter-houer (x)***	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (*y*)/ ***Aantal 5 liter-houers melk verkoop (y)***	48	30	26	44	23	32	39	29	42	33

	Solution/*Oplossing*	**Marks** ***Punte***
2.1		(3)

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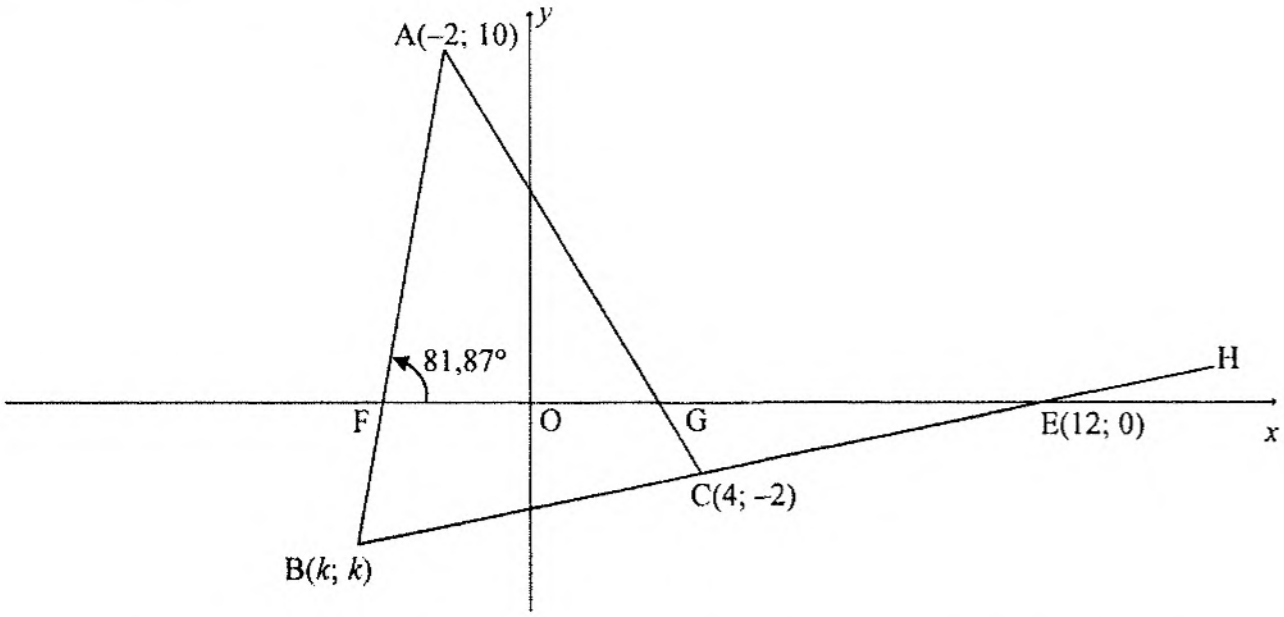
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	Solution/*Oplossing*	**Marks** ***Punte***
2.2		(3)
2.3		(2)
2.4		(2)
		[10]

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QUESTION/*VRAAG* 3



	Solution/*Oplossing*	**Marks** ***Punte***
3.1.1		(2)
3.1.2		(2)
3.2		(2)
3.3.1		(2)

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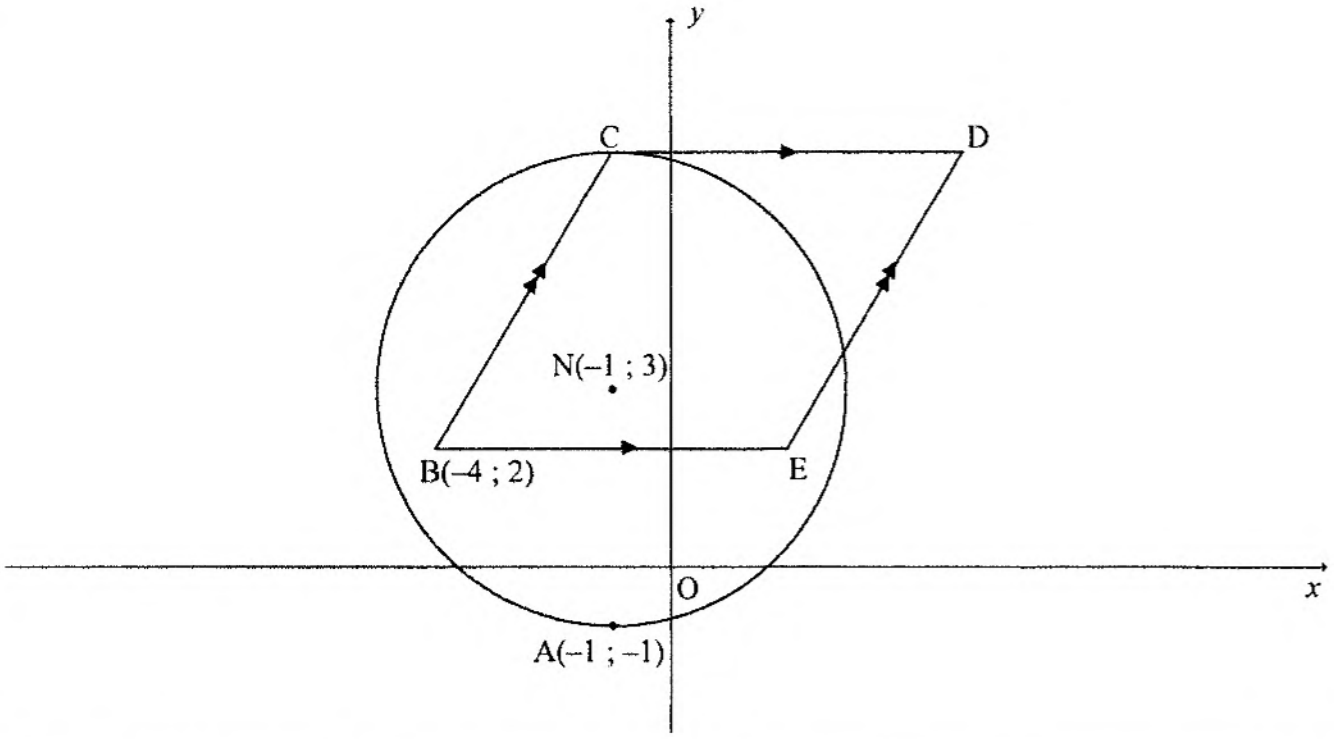
	Solution/*Oplossing*	Marks *Punte*
3.3.2		(4)
3.3.3		(2)
3.4.1		(5)
3.4.2(a)		(2)
3.4.2(b)		(3)
		[24]

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QUESTION/*VRAAG* 4



	Solution/*Oplossing*	**Marks** ***Punte***
4.1		(1)
4.2.1		(2)
4.2.2		(2)
4.2.3		(3)

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	Solution/*Oplossing*	Marks *Punte*
4.3.1		(3)
4.3.2		(4)
		[15]

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QUESTION/*VRAAG* 5

	Solution/*Oplossing*	**Marks** ***Punte***
5.1		(6)
5.2		(4)
5.3.1		(3)

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5.3.2		(4)
		[17]

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QUESTION/VRAAG 6

	Solution/*Oplossing*	Marks *Punte*
6.1.1		(3)
6.1.2		(5)

