



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE SENIOR
SERTIFIKAAT**

GRADE 12/GRAAD 12

MATHEMATICS P1/WISKUNDE V1

NOVEMBER 2019

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 18 pages.
*Hierdie nasienriglyne bestaan uit 18 bladsye.***

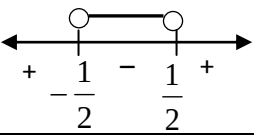
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.
- Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.

QUESTION/VRAAG 1

1.1.1	$x^2 + 5x - 6 = 0$ $(x + 6)(x - 1) = 0$ $x = -6 \text{ or } x = 1$	✓ factors ✓ $x = -6$ ✓ $x = 1$ (3)
1.1.2	$4x^2 + 3x - 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-5)}}{2(4)}$ $x = \frac{-3 \pm \sqrt{89}}{8}$ $x = -1,55 \text{ or } x = 0,8$	✓ substitution into the correct formula ✓ $x = -1,55$ ✓ $x = 0,8$ (3)
1.1.3	$4x^2 - 1 < 0$ $(2x + 1)(2x - 1) < 0$ $-\frac{1}{2} < x < \frac{1}{2}$ 	✓ factors ✓ method ✓ answer (3)
1.1.4	$(\sqrt{\sqrt{32} + x})(\sqrt{\sqrt{32} - x}) = x$ $\sqrt{32 - x^2} = x$ $32 - x^2 = x^2$ $-2x^2 = -32$ $x^2 = 16$ $x = \pm 4$ $\therefore x = 4$	✓ $\sqrt{32 - x^2}$ ✓ squaring both sides ✓ $x^2 = 16$ ✓ $x = 4$ (selection) (4)

1.2	$y + x = 12$ $y = -x + 12 \dots\dots\dots(1)$ $xy = 14 - 3x \dots\dots\dots(2)$ Sub (1) into (2) $x(-x + 12) = 14 - 3x$ $-x^2 + 12x - 14 + 3x = 0$ $-x^2 + 15x - 14 = 0$ $x^2 - 15x + 14 = 0$ $(x - 14)(x - 1) = 0$ $x = 14 \quad \text{or} \quad x = 1$ $y = -2 \quad \text{or} \quad y = 11$ OR/OF $y + x = 12$ $x = -y + 12 \dots\dots\dots(1)$ $xy = 14 - 3x \dots\dots\dots(2)$ Sub (1) into (2) $y(-y + 12) = 14 - 3(-y + 12)$ $12y - y^2 - 14 + 36 - 3y = 0$ $-y^2 + 9y + 22 = 0$ $y^2 - 9y - 22 = 0$ $(y + 2)(y - 11) = 0$ $y = -2 \quad \text{or} \quad y = 11$ $x = 14 \quad \text{or} \quad x = 1$	✓ y subject of the formula ✓ substitution ✓ simplification ✓ both values of x ✓ both values of y (5) OR/OF ✓ x subject of the formula ✓ substitution ✓ simplification ✓ both values of y ✓ both values of x (5)
1.3	$3 \quad 6 \quad 9 \quad 12 \quad 15 \quad 18 \quad 21 \quad 24 \quad 27 \quad 30$ $3 \quad 3 \quad 3^2 \quad 3 \quad 3 \quad 3^2 \quad 3 \quad 3 \quad 3^3 \quad 3$ $\therefore k = 14$	✓ identifying multiples of 3 ✓ ten multiples of 3 ✓ powers of 3 ✓ answer (4)
		[22]

