

I LOVE MATHS SERIES

BOOK 2

MATHEMATICS

GRADE 12

2021 REVISION MATERIAL

EUCLIDEAN GEOMETRY

**A collection of questions from previous
question papers (2018 to 2020).**

Prepared by T Faya

Take note: Coding of questions.

KZN J18 – KZN June 2018

KZN S18 – KZN Sept 2018

ECS18 – Eastern Cape Sept 2018

NM17 – National March 2017

And so on.....

This booklet is divided into **4 SECTIONS**.

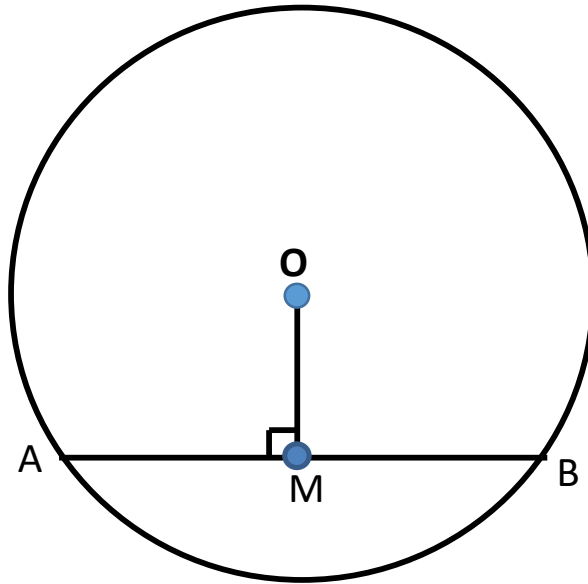
1. **SECTION A** – Questions on proofs of examinable theorems.
2. **SECTION B** – Questions on mostly level 1 and 2 problems.
3. **SECTION C** – Questions on mostly level 2 to 3 problems.
4. **SECTION D** – Mostly level 3 to level 4 questions

SECTION A

Examinable theorems.

1.1.

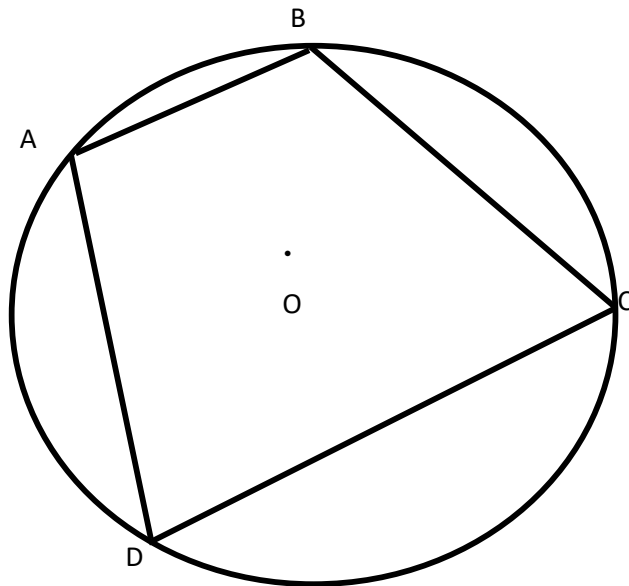
In the diagram O is the centre of the circle. Use the diagram to prove the theorem which states that if $OM \perp AB$ then $AM = MB$.



(6)

1.2

In the diagram below, O is the centre of the circle ABCD.

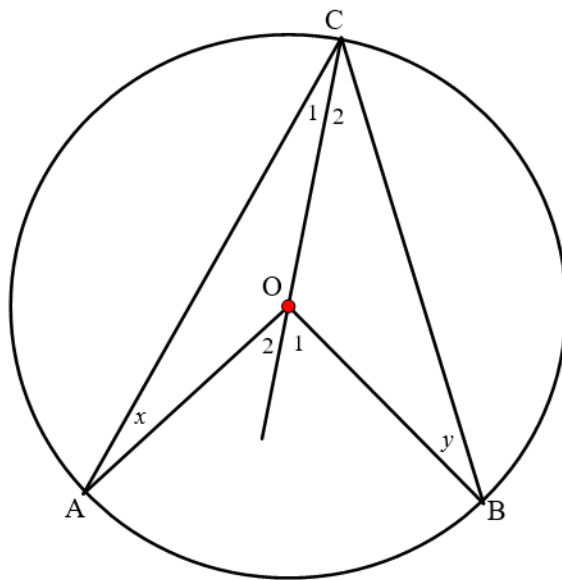


Prove the theorem that states that $\hat{B} + \hat{D} = 180^\circ$

(6)

1.3.

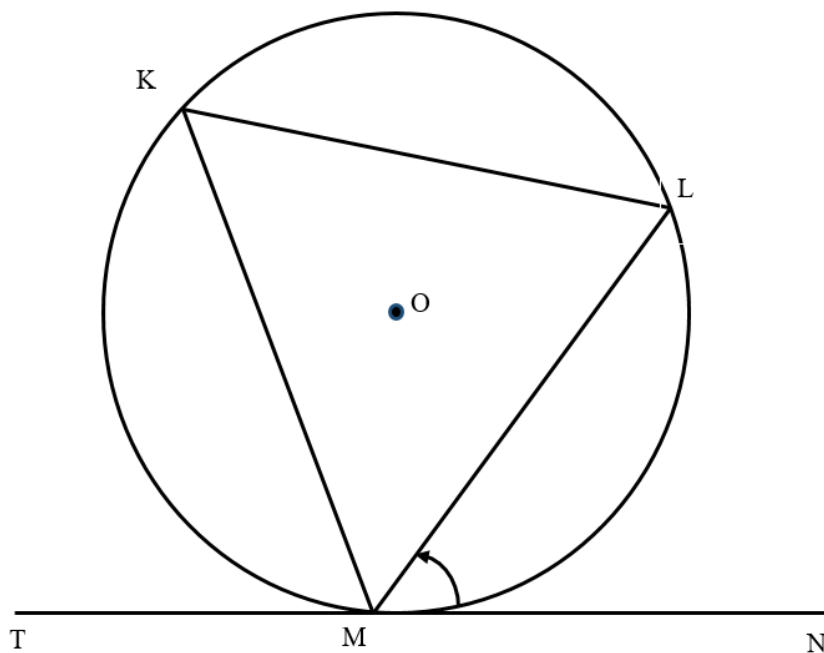
In the diagram, O is the centre of the circle. A, B and C are points on the circumference of the circle. Chords AC and BC and radii AO, BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$.



Use the diagram to prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference i.e. $\hat{AOB} = 2\hat{ACB}$. (6)

1.4.

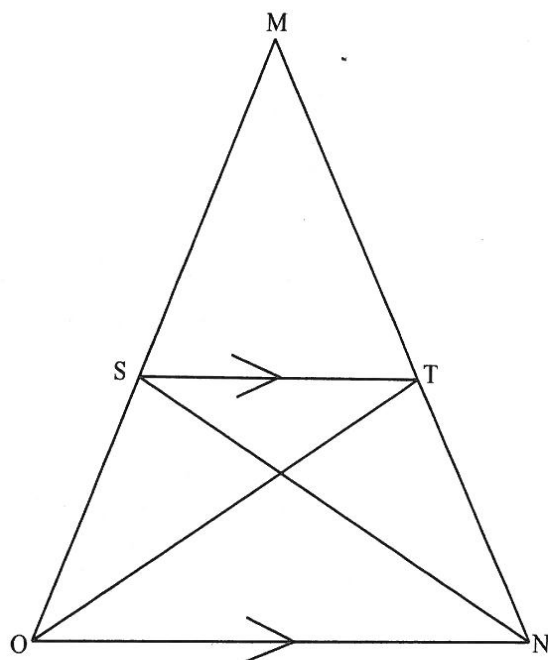
In the diagram, K, L and M are points on the circle centred at O. TM is a tangent to the circle at M. Prove the theorem which states that $\hat{LMN} = \hat{K}$.



(7)

1.5.

In the diagram, $\triangle MON$ is drawn. S is a point on MO and T is a point on MN such that $ST \parallel ON$. SN and OT are drawn.



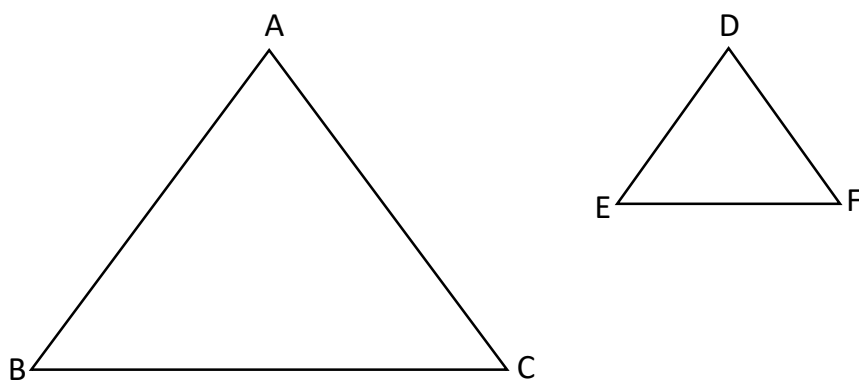
Use the diagram to prove the theorem which states that a line parallel to one side of a triangle divides the other two sides proportionally. In other words, prove that:

$$\frac{MS}{SO} = \frac{MT}{TN}.$$

(5)

1.6.

$\triangle ABC$ and $\triangle DEF$ with $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$ are shown below.



Prove that: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

(8)

SECTION B

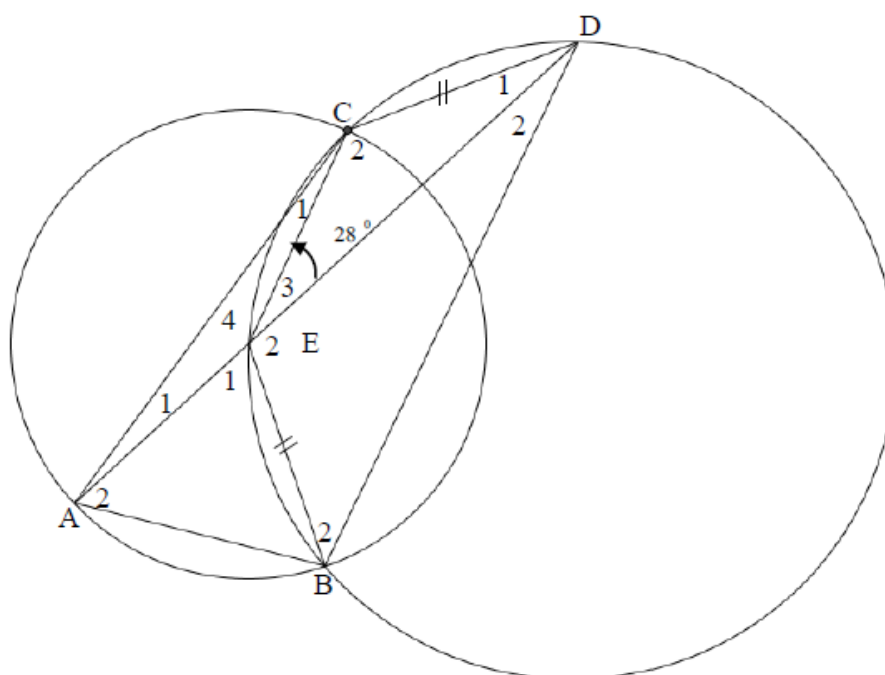
➤ Mostly Level 1 and 2 Questions on:

- Calculating angles.
- Finding angles equal to a specific angle.
- Finding angles equal to a given variable (usually x).

EC S18

QUESTION 8

In the diagram below, E is the centre of the circle passing through A, B and C. AE is produced to D so that DCEB is a cyclic quadrilateral of the larger circle such that $CD = EB$. CA is joined. $\angle CED = 28^\circ$

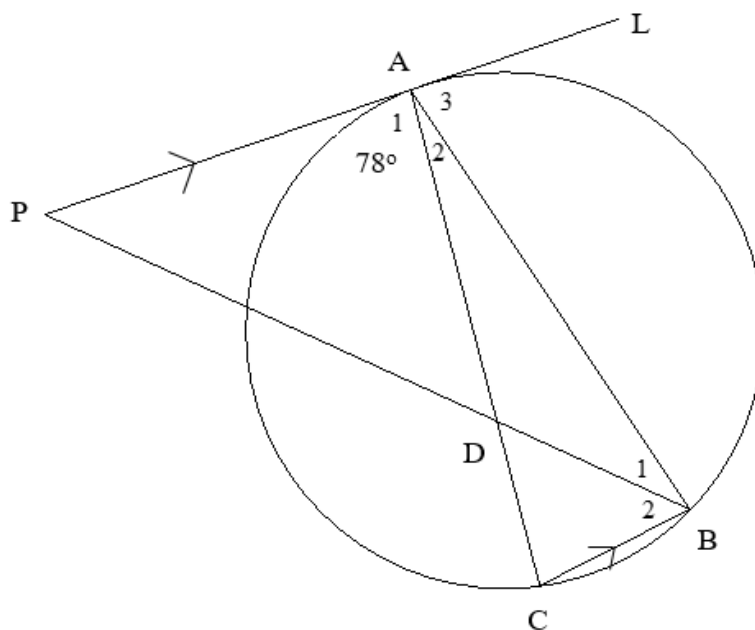


- 8.1 Determine the size of $\hat{D}_2$ (2)
- 8.2 Give a reason why $CE \parallel BD$ (1)
- 8.3 Show that $CD = EC$ (2)
- 8.4 Calculate, with reasons, the size of:
 - 8.4.1 $\hat{B}_2$ (4)
 - 8.4.2 $\angle BAC$ (3)

[12]

QUESTION 9 FS S18

- 9.1 In the diagram, PAL is a tangent to the circle at A. C and B are points on the circle such that $CB \parallel PAL$. AB, AC and PB are drawn. PB intersects AC at D. $\hat{A}_1 = 78^\circ$.

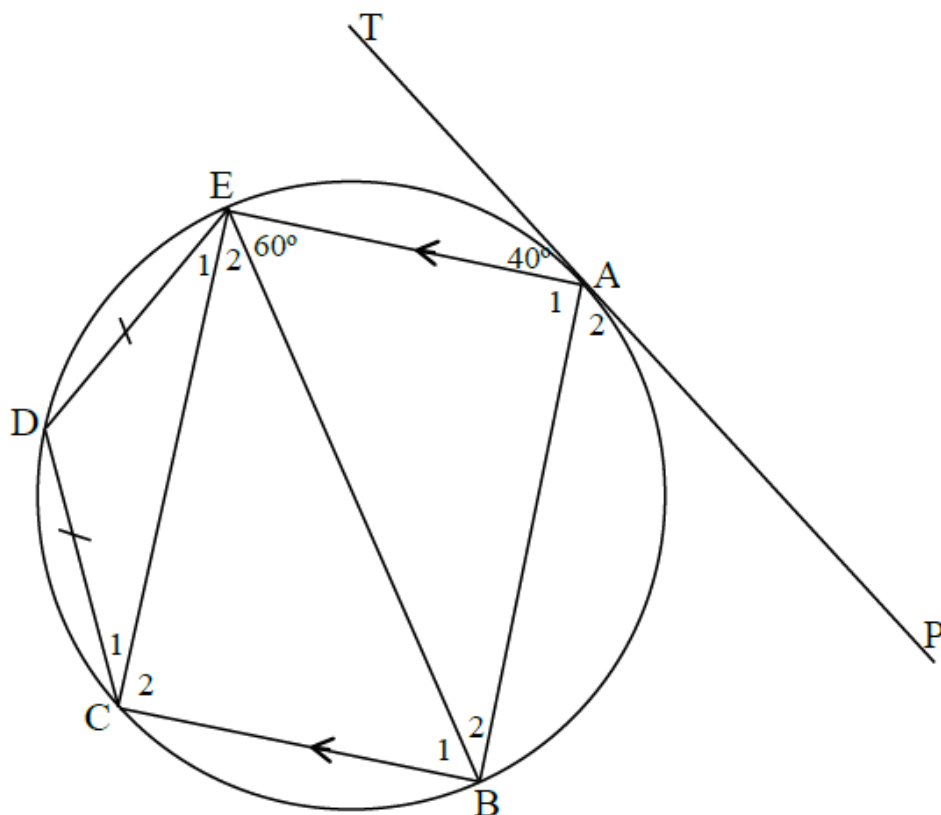


Determine the sizes of each of the following angles, with reasons:

- | | | |
|-------|-------------|-----|
| 9.1.1 | $\hat{C}$ | (1) |
| 9.1.2 | $\hat{ABC}$ | (2) |
| 9.1.3 | $\hat{A}_3$ | (1) |
| 9.1.4 | $\hat{A}_2$ | (1) |

QUESTION 8

In the diagram below, TAP is a tangent to circle ABCDE at A. $AE \parallel BC$ and $DC = DE$. $\hat{TAE} = 40^\circ$ and $\hat{AEB} = 60^\circ$.



8.1 Identify TWO cyclic quadrilaterals. (2)

8.2 Determine, with reasons, the size of the following angles:

8.2.1 $\hat{B}_2$ (2)

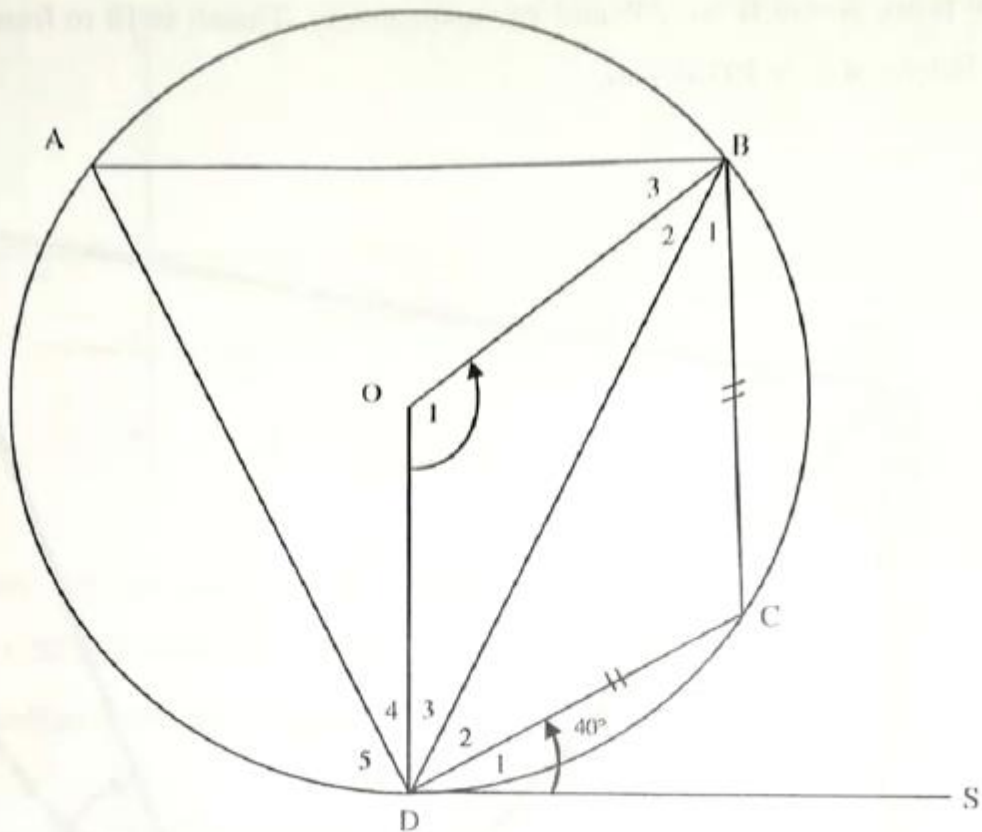
8.2.2 $\hat{B}_1$ (2)

8.2.3 $\hat{D}$ (2)

8.2.4 $\hat{E}_1$ (3)

QUESTION 8

- 8.1 In the figure below, ABCD is a cyclic quadrilateral in the circle centred at O and $\triangle DOB$ is drawn. DS is a tangent to the circle at D. $BC = DC$ and $\angle CDS = 40^\circ$.

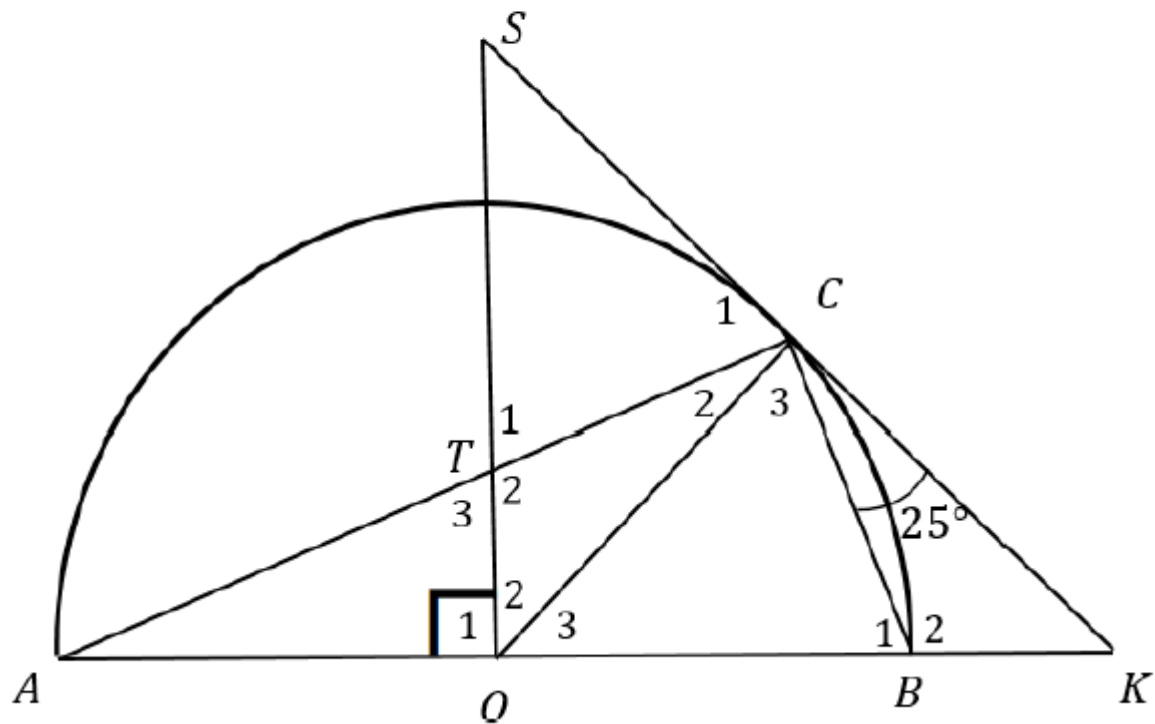


Calculate, with reasons, the size of:

- | | | |
|-------|--------------|-----|
| 8.1.1 | $\angle BDC$ | (3) |
| 8.1.2 | $\angle C$ | (2) |
| 8.1.3 | $\angle A$ | (2) |
| 8.1.4 | $\angle O_1$ | (2) |

QUESTION 9

ABC is a semicircle with centre O. AB is produced to K such that KCS is a tangent to the circle at C. $SO \perp AB$ and $\widehat{KCB} = 25^\circ$.



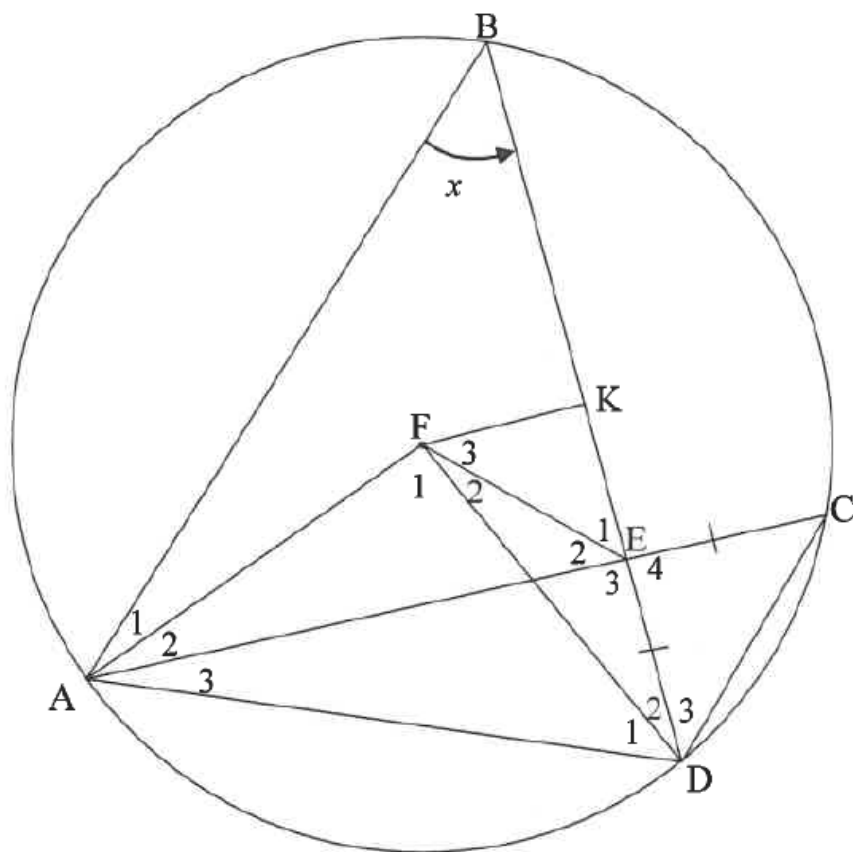
- 9.1 Give a reason why:
 - 9.1.1 $\widehat{CAB} = 25^\circ$ (1)
 - 9.1.2 $\widehat{OCA} = 25^\circ$ (1)
- 9.2 Prove that CTOB is a cyclic quadrilateral. (3)
- 9.3 Write down, with a reason, the size of $\widehat{COB}$. (2)
- 9.4 Prove that OK is a tangent to a circle passing through S, O and C. (4)

[11]

NM18

10.2

In the diagram, the circle with centre F is drawn. Points A , B , C and D lie on the circle. Chords AC and BD intersect at E such that $EC = ED$. K is the midpoint of chord BD . FK , AB , CD , AF , FE and FD are drawn. Let $\hat{B} = x$.



10.2.1 Determine, with reasons, the size of EACH of the following in terms of x :

(a) $\hat{F}_1$ (2)

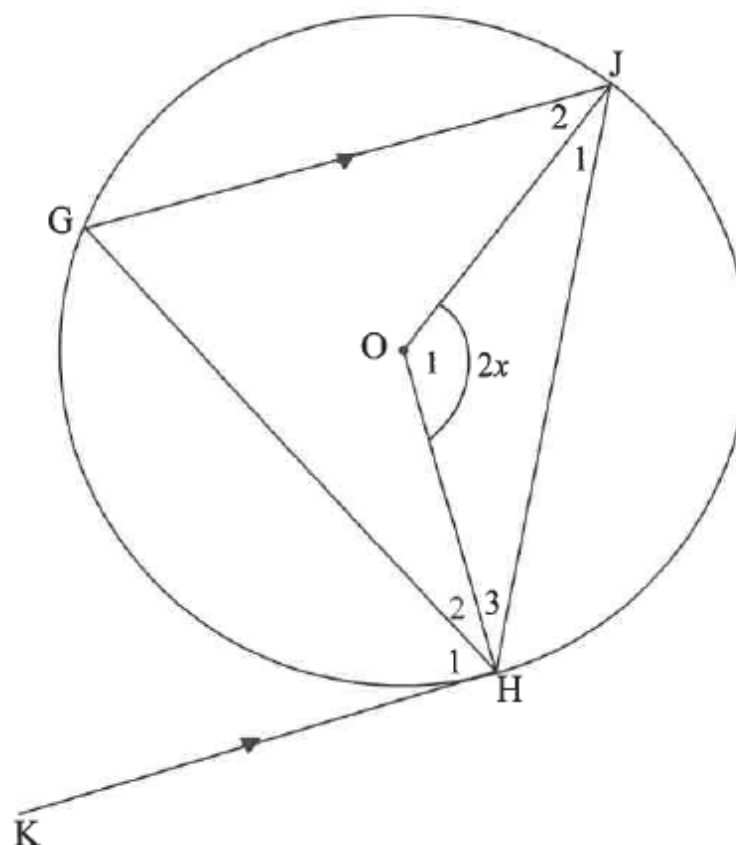
(b) $\hat{C}$ (2)

10.2.2 Prove, with reasons, that $AFED$ is a cyclic quadrilateral. (4)

10.2.3 Prove, with reasons, that $\hat{F}_3 = x$. (6)

QUESTION 8

- 8.1 In the diagram, O is the centre of the circle. Radii OH and OJ are drawn. A tangent is drawn from K to touch the circle at H . $\triangle HGJ$ is drawn such that $GJ \parallel KH$. $\hat{O}_1 = 2x$.

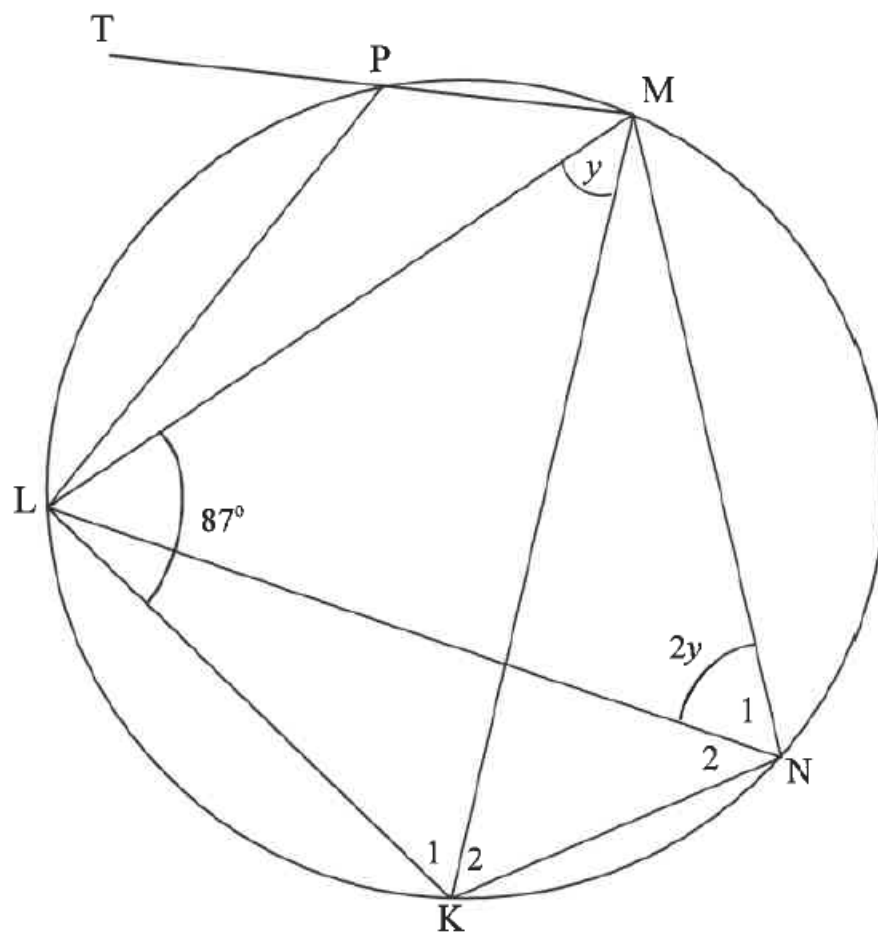


- 8.1.1 Name, giving reasons, THREE angles, each equal to x . (5)
- 8.1.2 Prove that $\hat{H}_2 = \hat{H}_3$. (3)

NJ18

8.2

In the diagram, $KLMN$ is a cyclic quadrilateral with $\angle KLM = 87^\circ$. Diagonals LN and MK are drawn. P is a point on the circle and MP is produced to T , a point outside the circle. Chord LP is drawn. $\angle LMK = y$ and $\angle N_1 = 2y$.



8.2.1 Name, giving a reason, another angle equal to y . (2)

8.2.2 Calculate, giving reasons, the size of:

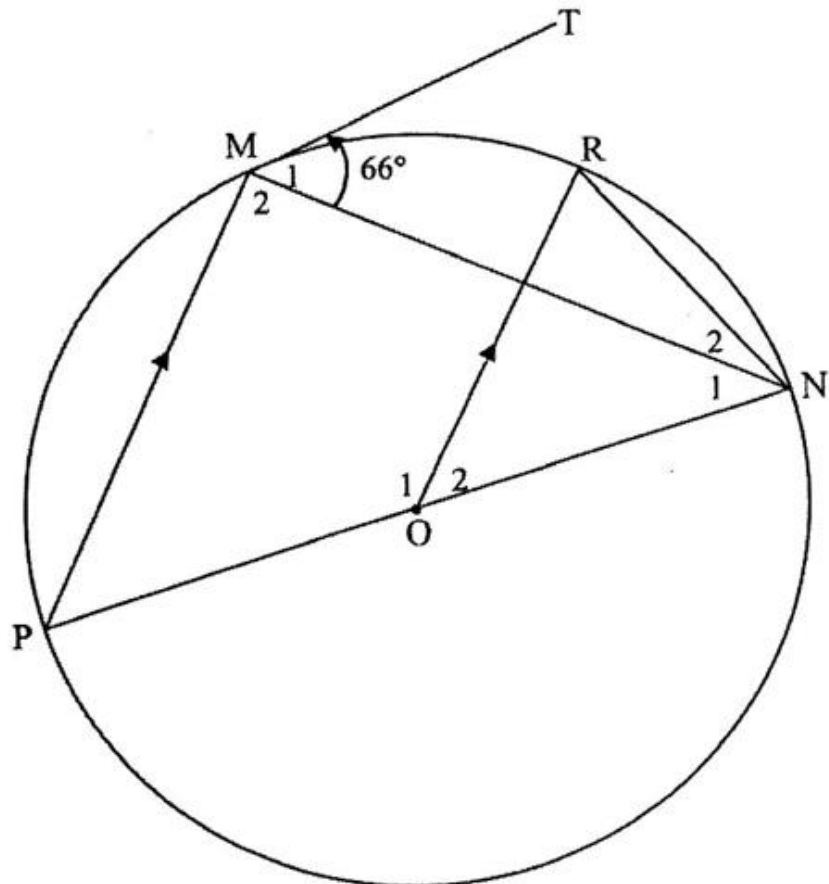
(a) y (3)

(b) $\angle TPL$ (2)

[15]

QUESTION 8

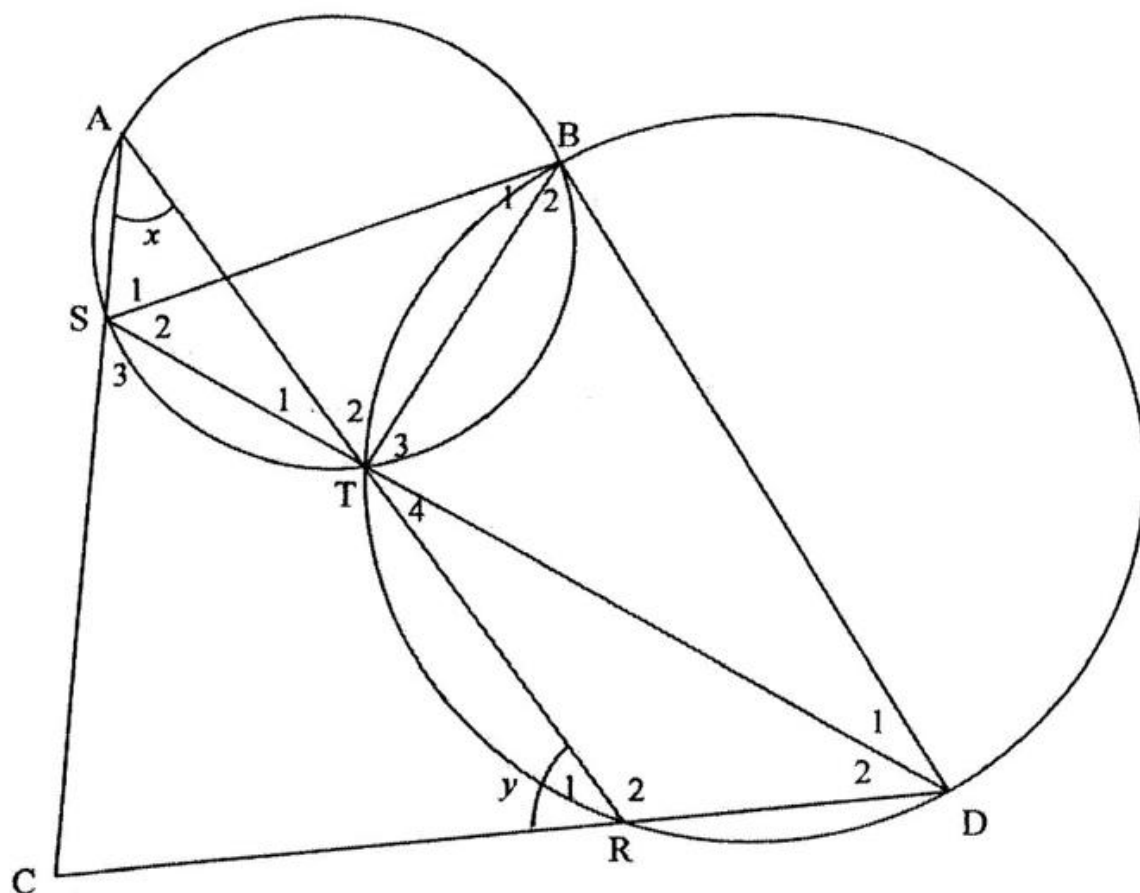
- 8.1 PON is a diameter of the circle centred at O . TM is a tangent to the circle at M , a point on the circle. R is another point on the circle such that $OR \parallel PM$. NR and MN are drawn. Let $\hat{M}_1 = 66^\circ$.