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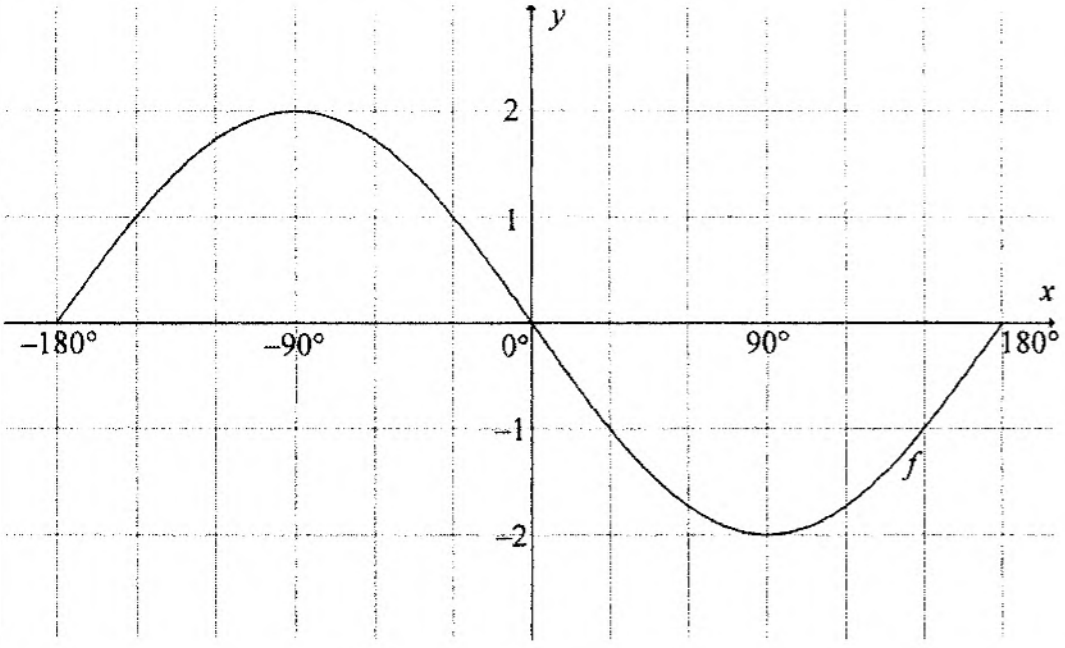
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	Solution/*Oplossing*	Marks *Punte*
6.2		(7)
		[15]

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QUESTION/*VRAAG* 7

	Solution/*Oplossing*	**Marks** ***Punte***
7.1		(3)
7.2		(2)
7.3		(1)
7.4		(2)
		[8]

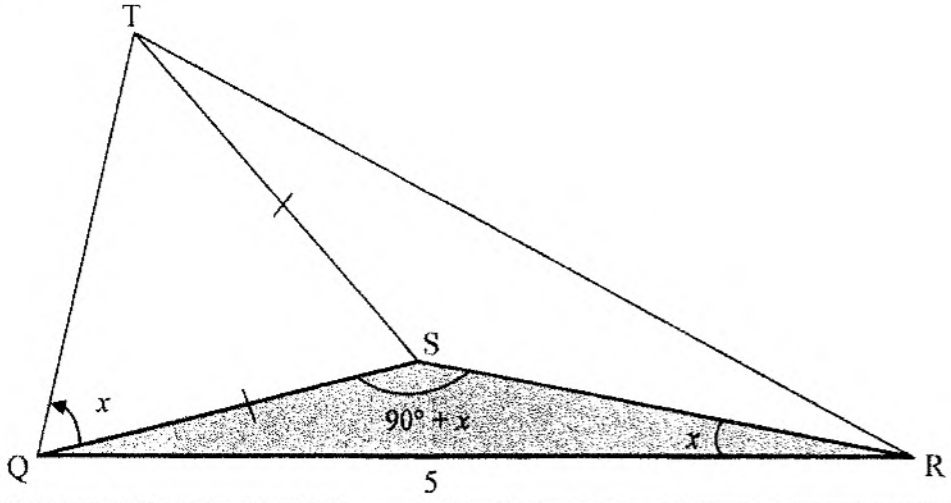
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Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.
Gee redes vir jou bewerings in VRAAG 8, 9, 10 en 11.

QUESTION/*VRAAG* 8



	Solution/*Oplossing*	**Marks** ***Punte***
8.1		(3)
8.2		(5)

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8.3		(2)
		[10]

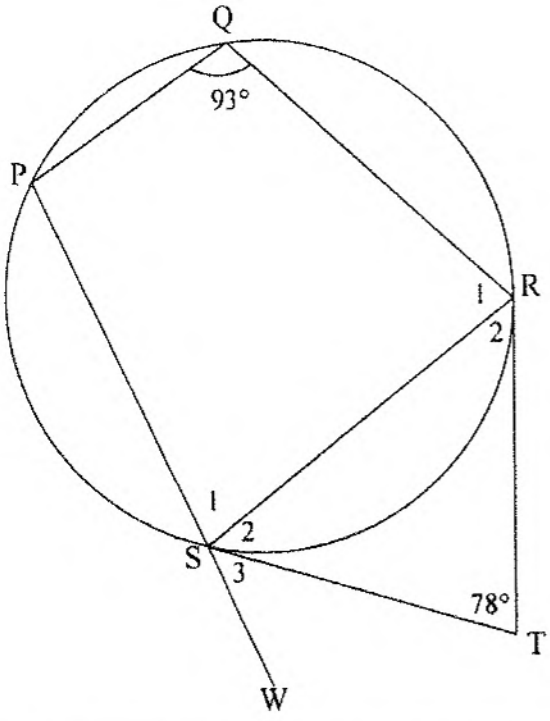
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QUESTION/*VRAAG* 9



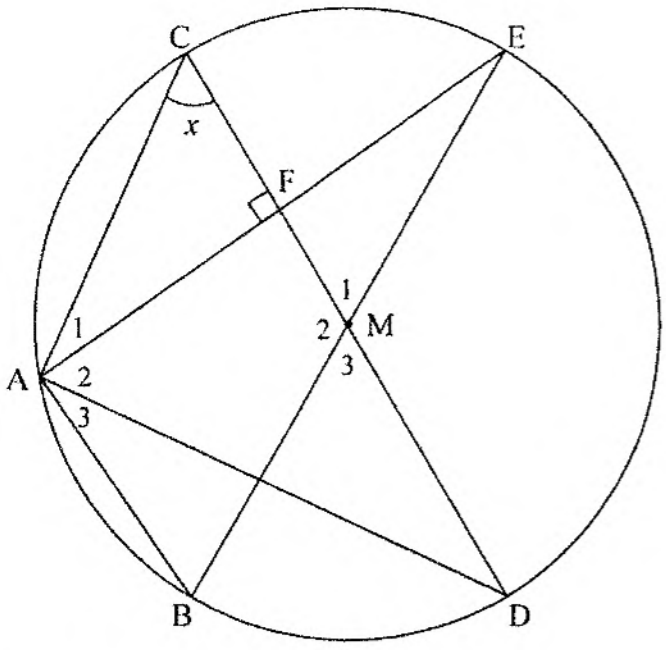
	Solution/*Oplossing*	**Marks** ***Punte***
9.1		(1)
9.2.1		(2)
9.2.2		(2)
		[5]