QUESTION/VRAAG 2

2.1.1	209 ; 186	✓209 ✓186 (2)
2.1.2	$ \begin{array}{ccccccc} & 321 & & 290 & & 261 & & 234 \\ & \diagdown & & \diagup & & \diagdown & & \diagup \\ 1^{st} \text{ diff} & & -31 & & -29 & & -27 & \\ & & \diagdown & & \diagup & & \diagdown & \\ 2^{nd} \text{ diff} & & & 2 & & 2 & & \end{array} $ $ \begin{array}{lll} 2a = 2 & 3a + b = -31 & a + b + c = 321 \\ a = 1 & 3(1) + b = -31 & 1 + (-34) + c = 321 \\ & b = -34 & c = 354 \end{array} $ $T_n = n^2 - 34n + 354$	✓ 2 nd diff = 2 ✓ a = 1 ✓ b = -34 ✓ c = 354 (4)
2.1.3	$n^2 - 34n + 354 = 74$ $n^2 - 34n + 280 = 0$ $(n-14)(n-20) = 0$ $n = 14 \quad \text{or} \quad n = 20$	✓equating T_n to 74 ✓standard form ✓14 ✓20 (4)
2.1.4	$f'(n) = 0$ $2n - 34 = 0$ $2n = 34$ $n = 17$ Term 17 will have the smallest value OR/OF $n = \frac{-b}{2a}$ $n = \frac{34}{2}$ $n = 17$ Term 17 will have the smallest value OR/OF $n = \frac{14+20}{2} = 17$ Term 17 will have the smallest value	✓ $2n - 34 = 0$ ✓answer (2) OR/OF ✓substitution ✓answer (2) OR/OF ✓substitution ✓answer (2)

2.2.1	$a = \frac{5}{8} \quad ; \quad r = \frac{1}{2} \quad ; \quad n = 21$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_{21} = \frac{\frac{5}{8} \left(1 - \left(\frac{1}{2} \right)^{21} \right)}{1 - \frac{1}{2}}$ $= 1,2499...$ $= 1,25$	✓ r ✓ substitution into the correct formula ✓ answer (3)
2.2.2	$T_n > \frac{5}{8192}$ $ar^{n-1} > \frac{5}{8192}$ $\frac{5}{8} \left(\frac{1}{2} \right)^{n-1} > \frac{5}{8192}$ $\left(\frac{1}{2} \right)^{n-1} > \frac{1}{1024}$ $\left(\frac{1}{2} \right)^{n-1} > \left(\frac{1}{2} \right)^{10} \quad \text{or} \quad 2^{-n+1} > 2^{-10}$ $\therefore n-1 < 10 \qquad \qquad -n+1 > -10$ $n < 11 \qquad \qquad \qquad n < 11$ $\therefore n = 10 \qquad \qquad \qquad \therefore n = 10$ OR/OF 8 ; 16 ; 32 ; ... ; 8192 $8 \cdot 2^{n-1} < 8192$ $2^{n-1} < 1024$ $2^{n-1} < 2^{10}$ $n-1 < 10$ $n < 11$ $\therefore n = 10$	✓ substitution into the correct formula ✓ method /same base or log ✓ calculating n ✓ answer (4) OR/OF ✓ substitution into the correct formula ✓ method ✓ calculating n ✓ answer (4)
		[19]