Calculate, with reasons, the size of EACH of the following angles:

- | | | |
|-------|-------------|-----|
| 8.1.1 | $\hat{P}$ | (2) |
| 8.1.2 | $\hat{M}_2$ | (2) |
| 8.1.3 | $\hat{N}_1$ | (1) |
| 8.1.4 | $\hat{O}_2$ | (2) |
| 8.1.5 | $\hat{N}_2$ | (3) |

In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)

9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

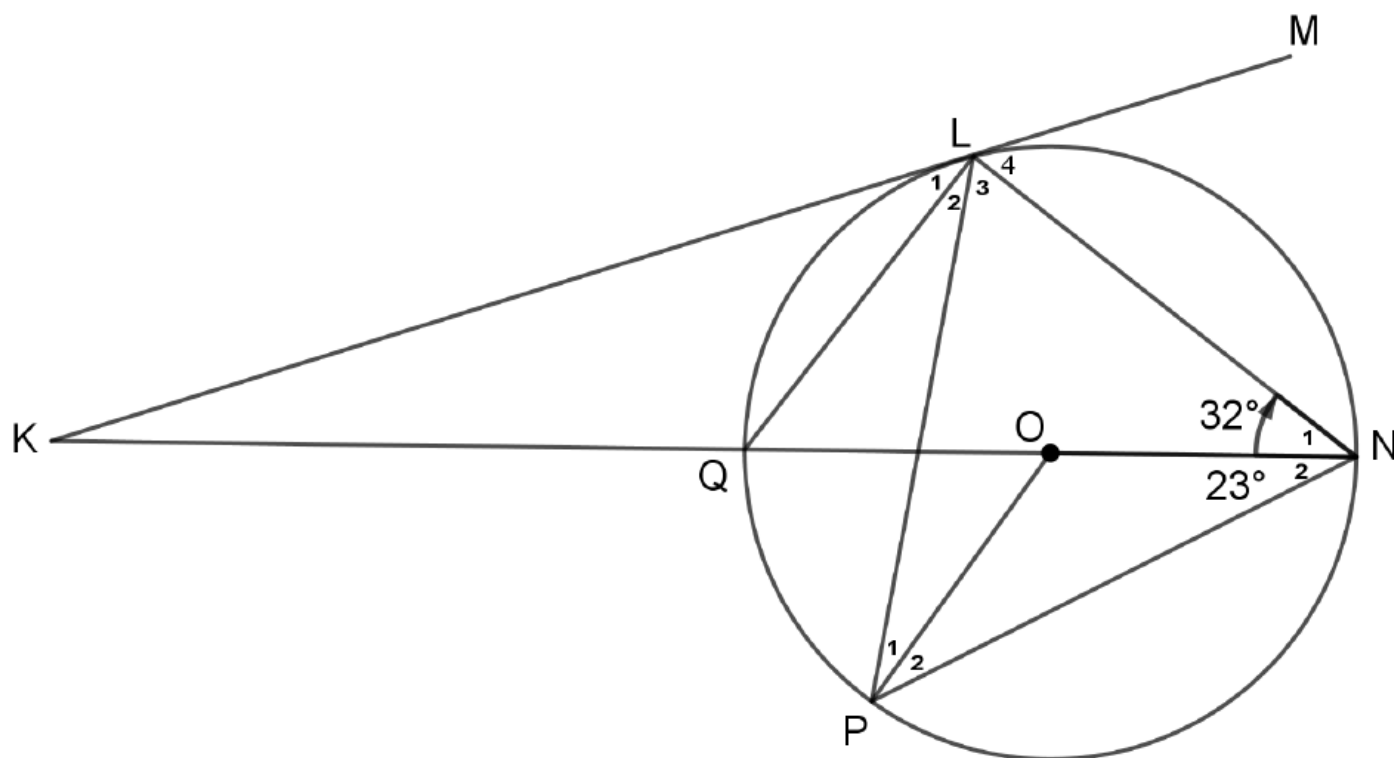
9.2.3 It is further given that $\hat{D}_2 = 30^\circ$ and $\hat{AST} = 100^\circ$.
Prove that SD is not a diameter of circle BDS. (4)
[16]

KZNM19

5.2 In the sketch below, circle centre O has a tangent KLM.

Diameter NQ produced meet the tangent in K.

$$\hat{N}_1 = 32^\circ \text{ and } \hat{N}_2 = 23^\circ.$$



Calculate, with reasons, the size of:

5.2.1 $\hat{P}_2$ (1)

5.2.2 $\hat{POQ}$ (2)

5.2.3 $\hat{L}_2$ (2)

5.2.4 $\hat{NLQ}$ (1)

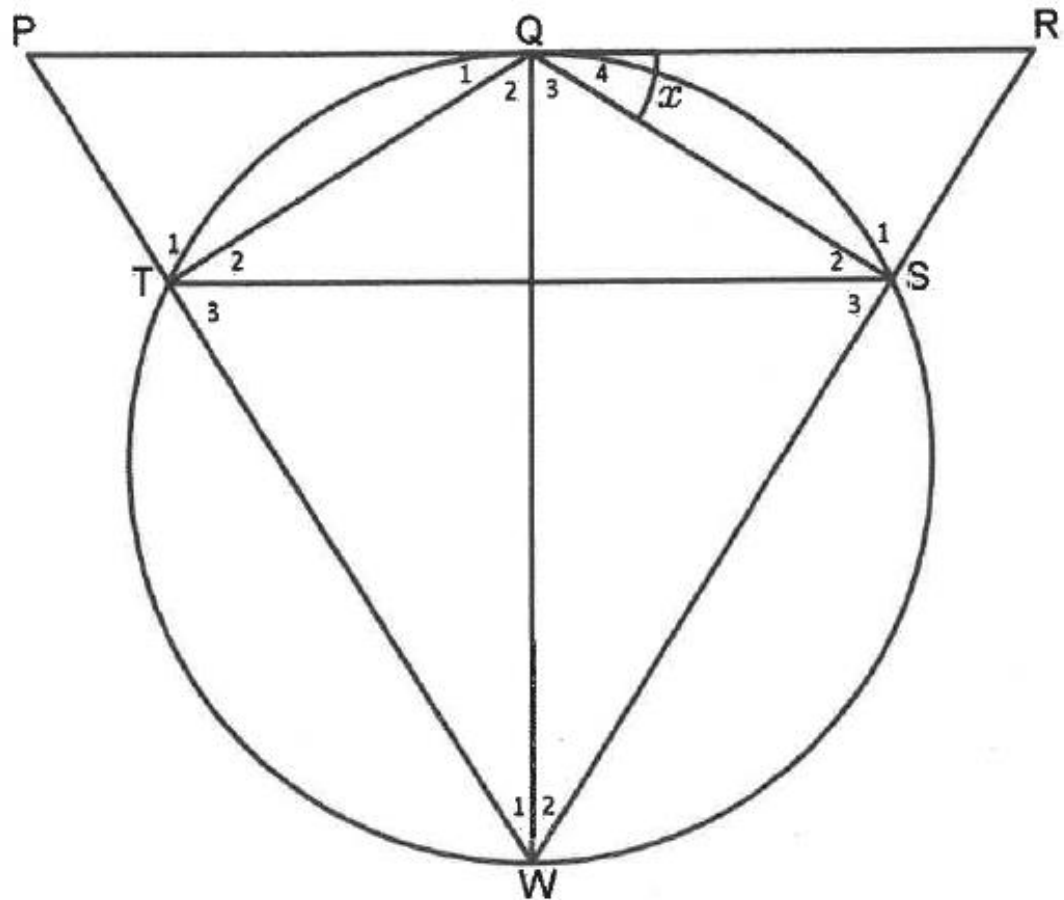
5.2.5 $\hat{L}_3$ (2)

5.2.6 $\hat{PLK}$ (2)

[11]

KZMJ19
QUESTION 8

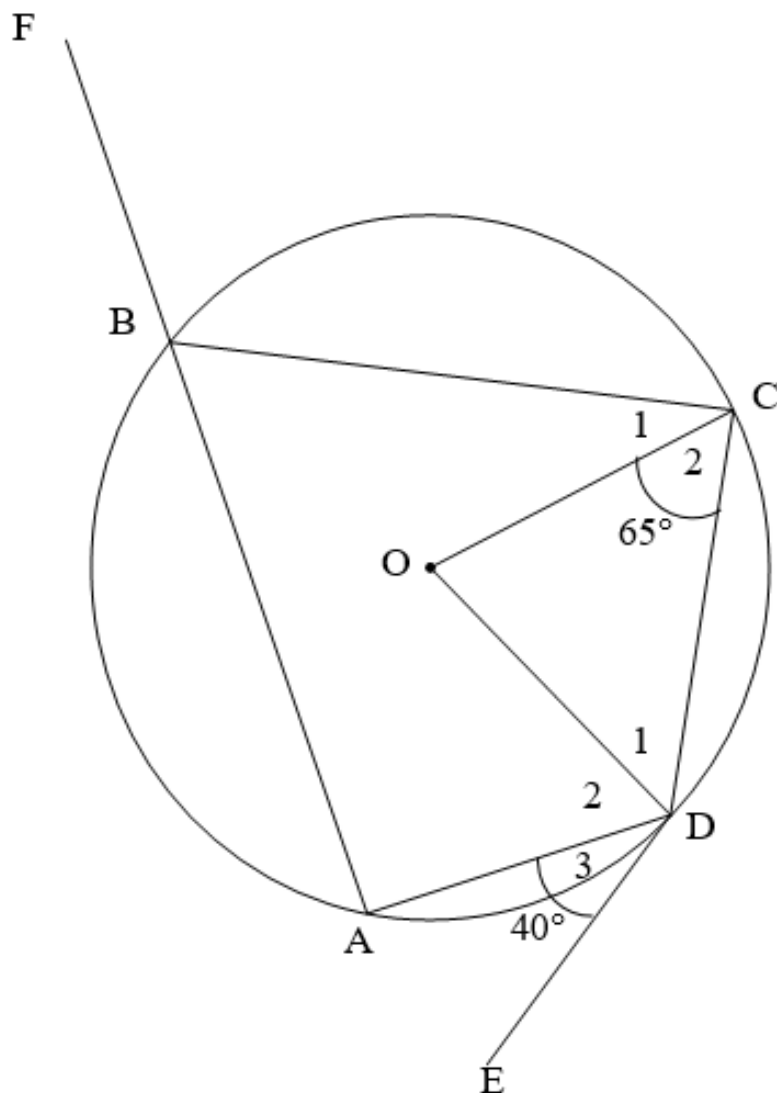
In the figure, $TQSW$ is a cyclic quadrilateral with tangent PR touching the circle at Q .
 WQ bisects $\widehat{PWR}$. $\widehat{Q}_4 = x$



- 8.1 Name with reasons 5 other angles each equal to x . (5)
- 8.2 Prove that:
- 8.2.1 $TS \parallel PR$ (2)
- 8.2.2 $\widehat{Q}_3 = \widehat{P}$ (3)
- 8.2.3 $\triangle TQS$ is an isosceles triangle (4)
- 8.2.4 $\widehat{WQP} = \widehat{T}_1$ (3)
- [17]

QUESTION 8 KZNS19

- 8.1 In the diagram, ABCD is a cyclic quadrilateral in the circle centered at O. ED is a tangent to the circle at D. Chord AB is produced to F. Radii OC and OD are drawn. $\hat{ADE} = 40^\circ$ and $\hat{C}_2 = 65^\circ$,

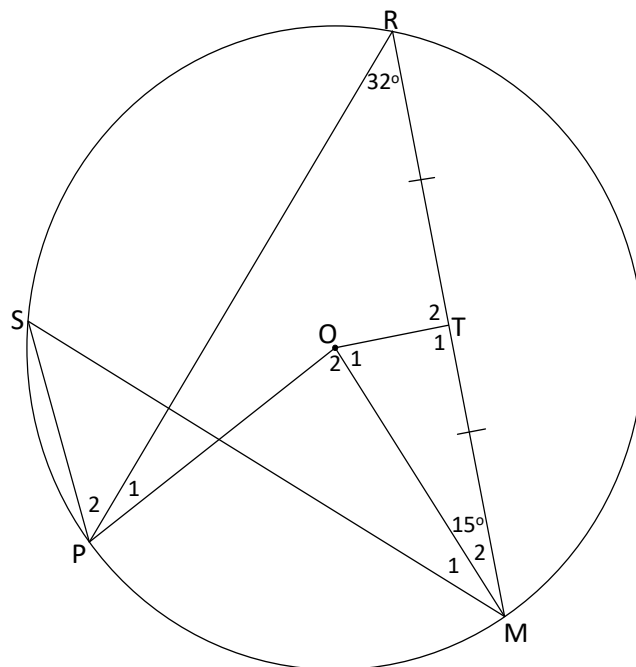


Determine, giving reasons, the size of each of the following angles:

8.1.1 $\hat{D}_2$ (3)

8.1.2 $\hat{FBC}$ (4)

- 8.2 In the diagram, O is the centre of the circle RMPS. OT bisects RM with T a point on RM. $\angle PRM = 32^\circ$. SP, SM and radii OP and OM are drawn. $\angle OMT = 15^\circ$.



Calculate, with reasons, the size of the angles:

8.2.1 $\hat{S}$ (2)

8.2.2 $\hat{O}_2$ (2)

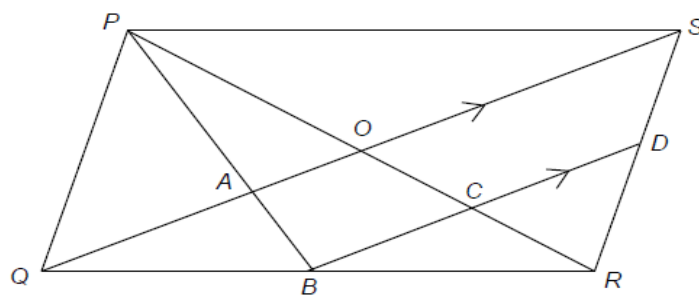
8.2.3 $\hat{O}_1$ (3)

[14]

6.2 LPM19

In the figure below PQRS is a parallelogram.

$QS \parallel BR$ and $\frac{QB}{BR} = \frac{2}{5}$.

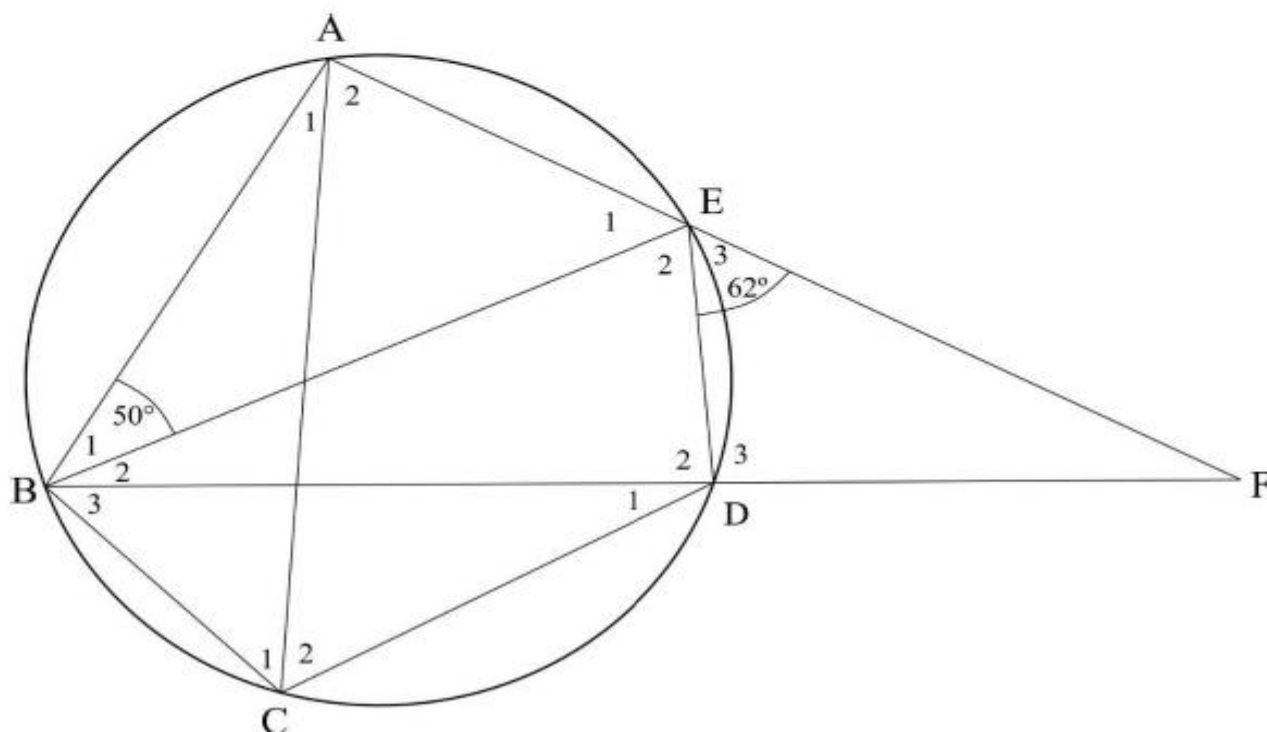


6.2.1 Determine $\frac{PQ}{OC}$. (4)

6.2.2 Determine $\frac{RC}{PQ}$. (3)

QUESTION 8

- 8.1 In the given diagram, A, B, C, D and E are points on the circle.
 BE is a diameter. $\widehat{E}_3 = 62^\circ$ and $\widehat{B}_1 = 50^\circ$.
 BD produced meets AE produced at F.



Determine, with reasons:

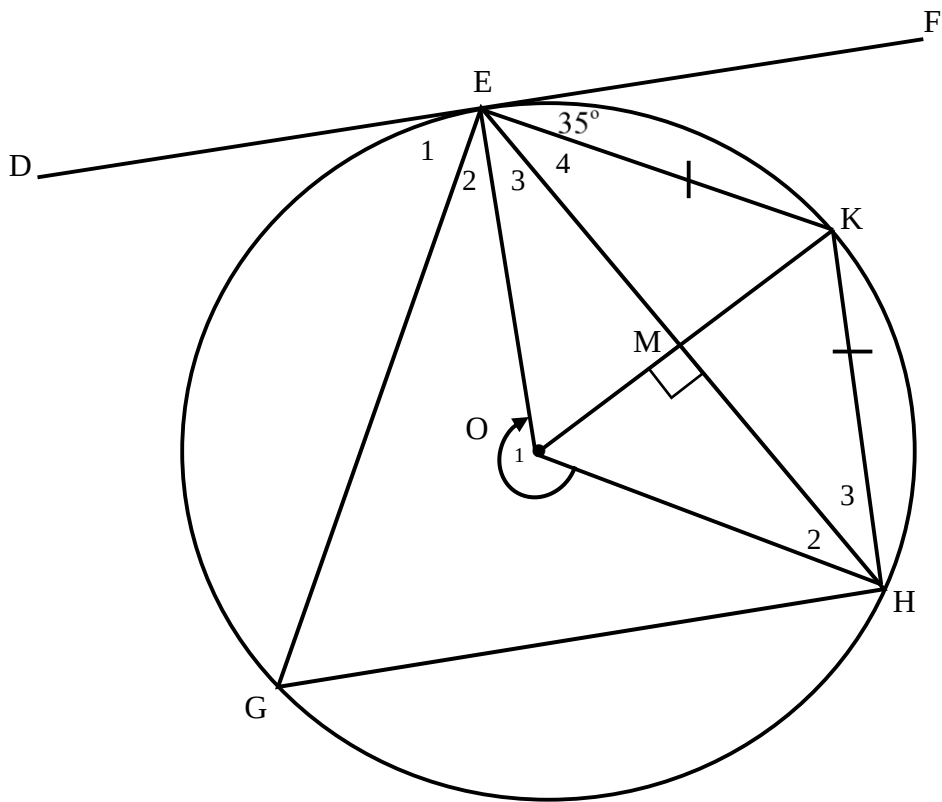
- 8.1.1 $\widehat{BAE}$ (2)
- 8.1.2 $\widehat{E}_1$ (2)
- 8.1.3 $\widehat{C}_1$ (2)
- 8.1.4 $\widehat{C}_2$ (2)
- 8.1.5 $\widehat{ABD}$ (2)
- 8.2 Complete the following theorem statement:

The angle between the tangent to a circle and the chord drawn from the point of contact is ...

(1)

QUESTION 8 FSS19

DF is a tangent to the circle at E. EKHG is a cyclic quadrilateral. $\hat{KEF} = 35^\circ$.
O is the centre of the circle. $OK \perp EH$ and $EK = HK$.



- 8.1 Determine, with reasons, the size of each of the following:
- 8.1.1 $\hat{E}_4$

(3)
- 8.1.2 $\hat{EKH}$

(2)
- 8.1.3 $\hat{G}$

(2)
- 8.1.4 $\hat{O}_1$

(2)
- 8.2 It is further given that $EH = 24$ units. $KM = 4$ units and the radius of the circle EKHG is x . Determine the value of x .

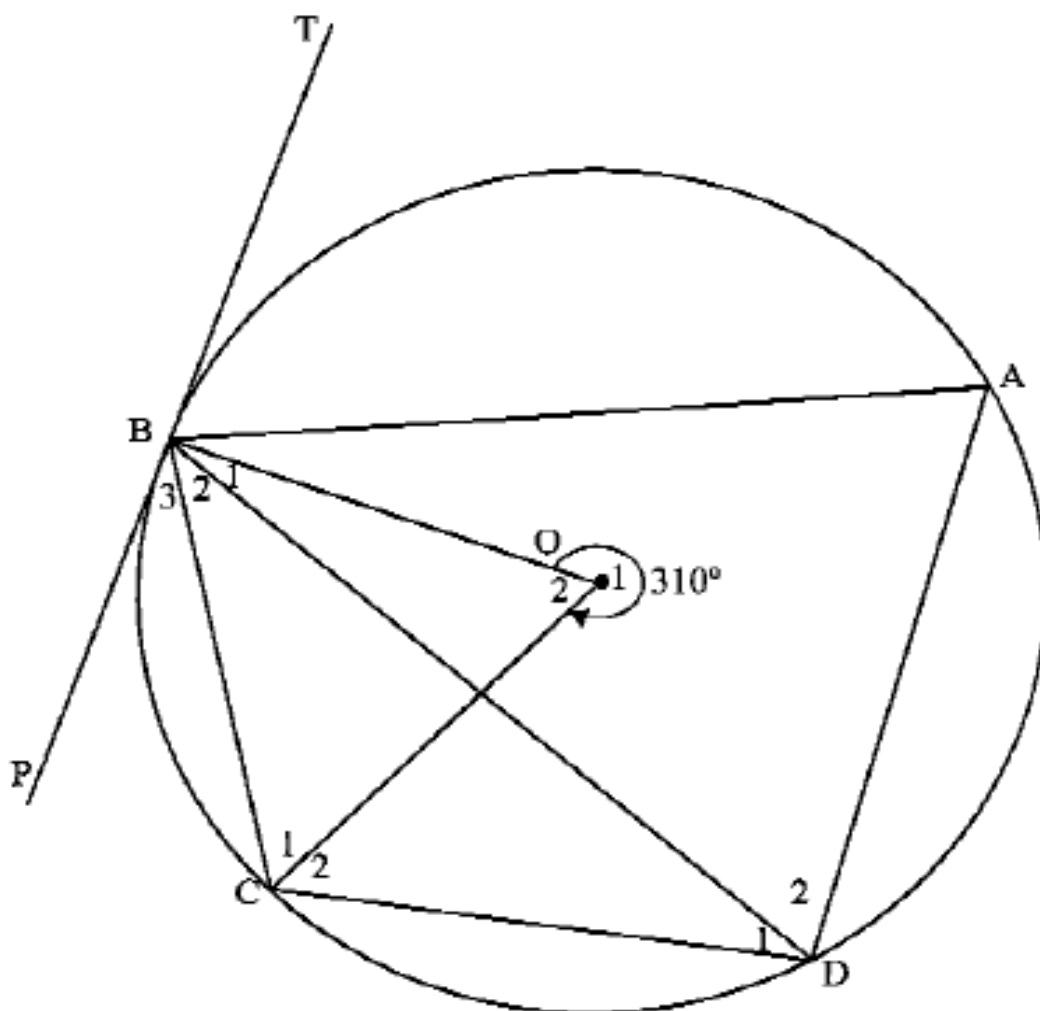
(4)

[13]

QUESTION 8

In the diagram below, A, B, C and D are points on a circle having centre O. PBT is a tangent to the circle at B.

Reflex $\hat{BOC} = \hat{O}_1 = 310^\circ$ as shown in the diagram below.



Calculate, giving reasons, the size of:

8.1 $\hat{D}_1$ (3)

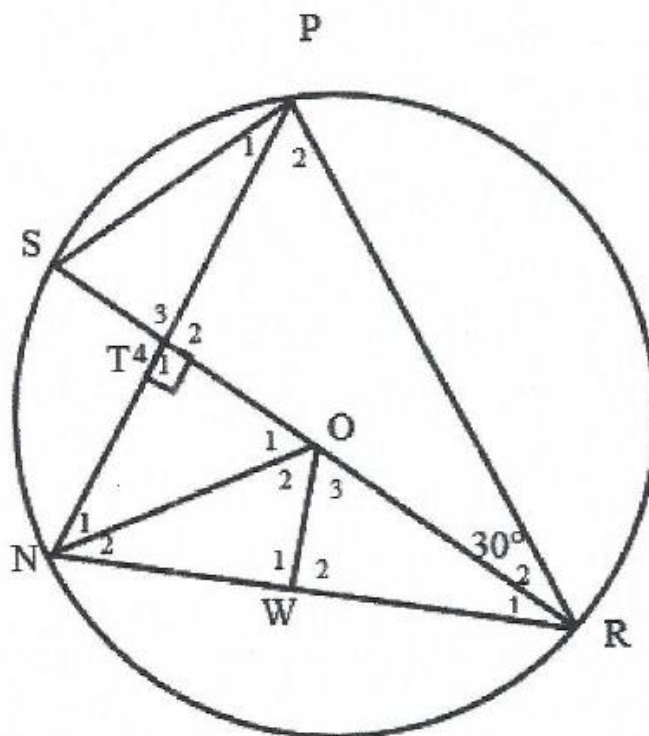
8.2 $\hat{B}_3$ (2)

8.3 $\hat{B}_1$, if it is given that $\hat{A} = 60^\circ$. (4)

[9]

QUESTION 8

In the diagram the vertices of $\triangle PNR$ lie on the circle with centre O . Diameter SR and chord NP intersect at T . $OT \perp NP$ and $\hat{R}_2 = 30^\circ$.



8.1 Determine, stating reasons, the size of:

8.1.1 $\hat{S}$ (3)

8.1.2 $\hat{R}_1$ (3)

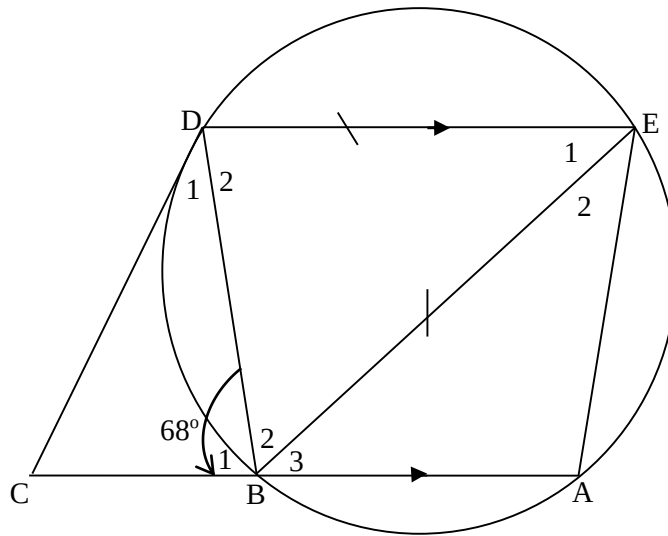
8.1.3 $\hat{N}_1$ (2)

8.2 If it is further given that $NW = WR$, prove that $TNWO$ is a cyclic quadrilateral. (4)

[12]

QUESTION 8 NWS19

In the diagram below, BAED is a cyclic quadrilateral with $BA \parallel DE$. $BE = DE$ and $\hat{D}BC = 68^\circ$. The tangent to the circle at D meets AB produced to C.



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{DEA}$ (2)

8.1.2 $\hat{A}$ (1)

8.1.3 $\hat{D}_2$ (2)

8.1.4 $\hat{B}_2$ (1)

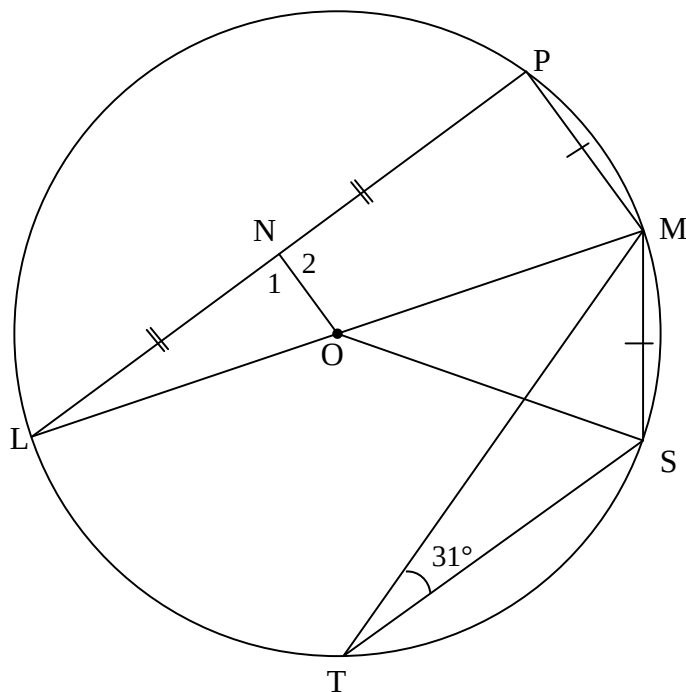
8.1.5 $\hat{D}_1$ (3)

8.2 Prove that $\triangle BDC$ is isosceles. (2)

8.3 Prove that DE is a tangent to the circle that passes through the points C, B and D at D. (2)
[13]

QUESTION 8 NJ19

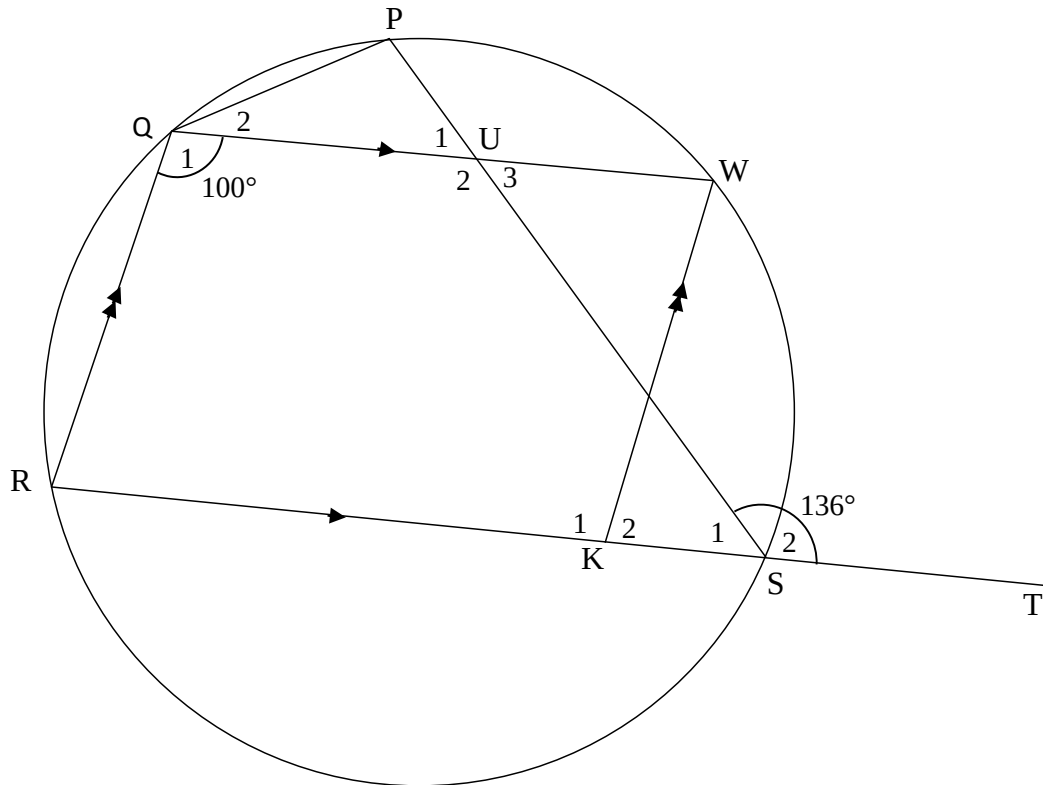
- 8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. $PM = MS$ and $\hat{M\hat{T}S} = 31^\circ$



- 8.1.1 Determine, with reasons, the size of each of the following angles:
- (a) $\hat{M\hat{O}S}$ (2)
 - (b) $\hat{L}$ (2)
- 8.1.2 Prove that $ON = \frac{1}{2} MS$. (4)

NN19
QUESTION 8

- 8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $\hat{PST} = 136^\circ$ and $\hat{Q}_1 = 100^\circ$.

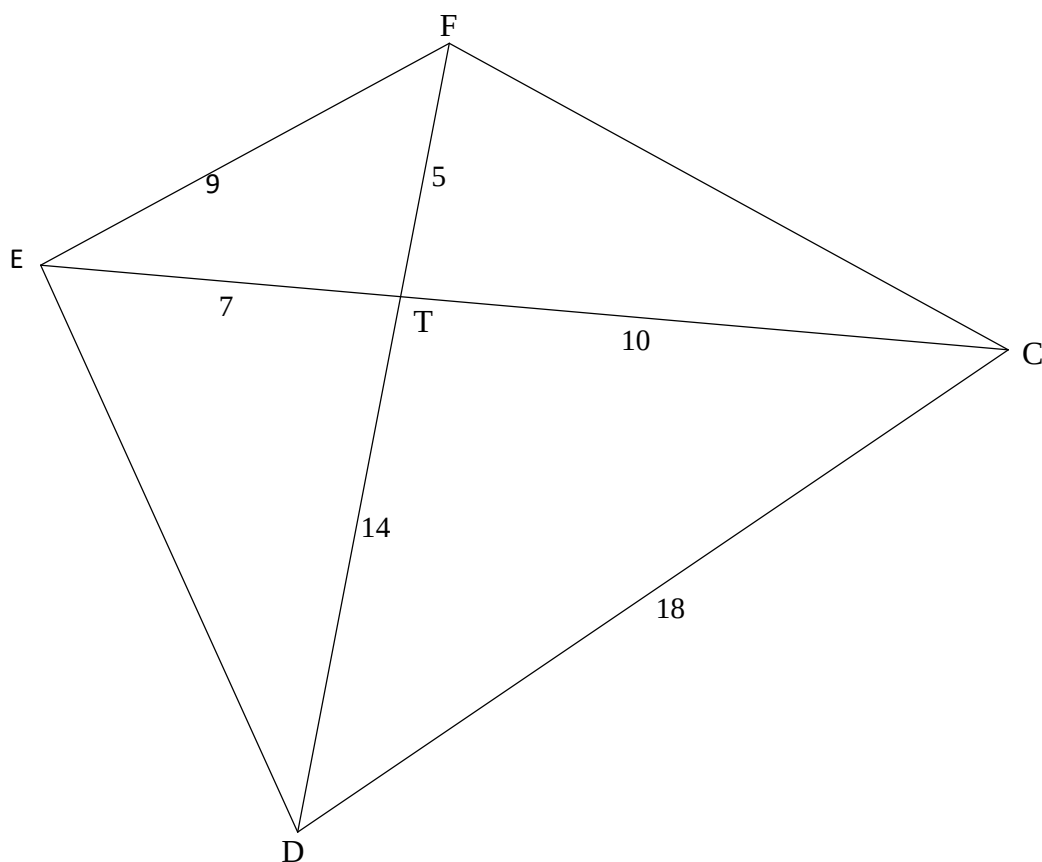


Determine, with reasons, the size of:

- | | | |
|-------|--------------|-----|
| 8.1.1 | $\hat{R}$ | (2) |
| 8.1.2 | $\hat{P}$ | (2) |
| 8.1.3 | $\angle PQW$ | (3) |
| 8.1.4 | $\hat{U}_2$ | (2) |

8.2

In the diagram, the diagonals of quadrilateral CDEF intersect at T.
 $EF = 9$ units, $DC = 18$ units, $ET = 7$ units, $TC = 10$ units, $FT = 5$ units and
 $TD = 14$ units.

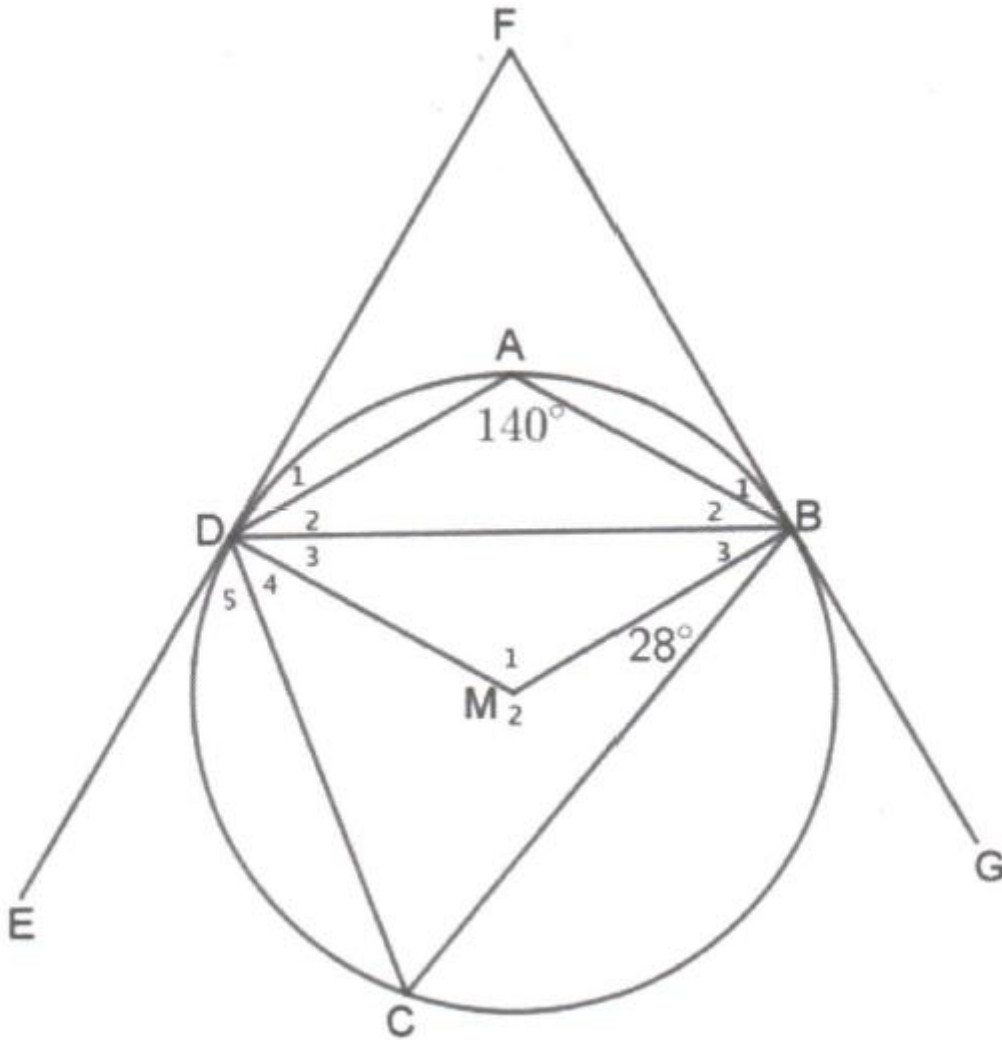


Prove, with reasons, that:

8.2.1 $\hat{EFD} = \hat{ECD}$ (4)

8.2.2 $\hat{DFC} = \hat{DEC}$ (3)
[16]

In the diagram below, M is the centre of the circle DABC. EDF is a tangent to the circle at D and FBG is another tangent to the circle at B.



Calculate the following angles, with reasons:

$$5.1 \quad \hat{C} \quad (2)$$

$$5.2 \quad \hat{M}_1 \quad (3)$$

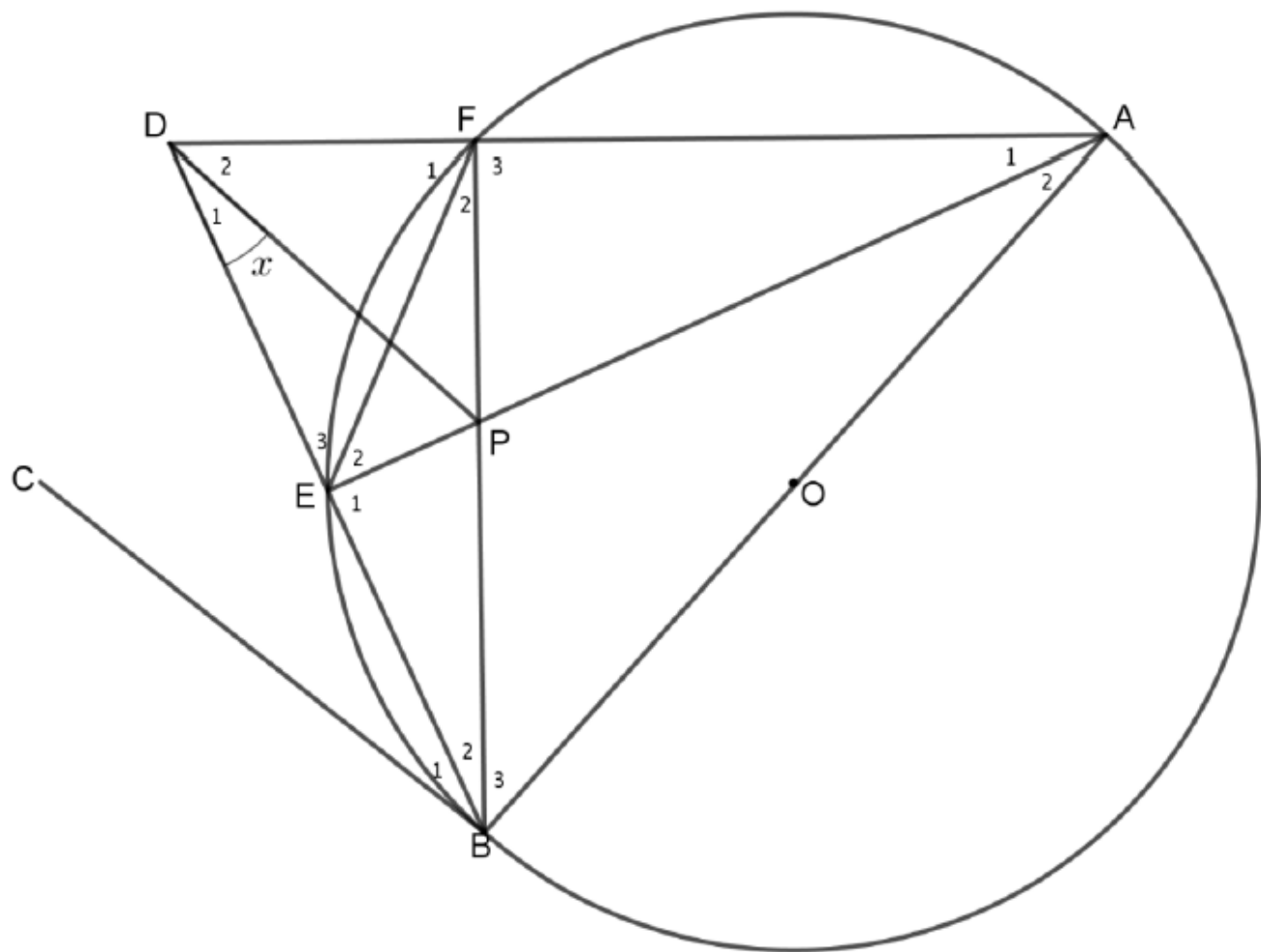
$$5.3 \quad \hat{B}_3 \quad (3)$$

$$5.4 \quad \hat{D}_5 \quad (2)$$

[10]

KZNJ20

7.2 In the diagram, AOB is a diameter of the circle and BC is a tangent to the circle. BE and AF produced cut each other at P. FE and PD are joined. $\widehat{D}_1 = x$.



