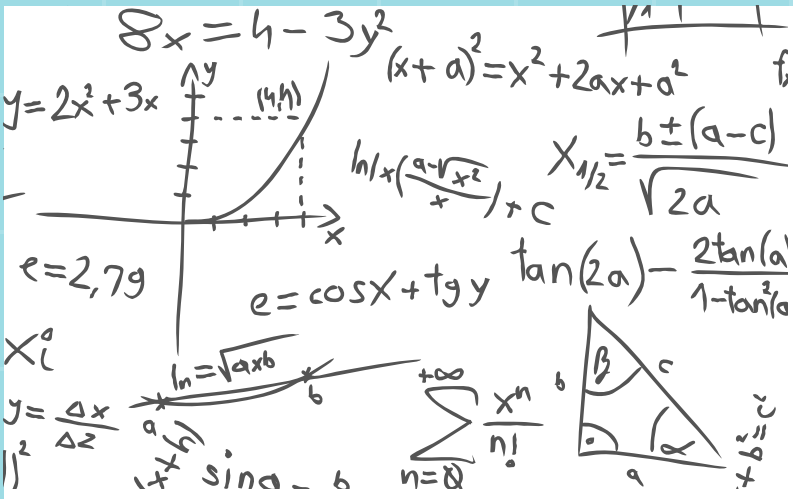
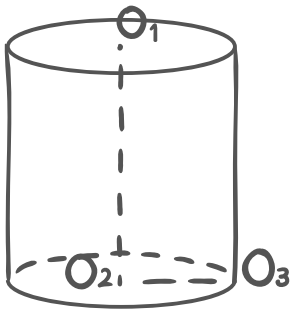
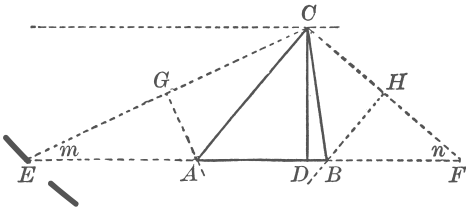
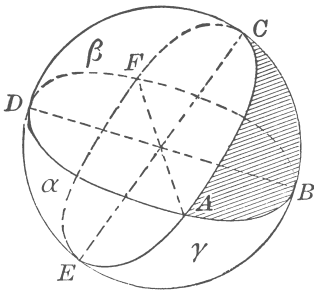


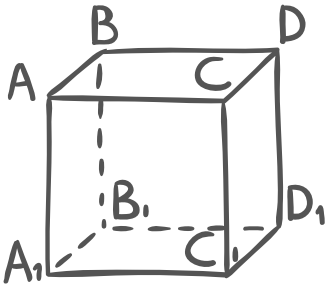


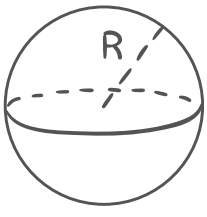




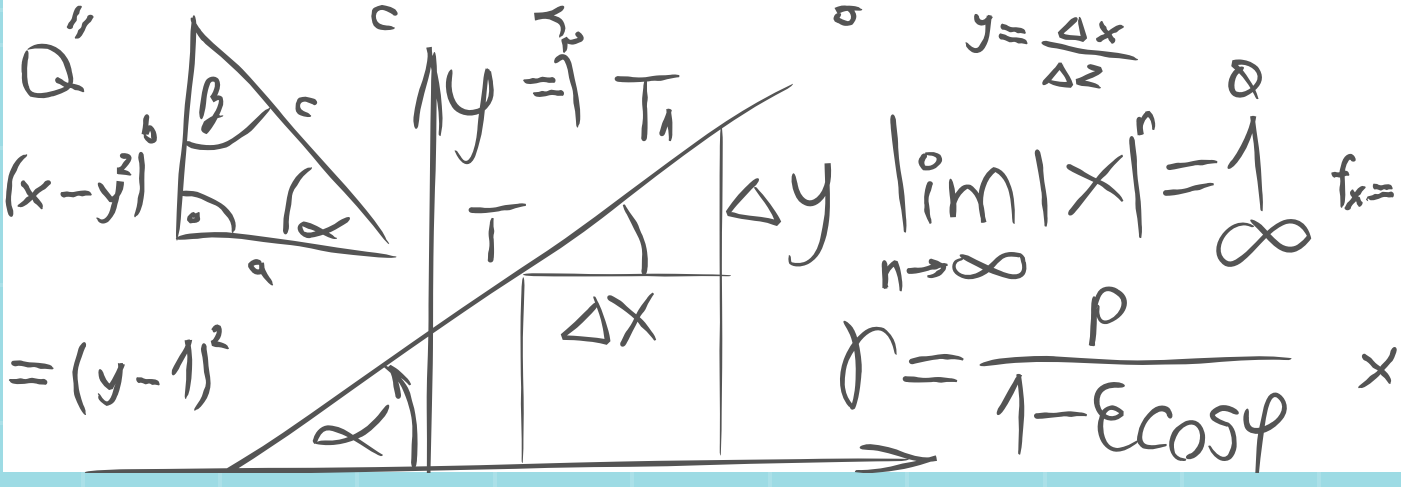
MATHEMATICS FINAL EXAM

PAPER 1





GRADE 12 MATHWITHMEGAN



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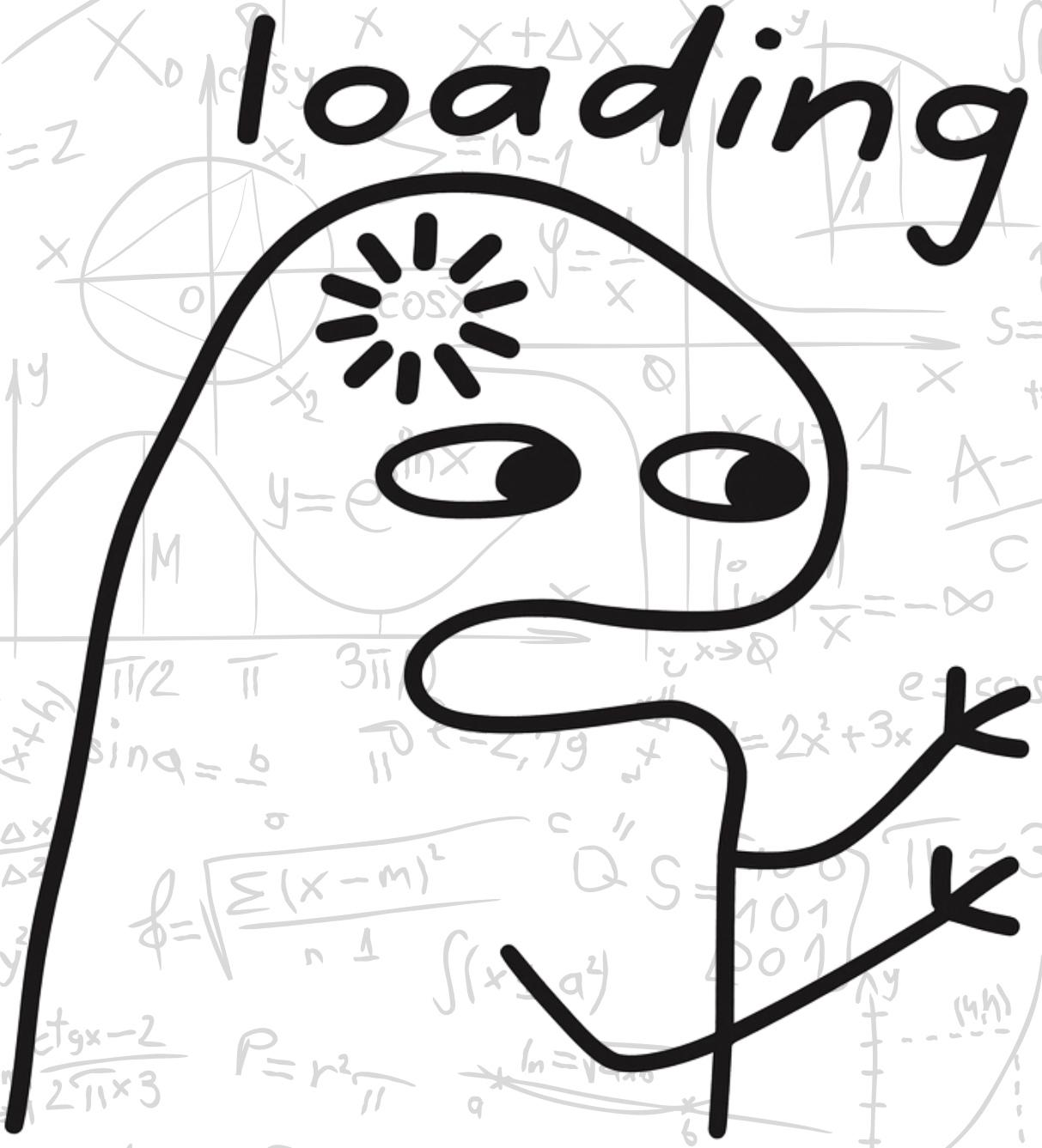
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ALGEBRA



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QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-3)=0$ (2)

1.1.2 $2x^2+1=4x$ (correct to TWO decimal places) (4)

1.1.3 $x^2-2x-3>0$ (4)

1.1.4 $2^{2x}-2^{x+2}-32=0$ (5)

1.1.5 $\sqrt{-2x+4}-x=2$ (4)

1.2 Solve for x and y simultaneously:

$$2x+y=3$$
$$y^2+xy=2$$ (5)

1.3 Consider the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots$

Each factor in the product is of the form $\left(1+\frac{1}{n}\right)$ for $n\geq 2$.

Determine ALL the values of n for which the product will be an integer value. (3)

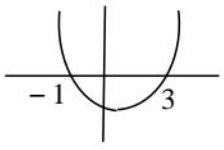
[27]

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- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION 1/*VRAAG* 1

1.1.1	$x(x-3)=0$ $x=0$ or $x=3$	✓ $x=0$ ✓ $x=3$ (2)
1.1.2	$2x^2-4x+1=0$ $x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$ $x=1{,}71$ or $x=0{,}29$ Answer only: $\frac{4}{4}$	✓ standard form ✓ correct substitution in correct formula ✓ answer ✓ answer (4)
1.1.3	$x^2-2x-3>0$ $(x-3)(x+1)>0$ CV: $x=3$ or $x=-1$  $x<-1$ or $x>3$ **OR/*OF*** $x\in(-\infty;-1)\cup(3;\infty)$ Answer only: $\frac{3}{4}$	✓factors ✓ critical values ✓✓ answer (4)
1.1.4	$2^{2x}-2^{x+2}-32=0$ $2^{2x}-2^2.2^x-32=0$ $(2^x-8)(2^x+4)=0$ $2^x=8$ or $2^x\neq-4$ $2^x=2^3$ no solution $x=3$ Answer only: $\frac{1}{5}$	✓use of exponential law ✓factors ✓ $2^x=8$ ✓ $2^x\neq-4$ ✓ answer (5)
1.1.5	$\sqrt{-2x+4}-x=2$ $\sqrt{-2x+4}=2+x$ $\left(\sqrt{-2x+4}\right)^2=(2+x)^2$ $-2x+4=4+4x+x^2$ $x^2+6x=0$ $x(x+6)=0$ $x=0$ or $x\neq-6$	✓ isolating the root ✓ squaring both sides ✓ standard form ✓ answers with selection (4)

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1.2	$y = 3 - 2x \quad \ldots (1)$ $y^2 + xy = 2 \quad \ldots (2)$ Subs (1) into (2): $(3-2x)^2 + (3-2x)x = 2$ $4x^2 - 12x + 9 - 2x^2 + 3x - 2 = 0$ $2x^2 - 9x + 7 = 0$ $(2x-7)(x-1) = 0$ $x = \frac{7}{2}$ or $x = 1$ $y = -4 \quad y = 1$ **OR/*OF*** $x = \frac{3-y}{2} \quad \ldots (1)$ $y^2 + xy = 2 \quad \ldots (2)$ Subs (1) into (2): $y^2 + \left(\frac{3-y}{2}\right)y = 2$ $2y^2 + 3y - y^2 - 4 = 0$ $y^2 + 3y - 4 = 0$ $(y+4)(y-1) = 0$ $y = -4$ or $y = 1$ $x = \frac{7}{2}$ or $x = 1$	✓ eq 1 ✓ substitution ✓ standard form ✓ both x-values ✓ both y-values (5) **OR/*OF*** ✓ eq 1 ✓ substitution ✓ standard form ✓ both y-values ✓ both x-values (5)
1.3	Product $= \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{n}{n-1} \times \frac{n+1}{n} = \frac{n+1}{2}$ The product will be an integer value when $n + 1$ is divisible by 2 i.e. $n \in \{3;5;7;\ldots\}$ Therefore, n must be an odd number, greater than 2. **OR/*OF*** For $n = 2$ the product is $\frac{3}{2}$ For $n = 3$ the product is $\left(\frac{3}{2}\right)\left(\frac{4}{3}\right) = 2$ For $n = 4$ the product is $\left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right) = \frac{5}{2}$ For $n = 5$ the product is $\left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)\left(\frac{6}{5}\right) = 3$ $n \in \{3;5;7;\ldots\}$ Therefore, n must be an odd number, greater than 2.	✓ $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{n+1}{n}$ ✓ statement $\left(\frac{n+1}{2}\right)$ ✓ n is odd for $n > 2$ (3) **OR/*OF*** ✓ generation of the four products ✓ identifying the pattern when n is odd ✓ n is odd for $n > 2$ (3)
		[27]

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QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (answers correct to TWO decimal places) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\dfrac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.3 Given: $P = (1-a)$ and $T = (1+a)(1+a^2)(1+a^4)..(1+a^{512})$

Determine the value of $P \times T$ in terms of a. (3)

[26]

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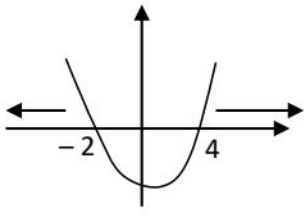
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QUESTION 1/*VRAAG* 1

1.1.1	$3x^2+5x=0$ $x(3x+5)=0$ $x=0$ or $x=-\frac{5}{3}$	✓ $x=0$ ✓ $x=-\frac{5}{3}$ (2)
1.1.2	$4x^2+3x-5=0$ $x=\frac{-(3)\pm\sqrt{(3)^2-4(4)(-5)}}{2(4)}$ $x=0{,}80$ or $x=-1{,}55$	✓ correct substitution into correct formula ✓ answer ✓ answer (3)
1.1.3	$(x-1)^2-9\geq 0$ $x^2-2x-8\geq 0$ $(x-4)(x+2)\geq 0$ $x=4$ or $x=-2$ $x\leq -2$ or $x\geq 4$ 	✓ standard form ✓ critical values ✓✓ $x\leq -2$ or $x\geq 4$ (4)
1.1.4	$5^{2x}-5^x=0$ $5^x(5^x-1)=0$ $5^x\neq 0$ or $5^x=1$ $x=0$ **OR/OF** $5^{2x}=5^x$ $2x=x$ $2x-x=0$ $x=0$	✓ common factor ✓ $5^x\neq 0$ ✓ $5^x=1$ ✓ $x=0$ (4) **OR/OF** ✓ $5^{2x}=5^x$ ✓ $2x=x$ ✓ $2x-x=0$ ✓ $x=0$ (4)

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1.1.5	$\frac{x}{\sqrt{20-x}}=1$ $x=\sqrt{20-x}$ $x^2=20-x$ $x^2+x-20=0$ $(x+5)(x-4)=0$ $x=4$ or $x\neq -5$	✓ isolating the surd ✓ squaring both sides ✓ standard form ✓ answers ✓ selection (5)
1.2	$2x^2-y^2=7$ …. (1) $x+y=9$ …. (2) $y=9-x$ $2x^2-(9-x)^2=7$ $2x^2-81+18x-x^2=7$ $x^2+18x-88=0$ $(x+22)(x-4)=0$ $x=-22$ or $x=4$ $y=31$ or $y=5$ **OR/*OF*** $2x^2-y^2=7$ …. (1) $x+y=9$ …. (2) $x=9-y$ $2(9-y)^2-y^2=7$ $2(81-18y+y^2)-y^2-7=0$ $162-36y+2y^2-y^2-7=0$ $y^2-36y+155=0$ $(y-31)(y-5)=0$ $y=31$ or $y=5$ $x=-22$ or $x=4$	✓ $y=9-x$ ✓ substitution ✓ standard form ✓ x-values ✓ y-values (5) **OR/*OF*** ✓ $x=9-y$ ✓ substitution ✓ standard form ✓ y-values ✓ x-values (5)
1.3	$P\times T=(1-a)(1+a)(1+a^2)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^2)(1+a^2)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^4)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^8)...(1+a^{512})$ $P\times T=(1-a^{512})(1+a^{512})$ $=1-a^{1024}$	✓ $(1-a^4)$ ✓ $(1-a^{512})$ ✓ $1-a^{1024}$ (3)
		[26]

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + x - 12 = 0$ (3)

1.1.2 $3x^2 - 2x = 6$ (answers correct to TWO decimal places) (4)

1.1.3 $\sqrt{2x+1} = x - 1$ (4)

1.1.4 $x^2 - 3 > 2x$ (4)

1.2 Solve for x and y simultaneously:

$x + 2 = 2y$ and $\frac{1}{x} + \frac{1}{y} = 1$ (5)

1.3 Given: $2^{m+1} + 2^m = 3^{n+2} - 3^n$ where m and n are integers.

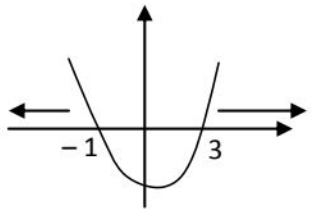
Determine the value of $m + n$. (4)
[24]

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QUESTION 1/*VRAAG* 1

1.1.1	$x^2+x-12=0$ $(x-3)(x+4)=0$ $x=3$ or $x=-4$	✓factors/formula ✓answer ✓answer (3)
1.1.2	$3x^2-2x=6$ $3x^2-2x-6=0$ $x=\frac{2\pm\sqrt{(-2)^2-4(3)(-6)}}{2(3)}$ $x=1{,}79$ or $x=-1{,}12$	✓ standard form ✓ substitution into correct formula ✓ answer ✓ answer (4)
1.1.3	$\sqrt{2x+1}=x-1$ $2x+1=(x-1)^2$ $2x+1=x^2-2x+1$ $x^2-4x=0$ $x(x-4)=0$ $x=0$ or $x=4$ $x\neq 0$ or $x=4$	✓ squaring both sides ✓ standard form ✓ both x-values ✓ valid answer (4)
1.1.4	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ CV's: $x=-1$; $x=3$  $x<-1$ or $x>3$	✓ standard form ✓ critical values/factors ✓✓answer (4)

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1.2	$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $x=2y-2$ $\frac{1}{2y-2}+\frac{1}{y}=1$ $y+2y-2=2y^2-2y$ $2y^2-5y+2=0$ $(2y-1)(y-2)=0$ $y=\frac{1}{2}$ or $y=2$ $x=-1$ or $x=2$ **OR/*OF*** $\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $y=\frac{x}{2}+1$ $\frac{1}{x}+\frac{1}{\frac{x}{2}+1}=1$ $\frac{1}{x}+\frac{2}{x+2}=1$ $x+2+2x=x^2+2x$ $x^2-x-2=0$ $(x+1)(x-2)=0$ $x=-1$ or $x=2$ $y=\frac{1}{2}$ or $y=2$	✓ $x=2y-2$ ✓substitution ✓standard form ✓y-values ✓x-values (5) **OR/*OF*** ✓ $y=\frac{x}{2}+1$ ✓substitution ✓standard form ✓x-values ✓y-values (5)

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1.3	$2^{m+1}+2^m=3^{n+2}-3^n$ $2^m(2+1)=3^n(3^2-1)$ $2^m(3)=3^n(8)$ $2^m(3)=3^n(2^3)$ $\therefore m=3$ and $n=1$ $\therefore m+n=4$ **OR/*OF*** $2^{m+1}+2^m=3^{n+2}-3^n$ $2^m(2+1)=3^n(3^2-1)$ $2^m(3)=3^n(8)$ $2^m(3)=3^n(2^3)$ $2^{m-3}=3^{n-1}$ Only true if $m-3=0$ and $n-1=0$ $\therefore m+n=4$	✓ factors ✓ $2^m(3)=3^n(2^3)$ (same bases) ✓ $m=3$ and $n=1$ ✓ $m+n=4$ (4) **OR/*OF*** ✓ factors ✓ $2^m(3)=3^n(2^3)$ (same bases) ✓ $m-3=0$ and $n-1=0$ ✓ $m+n=4$ (4)
		[24]

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 7x + 12 = 0$ (3)

1.1.2 $x(3x + 5) = 1$ (correct to TWO decimal places) (4)

1.1.3 $x^2 < -2x + 15$ (4)

1.1.4 $\sqrt{2(1-x)} = x - 1$ (4)

1.2 Solve for x and y simultaneously:

$3^{x+y} = 27$ and $x^2 + y^2 = 17$ (6)

1.3 Determine, **without the use of a calculator**, the value of:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}$$ (3)

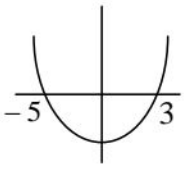
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QUESTION 1/*VRAAG* 1

1.1.1	$x^2-7x+12=0$ $(x-4)(x-3)=0$ $x=4$ or $x=3$ Answer only: Full Marks	✓ factors ✓ $x=4$ ✓ $x=3$ (3)
1.1.2	$3x^2+5x-1=0$ $x=\frac{-5\pm\sqrt{5^2-4(3)(-1)}}{2(3)}=\frac{-5\pm\sqrt{37}}{6}$ $\therefore x=0{,}18$ or $x=-1{,}85$	✓standard form ✓substitution into the correct formula ✓ $x=0{,}18$ ✓ $x=-1{,}85$ (4)
1.1.3	$x^2+2x-15<0$ $(x-3)(x+5)<0$ $x=3$ or $x=-5$ $-5<x<3$ 	✓standard form ✓ critical values ✓✓answer (4)
1.1.4	$\sqrt{2(1-x)}=x-1$ $\left(\sqrt{2(1-x)}\right)^2=(x-1)^2$ $2-2x=x^2-2x+1$ $x^2-1=0$ $\therefore x=1$ and $x\neq-1$	✓squaring both sides ✓simplification ✓standard form ✓ answer with selection (4)

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1.2	$3^{x+y}=27$	
	$x^2+y^2=17$	
	$3^{x+y}=3^3$	✓ $3^{x+y}=3^3$
	$x+y=3$ .....(1)	✓ $x+y=3$
	$y=3-x$	
	$x^2+(3-x)^2=17$	✓substitution
	$2x^2-6x-8=0$	
	$x^2-3x-4=0$	✓standard form
	$(x-4)(x+1)=0$	
	$x=4$ or $x=-1$	✓x-values
	$y=-1$ or $y=4$	✓y-values (6)
	OR/*OF*	**OR/*OF***
	$3^{x+y}=27$	
	$x^2+y^2=17$	
	$3^{x+y}=3^3$	✓ $3^{x+y}=3^3$
	$x+y=3$ .....(1)	✓ $x+y=3$
	$x=3-y$	
	$(3-y)^2+y^2=17$	✓substitution
	$9-6y+y^2+y^2-17=0$	
	$2y^2-6y-8=0$	✓standard form
	$y^2-3y-4=0$	
	$(y-4)(y+1)=0$	
	$y=-1$ or $y=4$	✓y-values
	$x=4$ or $x=-1$	✓x-values (6)