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QUESTION/*VRAAG* 10



	Solution/*Oplossing*	**Marks** ***Punte***
10.1		(1)
10.2		(3)
10.3		(4)
10.4		(5)
		[13]

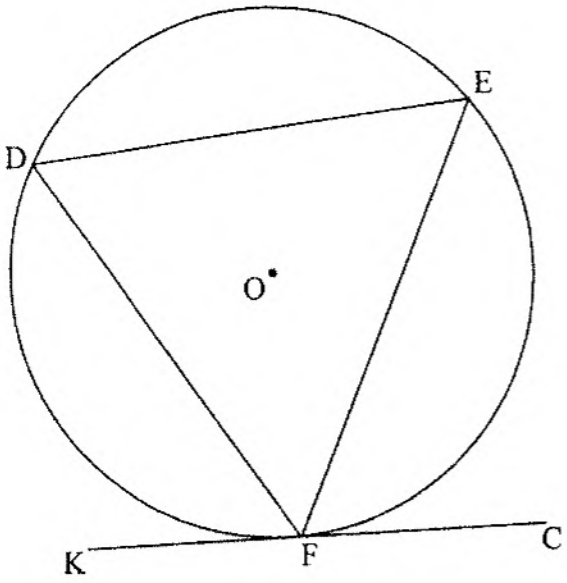
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QUESTION/*VRAAG* 11

11.1

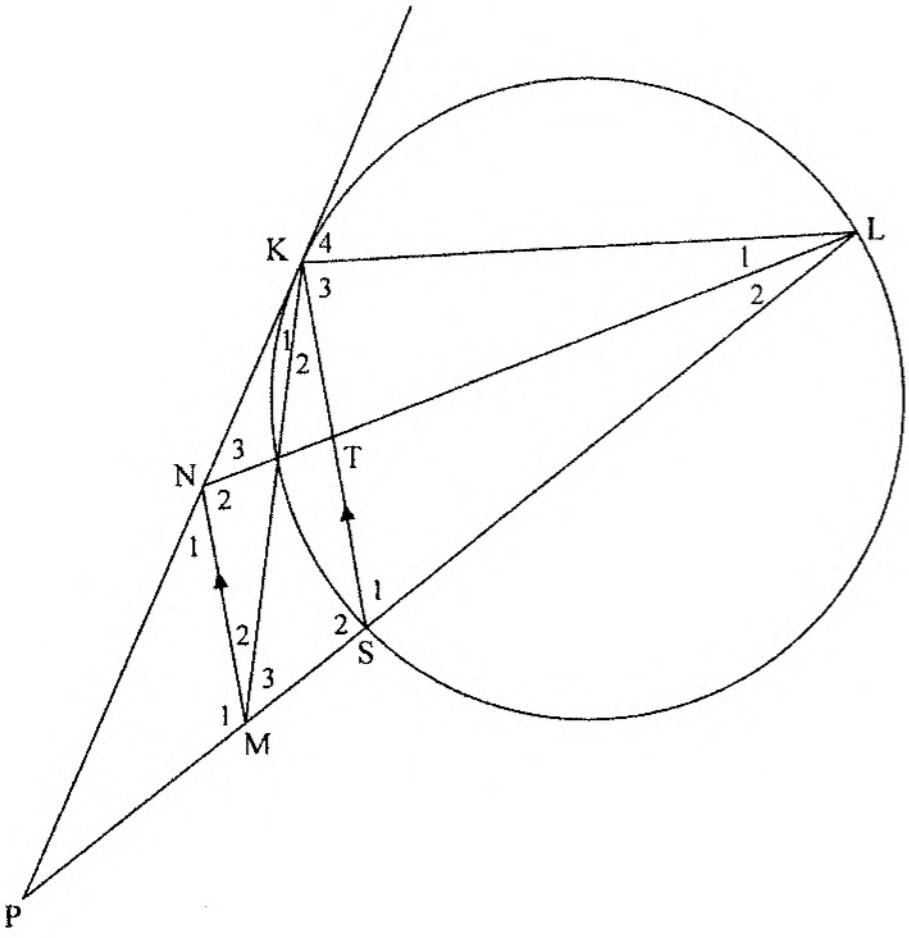


	Solution/*Oplossing*	**Marks** ***Punte***
		(5)

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11.2



	Solution/*Oplossing*	Marks *Punte*
11.2.1(a)		(4)
11.2.1(b)		(1)

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	Solution/*Oplossing*	Marks *Punte*
11.2.2		(5)
11.2.3		(4)
11.2.4		(4)
		[23]

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	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

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	Additional space/*Bykomende ruimte*	Marks/*Punte*

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	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

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	Additional space/*Bykomende ruimte*	Marks *Punte*

TOTAL/TOTAAL: 150

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RE-MARK/RE-CHECK *HERMERK/HERSIEN*				
Question *Vraag*	Marks *Punte*			Initials *Voorletters*
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
TOTAL *TOTAAL*				
HASH TOTAL *KAF-TOTAAL*				

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GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

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NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

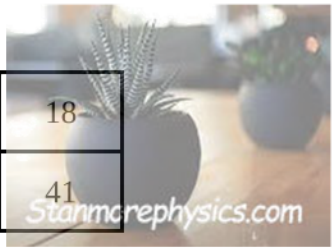
GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

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QUESTION/*VRAAG* 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41



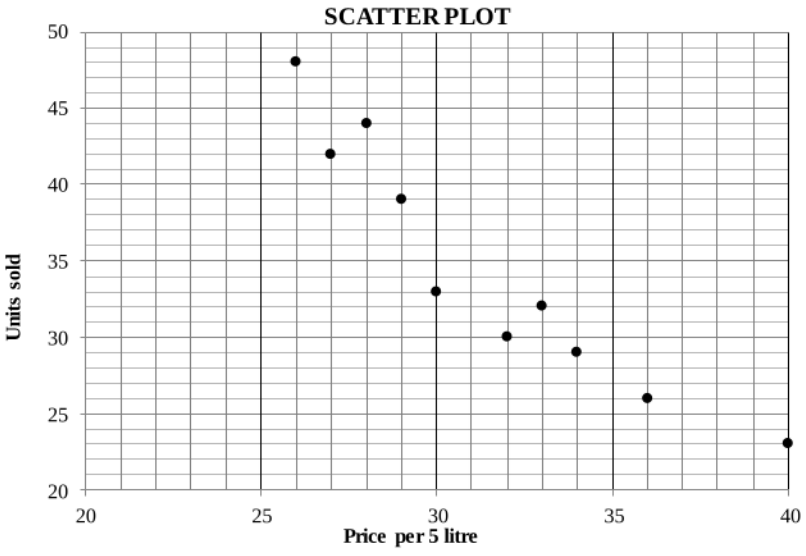
1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$ Answer only: Full marks *Slegs antw: Volpunte*	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10{,}1707 \approx 10{,}17$	✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore$ 5 days	✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** 22–20 = 2 $2 \times 18 = 36$	✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/*Opsie B* <u>Any one of the following reasons/*Enige een van die vlg redes*</u>: • Median/*Mediaan* = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*	✓ answer (1)
		[10]