Prove that:

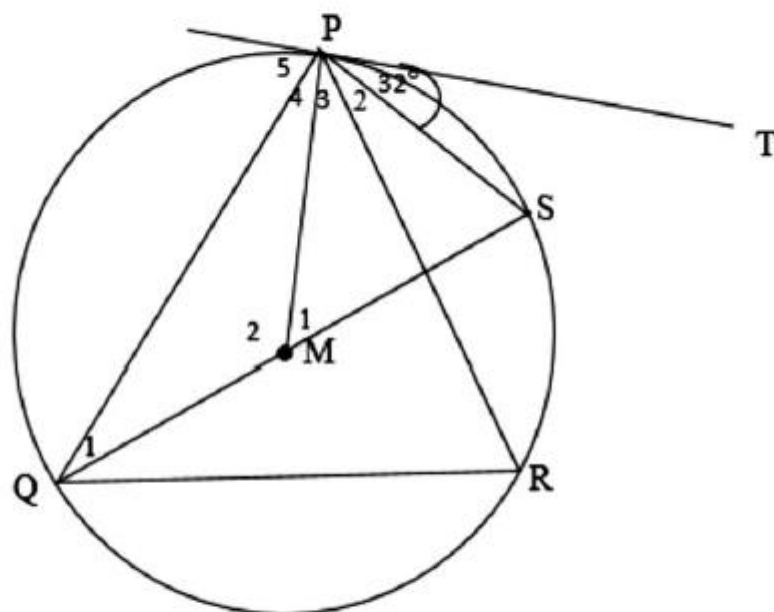
7.2.1 PFDE is a cyclic quadrilateral. (5)

7.2.2 Write down with reasons, three other angles to x . (6)

7.2.3 Show that $PD \parallel BC$. (2)

FSS20

- 9.3 In the diagram, TP is a tangent to the circle with centre M at P. QS is a diameter of the circle and R is on the circumference of the circle. $\angle T\hat{P}S = 32^\circ$.



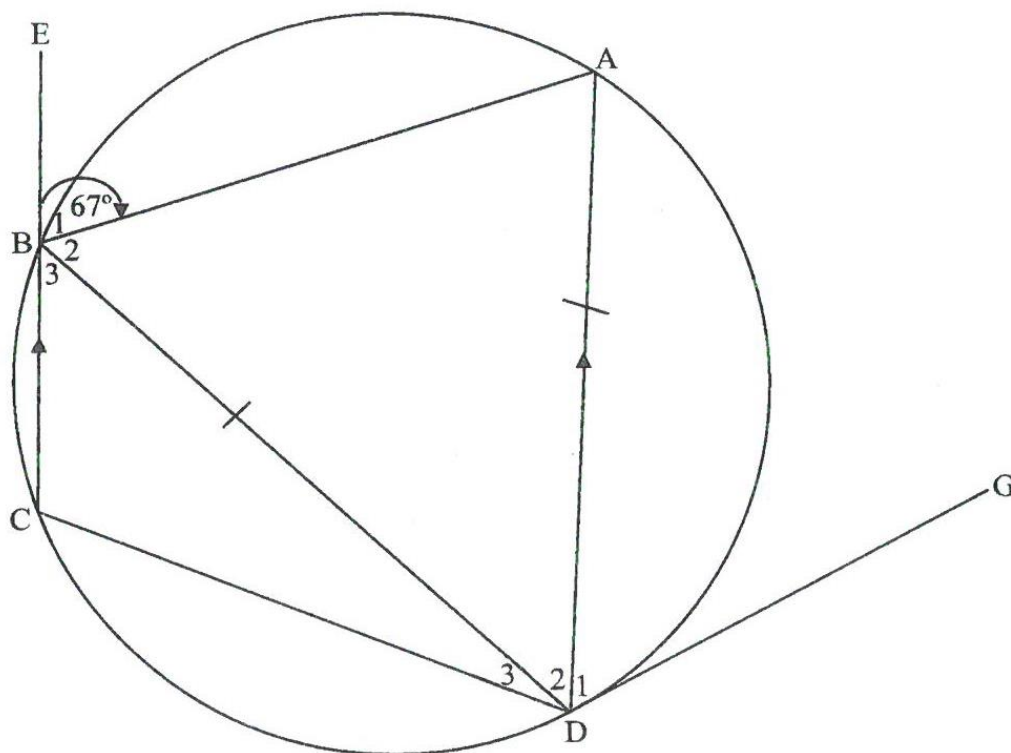
Calculate the following, giving reasons:

- | | | |
|-------|-------------|-----|
| 9.3.1 | $\hat{Q}_1$ | (2) |
| 9.3.2 | $\hat{P}_4$ | (2) |
| 9.3.3 | $\hat{M}_1$ | (2) |
| 9.3.4 | $\hat{R}$ | (4) |

QUESTION 8

In the diagram below, points A, B, C and D lie on the circumference of a circle with $AD \parallel EC$. CB is produced to E. GD is a tangent to the circle at D and $DB = AD$.

$$\hat{EBA} = 67^\circ$$



8.1 Calculate, with reasons, the size of the following angles:

8.1.1 $\hat{ADC}$ (2)

8.1.2 $\hat{C}$ (1)

8.1.3 $\hat{A}$ (1)

8.1.4 $\hat{D}_2$ (3)

8.1.5 $\hat{BDG}$ (2)

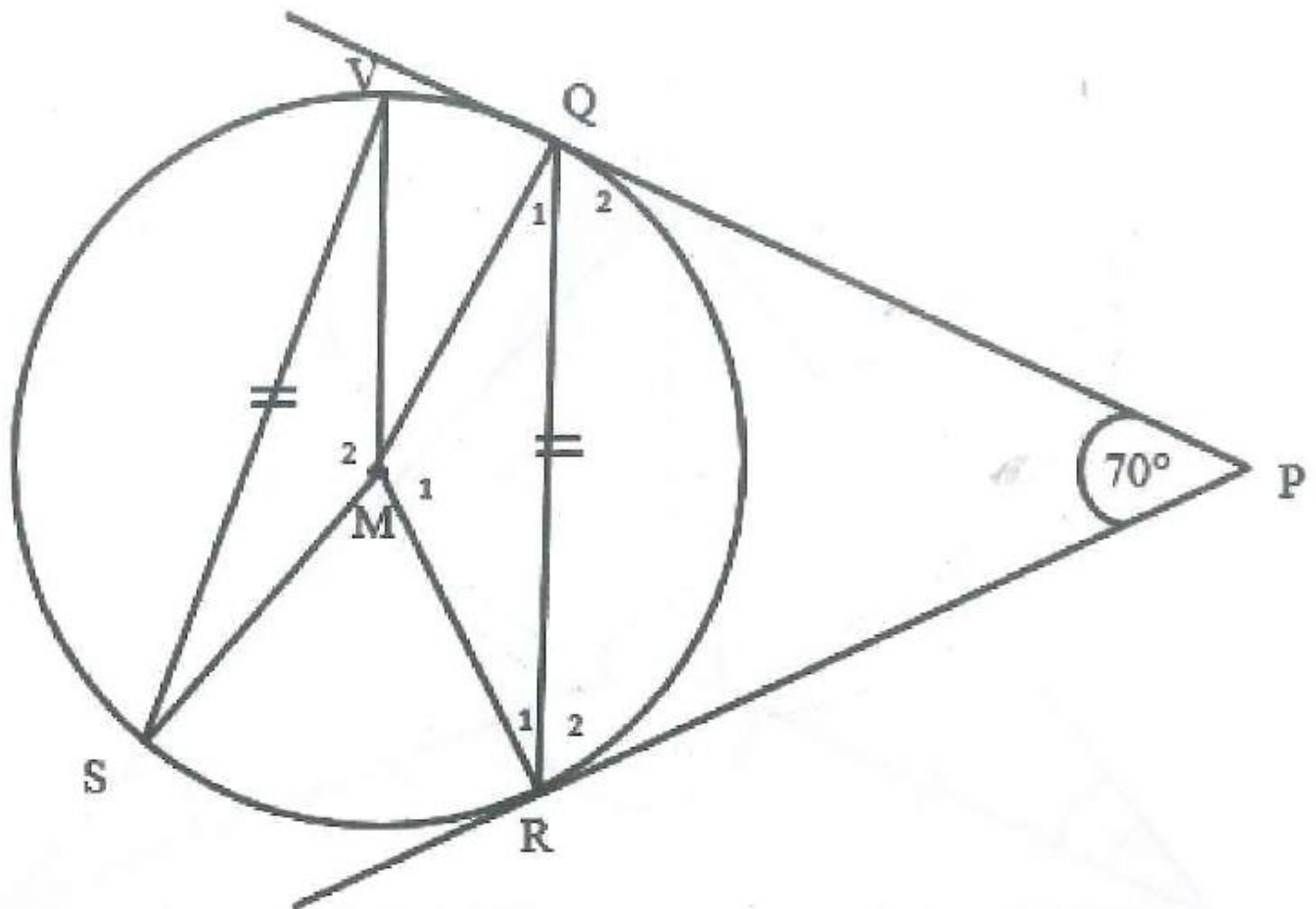
8.2 Prove that $AB = CD$. (2)

[11]

QUESTION 8

8.1 M is the centre of the circle SVQR and SV and QR are equal chords.

RP and QP are tangents to the circle at R and Q respectively. $\hat{R}PQ = 70^\circ$.



Calculate, giving reasons, the size of:

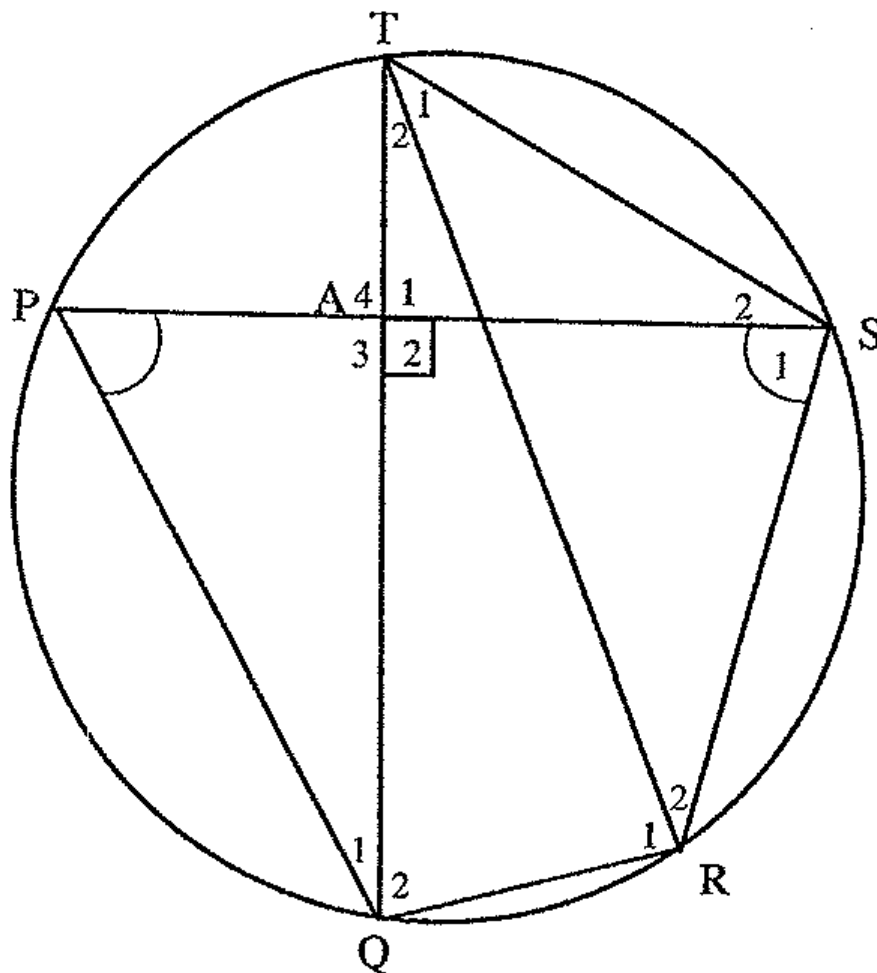
8.1.1 $\hat{R}_2$ (4)

8.1.2 $\hat{Q}_1$ (3)

8.1.3 $\hat{M}_2$ (3)

QUESTION 8

In the diagram, TR is a chord of circle PQRST. $QAT \perp PAS$. $QAT \perp PAS$ and $\hat{P} = \hat{S}_1$.
 In the following questions, give a reason for each statement.

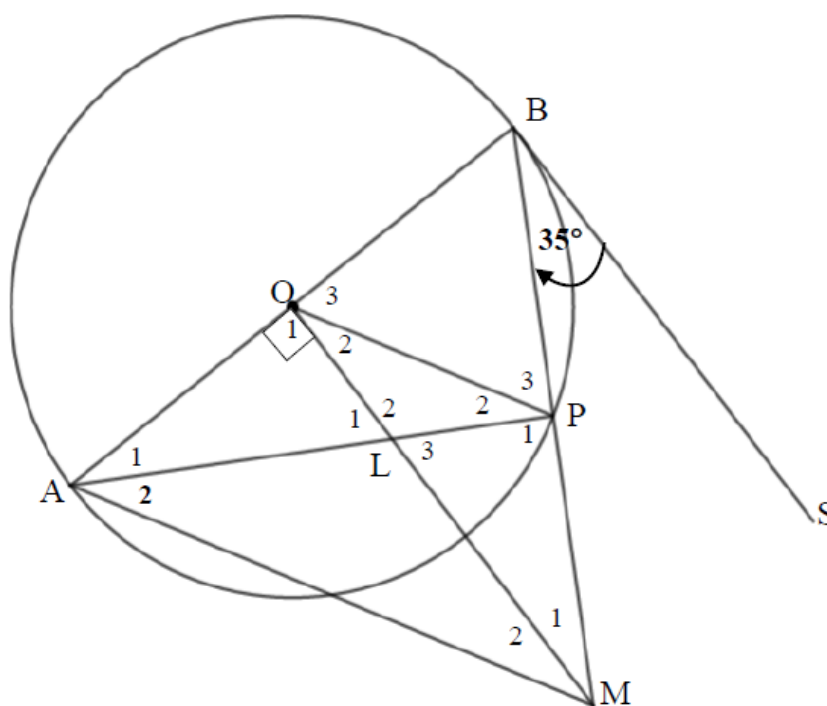


- 8.1 Name **three** angles with reasons, each equal to 60° . (4)
- 8.2 Calculate the size of $\hat{QRS}$. (2)
- 8.3 Prove that $PS \parallel QR$. (2)
- 8.4 Prove that TR is a diameter of the circle. (3)

[11]

WCS20

- 8.2 O is the centre of the circle in the diagram.
- BP is produced to M such that $OM \perp AB$.
 - AP intersects OM at L.
 - BS is a tangent to the circle at B.
 - $\widehat{MBS} = 35^\circ$



8.2.1 Calculate, with reasons, the size of:

- | | |
|---------------------|-----|
| (a) $\widehat{A}_1$ | (2) |
| (b) $\widehat{O}_3$ | (2) |
| (c) $\widehat{P}_3$ | (2) |
| (d) $\widehat{M}_1$ | (3) |

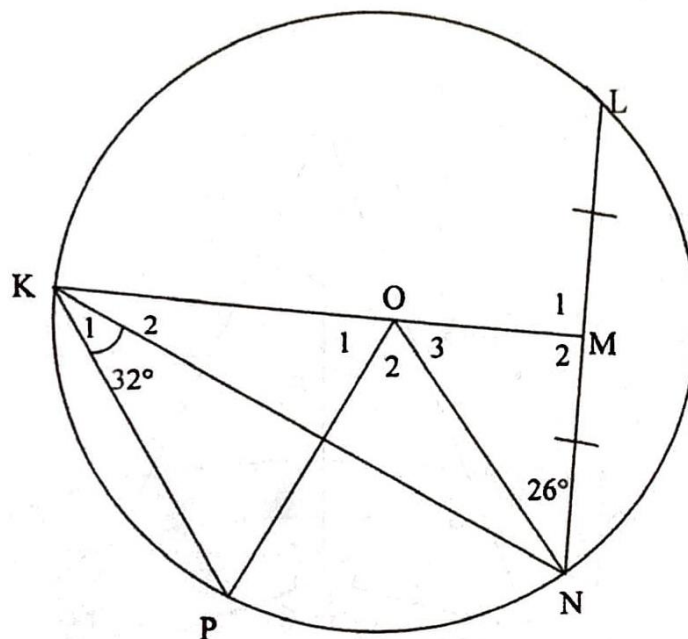
8.2.2 Prove that:

- | | |
|---|-----|
| (a) OLPB is a cyclic quadrilateral. | (3) |
| (b) $BS \parallel OM$. | (2) |
| (c) OP is a tangent to the circle through P, L and M. | (3) |

[22]

QUESTION 8

- 8.1 O is the centre of the circle.. KOM bisects chord LN and $\hat{MNO} = 26^\circ$. K and P are points on the circle with $\hat{NKP} = 32^\circ$. OP is drawn.



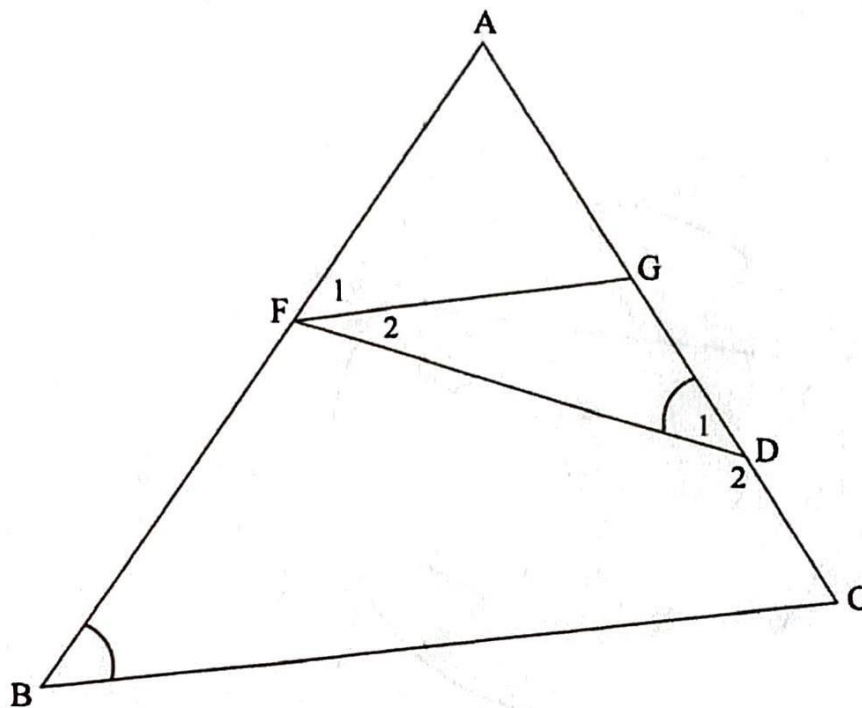
- 8.1.1 Determine, giving reasons, the size of:

(a) $\hat{O}_2$ (2)

(b) $\hat{O}_1$ (4)

- 8.1.2 Prove, giving reasons, that KN bisects $\hat{OKP}$. (3)

- 8.2 In $\triangle ABC$, F and G are points on sides AB and AC respectively. D is a point on GC such that $\hat{D}_1 = \hat{B}$.



- 8.2.1 If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that $FG \parallel BC$. (4)
- 8.2.2 If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x . (4)

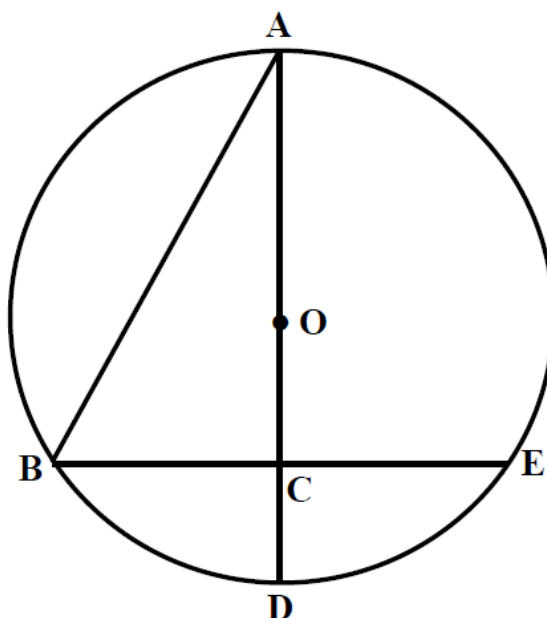
SECTION C

- Mostly level 2 and level 3 questions.

KZNS18

QUESTION 8

In the diagram below, $AOCD$ is a diameter of the circle with centre O and chord $BE = 30$ cm. $AOCD \perp BE$ and $OC = 2CD$.

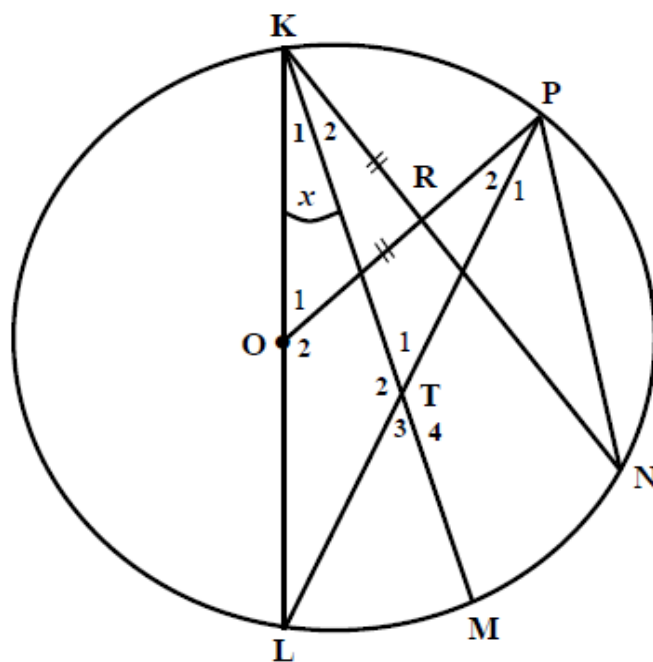


Calculate with reasons:

- 8.1 BC (2)
- 8.2 If $CD = a$ units, determine OC in terms of a . (1)
- 8.3 Calculate OB . (1)
- 8.4 AB (correct to one decimal place). (3)
- 8.5 the radius of the circle CAB . (2)

[9]

- 9.2 KOL is the diameter of the circle KPNML having centre O. R is the point on chord KN, such that $KR = RO$. OR is produced to P. Chord KM bisects $\angle LKN$ and cuts LP in T. $\angle K_1 = x$.



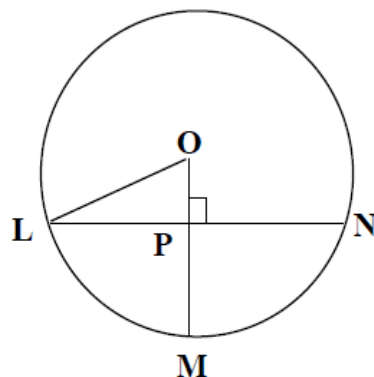
Prove with reasons that:

- | | | |
|-------|---------------------------------|-------------|
| 9.2.1 | $TK = TL$ | (5) |
| 9.2.2 | KOTP is a cyclic quadrilateral. | (3) |
| 9.2.3 | $PN \parallel MK$ | (3) |
| | | [16] |

KZNJ18

QUESTION 5

In the accompanying diagram alongside, LN is a chord of the circle with centre O. OP is drawn perpendicularly onto LN and meets the circle in M.

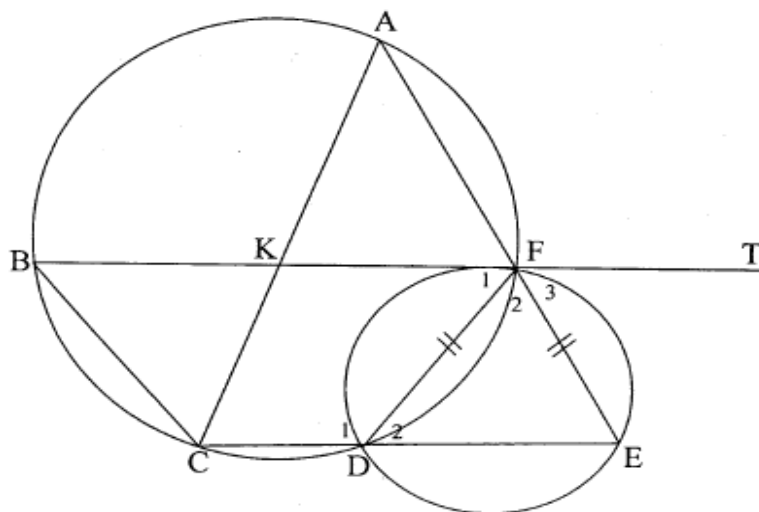


5.1 Prove that $MN^2 = 2 OM \cdot PM$. (4)

5.2 If $OL = 9$ units and $OP = 1$ unit, calculate the length of MN . (2)
[6]

QUESTION 6

In the figure below, two circles cut in points F and D. BFT is a tangent to the smaller circle at F. Straight line AFE is drawn so that $FE = FD$. CDE is a straight line and chords AC and BF cut at K.



Prove that:

6.1 $BT \parallel CE$. (4)

6.2 BCEF is a parallelogram. (5)

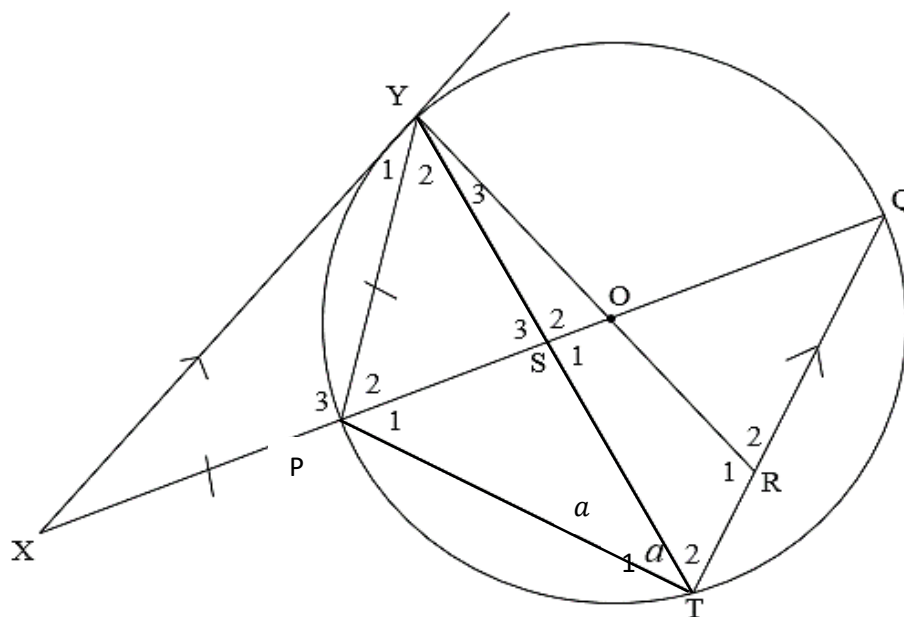
6.3 $AC = BF$. (6)

6.4 BF is a diameter if it is given that $AF = FE$. (7)

[22]

FS S18

- 9.2 XY is a tangent to the circle with centre O. XPQ, YOR, YST are straight lines.
 $PX = PY$, $XY \parallel TQ$ and $\hat{T}_1 = a$

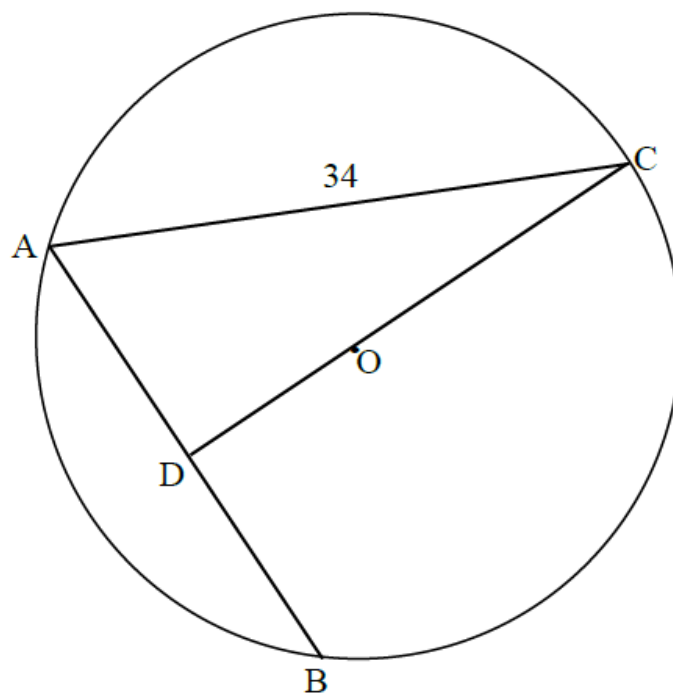


- 9.2.1 Write down, with reasons, FOUR other angles each equal to a . (6)
- 9.2.2 Prove that $\hat{T}_2 = 2\hat{T}_1$ (2)
- 9.2.3 Prove that $\hat{T}_2 = 90^\circ - a$. (2)
- 9.2.4 Prove that SORT is a cyclic quadrilateral. (2)
- 9.2.5 Determine the value of a . (2)
- 9.2.6 Show that $TR = RQ$. (3)

[17]

GP S18

- 8.3 In the diagram below, radius CO is produced to bisect chord AB at D.
 $CA = 34 \text{ mm}$ and $AB = 40 \text{ mm}$

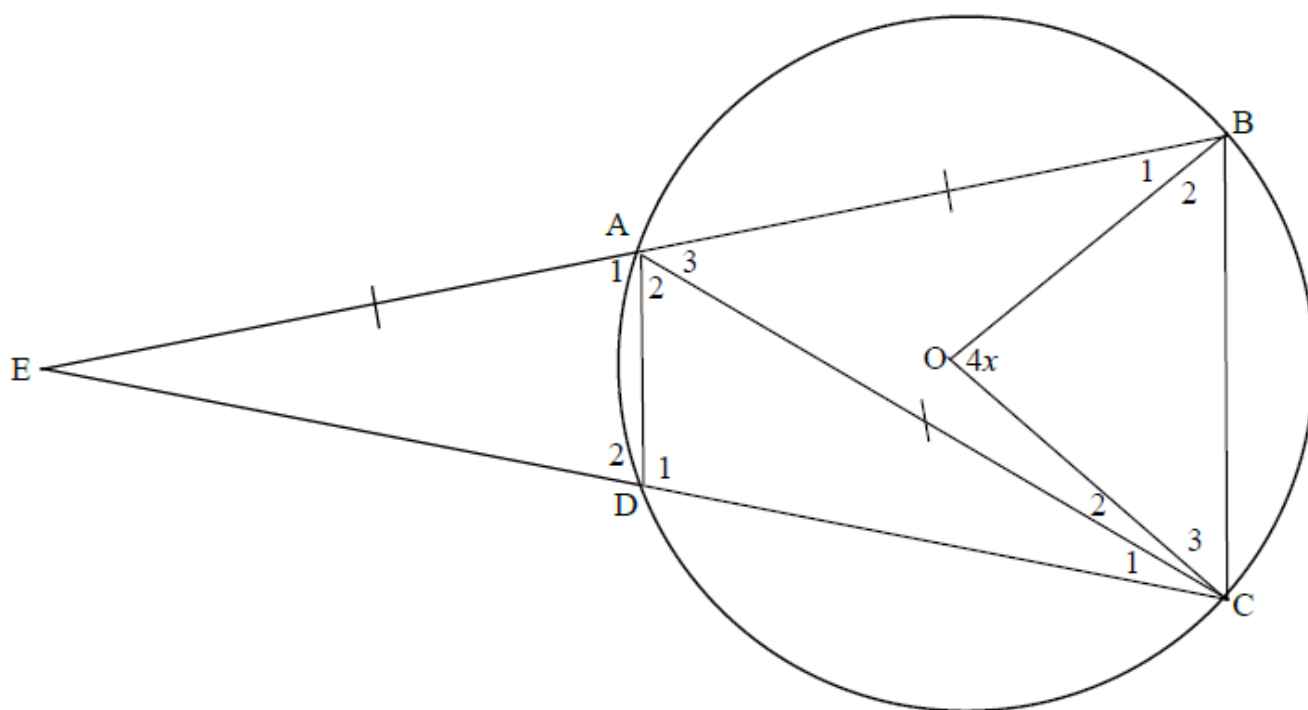


Calculate the size of $\hat{C}$.

(4)

QUESTION 9

In the diagram below, O is the centre of the circle. $ABCD$ is a cyclic quadrilateral. BA and CD are produced to intersect at E such that $AB = AE = AC$.



Determine in terms of x :

9.1 $\hat{B}_2$ (2)

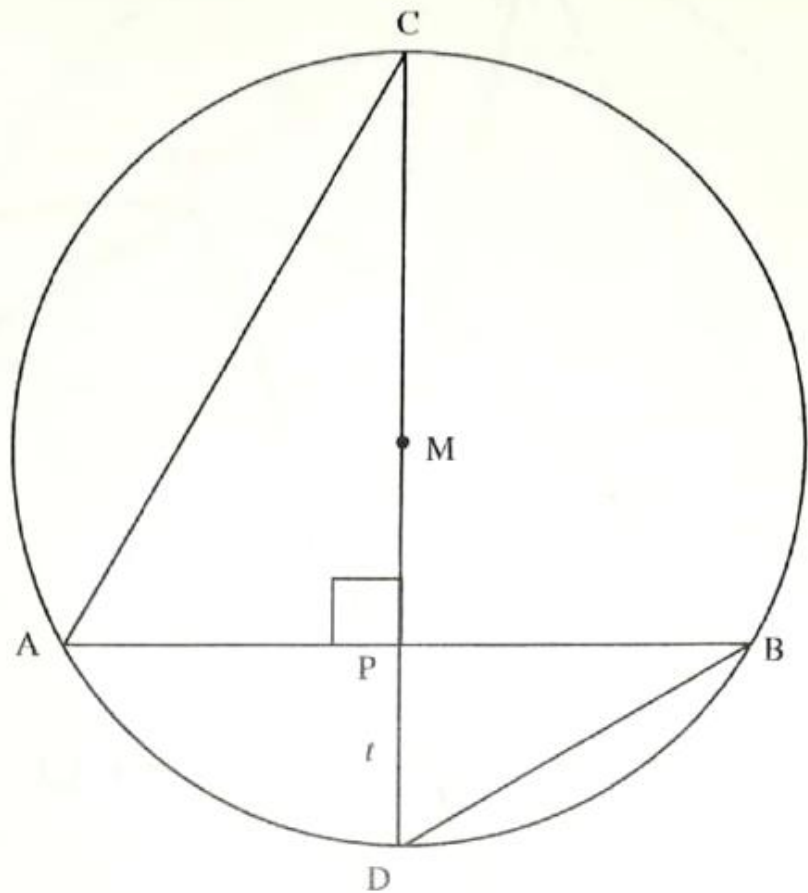
9.2 $\hat{E}$ (5)

9.3 $\hat{C}_2$ (3)

9.4 If $\hat{E} = \hat{C}_2 = x$, prove that ED is a diameter of circle AED . (4)
[14]

MP S18

8.2 In the diagram, M is the centre of circle and diameter CMPD is perpendicular to chord AB. $AB = 4t$, $PD = t$ and $CP = 15$ cm.



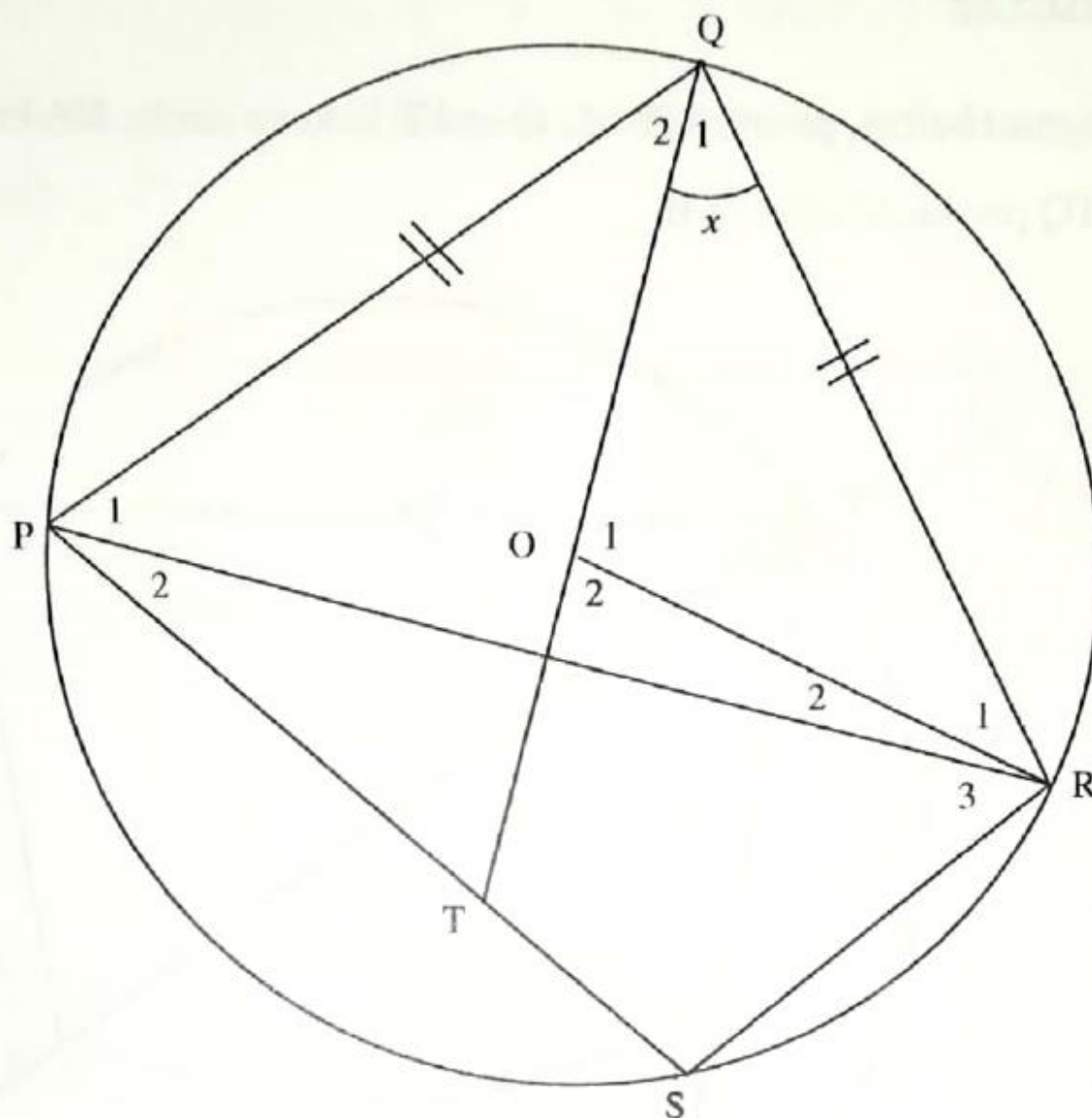
8.2.1 Give a reason why $AP = 2t$. (1)

8.2.2 If it is further given that $\triangle CAP \parallel \triangle BDP$, calculate:

- a) The value of t . (4)
- b) The length of the radius of the circle. (2)

MP S18

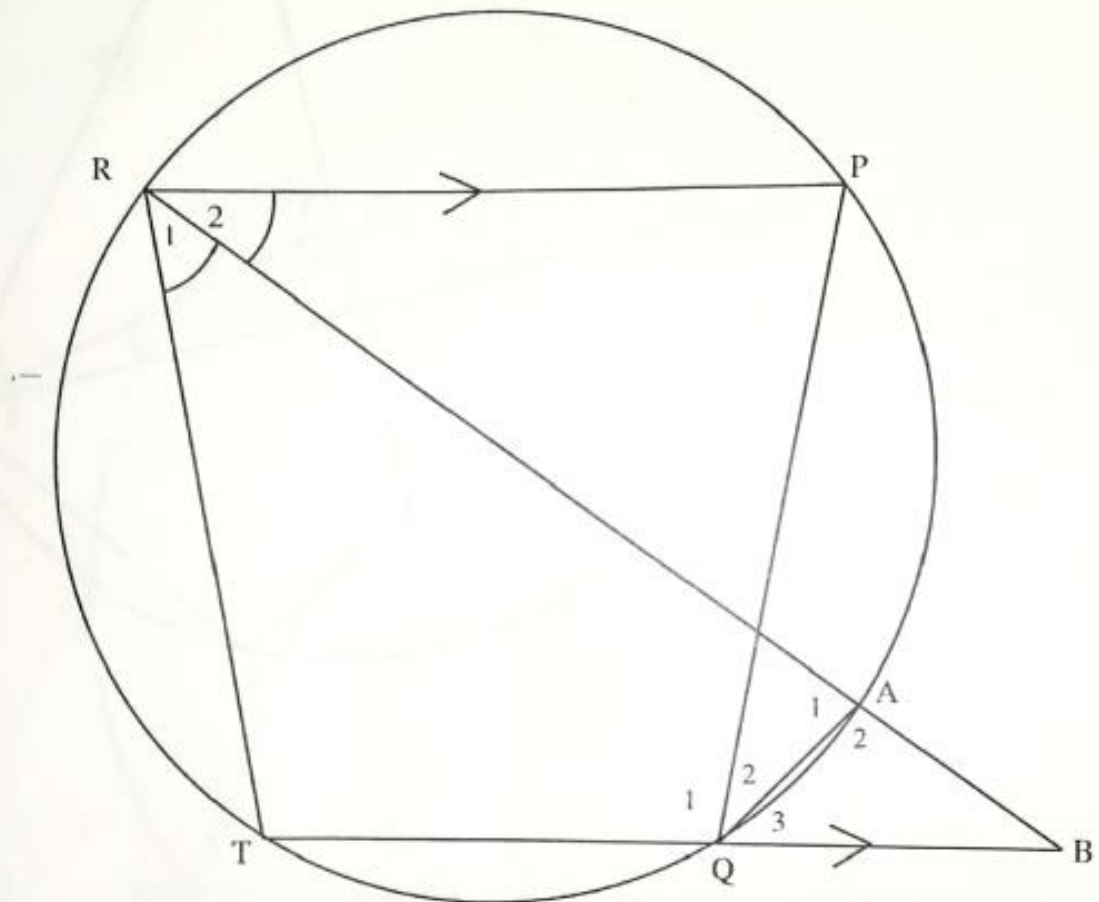
- 9.2 In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS. $PQ = QR$ and $\angle Q_1 = x$.