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1.3	$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}$ $=\frac{1}{\sqrt{1}+\sqrt{2}}\times\frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}}+...$ $+\frac{1}{\sqrt{99}+\sqrt{100}}\times\frac{\sqrt{99}-\sqrt{100}}{\sqrt{99}-\sqrt{100}}$ $=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+2...-\sqrt{99}+10$ $=-1+10$ $=9$	✓rationalisation ✓ simplification ✓ answer (3)
		[24]

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QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-6)(x+2)=0$ (2)

1.1.2 $2x^2-6x+1=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2-90>x$ (4)

1.1.4 $x-7\sqrt{x}=-12$ (4)

1.2 Solve for x and y simultaneously:

$$2x-y=2$$
$$xy=4$$ (5)

1.3 Show that $2.5^n-5^{n+1}+5^{n+2}$ is even for all positive integer values of n. (3)

1.4 Determine the values of x and y if: $\frac{3^{y+1}}{32}=\sqrt{96^x}$ (4)

[25]

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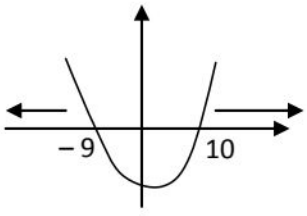
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QUESTION1/*VRAAG* 1

1.1.1	$(3x-6)(x+2)=0$ $x=2$ or $x=-2$	✓ $x=2$ ✓ $x=-2$ (2)
1.1.2	$2x^2-6x+1=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{6\pm\sqrt{(-6)^2-4(2)(1)}}{2(2)}$ $x=2{,}82$ or $x=0{,}18$	✓ correct substitution into correct formula ✓ 2,82 ✓ 0,18 (3)
1.1.3	$x^2-90>x$ $x^2-x-90>0$ $(x+9)(x-10)>0$ CV: $x=-9$ or $x=10$  $x<-9$ or $x>10$ **OR/OF** $(-\infty;-9)$ or $(10;\infty)$	✓ standard form ✓ critical values ✓✓ $x<-9$ or $x>10$ (4)

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1.1.4	$x-7\sqrt{x}=-12$ $x+12=7\sqrt{x}$ $(x+12)^2=\left(7\sqrt{x}\right)^2$ $x^2+24x+144=49x$ $x^2-25x+144=0$ $(x-16)(x-9)=0$ $x=16$ or $x=9$ **OR/*OF*** $x-7\sqrt{x}+12=0$ $\left(\sqrt{x}-3\right)\left(\sqrt{x}-4\right)=0$ or let $\sqrt{x}=k$ $\sqrt{x}=3$ or $\sqrt{x}=4$ $x=9$ or $x=16$	✓ isolating the root ✓ squaring both sides ✓ standard form ✓ both answers (4) **OR/*OF*** ✓ standard form ✓ factors ✓ answers ✓ both answers for x (4)
1.2	$2x-y=2$ $y=2x-2$ .....(1) $xy=4$ .....(2) (1) in (2): $x(2x-2)=4$ $2x^2-2x-4=0$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2$ or $x=-1$ $y=2$ $y=-4$	✓ eq 1 ✓ substitution ✓ standard form ✓ x-values ✓ y-values (5)

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	OR/*OF*	**OR/*OF***
	$2x-y=2$	
	$x=\frac{1}{2}y+1$ .....(1)	✓ eq 1
	$xy=4$ .....(2)	
	(1) in (2):	
	$y\left(\frac{1}{2}y+1\right)=4$	✓ substitution
	$\frac{1}{2}y^2+y-4=0$	✓ standard form
	$y^2+2y-8=0$	
	$(y+4)(y-2)=0$	
	$y=-4$ or $y=2$	✓ *y*-values
	$x=-1$ $x=2$	✓ *x*-values (5)
	OR/OF	**OR/OF**
	$2x-y=2$ .....(1)	
	$y=\frac{4}{x}$ .....(2)	✓ eq 2
	(2) in (1):	
	$2x-\frac{4}{x}=2$	✓ substitution
	$2x^2-2x-4=0$ $x^2-x-2=0$	✓ standard form
	$(x-2)(x+1)=0$	
	$x=2$ or $x=-1$ $y=2$ $y=-4$	✓ *x*-values ✓ *y*-values (5)

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	OR/OF $2x-y=2$ .....(1) $x=\frac{4}{y}$ .....(2) (2)in (1): $2\left(\frac{4}{y}\right)-y=2$ $8-y^2-2y=0$ $y^2+2y-8=0$ $(y+4)(y-2)=0$ $y=-4$ or $y=2$ $x=-1$ $x=2$	**OR/OF** ✓ eq 2 ✓ substitution ✓ standard form ✓ y-values ✓ x-values (5)
1.3	$2.5^n-5^{n+1}+5^{n+2}=2.5^n-5^n.5^1+5^n.5^2$ $=5^n(2-5+25)$ $=5^n(22)$ $2\left(5^n(11)\right)$ **OR/OF** Any integer multiplied by an even number will be even	✓ exp law ✓common factor ✓answer/explanation (3)
1.4	$\frac{3^{y+1}}{32}=\sqrt{96^x}$ $\frac{3^{y+1}}{2^5}=(96)^{\frac{x}{2}}$ $3^{y+1}.2^{-5}=2^{\frac{5x}{2}}.3^{\frac{x}{2}}$ $-5=\frac{5x}{2}$ $\therefore x=-2$ $y+1=\frac{x}{2}$ $y+1=\frac{-2}{2}$ $\therefore y=-2$	✓ $\frac{3^{y+1}}{2^5}=(96)^{\frac{x}{2}}$ ✓ $3^{y+1}.2^{-5}=2^{\frac{5x}{2}}.3^{\frac{x}{2}}$ ✓ $x=-2$ ✓ $y=-2$ (4)

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	OR/OF $\frac{3^{y+1}}{32} = \sqrt{96^x}$ $\left(\frac{3^{y+1}}{2^5}\right)^2 = \left(\sqrt{(96)^x}\right)^2$ $\frac{3^{2y+2}}{2^{10}} = 2^{5x}.3^x$ $3^{2y+2}.2^{-10} = 2^{5x}.3^x$ $-10 = 5x$ $\therefore x = -2$ $2y + 2 = -2$ $\therefore y = -2$	**OR/OF** ✓ $\left(\frac{3^{y+1}}{2^5}\right)^2 = \left(\sqrt{(96)^x}\right)^2$ ✓ $3^{2y+2}.2^{-10} = 2^{5x}.3^x$ ✓ $x = -2$ ✓ $y = -2$ (4)
		[25]

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 2x - 15 = 0$ (3)

1.1.2 $5x^2 - x - 9 = 0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.3 $x^2 \leq 3x$ (4)

1.2 Given: $a + \frac{64}{a} = 16$

1.2.1 Solve for a. (3)

1.2.2 Hence, solve for x: $2^x + 2^{6-x} = 16$ (3)

1.3 **Without using a calculator**, calculate the value of $\sqrt{\frac{2^{1002} + 2^{1006}}{17(2)^{998}}}$ (4)

1.4 Solve for x and y simultaneously:

$2x - y = 2$ and $\frac{1}{x} - 3y = 1$ (6)

[26]

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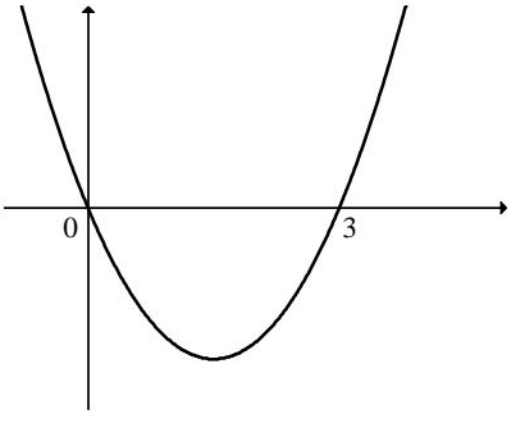
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QUESTION/*VRAAG* 1

1.1.1	$x^2+2x-15=0$ $(x+5)(x-3)=0$ $x=-5$ or $x=3$	✓ factors ✓ $x=-5$ ✓ $x=3$ (3)
1.1.2	$5x^2-x-9=0$ $x=\frac{-(-1)\pm\sqrt{(-1)^2-4(5)(-9)}}{2(5)}$ $x=\frac{1\pm\sqrt{181}}{10}$ $x=1{,}45$ or $x=-1{,}25$	✓substitution into the correct formula ✓ $x=1{,}45$ ✓ $x=-1{,}25$ (3)
1.1.3	$x^2\le 3x$ $x^2-3x\le 0$ $x(x-3)\le 0$  $0\le x\le 3$ OR $x\in[0;3]$	✓ standard form ✓ factors ✓✓ answer (4)
1.2.1	$a+\frac{64}{a}=16$ $a^2-16a+64=0$ $(a-8)^2=0$ $a=8$	✓ standard form ✓ factors ✓ answer (3)

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1.2.2	$2^x + 2^{6-x} = 16$ $2^x + \frac{64}{2^x} = 16$ $2^x = 8$ (from 1.2.1) $2^x = 2^3$ $x = 3$	 ✓ exp law ✓ $2^x = 8$ ✓ answer (3)
1.3	$\sqrt{\frac{2^{1002}\left(1+2^4\right)}{17(2)^{998}}}$ $= \sqrt{\frac{2^4(17)}{17}}$ $= \sqrt{2^4}$ $= 2^2$ $= 4$	✓common factor ✓second factor ✓simplification ✓answer (4)
1.4	$2x - y = 2 \quad \ldots(1)$ $\frac{1}{x} - 3y = 1 \quad \ldots(2)$ $y = 2x - 2$ $\frac{1}{x} - 3(2x-2) = 1$ $\frac{1}{x} - 6x + 6 - 1 = 0$ $1 - 6x^2 + 6x - x = 0$ $-6x^2 + 5x + 1 = 0$ $6x^2 - 5x - 1 = 0$ $(6x+1)(x-1) = 0$ $x = -\frac{1}{6}$ or $x = 1$ $y = 2\left(-\frac{1}{6}\right) - 2$ or $y = 2(1) - 2$ $y = -\frac{7}{3}$ or $y = 0$	 ✓ $y = 2x - 2$ ✓substitution ✓simplification ✓standard form ✓x-values ✓y-values (6)

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OR/*OF*

$x = \frac{2+y}{2} \quad \ldots(1)$

$\frac{1}{x} - 3y = 1 \quad \ldots(2)$

$\frac{1}{\frac{2+y}{2}} - 3y = 1$

$\frac{2}{2+y} - 3y = 1$

$\frac{2-6y-3y^2}{2+y} = 1$

$2 - 6y - 3y^2 = 2 + y$

$-3y^2 - 7y = 0$

$-y(3y+7) = 0$

$y = 0 \quad \text{or} \quad y = -\frac{7}{3}$

$x = 1 \quad \text{or} \quad x = -\frac{1}{6}$

OR/*OF*

✓ $x = \frac{2+y}{2}$

✓substitution

✓simplification

✓standard form

✓*y*-values

✓*x*-values

(6)
[26]

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 2x - 24 = 0$ (3)

1.1.2 $2x^2 - 3x - 3 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 + 5x \leq -4$ (4)

1.1.4 $\sqrt{x+28} = 2 - x$ (4)

1.2 Solve simultaneously for x and y in:

$2y = 3 + x$ and $2xy + 7 = x^2 + 4y^2$ (6)

1.3 The roots of an equation are $x = \dfrac{-n \pm \sqrt{n^2 - 4mp}}{2m}$ where m, n and p are positive real numbers. The numbers m, n and p, in that order, form a geometric sequence. Prove that x is a non-real number. (4)
[24]

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QUESTION/VRAAG 1

1.1.1	$x^2-2x-24=0$ $(x-6)(x+4)=0$ $x=6$ or $x=-4$	✓factors ✓$x=6$ ✓$x=-4$ (3)
1.1.2	$2x^2-3x-3=0$ $x=\frac{3\pm\sqrt{(-3)^2-4(2)(-3)}}{2(2)}$ $x=\frac{3\pm\sqrt{33}}{4}$ $x=2{,}19$ or $x=-0{,}69$	✓substitution into the correct formula ✓$x=2{,}19$ ✓$x=-0{,}69$ (3)
1.1.3	$x^2+5x\le -4$ $x^2+5x+4\le 0$ $(x+4)(x+1)\le 0$ Critical values: $x=-4$ or $x=-1$  $-4\le x\le -1$ **OR/*OF*** $x\in[-4\,;-1]$	✓ standard form ✓ critical values ✓✓answer (4)
1.1.4	$\sqrt{x+28}=2-x$ $\left(\sqrt{x+28}\right)^2=(2-x)^2$ $x+28=4-4x+x^2$ $x^2-5x-24=0$ $(x-8)(x+3)=0$ $x\neq 8$ or $x=-3$	✓ squaring both sides ✓ standard form ✓ factors ✓ answers with selection (4)

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1.2	$2y=3+x$ $x=2y-3 \quad \ldots (1)$ $2xy+7=x^2+4y^2 \quad \ldots (2)$ $2y(2y-3)+7=(2y-3)^2+4y^2$ $4y^2-6y+7=4y^2-12y+9+4y^2$ $4y^2-6y+2=0$ $2y^2-3y+1=0$ $(2y-1)(y-1)=0$ $y=\frac{1}{2}$ or $y=1$ $x=-2$ or $x=-1$ **OR/*OF*** $2y=3+x$ $y=\frac{3}{2}+\frac{x}{2} \quad \ldots (1)$ $2xy+7=x^2+4y^2 \quad \ldots (2)$ $2x\left(\frac{3}{2}+\frac{x}{2}\right)+7=x^2+4\left(\frac{3}{2}+\frac{x}{2}\right)^2$ $3x+x^2+7=x^2+9+6x+x^2$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-2$ or $x=-1$ $y=\frac{1}{2}$ or $y=1$	✓equation 1 ✓substitution ✓simplification ✓standard form ✓y – values ✓x – values (6) **OR/*OF*** ✓equation 1 ✓substitution ✓simplification ✓standard form ✓x – values ✓y – values (6)
1.3	$\frac{n}{m}=\frac{p}{n}$ $n^2=mp$ $\Delta=b^2-4ac$ $\Delta=n^2-4mp$, but $n^2=mp$ $\Delta=n^2-4n^2$ **OR/*OF*** $\Delta=mp-4mp$ $\Delta=-3n^2$ $\Delta=-3mp$ $n^2>0$ $mp>0$ $\therefore -3n^2<0$ $\therefore -3mp<0$ $\therefore \Delta<0 \Rightarrow x$ is a non-real number	✓$\frac{n}{m}=\frac{p}{n}$ ✓$n^2=mp$ ✓$\Delta=-3n^2$ or $-3mp$ ✓$\Delta<0$ (4)
		[24]

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 20 = 0$ (3)

1.1.2 $3x^2 - 2x - 6 = 0$ (correct to TWO decimal places) (4)

1.1.3 $(x-1)^2 > 9$ (4)

1.1.4 $2\sqrt{x+6} + 2 = x$ (4)

1.2 Solve simultaneously for x and y:

$4x + y = 2$ and $4x + y^2 = 8$ (5)

1.3 If it is given that $2^x \times 3^y = 24^6$, determine the numerical value of $x - y$. (4)
[24]

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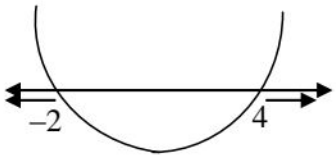
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QUESTION/*VRAAG* 1

1.1.1	$x^2 - x - 20 = 0$ $(x-5)(x+4) = 0$ $x = 5$ or $x = -4$	✓factors ✓ $x = 5$ ✓ $x = -4$ (3)
1.1.2	$3x^2 - 2x - 6 = 0$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$ $x = \frac{1 \pm \sqrt{19}}{3}$ $x = -1{,}12$ or $x = 1{,}79$	✓substitution ✓simplification ✓ $x = -1{,}12$ ✓ $x = 1{,}79$ (4)
1.1.3	$(x-1)^2 > 9$ $x^2 - 2x - 8 > 0$ $(x-4)(x+2) > 0$ Critical values: $x = 4$ or $x = -2$  $x < -2$ or $x > 4$ **OR/OF** $(-\infty;-2)$ or $(4;\infty)$ **OR/OF** $x - 1 > 3$ or $x - 1 < -3$ $x > 4$ or $x < -2$	✓standard form ✓critical values ✓✓ $x < -2$ or $x > 4$ (4) **OR/OF** ✓✓ $(-\infty;-2)$ or $(4;\infty)$ **OR/OF** ✓ $x - 1 > 3$ ✓ $x - 1 < -3$ ✓✓ $x > 4$ or $x < -2$ (4)

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1.1.4	$2\sqrt{x+6}+2=x$ $2\sqrt{x+6}=x-2$ $4(x+6)=(x-2)^2$ $4x+24=x^2-4x+4$ $x^2-8x-20=0$ $(x-10)(x+2)=0$ $x=10$ or $x\neq -2$	✓ isolating the surd ✓ $4x+24=x^2-4x+4$ ✓ $x=10$ ✓ $x\neq -2$ (4)
1.2	$4x=2-y$ . . . (1) $4x+y^2=8$ . . . (2) $\therefore 2-y+y^2=8$ $y^2-y-6=0$ $(y-3)(y+2)=0$ $y=3$ or $y=-2$ $x=-\frac{1}{4}$ or $x=1$ **OR/OF** $y=-4x+2$ . . . (1) $4x+y^2=8$ . . . (2) $4x+(-4x+2)^2=8$ $4x+16x^2-16x+4-8=0$ $16x^2-12x-4=0$ $4x^2-3x-1=0$ $(4x+1)(x-1)=0$ $x=-\frac{1}{4}$ or $x=1$ $y=3$ or $y=-2$	✓ $4x=2-y$ ✓substitution ✓standard form ✓y-values ✓x-values (5) **OR/OF** ✓ $y=-4x+2$ ✓substitution ✓standard form ✓x-values ✓y-values (5)
1.3	$2^x\times 3^y=(2^3\times 3)^6$ $2^x\times 3^y=2^{18}\times 3^6$ $2^x=2^{18}$ and $3^y=3^6$ $x=18$ and $y=6$ $\therefore x-y=18-6$ $\therefore x-y=12$	✓ $2^3\times 3$ ✓ 2^{18} or 3^6 ✓ $x=18$ or $y=6$ ✓ answer (A) (4)
		[24]

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 6x = 0$ (2)

1.1.2 $x^2 + 10x + 8 = 0$ (correct to TWO decimal places) (3)

1.1.3 $(1 - x)(x + 2) < 0$ (3)

1.1.4 $\sqrt{x+18} = x - 2$ (5)

1.2 Solve simultaneously for x and y:

$x + y = 3$ and $2x^2 + 4xy - y = 15$ (6)

1.3 If n is the largest integer for which $n^{200} < 5^{300}$, determine the value of n. (3)

[22]

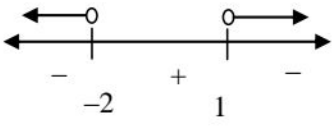
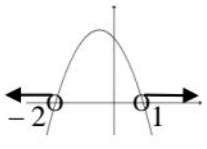
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REMEMBER: (A) next o the description of a tick implies accuracy mark
ONTHOU: (A) langs die beskrywing van `n regmerk impliseer akkuraatheids-punt
QUESTION/*VRAAG* 1

1.1.1	$x^2 - 6x = 0$ $x(x-6) = 0$ $x = 0$ or $x = 6$	✓ common factor ✓ both answers (2)
1.1.2	$x^2 + 10x + 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-10 \pm \sqrt{10^2 - 4(1)(8)}}{2(1)}$ $= \frac{-10 \pm \sqrt{68}}{2}$ $x = -0{,}88$ or $x = -9{,}12$	✓ substitution into the correct formula ✓ $-0{,}88$ ✓ $-9{,}12$ (3)
1.1.3	$(1-x)(x+2) < 0$ Critical values: $x = 1$ or $x = -2$  or  $x < -2$ or $x > 1$	✓ critical values ✓ correct method ✓ answer (3)
1.1.4	$\sqrt{x+18} = x-2$ $x+18 = x^2 - 4x + 4$ $0 = x^2 - 5x - 14$ $(x-7)(x+2) = 0$ $x = 7$ or $x \neq -2$	✓ squaring both sides (m) ✓ standard form ✓ factors ✓ both answers ✓ rejection of $x = -2$ (5)

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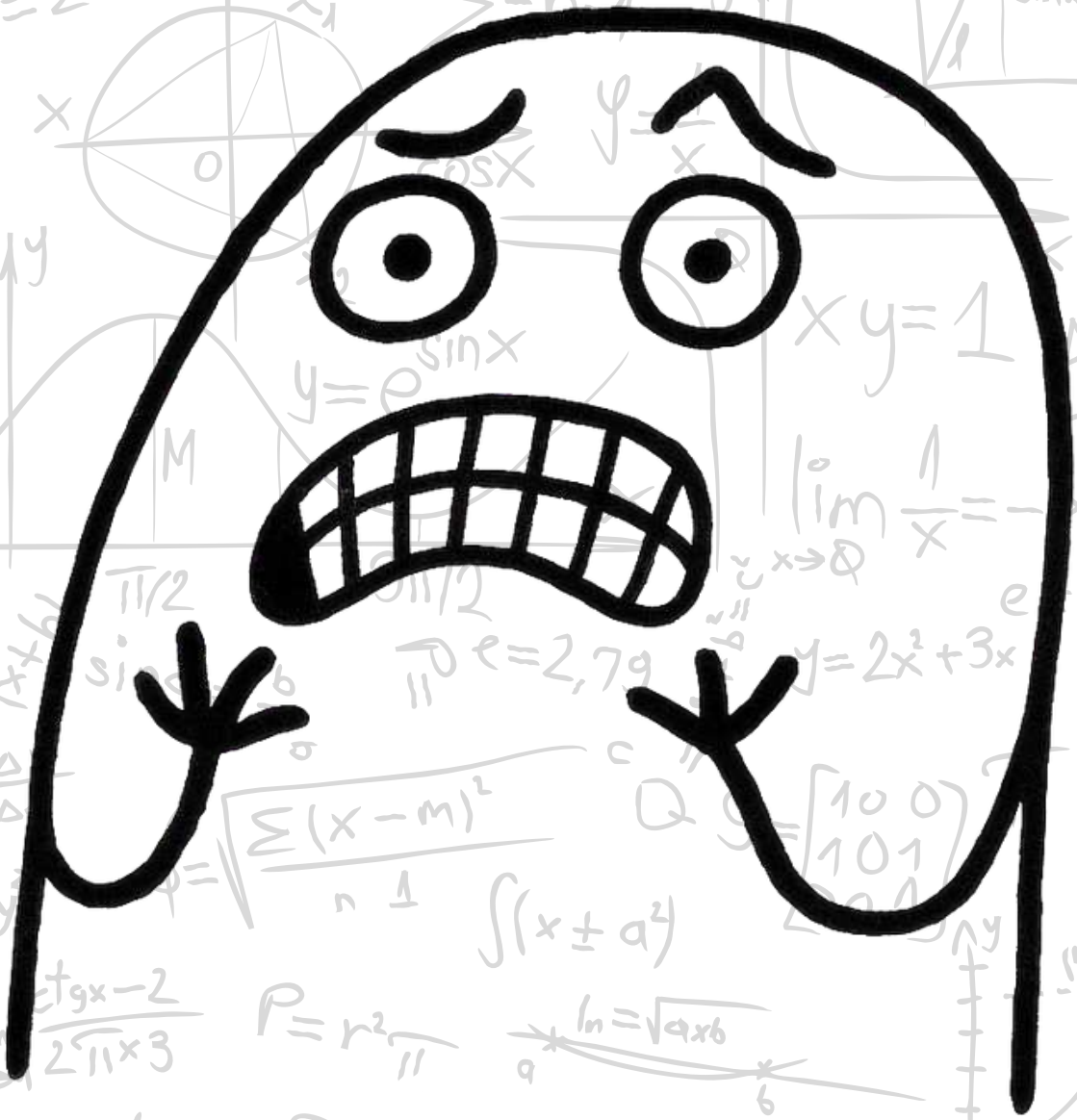
1.2	$x+y=3$ $y=3-x$ …….. (1) $2x^2+4xy-y=15$ …….. (2) Substitute (1) into (2): $2x^2+4x(3-x)-(3-x)=15$ $2x^2+12x-4x^2-3+x-15=0$ $-2x^2+13x-18=0$ $2x^2-13x+18=0$ $(2x-9)(x-2)=0$ $x=\frac{9}{2}$ or $x=2$ $y=-\frac{3}{2}$ or $y=1$ **OR** $x+y=3$ $x=3-y$ …….. (1) $2x^2+4xy-y=15$ …….. (2) Substitute (1) into (2): $2(3-y)^2+4(3-y)y-y=15$ $2y^2-12y+18-4y^2+12y-y-15=0$ $-2y^2-y+3=0$ $2y^2+y-3=0$ $(2y+3)(y-1)=0$ $y=-\frac{3}{2}$ or $y=1$ $x=\frac{9}{2}$ or $x=2$	✓ y subject of the formula ✓ substitution ✓ standard form ✓ factors ✓ x-values ✓ y-values (6) **OR** ✓ x subject of the formula ✓ substitution ✓ standard form ✓ factors ✓ y-values ✓ x-values (6)
1.3	$n^{200}<5^{300}$ $(n^2)^{100}<(5^3)^{100}$ $(n^2)^{100}<(125)^{100}$ $n^2<125$ Maximum value of n is 11. **OR** $200\log n<300\log 5$ $n<10^{\frac{3}{2}\log 5}$ $n<11{,}18$ $\therefore n=11$ **OR**	✓ $(n^2)^{100}<(5^3)^{100}$ ✓ $n^2<125$ ✓ 11 (3) **OR** ✓ use of logs ✓ $n<11{,}18$ ✓11 (3) **OR**

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	$n^{200} < 5^{300}$ $(n^2)^{100} < (5^3)^{100}$ $\sqrt{n^2} < \sqrt{5^3}$ $n < 5^{\frac{3}{2}}$ $n < 11{,}18$ $\therefore n = 11$ **OR** $n^{200} < 5^{300}$ $n < 5^{\frac{300}{200}}$ $n < 11{,}18$ $\therefore n = 11$	✓ $(n^2)^{100} < (5^3)^{100}$ ✓ $n < 11{,}18$ ✓ 11 (3) **OR** ✓ $n < 5^{\frac{300}{200}}$ ✓ $n < 11{,}18$ ✓ $n = 11$ (3)
		[22]

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SEQUENCES & SERIES



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QUESTION 2

2.1 The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

2.1.1 Calculate the sum of this series. (2)

2.1.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4)

2.2 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; ...