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QUESTION/*VRAAG* 2

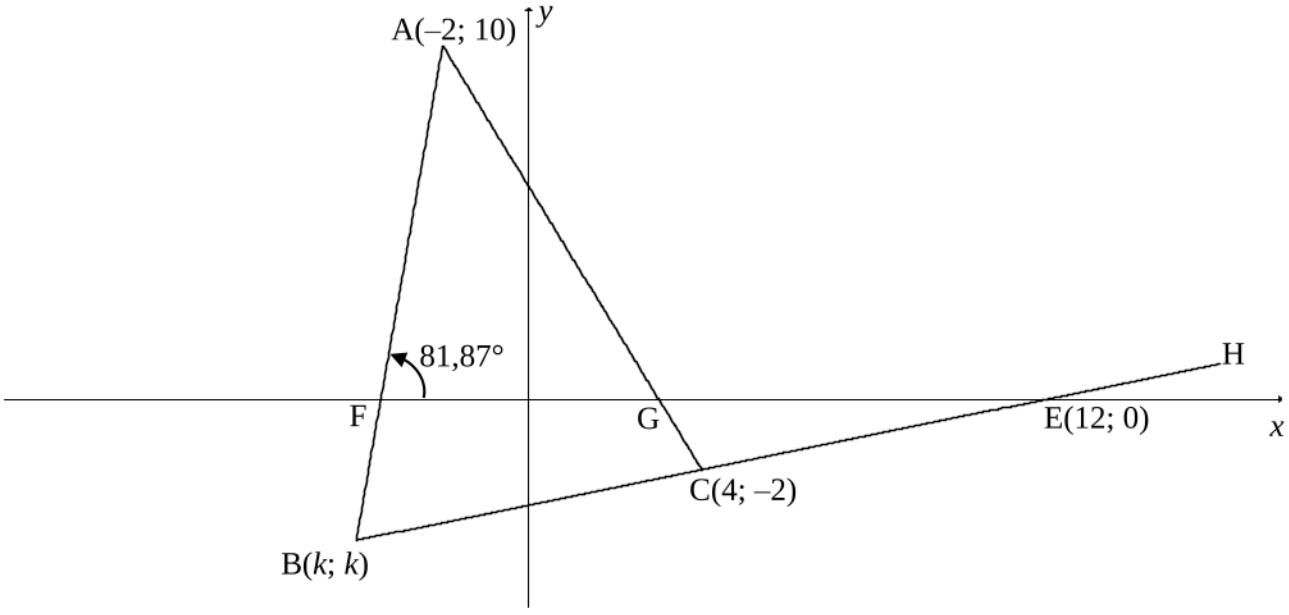
Price of milk in rands per 5-litre container (*x*) ***Prys van melk in rand, per 5 liter-houer (x)***	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (*y*) ***Aantal 5 liter-houers melk verkoop (y)***	48	30	26	44	23	32	39	29	42	33

2.1		1 mark: 3 to 5 points plotted correctly 2 marks: 6 to 9 points plotted correctly 3 marks: all points plotted correctly (3)
2.2	$a = 90{,}478... \approx 90{,}48$ $b = -1{,}773... \approx -1{,}77$ $\hat{y} = 90{,}48 - 1{,}77x$ Answer only: Full marks *Slegs antw: Volpunte*	✓ *a* ✓ *b* ✓ equation (3)
2.3	$y = 23{,}069\ldots \approx 23{,}07$ units/*eenhede* (calculator/*sakrekenaar*) **OR/*OF*** $y = 90{,}48 - 1{,}77(38)$ $y = 23{,}22$ units/*eenhede*	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	$r = -0{,}94$ The value of *r* indicates a strong relationship between the cost per 5 litre and the number of units sold ∴ there is a good chance of the prediction being accurate./ *Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan ∴ daar is 'n goeie kans dat die voorspelling akkuraat is*	✓ value of *r* **OR/OF** strong relationship/ *sterk verwantskap* ✓ accurate/*akkuraat* (2)
		[10]

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QUESTION/*VRAAG* 3



3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

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3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore$ B(–4; –4) **OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $-4k=12-k$ $k=-4$ $\therefore$ B(–4; –4) **OR/*OF*** $m_{AB}=\tan 81{,}87°$ $m_{AB}=7$ $m_{AB}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore$ B(–4; –4) **OR/*OF*** EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore$ B(–4; –4)	**OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $4k=k-12$ $k=-4$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2) ✓ equating EB & AB ✓ answer (2)

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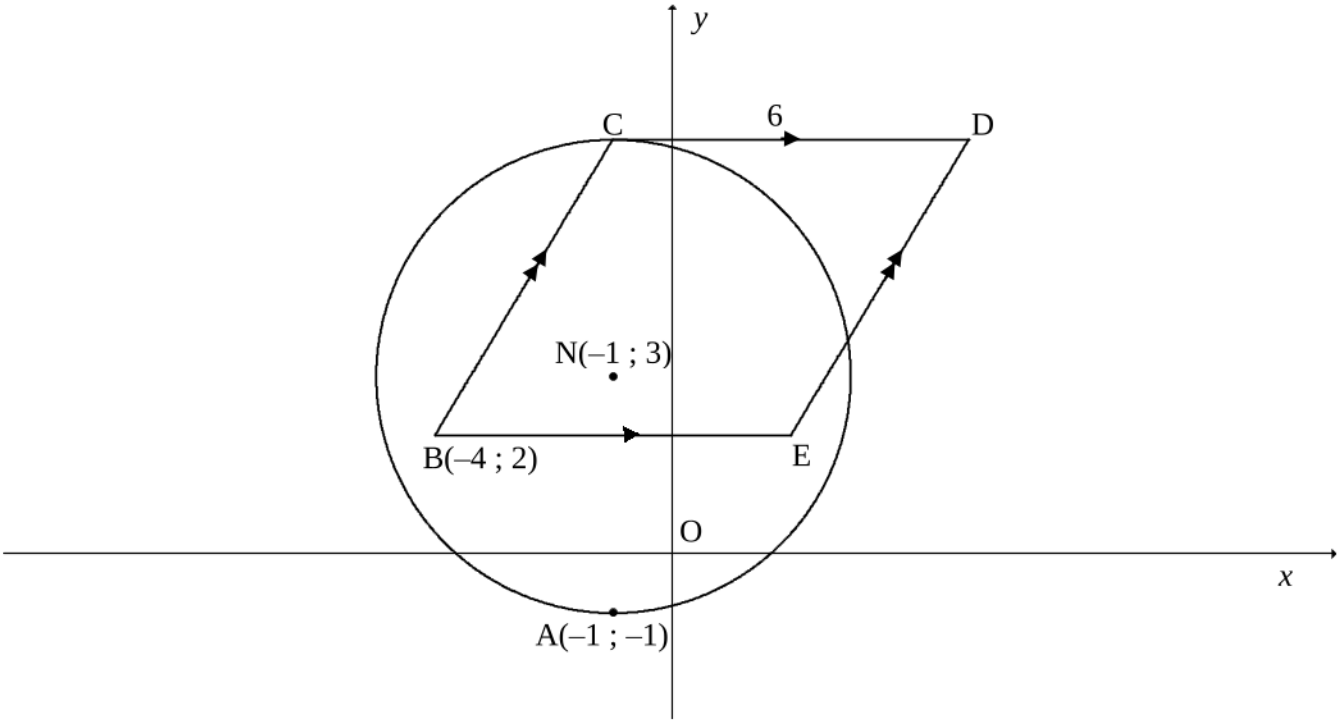
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3.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63,43...°$ $\therefore \theta = 116,57°$ $\therefore \hat{A} = 116,57° - 81,87°$ [ext $\angle$ of Δ] $\therefore \hat{A} = 34,70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0,822...$ $\therefore A = 34,7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116,57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ *x*-value ✓ *y*-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$ T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