- 9.2.1 Calculate, with reasons, $\hat{P}_1$ in terms of x . (4)
- 9.2.2 Prove that TQ bisects $\hat{PQR}$. (3)
- 9.2.3 Prove that STOR is a cyclic quadrilateral. (5)

QUESTION 10

In the diagram below, points R, P, A, Q and T lie on a circle. RA bisects $\hat{R}$ and $RP \parallel TB$. RA and TQ produced meet at B.



Prove that:

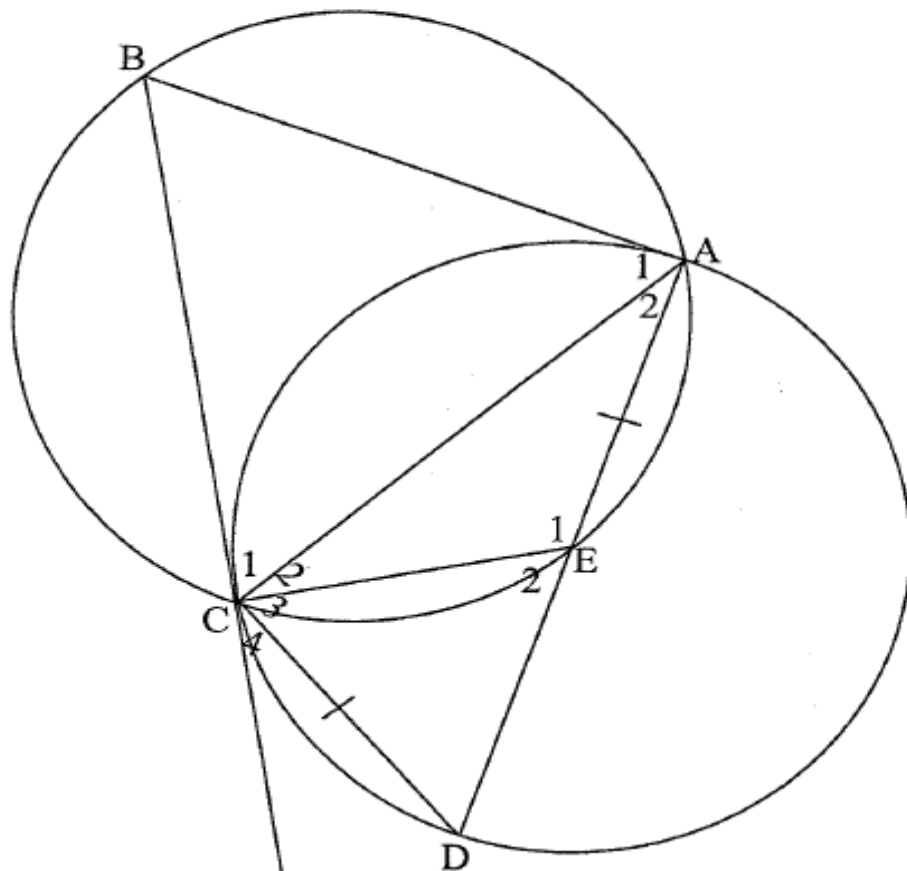
10.1 AQ bisects $\hat{PQB}$. (5)

10.2 $\hat{TRP} = \hat{A}$ (4)

[9]

QUESTION 9

Two equal circles cut each other in A and C. BA and BC are tangents to one circle at A and C respectively and they are chords of the other circle. E is a point on the circumference of one circle and AE produced cuts the other circle in D. Chords AE and CD are equal.

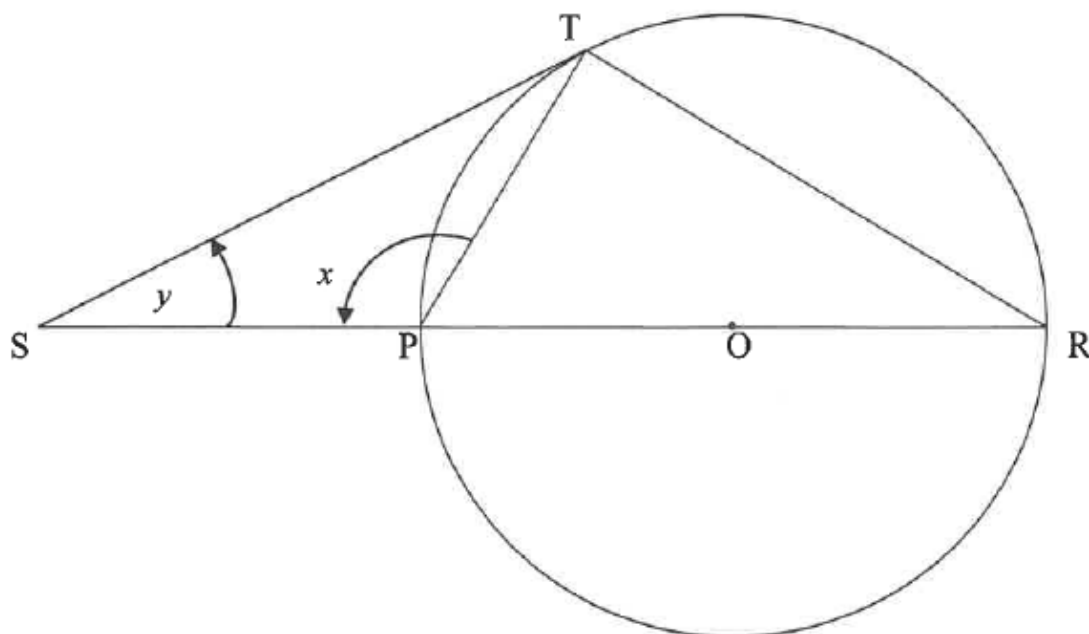


Prove that:

- 9.1 $\hat{C}_2 = \hat{C}_4$ (4)
 - 9.2 $\hat{C}_3 = \hat{A}_1$ (3)
 - 9.3 E is the centre of the circle that passes through A, C and D. (4)
 - 9.4 $\triangle ECD$ is equilateral. (2)
- [13]**

QUESTION 8

In the diagram, PR is a diameter of the circle with centre O . ST is a tangent to the circle at T and meets RP produced at S . $\hat{SPT} = x$ and $\hat{S} = y$.

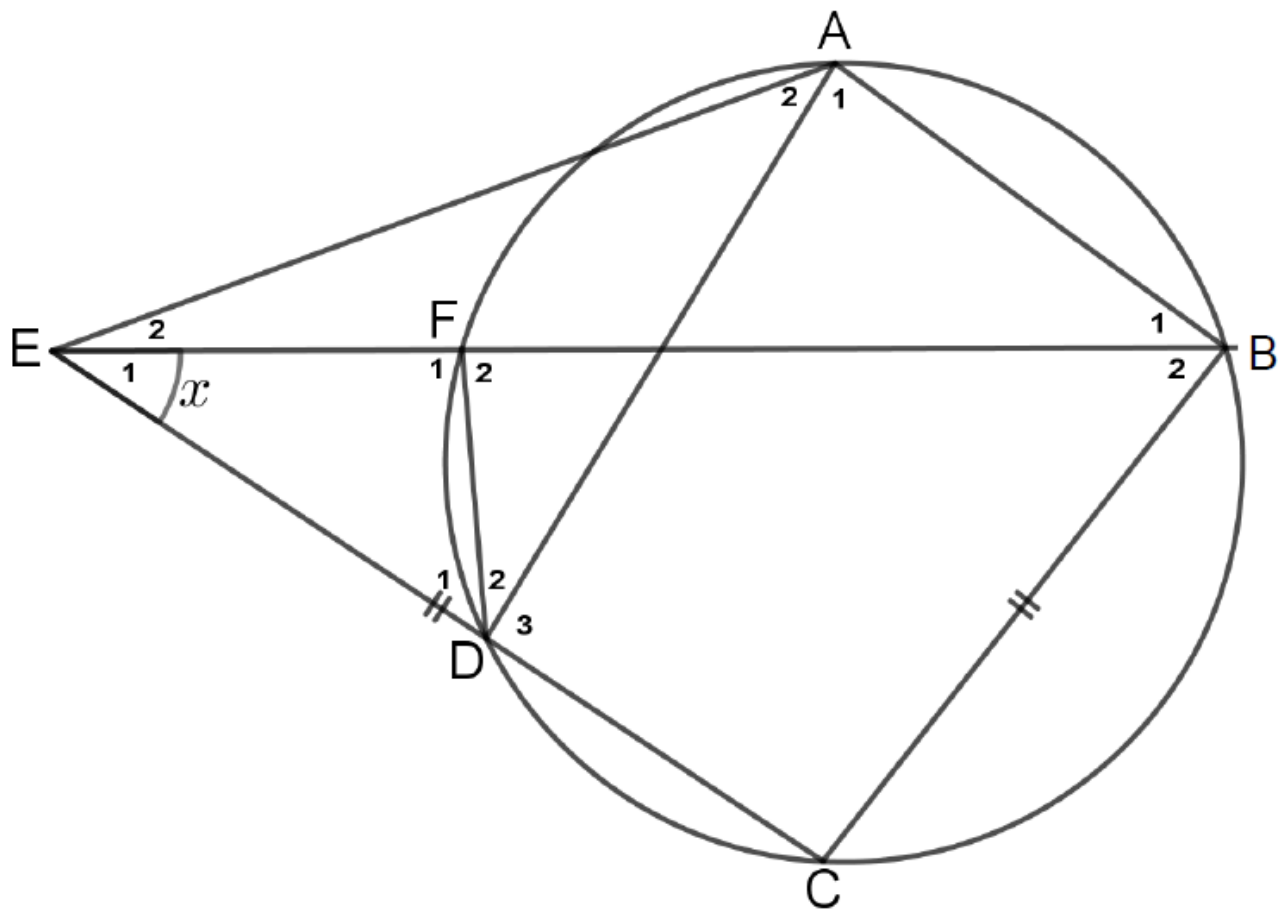


Determine, with reasons, y in terms of x .

[6]

KZNM19
QUESTION 4

In the diagram below, $BC = CE$; $\hat{E}_1 = x$ and $\hat{D}_1 = \hat{D}_2$.

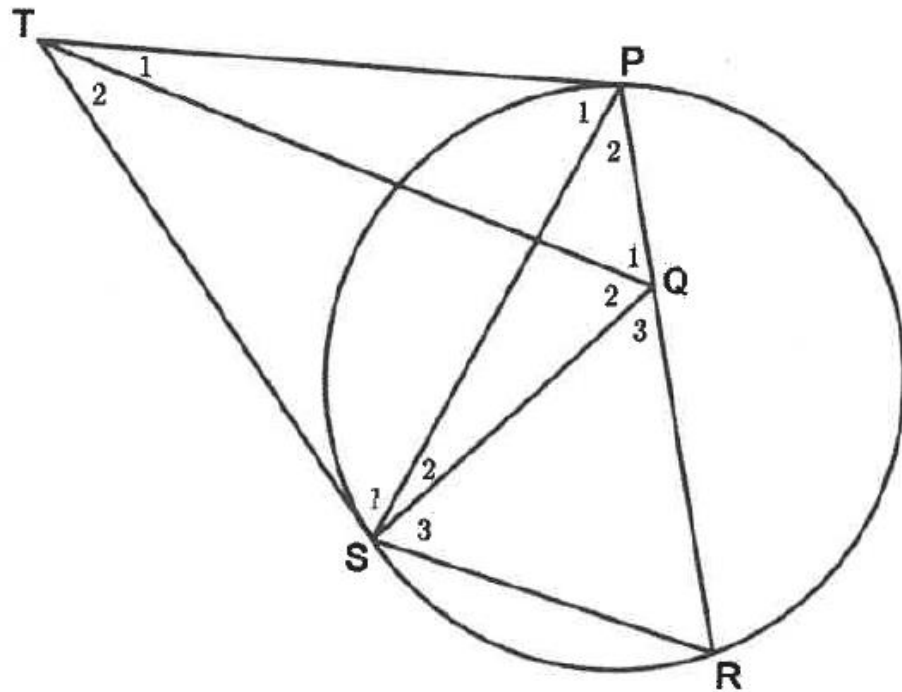


- 4.1 Name, with reasons, TWO other angles each equal to x and show that $FD = FE$. (4)
- 4.2 Prove that BF bisects $\hat{CBA}$. (4)
- 4.3 Hence, or otherwise, prove that $\hat{A}_1 = \hat{CBA}$. (4)

[12]

QUESTION 9

- 9.1 In the figure, TP and TS are tangents to the circle. R is a point on the circle and RS are drawn. Q is a point on PR such that $\widehat{TQP} = \widehat{TPS}$. SQ is drawn.

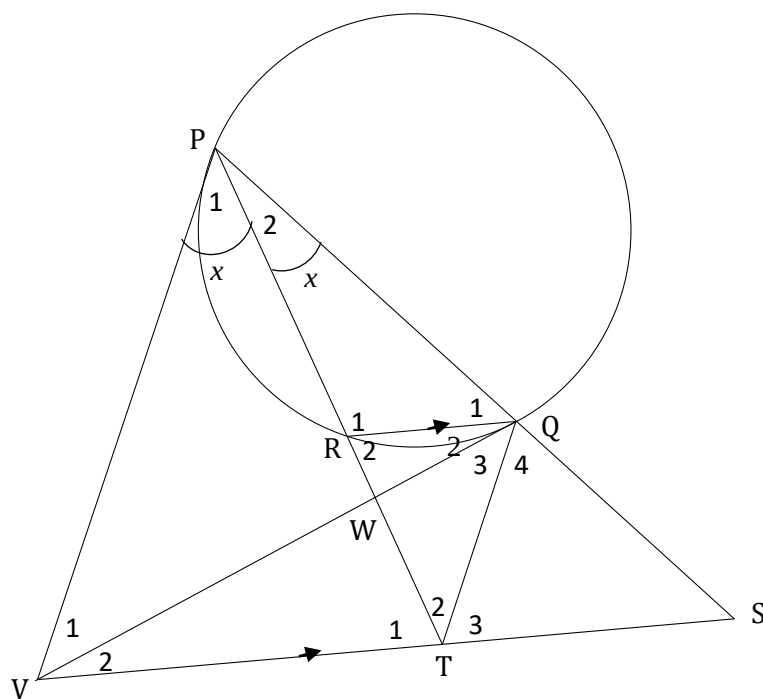


Prove that:

- 9.1.1 $TQ \parallel SR$ (3)
- 9.1.2 QPTS is a cyclic quadrilateral (3)
- 9.1.3 TQ bisects $\widehat{SQP}$. (3)

QUESTION 9 KZNS19

In the diagram, PV and VQ are tangents to the circle at P and Q . PQ is produced to S and chord PR is produced to T such that $VT \parallel RQ$. VQ and RT intersect at W . $\hat{P}_1 = \hat{P}_2 = x$.



Prove that:

9.1 $\hat{S} = x$ (4)

9.2 $PQTV$ is a cyclic quadrilateral (5)

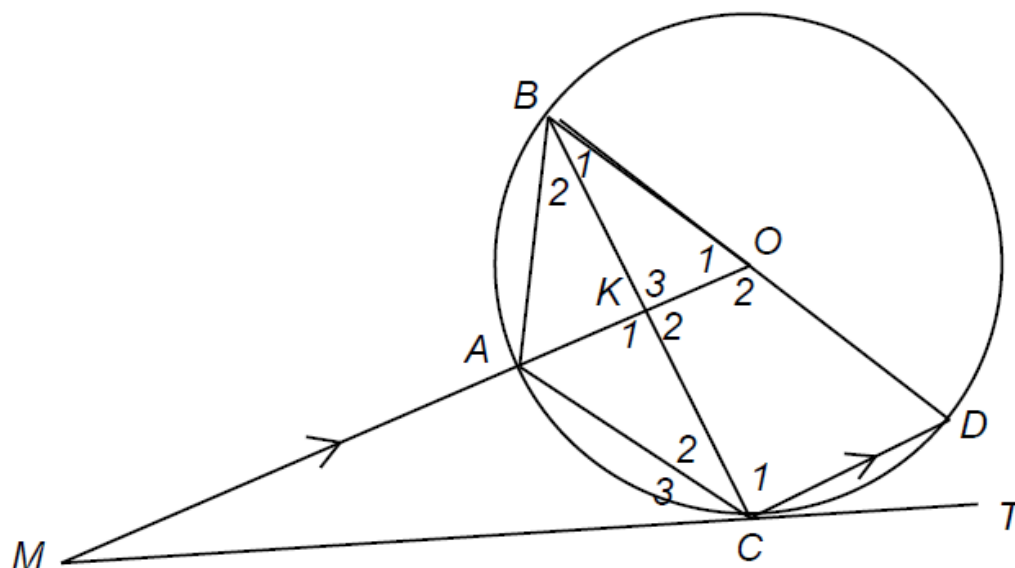
9.3 TQ is a tangent to the circle passing through Q , W and P (3)

[12]

LPM19

6.3 In the figure O is the centre of the circle.

MCT is a tangent touching the circle at C , $OM \parallel DC$.



6.3.1 Give the correct reasons for the following statements:

	STATEMENT	REASON
(a)	$\hat{C}_1 = 90^\circ$	
(b)	$\therefore \hat{K}_2 = 90^\circ$	
(c)	$\therefore BK = KC$	
(d)	$\hat{C}_2 + \hat{C}_3 = \hat{D}$	
(e)	But $\hat{D} = \hat{O}_1$	
(f)	$\therefore \hat{O}_1 = \hat{C}_2 + \hat{C}_3$ $\therefore BOCM$ is a cyclic quad.	

(6)

(3)

6.3.2 Prove $\triangle BOK \parallel \triangle MCK$

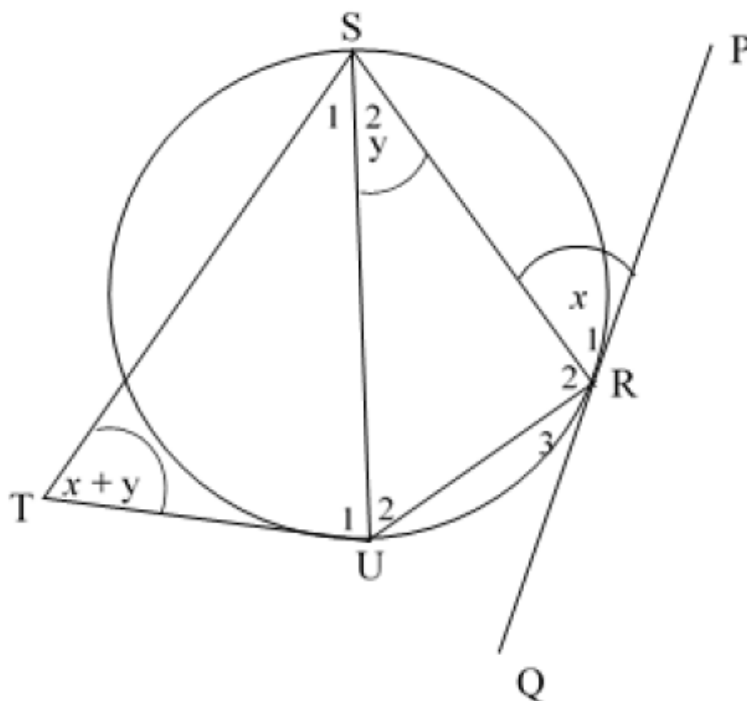
6.3.3 Determine BO if $OK = 1$ unit and $MK = 9$ units.

(5)

ECS19

- 8.3 In the diagram below, PRQ is a tangent to the circle SUR at R. SU, SR and UR are drawn. Lines from S and U produced meet at T outside the circle.

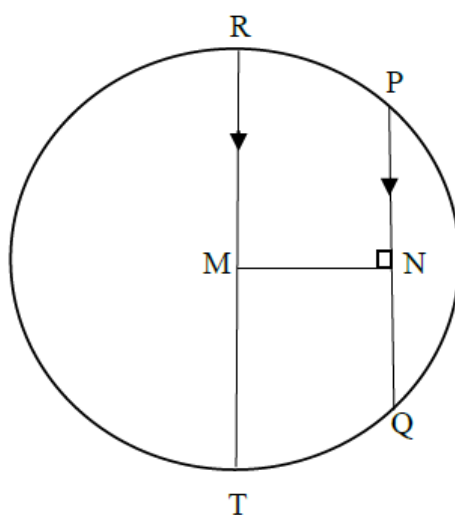
$$\hat{R}_1 = x; \hat{S}_2 = y \text{ and } \hat{STU} = x + y$$



Prove that STUR is a cyclic quadrilateral.

(5)

- 8.4 The diagram below shows a circle centre M passing through the points R, P, Q and T. RT is the diameter. PQ is a chord such that $PQ \parallel RT$ and $MN \perp PQ$. $PQ = 16$ units, $MN = 6$ units.

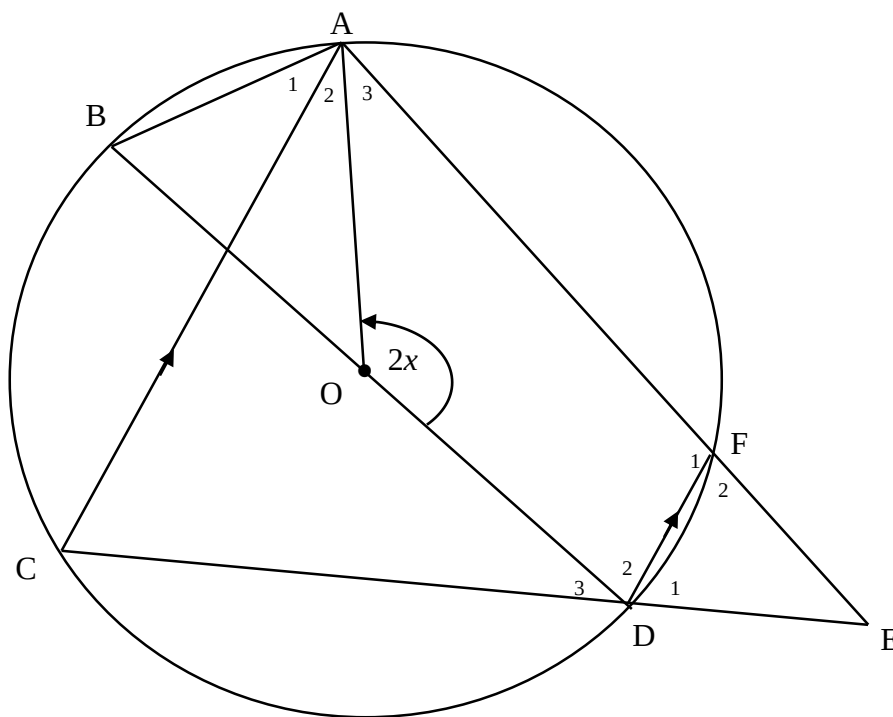


Determine the length of RT.

(4)

QUESTION 9 FSS19

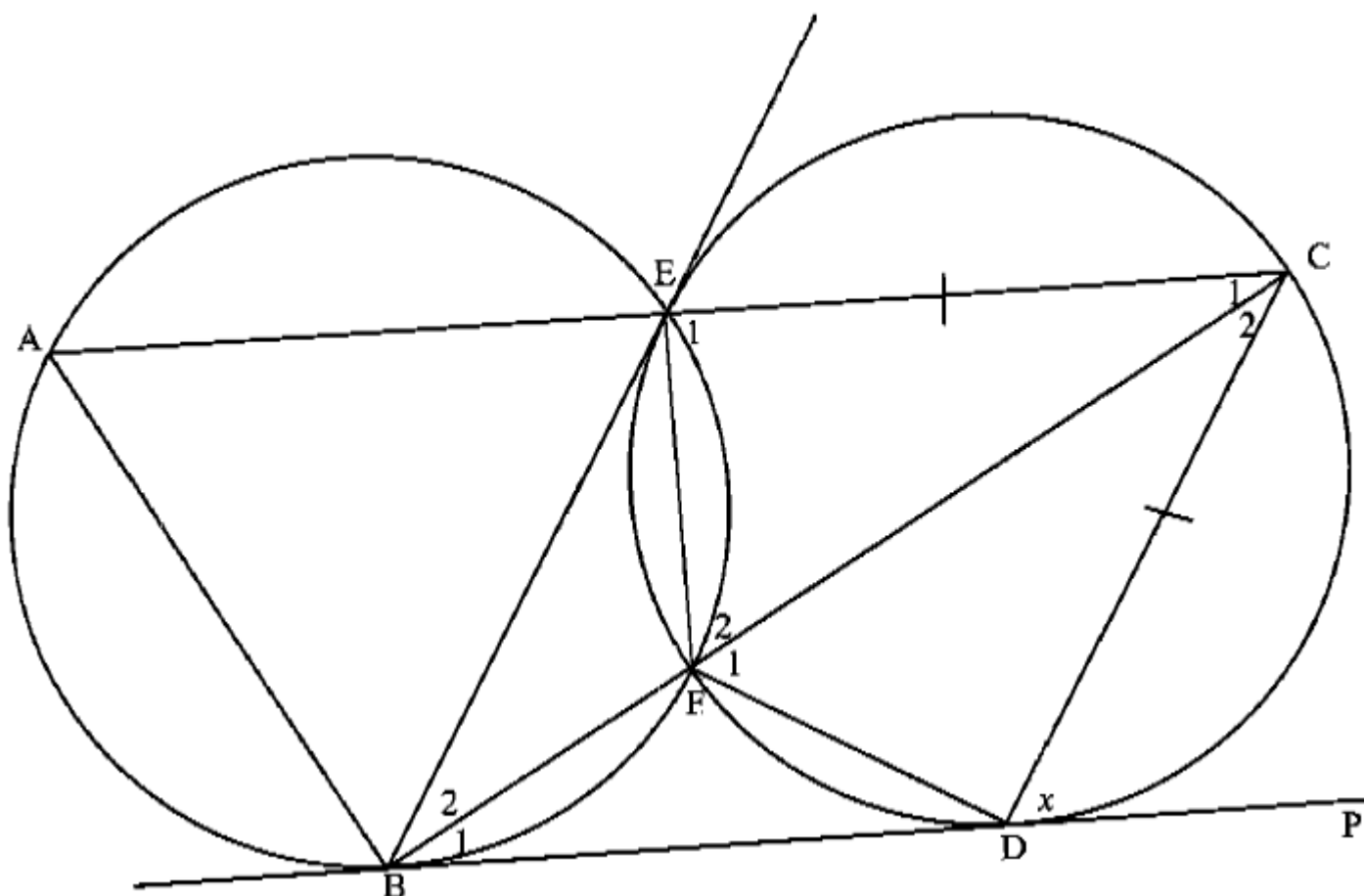
- 9.1 A circle with centre O is given below. Lines CD and AF are produced to E.
 $\angle AOD = 2x$ and BD is the diameter. $AC \parallel FD$.



- 9.1.1 Determine, with reasons, four other angles that are each equal to x . (6)
- 9.1.2 Express $\hat{E}$ in terms of x . (2)
- 9.1.3 Prove that AODE is a cyclic quadrilateral. (2)
- 9.2 It is further given that $ED : DC = 8 : 12$ and $FE = 10$. Calculate the length of AF. (3)
- [13]**

QUESTION 9

- 9.1 Complete the statement so that it is TRUE.
Angles subtended by a chord of a circle, on the same side of a chord, are ... (1)
- 9.2 In the diagram below, ABFE and EFDC are cyclic quadrilaterals in two equal circles that intersect at E and F. BFC and AEC are straight lines. BD is a common tangent to the circles at B and D respectively. EC = CD.
Let $\hat{CDP} = x$

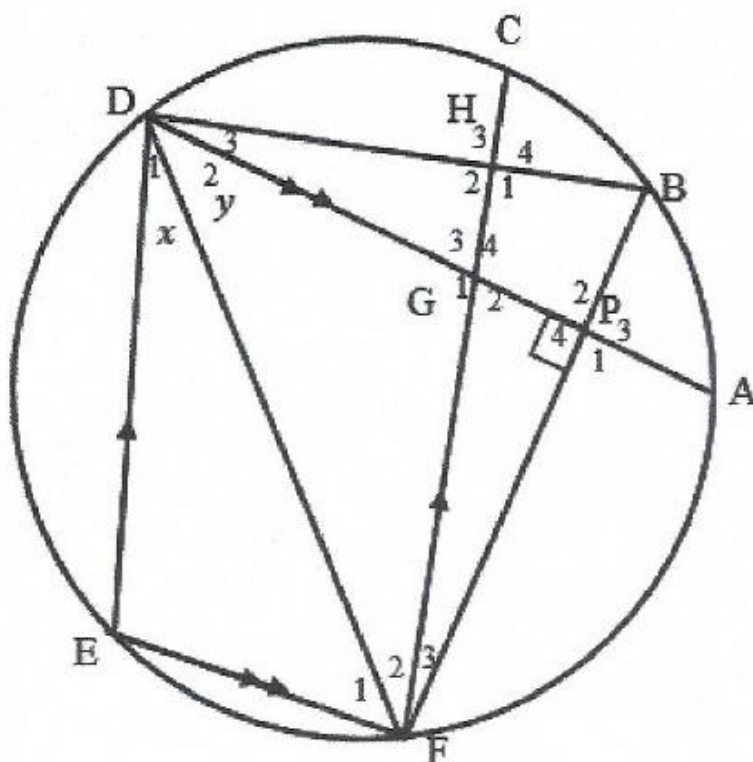


Prove, giving reasons, that:

- 9.2.1 $\hat{F}_1 = \hat{F}_2$. (3)
- 9.2.2 ABDC is a cyclic quadrilateral. (3)
- 9.2.3 $BE \parallel CD$. (2)
- 9.2.4 FC is a diameter of circle FDCE if it is given that EBDC is a rhombus. (5)
- [14]

QUESTION 9

- 9.1 A, B, C, D, E and F are points on the circumference of the circle. $ED \parallel FC$ and $AD \parallel FE$. $DA \perp FB$. FC intersects DA at G and DB at H.



Let $\hat{EDF} = x$ and $\hat{ADF} = y$.

Prove that:

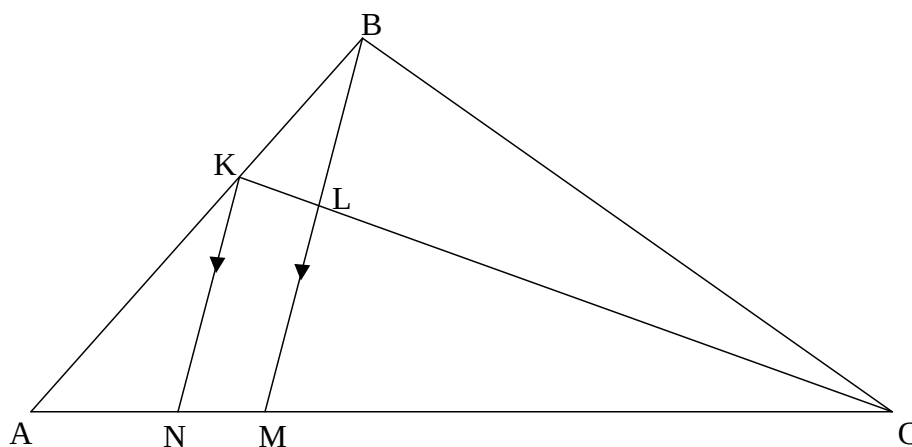
9.1.1 $\hat{B} = x + y$ (4)

9.1.2 $\hat{H}_1 = 90^\circ$ (6)

NJ19

8.2

In $\triangle ABC$ in the diagram, K is a point on AB such that $AK : KB = 3 : 2$.
 N and M are points on AC such that $KN \parallel BM$. BM intersects KC at L .
 $AM : MC = 10 : 23$.



Determine, with reasons, the ratio of:

8.2.1 $\frac{AN}{AM}$ (2)

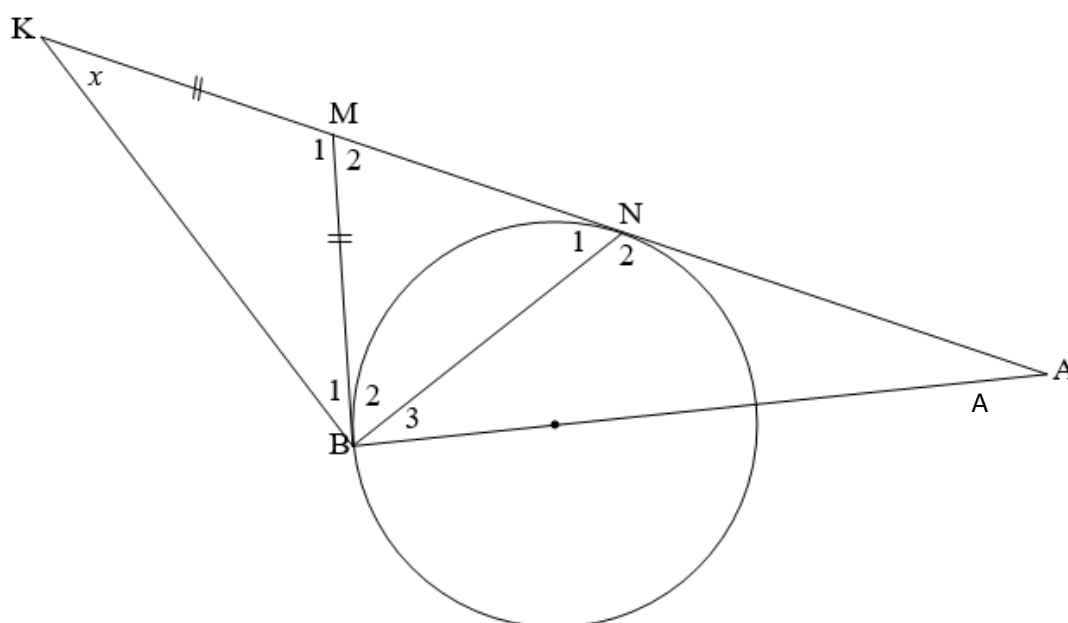
8.2.2 $\frac{CL}{LK}$ (3)

[13]

QUESTION 9 NJ19

In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N . The straight line from B passing through the centre of the circle meets MN produced in A .
 NM is produced to K such that $BM = MK$. BK and BN are drawn.

Let $\hat{K} = x$.



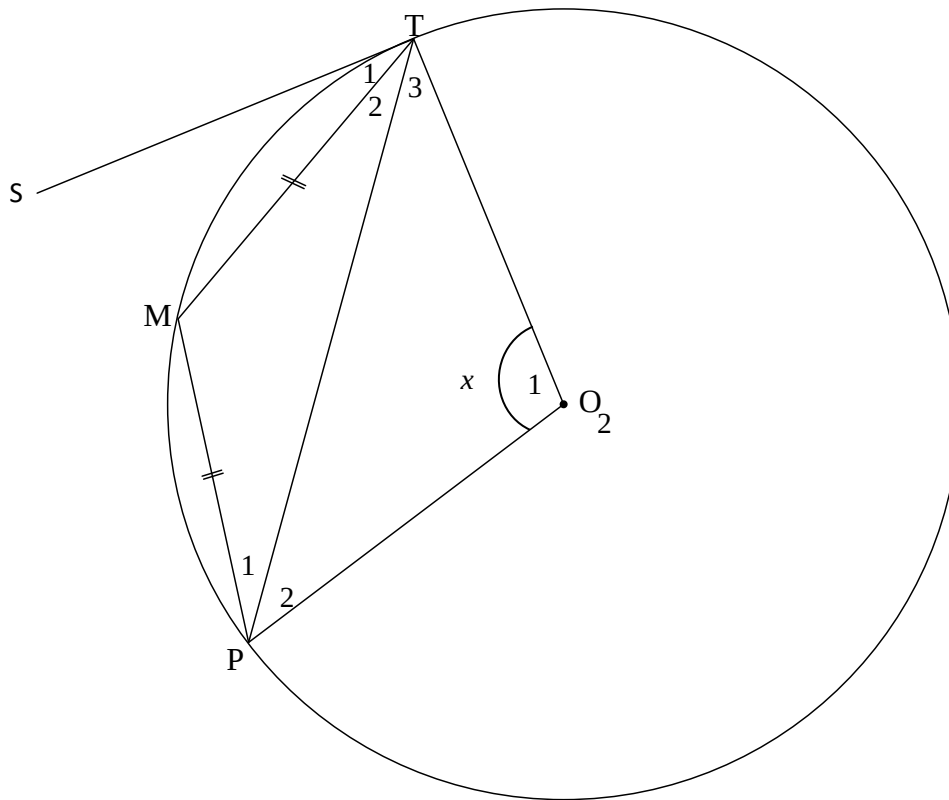
9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x . (6)

9.2 Prove that BA is a tangent to the circle passing through K , B and N . (5)

[11]

QUESTION 9 NN19

In the diagram, O is the centre of the circle. ST is a tangent to the circle at T . M and P are points on the circle such that $TM = MP$. OT , OP and TP are drawn. Let $\hat{O}_1 = x$.

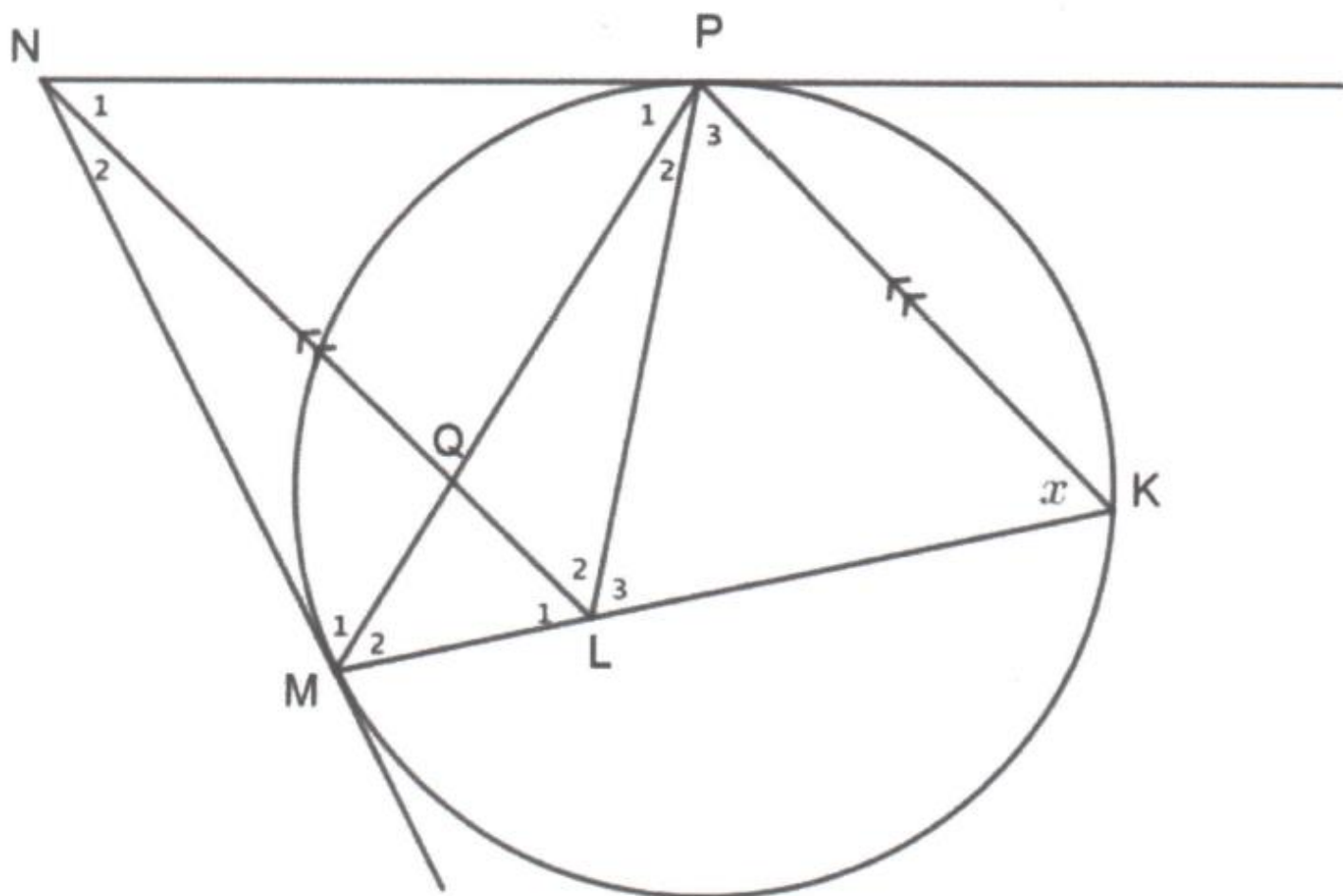


Prove, with reasons, that $\hat{S}TM = \frac{1}{4}x$.