2.2.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3)

2.2.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5)
[14]

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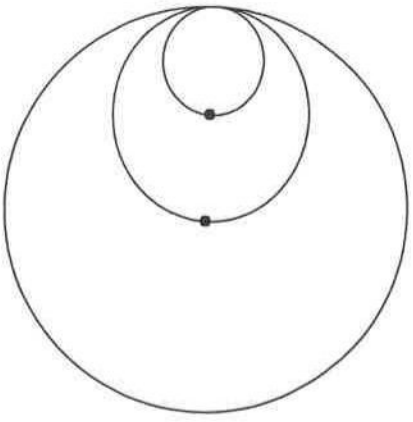
QUESTION/*VRAAG* 2

2.1.1	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{20} = \frac{20}{2}[2(7) + (20-1)(5)]$ $S_{20} = 1\,090$	✓ substitution ✓ answer (2)
2.1.2	$T_n = 7 + (n-1)(5)$ $T_n = 5n + 2$ $S_{75} - S_{20} = 14\,400 - 1\,090 = 13\,310$ $\sum_{n=1}^{75}(5n+2) - \sum_{n=1}^{20}(5n+2) = 13\,310$ $\therefore \sum_{n=21}^{75}(5n+2) = 13\,310$ **OR/*OF*** $T_{21} + T_{22} + T_{23} + ... + T_{75} = 14\,400 - 1\,090$ $107 + 112 + 117 + ... = 13\,310$ $T_n = 102 + 5n$ $\therefore \sum_{1}^{55}(5n+102) = 13\,310$	✓ general term ✓ $S_{75} - S_{20}$ ✓ $\sum_{n=21}^{75}$ ✓ $= 13\,310$ (4) **OR/*OF*** ✓ $S_{75} - S_{20}$ ✓ general term ✓ $\sum_{n=1}^{55}$ ✓ $= 13\,310$ (4)
2.2.1	$T_n = 2n - 1$ 98^{th} first difference = $2(98) - 1$ $= 195$ Quadratic: $T_{98} = 9\,632 - 195$ $= 9\,437$	✓answer ✓method ✓answer (3)
2.2.2	28 ; 29 ; 32 1 3 5 7 2 2 2 $3a + b = 1$ $2a = 2$ $3(1) + b = 1$ $a = 1$ $b = -2$ $9a + 3b + c = 32$ $T_3 = 1(3)^2 - 2(3) + c = 32$ $c = 29$ **OR/*OF*** $1 - 2 + c = 28$ $c = 29$ $\therefore T_n = n^2 - 2n + 29$	✓ second difference ✓ $a = 1$ ✓ $b = -2$ ✓ method to calculate c ✓ $c = 29$ (5)
		[14]

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QUESTION 3

A circle with radius 6 cm is drawn.
A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram.
A third, smaller circle is drawn through the centre of the second circle and touches the second circle internally. The process of drawing circles continues and forms a geometric pattern.



3.1 Write down the radius of the 3^{rd} circle. (2)

3.2 Calculate the sum of the areas of the first 10 circles. (4)

3.3 Which circle has a diameter of $\frac{3}{128}$ cm? (4)

[10]

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QUESTION/*VRAAG* 3

3.1	2^{nd} circle: $\frac{6}{2}=3\text{ cm}$ 3^{rd} circle: $\frac{3}{2}=1{,}5\text{ cm}$ **OR/*OF*** 3^{rd} circle $=6\left(\frac{1}{2}\right)^2=1{,}5$	✓✓ radius 3^{rd} circle (2) **OR/*OF*** ✓✓ radius 3^{rd} circle (2)
3.2	$36\pi\ ;\ 9\pi\ ;\ \frac{9}{4}\pi\ ;\ ...$ $r=\frac{1}{4}\ \ ;\ \ a=36\pi$ $S_{10}=\frac{36\pi\left(\left(\frac{1}{4}\right)^{10}-1\right)}{\frac{1}{4}-1}$ $S_{10}=150{,}80$	✓ a ✓ r ✓ substitution ✓ answer (4)
3.3	$6\ ;\ 3\ ;\ \frac{3}{2}\ ...$ $\text{radius}=\frac{1}{2}\left(\frac{3}{128}\right)=\frac{3}{256}$ $\frac{3}{256}=6\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{512}=\left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^9=\left(\frac{1}{2}\right)^{n-1}$ **or/*of*** $n-1=\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{512}\right)$ $\therefore n-1=9$ $\quad$ $\therefore n-1=9$ $\therefore n=10$ $\quad$ $\therefore n=10$	✓ $\frac{3}{256}$ ✓ equating to radius general term ✓ simplification ✓ answer (4)

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	OR/*OF* 12 ; 6 ; 3 ; … $\text{diameter} = \frac{3}{128}$ $\frac{3}{128} = 12\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^{9} = \left(\frac{1}{2}\right)^{n-1}$ **or/*of*** $n-1 = \log_{\left(\frac{1}{2}\right)}\left(\frac{1}{512}\right)$ $\therefore n-1 = 9$ $\therefore n-1 = 9$ $\therefore n = 10$ $\therefore n = 10$	**OR/*OF*** ✓ $12\left(\frac{1}{2}\right)^{n-1}$ ✓ equating general term of diameter to $\frac{3}{128}$ ✓simplification ✓ answer (4)
		[10]

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QUESTION 2

2.1 Consider the geometric series: $4 + 2 + 1 + \frac{1}{2} + ...$

2.1.1 Does this series converge? Justify your answer. (2)

2.1.2 Calculate S_∞. (2)

2.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Calculate the value of k. (5)

[9]

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QUESTION 2/*VRAAG* 2

2.1.1	$r = \frac{1}{2}$ Yes, because $-1 < \frac{1}{2} < 1$	✓ $r = \frac{1}{2}$ ✓ answer with reason (2)
2.1.2	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{4}{1-\frac{1}{2}}$ $\therefore S_\infty = 8$	✓ substitution ✓ answer (2)
2.2	$\sum_{p=k}^{10} 3^{p-1} = 3^{k-1} + 3^{k+1-1} + 3^{k+2-1} + \ldots + 3^9$ $= 3^{k-1} + 3^k + 3^{k+1} + \ldots 3^9$ $S_n = \frac{a(r^n - 1)}{r-1}$ $29\,520 = \frac{3^{k-1}(3^{11-k} - 1)}{3-1}$ $3^{10} - 3^{k-1} = 59\,040$ $3^{k-1} = 9$ $k - 1 = 2$ $\therefore k = 3$	✓ $a = 3^{k-1}$ ✓ $r = 3$ ✓ $n = 11 - k$ ✓ substitution ✓ answer (5)
		[9]

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QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; ...

3.1.1 Show that the general term of this number pattern is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\,527$? (4)

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41}. (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)
[15]

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QUESTION 3/*VRAAG* 3

3.1.1	3 ; 7 ; 12 ; 18 ∨ ∨ ∨ First diff: 4 ; 5 ; 6 ∨ ∨ Second diff: 1 ; 1 $2a=1$ $a=\frac{1}{2}$ $3a+b=4$ $3\left(\frac{1}{2}\right)+b=4$ $b=\frac{5}{2}$ $a+b+c=3$ $\frac{1}{2}+\frac{5}{2}+c=3$ $c=0$ $T_n=\frac{1}{2}n^2+\frac{5}{2}n$	✓ $2a=1$ ✓ $3\left(\frac{1}{2}\right)+b=4$ ✓ $\frac{1}{2}+\frac{5}{2}+c=3$ (3)
3.1.2	$13\,527=\frac{1}{2}n^2+\frac{5}{2}n$ $n^2+5n-27\,054=0$ $(n-162)(n+167)=0$ $n=162$ or $n=-167$ $T_{161}=13\,363$ $\therefore T_{161}+164=13\,527$ 164 must be added. **OR/*OF*** $T_n=3+$ sum of 1st differences $13527=3+4+5+...+n$ $\frac{n-3+1}{2}[3+n]=13527$ $n^2+n-27060=0$ $(n+165)(n-167)=0$ $n=164$	✓ $13\,527=\frac{1}{2}n^2+\frac{5}{2}n$ ✓ standard form ✓ answers for n ✓ 164 (4) **OR/OF** ✓ $13527=3+4+5+...+n$ ✓ $n^2+n-27060=0$ ✓ answers for n ✓ 164 (4)

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3.2.1	$T_n = 8+(n-1)(3)$ $T_n = 3n+5$ $41 = 3n+5$ $36 = 3n$ $n = 12$	✓ $T_n = 3n+5$ ✓ $T_n = 41$ ✓ answer (3)
3.2.2a	$P_{41} = 12$	✓ answer (1)
3.2.2b	$P_8 = a+7d = 1$ $P_{11} = a+10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a+7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$	✓ $a+7d = 1$ ✓ $a+10d = 2$ ✓ value of d ✓ value of a (4)
		[15]

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QUESTION 2

2.1 Given the arithmetic series: $7 + 12 + 17 + \ldots$

2.1.1 Determine the value of T_{91} (3)

2.1.2 Calculate S_{91} (2)

2.1.3 Calculate the value of n for which $T_n = 517$ (3)

2.2 The following information is given about a quadratic number pattern:

$T_1 = 3$, $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)
[16]

QUESTION 3

3.1 Given the geometric series: $3 + 6 + 12 + \ldots$ to n terms.

3.1.1 Write down the general term of this series. (1)

3.1.2 Calculate the value of k such that: $\sum_{p=1}^{k} \frac{3}{2}(2)^p = 98\,301$ (4)

3.2 A geometric sequence and an arithmetic sequence have the same first term.

- The common ratio of the geometric sequence is $\frac{1}{3}$
- The common difference of the arithmetic sequence is 3
- The sum of 22 terms of the arithmetic sequence is 734 more than the sum to infinity of the geometric sequence.

Calculate the value of the first term. (5)
[10]

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QUESTION 2/*VRAAG* 2

2.1.1	$7 + 12 + 17 + \ldots..$ $T_n = a + (n-1)d$ $T_{91} = 7 + (91-1)(5)$ $T_{91} = 457$ **OR/*OF*** $d = 5$ $T_n = 5n + 2$ $T_{91} = 5(91) + 2$ $T_{91} = 457$	✓$d = 5$ ✓substitution into correct formula ✓answer (3) **OR/*OF*** ✓$d = 5$ ✓substitution $n = 91$ ✓answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$ $S_9 = 21\ 112$ **OR/*OF*** $S_n = \frac{n}{2}(a + l)$ $S_{91} = \frac{91}{2}(7 + 457)$ $S_{91} = 21\ 112$	✓substitution into correct formula ✓answer (2) **OR/*OF*** ✓substitution into correct formula ✓answer (2)
2.1.3	$T_n = 7 + (n-1)(5)$ $5n + 2 = 517$ $5n = 515$ $n = 103$	✓substitution into correct formula ✓equate ✓answer (3)
2.2.1	$T_1 = 3\,;\, T_2 - T_1 = 9$ and $T_3 - T_2 = 21$ 3 12 33 66 111 9 21 33 45 12 12 12 $\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$ **OR/*OF*** $2a = 12$ $a = 6$ $3(6) + b = 9$ $b = -9$ $6 - 9 + c = 3$ $T_5 = 6(5)^2 - 9(5) + 6 = 111$	✓ constant second diff = 12 ✓ first differences : 33 and 45 (2) **OR/*OF*** ✓ constant second diff = 12 ✓substitute 5 (2)

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2.2.2	$2a=12$ $a=6$ $3(6)+b=9$ or $5\times6+b=21$ $b=-9$ $6-9+c=3$ $c=6$ $T_n=6n^2-9n+6$	✓ $2a=12$ ✓ $3(6)+b=9/5\times6+b=21$ ✓ $6-9+c=3$ (3)
2.2.3	$T_n^{/}=12n-9>0$ $n>\frac{3}{4}$ $\therefore\ T_n$ is increasing for $n\in N$ **OR/*OF*** $n=-\frac{b}{2a}=-\frac{-9}{2(6)}$ $n=\frac{3}{4}$ $\therefore$ min at $n=1$ for $n\in N$ $\therefore\ T_n$ is increasing for $n\in N$	✓ $T_n^{/}=12n-9$ ✓ $n>\frac{3}{4}$ ✓ increasing for $n\in N$ (3) **OR/*OF*** ✓ $n=-\frac{b}{2a}=\frac{9}{2(6)}$ ✓ $n=\frac{3}{4}$ ✓ increasing for $n\in N$ (3)
		[16]

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QUESTION 3/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_n = 3(2)^{n-1}$	✓ $T_n = 3(2)^{n-1}$ (1)
3.1.2	$\sum_{p=1}^{k} \frac{3}{2}.2^p = 98\ 301$ $\sum_{p=1}^{k} \frac{3}{2}.2^p = 3+6+12+...$ $n = k$ $\frac{3\left[(2)^k - 1\right]}{2-1} = 98\ 301$ $(2)^k = 32\ 768$ $2^k = 2^{15}$ **OR/*OF*** $k = \log_2 32\ 768$ $\therefore k = 15$	✓expansion ✓ $n = k$ ✓substitution into correct formula ✓$k = 15$ (4)
3.2	$S_{22} = \frac{22}{2}\left[2a + 21(3)\right]$ $S_{22} = 22a + 693$ $S_\infty = \frac{a}{1-\frac{1}{3}}$ $= \frac{3a}{2}$ $\therefore 22a + 693 = \frac{3a}{2} + 734$ $44a + 1386 = 3a + 1468$ $41a = 82$ $a = 2$	✓ substitution into S_n ✓ $S_{22} = 22a + 693$ ✓substitution into S_∞ ✓ $S_{22} = S_\infty + 734$ ✓ answer (5)
		[10]

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QUESTION 2

2.1 Given the geometric series: $\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+...$

2.1.1 Is this a convergent geometric series? Justify your answer with the necessary calculations. (2)

2.1.2 Calculate the sum to infinity of this series. (2)

2.2 An arithmetic and a geometric sequence are combined to form the pattern, which is given by: $P_n = x\,;\frac{1}{3}\,;2x\,;\frac{1}{9}\,;3x\,;\frac{1}{27}\,;...$

2.2.1 Write down the next TWO terms of the pattern. (2)

2.2.2 Determine the general term (T_n) for the odd terms of this pattern. Write down your answer in terms of x. (2)

2.2.3 Calculate the value of P_{26}. (3)

2.2.4 If $\sum_{n=1}^{21} P_n = 33,5$, determine the value of x. (6)
[17]

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QUESTION 2/*VRAAG* 2

2.1.1	$\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+...$ $r=\frac{\frac{1}{15}}{\frac{1}{5}}=\frac{1}{3}$ $-1<\frac{1}{3}<1$ $\therefore$ the series is convergent.	✓ $r=\frac{1}{3}$ ✓ answer (any indicator of convergence) (2)
2.1.2	$S_\infty=\frac{a}{1-r}$ $=\frac{\frac{1}{5}}{1-\frac{1}{3}}$ $=\frac{3}{10}$	✓ substitution ✓ answer (2)
2.2.1	$4x\ ;\ \frac{1}{81}$	✓ $4x$ ✓ $\frac{1}{81}$ (2)
2.2.2	$T_n=x+(n-1)x$ $=x+xn-x$ $=xn$ Answer only: Full Marks	✓ substitution ✓ answer (2)
2.2.3	$T_n=ar^{n-1}$ $T_{13}=\frac{1}{3}\left(\frac{1}{3}\right)^{13-1}$ $T_{13}=\left(\frac{1}{3}\right)^{13}$ or $\frac{1}{1\,594\,323}$ or $6{,}27\times10^{-7}$ or 3^{-13}	✓ $n=13$ ✓ $r=\frac{1}{3}$ ✓ answer (3)
2.2.4	$\sum_{n=1}^{21}P_n=S_{11}+S_{10}$ $=\frac{11}{2}[2x+10x]+\frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^{10}\right]}{1-\frac{1}{3}}$ $=66x+0{,}5$ $33{,}5=66x+0{,}5$ $\therefore x=\frac{1}{2}$	✓ S_{11} ✓ $+S_{10}$ ✓ arithmetic sum ✓ geometric sum ✓ $66x+0{,}5$ (A) ✓ answer (6)
		[17]

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QUESTION 3

A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

3.1 Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$. (6)

3.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)
[9]

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QUESTION 3/*VRAAG* 3

3.1	$x \;;\; x \;;\; T_3 \;;\; \ldots$ $0 \quad T_3 - x$ 10	
	$2a = 10 \qquad 3a + b = 0$ $a = 5 \qquad\quad b = -15$	✓ $2a = 10$ ✓ $3a + b = 0$
	$T_3 - x - 0 = 10$ $\therefore T_3 = x + 10$	✓ $T_3 = x + 10$
	$2x + T_3 = 28$ $2x + x + 10 = 28$ $3x = 18$ $x = 6$	✓ $2x + T_3 = 28$ ✓ $x = 6$
	$a + b + c = 6$ $5 - 15 + c = 6$ $c = 16$ $\therefore T_n = 5n^2 - 15n + 16$	✓ $5 - 15 + c = 6$ (6)
	OR/*OF*	**OR/*OF***
	$2a = 10$ $\therefore a = 5$	✓ $2a = 10$
	$T_1 = a + b + c \quad T_2 = 4a + 2b + c \quad T_3 = 9a + 3b + c$ $= 5 + b + c \qquad = 20 + 2b + c \qquad = 45 + 3b + c$	
	$5 + b + c = 20 + 2b + c$ $b = -15$	✓ $5 + b + c = 20 + 2b + c$
	$T_1 = -10 + c \qquad T_2 = -10 + c \qquad T_3 = c$	✓ $T_1 = -10 + c$ ✓ $T_2 = -10 + c$
	$T_1 + T_2 + T_3 = -10 + c - 10 + c + c$ $28 = 3c - 20$ $c = 16$	✓ $28 = 3c - 20$ ✓ $c = 16$ (6)

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3.2	$T_n = 5n^2 - 15n + 16$ $216 = 5n^2 - 15n + 16$ $5n^2 - 15n - 200 = 0$ $n^2 - 3n - 40 = 0$ $(n-8)(n+5) = 0$ $n = 8 \quad \text{or} \quad n \neq -5$ $\therefore T_8 = 216$	✓ equating ✓ standard form ✓ $n = 8$ (3)
		[9]

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QUESTION 2

2.1 The first term of a geometric series is 14 and the 6^{th} term is 448.

2.1.1 Calculate the value of the constant ratio, r. (2)

2.1.2 Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of 114 674. (4)

2.1.3 If the first term of another series is 448 and the 6^{th} term is 14, calculate the sum to infinity of the new series. (3)

2.2 If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$, determine the value of k. (5)
[14]

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QUESTION 2/*VRAAG* 2

2.1.1	$a=14$ $T_6=14r^5=448$ $r^5=32$ $\therefore r=2$ Answer only: full marks	✓ $T_6=14r^5=448$ ✓ $r=2$ (2)
2.1.2	$T_n=14(2)^{n-1}$ $S_n=\frac{14(2^6-1)}{2-1}$ $S_6=882$ $114\,674-882=113\,792$ $113\,792=896\left(2^n-1\right)$ $128=2^n$ $n=7$ **OR/OF** $S_n=\frac{a(r^n-1)}{r-1}$ $114\,674=\frac{14(2^n-1)}{2-1}$ $8\,191=2^n-1$ $2^n=8\,192$ $n=\log_2 8\,192$ $n=13$ $\therefore$ 7 more terms must be added to the first 6 terms.	✓ substitution into correct formula ✓ $S_6=882$ ✓ $128=2^n$ ✓ 7 (4) **OR/OF** ✓ substitution into correct formula ✓ $2^n=8\,192$ ✓ $n=13$ ✓ 7 (4)
2.1.3	$r=\frac{1}{2}$ OR $448r^5=14$ $\therefore r=\frac{1}{2}$ $S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{448}{1-\frac{1}{2}}$ $S_\infty=896$	✓ $r=\frac{1}{2}$ ✓ substitution ✓ answer (3)

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2.2	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$	
	$T_1=\frac{1}{6}\qquad T_2=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}$	✓ $T_1=\frac{1}{6}$
	$d=\frac{3}{6}-\frac{1}{6}=\frac{1}{3}$	✓ d
	$\frac{121}{6}=\frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$	✓ substitution
	$\frac{121}{3}=n\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$	
	$\frac{121}{3}=\frac{1}{3}n^2$ $121=n^2$	
	$n=11$ $\therefore k=10$	✓ value of n ✓ value of k (5)
	OR/*OF*	**OR/*OF***
	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$	
	$a=\frac{1}{6}$	✓ $a=\frac{1}{6}$
	$l=\frac{1}{3}k+\frac{1}{6}$	✓ l
	$n=k+1$	✓ $n=k+1$
	$S_n=\frac{n}{2}[a+l]$	
	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{6}+\frac{1}{3}k+\frac{1}{6}\right]$	
	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{3}k+\frac{1}{3}\right]$	
	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{k+1}{3}\right]$	
	$\frac{121}{6}=\frac{(k+1)^2}{6}$	✓ $\frac{121}{6}=\frac{(k+1)^2}{6}$
	$k+1=\pm\sqrt{121}$ $k+1=11$ $k=10$	✓ value of k (5)
		[14]

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QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

3.2 Determine the value of the 60^{th} term of this number pattern. (2)

3.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. (3)

3.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)
[10]

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QUESTION 3/*VRAAG* 3

3.1	$3a+b=7$ $3+b=7$ $b=4$ **OR/OF** $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$	✓ $3a+b=7$ ✓ $3+b=7$ (2) **OR/OF** ✓ $T_2-T_1=7$ ✓ substitution (2)
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$ Answer only: full marks	✓ substitution ✓ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ Answer only: full marks **OR/*OF*** First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$	✓first difference ✓ 2 ✓ $2p+5$ (3) **OR/*OF*** ✓first difference ✓ 2 ✓ $2p+5$ (3)
3.4	$157=2p+5$ $p=76$ ∴ Between T_{76} and T_{77} **OR/OF** $T_{n+1}-T_n=157$ $(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$ $n^2+2n+1+4n+4+9-n^2-4n-9=157$ $2n=152$ $n=76$ ∴ Between T_{76} and T_{77}	✓ $157=2p+5$ ✓ $p=76$ ✓ T_{76} and T_{77} (3) **OR/OF** ✓ $T_{n+1}-T_n=157$ ✓ $n=76$ ✓ T_{76} and T_{77} (3)
		[10]

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QUESTION 2

2.1 The first term of an arithmetic sequence is -1 and the 7^{th} term is 35.

Determine:

2.1.1 The common difference of the sequence (2)

2.1.2 The number of terms in the sequence if the last term of the sequence is 473 (3)

2.1.3 The sum of the first 40 terms in this sequence (2)

2.2 75 ; 53 ; 35 ; 21 ; ... is a quadratic number pattern.