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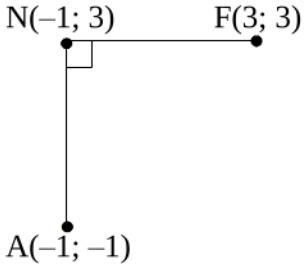
QUESTION/*VRAAG* 4



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\,;\,7)$	✓*x* value ✓ *y* value (2)
4.2.2	CD = 6 units $\therefore D(5\,;\,7)$	✓*x* value ✓ *y* value (2)
4.2.3	$\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

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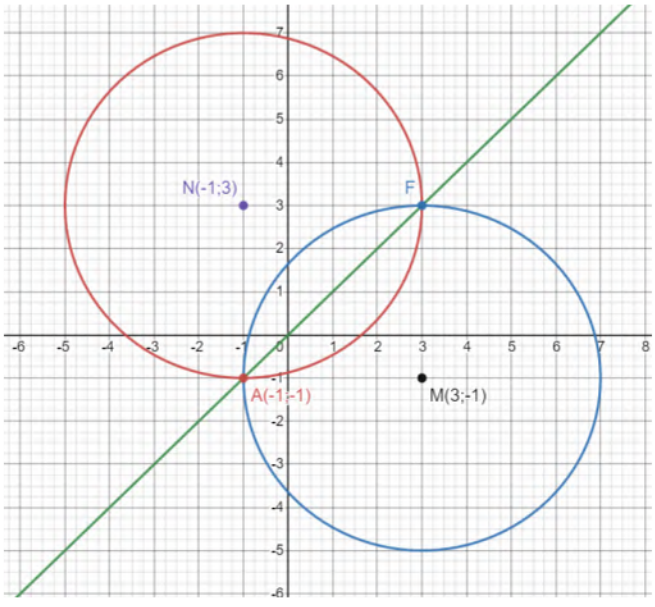
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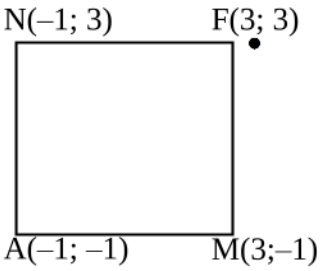
	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...°$ $B\hat{C}D = 180° - \alpha$ $B\hat{C}D = 180° - 59{,}036...°$ $B\hat{C}D = 120{,}96°$ Area $\Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96°$ $= 15$ units^2	✓ $B\hat{C}D = 120{,}96°$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $c = 2$ $\therefore y = -x + 2$ $y - 3 = -1(x+1)$ $y - 3 = -x - 1$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1)	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4)
	OR/*OF* N(–1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2}; \frac{-1+3}{2}\right)$ = (1 ; 1)  	✓✓ coordinates of F ✓x value ✓ y value (4)

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OR/*OF*





NAMF is a square (NA=NF=AM=MF and NA ⊥ AM)

Midpoint NM = (1 ; 1)

= Midpoint of AF

✓ NAMF = square

✓*x* ✓ *y* of midpt NM
✓ midpt AF

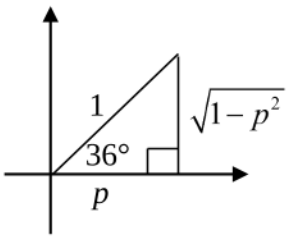
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[15]

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QUESTION/*VRAAG* 5

5.1	$\frac{\sin 140°.\sin(360°-x)}{\cos 50°.\tan(-x)}$ $=\frac{\sin 40°(-\sin x)}{\sin 40°(-\tan x)}$ $=\frac{-\sin x}{-\frac{\sin x}{\cos x}}$ $=\cos x$	✓ sin40° ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ answer (6)
5.2	$\text{LHS}=\frac{-2\sin^2 x+\cos x+1}{1-\cos(540°-x)}$ $\quad$ $\text{RHS}=2\cos x-1$ $\text{LHS}=\frac{-2(1-\cos^2 x)+\cos x+1}{1-(-\cos x)}$ $\text{LHS}=\frac{-2+2\cos^2 x+\cos x+1}{1+\cos x}$ $\text{LHS}=\frac{2\cos^2 x+\cos x-1}{1+\cos x}$ $\text{LHS}=\frac{(2\cos x-1)(\cos x+1)}{1+\cos x}$ $\text{LHS}=2\cos x-1$ $\therefore \text{LHS}=\text{RHS}$	✓ identity i. t. o. $\cos x$ ✓ $\cos(540°-x)=-\cos x$ ✓ standard form ✓ factors (4)
5.3.1	$\sin 36°=\sqrt{1-p^2}$ $\tan 36°=\frac{\sqrt{1-p^2}}{p}$ **OR/*OF*** $\cos^2 36°=1-\sin^2 36°$ $\cos 36°=\sqrt{1-(1-p^2)}$ $=p$ $\tan 36°=\frac{\sin 36°}{\cos 36°}$ $=\frac{\sqrt{1-p^2}}{p}$	 ✓ method ✓ value of p ✓ answer (3) ✓ method ✓ $\cos 36°=p$ ✓ answer (3)

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5.3.2	$\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(2\cos^2 36° - 1)$ $= -2p^2 + 1$ **OR/*OF*** $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(1 - 2\sin^2 36°)$ $= -1 + 2\left(\sqrt{1-p^2}\right)^2$ $= -1 + 2(1-p^2)$ $= -2p^2 + 1$ **OR/*OF*** $\cos 108°$ $= -\cos 72°$ $= -\cos(2\times 36°)$ $= -(\cos^2 36° - \sin^2 36°)$ $= -\left(p^2 - \left(\sqrt{1-p^2}\right)^2\right)$ $= -\left(p^2 - (1-p^2)\right)$ $= -2p^2 + 1$ **OR/*OF*** $\cos 108°$ $= \cos(2\times 54°)$ $= 2\cos^2 54° - 1$ $= 2(1-p^2) - 1$ $= 1 - 2p^2$ **OR/*OF*** $\cos 108° = \cos(72° + 36°)$ $= \cos 72°\cos 36° - \sin 72°\sin 36°$ $= (2\cos^2 36° - 1)\cos 36° - (2\sin 36°\cos 36°)\sin 36°$ $= 2\cos^3 36° - \cos 36° - 2\cos 36°\sin^2 36°$ $= 2p^3 - p - 2p\left(\sqrt{1-p^2}\right)^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4) ✓ expansion ✓ both double angle identities ✓ value of sin 36° ✓ answer i. t. o. p (4)
		[17]