[7]

QUESTION 6

In the diagram below, NP and NM are tangents to the circle at P and M. K is a point on the circle KP, KM and PM are chords so that $\widehat{K} = x$. L is a point on KM so that $KP \parallel LN$, L and P are joined.



6.1 Prove that NMLP is a cyclic quadrilateral. (4)

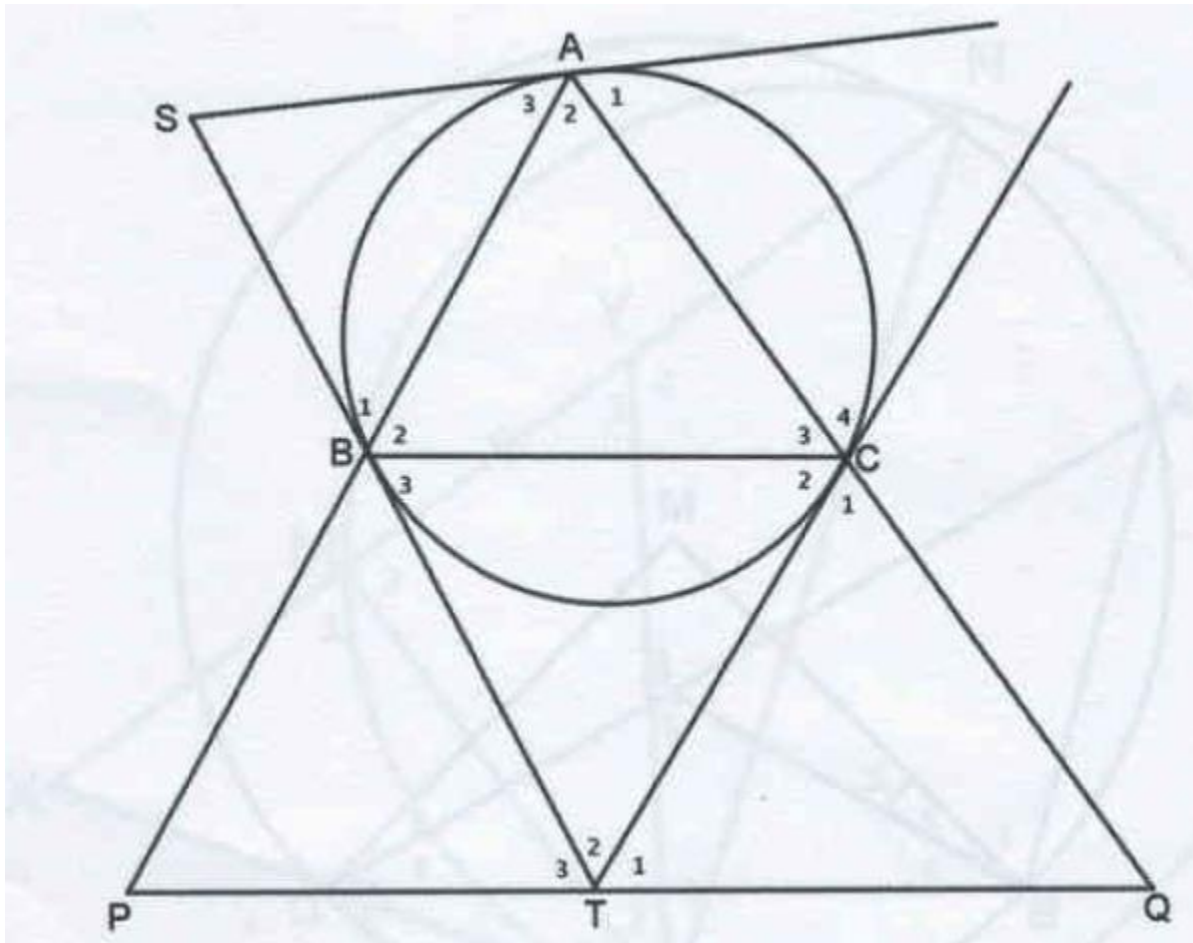
6.2 Prove that $\triangle KLP$ is isosceles. (6)

[10]

KZNS20

- 8.2 In the diagram, SBT, SA and TC are tangents to the circle at B, A and C respectively. AB is produced to P and AC is produced to Q such that T lies on the line PQ.

In $\triangle APQ$, $\frac{AB}{AP} = \frac{AC}{AQ}$.

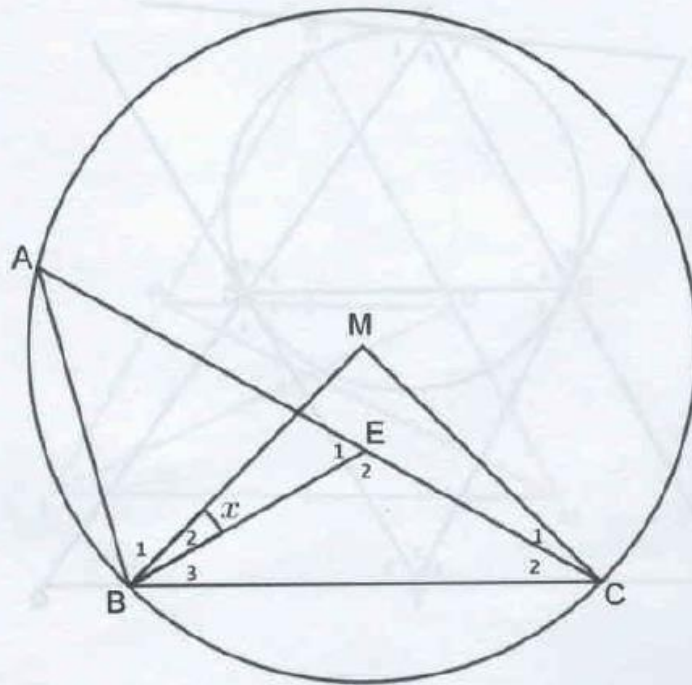


Use the above information to prove:

- 8.2.1 $\angle A_2 = \angle T_1$ (4)
- 8.2.2 $\triangle ABC \sim \triangle TCQ$ (4)
- 8.2.3 ABTQ is a cyclic quadrilateral. (4)
- 8.2.4 Prove that TQ is a tangent to circle TBC at T. (5)

QUESTION 9

In the diagram, M is the centre of the circle through A , B and C . E is on AC . AC bisects $\widehat{MCB}$ and EB bisects $\widehat{MBC}$. $\widehat{B}_2 = x$



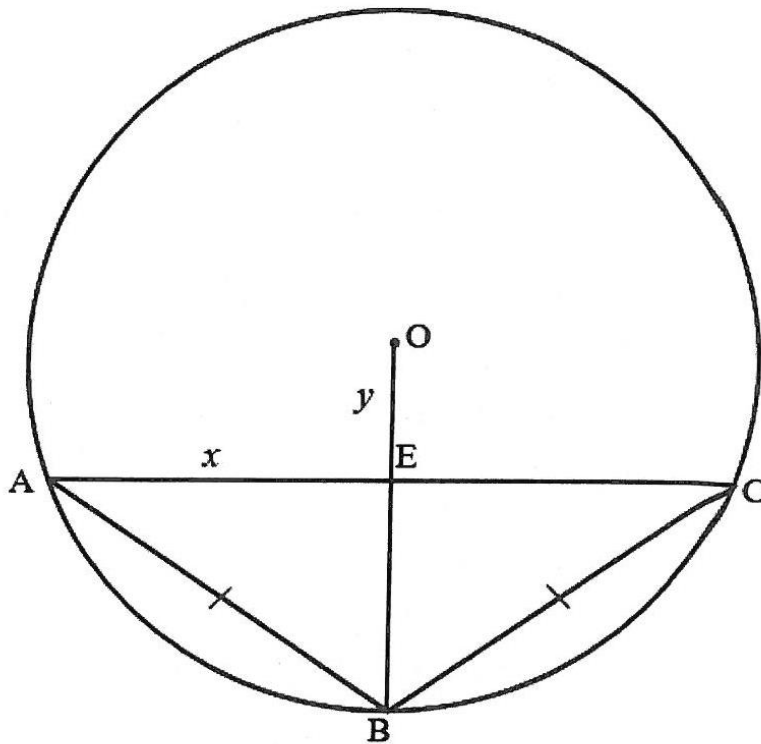
- 9.1 Determine the size of $\widehat{E}_2$ in terms of x . (4)
- 9.2 Show $\widehat{BAC} = 90^\circ - 2x$ (3)
- 9.3 Prove that AE is a diameter of circle ABE . (5)

QUESTION 8

8.1 Complete the following theorem statement:

The line drawn from the centre of a circle perpendicular to a chord ... (1)

8.2 In the diagram below, circle ABC with centre O is given. $OB = 8$ units and $AB = BC = 10$ units. E is the midpoint of AC. Let $OE = y$ and $AE = x$.



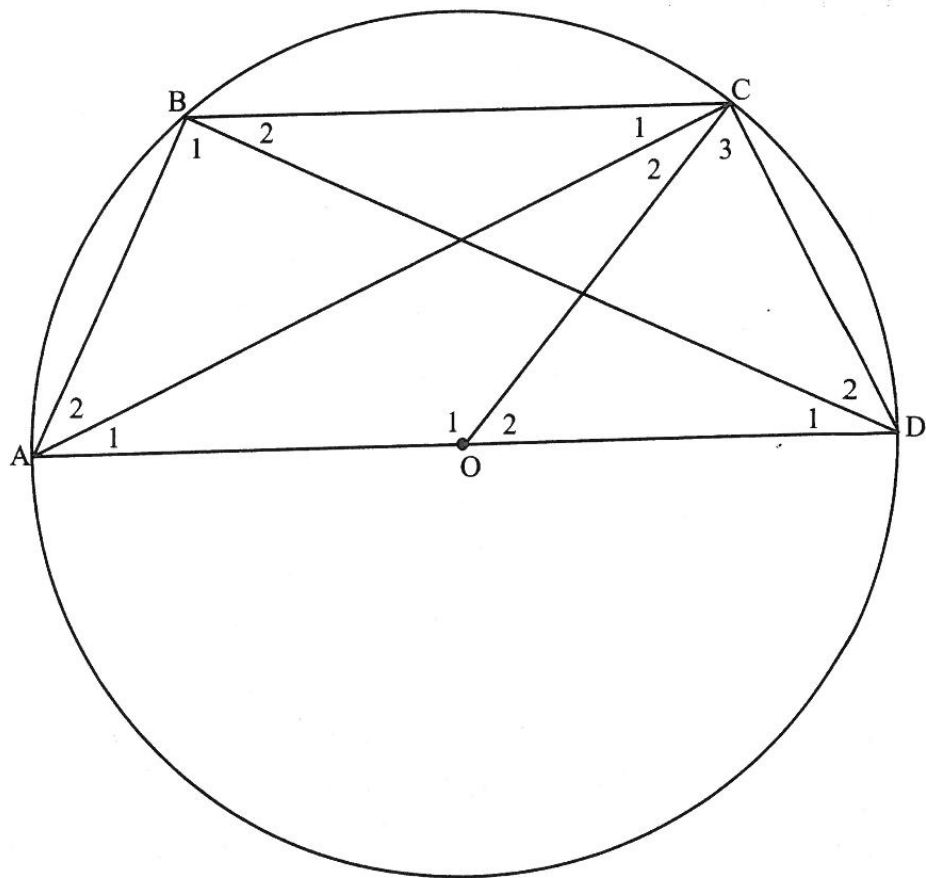
Calculate, with reasons, the length of OE. (5)

8.3 Complete the following theorem statement:

The angle subtended by an arc at the centre of a circle is ... at the circle (on the same side of the chord as the centre). (1)

ECS20

8.4 In the diagram, O is the centre of a circle ABCD. AOD is the diameter and OC is a radius. AB, BC, CD, AC and BD are straight lines.



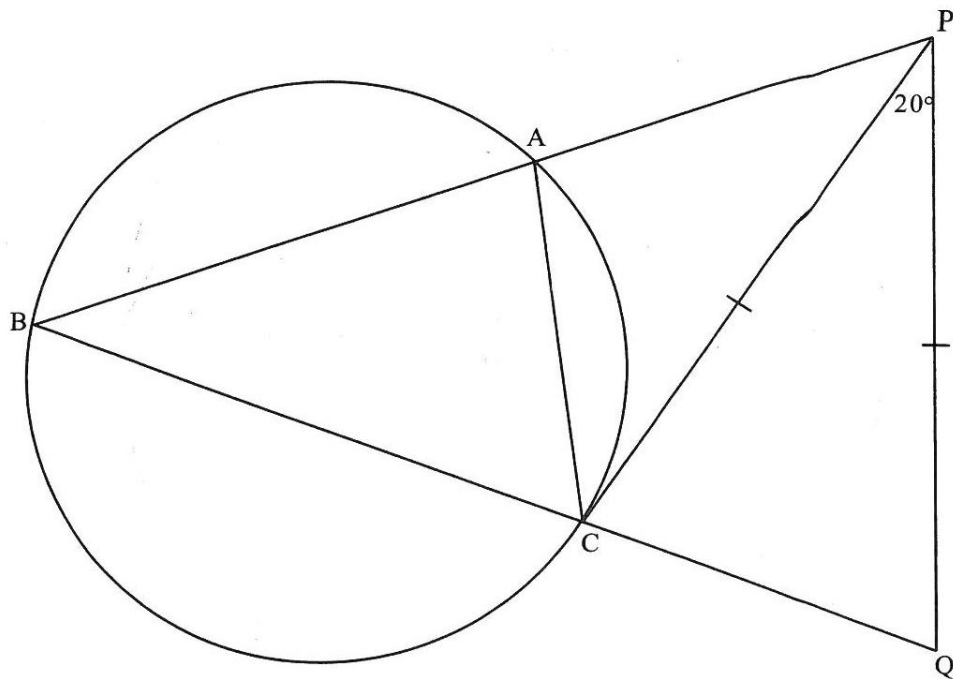
Write down, with reasons, an equation that expresses the relationship between each of the given groups of angles.

	ANGLES	EQUATION / RELATIONSHIP	REASON
e.g.	$\widehat{M}_3; \widehat{P}$	$\widehat{M}_3 = 2 \times \widehat{P}$	$\angle$ at centre = $2 \times \angle$ at circum.
8.4.1	$\widehat{O}_2; \widehat{B}_2$		
8.4.2	$\widehat{D}_1; \widehat{C}_3; \widehat{D}_2$		
8.4.3	$\widehat{B}_1; \widehat{B}_2; \widehat{D}_1; \widehat{D}_2$		
8.4.4	$\widehat{D}_1; \widehat{C}_1$		

(8)

QUESTION 9

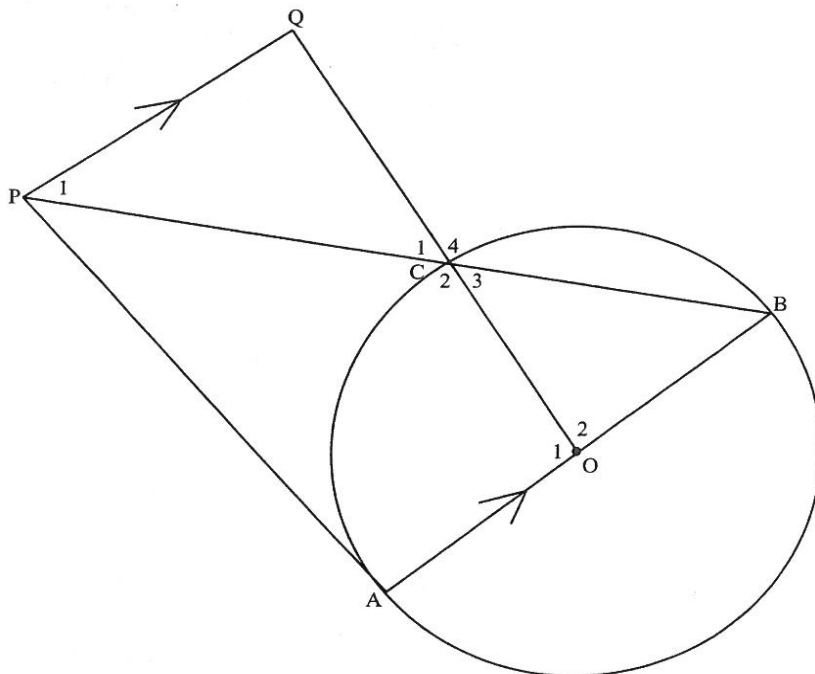
- 9.1 Given that PC is a tangent to the circle ACB; BAP and BCQ are straight lines.
 $PC = PQ$ and $\angle CPQ = 20^\circ$.



Prove, stating reasons, that BC is NOT a diameter.

(5)

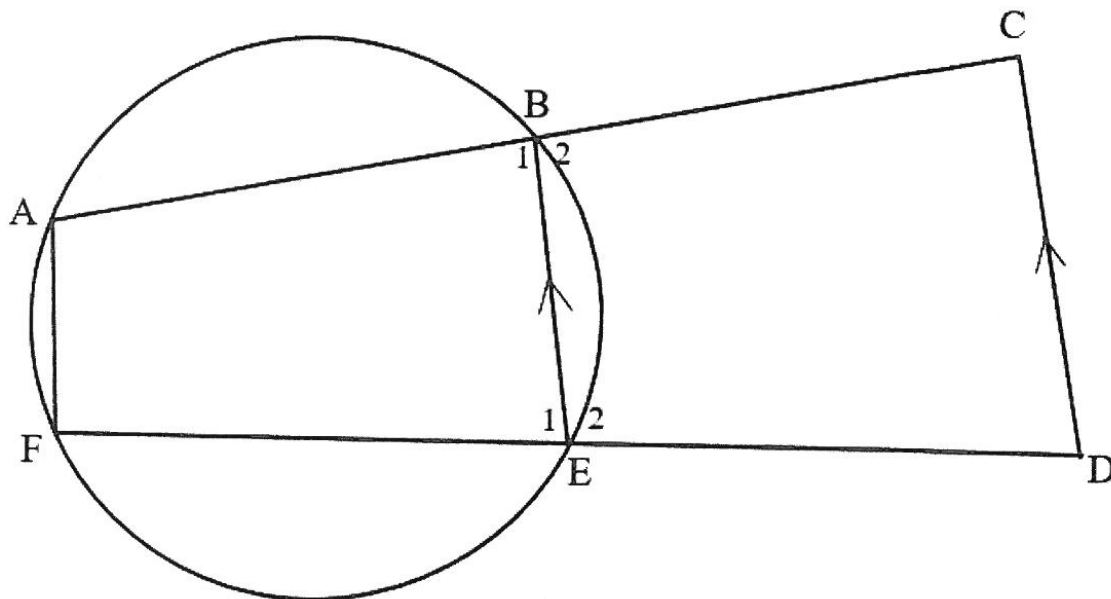
- 9.2 In the diagram below, O is the centre of circle ABC . The tangent PA to the circle and diameter AB meets at A . OCQ and BCP are straight lines. $PQ \parallel AB$.



Prove, stating reasons, that $PQ = QC$.

(6)

- 9.3 In the diagram below, chords AB and FE of circles with centre O are produced to points C and D . $BE \parallel CD$.

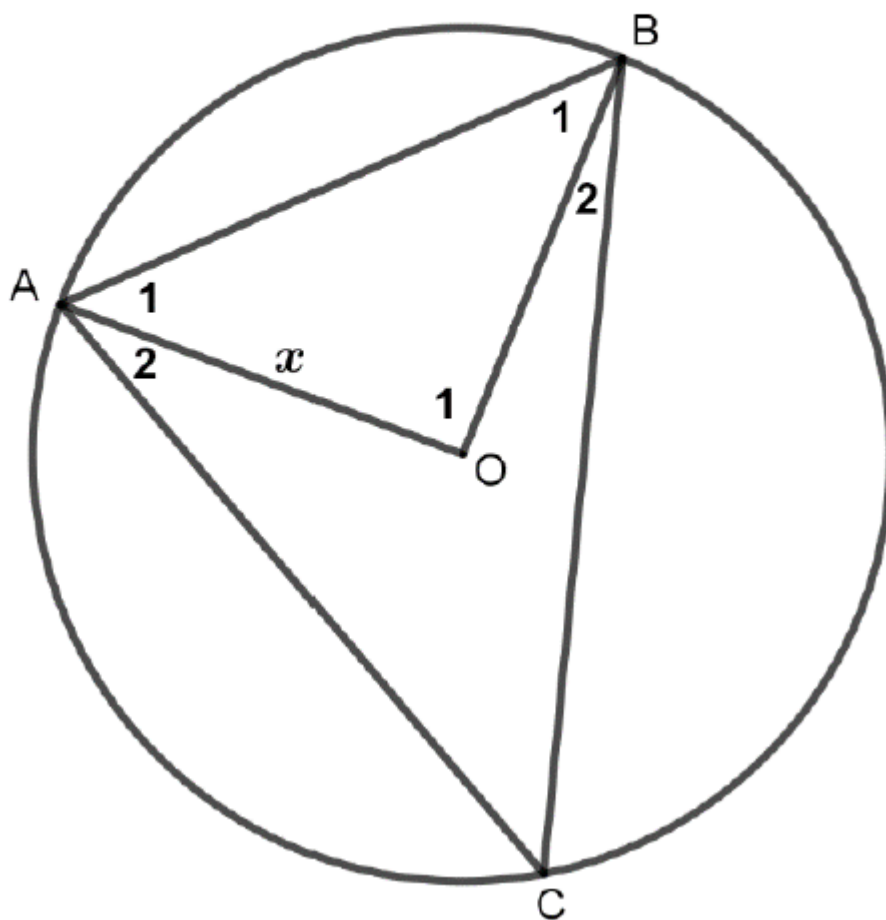


Prove that $ACDF$ is a cyclic quadrilateral.

(5)

QUESTION 9

A, B and C are three points on the circle with centre O such that $AB = BC = \frac{3}{2}AO$.
 $AO = x$.

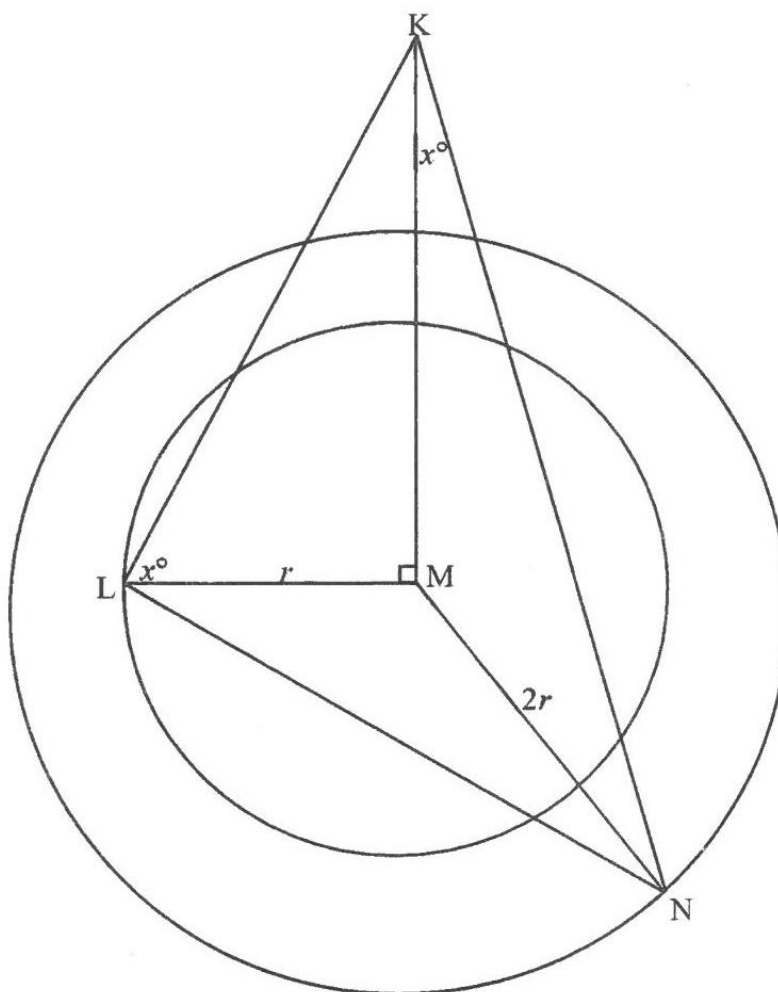


- 9.1 Calculate the size of $\hat{O}_1$ rounded off to the nearest degree. (5)
- 9.2 If $\hat{O}_1 = 97^\circ$ and $x = 10$ cm, calculate the length of AC correct to two decimal places. (4)

GPS20

QUESTION 7

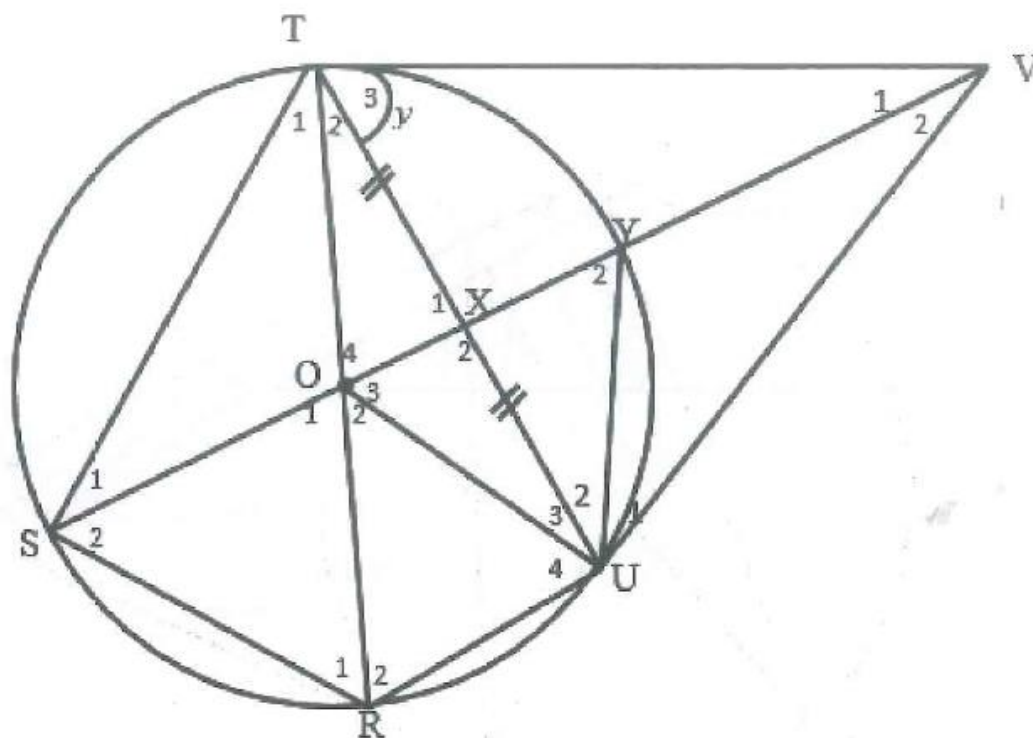
In the figure below, KM is a vertical flag post set in the centre of two circles which lie on the same horizontal plane. $\hat{MKN} = \hat{MLK} = x^\circ$. The radius of the inner circle $ML = r$ units and the radius of the outer circle $MN = 2r$ units.



- 7.1 Calculate the value of x . (6)
- 7.2 If $r = 5\text{ m}$ and $\hat{LMN} = 110^\circ$, calculate the length of LN . (2)
- [8]

LPS20

- 8.2 TV and VU are tangents to the circle with centre O at T and U respectively. TSRUY are points on the circle such that RT is the diameter. X is the midpoint of chord TU. $\hat{T}_3 = y$.



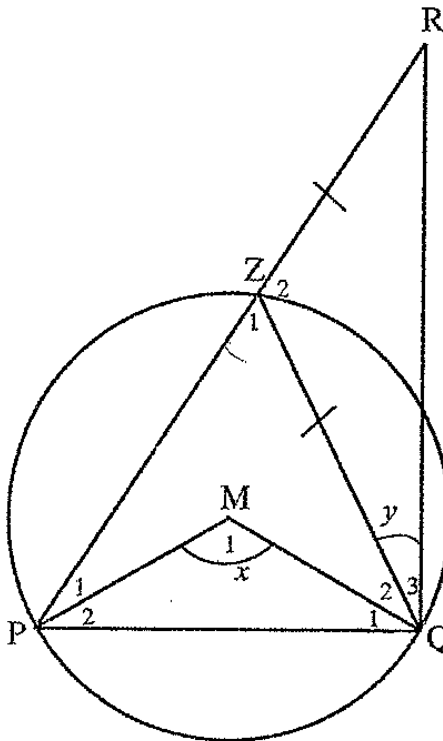
Prove that:

8.2.1 $RU \parallel SY$ (5)

8.2.2 $\hat{T}_1 = \frac{1}{2}y$ (5)

QUESTION 9

M is the centre of the circle. $\hat{M}_1 = x$ and $\hat{Q}_3 = y$. $ZR = ZQ$.



9.1 Prove that $x = 4y$. (5)

9.2 If it is further given that $\hat{P}_2 = \hat{R}$, calculate the value of x and y . (6)

[11]

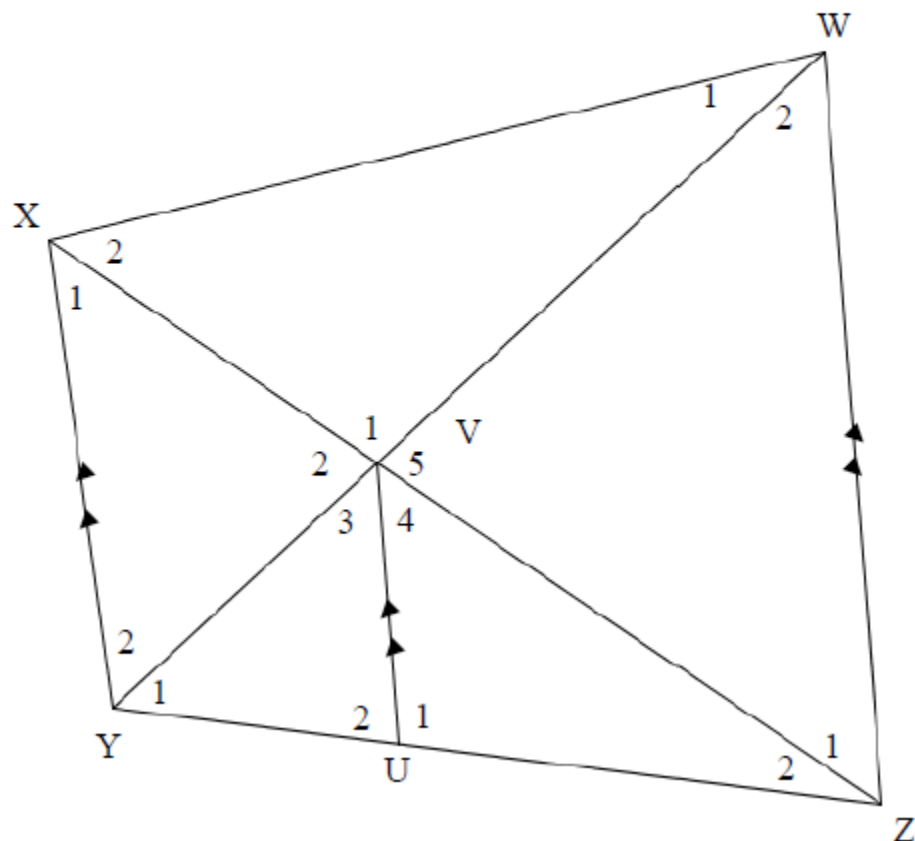
SECTION D

- Mostly level 3 and 4 questions

EC S18

QUESTION 9

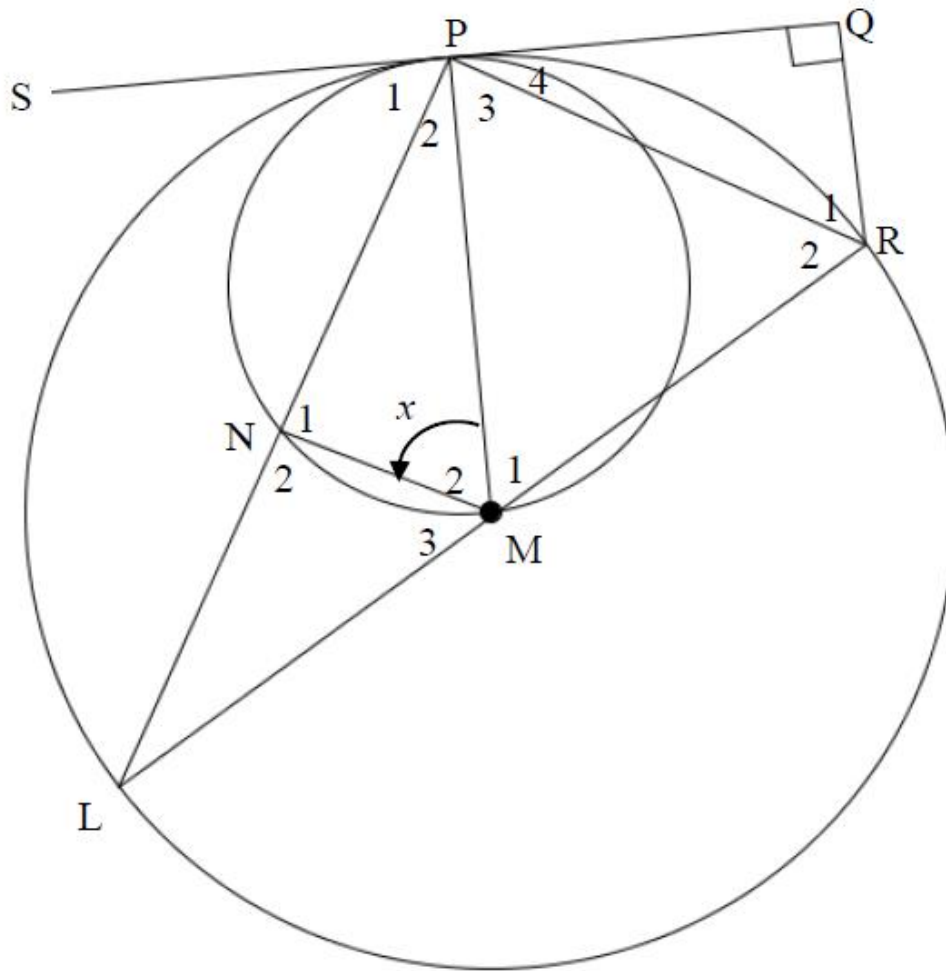
In the diagram below, WXYZ is a trapezium, with $XY \parallel UV \parallel WZ$. $UZ : YZ = 4 : 7$. $\hat{V}_3 = \hat{V}_4$



- 9.1 Prove that $\frac{YV}{VW} = \frac{XV}{VZ}$ (3)
- 9.2 Determine the numerical value of $\frac{\text{Area of } \triangle XVY}{\text{Area of } \triangle WVZ}$ in a simplified form. (4)
- 9.3 Show that WXYZ is a cyclic quadrilateral. (3)
- 9.4 Prove that UV is a tangent to the circle passing through X, V and Y. (2)
- [12]**

EC S18

10.2 SPQ is a common tangent for both circles. M is the centre of the bigger circle. PM is the diameter of the smaller circle. $RQ \perp PQ$. $\hat{PMN} = x$



Prove that:

10.2.1 $LN = NP$ (3)

10.2.2 PR bisects $\hat{QL}$ (4)

10.2.3 $\triangle PNM \parallel \triangle PQR$ (4)

10.3 If it is given that, $PR = 30$ cm and $QR = 15$, calculate the:

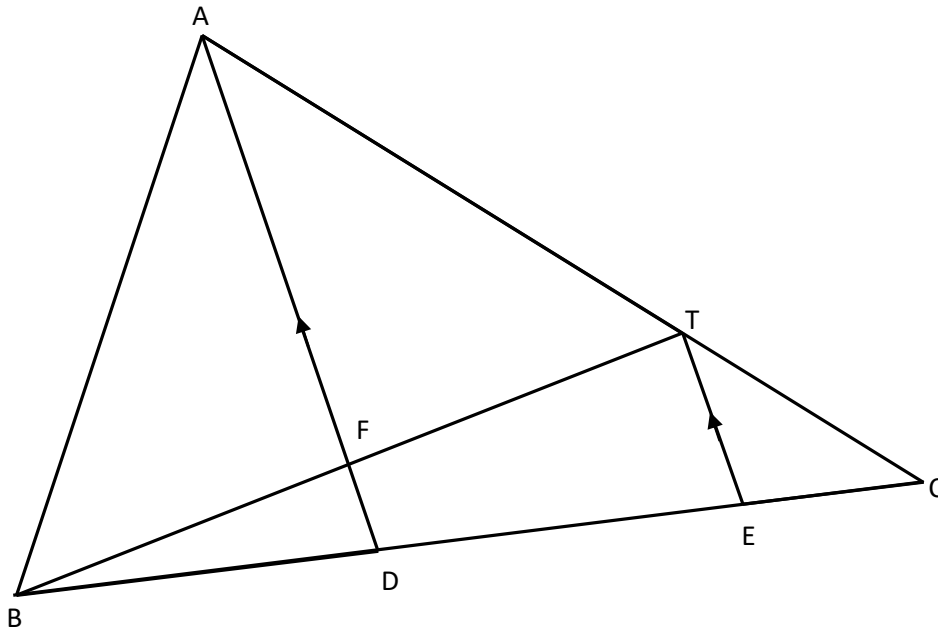
10.3.1 length of LR. (5)

10.3.2 value of x (4)

[26]

FS S18

- 10.2 In the figure below, D is a point on side BC of $\triangle ABC$ such that $BD = 6 \text{ cm}$ and $DC = 9 \text{ cm}$. T and E are points on AC and DC respectively and $TE \parallel AD$ and $AT:TC = 2:1$

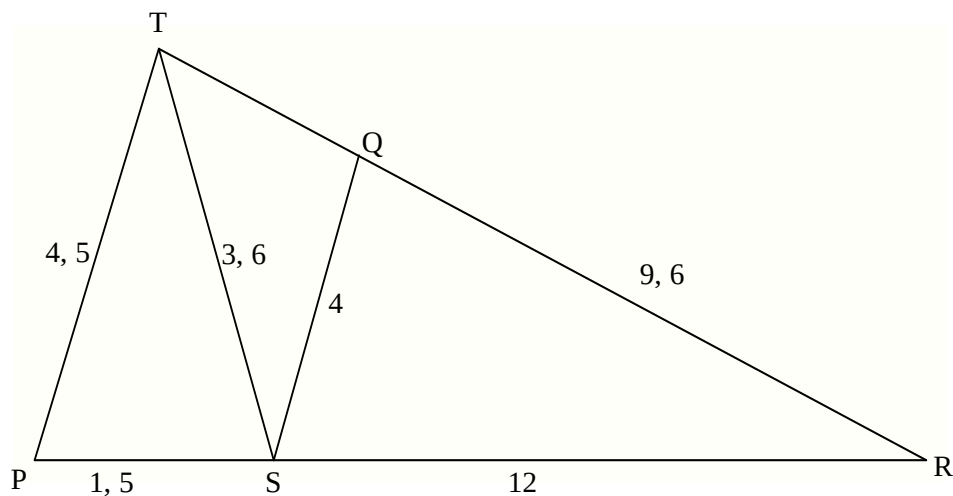


- 10.2.1 Show that D is the midpoint of BE. (3)
- 10.2.2 If $FD = 2 \text{ cm}$, calculate the length of TE. (3)
- 10.2.3 Calculate the numerical value of:
- 10.2.3(a) $\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$ (2)
- 10.2.3(b) $\frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC}$ (3)

[16]

QUESTION 11 FS S18

In the diagram, TPR is a triangle with TP = 4,5 units. Q and S are points on TR and PR respectively. QR = 9,6 units, QS = 4 units, TS = 3,6 units, PS = 1,5 units and SR = 12 units.



11.1 Prove that PT is a tangent to the circle which passes through the points T, S and R.

(7)

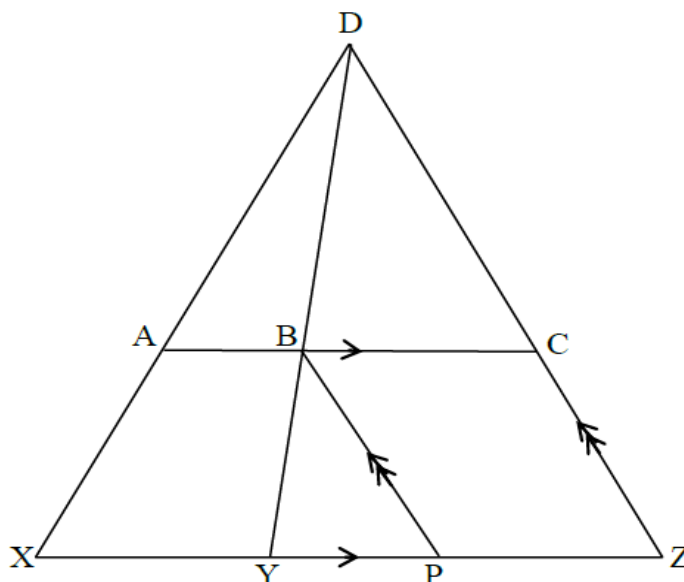
11.2 Calculate the length of TQ.