QUESTION/VRAAG 3

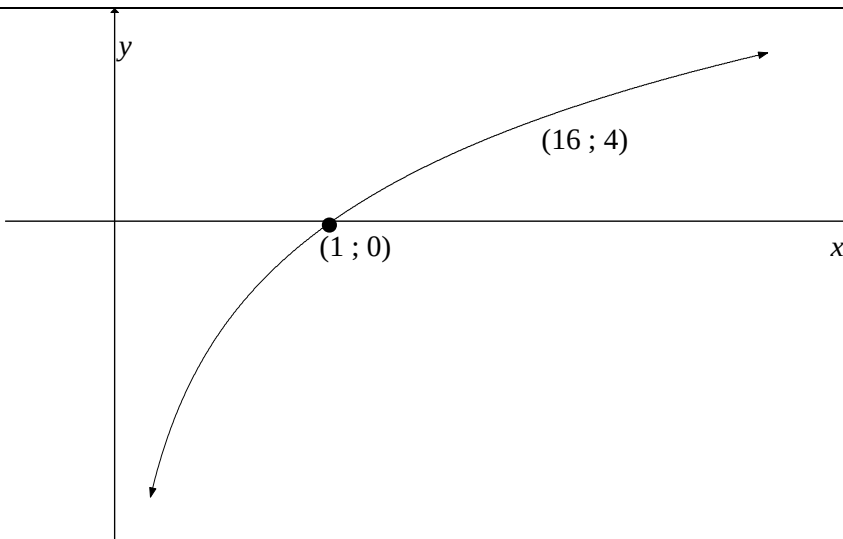
3.1	$\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ $= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} \right) - \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} + \frac{1}{9} \right)$ $= 1 - \frac{1}{9}$ $= \frac{8}{9}$	$\checkmark \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} \right)$ $\checkmark \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} + \frac{1}{9} \right)$ $\checkmark \text{ answer} \quad (3)$
3.2	$\left(\frac{1}{3} \times \frac{2}{3} \right) + \left(\frac{2}{3} \times \frac{2}{3} \right) + \left(1 \times \frac{2}{3} \right) + \dots + \left(4 \times \frac{2}{3} \right)$ $= \frac{2}{9} + \frac{4}{9} + \frac{2}{3} + \dots + \frac{8}{3}$ $a = \frac{2}{9} \quad \text{and} \quad d = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$ $S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{OR} \quad S_n = \frac{n}{2} (a + l)$ $S_{12} = \frac{12}{2} \left[2 \left(\frac{2}{9} \right) + (12-1) \frac{2}{9} \right] \quad S_{12} = \frac{12}{2} \left(\frac{2}{9} + \frac{8}{3} \right)$ $= \frac{52}{3} \text{ m}^2 \quad \quad \quad = \frac{52}{3} \text{ m}^2$ $\therefore \text{ for both sides} = 2 \times \frac{52}{3} = \frac{104}{3} = 34,67 \text{ m}^2$ OR/OF $\frac{2}{9} \times (1+2+3+4+5+6+7+8+9+10+11+12) \times 2$ $= 34,67 \text{ m}^2$ OR/OF $T_1 = \frac{2}{9} \times 12 = \frac{8}{3} \quad \quad \quad l = \frac{2}{9} \times 1 = \frac{2}{9}$ $2S_{12} = 2 \left(\frac{12}{2} \right) \left(\frac{8}{3} + \frac{2}{9} \right)$ $= 34,67 \text{ m}^2$	$\checkmark \checkmark a$ $\checkmark d$ $\checkmark \text{ substitution into the correct formula}$ $\checkmark \text{ answer}$ $\checkmark \text{ answer for both sides} \quad (6)$ OR/OF $\checkmark \checkmark a$ $\checkmark \checkmark (1 + \dots + 12)$ $\checkmark \times 2$ $\checkmark \text{ answer} \quad (6)$ OR/OF $\checkmark \checkmark a$ $\checkmark T_1 = \frac{8}{3} \quad \checkmark l = \frac{2}{9}$ $\checkmark \text{ substitution into correct formula}$ $\checkmark \text{ answer} \quad (6)$
		[9]

QUESTION/VRAAG 4

4.1	$p = -1$	✓ $p = -1$ (1)
4.2	$y = \frac{a}{x-1}$ $-3 = \frac{a}{0-1}$ $a = 3$ $y = x^2 + bx - 3$ $0 = (1)^2 + (1)b - 3$ $b = 2$	✓ coordinates D(0 ; -3) ✓ substitute (0 ; -3) ✓ substitute (1 ; 0) (3)
4.3	$y = x^2 + 2x - 3$ axis of sym: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)}$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4) OR/OF $\frac{dy}{dx} = 0$ $2x + 2 = 0$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4)	✓ substitution ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4) OR/OF ✓ derivative ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4)
4.4	$y \in [-4; \infty)$ or $y \geq -4$	✓ -4 ✓ answer (2)
4.5	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	✓ gradient ✓ subs m and $(-1; -4)$ ✓ equation (3)
4.6	No, the line passes through C and D OR/OF No, a tangent through turning point C will have a gradient of 0	✓ No ✓ reason (2) OR/OF ✓ No ✓ reason (2)