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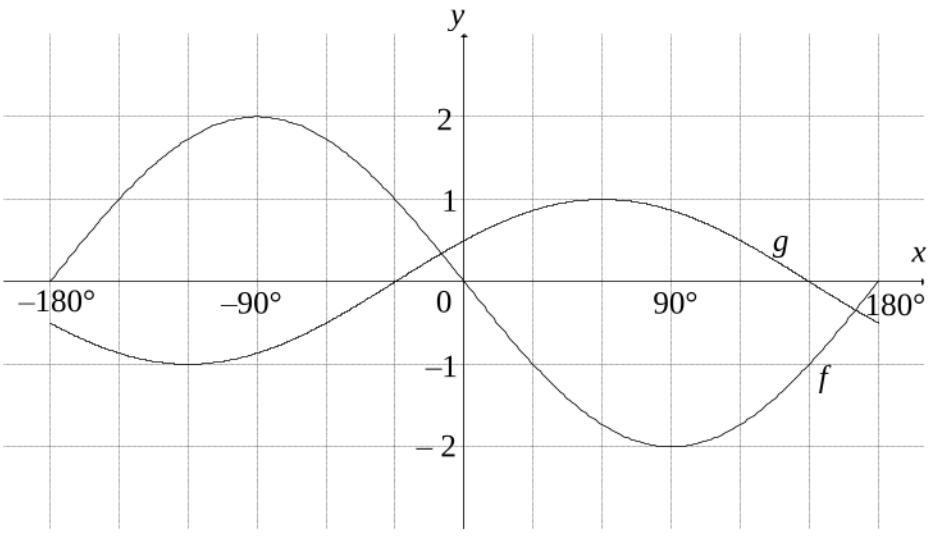
QUESTION/*VRAAG* 6

6.1.1	$\cos(\alpha+\beta)$ $=\cos(\alpha-(-\beta))$ $=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)$ $=\cos\alpha\cos\beta+\sin\alpha(-\sin\beta)$ $=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	✓ $\cos(\alpha-(-\beta))$ ✓ expansion ✓ reduction (3)
6.1.2	$2\cos 6x\cos 4x-\cos 10x+2\sin^2 x$ $=2\cos 6x\cos 4x-\cos(6x+4x)+2\sin^2 x$ $=2\cos 6x\cos 4x-(\cos 6x\cos 4x-\sin 6x\sin 4x)+2\sin^2 x$ $=\cos 6x\cos 4x+\sin 6x\sin 4x+2\sin^2 x$ $=\cos 2x+2\sin^2 x$ $=1-2\sin^2 x+2\sin^2 x$ $=1$	✓ $\cos 10x=\cos(6x+4x)$ ✓ expansion of $\cos(6x+4x)$ ✓ $\cos 2x$ ✓ $1-2\sin^2 x$ ✓ answer (5)
6.2	$\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=2(2\sin x\cos x)$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $\sin x(4\cos^2 x-1)=0$ $\sin x=0$ or $\cos^2 x=\frac{1}{4}$ $\cos x=-\frac{1}{2}$ $x=180°+k.360°;\ k\in Z$ or $x=120°+k.360°;\ k\in Z$ $x=240°+k.360°;\ k\in Z$ **OR/*OF*** $\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=4\sin x\cos x$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $4\sin x(1-\sin^2 x)-\sin x=0$ $3\sin x-4\sin^3 x=0$ $\sin x(3-4\sin^2 x)=0$ $\sin x=0$ or $\sin^2 x=\frac{3}{4}$ $\sin x=\frac{\sqrt{3}}{2}$ or $\sin x=-\frac{\sqrt{3}}{2}$ $x=180^\circ+k.360^\circ,k\in Z$ or $x=120^\circ+k.360^\circ,k\in Z$ or $x=240^\circ+k.360^\circ,k\in Z$	✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm 120°$ ✓ $k.360°;\ k\in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm 120°$ ✓ $k.360°;\ k\in Z$ (7)
		[15]

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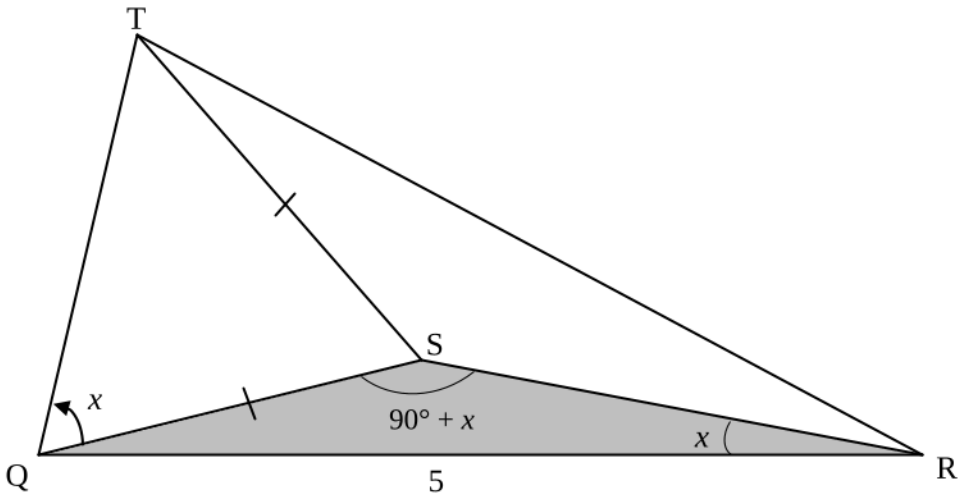
QUESTION/*VRAAG* 7

7.1		✓ both turning points ✓ both *x* intercepts (–30° & 150°) ✓ shape (3)
7.2	Period = 120°	✓✓ answer (2)
7.3	$x = -30°$	✓ answer (1)
7.4	Range of/*waardeversameling van g*: $y \in [-1;1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2};\frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2};\frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $\frac{1}{2} \le y \le \frac{3}{2}$	✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
		[8]

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QUESTION/*VRAAG* 8



8.1	In ΔSQR: $\frac{QS}{\sin x}=\frac{QR}{\sin(90°+x)}$ $\frac{QS}{\sin x}=\frac{5}{\cos x}$ $QS=\frac{5\sin x}{\cos x}$ $QS=5\tan x$	✓ correct use of sine rule ✓ $\sin(90°+x)=\cos x$ ✓ $QS=\frac{5\sin x}{\cos x}$ (3)
8.2	$\frac{QT}{\sin(180°-2x)}=\frac{TS}{\sin x}$ $\frac{QT}{\sin 2x}=\frac{5\tan x}{\sin x}$ $QT=\frac{5\tan x\sin 2x}{\sin x}$ $QT=\frac{5\left(\frac{\sin x}{\cos x}\right)(2\sin x\cos x)}{\sin x}$ $QT=\frac{5\sin x(2\sin x)}{\sin x}$ $QT=10\sin x$	✓ correct use of sine rule ✓ TS = QS = 5tan*x* ✓ $QT=\frac{5\tan x\sin 2x}{\sin x}$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\sin 2x=2\sin x\cos x$ (5)