2.2.1 Write down the FIFTH term of the number pattern. (1)

2.2.2 Determine the n^{th} term of the number pattern. (4)

2.2.3 Determine the maximum value of the following number pattern:

$$-15\ ;-\frac{53}{5}\ ;-7\ ;-\frac{21}{5}\ ;\ldots$$ (4)

[16]

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QUESTION/*VRAAG* 2

2.1.1	$a+6d=35$ $-1+6d=35$ $6d=36$ $d=6$ **OR/OF** $\frac{35-(-1)}{7-1}=6$ ANSWER ONLY: FULL MARKS	✓ substitution ✓answer (2) **OR/OF** ✓ substitution ✓answer (2)
2.1.2	$T_n=a+(n-1)d$ $473=-1+(n-1)(6)$ $79=n-1$ $\therefore n=80$ ANSWER ONLY: FULL MARKS	✓substitution into the correct formula ✓ equating to 473 ✓ answer (3)
2.1.3	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{40}=\frac{40}{2}[2(-1)+(40-1)(6)]$ $\therefore S_{40}=4\,640$ **OR/*OF*** $T_{40}=6(40)-7$ $=233$ $S_n=\frac{n}{2}(a+l)$ $=\frac{40}{2}(-1+233)$ $=4\,640$	✓ substitution ✓answer (2) **OR/*OF*** ✓ substitution ✓answer (2)
2.2.1	75 53 35 21 −22 −18 −14 4 4 $T_5=11$	✓answer (A) (1)
2.2.2	$T_n=an^2+bn+c$ $2a=4$ $a=2$ $3a+b=-22$ $6+b=-22$ $b=-28$ $a+b+c=75$ $2-28+c=75$ $c=101$ $\therefore T_n=2n^2-28n+101$	✓ $T_n=an^2+bn+c$ ✓ $a=2$ ✓ $b=-28$ ✓ $c=101$ (4)

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2.2.3 Minimum value of T_n

$n = -\frac{b}{2a} = -\frac{(-28)}{2(2)}$

$n = 7$ ✓ $n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓ min value = 3

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓ $-\frac{1}{5}$ value of term of old pattern

Maximum value of new pattern $= -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$ (4)

OR/*OF*

$T_n' = 4n - 28$

$4n - 28 = 0$

$4n = 28$

$n = 7$ ✓$n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓ min value = 3

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓ $-\frac{1}{5}$ value of term of old pattern

Maximum value of new pattern $= -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$ (4)

OR/OF

$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$ ✓✓ $T_n \div (-5)$

$n = -\frac{b}{2a} = \frac{-\frac{28}{5}}{2\left(\frac{-2}{5}\right)}$

$= 7$ ✓$n = 7$

$T_7 = -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$ (4)

OR/OF **OR/OF**

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$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$

$T'_n = -\frac{4}{5}n + \frac{28}{5}$ ✓✓ $T_n \div (-5)$

$-\frac{4}{5}n + \frac{28}{5} = 0$

$-4n = -28$

$n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓$n = 7$

Maximum value of new pattern $= -\frac{3}{5}$

✓ max value $= -\frac{3}{5}$

(4)

[16]

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QUESTION 3

3.1 Consider the following geometric sequence: 1 024 ; 256 ; 64 ; ...

Calculate:

3.1.1 The 10^{th} term of the sequence (2)

3.1.2 $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ (4)

3.2 The first two terms of a geometric sequence are:

$-t^2-6t-9$ and $\frac{t^3+9t^2+27t+27}{2}$

Determine the values of t for which the sequence will converge. (5)
[11]

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QUESTION/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_{10} = 1\,024\left(\frac{1}{4}\right)^{10-1}$ $\therefore T_{10} = \frac{1}{256}$ ANSWER ONLY: FULL MARKS	✓substitution into the correct formula ✓ answer (2)
3.1.2	$\sum_{p=0}^{8} 256\left(4^{1-p}\right) = 1\,024 + 256 + 64 + \ldots$ $S_n = \frac{a\left[1-r^n\right]}{1-r}$ $S_9 = \frac{1024\left[1-\left(\frac{1}{4}\right)^9\right]}{1-\frac{1}{4}}$ $S_9 = \frac{87\,381}{64}$ $= 1\,365{,}33$ **OR/OF** $\sum_{p=0}^{8} 256\left(4^{1-p}\right)$ $= 1\,024 + 256 + 64 + 16 + 4 + 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$ $S_9 = \frac{87\,381}{64}$ $= 1\,365{,}33$	✓ 1024 ✓ $n = 9$ ✓substitution into the correct formula ✓answer (4) **OR/OF** ✓ 1024 ✓ rest of expansion ✓ $n = 9$ terms ✓ answer (4)
3.2	$-t^2 - 6t - 9\,;\, \frac{t^3 + 9t^2 + 27t + 27}{2}$ $-\left(t^2 + 6t + 9\right);\ \frac{1}{2}(t+3)\left(t^2 + 6t + 9\right)$ $-(t+3)^2\ ;\ \frac{1}{2}(t+3)^3$ $r = \frac{-(t+3)}{2}$ $-1 < \frac{-t-3}{2} < 1$ $-2 < -t - 3 < 2$ $1 < -t < 5$ $-5 < t < -1$	✓ $r = \frac{\frac{t^3 + 9t^2 + 27t + 27}{2}}{-t^2 - 6t - 9}$ ✓ $-(t^2 + 6t + 9)$ ✓ $\frac{1}{2}(t+3)\left(t^2 + 6t + 9\right)$ ✓ $-1 < \frac{-t-3}{2} < 1$ ✓ answer (5)
		[11]

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QUESTION 2

Given the geometric series: $x + 90 + 81 + \ldots$

2.1 Calculate the value of x. (2)

2.2 Show that the sum of the first n terms is $S_n = 1\,000\left(1-(0,9)^n\right)$. (2)

2.3 Hence, or otherwise, calculate the sum to infinity. (2)
[6]

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QUESTION/VRAAG 2

2.1	$\frac{90}{x}=\frac{81}{90}$ $81x=8\,100$ $x=100$ **OR/OF** $x=90\times\frac{10}{9}$ $x=100$	✓ $\frac{90}{x}=\frac{81}{90}$ ✓ answer (2) **OR/OF** ✓ $\frac{10}{9}$ ✓ answer (2)
2.2	$S_n=\frac{a\left(1-r^n\right)}{1-r}$ $S_n=\frac{100\left(1-(0{,}9)^n\right)}{1-0{,}9}$ $S_n=\frac{100\left(1-(0{,}9)^n\right)}{0{,}1}$ $\therefore S_n=1\,000\left(1-(0{,}9)^n\right)$	✓$r=0{,}9$ ✓substitution into correct formula (2)
2.3	$S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{100}{1-\frac{9}{10}}$ $S_\infty=1000$ **OR/OF** $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0{,}9)^n\right)\right]$ $S_\infty=1000$	✓ substitution ✓ answer (2) **OR/OF** ✓ $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0{,}9)^n\right)\right]$ ✓ answer (2)
		[6]

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QUESTION 3

Consider the quadratic number pattern: $-145\ ;\ -122\ ;\ -101\ ;\ \ldots$

3.1 Write down the value of T_4. (1)

3.2 Show that the general term of this number pattern is $T_n = -n^2 + 26n - 170$. (3)

3.3 Between which TWO terms of the quadratic number pattern will there be a difference of -121? (4)

3.4 What value must be added to each term in the number pattern so that the value of the maximum term in the new number pattern formed will be 1? (3)

[11]

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QUESTION 3

3.1	-82	✓ answer (1)
3.2	$-145\ ;\ -122\ ;\ -101\ ;\ \ldots$ 23 21 -2 -2 $2a=-2 \quad \therefore\ a=-1$ $3a+b=23 \quad \therefore\ 3(-1)+b=23 \quad \therefore\ b=26$ $a+b+c=-145\ \therefore\ -1+26+c=-145 \quad \therefore\ c=-170$ $\therefore T_n=-n^2+26n-170$ **OR/OF** $2a=-2 \quad \therefore\ a=-1$ $c=-145+(-2)-23=-170$ $\therefore T_n=-n^2+bn-170$ $-145=-1+b-170$ $b=26$ $\therefore T_n=-n^2+26n-170$	✓ $2a=-2$ ✓ $3(-1)+b=23$ ✓ $-1+26+c=-145$ (3) **OR/OF** ✓ $2a=-2$ ✓ $c=-145+(-2)-23$ ✓ $-145=-1+b-170$ (3)
3.3	$T_n=bn+c$ or $T_n=a+(n-1)d$ $T_n=-2n+25$ $=23+(n-1)(-2)$ $=25-2n$ $-2n+25=-121$ $-2n=-146$ $n=73$ Between T_{73} and T_{74} **OR/*OF*** $T_{n+1}-T_n=-(n+1)^2+26(n+1)-170-(-n^2+26n-170)$ $-121=-2n+25$ $n=73$ Between T_{73} and T_{74}	✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓answer (4) **OR/*OF*** ✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓ answer (4)
3.4	$n=\dfrac{-b}{2a}=\dfrac{-26}{2(1)}=13$ $T_{13}=-1$ $\therefore$ add 2 **OR/*OF*** $T'_n=-2n+26=0$ $n=13$ $T_{13}=-(13)^2+26(13)-170=-1$ $\therefore$ add 2	✓13 ✓ $T_{13}=-1$ ✓ add 2 (3) **OR/*OF*** ✓13 ✓ $T_{13}=-1$ ✓ add 2 (3)
		[11]

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QUESTION 2

2.1 Consider the quadratic sequence: 72 ; 100 ; 120 ; 132 ; …

2.1.1 Determine T_n, the n^{th} term of the quadratic sequence. (4)

2.1.2 A term in the quadratic sequence 72 ; 100 ; 120 ; 132 ; … is equal to the twelfth term of the sequence of first-differences. Determine the position of this term in the quadratic sequence. (5)

2.1.3 Determine the maximum value of the quadratic sequence. (3)

2.1.4 Determine the maximum value of the sequence:

–23 ; 5 ; 25 ; 37 ; … (1)

2.2 Consider the sequence: $-11 ; 2\sin 3x ; 15 ; \ldots$
Determine the values of x in the interval $[0°; 90°]$ for which the sequence will be arithmetic. (4)
[17]

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QUESTION/*VRAAG* 2

2.1.1	72 ; 100 ; 120 ; 132 ; … 28 20 12 −8 − 8 $2a = -8$ $a = -4$ $3a + b = 28$ $b = 40$ $a + b + c = 72$ $c = 36$ $T_n = -4n^2 + 40n + 36$	✓ second differences = – 8 ✓ $a = -4$ ✓ $b = 40$ ✓ $c = 36$ (4)
2.1.2	$T_{12} = 36 - 8n = 36 - 8(12) = -60$ $-4n^2 + 40n + 36 = -60$ $n^2 - 10n - 24 = 0$ $(n-12)(n+2) = 0$ $\therefore n = 12$	✓ $36 - 8n$ ✓ -60 ✓ standard form ✓factors ✓ $n = 12$ (5)
2.1.3	$T_n' = -8n + 40 = 0$ $n = 5$ $T_n = -4(5)^2 + 40(5) + 36 = 136$ **OR/OF** $\frac{-b}{2a} = \frac{40}{8} = 5$ $T_n = -4(5)^2 + 40(5) + 36$ $= 136$	✓ $-8n + 40$ ✓ $n = 5$ ✓136 (3) **OR/OF** ✓substitution ✓ $n = 5$ ✓136 (3)
2.1.4	Maximum value $= 41$	✓value (1)
2.2	$2\sin 3x - (-11) = 15 - 2\sin 3x$ $4\sin 3x = 4$ $\sin 3x = 1$ $3x = 90^\circ$ $\therefore x = 30^\circ$	✓equating ✓ $4\sin 3x = 4$ ✓ $\sin 3x = 1$ ✓answer (4)
		[17]

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QUESTION 3

3.1 If $r = \frac{1}{5}$ and $a = 2\,000$, determine:

3.1.1 T_n, the general term of the series (1)

3.1.2 T_7 (1)

3.1.3 Which term of the series will have a value of $\frac{16}{15\,625}$ (3)

3.2 Consider the geometric series where $\sum_{n=1}^{\infty} T_n = 27$ and $S_3 = 26$.

Calculate the value of the constant ratio (r) of the series. (4)
[9]

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QUESTION/*VRAAG* 3

3.1.1	$T_n = ar^{n-1} = 2000\left(\frac{1}{5}\right)^{n-1}$	✓ $2000\left(\frac{1}{5}\right)^{n-1}$ (1)
3.1.2	$T_7 = 2000\left(\frac{1}{5}\right)^{7-1} = \frac{16}{125}$	✓ $\frac{16}{125}$ (1)
3.1.3	$\frac{16}{15625} = 2000\left(\frac{1}{5}\right)^{n-1}$ $\frac{1}{1953125} = \left(\frac{1}{5}\right)^{n-1}$ $\left(\frac{1}{5}\right)^{9} = \left(\frac{1}{5}\right)^{n-1}$ **OR** $n-1 = \log_{\frac{1}{5}} \frac{1}{1953125}$ $n-1=9$ $n=10$	✓ equating ✓ same base / use of log ✓ answer (3)
3.2	$S_\infty = 27 = \frac{a}{1-r}$ $S_3 = \frac{a(1-r^3)}{1-r} = 26$ $27(1-r^3) = 26$ $1-r^3 = \frac{26}{27}$ $r^3 = \frac{1}{27}$ $\therefore r = \frac{1}{3}$ **OR/OF** $S_\infty = 27 = \frac{a}{1-r}$ $a = 27(1-r)$ But $a + ar + ar^2 = 26$ $a(1+r+r^2) = 26$ $27(1-r)(1+r+r^2) = 26$ $(1-r)(1+r+r^2) = \frac{26}{27}$ $r^2 + r + 1 - r^3 - r^2 - r = \frac{26}{27}$ $-r^3 + 1 = \frac{26}{27}$ $r^3 = \frac{1}{27}$ $\therefore r = \frac{1}{3}$	✓ $S_\infty = 27 = \frac{a}{1-r}$ ✓ $S_3 = \frac{a(1-r^3)}{1-r} = 26$ ✓substitution ✓ $r = \frac{1}{3}$ (4) **OR/OF** ✓ $a = 27(1-r)$ ✓ $a + ar + ar^2 = 26$ ✓substitution ✓ $r = \frac{1}{3}$ (4)
		[9]

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QUESTION 2

2.1 $7 ; x ; y ; -11 ; \ldots$ is an arithmetic sequence. Determine the values of x and y. (4)

2.2 Given the quadratic number pattern: $-3 ; 6 ; 27 ; 60 ; \ldots$

2.2.1 Determine the general term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.2.2 Calculate the value of the 50^{th} term of the pattern. (2)

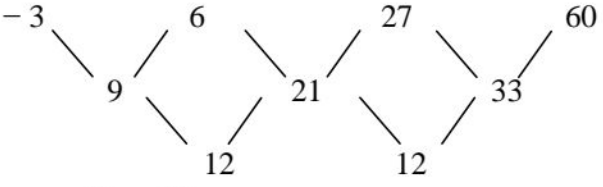
2.2.3 Show that the sum of the first n first-differences of this pattern can be given by $S_n = 6n^2 + 3n$. (3)

2.2.4 How many consecutive first-differences were added to the first term of the quadratic number pattern to obtain a term in the quadratic number pattern that has a value of 21 060? (4)
[17]

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QUESTION/*VRAAG* 2

2.1	$7\ ;\ x\ ;\ y\ ;\ -11\ ;\ \ldots$ $a = 7$ $a + 3d = -11$ $7 + 3d = -11$ $d = -6$ $x = a + d = 7 + (-6) = 1$ $y = a + 2d = 7 + 2(-6) = -5$	✓ $7 + 3d = -11$ ✓ $d = -6$ ✓ value of x ✓ value of y (4)
	OR $a + 3d = -11$ $3d = -11 - 7$ $3d = -18$ $d = -6$ $x = 1$ $y = -5$	**OR** ✓ $3d = -11 - 7$ ✓ $d = -6$ ✓ $x = 1$ ✓ $y = -5$ (4)
	OR $x - 7 = y - x$ and $y - x = -11 - y$ $2x - 7 = y$ ....(1) $2y = -11 + x$...(2) (1) into (2) $2(2x - 7) = -11 + x$ $4x - 14 = -11 + x$ $3x = 3$ $x = 1$ $y = 2(1) - 7 = -5$	**OR** ✓ 2 equations ✓ substitution ✓ value of x ✓ value of y (4)
2.2.1	$-3\ ;\ 6\ ;\ 27\ ;\ 60\ ;\ \ldots$  $2a = 12$ $a = 6$ $3a + b = 9$ $3(6) + b = 9$ $b = -9$ $a + b + c = -3$ $6 - 9 + c = -3$ $c = 0$ $T_n = 6n^2 - 9n$	✓ second difference ✓ $a = 6$ ✓ $b = -9$ ✓ $c = 0$ (4)

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2.2.2	$T_{50} = 6(50)^2 - 9(50)$ $= 14\ 550$	**Answer Only: Full Marks**	✓ substitute 50 ✓ answer (2)
2.2.3	9 ; 21 ; 33 ; … $a = 9$ $d = 12$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}[2(9) + (n-1)(12)]$ $= \frac{n}{2}[18 + 12n - 12]$ $= \frac{n}{2}[12n + 6]$ $= 6n^2 + 3n$		✓ a and d ✓ substitution into the correct formula ✓ $\frac{n}{2}[12n + 6]$ (3)
2.2.4	$-3 + S_n = 21060$ $S_n = 21063$ $6n^2 + 3n = 21063$ $6n^2 + 3n - 21063 = 0$ $2n^2 + n - 7021 = 0$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $n = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-7021)}}{2(2)}$ $n = 59 \text{ or } n \neq \frac{-119}{2}$ $\therefore\ n = 59$ **OR** $T_n = 21060$ $6n^2 - 9n - 21060 = 0$ $2n^2 - 3n - 7020 = 0$ $n = 60$ $\therefore 59$ first differences must be added.		✓ $-3 + S_n = 21060$ ✓ equation ✓ standard form ✓ answer (4) ✓ equation ✓ standard form ✓✓ answer (4)
			[17]

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QUESTION 3

3.1 Prove that $\sum_{k=1}^{\infty} 4.3^{2-k}$ is a convergent geometric series. Show ALL your calculations. (3)

3.2 If $\sum_{k=p}^{\infty} 4.3^{2-k} = \frac{2}{9}$, determine the value of p. (5)

[8]

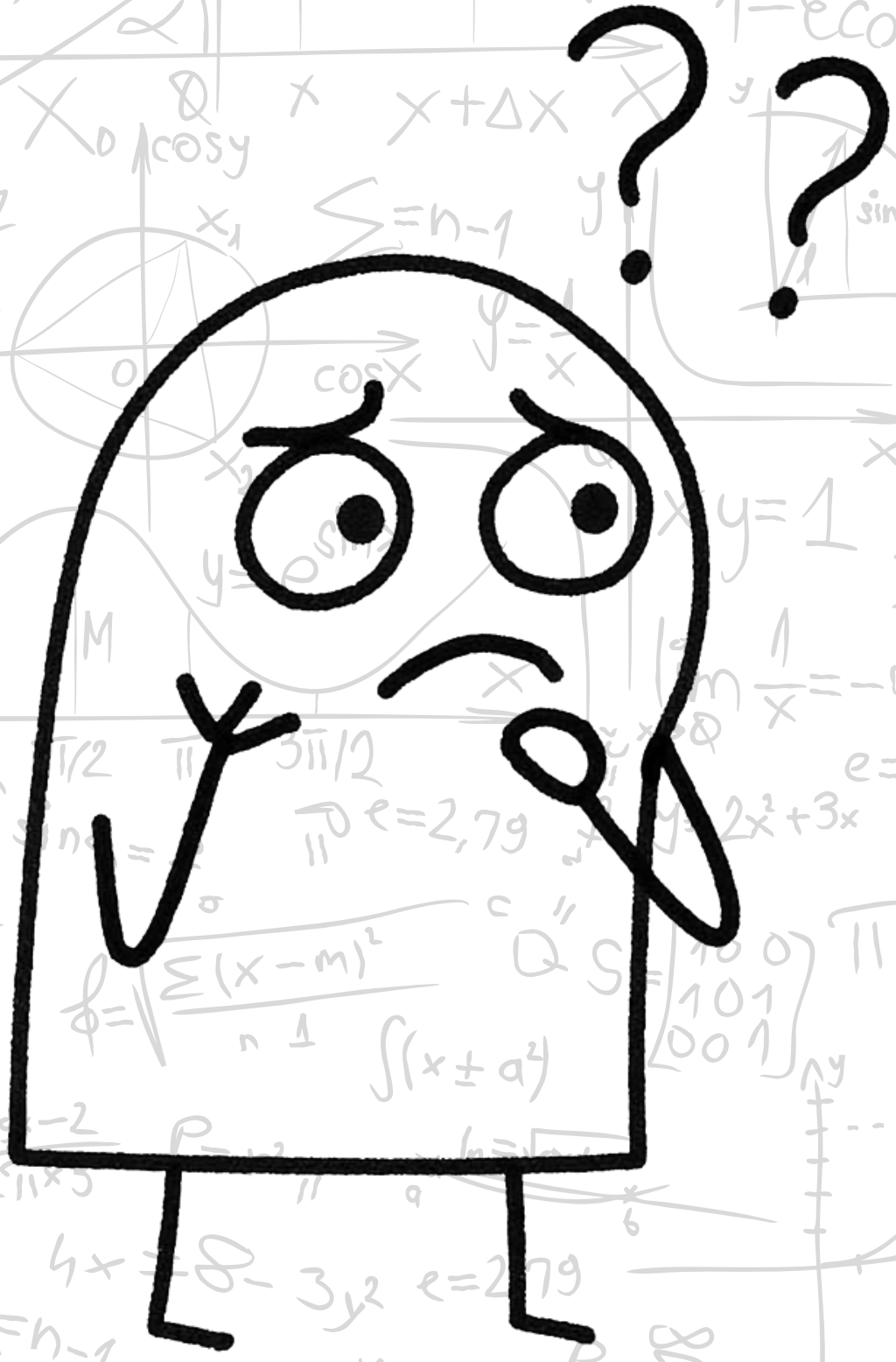
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QUESTION/*VRAAG* 3

3.1	$\sum_{k=1}^{\infty} 4.3^{2-k} = 12+4+\frac{4}{3}+...$ $r = \frac{4}{12} = \frac{1}{3}$ $-1 < \frac{1}{3} < 1$ $\therefore$ series is convergent $(-1 < r < 1)$	✓ $12+4+\frac{4}{3}+..$ or $36\left(\frac{1}{3}\right)^k$ ✓ value of r ✓ $-1 < r < 1$ (3)
3.2	$\sum_{k=p}^{\infty} 4.3^{2-k} = 4.3^{2-p} + 4.3^{1-p} + 4.3^{-p} + ...$ $a = 4.3^{2-p}$ $r = \frac{1}{3}$ $S_{\infty} = \frac{a}{1-r}$ $\frac{2}{9} = \frac{4.3^{2-p}}{1-\frac{1}{3}}$ $4.3^{2-p} = \frac{4}{27}$ $3^{2-p} = 3^{-3}$ $2-p = -3$ $p = 5$	✓ expression for a ✓ substitution of a, r and S_{∞} ✓ simplification $\left(4.3^{2-p} = \frac{4}{27}\right)$ ✓ $3^{2-p} = 3^{-3}$ ✓ answer (5)
		[8]

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FUNCTIONS



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QUESTION 4

Given: $f(x) = a^x - 1$ for $a > 0$. B$\left(2; \frac{-5}{9}\right)$ is a point on f.