4.7	$f(m-x) = f[-(x-m)]$ f is reflected in the y -axis and translated 1 unit to the left and 4 units upwards. Therefore: $m = -1$ $q = 4$ OR/OF Substitute $x = 0$ and $q = 4$ for one x - intercept $h(x) = (m-x)^2 + 2(m-x) - 3 + q$ $h(0) = (m-0)^2 + 2(m-0) - 3 + 4$ $0 = m^2 + 2m + 1$ $0 = (m+1)^2$ $m = -1$ $q = 4$	✓✓ value of m ✓✓ value of q (4) OR/OF ✓✓ value of m ✓✓ value of q (4)
		[19]

QUESTION/VRAAG 5

5.1	$f(x) = k^x$ $16 = k^4$ $k = 2$	✓ substitution (4 ; 16) ✓ answer (2)
5.2	$f : y = 2^x$ $f^{-1} : x = 2^y$ $y = \log_2 x$	✓ $x = 2^y$ ✓ $y = \log_2 x$ (2)
5.3		✓ asymptote ✓ shape ✓ for any two valid points eg.(16 ; 4) or (2 ; 1) or (4 ; 2) or (1 ; 0) (4)
5.4.1	$x \in (1 ; \infty)$ or $x > 1$	✓ 1 ✓ answer (2)
5.4.2	$0 < x \leq \frac{1}{2}$ or $x \in \left(0; \frac{1}{2}\right]$	✓ $\frac{1}{2}$ ✓ answer (2)

QUESTION/VRAAG 6

6.1	Kuda : $A = P(1 + in)$ $= 5\,000(1 + 0,083 \times 4)$ $= R6\,660,00$ Final Answer : $R6\,660,00 + R266,40$ $= R6\,926,40$ OR/OF Kuda : $A = P(1 + in) \times 1,04$ $= 5\,000(1 + 0,083 \times 4) \times 1,04$ $= R6\,926,40$ Thabo : $A = P(1 + i)^n$ $= 5\,000 \left(1 + \frac{0,081}{12}\right)^{12 \times 4}$ $= R6\,905,71$ Kuda will have a better investment	✓ substitution into the correct formula ✓ final answer OR/OF ✓ substitution into the correct formula ✓ final answer ✓ substitution into the correct formula ✓ answer ✓ conclusion (5)
6.2.1	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $525\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,1}{12}\right)^{-n}\right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1 - \left(1 + \frac{0,1}{12}\right)^{-n}$ $-n \log \left(1 + \frac{0,1}{12}\right) = \log \frac{13}{48}$ $-n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12}\right)}$ $n = 157,40$ $n = 158 \text{ payments}$ OR/OF	✓ $\frac{0,1}{12}$ ✓ substitution into the correct formula ✓ simplification ✓ use of logs ✓ answer (5) OR/OF

	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $525\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,1}{12} \right)^{-12n} \right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1 - \left(1 + \frac{0,1}{12} \right)^{-12n}$ $-12n \log \left(1 + \frac{0,1}{12} \right) = \log \frac{13}{48}$ $-12n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12} \right)}$ $n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12} \right)} \times \frac{1}{12}$ $n = 13,11686841$ Number of payments = $13,11686841 \times 12 = 157,40$ $n = 158$ payments	✓ $\frac{0,1}{12}$ ✓ substitution into the correct formula ✓ simplification ✓ use of logs ✓ answer (5)
6.2.2	Difference: $R6\,000 - R5\,066,36 = R933,64$ $F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{933,64 \left[\left(1 + \frac{0,1}{12} \right)^{108} - 1 \right]}{\frac{0,1}{12}}$ $= R162\,503,51$ OR/OF	✓ $R933,64$ ✓ $n = 108$ ✓ substitution into the correct formula ✓ answer (4) OR/OF