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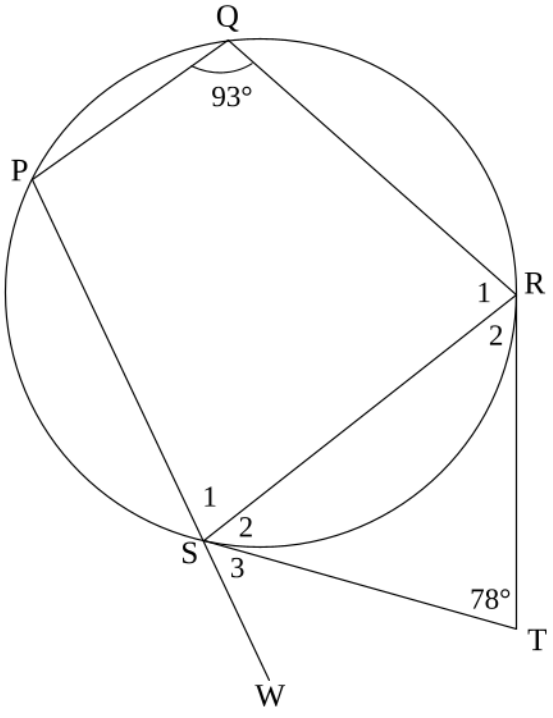
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	OR/*OF*	
	$QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$ $QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$ $QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$ $QT^2 = 50\tan^2 x(1+\cos 2x)$ $QT^2 = 50\tan^2 x(1+2\cos^2 x-1)$ $QT^2 = 50\tan^2 x(2\cos^2 x)$ $QT^2 = 100\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)$ $QT^2 = 100\sin^2 x$ $QT = 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan*x* ✓ $\cos 2x = 2\cos^2 x - 1$ & reduction ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $QT^2 = 100\sin^2 x$ (5)
	OR/*OF*	
	$TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ[TQ - 10\tan x.\cos x]$ $TQ = 10\tan x.\cos x \quad (TQ \neq 0)$ $= 10\dfrac{\sin x}{\cos x}.\cos x$ $= 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan*x* ✓ quadratic equation ito TQ ✓ TQ = 10tan *x* . cos *x* ✓ $\tan x = \dfrac{\sin x}{\cos x}$ (5)
8.3	Area of $\Delta TQR = \dfrac{1}{2}.TQ.QR\sin T\hat{Q}R$ $= \dfrac{1}{2}(10\sin 25°)(5)(\sin 70°)$ $= 9{,}93\text{ unit}^2$	✓ correct substitution into the area rule ✓ answer (2)
		[10]

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QUESTION/VRAAG 9

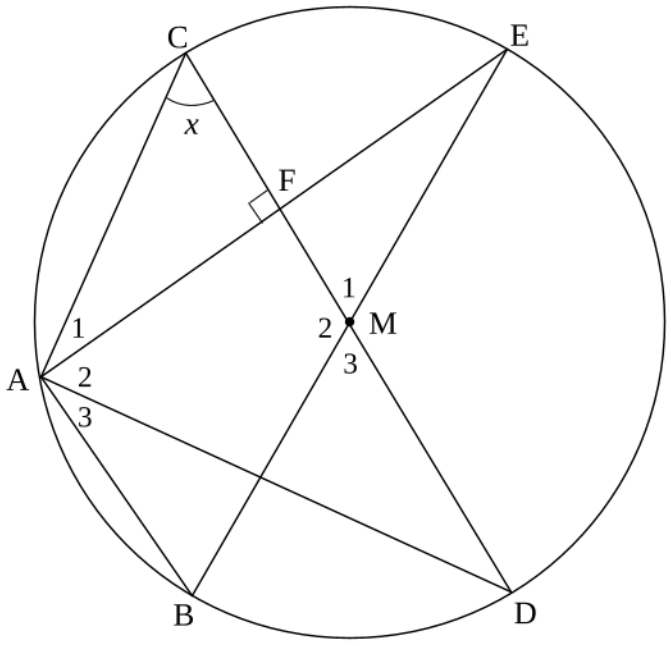


9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp ∠s of cyclic quad/*teenoorst ∠e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [∠s on a str line/*∠e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

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QUESTION/*VRAAG* 10



10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt ⊥ op koord*	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts∠*= 2×omtreks∠]	✓ S ✓ S ✓ R (3)
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/*∠ in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl ∠e*] $\therefore \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts∠*= 2 ×omtreks∠] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $\hat{M}_3 = 180^\circ - 2x$ [vert. opp/ *regoorstaande ∠e*] $\therefore \hat{A}_3 = 90^\circ - x$ [∠at centre= 2×∠ at circumf/*midpts∠*= 2 ×omtreks∠] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/*∠ in halfsirkel*] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ S ✓ R ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

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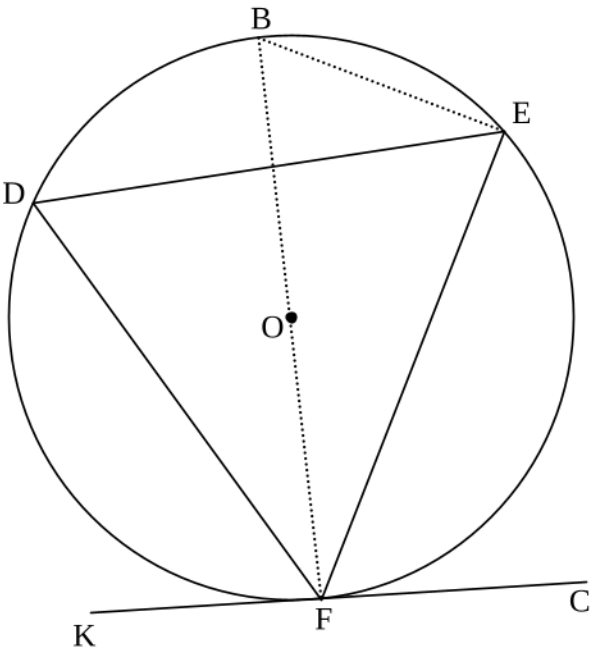
	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x =$ C ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠ in halfsirkel*] ∴ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)
10.4	AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] ∴ EB = 36 units [diameter = 2 radius] ∴ $AE^2 = (36)^2 - (24)^2$ [Pythagoras] AE = $12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] ∴ $FE^2 = (18)^2 - (12)^2$ [Pythagoras] FE = $6\sqrt{5}$ AE = $12\sqrt{5}$ or 26,83 units	✓ FM = 12 ✓ R ✓ EB = 36 ✓ using Pyth correctly ✓ answer (5) ✓ FM = 12 ✓ R ✓ EM = 18 ✓ using Pyth correctly ✓ answer (5)
		[13]

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QUESTION/*VRAAG* 11

11.1

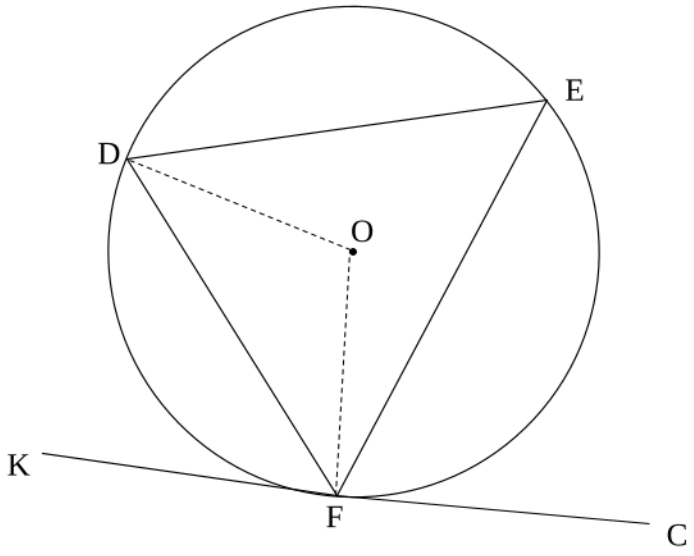


	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$	
	$= 90° - B\hat{F}D$ [∠s same segment/*∠e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