4.1 Calculate the value of a. (2)

4.2 Write down the range of f. (1)

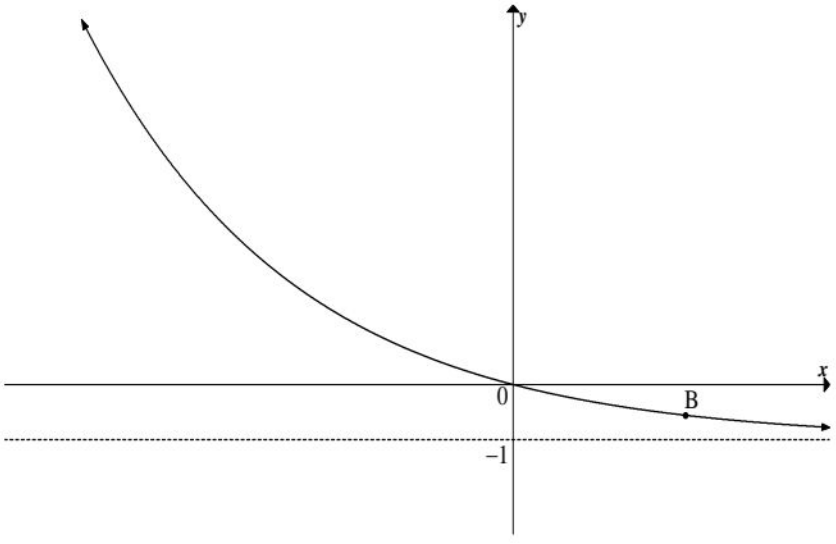
4.3 Sketch the graph of f. Clearly show the intercepts with the axes and asymptotes, if any. (3)

4.4 It is further given that C is a point on f at $y = \frac{19}{8}$.

Determine the coordinates of C$'$, the image of C, when C is reflected about the line $y = x$. (3)
[9]

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QUESTION/*VRAAG* 4

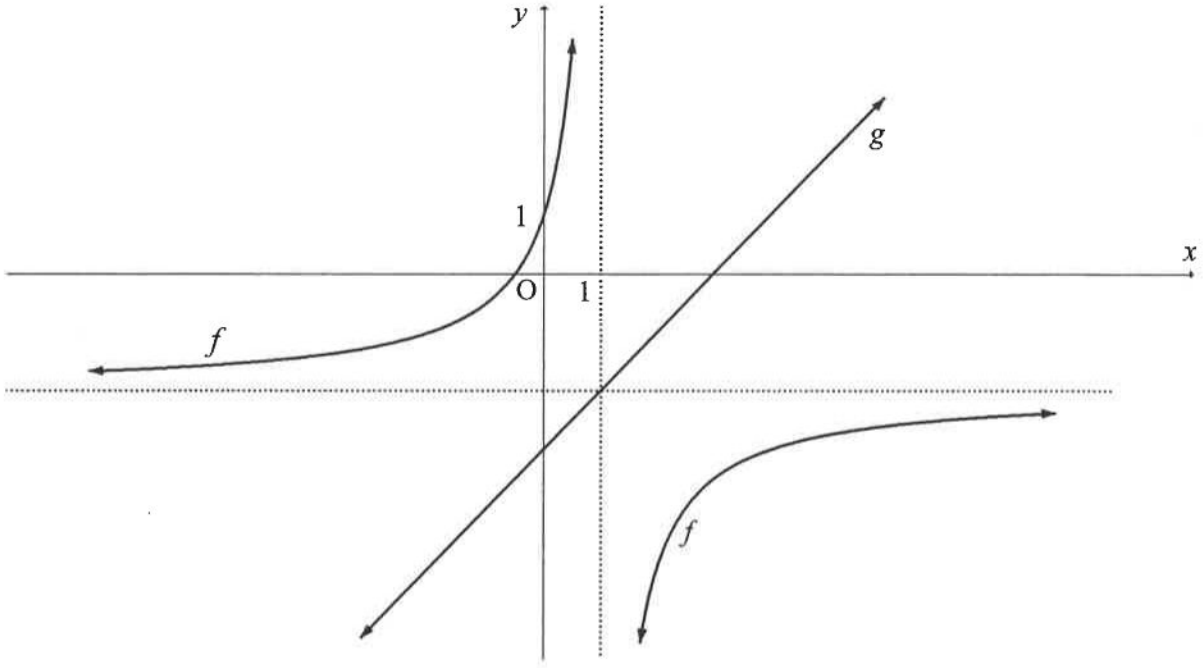
4.1	$f(x) = a^x - 1$ $-\frac{5}{9} = a^2 - 1$ $a^2 = \frac{4}{9}$ $a = \frac{2}{3}$	✓ substitution ✓ answer (2)
4.2	$y > -1$ **OR/*OF*** $y \in (-1\,;\infty)$	✓ answer (1) ✓ answer (1)
4.3		✓(0 ; 0) ✓ asymptote ✓decreasing shape (3)
4.4	$\frac{19}{8} = \left(\frac{2}{3}\right)^x - 1$ $\frac{27}{8} = \left(\frac{2}{3}\right)^x$ $\left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{-x}$ $\therefore x = -3$ $\therefore \text{C}\left(-3\,;\frac{19}{8}\right)$ $\text{C}'\left(\frac{19}{8}\,;-3\right)$	✓substitution ✓ answer ✓ answer (3)
		[9]

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QUESTION 5

Sketched below is the graph of $f(x)=\frac{a}{x+p}+q$ having the domain $(-\infty;1)\cup(1;\infty)$.

The graph of f cuts the y-axis at $(0;1)$. A line of symmetry of f is given by $g(x)=x-3$.



5.1 Write down the value of p. (1)

5.2 Determine the equation of the horizontal asymptote of f. (2)

5.3 Calculate the value of a. (2)

5.4 For which values of x is $f(x)\geq 0$? (3)

5.5 Graph f undergoes a transformation to h where:

- The domain and range of h are the same as that of f
- $h'(x)$, the derivative of h, is negative on its domain

Describe a possible transformation that f could have undergone to result in h. (2)
[10]

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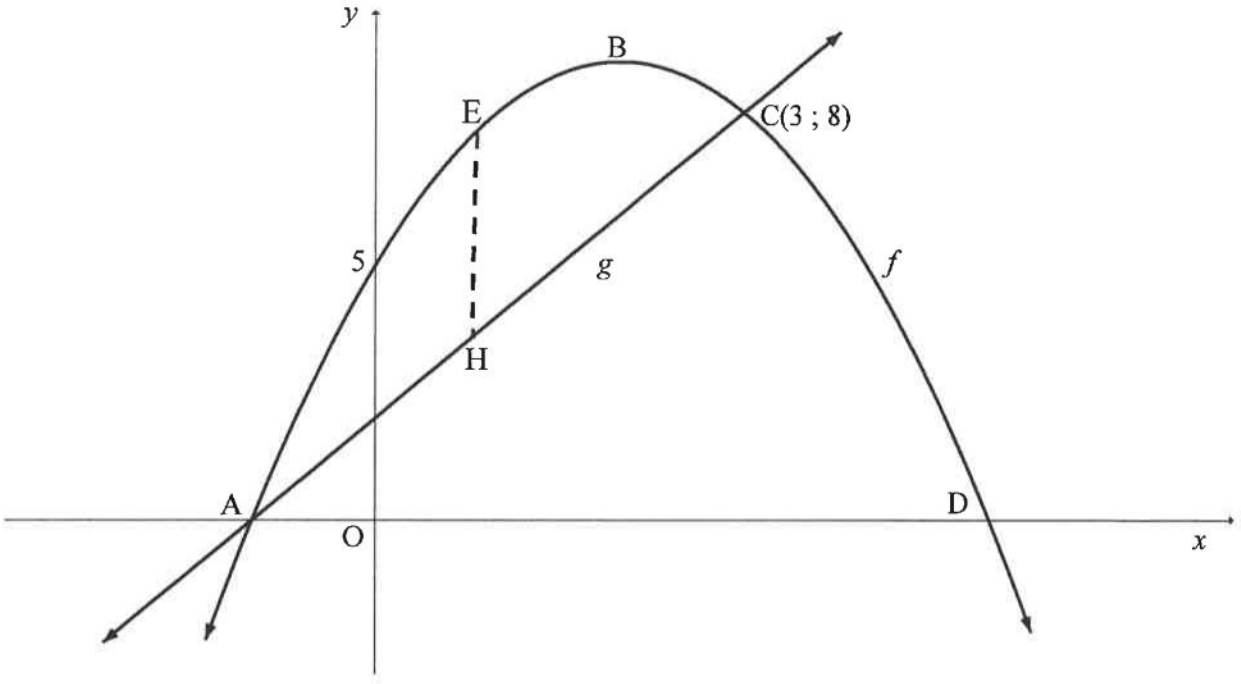
QUESTION/*VRAAG* 5

5.1	$p = -1$	✓ $p = -1$ (1)
5.2	$y = x - 3$ $y = 1 - 3$ $y = -2$ Answer only: $\frac{2}{2}$	✓ substitution ✓ answer (2)
5.3	$1 = \frac{a}{0-1} - 2$ $3 = \frac{a}{-1}$ $a = -3$	✓ substitute (0 ; 1) ✓ answer (2)
5.4	$\frac{-3}{x-1} - 2 = 0$ $-3 = 2x - 2$ $2x = -1$ $\therefore x = -\frac{1}{2}$ $x \in \left[-\frac{1}{2}; 1\right)$ **OR/*OF*** $-\frac{1}{2} \leq x < 1$	✓x-intercept ✓✓ answer (3)
5.5	f is reflected in the y-axis, then translated 2 units to the right **OR/*OF*** f is reflected in the x-axis, then translated 4 units down **OR/*OF*** f is reflected about the line $x = 1$ **OR/*OF*** f is reflected about the line $y = -2$	✓ reflection in y-axis ✓ translation 2 units right (2) **OR/*OF*** ✓ reflection in x-axis ✓ translation 4 units down (2) **OR/*OF*** ✓✓ reflection about line $x = 1$ (2) **OR/*OF*** ✓✓ reflection about line $y = -2$ (2)
		[10]

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QUESTION 6

In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and g, a straight line, are drawn. $C(3\,;8)$ is a point of intersection of f and g. EH is drawn parallel to the y-axis, with E a point on f and H a point on g.



6.1 Calculate the coordinates of B, the turning point of f. (3)

6.2 Show that the equation of the line through A and C is given by $g(x) = 2x + 2$. (3)

6.3 Calculate the maximum length of EH for $f > g$. (4)

6.4 Given: $k(x) = f(x+m) = -x^2 - 2mx - m^2 + 4x + 4m + 5$

Determine the value of m such that g is a tangent to k. (5)
[15]

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QUESTION/*VRAAG* 6

6.1	$x = \frac{-4}{2(-1)} = 2$ **OR/*OF*** $f'(x) = -2x + 4 = 0$ $x = 2$ $y = -(2)^2 + 4(2) + 5$ $= 9$ $\therefore B(2\ ;\ 9)$	✓ method for x ✓ value of x ✓ value for y (3)
6.2	$x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = 5$ or $x = -1$ $\therefore A(-1\ ;\ 0)$ Equation of AC: $\frac{8-0}{3+1} = \frac{y-0}{x+1}$ **OR/*OF*** $m = \frac{8-0}{3+1} = 2$ $8x + 8 = 4y$ $y - 0 = 2(x+1)$ $2x + 2 = y$ $y = 2x + 2$	✓ $A(-1;0)$ ✓ method for gradient ✓ substitute point A or C into equation of a straight line (3)
6.3	$EH = -x^2 + 4x + 5 - (2x + 2)$ $EH = -x^2 + 2x + 3$ Max EH at: $-2x + 2 = 0$ **OR/*OF*** $x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)}$ $x = 1$ $x = 1$ Max length of EH $= -(1)^2 + 2(1) + 3$ $= 4$ Answer only: $\frac{1}{4}$	✓ difference ✓ $EH = -x^2 + 2x + 3$ ✓ x - value ✓ length (4)
6.4	$k(x) = f(x+m)$ $= -x^2 - 2mx - m^2 + 4x + 4m + 5$ $k'(x) = -2x - 2m + 4$ $k'(x) = 2$ $-2x - 2m + 4 = 2$ $-2x = 2m - 2$ $x = 1 - m$ Answer only: $\frac{0}{5}$ $k(1-m) = g(1-m)$ $-(1-m)^2 - 2m(1-m) - m^2 + 4(1-m) + 4m + 5 = 2(1-m) + 2$ $-1 + 2m - m^2 - 2m + 2m^2 - m^2 + 4 - 4m + 4m + 5 = 2 - 2m + 2$ $-2m - 4 = 0$ $\therefore m = -2$	✓ $k'(x)$ ✓ $k'(x) = 2$ ✓ $x = 1 - m$ ✓ substitution ✓ answer (5)

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	OR/*OF*	**OR/*OF***
	$k(x) = f(x+m) = g(x)$ $-x^2 - 2mx - m^2 + 4x + 4m + 5 = 2x + 2$ $0 = x^2 + (2m-2)x + m^2 - 4m - 3$	✓ equating ✓ simplification
	$\Delta = (2m-2)^2 - 4(1)(m^2 - 4m - 3)$ $\Delta = 4m^2 - 8m + 4 - 4m^2 + 16m + 12$ $\Delta = 8m + 16 = 0$ $\therefore m = -2$	✓ substitution into Δ ✓ $\Delta = 0$ ✓ answer (5)
	OR/*OF*	**OR/*OF***
	$f(x) = -x^2 + 4x + 5$ $f'(x) = -2x + 4 = 2$ $-2x = -2$ $x = 1$	✓ $f'(x) = 2$ ✓ x-value
	$f(1) = -(1)^2 + 4(1) + 5$ $\therefore y = 8$ $\therefore$ translate 2 units to the right $\therefore m = -2$	✓ y-value ✓ 2 units to the right ✓ answer (5)
		[15]

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QUESTION 4

Given: $g(x) = \frac{1}{x-1} + 2$

4.1 Write down the equations of the asymptotes of g. (2)

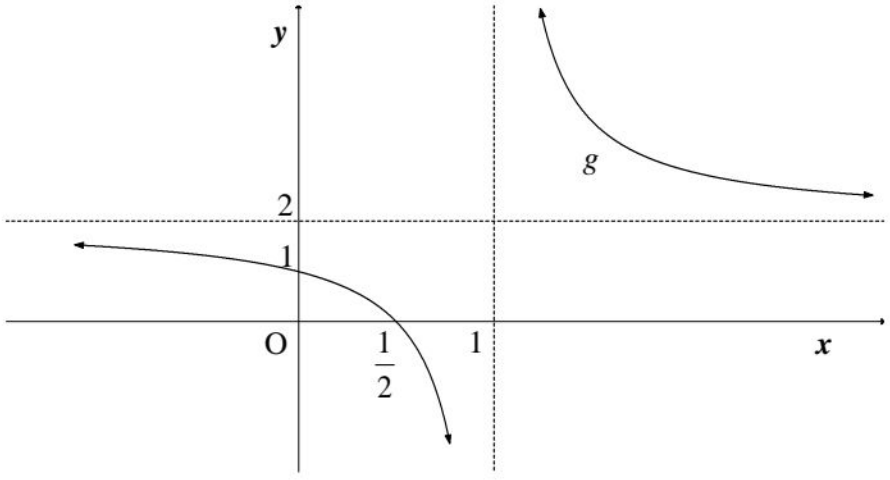
4.2 Draw a graph of g, indicating any intercepts with the axes and asymptotes. (4)

4.3 Determine the values of x where $g(x) > 0$. (2)

4.4 Determine the equation of the axis of symmetry of g which has a negative gradient. (2)
[10]

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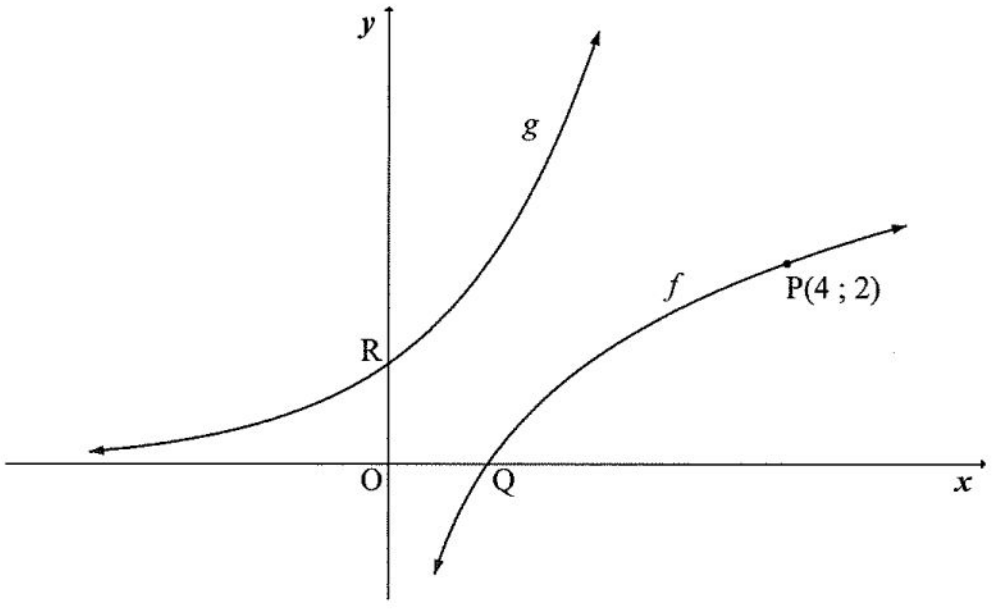
QUESTION 4/*VRAAG* 4

4.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.2		✓ x-intercept ✓ y-intercept ✓ asymptotes ✓ shape (4)
4.3	$x < \frac{1}{2}$ or $x > 1$	✓ $x < \frac{1}{2}$ ✓ $x > 1$ (2)
4.4	$y = -(x-1) + 2$ $y = -x + 3$ **OR/*OF*** $y - 2 = -(x-1)$ $y = -x + 3$ **OR/OF** $y = -x + c$ $2 = -(1) + c$ $c = 3$ $\therefore y = -x + 3$	✓ substitution of (1 ; 2) ✓ answer (2) **OR/*OF*** ✓ substitution of (1 ; 2) ✓ answer (2) **OR/OF** ✓ substitution of (1 ; 2) ✓ answer (2)
		[10]

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QUESTION 5

In the diagram, the graphs of $f(x) = \log_a x$ and g are drawn. Graph g is the reflection of f in the line $y = x$. Graph f passes through the point P(4 ; 2). Q is the x-intercept of f and R is the y-intercept of g.



5.1 Write down the coordinates of P$'$, the image of P on g. (2)

5.2 Show that $a = 2$. (2)

5.3 Write down the equation of g in the form $y = \ldots$ (1)

5.4 T is a point on f in the first quadrant where TR is parallel to the x-axis. Calculate the area of $\Delta RTP'$. (4)
[9]

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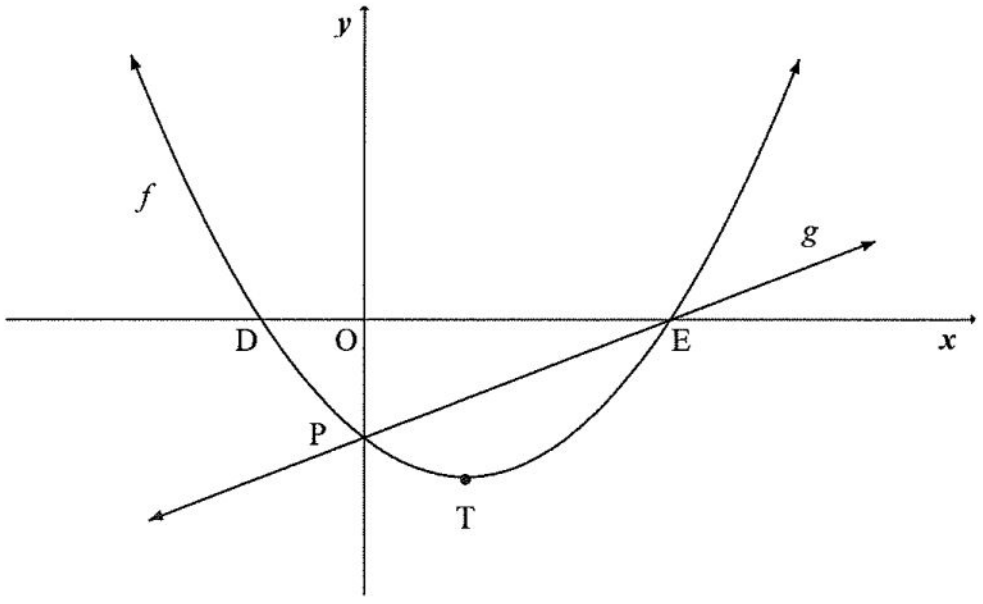
QUESTION5/*VRAAG* 5

5.1	$P'(2;4)$	✓ $x=2$ ✓ $y=4$ (2)
5.2	$f(x)=\log_a x$ $2=\log_a 4$ $a^2=4$ $a=2$	✓substitute (4 ; 2) ✓ $a^2=4$ (2)
5.3	$y=2^x$	✓ $y=2^x$ (1)
5.4	$1=\log_2 x$ $\therefore x=2$ T(2 ; 1) RT = 2 units P'T = 3 units Area of $\Delta RTP' = \frac{1}{2}.RT.TP'$ $= \frac{1}{2}\times 2\times 3 = 3 \text{ units}^2$	✓ $x=2$ ✓ RT = 2 units ✓ P'T = 3 units ✓ answer (4)
		[9]

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QUESTION 6

The graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is T(1 ; – 4). The graphs of f and g intersect at P and E.