	$F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{6000 \left[\left(1 + \frac{0,1}{12} \right)^{108} - 1 \right]}{\frac{0,1}{12}}$ $= R1\,044\,322,28$ $F = \frac{5\,066,36 \left[\left(1 + \frac{0,1}{12} \right)^{108} - 1 \right]}{\frac{0,1}{12}}$ $F = R881\,818,77.....$ Amount available for withdrawal $= R1\,044\,322,28 - R881\,818,77$ $= R162\,503,51$ OR/OF Outstanding balance with monthly repayment of R5 066,35 $= 525\,000 \left(1 + \frac{0,1}{12} \right)^{108} - \frac{5\,066,36 \left[\left(1 + \frac{0,1}{12} \right)^{108} - 1 \right]}{\frac{0,1}{12}}$ $= R404\,666,23$ Outstanding balance with monthly repayment of R6 000 $= 525\,000 \left(1 + \frac{0,1}{12} \right)^{108} - \frac{6\,000 \left[\left(1 + \frac{0,1}{12} \right)^{108} - 1 \right]}{\frac{0,1}{12}}$ $= R242\,162,72$ Amount available for withdrawal $R404\,666,23 - R242\,162,72 = R162\,512,18$	✓ $n = 108$ ✓ substitution into correct formula ✓ substitution into correct formula ✓ final answer (4) OR/OF ✓ $n = 108$ ✓ substitution into the correct formula ✓ substitution into the correct formula ✓ final answer (4)
		[14]

QUESTION/VRAAG 7

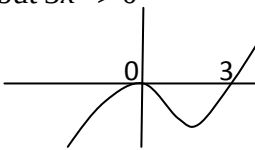
7.1	$f(x) = 4 - 7x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4 - 7(x+h) - (4 - 7x)}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-7)}{h}$ $= -7$	✓ $4 - 7(x+h)$ ✓ substitution ✓ simplification ✓ answer (4)
7.2	$y = 4x^8 + \sqrt{x^3}$ $= 4x^8 + x^{\frac{3}{2}}$ $\frac{dy}{dx} = 32x^7 + \frac{3}{2}x^{\frac{1}{2}}$	✓ $x^{\frac{3}{2}}$ ✓ $32x^7$ ✓ $\frac{3}{2}x^{\frac{1}{2}}$ (3)
7.3.1	$y = ax^2 + a$ $\frac{dy}{dx} = 2ax + 0$ $\frac{dy}{dx} = 2ax$	✓ $2ax$ (1)
7.3.2	$y = ax^2 + a$ $\frac{dy}{da} = x^2 + 1$	✓ ✓ answer (2)

7.4	Substitute (2 ; b) in $y = x + \frac{12}{x}$ $b = 2 + \frac{12}{2}$ $b = 8$ $m_{\text{tangent}} = \frac{dy}{dx}$ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ $m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$ $m_{\text{perp}} = \frac{1}{2}$ Equation of perpendicular line: $y - y_1 = m(x - x_1) \quad \text{OR} \quad y = mx + c$ $y - 8 = \frac{1}{2}(x - 2) \quad \quad 8 = \frac{1}{2}(2) + c$ $y = \frac{1}{2}x + 7 \quad \quad c = 7$ $y = \frac{1}{2}x + 7$	✓ value of b ✓ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ ✓ gradient of perpendicular line ✓ equation (4)
		[14]

QUESTION/VRAAG 8

8.1	36cm	✓ answer (1)
8.2	$\therefore t = 6$ ($-2t^2 + 3t - 6$) have no real roots Insect reaches the floor only once.	✓✓✓ only once (3)
8.3	$h(t) = -2t^3 + 15t^2 - 24t + 36$ $h'(t) = -6t^2 + 30t - 24$ $-6t^2 + 30t - 24 = 0$ $t^2 - 5t + 4 = 0$ $(t - 4)(t - 1) = 0$ $t = 4$ or $t = 1$ Only $t = 4$ because maximum value required $h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52 \text{ cm}$	✓ expansion ✓ $-6t^2 + 30t - 24 = 0$ ✓ both values ✓ answer (4)
		[8]