OR/*OF*

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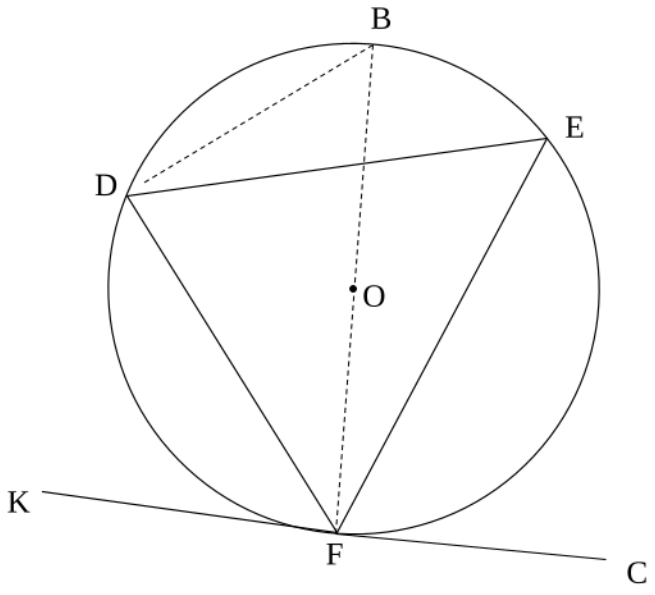


	Construction: Draw radii DO and OF *Konstruksie: Trek radii DO en OF* $O\hat{F}K = 90°$ or $D\hat{F}K = 90° - O\hat{F}D$ radius⊥ tangent/*raaklyn*] $O\hat{D}F = O\hat{F}D$ [∠s opp = sides/*∠e teenoor = sye*] $\therefore D\hat{O}F = 180° - 2O\hat{F}D$ [∠s of Δ/*∠e van Δ*] $D\hat{E}F = 90° - O\hat{F}D$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore D\hat{F}K = D\hat{E}F$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

OR/OF

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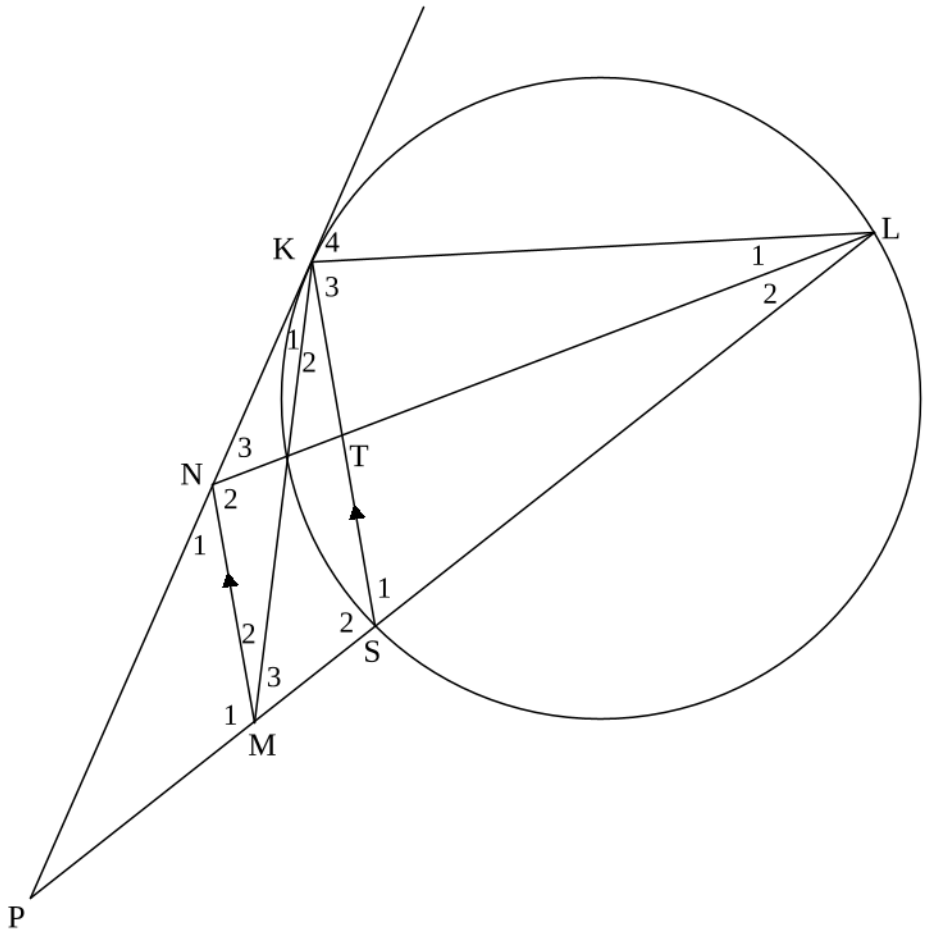


	Construction: Draw diameter BF and join BD. *Konstruksie: Trek middellyn BF en verbind BD.* $B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*] $F\hat{D}B = 90°$ [∠ in half circle/*semi-sirkel*] $\hat{B} = 90° - B\hat{F}D$ $\therefore D\hat{F}K = \hat{B}$ but $\hat{B} = \hat{E}$ [∠s same segment/*∠e dieselfde segment*] $\therefore D\hat{F}K = \hat{E}$	✓ construction ✓ S ✓/R ✓ S ✓ S/R (5)

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11.2



11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp ∠s; / *ooreenk ∠s;* MN || KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)
11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$ $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF** $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp ∠s; / *ooreenk ∠s;* MN || KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*] $\hat{N}_1 = K\hat{L}S$ $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF**	✓ R (1) ✓ R (1)

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	$NKL = 180^\circ - K_4$ [adj. suppl.] $\therefore NKL = 180^\circ - NML$ [proved] $\therefore$ KLMN is a cyclic quad [opp.$\angle$s supplementary]	✓ R (1)
11.2.2	In $\Delta LKN \| \| \| \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $\hat{L}_1 = \hat{M}_2$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $= \hat{K}_2$ [alt $\angle$s; / *verw$\angle$e;* MN||KS] $N\hat{K}L = M\hat{S}K$ [$\angle$s of Δ / *$\angle$e van Δ*] $\Delta LKN \| \| \| \Delta KSM$	✓ S ✓ R ✓ S ✓ S/R ✓ S (5)
	OR/OF In $\Delta LKN \| \| \| \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $N\hat{K}L = \hat{M}_1$ [ext $\angle$ of cyclic quad/*buite$\angle$ van koordevh*] $= \hat{S}_2$ [corresp $\angle$s/*ooreenk $\angle$e*; KS || NM] $\Delta LKN \| \| \| \Delta KSM$ [$\angle$,$\angle$,$\angle$]	✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
	OR/OF In $\Delta LKN \| \| \| \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / *$\angle$e in dieselfde sirkel segm*] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [$\angle$s on straight line/*$\angle$e op reguitlyn*] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\Delta LKN \| \| \| \Delta KSM$ [$\angle$,$\angle$,$\angle$]	✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\Delta LKN \| \| \| \Delta KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line || one side of Δ / *lyn || een sy v Δ*] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/*TOTAAL*: 150

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