(5)

[12]

GP S18

10.2 In $\triangle DXZ$ below, $AC \parallel XZ$ and $BP \parallel DZ$. DY is drawn to intersect AC at B.



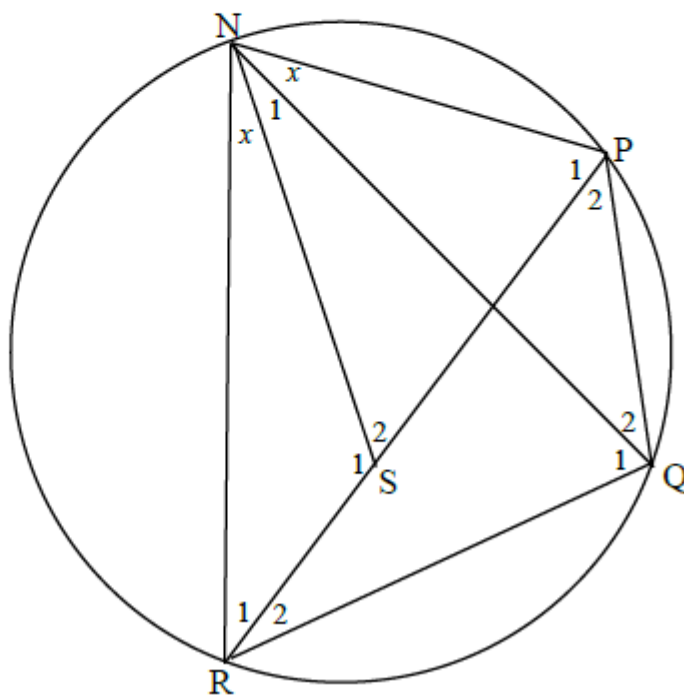
Prove that: $\frac{BC}{YZ} = \frac{DA}{DX}$

(5)

GP S18

QUESTION 11

In the diagram below, NPQR is a cyclic quadrilateral with S a point on chord PR. N and S are joined and $\hat{RNS} = \hat{PNQ} = x$.

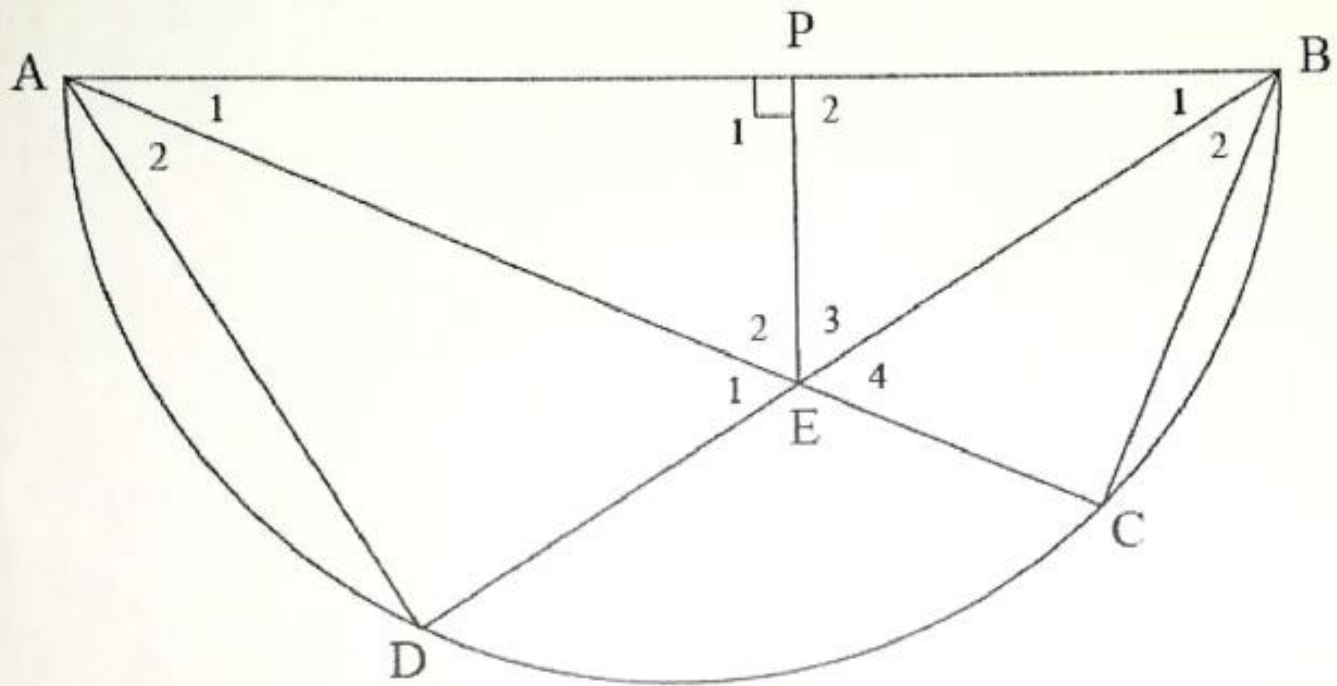


Prove that:

- 11.1 $\triangle NSR \parallel \triangle NPQ$ (3)
 - 11.2 $\triangle NQR \parallel \triangle NPS$ (3)
 - 11.3 $NR \cdot PQ + NP \cdot QR = NQ \cdot PR$ (4)
- [10]**

QUESTION 11

In the diagram below, AB is the diameter of semi-circle ADCB. Chords AC and BD intersect at E. EP is perpendicular to AB.



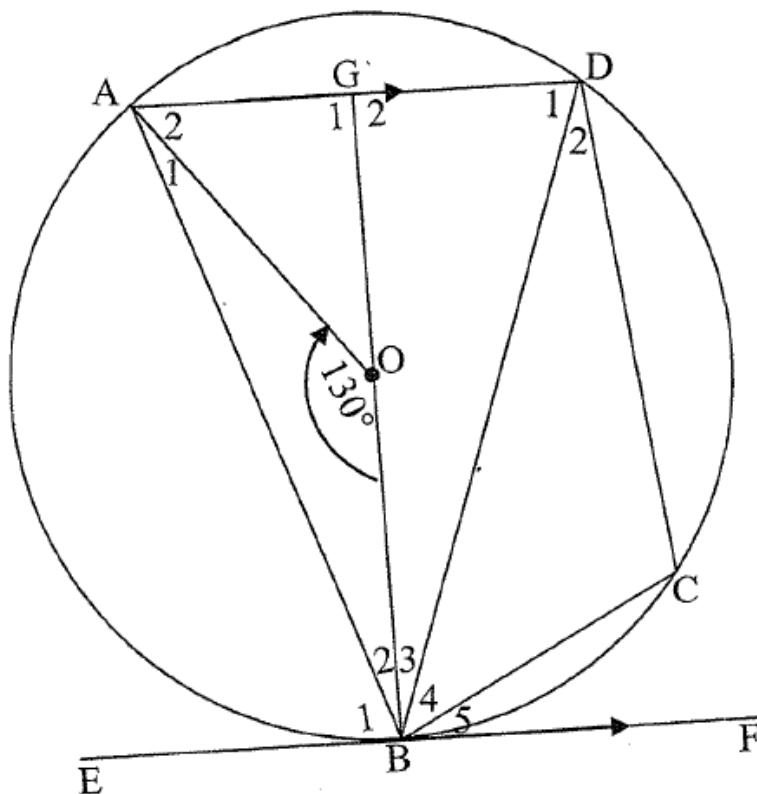
11.1 Prove that $\triangle BPE \parallel \triangle BDA$. (4)

11.2 Prove that $AB^2 = BD^2 + \frac{BD^2 \cdot PE^2}{BP^2}$ (6)

[10]

NW S18

- 8.2 In the diagram below, the circle having centre O, passes through A, B, C and D, with $\hat{AOB} = 130^\circ$. EBF is a tangent to the circle at B with $EF \parallel AD$. BOG is a straight line.



Calculate, with reasons, the size of:

- 8.2.1 $\hat{D}_1$ (2)
- 8.2.2 $\hat{B}_1$ (2)
- 8.2.3 $\hat{BAD}$ (1)
- 8.2.4 $\hat{C}$ (2)
- 8.2.5 $\hat{B}_3$ (3)
- 8.2.6 Calculate the length of GD, if $AD = \frac{\sqrt{7}}{2}$ units. (3)

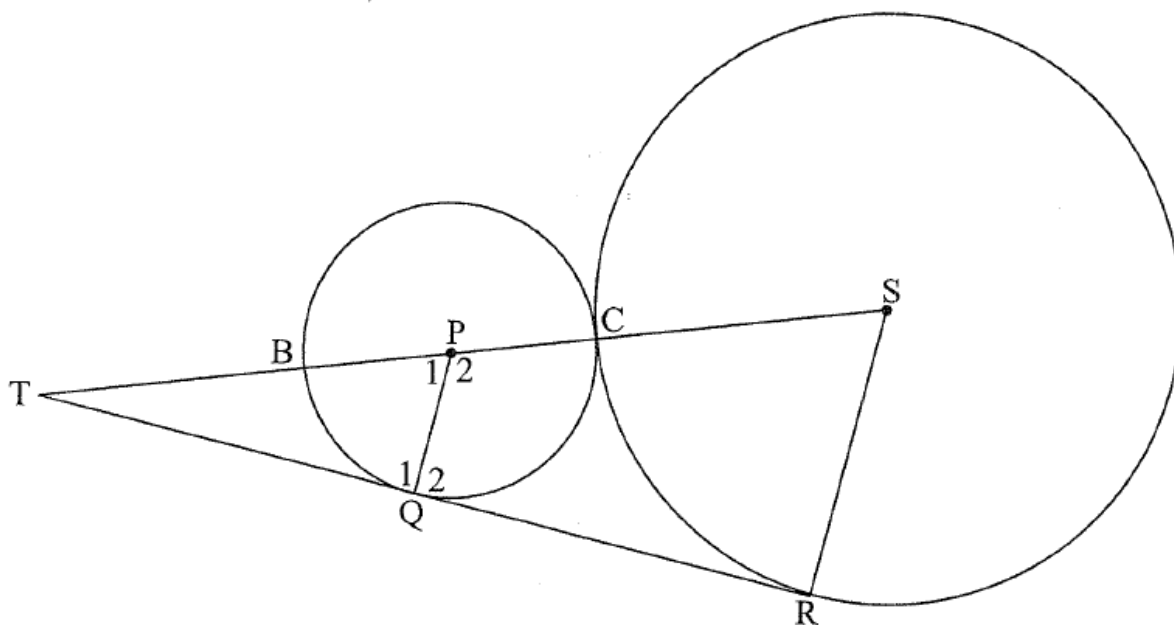
[18]

NW S18
QUESTION 10

10.1 Complete the following theorem to make the statement TRUE:

If a line divides two sides of a triangle in the same proportion, then the line is ... (1)

10.2 Two circles with centres P and S touch each other externally at C. SP produced intersects circle P at B. A common tangent at R and Q meets SB produced at T.



Prove that:

10.2.1 $PQ \parallel SR$ (4)

10.2.2 $TP = \frac{TQ(BP + SR)}{QR}$ (4)

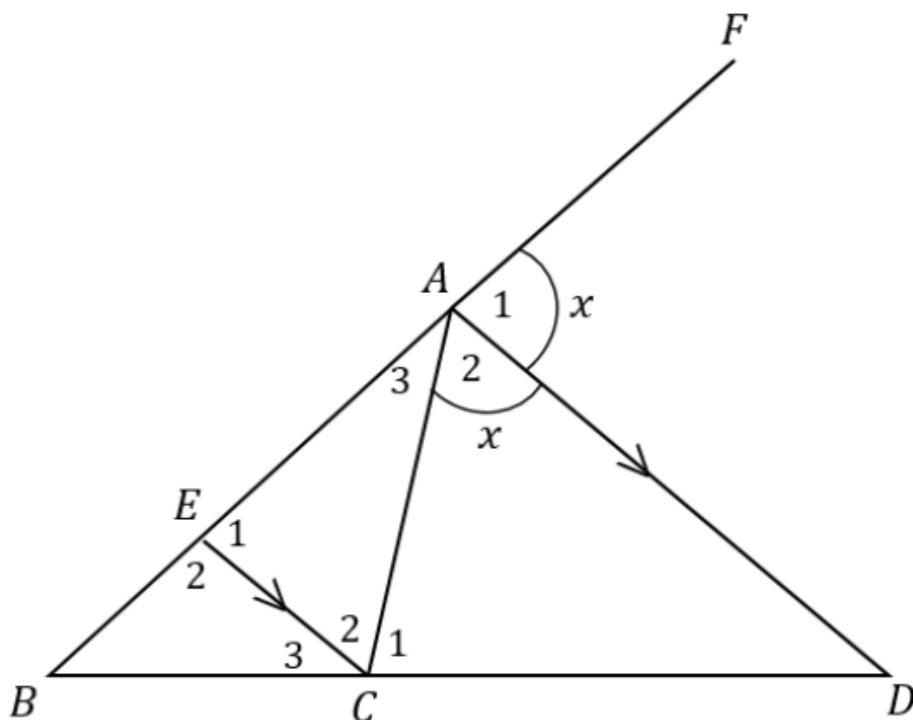
10.2.3 $\Delta TQP \parallel \Delta TRS$ (3)

10.2.4 $\sqrt{TS^2 - CS^2} = \frac{\sqrt{(TP^2 + BP^2 - 2TP \cdot BP \cos S)} \cdot CS}{BP}$ (6)

[18]

WCS18

- 10.2 In $\triangle ABD$, BA is produced to F. AC is drawn with C on BD such that $\widehat{A_1} = \widehat{A_2} = x$. $CE \parallel DA$ where E is on AB.



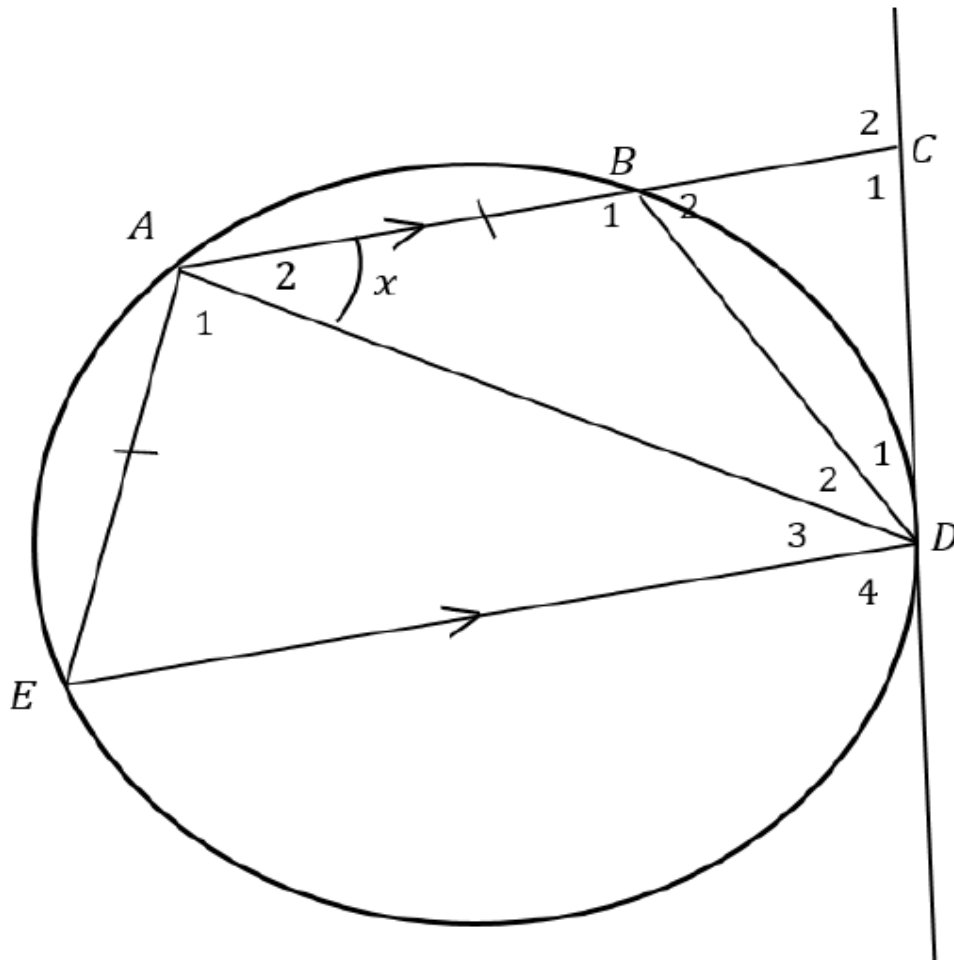
10.2.1 Give two more angles equal to x . (3)

10.2.2 Prove that $\frac{BD}{DC} = \frac{BA}{AC}$. (4)

WCS18
QUESTION 11

In the diagram, a circle with tangent CD is drawn. A , B , D and E are points on the circumference of the circle. $AE = AB$ and AB is parallel to ED .

$$\hat{A}_2 = x$$

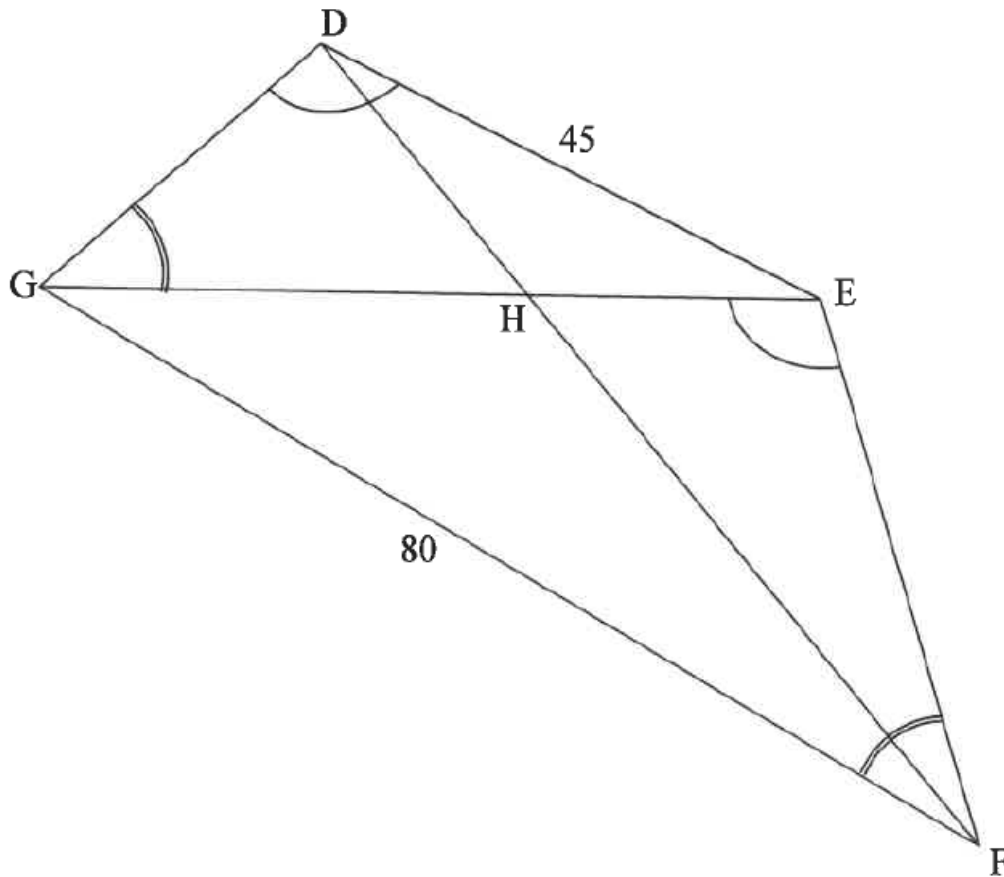


- 11.1 Give, with reasons, three more angles equal to x . (6)
- 11.2 Prove that $\triangle DEA \parallel \triangle DBC$ (4)
- 11.3 Prove that $BC \cdot ED = AE^2$. (5)
- 11.4 Calculate the value of x if it is given that $\hat{A}_1 = 75^\circ$. (3)

[18]

NM18
QUESTION 9

In the diagram, DEFG is a quadrilateral with $DE = 45$ and $GF = 80$. The diagonals GE and DF meet in H. $\hat{GDE} = \hat{FEG}$ and $\hat{DGE} = \hat{EFG}$.

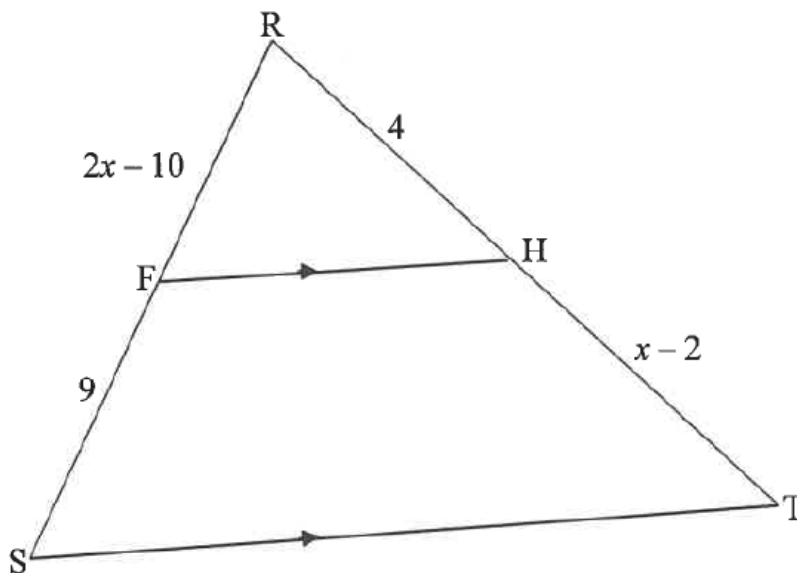


- 9.1 Give a reason why $\triangle DEG \parallel \triangle EGF$. (1)
 - 9.2 Calculate the length of GE. (3)
 - 9.3 Prove that $\triangle DEH \parallel \triangle FGH$. (3)
 - 9.4 Hence, calculate the length of GH. (3)
- [10]**

NJ18

9.2

In $\triangle RST$, F is a point on RS and H is a point on RT such that $FH \parallel ST$.
 $RF = 2x - 10$, $FS = 9$, $RH = 4$ and $HT = x - 2$.

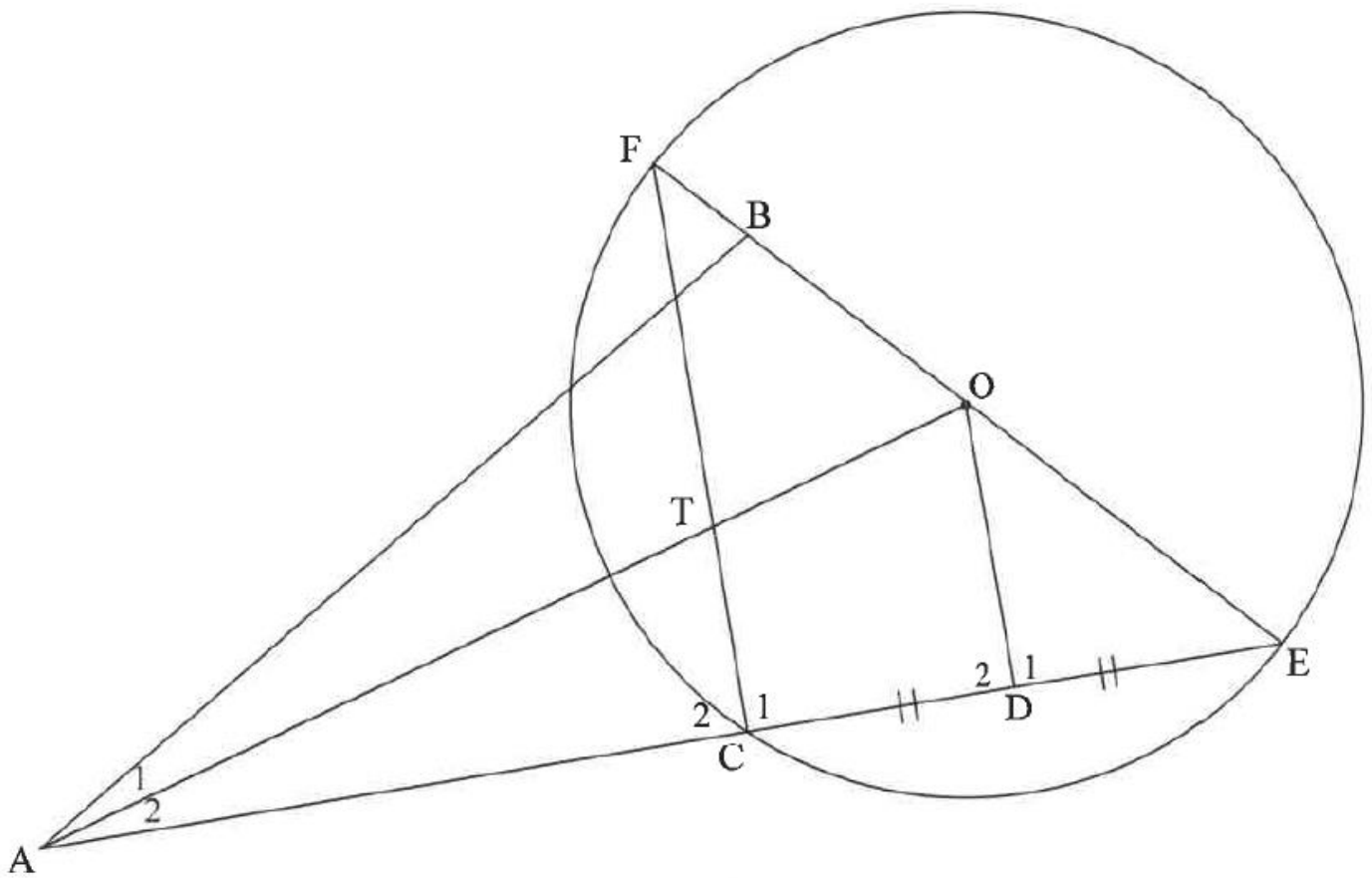


9.2.1 Determine, giving a reason, the value of x . (5)

9.2.2 Determine the ratio: $\frac{\text{area } \triangle RFH}{\text{area } \triangle RST}$. (4)

QUESTION 10

In the diagram, $FBOE$ is a diameter of a circle with centre O . Chord EC produced meets line BA at A , outside the circle. D is the midpoint of CE . OD and FC are drawn. $AFBC$ is a cyclic quadrilateral.



10.1 Prove, giving reasons, that:

10.1.1 $FC \parallel OD$ (5)

10.1.2 $\hat{DOE} = \hat{BAE}$ (4)

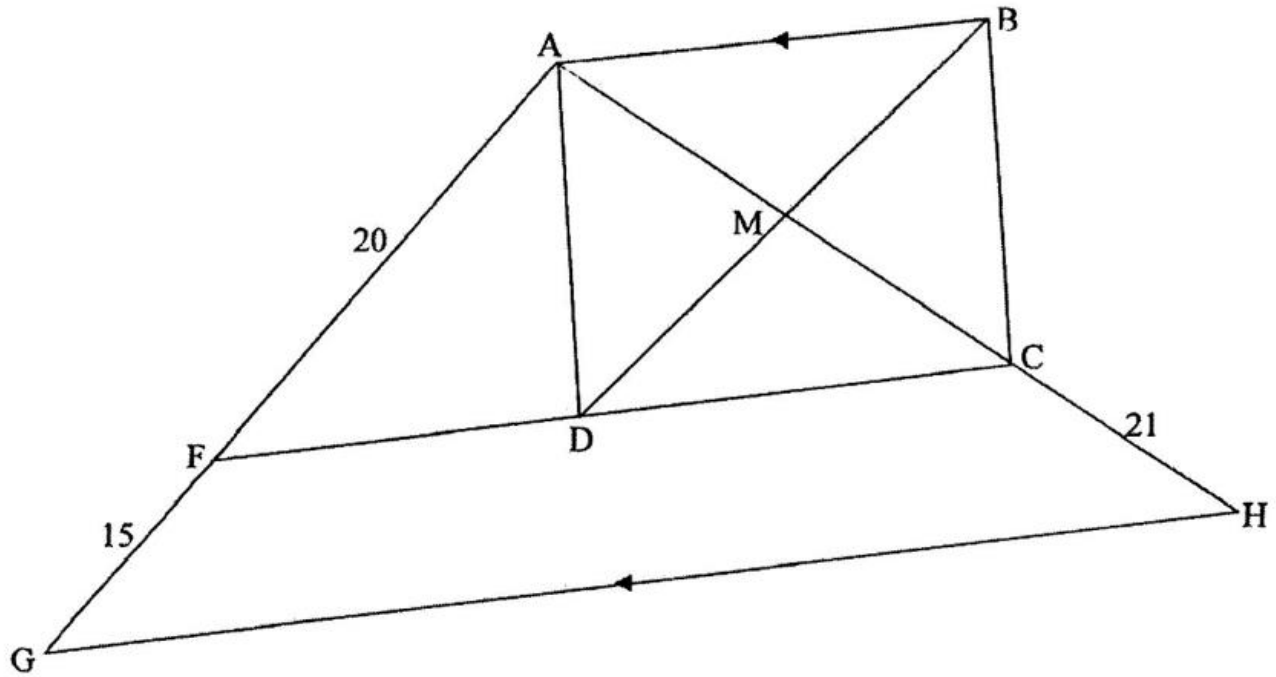
10.1.3 $AB \times OF = AE \times OD$ (7)

10.2 If it is further given that $AT = 3TO$, prove that $5CE^2 = 2BE \cdot FE$ (5)
[21]

NN18

8.2

In the diagram, $\triangle AGH$ is drawn. F and C are points on AG and AH respectively such that $AF = 20$ units, $FG = 15$ units and $CH = 21$ units. D is a point on FC such that $ABCD$ is a rectangle with AB also parallel to GH . The diagonals of $ABCD$ intersect at M , a point on AH .

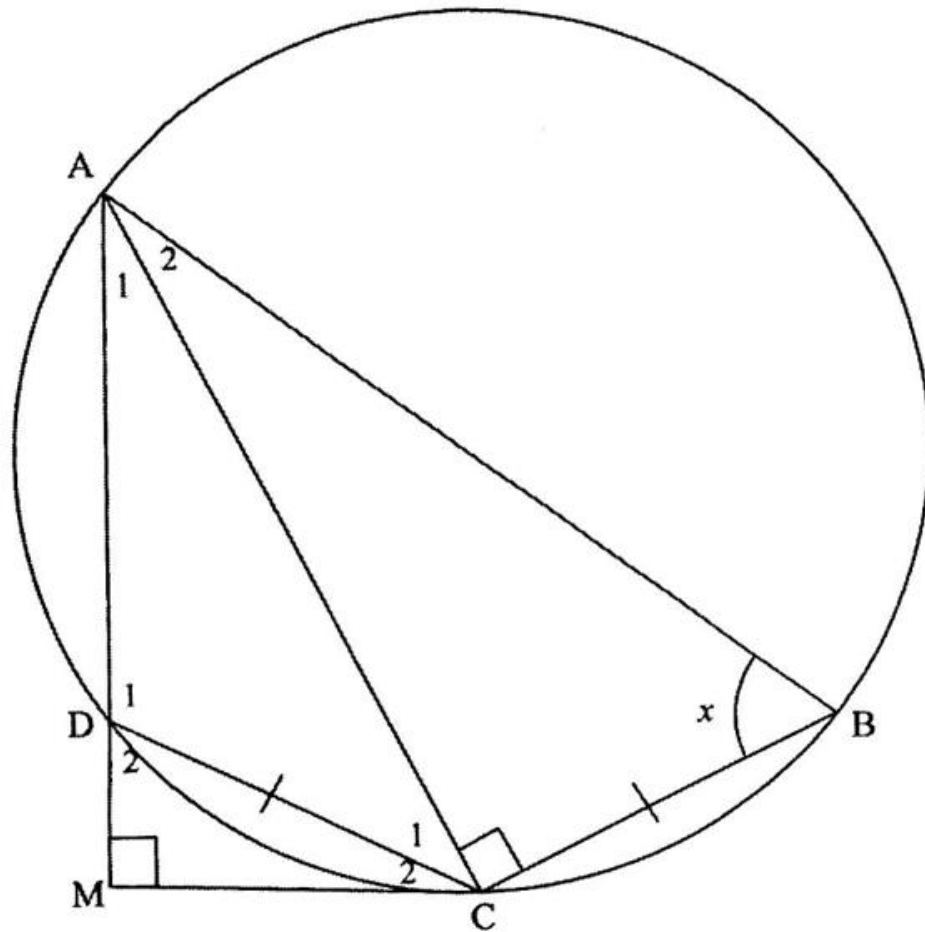


8.2.1 Explain why $FC \parallel GH$. (1)

8.2.2 Calculate, with reasons, the length of DM . (5)

QUESTION 10

In the diagram, $ABCD$ is a cyclic quadrilateral such that $AC \perp CB$ and $DC = CB$. AD is produced to M such that $AM \perp MC$. Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C

(5)

10.1.2 $\triangle ACB \parallel \triangle CMD$

(3)

10.2 Hence, or otherwise, prove that:

10.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$

(6)

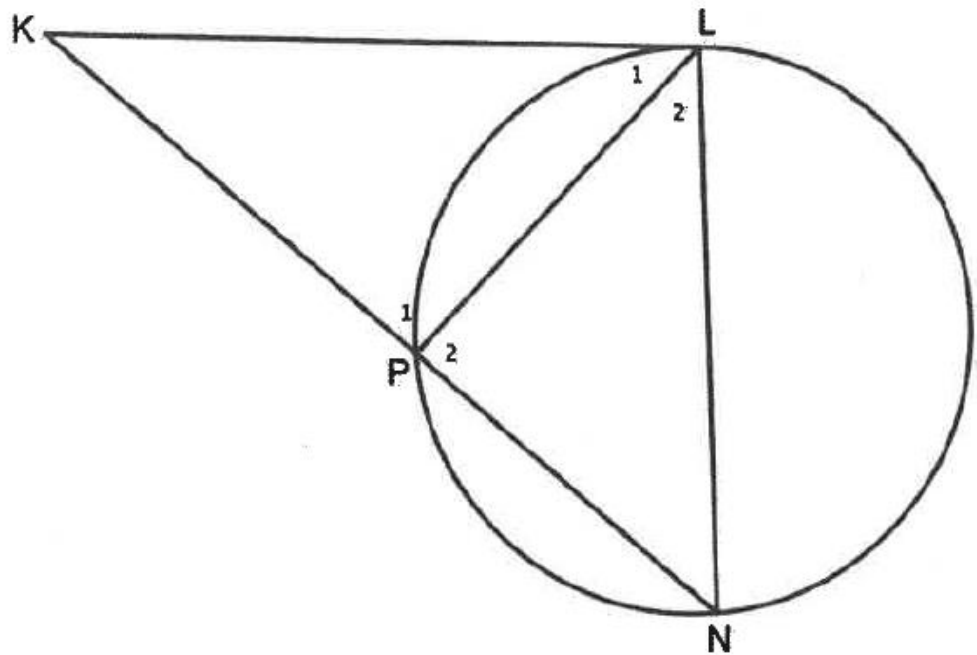
10.2.2 $\frac{AM}{AB} = \sin^2 x$

(2)

[16]

KZNJ19

9.2 In the figure, LN is the diameter of the circle. KL is the tangent to the circle at L.



9.2.1 Prove that $\triangle LPK \sim \triangle NPL$ (4)

9.2.2 Show that $PL^2 = NP \cdot PK$ (3)

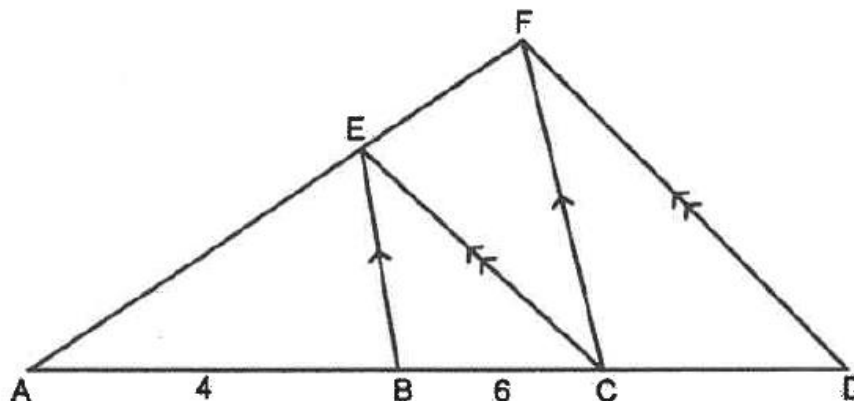
9.2.3 Name another triangle which is similar to $\triangle NPL$ (1)

9.2.4 Calculate the area of the circle if $NP = 10$ cm and $PK = 6$ cm (6)

[23]

QUESTION 10

In $\triangle ADF$, $DF \parallel CE$ and $CF \parallel BE$. If $AB = 4$ units and $BC = 6$ units, then calculate



10.1 the length of CD (3)

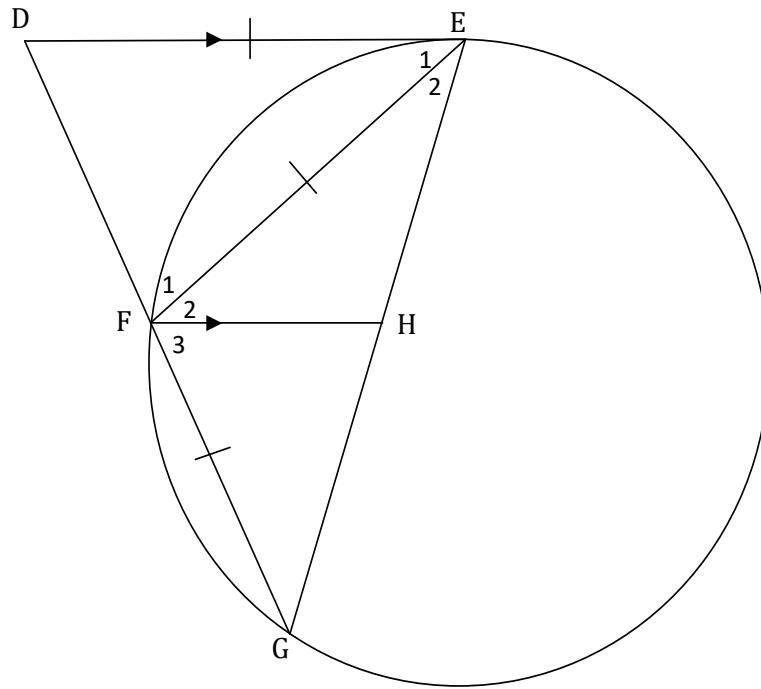
10.2 the numerical value of

$$\frac{\text{area of } \triangle FEC}{\text{area of } \triangle FAD} \quad (4)$$

[7]

KZNS19

10.2 In the diagram, DE is a tangent to the circle at E and DFG is a secant intersecting the circle at F and G. $DE = EF = FG$. H is a point on EG such that $FH \parallel DE$.



10.2.1 Determine, giving reasons, 3 angles each equal to $\hat{DEF}$. (4)

10.2.2 Prove that:

a) $\triangle DEF \sim \triangle DGE$ (3)

b) $\hat{D} = 72^\circ$. (5)

10.2.3 If it is further given that $DF = k$ units and $FG = 2$ units, prove that $k^2 + 2k = 4$. (3)

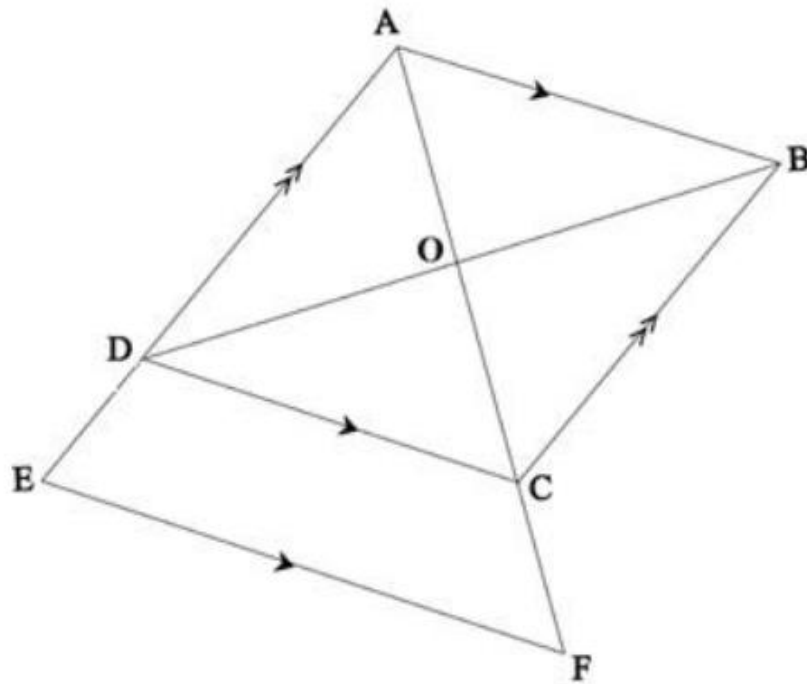
10.2.4 Determine, giving reasons, the ratio of $\frac{GH}{GE}$ in terms of k . (2)

[24]

QUESTION 9

In the diagram below, ABCD is a parallelogram. AD and AC are produced to E and F respectively so that $EF \parallel DC$. AF and DB intersect at O.