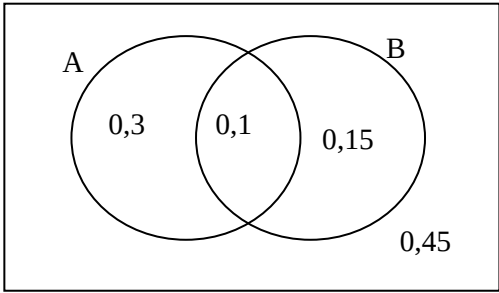
QUESTION/VRAAG 9

9.1	$f'(x) = 9x^2$ $3x^3 = 9x^2$ $3x^3 - 9x^2 = 0$ $3x^2(x - 3) = 0$ $x = 0$ or $x = 3$	$\checkmark f'(x) = 9x^2$ $\checkmark x = 0$ $\checkmark x = 3$ (3)
9.2.1	For f and f'	$\checkmark$ answer (1)
9.2.2	The point (0 ; 0) is : A point of inflection of f A turning point of f'	$\checkmark f$: inflection point $\checkmark f'$: turning point (2)
9.3	$f''(x) = 18x$ Distance = $f''(1) - f'(1)$ $= 18(1) - 9(1)^2$ $= 9$	$\checkmark f''(x) = 18x$ $\checkmark$ substitution $\checkmark$ answer (3)
9.4	$3x^3 - 9x^2 < 0$ $3x^2(x - 3) < 0$ but $3x^2 > 0$  $\therefore x - 3 < 0$ $\therefore x < 3, x \neq 0$	$\checkmark 3x^3 - 9x^2 < 0$ $\checkmark$ factors $\checkmark x < 3$ $\checkmark x \neq 0$ (4)
		[13]

QUESTION/VRAAG 10

10.1	$P(\text{same day}) = \frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%	$\checkmark$ 4 numerator $\checkmark$ 16 denominator (2)
10.2	$P(2 \text{ consecutive days}) = \frac{3 \times 2}{16} = \frac{3}{8}$	$\checkmark 3 \checkmark \times 2$ $\checkmark$ answer (3)
		[5]

QUESTION/VRAAG 11

11.1.1	$P(A) \times P(B)$ independent events $= 0,40 \times 0,25 = 0,1$ 	✓ 0,1 ✓ 0,15 and 0,3 ✓ 0,45 (3)
11.1.2	$P(A \text{ or not } B) = P(A) + P(\text{not } B) - P(A \text{ and not } B)$ $= 0,4 + 0,75 - 0,3$ $= 0,85$ OR/OF $P(A \text{ or not } B) = 1 - P(\text{only } B)$ $= 1 - 0,15$ $= 0,85$ OR/OF From Venn diagram: $0,3 + 0,1 + 0,45 = 0,85$	✓ substitution ✓ answer (2) OR/OF ✓ 1 – 0,15 ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
11.2	$(5 \times 1 \times 5) + (5 \times 1 \times 6) + (5 \times 1 \times 6) + (5 \times 1 \times 5) = 110$ $110 \times 5 = 550 > 500$ Not possible, because not enough space OR/OF $(5 \times 2 \times 5) + (5 \times 2 \times 6) = 110$ $110 \times 5 = 550 > 500$ Not possible because not enough space OR/OF	✓ $5 \times 1 \times 5$ ✓ $5 \times 1 \times 6$ ✓ $5 \times 1 \times 6$ ✓ $5 \times 1 \times 5$ ✓ 110 ✓ conclusion (6) OR/OF ✓✓ $5 \times 2 \times 5$ ✓✓ $5 \times 2 \times 6$ ✓ 110 ✓ conclusion (6) OR/OF

	$5 \times 4 \times 6 = 120$ $5 \times 2 = 10$ $\therefore 120 - 10 = 110$ $110 \times 5 = 550 > 500$ Not possible because not enough space	$\checkmark\checkmark 5 \times 4 \times 6 = 120$ $\checkmark 5 \times 2 = 10$ $\checkmark 120 - 10$ $\checkmark 110$ $\checkmark$ conclusion (6)
		[11]

TOTAL/TOTAAL: 150