6.1 Write down the range of f. (1)

6.2 Calculate the coordinates of D and E. (3)

6.3 Determine the equation of g. (2)

6.4 Write down the values of x for which $f(x) - g(x) > 0$. (2)

6.5 Determine the maximum vertical distance between h and g if $h(x) = -f(x)$ for $x \in [-2 ; 3]$. (5)

6.6 Given: $k(x) = g(x) - n$.

Determine n if k is a tangent to f. (5)
[18]

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QUESTION 6/*VRAAG 6*

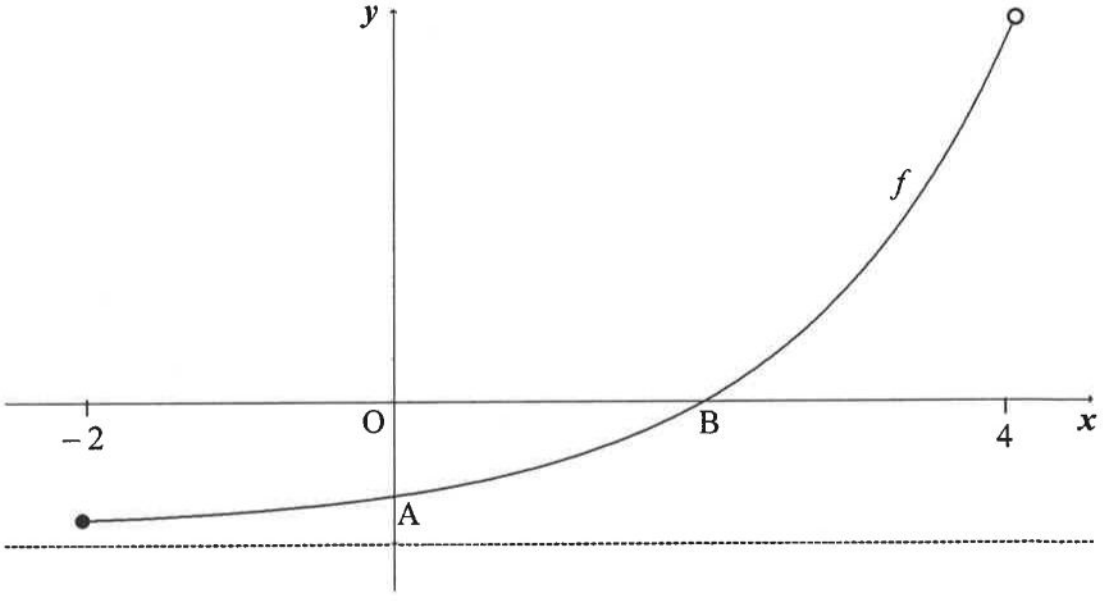
6.1	$y \geq -4$ or $y \in [-4;\infty)$	✓ answer (1)
6.2	$x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $\therefore$ E(3 ; 0) and D(−1 ; 0)	✓ = 0 ✓ both x-values ✓ correct identification of coordinates (3)
6.3	P(0 ; − 3) $\therefore m_{PE} = 1$ $\therefore g(x) = x - 3$	✓ $m_{PE} = 1$ ✓ $g(x) = x - 3$ (2)
6.4	$f(x) > g(x)$ $x < 0$ or $x > 3$	✓ $x < 0$ ✓ $x > 3$ (2)
6.5	Distance $= -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$ $D' = -2x + 1 = 0$ or/of $x = -\frac{b}{2a}$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$ $D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $= \frac{25}{4} = 6{,}25$	✓ $D = -x^2 + x + 6$ ✓ method ✓ $x = \frac{1}{2}$ ✓ substitution ✓ answer (5)
6.6	$f'(x) = m$ $1 = 2x - 2$ $x = \frac{3}{2}$ Point on f: $\left(\frac{3}{2}; \frac{-15}{4}\right)$ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ $\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2{,}25$ **OR/OF** $f(x) = k(x)$ $x^2 - 2x - 3 = x - 3 - n$ $x^2 - 3x + n = 0$ $\Delta = b^2 - 4ac$ $= (-3)^2 - 4(1)(n)$ To touch : $\Delta = 0$ $0 = 9 - 4n$ $4n = 9$ $n = \frac{9}{4}$	✓ $1 = 2x - 2$ ✓ $x = \frac{3}{2}$ ✓ $\frac{-15}{4}$ ✓ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ ✓ answer (5) **OR/OF** ✓ equating ✓ standard form ✓ substitution into Δ ✓ $\Delta = 0$ ✓ answer (5)
		[18]

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QUESTION 4

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2 ; 4)$.
A and B are respectively the y- and x-intercepts of f.



4.1 Write down the equation of the asymptote of f. (1)

4.2 Determine the coordinates of B. (2)

4.3 Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$ (3)

4.4 Calculate the vertical distance between k and f at $x = 1$ (3)

4.5 Write down the equation of g if it is given that $g(x) = f(x) + 4$ (1)

4.6 Write down the domain of g^{-1}. (2)

4.7 Write down the equation of g^{-1} in the form $y = \ldots$ (2)
[14]

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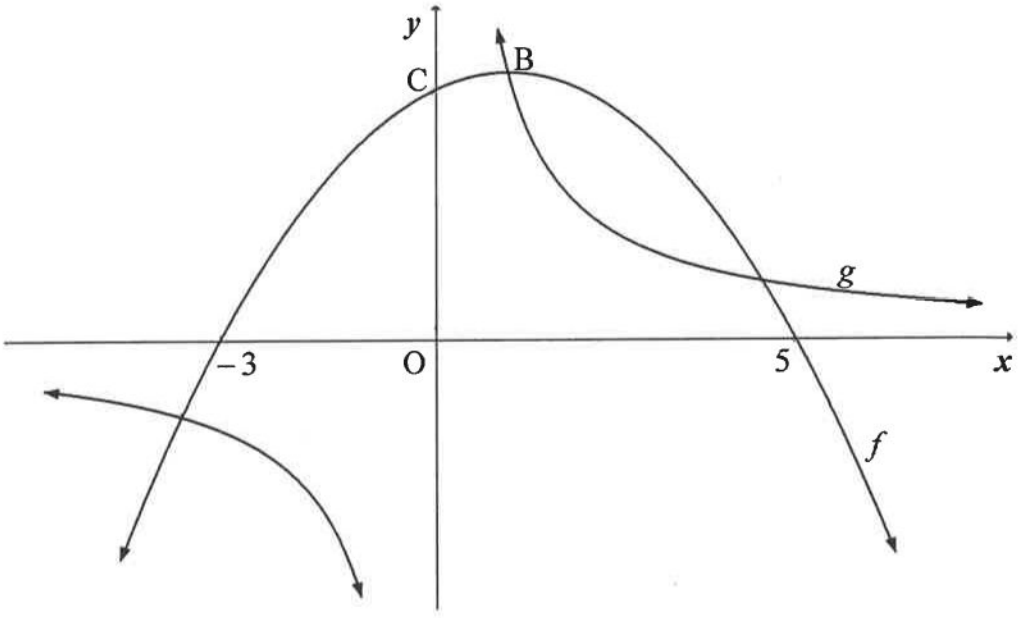
QUESTION 4/*VRAAG* 4

4.1	$y=-4$	✓ $y=-4$ (1)
4.2	$x-$ intercept: $0=2^x-4$ $4=2^x$ $x=2$ $\therefore \text{B}(2;0)$	✓$y=0$ ✓$x=2$ (2)
4.3	$y=2^0-4=-3$ $\therefore \text{A}(0;-3)$ $y=mx+c$ $m=\frac{3}{2}$ $k(x)=\frac{3}{2}x-3$	✓$y=-3$ ✓gradient ✓equation (3)
4.4	$k(1)=\frac{3}{2}(1)-3=\frac{-3}{2}$ $f(1)=2^1-4=-2$ Vertical distance $=-\frac{3}{2}-(-2)=\frac{1}{2}$ units	✓$k(1)$ ✓$f(1)=-2$ ✓answer (3)
4.5	$g(x)=f(x)+4$ $g(x)=2^x\ ;\ x\in[-2\ ;4)$	✓ $g(x)=2^x$ (1)
4.6	Range of g: $y\in\left[\frac{1}{4};16\right)$ Domain of g^{-1}: $x\in\left[\frac{1}{4};16\right)$ or/*of* $\frac{1}{4}\le x<16$	✓✓$x\in\left[\frac{1}{4};16\right)$ (2)
4.7	$g: y=2^x$ $g^{-1}: x=2^y$ $g^{-1}(x)=\log_2 x,\ x\in\left[\frac{1}{4};16\right)$	✓swop x and y ✓ equation (2)
		[14]

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QUESTION 5

The graphs of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f. The graph f has x-intercepts at $(-3\ ;\ 0)$ and $(5\ ;\ 0)$ and a y-intercept at C.



5.1 Write down the coordinates of the turning point of f. (2)

5.2 Calculate the coordinates of C. (2)

5.3 Calculate the value of d. (1)

5.4 Write down the range of g. (1)

5.5 For which values of x will $f(x).g(x) \leq 0$? (3)

5.6 Calculate the values of k so that $h(x) = -2x + k$ will not intersect the graph of g. (5)

5.7 h is a tangent to g at R, a point in the first quadrant. Calculate t such that $y = f(x) + t$ intersects g at R. (4)
[18]

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QUESTION 5/*VRAAG* 5

5.1	$(1\ ;\ 8)$	✓ $x=1$ ✓ $y=8$ (2)
5.2	$y=-\frac{1}{2}(0-1)^2+8$ $=7\frac{1}{2}$ $C\left(0;\frac{15}{2}\right)$	✓$x=0$ ✓ answer (2)
5.3	$8=\frac{d}{1}$ $\therefore d=8$	✓substitution $(1\ ;\ 8)$ (1)
5.4	$y\in R\ ;\ y\neq 0$	✓ $y\neq 0$ (1)
5.5	$-3\leq x<0$ or $x\geq 5$ **OR/*OF*** $x\in[-3\ ;\ 0)\cup[5\ ;\infty)$	✓✓ $-3\leq x<0$ ✓ $x\geq 5$ (3)
5.6	$-2x+k=\frac{8}{x}$ $-2x^2+kx-8=0$ $\Delta=(k)^2-4(-2)(-8)$ $k^2-64<0$ $CV: k=8\ ;\ k=-8$ $\therefore -8<k<8$ **or/of** $k\in(-8\ ;8)$ **OR/*OF*** $g'(x)=h'(x)$ $-\frac{8}{x^2}=-2$ $-8=-2x^2$ $x=\pm 2$ $y=\pm 4$ $\therefore$ B(2 ; 4) and A(– 2 ; – 4) For tangents: $h(x)=-2x+k$ or $h(x)=-2x+k$ $4=-2(2)+k$ $-4=-2(-2)+k$ $k=8$ $k=-8$ $\therefore -8<k<8$ or/*of* $k\in(-8\ ;8)$	✓ $-2x+k=\frac{8}{x}$ ✓ standard form ✓substitution into Δ ✓ $\Delta<0$ or $\Delta=0$ ✓inequality (5) **OR/*OF*** ✓ $-\frac{8}{x^2}$ ✓ $=-2$ ✓ x-values ✓ y-values ✓inequality (5)

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5.7	$h(x) = -2x + 8$ $-2x + 8 = \frac{8}{x}$ $-2x^2 + 8x = 8$ $-2x^2 + 8x - 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $\therefore x = 2$	✓ $x = 2$
	$f(2) = \frac{15}{2}$ $h(2) = 4$ $4 = \frac{15}{2} + t$ $\therefore t = -\frac{7}{2}$	✓ $f(2)$ ✓ $h(2)$ ✓answer (4)
	OR/*OF* $f(2) = \frac{15}{2}$ Tangent point of contact (2 ; 4) $\therefore 4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $4 = \frac{15}{2} + t$ $\therefore t = -\frac{7}{2}$	**OR/*OF*** ✓ $x = 2$ ✓ $f(2)$ ✓ $h(2)$ ✓answer (4)
	OR/*OF* $g(x) = 8x^{-1}$ $g'(x) = -8x^{-2}$ $-2 = -8x^{-2}$ $\frac{1}{4} = \frac{1}{x^2}$ $x = 2$ $y = \frac{8}{2} = 4$ $R(2 ; 4)$ $y = -\frac{1}{2}(x-1)^2 + 8 + t$ $4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $t = -\frac{7}{2}$	**OR/*OF*** ✓ $x = 2$ ✓ $h(2)$ ✓ $f(2)$ ✓answer (4)
		[18]

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QUESTION 4

4.1 Given the function $p(x) = \left(\frac{1}{3}\right)^x$.

4.1.1 Is p an increasing or decreasing function? (1)

4.1.2 Determine p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

4.1.3 Write down the domain of p^{-1}. (1)

4.1.4 Write down the equation of the asymptote of $p(x) - 5$. (1)

4.2 Given: $f(x) = \frac{4}{x-1} + 2$

4.2.1 Write down the equations of the asymptotes of f. (2)

4.2.2 Calculate the x-intercept of f. (2)

4.2.3 Sketch the graph of f, label all asymptotes and indicate the intercepts with the axes. (4)

4.2.4 Use your graph to determine the values of x for which $\frac{4}{x-1} \geq -2$. (2)

4.2.5 Determine the equation of the axis of symmetry of $f(x-2)$, that has a negative gradient. (3)

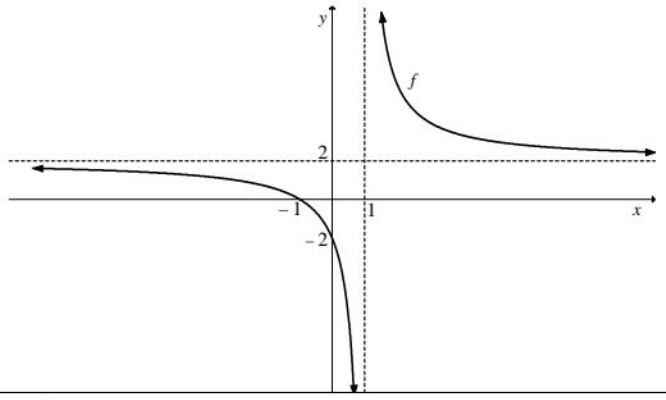
[18]

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QUESTION 4/*VRAAG* 4

4.1.1	decreasing	✓ decreasing (1)
4.1.2	$y=\left(\frac{1}{3}\right)^x$ $x=\left(\frac{1}{3}\right)^y$ $\therefore y=\log_{\frac{1}{3}} x$ **OR/OF** $y=3^{-x}$ $x=3^{-y}$ $\therefore y=-\log_3 x$	✓ swop x and y ✓ answer (2) **OR/OF** ✓ swop x and y ✓ answer (2)
4.1.3	$x>0; x\in R$	✓ answer (1)
4.1.4	$y=-5$	✓ answer (1)
4.2.1	$x=1$ $y=2$	✓ $x=1$ ✓ $y=2$ (2)
4.2.2	$\frac{4}{x-1}+2=0$ $4=-2x+2$ $2x=-2$ $x=-1$	✓ let $y=0$ ✓ $x=-1$ (2)

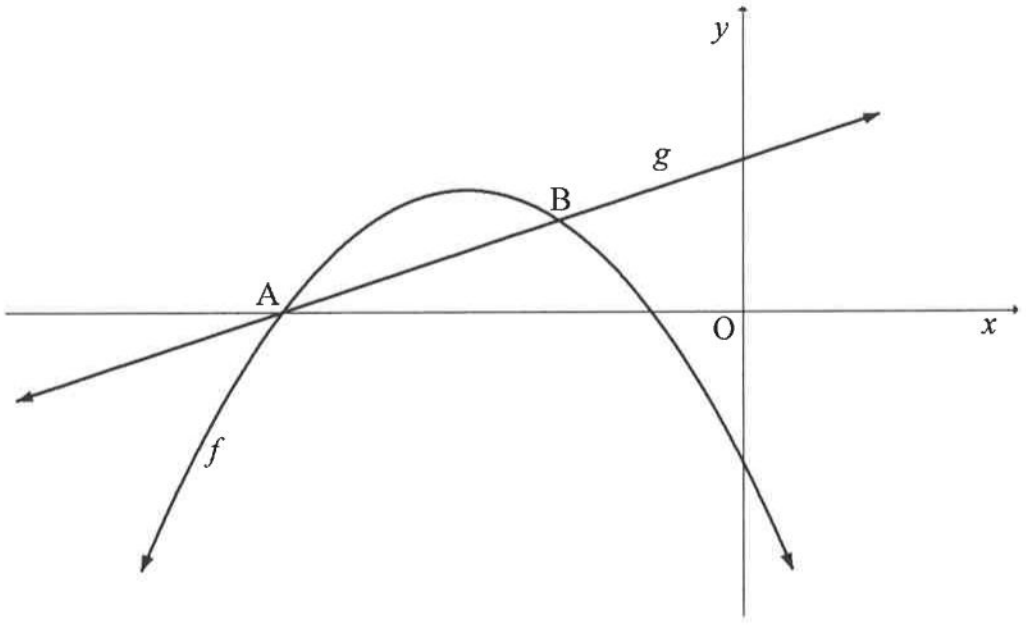
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4.2.3		✓ asymptotes ✓ x-intercept ✓ y-intercept ✓ shape (4)
4.2.4	$\frac{4}{x-1} \geq -2$ $\frac{4}{x-1} + 2 \geq 0$ $x \leq -1$ or $x > 1$	✓ $x \leq -1$ ✓ $x > 1$ (2)
4.2.5	$y = -x + c$ $2 = -3 + c$ $c = 5$ $y = -x + 5$ **OR/*OF*** $y = -x + c$ $2 = -1 + c$ $c = 3$ $y = -x + 3$ $y = -(x-2) + 3$ $y = -x + 5$ **OR/OF** $y = -(x+p) + q$ $y = -((x-2) + (-1)) + 2$ $y = -x + 5$	✓ intersection of axes at (3 ; 2) ✓ subst (3 ; 2) and $m = -1$ ✓ $y = -x + 5$ (3) **OR/*OF*** ✓✓ $-(x-2)+3$ ✓ $y = -x + 5$ (3) **OR/*OF*** ✓✓ $y = -((x-2) + (-1)) + 2$ ✓ $y = -x + 5$ (3)
		[18]

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QUESTION 5

The graphs of the functions $f(x) = -(x+3)^2 + 4$ and $g(x) = x + 5$ are drawn below.
The graphs intersect at A and B.



5.1 Write down the coordinates of the turning point of f. (2)

5.2 Write down the range of f. (1)

5.3 Show that the x-coordinates of A and B are -5 and -2 respectively. (4)

5.4 Hence, determine the values of c for which the equation $-(x+c+3)^2 + 4 = (x+c) + 5$ has ONE negative and ONE positive root. (2)

5.5 The maximum distance between f and g in the interval $x_A < x < x_B$ is k.
If $h(x) = g(x) + k$, determine the equation of h in the form $h(x) = ...$ (5)
[14]

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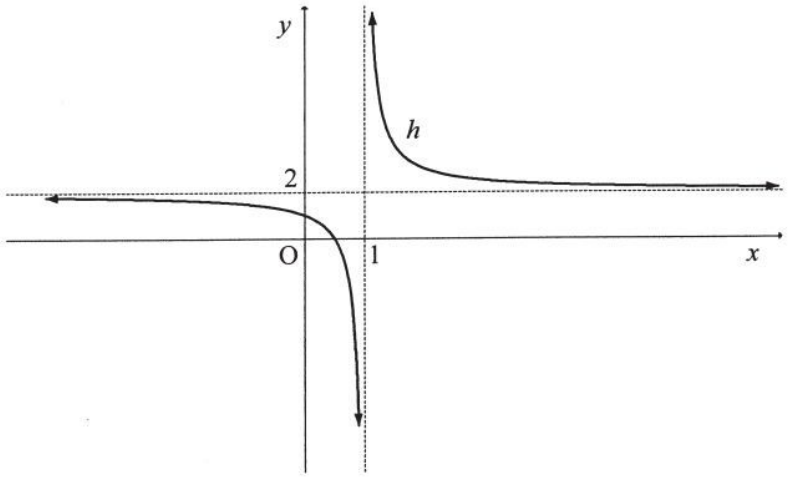
QUESTION 5/*VRAAG* 5

5.1	$\text{T.P}(-3;4)$	✓ -3 ✓ 4 (2)
5.2	$y \le 4$ or $y \in (-\infty;4]$	✓answer (1)
5.3	$f(x) = g(x)$ $-(x+3)^2 + 4 = x+5$ $-x^2 - 6x - 9 + 4 = x + 5$ $-x^2 - 7x - 10 = 0$ $x^2 + 7x + 10 = 0$ $(x+5)(x+2) = 0$ $x = -5$ or $x = -2$	✓ equating ✓ $-x^2 - 6x - 9$ ✓ standard form ✓ factors (4)
5.4	The graph must shift more than 2 and less than 5 units to the right $\therefore -5 < c < -2$	✓✓answer (2)
5.5	$D(x) = f(x) - g(x) = -x^2 - 7x - 10$ $\text{Max}: -2x - 7 = 0$ **OR/OF** $x = \frac{-(-7)}{2(-1)}$ $x = -\frac{7}{2}$ $D\left(-\frac{7}{2}\right) = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10 = 2{,}25$ $\therefore k = 2{,}25$ $\therefore h(x) = x + 7{,}25$	✓distance ✓ $-2x - 7 = 0$ ✓ $x = -\frac{7}{2}$ ✓ $k = 2{,}25$ ✓ $h(x) = x + 7{,}25$ (5)
		[14]