AD = 12 units; DE = 3 units; DC = 14 units; CF = 5 units.



9.1 Calculate, giving reasons, the length of:

9.1.1 AC (3)

9.1.2 AO (1)

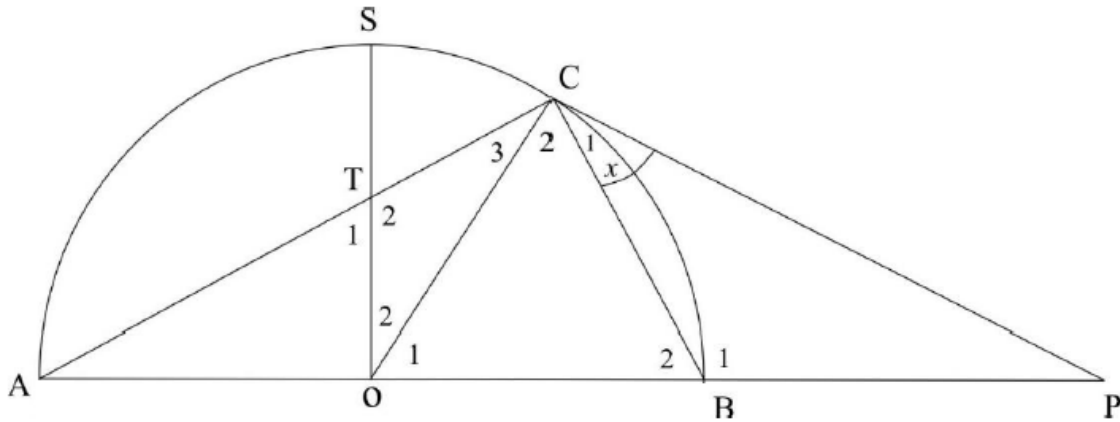
9.1.3 EF (3)

9.2 Prove that $\frac{\text{area } \triangle ADO}{\text{area } \triangle AEF} = \frac{8}{25}$ (3)

[10]

ECS19

- 10.2 In the diagram below, O is the centre of a semi-circle ACB. S is a point on the circumference and T lies on AC such that $STO \perp AB$. Diameter AB is produced to P, such that PC is a tangent to the semi-circle at C. Let $\hat{C}_1 = x$.



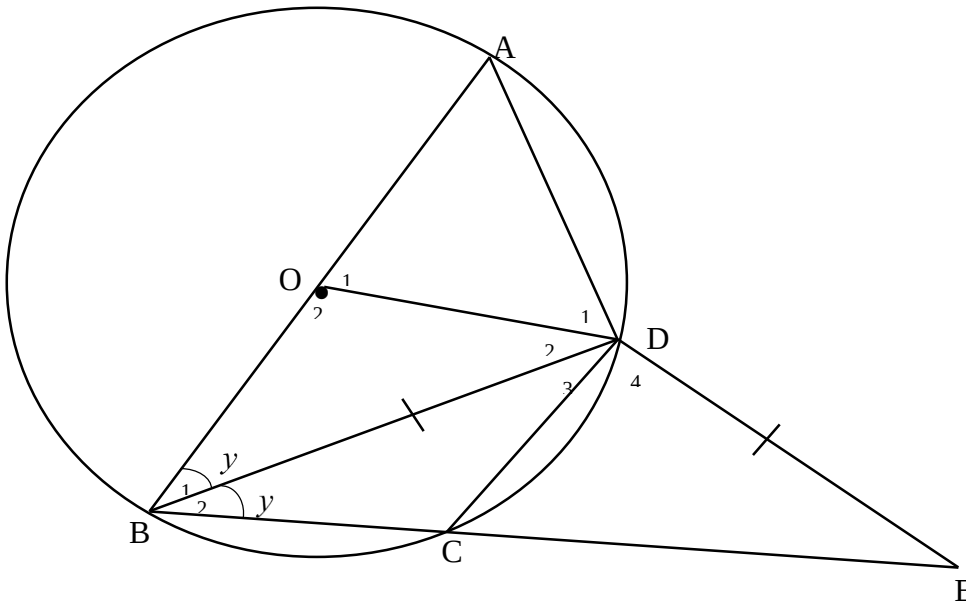
- 10.2.1 Write down, with reasons, 2 other angles equal to x . (3)
- 10.2.2 Prove that $\triangle TOC \parallel \triangle BPC$ (5)
- 10.2.3 Prove that $TO \cdot PC = OB \cdot BP$ (2)
- 10.2.4 If $BP = OB$, show that $3OC^2 = PC^2$ (3)

[19]

FSS19

10.2

In the diagram below, AB is the diameter of a circle with centre O. BD and BC are chords. $BD = DE$. BCE is a line. $\hat{B}_1 = \hat{B}_2 = y$.



Prove that:

$$10.2.1 \quad \hat{D}_4 = 90^\circ \quad (5)$$

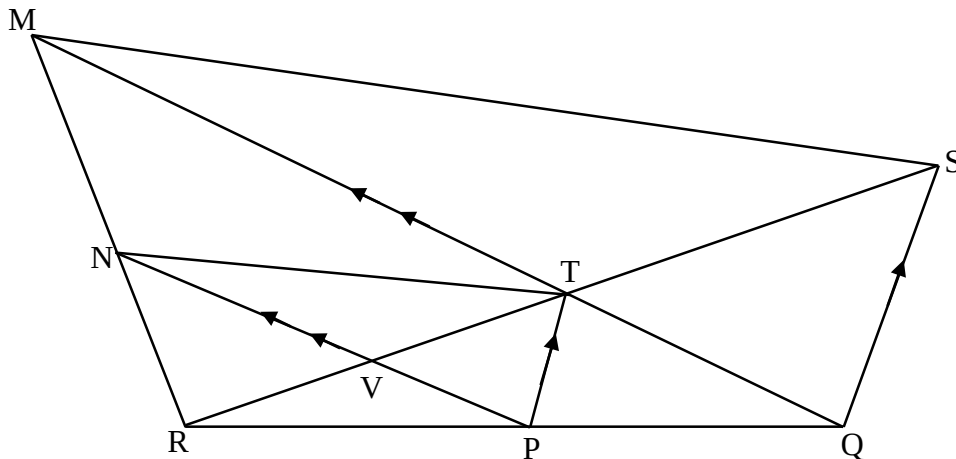
$$10.2.2 \quad \triangle BOD \parallel \triangle BDE \quad (3)$$

$$10.2.3 \quad DE^2 = BE \cdot OD \quad (4)$$

[17]

QUESTION 11 FSS19

In the diagram below, RQSM is a quadrilateral. N and P are points on MR and RQ respectively such that $MN \parallel RP$. The diagonals intersect at T. P is a point on RQ such that $TP \parallel SQ$. TR and NP intersect at V.



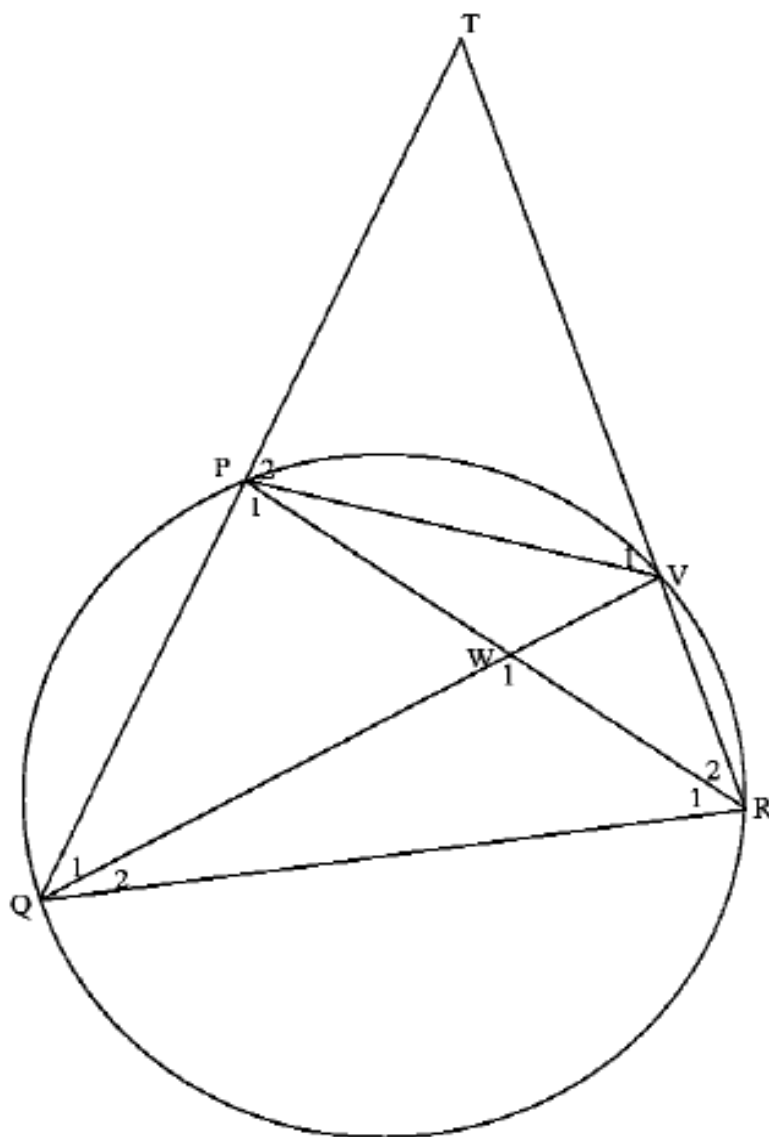
$$11.1 \quad \text{Prove that } NT \parallel MS. \quad (4)$$

$$11.2 \quad \text{If } RN = \frac{3}{5} NM \text{ and } RS = 32, \text{ determine } VT. \quad (4)$$

GPS19

10.2

In the diagram below, ΔPQR is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T . PR and QV intersect at W .



Prove, giving reasons, that:

10.2.1 $\hat{W}_1 = \hat{T}RQ$ (3)

10.2.2 $\Delta TQR \parallel \Delta QRW$ (3)

10.2.3 $\frac{PT}{QW} = \frac{PV}{WR}$ (6)

[17]

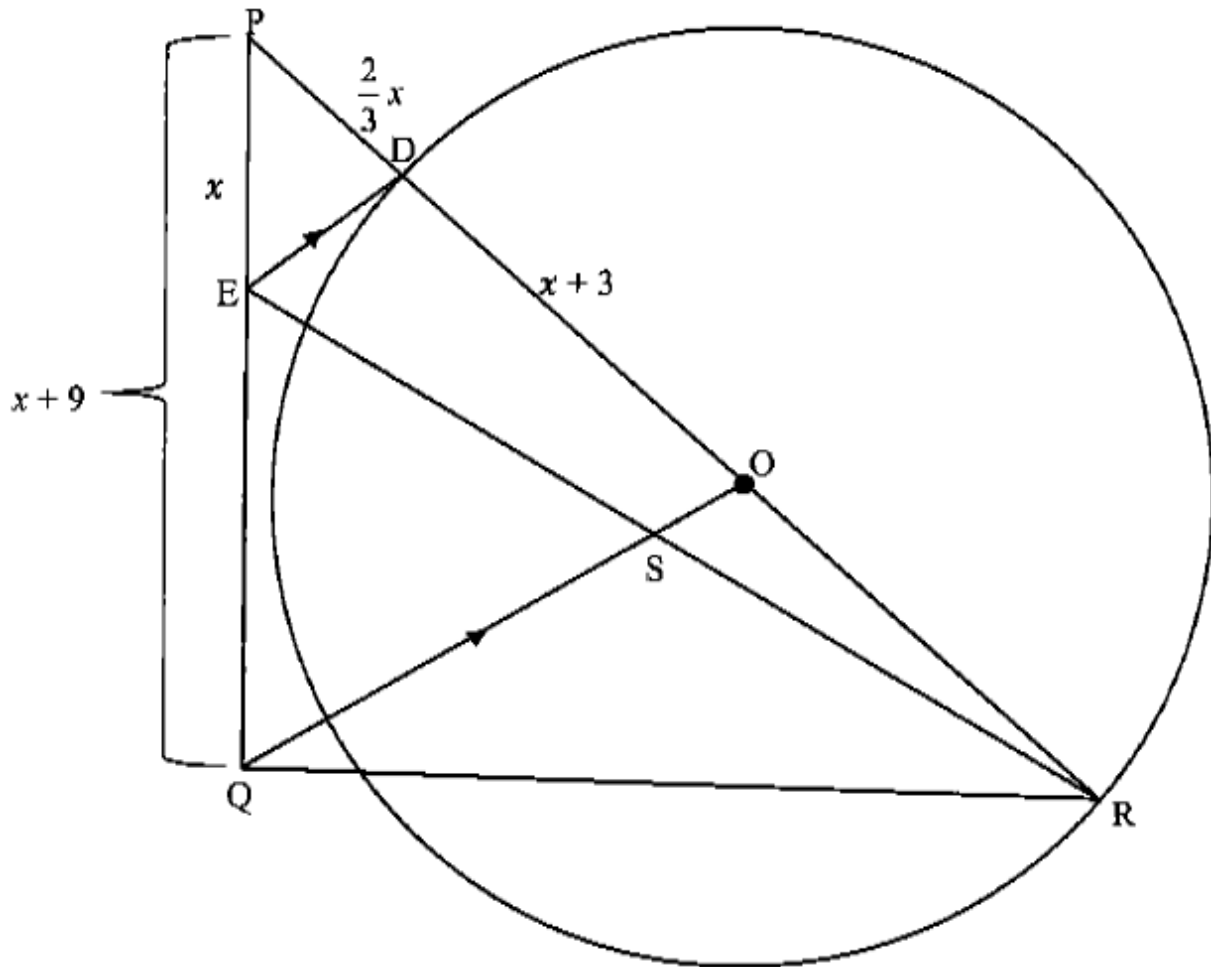
GPS19

QUESTION 11

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ.

RD is produced to P and PQ is joined.

$PE = x$ units, $PQ = x + 9$ units, $PD = \frac{2}{3}x$ units and $DO = x + 3$ units.

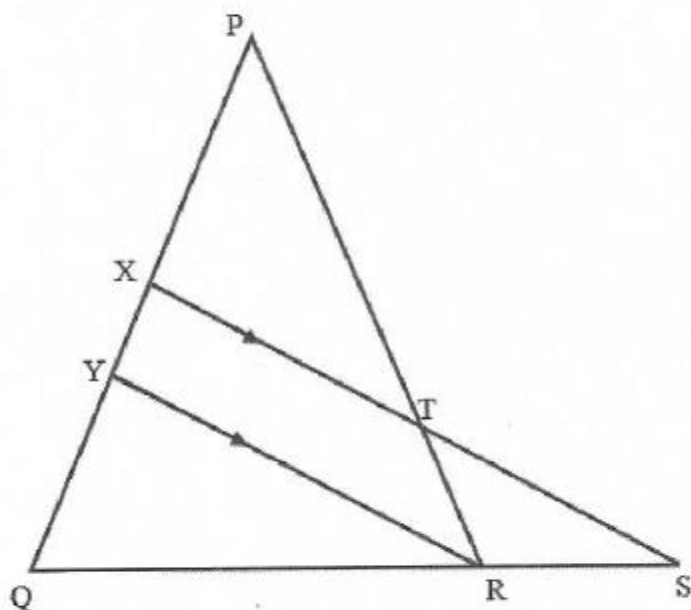


- 11.1 Calculate the length of RO. (4)
- 11.2 If $OS = 1,4$ units and S is the midpoint of ER, determine the length of DE. (2)
- 11.3 If the area of $\triangle PED = 2,7 \text{ units}^2$, find the area of $\triangle PER$. (4)
- [10]

LPS19

9.2 In the diagram QR of ΔPQR is produced such that $XS \parallel YR$.

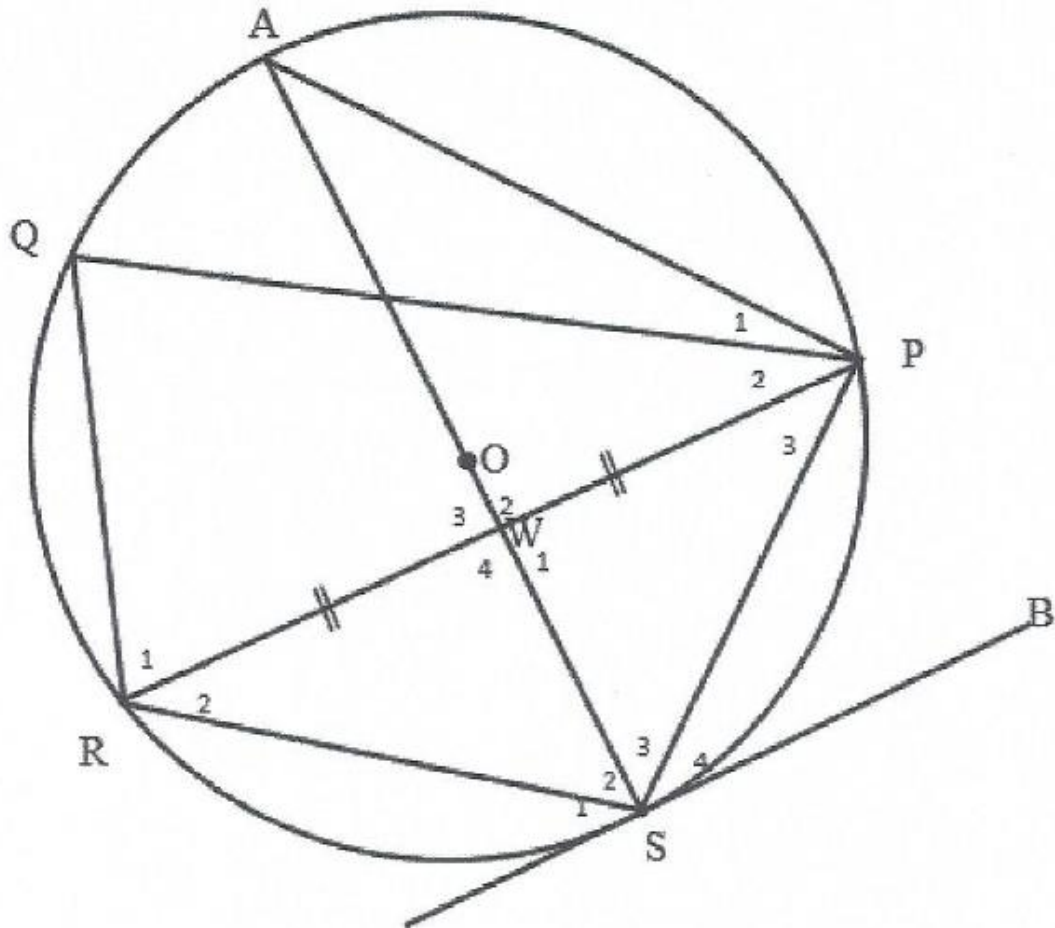
$$\frac{PX}{XQ} = \frac{2}{5} \text{ and } \frac{PT}{TR} = \frac{3}{2}.$$



Calculate, giving reasons, the value of $\frac{QR}{RS}$. (6)

LPS19

10.2 P, A, Q, R and S lie on the circle with centre O. SB touches the circle at S and $RW = WP$. AS and RP are straight lines.



Prove that:

$$10.2.1 \quad SB \parallel RP \quad (5)$$

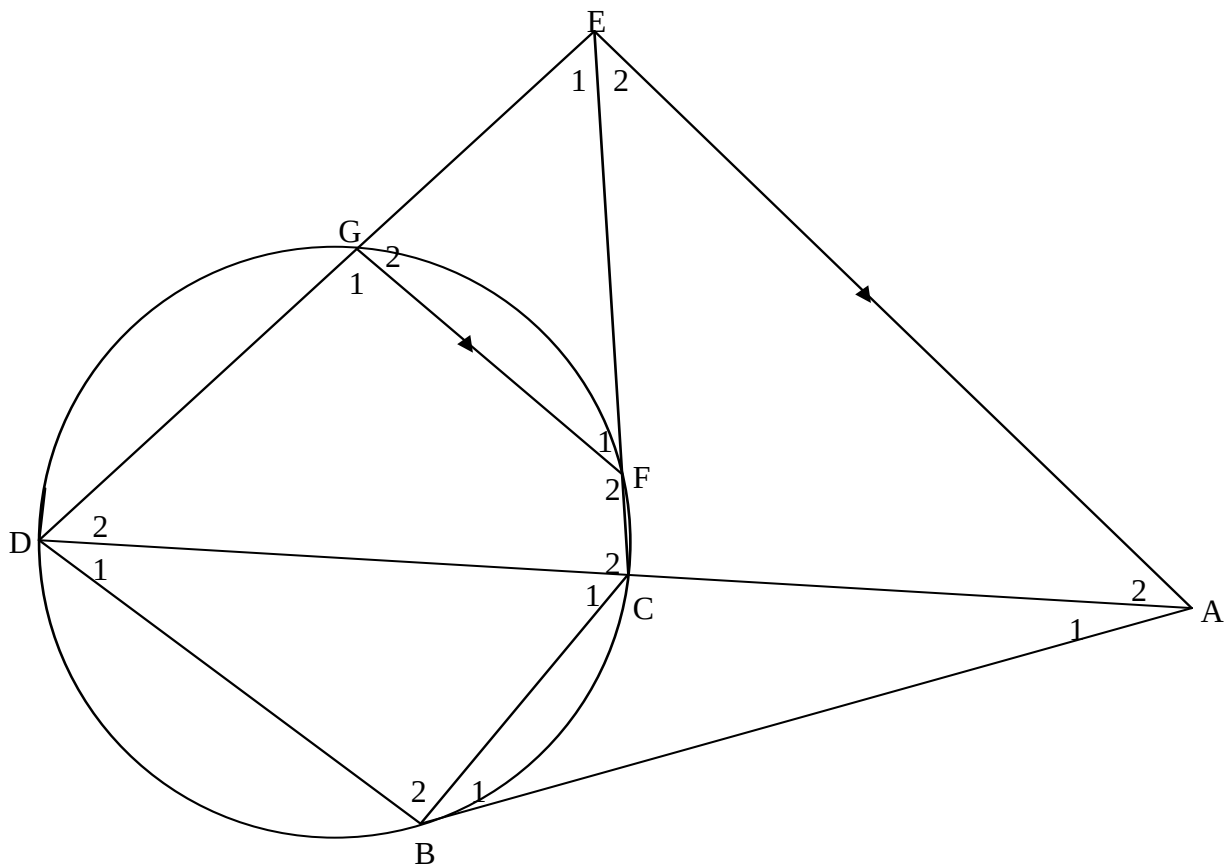
$$10.2.2 \quad \triangle APS \sim \triangle RWS \quad (4)$$

$$10.2.3 \quad RS^2 = WS \cdot AS \quad (4)$$

$$10.2.4 \quad AS = \frac{RW^2}{WS} + WS \quad (4)$$

QUESTION 9 NWS19

In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG produced and CF produced meet in E and DC is produced to A. $EA \parallel GF$



9.1 Give a reason why $\hat{B}_1 = \hat{D}_1$. (1)

9.2 Prove $\triangle ABC \parallel \triangle ADB$. (3)

9.3 Prove $\hat{E}_2 = \hat{D}_2$ (4)

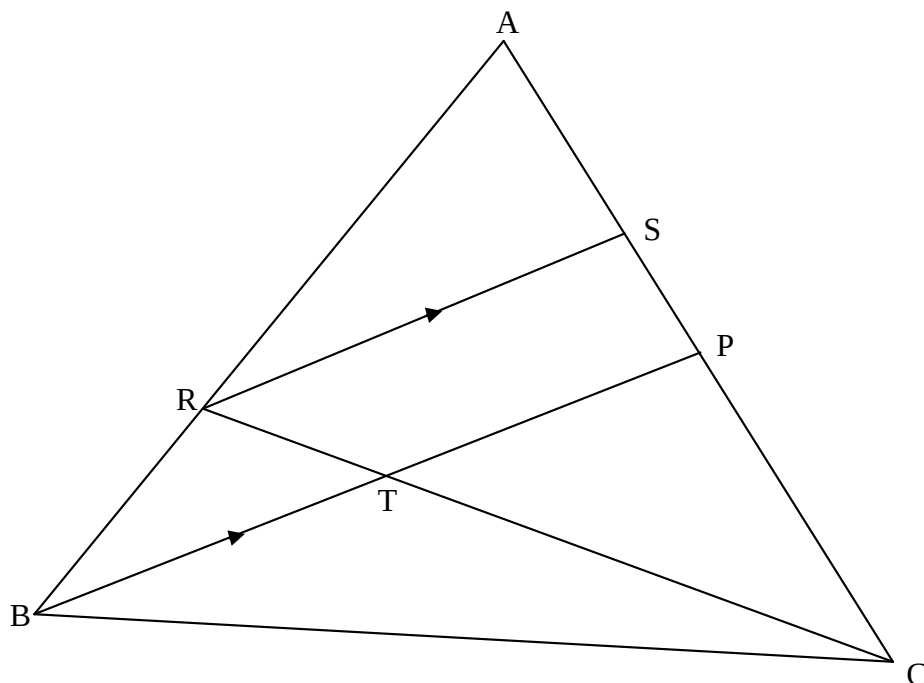
9.4 Prove $AE = \sqrt{AD \times AC}$. (5)

9.5 Hence, show that $AE = AB$. (3)

[16]

NWS19

- 10.2 In the diagram below, P is the midpoint of AC in $\triangle ABC$. R is a point on AB such that $RS \parallel BP$ and $\frac{AR}{AB} = \frac{3}{5}$. RC intersects BP in T.



Determine, with reasons, the following ratios:

10.2.1 $\frac{AS}{SC}$ (4)

10.2.2 $\frac{RT}{TC}$ (3)

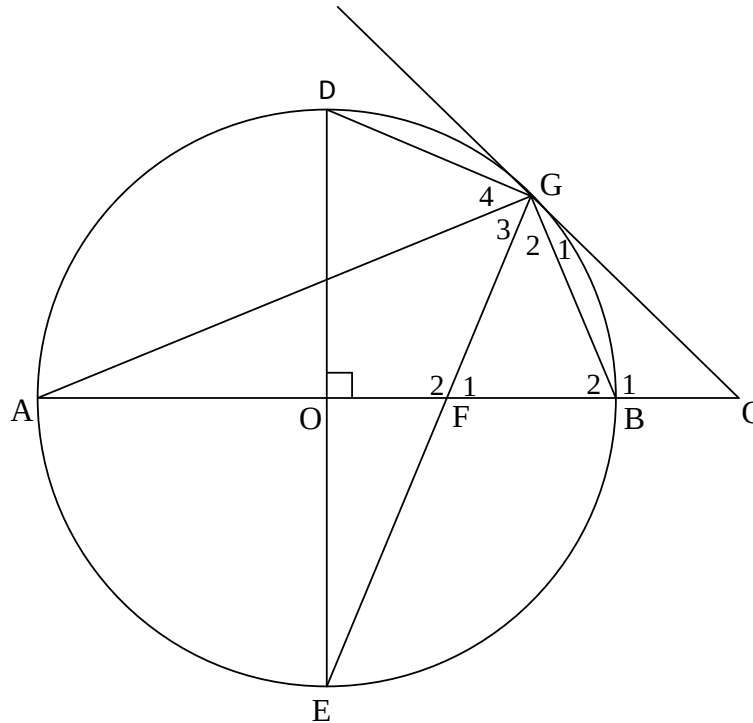
10.2.3 $\frac{\text{Area of } \triangle RAS}{\text{Area of } \triangle RSC}$ (2)

10.2.4 $\frac{\text{Area of } \triangle TPC}{\text{Area of } \triangle RSC}$ (3)

[18]

NJ19

- 10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G . The straight line from C passing through O cuts the circle at A and B . Diameter DOE is perpendicular to CA . GE and CA intersect at F . Chords DG , BG and AG are drawn.



10.2.1 Prove that:

(a) $DGFO$ is a cyclic quadrilateral (3)

(b) $GC = CF$ (5)

10.2.2 If it is further given that $CO = 11$ units and $DE = 14$ units, calculate:

(a) The length of BC (3)

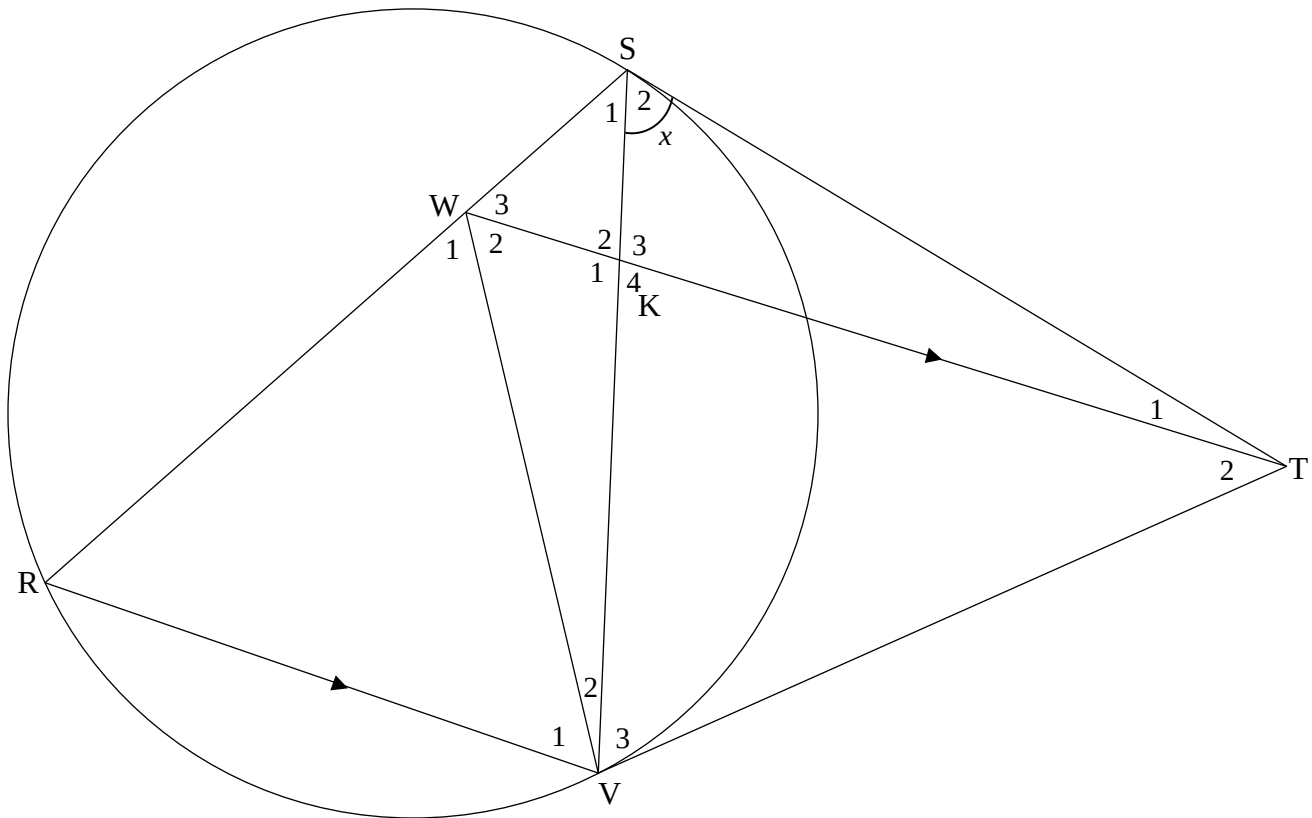
(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)

NN19

10.2

In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV . SV and WV are drawn. WT intersects SV at K . Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x . (6)

10.2.2 Prove, with reasons, that:

(a) $WSTV$ is a cyclic quadrilateral (2)

(b) $\triangle WRV$ is isosceles (4)

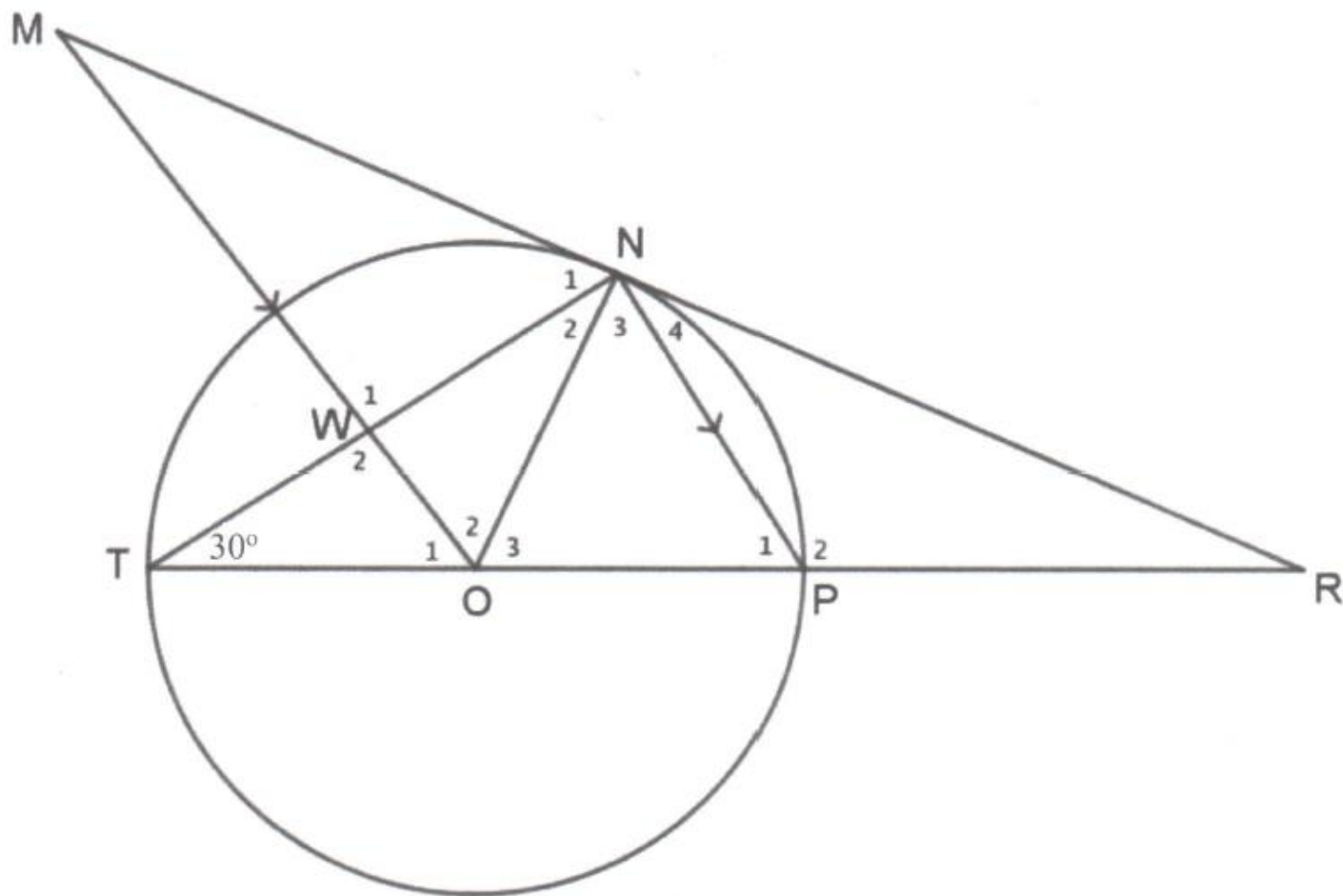
(c) $\triangle WRV \parallel \triangle TSV$ (3)

(d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

QUESTION 8

In the diagram below, TP is a diameter in the circle with centre O. TP is extended to R. RM is a tangent to the circle at N. MO intersects chord NT at W. $NP \parallel MO$. $\angle W\hat{T}O = 30^\circ$



8.1 Give, with reasons, THREE other angles each equal to 30° . (3)

8.2 Determine $\angle R\hat{N}T$ (2)

8.3 Prove that:

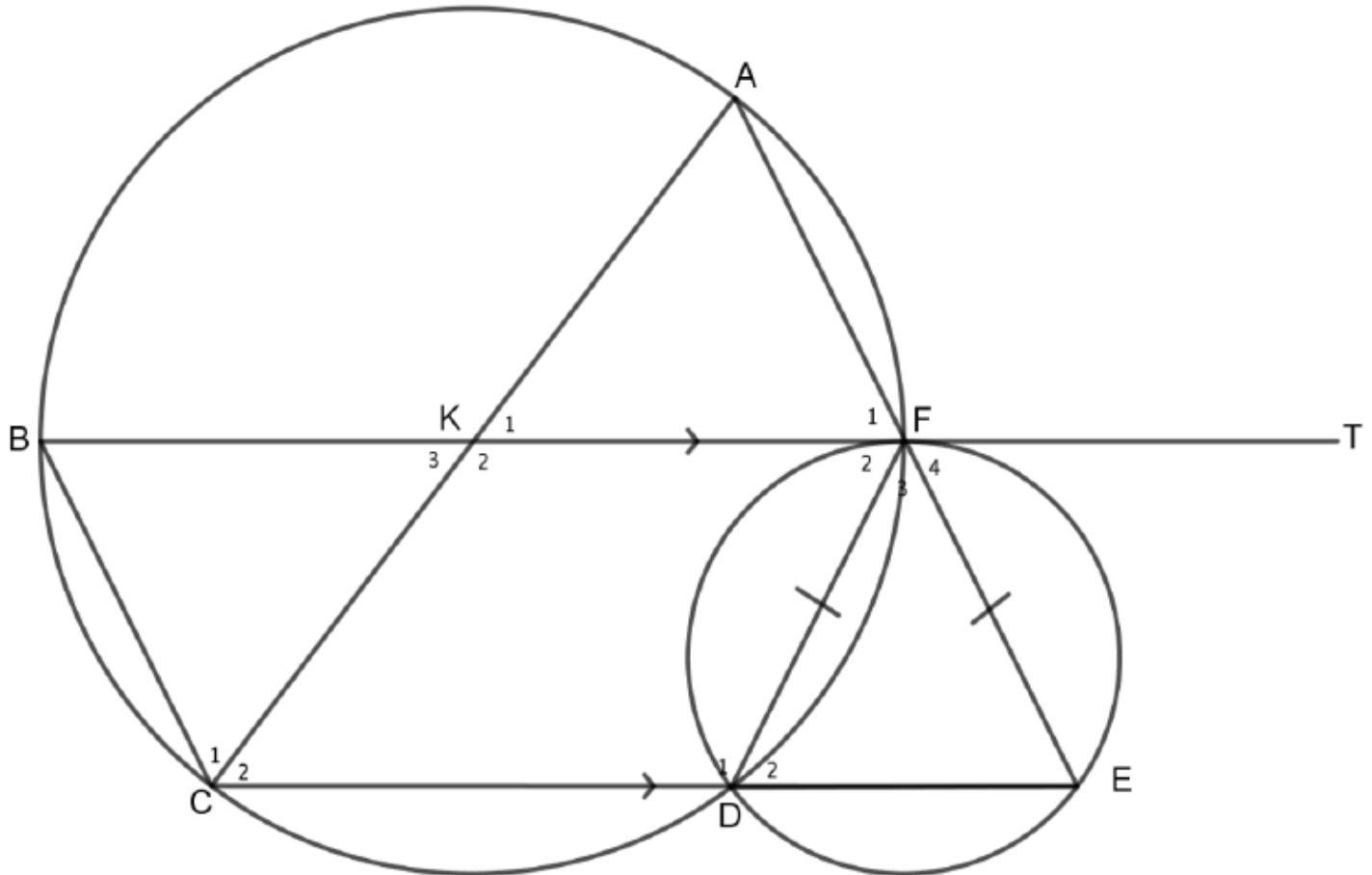
8.3.1 $\triangle RNP \parallel \triangle RTN$ (3)

8.3.2 $TW \cdot RN = \frac{1}{2} RT \cdot NP$ (5)

[13]

QUESTION 8

In the figure, two circles intersect in F and D. BFT is a tangent to the small circle at F. Straight line AFE is drawn so that $FE = FD$. CDE is a straight line and chords AC and BF intersect in K. $BT \parallel CE$.



Prove that:

8.1 BCEF is a parallelogram (6)

8.2 $AC = BF$ (3)

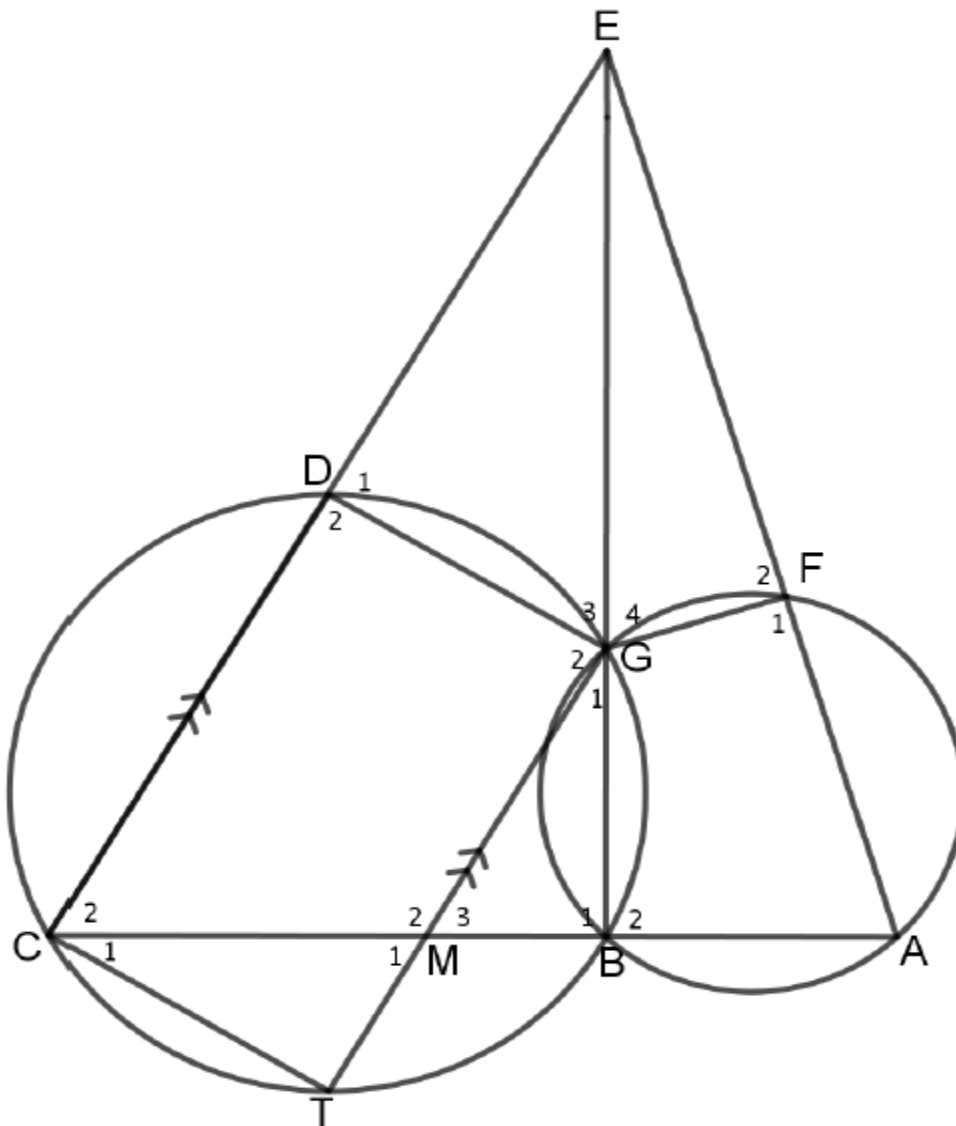
8.3 $BK = \frac{BC}{FA} \cdot AK$ (5)

8.4 If $BC = 24$ units, $AF = 12$ units and $AC = 15$ units find CK (6)

[20]

QUESTION 9

In the diagram, two circles intersect at B and G, ABC is a straight line. AF is a chord of the smaller circle and CD is a chord of the bigger circle, both produced, meet BG produced at E. Chord GT of the bigger circle is parallel to EC. GD and FG are drawn.

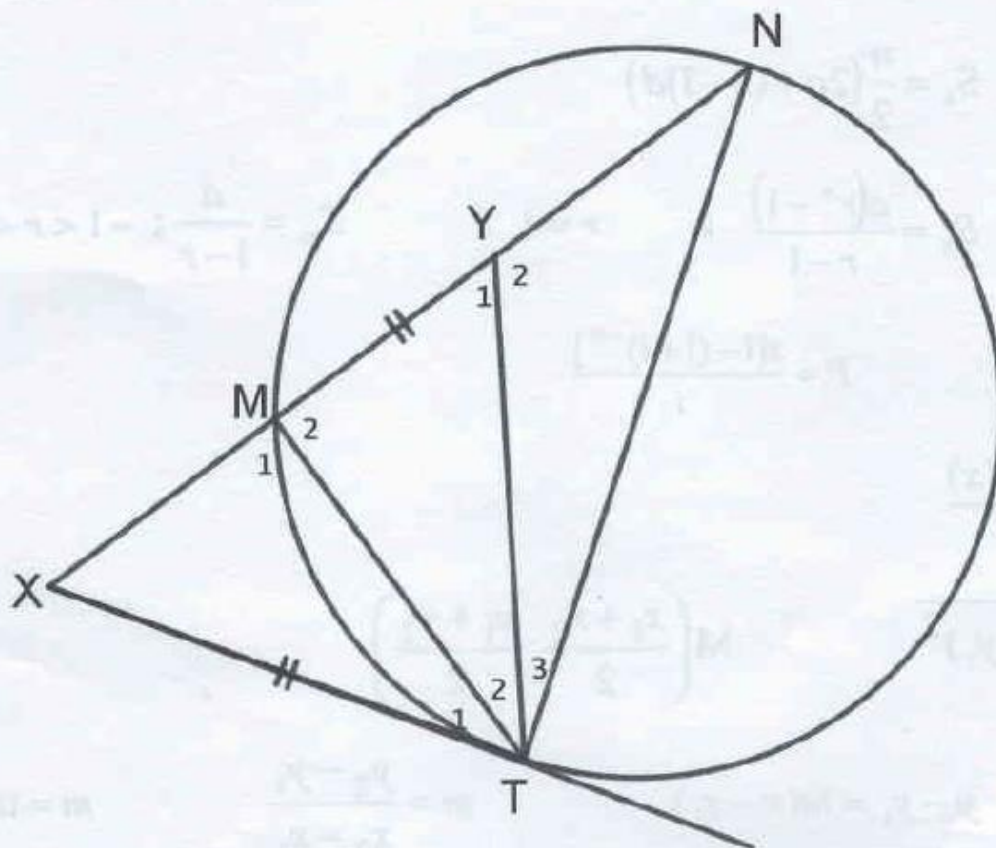


Prove that:

- 9.1 FGDE is a cyclic quadrilateral (3)
 - 9.2 TC is a tangent to circle CBE (3)
 - 9.3 $\hat{G}_3 = \hat{M}_1$ (2)
 - 9.4 $TM \cdot GE = MC \cdot DG$ (5)
- [13]

QUESTION 10

- 10.1 In the diagram XMN is a straight line and XT is a tangent to the circle. Y is a point on XN so that $XY = XT$.



Prove that:

10.1.1 YT bisect $\hat{MTN}$. (5)

10.1.2 $\frac{XM}{XT} = \frac{XT}{XN}$ (6)

- 10.2 Given that $MY = 20$ mm, $YN = 50$ mm and $XT = k$ mm:

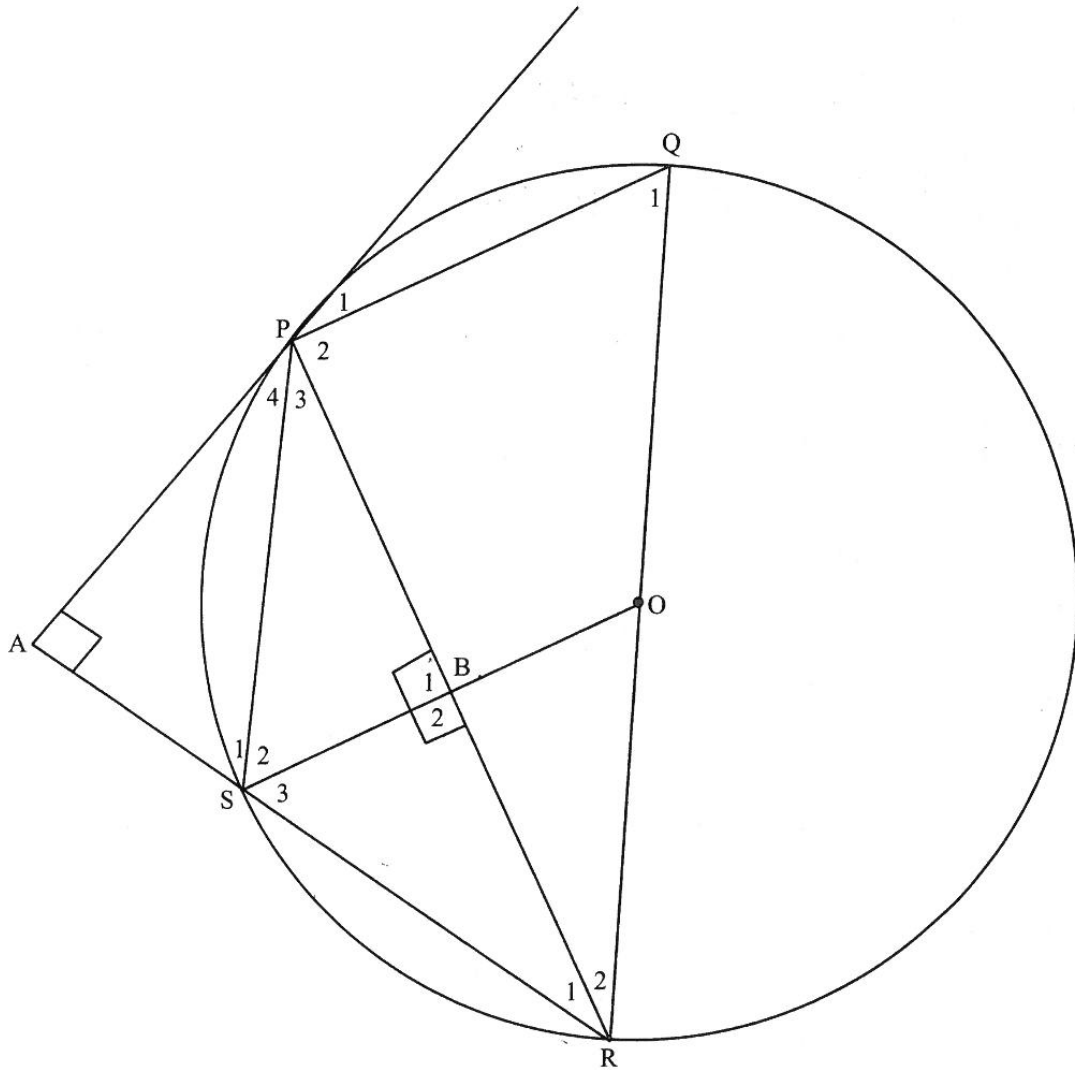
10.2.1 Express XM in terms of k . (3)

10.2.2 Calculate the length of k . (4)

[18]

ECS20

- 10.2 In the diagram, O is the centre of the circle. PQRS is a cyclic quadrilateral. The tangent through P intersects RS produced at A. $OB \perp PR$ and $PA \perp AS$.



Prove that:

$$10.2.1 \quad \triangle APS \parallel \triangle BRS \quad (3)$$

$$10.2.2 \quad AP \cdot RS = BR \cdot PS \quad (1)$$

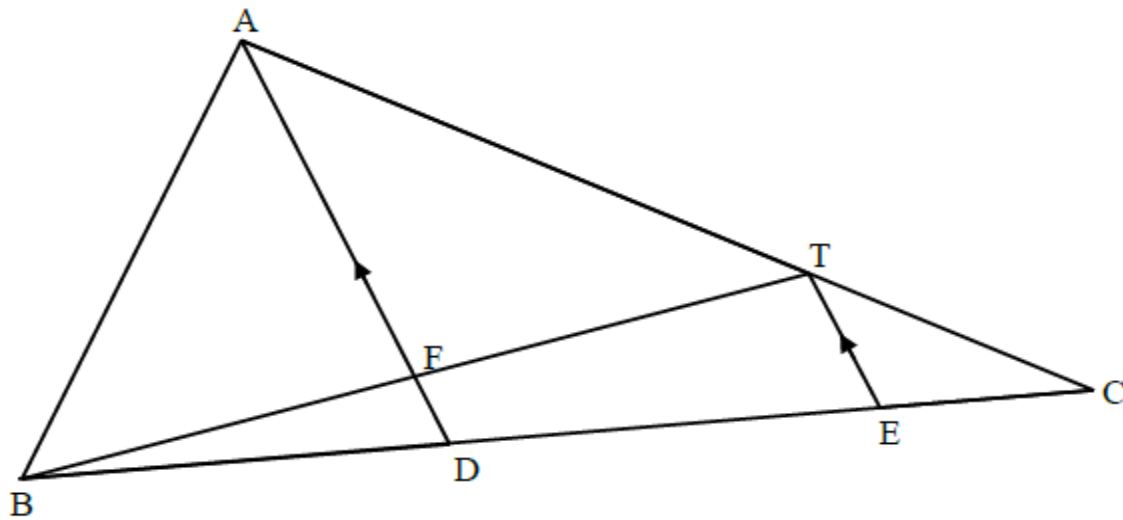
$$10.2.3 \quad \hat{P}_4 = \hat{R}_2 \quad (4)$$

$$10.2.4 \quad BR \cdot RQ = RS \cdot RP \quad (6)$$

[19]

FSS20

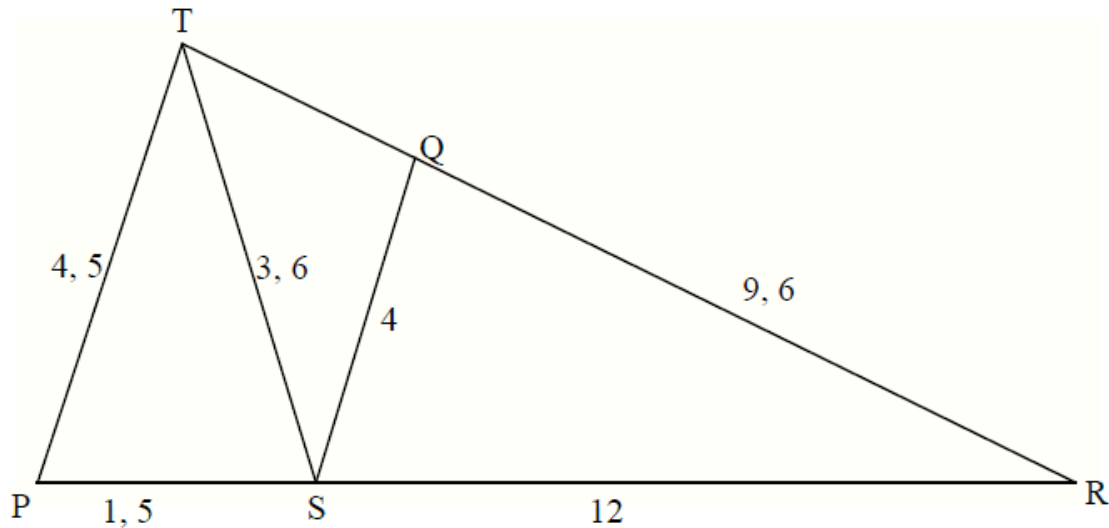
- 10.3 In the figure, D is a point on side BC of $\triangle ABC$ such that $BD = 6$ cm and $DC = 9$ cm. T and E are points on AC and DC respectively such that $TE \parallel AD$ and $AT : TC = 2 : 1$



- 10.3.1 Show that D is the midpoint of BE. (3)
- 10.3.2 If $FD = 2$ cm, calculate the length of TE. (3)
- 10.3.3 Calculate the numerical value of:
- (a) $\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$ (3)
- (b) $\frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC}$ (3)

QUESTION 11

In the diagram, $\triangle TPR$ is a triangle with $TP = 4,5$ units. Points Q and S are on TR and PR respectively such that $QR = 9,6$ units, $QS = 4$ units, $TS = 3,6$ units, $PS = 1,5$ units and $SR = 12$ units.



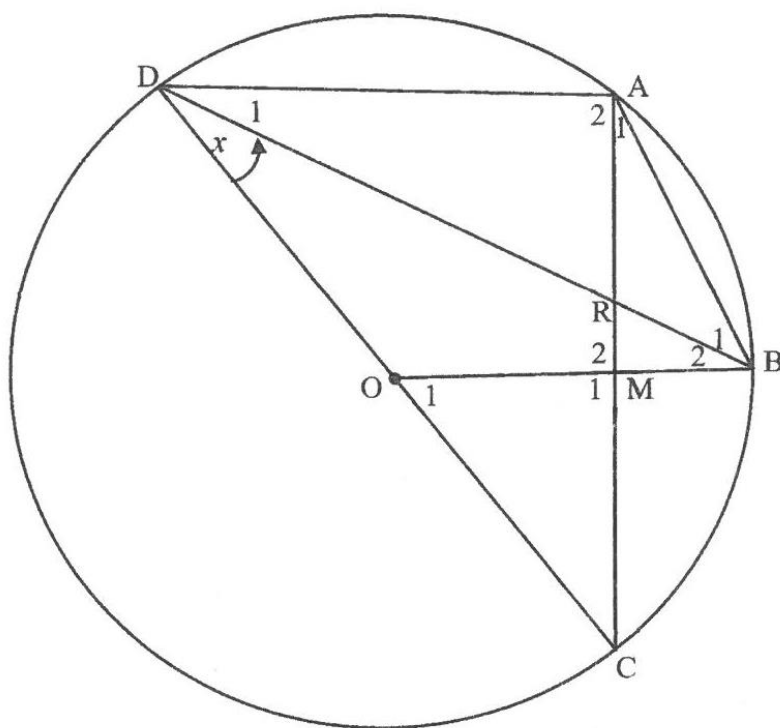
- 11.1 Prove that PT is a tangent to the circle which passes through the points T, S and R . (7)
- 11.2 Calculate the length of TQ . (5)

[12]

GPS20

QUESTION 9

- 9.1 In the diagram below, A, B, C and D are points on a circle with centre O. OB intersects AC at M, the midpoint of chord AC. Let $\hat{BDC} = x$.



- 9.1.1 Determine, with reasons, in terms of x :

(a) $\hat{O}_1$ (1)

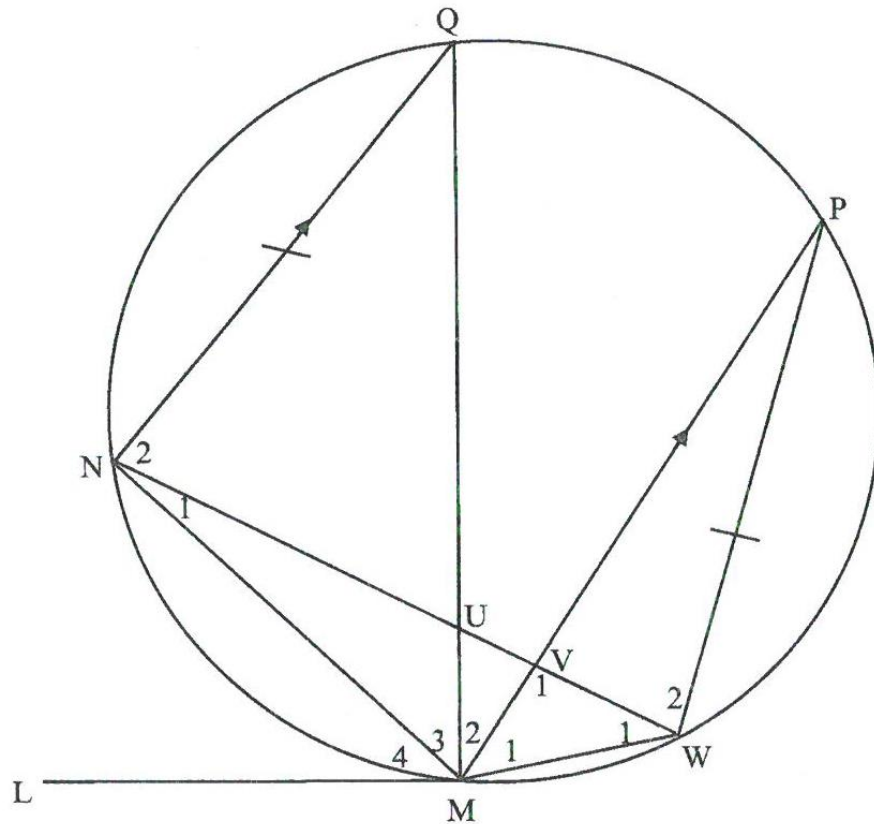
(b) $\hat{ABO}$ (4)

- 9.1.2 Prove that AB is a tangent to the circle that passes through points A, D and R. (6)

9.1.3 Prove that $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$. (4)

9.2

In the diagram below, LM is a tangent to circle QNMWP at M. NW cuts QM and PM at U and V respectively.
 $NQ = WP$ and $NQ \parallel MP$.



9.2.1 State, with reasons, THREE angles equal to $\hat{M}_2$. (3)

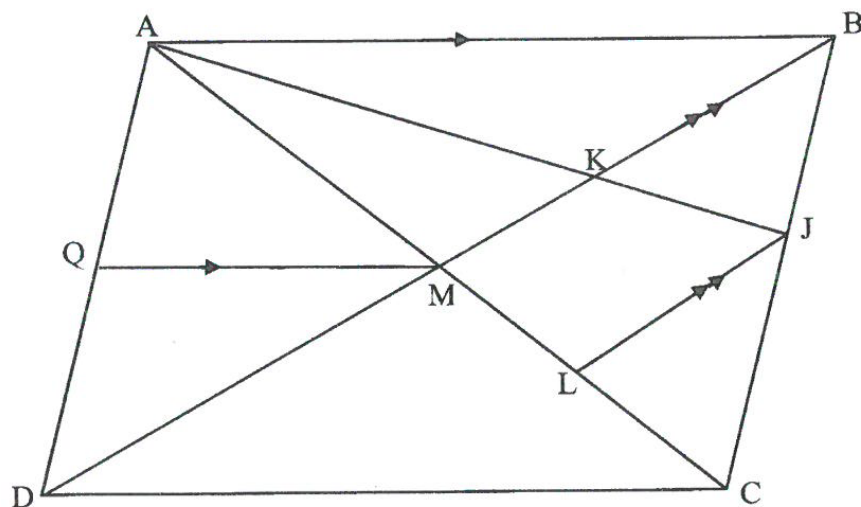
9.2.2 Prove that $\triangle WMV \parallel \triangle QMN$. (3)

9.2.3 Prove that $\frac{MV}{WV} = \frac{MN}{PW}$. (3)

GPS20

10.2

ABCD is a parallelogram with diagonals that intersect at M. J is a point on BC. BJ : JC is 2 : 3. AJ meets BD at K. $BD \parallel JL$ and JL meets AC at L. Q is a point on AD such that $AB \parallel QM$.



10.2.1 Determine, with reasons, the following ratios:

(a) $\frac{ML}{LC}$ (2)

(b) $\frac{AK}{KJ}$ (3)

10.2.2 If $AB = \sqrt{10}$ units and $BC = \frac{2}{3}AB$.

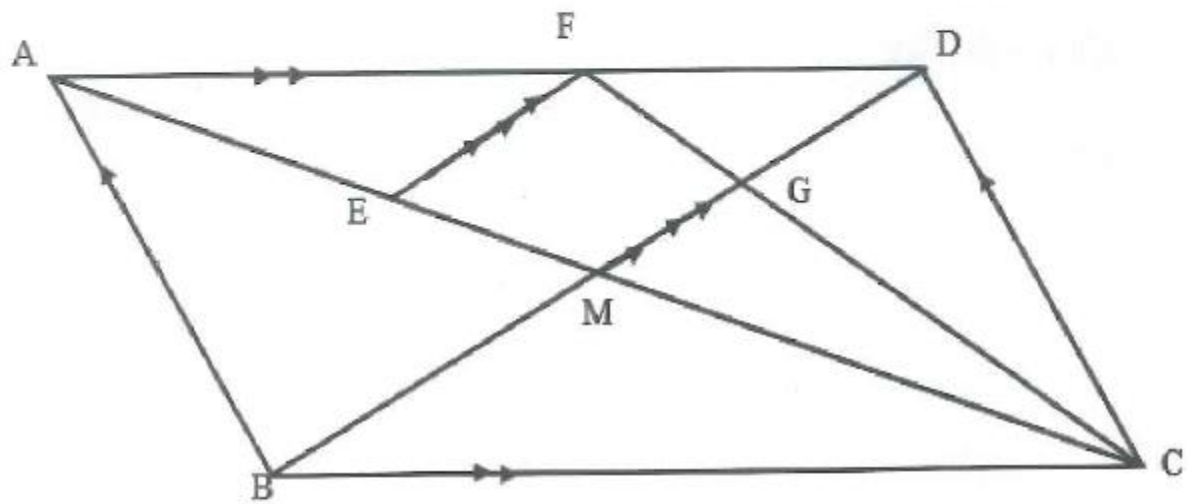
Calculate the length of AQ.

(4)

[15]

LPS20

- 9.2 In the diagram ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that $AF:FD = 4:3$. E is a point on AM such that $EF \parallel BD$. FC and MD intersect in G.



Calculate, giving reasons:

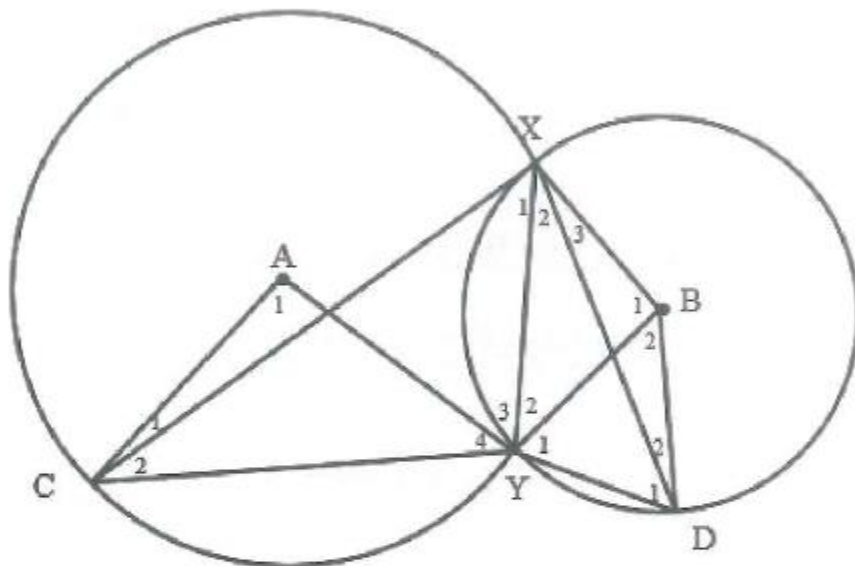
9.2.1 $\frac{EM}{AM}$ (3)

9.2.2 $\frac{CM}{ME}$ (3)

9.2.3 $\frac{\text{area } \triangle FDC}{\text{area } \triangle BDC}$ (4)

QUESTION 10

Two circles with centres A and B, intersect at X and Y. The radius of circle A is R and the radius of circle B is r. CX is a tangent to circle B at X and DX is a tangent to circle A at X.



Prove that:

$$10.1 \quad \triangle X Y C \parallel \triangle D Y X \text{ and hence deduce that } XY^2 = DY \cdot YC \quad (5)$$

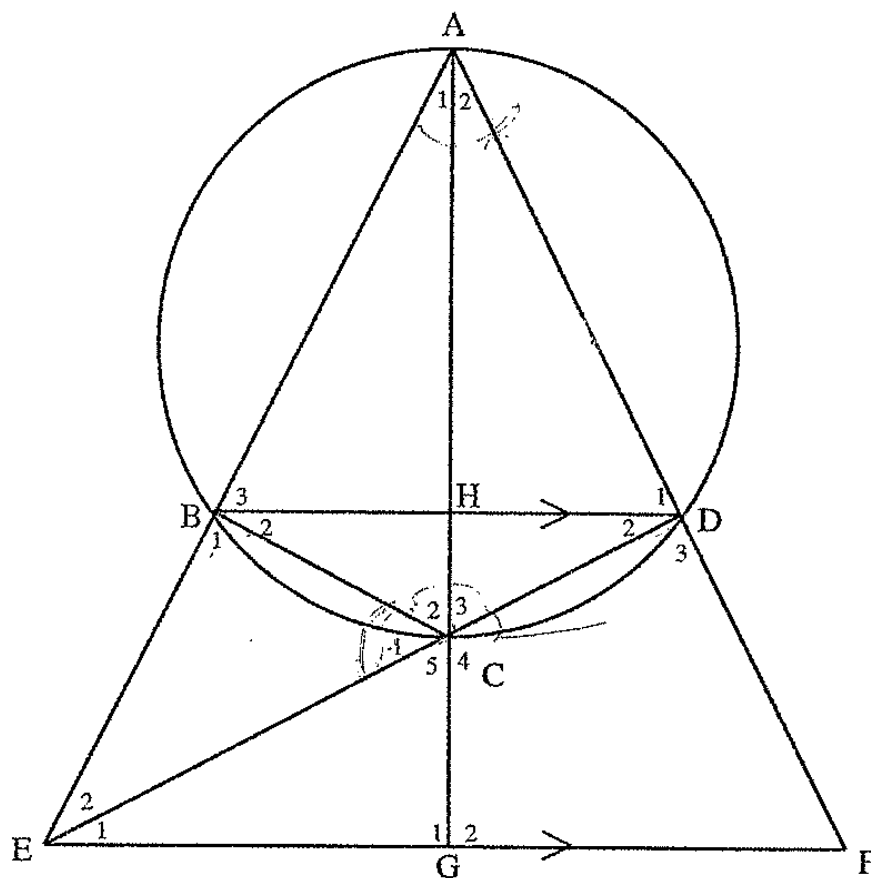
$$10.2 \quad \hat{A}_1 = \hat{B}_1 \quad (4)$$

$$10.3 \quad \text{If } \triangle C A Y \parallel \triangle Y B X, \text{ then } \frac{r^2}{R^2} = \frac{DY}{CY} \quad (5)$$

[14]

MPS20

10.2 In the figure ABCD is a cyclic quadrilateral. $EF \parallel BD$, ACG and DCE are straight lines and BC is joined.

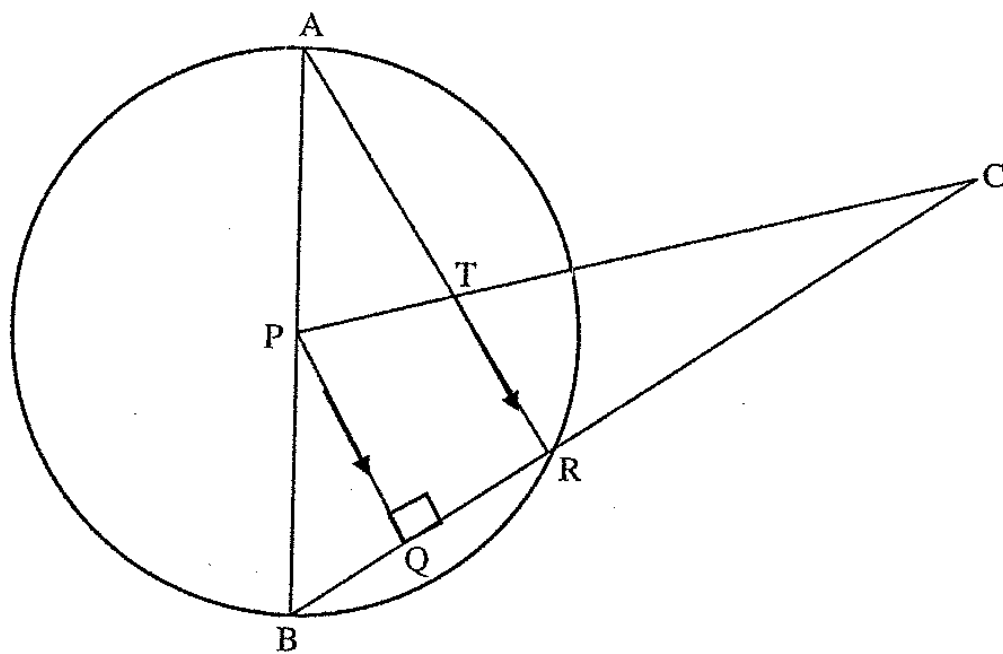


Prove that:

- $$\begin{array}{ll} 10.2.1 & \text{EF is a tangent to circle EAC.} \\ 10.2.2 & \triangle ABC \parallel \triangle EDF \\ 10.2.3 & \frac{BE.AH}{GH} = \frac{ED.BC}{DF} \end{array}$$

QUESTION 11

AB is a diameter and BR is a chord of a circle centred at P. BR is produced to C, PC and AR intersect at T and $PQ \perp BR$. Calculate the length of BC if $BQ = 24 \text{ mm}$ and $PT = \frac{1}{3}TC$.



[5]

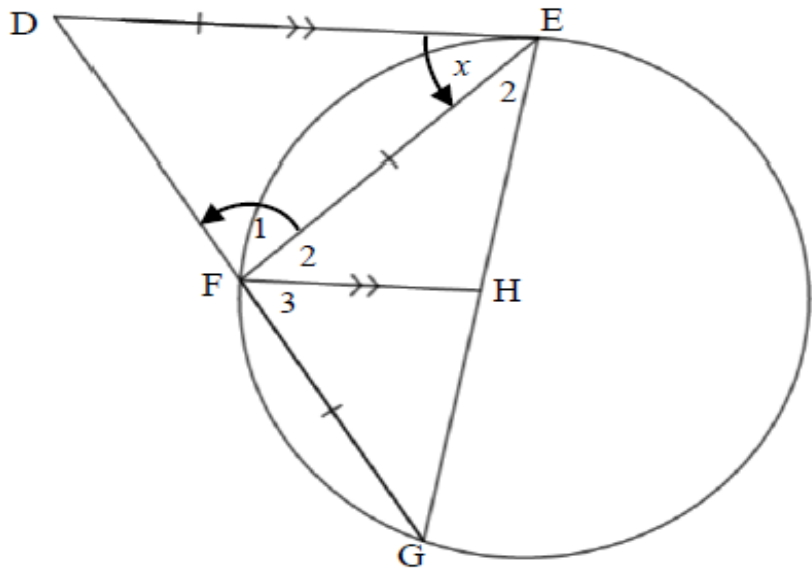
QUESTION 9

In the diagram is:

- DE a tangent to the circle at E .
- DFG is a straight line.
- $DE = EF = FG$
- $HF \parallel DE$.

It is further given that $\frac{DF}{DE} = y$.

- Let $\widehat{DEF} = x$.



- 9.1 Give, with reasons, THREE other angles equal to x . (6)

- 9.2 Prove that:

$$9.2.1 \quad \frac{EH}{HG} = y \quad (3)$$

$$9.2.2 \quad \widehat{D} = 72^\circ \quad (5)$$

$$9.2.3 \quad \triangle DGE \parallel \triangle DEF \quad (3)$$

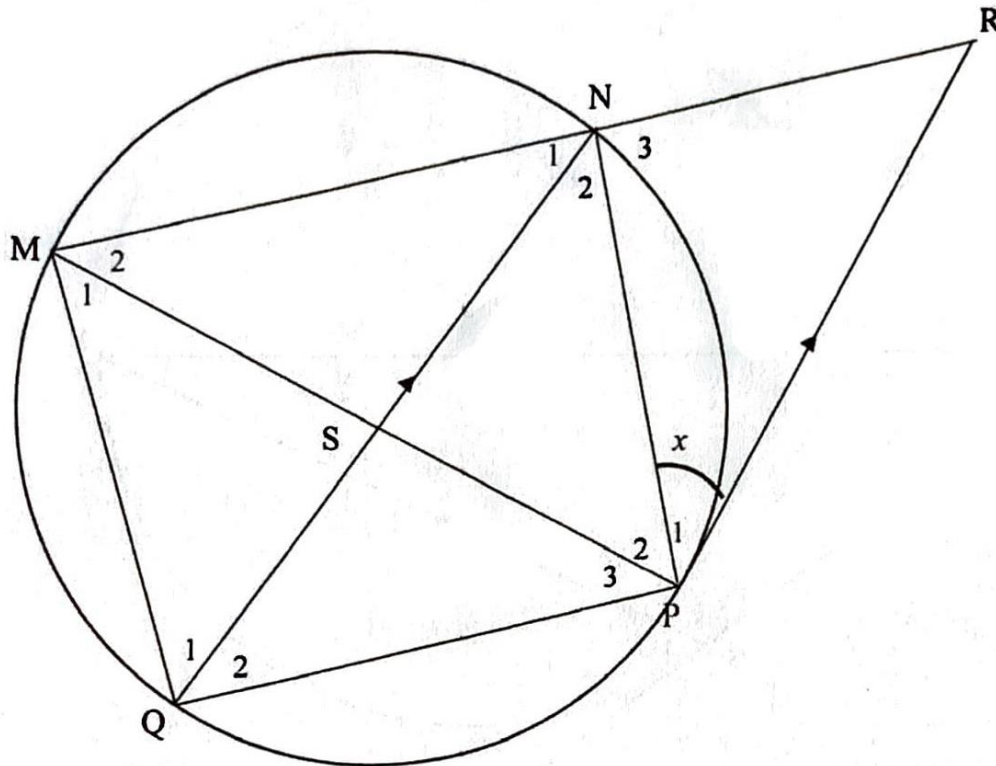
$$9.2.4 \quad DE^2 = DF \cdot DG \quad (2)$$

$$9.2.5 \quad y^2 + y = 1 \tag{5}$$

$$9.2.6 \quad \cos 72^\circ = \frac{1}{2}y \quad (3)$$

[27]

Chord QN bisects $\hat{MNP}$ and intersects chord MP at S. The tangent at P meets MN produced at R such that $QN \parallel PR$. Let $\hat{P}_1 = x$.



9.2.1 Determine the following angles in terms of x . Give reasons

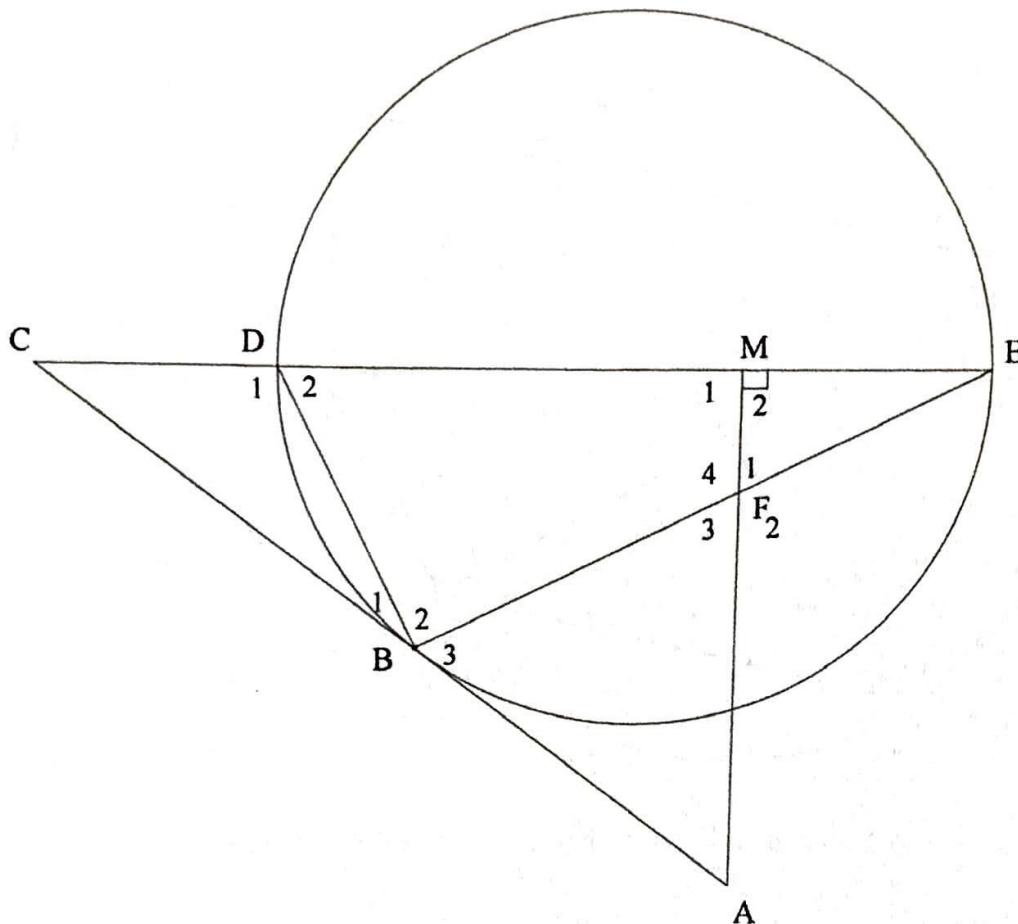
(a) $\hat{N}_2$ (2)

(b) $\hat{Q}_2$ (2)

9.2.2 Prove, giving reasons, that $\frac{MN}{NR} = \frac{MS}{SQ}$ (6)

QUESTION 10

In the diagram, a circle passes through D, B and E. Diameter ED of the circle is produced to C and AC is a tangent to the circle at B. M is a point on DE such that $AM \perp DE$. AM and chord BE intersect at F.



10.1 Prove, giving reasons, that:

10.1.1 FBDM is a cyclic quadrilateral (3)

10.1.2 $\hat{B}_3 = \hat{F}_1$ (4)

10.1.3 $\triangle CDB \parallel \triangle CBE$ (3)

10.2 If it is further given that $CD = 2$ units and $DE = 6$ units, calculate the length of:

10.2.1 BC (3)

10.2.2 DB (4)

[17]