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QUESTION 4

4.1 Sketched below is the graph of $h(x)=\frac{1}{x+p}+q$. The asymptotes of h intersect at $(1\ ;2)$.



4.1.1 Write down the values of p and q. (2)

4.1.2 Calculate the coordinates of the x-intercept of h. (2)

4.1.3 Write down the x-coordinate of the x-intercept of g if $g(x)=h(x+3)$. (2)

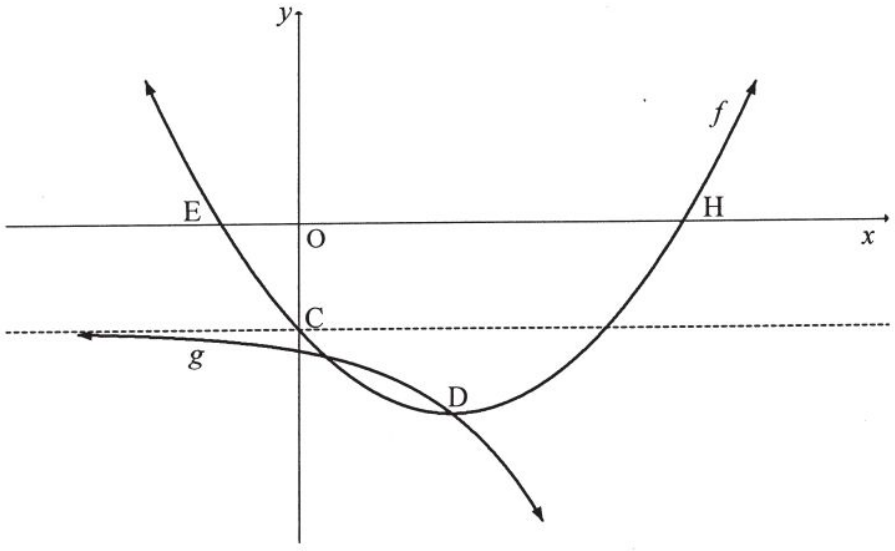
4.1.4 The equation of an axis of symmetry of h is $y=x+t$. Determine the value of t. (2)

4.1.5 Determine the values of x for which $-2\leq\frac{1}{x-1}$. (3)

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4.2 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



4.2.1 Write down the y-coordinate of C. (1)

4.2.2 Determine the coordinates of D. (2)

4.2.3 Determine the values of a and q. (3)

4.2.4 Write down the range of g. (1)

4.2.5 Determine the values of k for which the value of $f(x) - k$ will always be positive. (2)
[20]

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QUESTION 4/*VRAAG* 4

4.1.1	$p=-1$ and $q=2$	✓ $p=-1$ ✓ $q=2$ (2)
4.1.2	$\frac{1}{x-1}+2=0$ $-2x+2=1$ $x=\frac{1}{2}$ $\left(\frac{1}{2};0\right)$	✓ $=0$ ✓ answer (2)
4.1.3	$x=\frac{1}{2}-3$ $=\frac{-5}{2}$ Answer only: full marks	✓ –3 ✓ $x=\frac{-5}{2}$ (2)
4.1.4	$y=x+t$ $2=1+t$ $t=1$	✓subst (1 ; 2) ✓ $t=1$ (2)
4.1.5	$-2\le\frac{1}{x-1}$ Answer only: full marks $\frac{1}{x-1}+2\ge 0$ $\therefore x\le\frac{1}{2}$ or $x>1$ **OR/*OF*** $x\in\left(-\infty;\frac{1}{2}\right]$ or $(1;\infty)$	 ✓ $\frac{1}{x-1}+2\ge 0$ ✓ $x\le\frac{1}{2}$ ✓ $x>1$ (3)
4.2.1	$y=-5$	✓ answer (1)
4.2.2	$x=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=2$ $f(2)=2^2-4(2)-5=-9$ $\therefore$ D(2 ; –9) **OR/*OF*** $f'(x)=2x-4$ $2x-4=0$ $x=2$ $f(2)=2^2-4(2)-5=-9$ $\therefore$ D(2 ; –9)	✓ $x=2$ ✓ $y=-9$ (2) **OR/*OF*** ✓ $x=2$ ✓ $y=-9$ (2)

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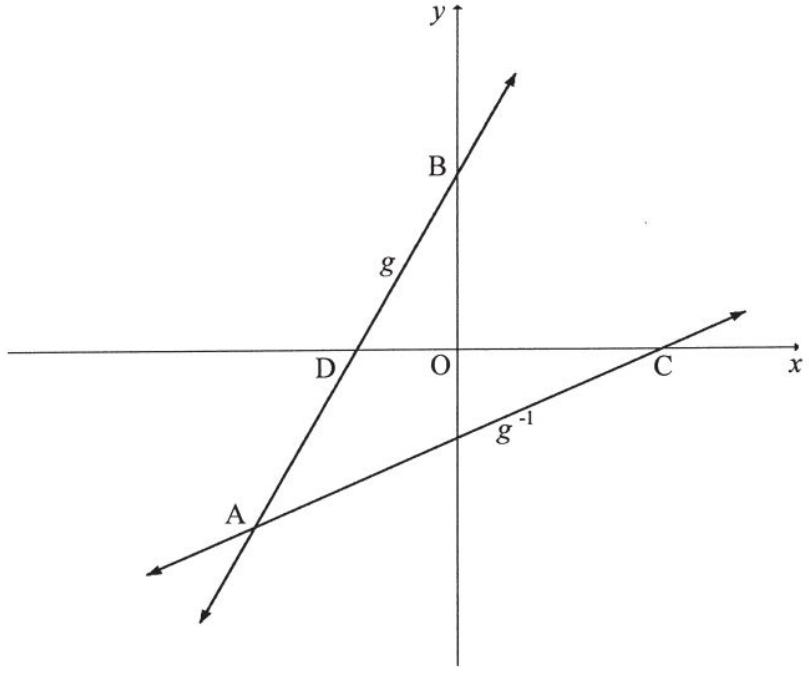
4.2.3	$q = -5$ $-9 = a(2)^2 - 5$ $-4 = 4a$ $a = -1$ $\therefore g(x) = -2^x - 5$	✓ $q = -5$ ✓ substitution of $(2\ ;-9)$ ✓ $a = -1$ (3)
4.2.4	$y \in (-\infty\ ;-5)$ OR $y < -5; y \in R$	✓answer (1)
4.2.5	$k < -9$	✓-9 ✓ $k < -9$ (2)
		[20]

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QUESTION 5

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



5.1 Write down the y-coordinate of B. (1)

5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

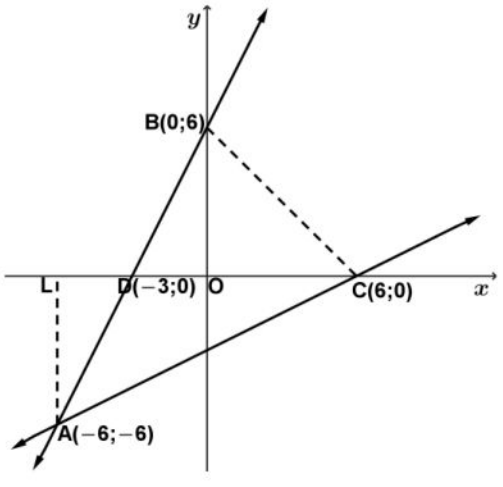
5.3 Determine the coordinates of A. (3)

5.4 Calculate the length of AB. (2)

5.5 Calculate the area of ΔABC. (5)
[13]

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QUESTION 5/*VRAAG* 5

5.1	$g(x) = 2x + 6$ $y = 6$	✓ $y = 6$ (1)
5.2	$y = 2x + 6$ $x = 2y + 6$ $y = \frac{1}{2}x - 3$ Answer only: Full marks	✓ swop x and y ✓ equation (2)
5.3	$\frac{1}{2}x - 3 = 2x + 6$ $x - 6 = 4x + 12$ $3x = -18$ $x = -6$ $A(-6;-6)$ **OR/*OF*** $2x + 6 = x$ $x = -6$ $y = -6$	✓equating ✓$x = -6$ ✓ $y = -6$ (3) **OR/*OF*** ✓equating ✓$x = -6$ ✓$y = -6$ (3)
5.4	$AB = \sqrt{(6)^2 + (12)^2}$ $= \sqrt{180} = 6\sqrt{5} = 13{,}42$	✓substitution ✓answer (2)
		

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5.5	$BC = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$ $AB = AC = \sqrt{180}$ symmetry of g and g^{-1} $\perp h = (\sqrt{180})^2 - \left(\frac{\sqrt{72}}{2}\right)^2$ $= \sqrt{162} = 9\sqrt{2}$ area of $\Delta ABC = \frac{1}{2} BC \times h$ $= \frac{1}{2} \times \sqrt{72} \times \sqrt{162} = 54 \text{ units}^2$ **OR/*OF*** $\tan B\hat{D}C = 2$ $\therefore B\hat{D}C = 63{,}43°$ $\tan D\hat{C}A = \frac{1}{2}$ $\therefore D\hat{C}A = 26{,}57°$ $\therefore D\hat{A}C = 36{,}86°$ (ext angle triangle) Area of $\Delta ABC = \frac{1}{2}\left(\sqrt{180}\right)\left(\sqrt{180}\right)\sin 36{,}86°$ $= 53{,}99 \text{ units}^2$ **OR/OF** Area of ΔABC = Area of ΔBDC + Area of ΔADC $= \frac{1}{2} DC.BO + \frac{1}{2} DC.height$ $= \frac{1}{2}(9)(6) + \frac{1}{2}(9)(6)$ $= 54 \text{ units}^2$	✓ BC ✓ AB = AC /midpoint (3 ; 3) ✓ $\perp h$ (A) ✓ substitution ✓ answer (A) (5) **OR/*OF*** ✓ $B\hat{D}C = 63{,}43°$ ✓ $D\hat{A}C = 36{,}86°$ ✓ AC = $\sqrt{180}$ ✓ substitution into the correct formula ✓ answer (A) (5) **OR/OF** ✓ Areas ($\Delta BDC + \Delta ADC$) ✓ $\frac{1}{2} DC.BO$ ✓ $\frac{1}{2} DC.height$ ✓ substitution ✓ answer (A) (5)
		[13]

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QUESTION 4

The graph of $g(x) = a\left(\frac{1}{3}\right)^x + 7$ passes through point E(–2 ; 10).

4.1 Calculate the value of a. (3)

4.2 Calculate the coordinates of the y-intercept of g. (2)

4.3 Consider: $h(x) = \left(\frac{1}{3}\right)^x$

4.3.1 Describe the translation from g to h. (2)

4.3.2 Determine the equation of the inverse of h, in the form $y = \ldots$ (2)
[9]

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QUESTION 4

4.1	$10 = a\left(\frac{1}{3}\right)^{-2} + 7$ $3 = 9a$ $\therefore a = \frac{1}{3}$	✓ subs (–2 ; 10) ✓ simplification ✓ answer (3)
4.2	$y = g(0)$ $y = \frac{1}{3} \times \left(\frac{1}{3}\right)^{0} + 7$ $y = \frac{22}{3} = 7,33$ $\therefore \left(0 ; \frac{22}{3}\right)$ ANSWER ONLY: FULL MARKS	✓ substitution of $x = 0$ ✓ answer (2)
4.3.1	Translation by 1 unit to the right and 7 units downwards	✓ 1 unit right ✓ 7 units downwards (2)
4.3.2	$h(x) = \left(\frac{1}{3}\right)^{x}$ $h^{-1}: \quad x = \left(\frac{1}{3}\right)^{y}$ $y = \log_{\frac{1}{3}}(x)$ **OR/*OF*** $y = -\log_{3}(x)$ ANSWER ONLY: FULL MARKS	✓ swap x and y ✓ answer (2)
		[9]

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QUESTION 5

Consider: $g(x) = \frac{a}{x+p} + q$

The following information of g is given:

- Domain: $x \in \mathrm{R};\ x \neq -2$
- x-intercept at K(1 ; 0)
- y-intercept at $\mathrm{N}\left(0\ ;-\frac{1}{2}\right)$

5.1 Show that the equation of g is given by: $g(x) = \frac{-3}{x+2} + 1$ (6)

5.2 Write down the range of g. (1)

5.3 Determine the equation of h, the axis of symmetry of g, in the form $y = mx + c$, where $m > 0$. (3)

5.4 Write down the coordinates of K', the image of K reflected over h. (2)
[12]

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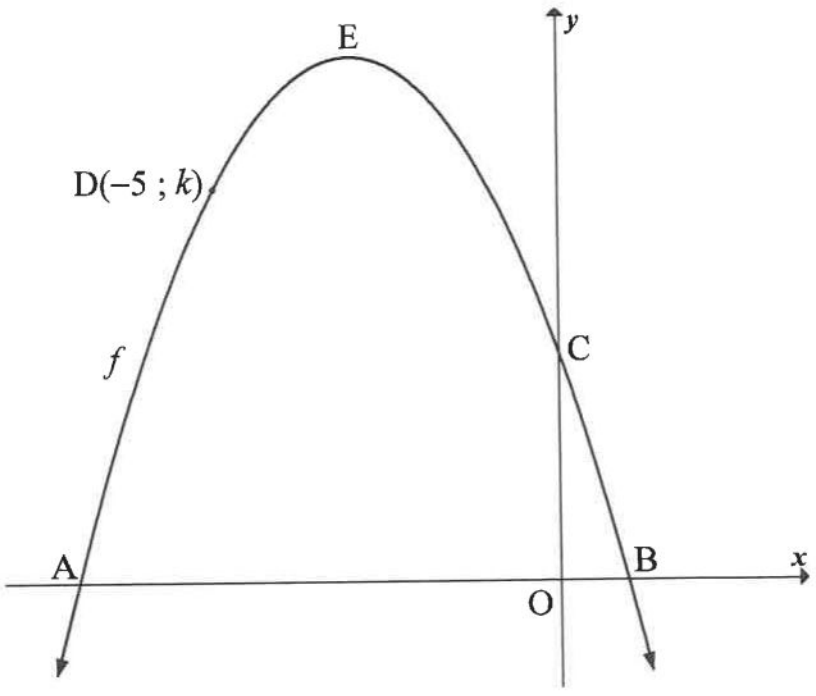
QUESTION 5

5.1	$g(x)=\frac{a}{x+2}+q$ Subs (1 ; 0): $0=\frac{a}{1+2}+q$ $0=a+3q$ Subs $\left(0\ ;-\frac{1}{2}\right)$ $-\frac{1}{2}=\frac{a}{0+2}+q$ $-1=a+2q$ Solving simultaneously: $q=1$ $a=-3$ $\therefore g(x)=\frac{-3}{x+2}+1$	✓ $g(x)=\frac{a}{x+2}+q$ ✓ $0=a+3q$ ✓ $-1=a+2q$ ✓ solving simultaneously ✓ $q=1$ ✓ $a=-3$ (6)
5.2	$y \in \mathrm{R};\ y \neq 1$ **OR/*OF*** $(-\infty;1)$ or $(1;\infty)$ **OR/*OF*** $y<1$ or $y>1$	✓ answer (1)
5.3	$y-1=1(x+2)$ **OR/*OF*** $1=1(-2)+c$ ANSWER ONLY: FULL MARKS $y=x+3$ $c=3$ $y=x+3$	✓ $m=1$ ✓ subs point $(-2\ ;\ 1)$ ✓ answer (3)
5.4	K$'$ (–3 ; 4)	✓ x-value ✓ y-value (2)
		[12]

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QUESTION 6

The sketch below shows the graph of $f(x) = -x^2 - 6x + 7$.
C is the y-intercept of f.
A and B are the x-intercepts of f.
D$(-5 ; k)$ is a point on f.



6.1 Calculate the coordinates of E, the turning point of f. (3)

6.2 Write down the value of k. (1)

6.3 Determine the equation of the straight line passing through C and D. (4)

6.4 A tangent, parallel to CD, touches f at P. Determine the coordinates of P. (4)

6.5 For which values of x will $f(x) - 12 > 0$? (2)
[14]

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QUESTION 6

6.1	$f(x) = -x^2 - 6x + 7$ $f'(x) = -2x - 6$ $-2x - 6 = 0$ **OR/*OF*** $x = -\frac{(-6)}{2(-1)}$ $x = -3$ E(–3 ; 16) ANSWER ONLY: FULL MARKS	✓ method ✓ *x*-value ✓ *y*-value (3)
6.2	$k = f(-5)$ $k = -(-5)^2 - 6(-5) + 7$ $\therefore k = 12$	✓ answer (A) (1)
6.3	C(0 ; 7) D(–5 ; 12) $m_{CD} = \frac{12 - 7}{-5 - 0}$ $m_{CD} = -1$ Equation of CD: $y = -x + 7$	✓ coordinates of C ✓ substitution ✓ *m* ✓ answer (4)
6.4	$-2x - 6 = -1$ $-2x = 5$ $x = -\frac{5}{2}$ $y = f\left(\frac{-5}{2}\right) = -\left(\frac{-5}{2}\right)^2 - 6\left(\frac{-5}{2}\right) + 7 = \frac{63}{4} = 15,75$ $\therefore \text{P}\left(-\frac{5}{2}; \frac{63}{4}\right)$	✓ $f'(x) = -2x - 6$ ✓ equating to -1 ✓ *x*-value ✓ *y*-value (A) (4)
6.5	Point by symmetry: (–1 ; 12) $-5 < x < -1$ **OR/*OF*** $-x^2 - 6x + 7 > 12$ $-x^2 - 6x - 5 > 0$ $x^2 + 6x + 5 < 0$ $(x + 1)(x + 5) < 0$ $-5 < x < -1$ ANSWER ONLY: FULL MARKS	✓ –1 ✓ answer (2) ✓ –1 ✓ answer (2)
		[14]

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QUESTION 5

Given: $f(x)=\frac{-1}{x-3}+2$

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Write down the domain of f. (1)

5.3 Determine the coordinates of the x-intercept of f. (2)

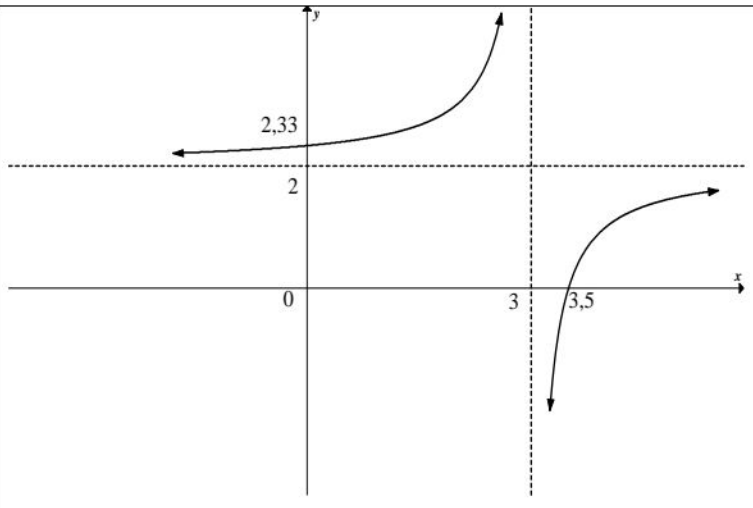
5.4 Write down the coordinates of the y-intercept of f. (2)

5.5 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes. (3)
[10]

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QUESTION/VRAAG 5

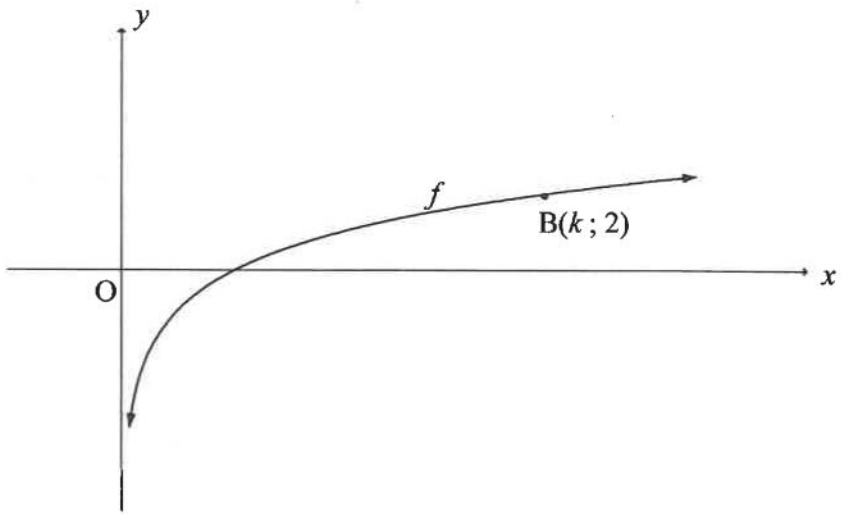
5.1	$x = 3$ $y = 2$	✓ $x = 3$ ✓ $y = 2$ (2)
5.2	$x \in R,\ x \neq 3$ **OR/*OF*** $x \in (-\infty\ ;3) \cup (3\ ;\infty)$ **OR/OF** $x < 3$ or $x > 3$	✓answer (1) **OR/OF** ✓answer (1) **OR/OF** ✓answer (1)
5.3	$0 = \frac{-1}{x-3} + 2$ $-2x + 6 = -1$ $x = \frac{7}{2}$ *x*-int: $\left(\frac{7}{2}\ ;\ 0\right)$	✓ $y = 0$ ✓ answer (2)
5.4	*y*-int: $\left(0\ ;\ \frac{7}{3}\right)$	✓ $x = 0$ ✓ $\frac{7}{3}$ (2)
5.5		✓ asymptotes ✓ intercepts with the axes ✓ shape (3)
		[10]

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QUESTION 6

The graph of $f(x) = \log_4 x$ is drawn below.

B(k ; 2) is a point on f.



6.1 Calculate the value of k. (2)

6.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)

6.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

6.4 For which values of x will $x.f^{-1}(x) < 0$? (2)

[8]

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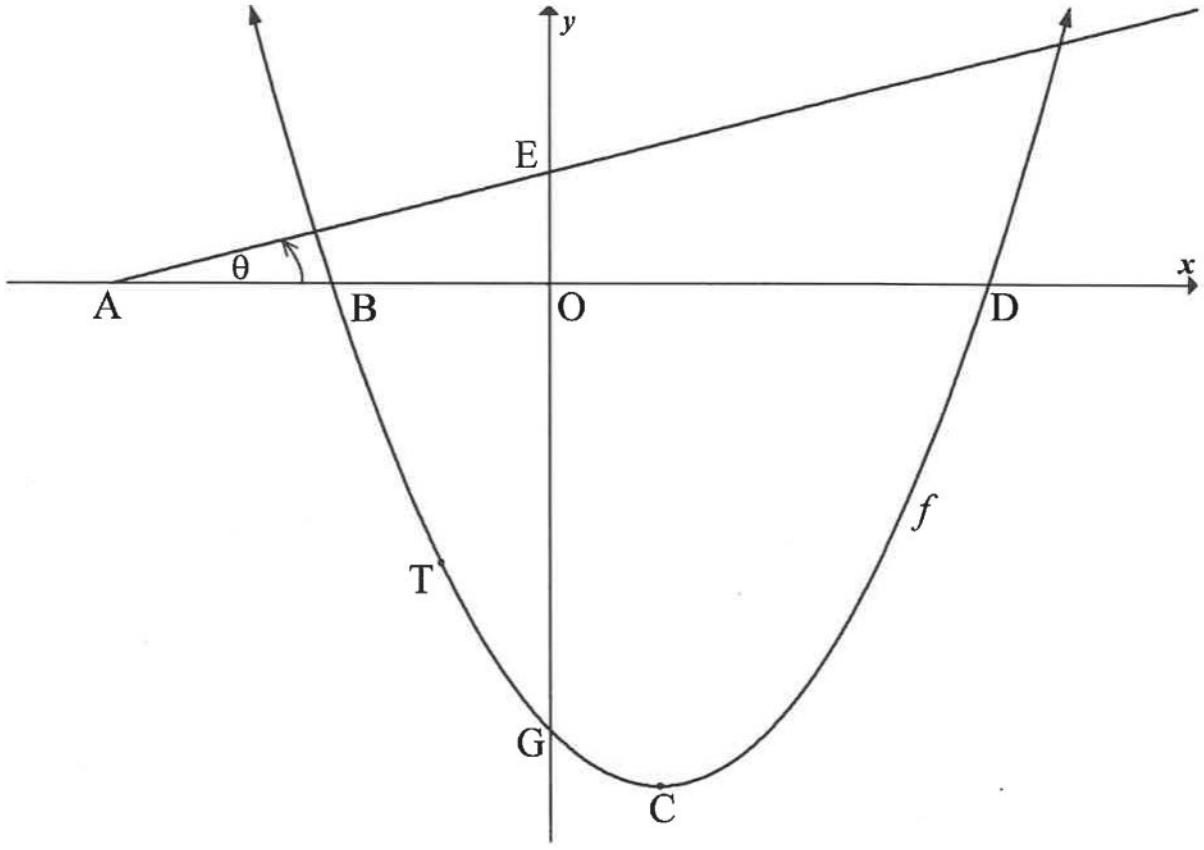
QUESTION/VRAAG 6

6.1	$f(x)=\log_4 x$ $2=\log_4 k$ $4^2=k$ $\therefore k=16$	✓ substitution of $(k\ ;\ 2)$ ✓ answer (2)
6.2	$-1=\log_4 x \quad \therefore\ x=\frac{1}{4}$ $\frac{1}{4}\le x\le 16$ or/of $x\in\left[\frac{1}{4}\ ;\ 16\right]$	✓ $x=\frac{1}{4}$ ✓ answer (2)
6.3	$f(x)=\log_4 x$ $y=\log_4 x$ $x=\log_4 y$ $y=4^x$	✓swopping x and y ✓answer (2)
6.4	$x<0$ **OR/*OF*** $x\in(-\infty\ ;\ 0)$	✓✓answer (2) **OR/*OF*** ✓✓answer (2)
		[8]

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QUESTION 7

The graph of $f(x) = (x+4)(x-6)$ is drawn below.
The parabola cuts the x-axis at B and D and the y-axis at G.
C is the turning point of f.
Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.
T is a point on f between B and G.



7.1 Write down the coordinates of B and D. (2)

7.2 Calculate the coordinates of C. (2)

7.3 Write down the range of f. (1)

7.4 Given that $\theta = 14{,}04°$ and the tangent to f at T is perpendicular to AE.

7.4.1 Calculate the gradient of AE, correct to TWO decimal places. (1)

7.4.2 Calculate the coordinates of T. (5)

7.5 A straight line, g, parallel to AE, cuts f at K(–3 ; –9) and R.
Calculate the x-coordinate of R. (6)
[17]

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QUESTION 7

7.1	B(–4 ; 0) D(6 ; 0)	✓ B(–4 ; 0) ✓ D(6 ; 0) (2)
7.2	$f(x)=x^2-2x-24$ $x_{tp}=\frac{-b}{2a}$ **OR/*OF*** $2x-2=0$ **OR/OF** $x=\frac{-4+6}{2}$ $x=\frac{-(-2)}{2(1)}$ $\therefore\ x_{tp}=1$ $y_{tp}=f(1)$ $=1^2-2(1)-24$ $=-25$ C(1 ; –25)	✓ $x_{tp}=1$ ✓ $y_{tp}=-25$ (2)
7.3	$y\geq -25$ **OR/*OF*** $y\in[-25\ ;\ \infty)$	✓ answer (1) **OR/OF** ✓ answer (1)
7.4.1	$m_{AE}=\tan 14,04°=0,25=\frac{1}{4}$	✓ answer (1)
7.4.2	$m_{\text{tang}}=-4$ $f'(x)=2x-2$ $2x-2=-4$ $x_T=-1$ $y_T=-21$	✓ $m_{\text{tang}}=-4$ ✓ $f'(x)=2x-2$ ✓ equating ✓ $x_T=-1$ ✓ $y_T=-21$ (5)
7.5	$m_{line}=\frac{1}{4}$ $y+9=\frac{1}{4}(x+3)$ **OR/OF** $-9=\frac{1}{4}(-3)+c$ $y+9=\frac{1}{4}x+\frac{3}{4}$ $c=-\frac{33}{4}=-8,25$ $y=\frac{1}{4}x-\frac{33}{4}$ **OR/OF** $y=0,25x-8,25$ $x^2-2x-24=\frac{1}{4}x-\frac{33}{4}$ $4x^2-8x-96=x-33$ $4x^2-9x-63=0$ $(4x-21)(x+3)=0$ $\therefore x=\frac{21}{4}=5,25$ or $x\neq -3$	✓ $m_{line}=\frac{1}{4}$ ✓ substitution *m* and K(– 3 ; – 9) ✓ $y=\frac{1}{4}x-\frac{33}{4}$ ✓ equating ✓ standard form ✓ answer with selection (6)
		[17]

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QUESTION 4

The lines $y = x + 1$ and $y = -x - 7$ are the axes of symmetry of the function $f(x) = \frac{-2}{x+p} + q$.

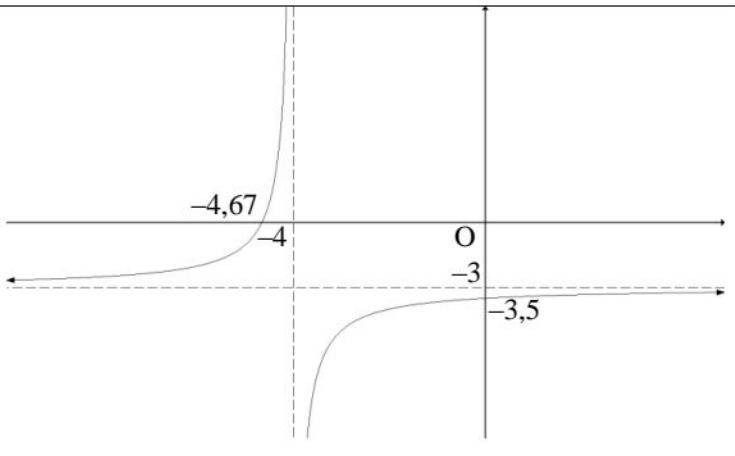
4.1 Show that $p = 4$ and $q = -3$. (4)

4.2 Calculate the x-intercept of f. (2)

4.3 Sketch the graph of f. Clearly label ALL intercepts with the axes and the asymptotes. (4)
[10]

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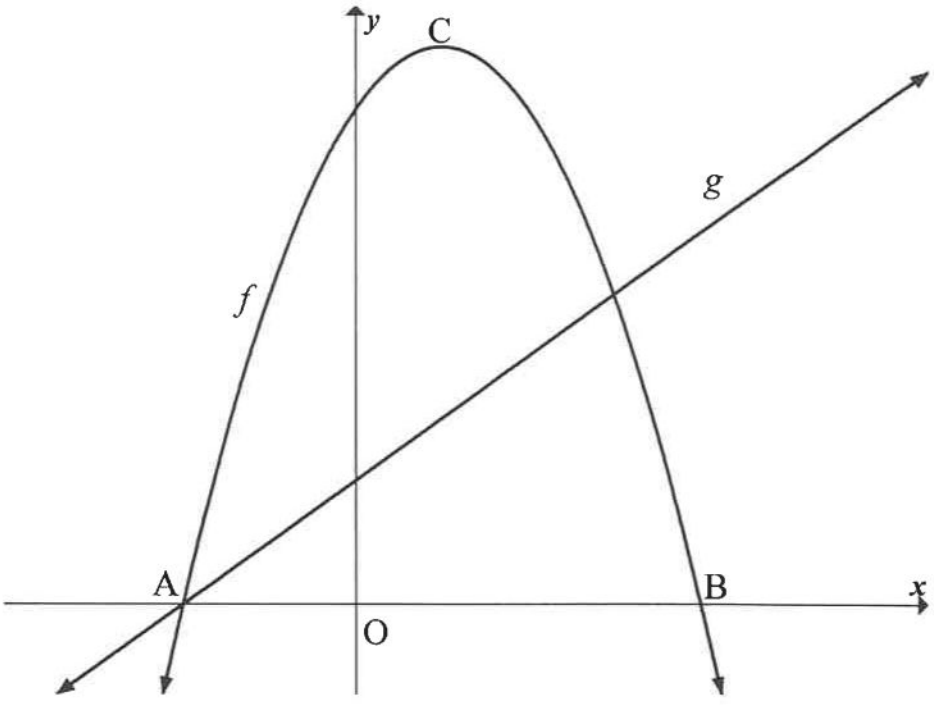
QUESTION/*VRAAG* 4

4.1	$x+1=-x-7$ $2x=-8$ $x=-4$ $\therefore y=-3$ $\therefore f(x)=\frac{-2}{x+4}-3$ $\therefore p=4$ and $q=-3$ **OR/OF** $p+q=1$ ……(1) $-p+q=-7$ $q=p-7$ …….(2) subs. (2) into (1) $p+p-7=1$ $2p=8$ $p=4$ $q=-3$	✓ $x+1=-x-7$ ✓ $2x=-8$ ✓ $x=-4$ ✓ $y=-3$ (4) **OR/OF** ✓ $p+q=1$ ✓ $q=p-7$ ✓ substitution ✓ simplification (4)
4.2	$y=\frac{-2}{x+4}-3$ $0=\frac{-2}{x+4}-3$ $-2-3(x+4)=0$ $-3x-14=0$ $\therefore x=-\frac{14}{3}$	✓ $y=0$ ✓ $x=-\frac{14}{3}$ (2)
4.3		✓ horizontal asymptote ✓ vertical asymptote ✓ y intercept ✓ shape (4)
		[10]

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QUESTION 5

Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x-intercepts of f. C is the turning point of f.



5.1 Calculate the coordinates of A and B. (3)

5.2 Determine the coordinates of C, the turning point of f. (2)

5.3 Write down the range of f. (1)

5.4 The graph of $h(x) = f(x + p) + q$ has a maximum value of 15 at $x = 2$.
Determine the values of p and q. (3)

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

5.6 For which value(s) of x will $g^{-1}(x).g(x) = 0$? (2)

5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect. (5)
[18]

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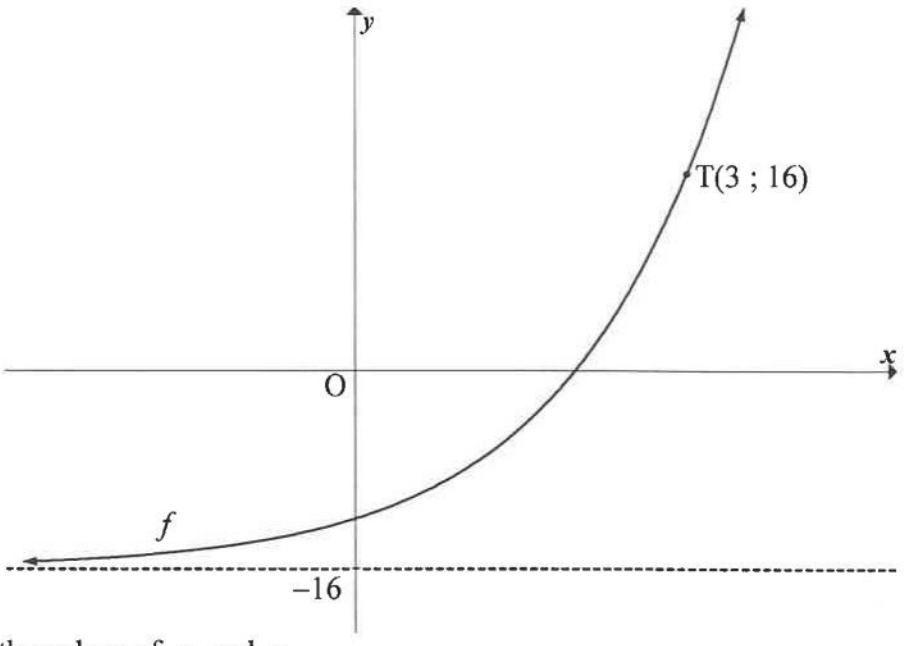
QUESTION 6

6.1 Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g. (2)

6.2 Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q. (4)
[8]

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QUESTION/*VRAAG* 5

5.1	$-2x^2+4x+16=0$ $x^2-2x-8=0$ $(x-4)(x+2)=0$ $x=4$ or $x=-2$ $\therefore$ A$(-2;0)$ and B$(4;0)$	✓ factors ✓ $x=-2$ ✓ $x=4$ (3)
5.2	$f(x)=-2x^2+4x+16$ $-\frac{b}{2a}=-\frac{-4}{-2(2)}=1$ $f(1)=-2(1)^2+4(1)+16=18$ $\therefore$ C$(1;18)$ **OR/OF** $f(x)=-2x^2+4x+16$ $f'(x)=-4x+4$ $-4x+4=0$ $x=1$ $f(1)=-2(1)^2+4(1)+16=18$ $\therefore$ C$(1;18)$	✓ 1 ✓ 18 (2) **OR/OF** ✓ 1 ✓ 18 (2)
5.3	$y \leq 18$ **OR/OF** $y\in(-\infty;18]$	✓ $y \leq 18$ (1) **OR/OF** ✓ $y\in(-\infty;18]$ (1)
5.4	TP (1 ; 18) for f TP (2 ; 15) for h $\therefore p=-1 \quad q=-3$	✓ TP for h at (2 ; 15) ✓ $p=-1$ ✓ $q=-3$ (3)
5.5	$y=2x+4$ $x=2y+4$ $\therefore y=\frac{1}{2}x-2$	✓swop x and y ✓ $y=\frac{1}{2}x-2$ (2)
5.6	$g(x)=0$ or $g^{-1}(x)=0$ $x=4$ or $x=-2$ (product 0 at x-intercepts)	✓ $x=4$ ✓ $x=-2$ (2)

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5.7	$-2x^2+4x+16+k=2x+4$ $-2x^2+2x+12+k=0$ $b^2-4ac<0$ $(2)^2-4(-2)(12+k)<0$ $4+8(12+k)<0$ $100+8k<0$ $k<-12{,}5$ **OR/OF** $g'(x)=2$ $f'(x)=-4x+4=2$ $x=\frac{1}{2}$ $f\left(\frac{1}{2}\right)=17{,}5$ $g\left(\frac{1}{2}\right)=5$ $\therefore k<-12{,}5$	✓equating ✓ standard form ✓ $b^2-4ac<0$ ✓ substitution ✓ answer (5) **OR/OF** ✓ $g'(x)=2$ ✓ $f'(x)=-4x+4$ ✓ $f\left(\frac{1}{2}\right)=17{,}5$ ✓ $g\left(\frac{1}{2}\right)=5$ ✓ answer (5)
		[18]

QUESTION/*VRAAG* 6

6.1.1	$y=3^x$ $x=3^y$ $y=\log_3 x$	✓ swop x and y ✓ equation (2)
6.1.2	$h(x)=3^{x-4}+2$ Transformation: 4 units left, 2 units down $P'(2;9)$	✓ $x=2$ (A) ✓ $y=9$ (A) (2)
6.2	$f(x)=2^{x+p}+q$ $q=-16$ $16=2^{p+3}-16$ $2^{p+3}=32$ $2^{p+3}=2^5$ $\therefore p+3=5$ $p=2$	✓ $q=-16$ ✓ substitute (3 ; 16) ✓ $2^{p+3}=2^5$ or $p+3=\log_2 32$ ✓ $p=2$ (4)
		[8]

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QUESTION 4

4.1 Given: $h(x)=\frac{-3}{x-1}+2$

4.1.1 Write down the equations of the asymptotes of h. (2)

4.1.2 Determine the equation of the axis of symmetry of h that has a negative gradient. (2)

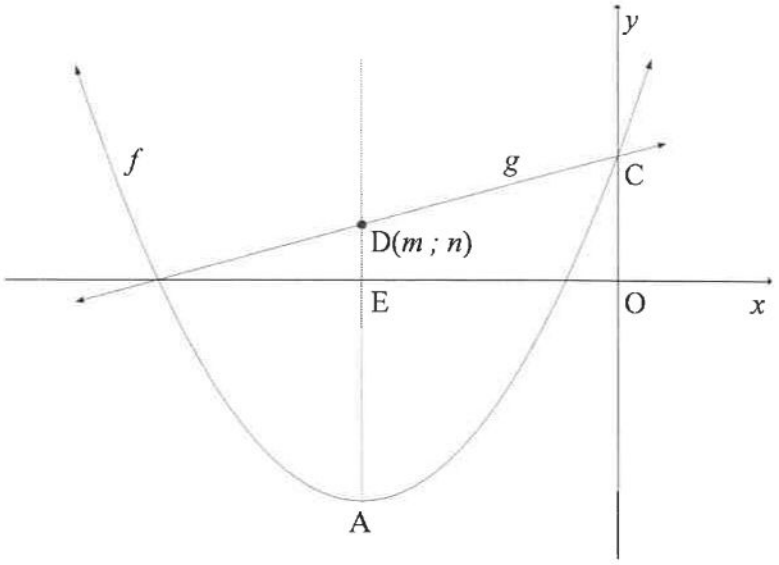
4.1.3 Sketch the graph of h, showing the asymptotes and the intercepts with the axes. (4)

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4.2 The graphs of $f(x)=\frac{1}{2}(x+5)^2-8$ and $g(x)=\frac{1}{2}x+\frac{9}{2}$ are sketched below.

- A is the turning point of f.
- The axis of symmetry of f intersects the x-axis at E and the line g at D(m ; n).
- C is the y-intercept of f and g.



4.2.1 Write down the coordinates of A. (2)

4.2.2 Write down the range of f. (1)

4.2.3 Calculate the values of m and n. (3)

4.2.4 Calculate the area of OCDE. (3)

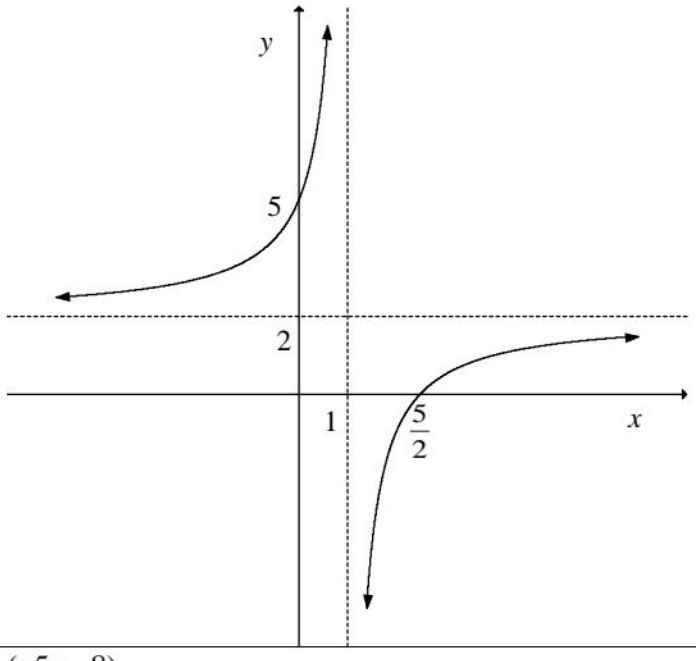
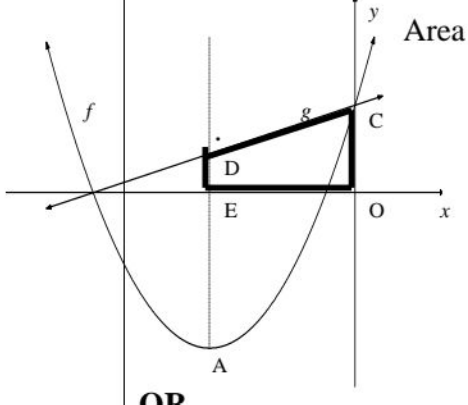
4.2.5 Determine the equation of g^{-1}, the inverse of g, in the form $y=\ldots$ (2)

4.2.6 If $h(x)=g^{-1}(x)+k$ is a tangent to f, determine the coordinates of the point of contact between h and f. (4)

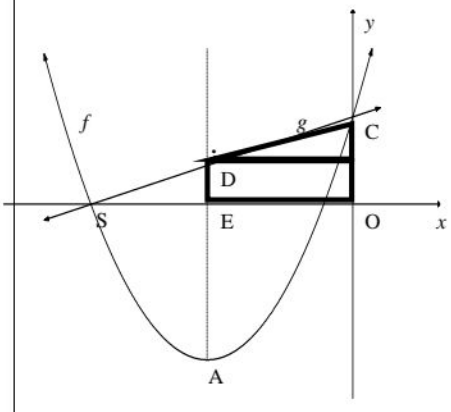
[23]

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QUESTION/*VRAAG* 4

4.1.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.1.2	$y = mx + c$ **or** $y - y_1 = m(x - x_1)$ **or** $y = -(x - p) + q$ $2 = -1 + c$ **or** $y - 2 = -1(x - 1)$ **or** $= -(x - 1) + 2$ $c = 3$ $y - 2 = -x + 1$ $y = -x + 3$ $y = -x + 3$ $y = -x + 3$	✓ substitution of $m = -1$ and (1 ; 2) ✓ answer (2)
4.1.3		✓ vertical asymptote: $x = 1$ and horizontal asymptote: $y = 2$ ✓ x-intercept: $\frac{5}{2}$ ✓ y-intercept: 5 ✓ shape (A) (4)
4.2.1	$(-5 ; -8)$	✓ $x = -5$ ✓ $y = -8$ (2)
4.2.2	$y \geq -8$ **or** $[-8; \infty)$	✓ answer (1)
4.2.3	$m = -5$ $n = g(-5)$ $= \frac{1}{2}(-5) + \frac{9}{2}$ $= 2$	✓ $m = -5$ ✓ substitution ✓ $n = 2$ (3)
4.2.4		
	 **OR**	Area trapezium $= \frac{1}{2}(DE + OC) \times OE$ $= \frac{1}{2}(2 + 4{,}5) \times 5$ $= \frac{65}{4}$ or 16,25 ✓ method ✓ correct substitution ✓ answer (3) **OR**

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$$\text{Area } \Delta = \frac{1}{2}b.h$$
$$= \frac{1}{2}(5)\left(\frac{5}{2}\right)$$
$$= \frac{25}{4}$$
$$\text{Area rect} = b.h$$
$$= (5)(2)$$
$$= 10$$

✓ method

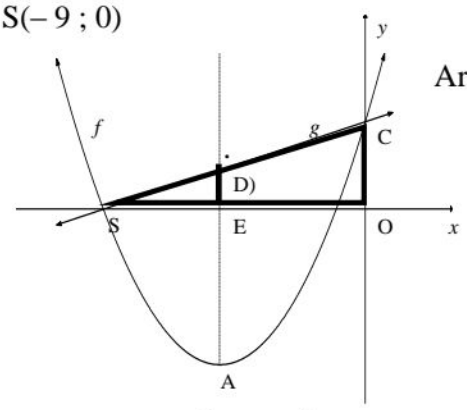
✓correct substitution
✓answer (3)

$$\text{Area trapezium} = \frac{25}{4} + 10 = \frac{65}{4} \text{ or } 16{,}25$$

OR

OR

S(– 9 ; 0)



$$\text{Area } \Delta \text{ SOC} = \frac{1}{2}b.h$$
$$= \frac{1}{2}(9)\left(\frac{9}{2}\right)$$
$$= \frac{81}{4}$$

$$\text{Area } \Delta \text{ SED} = \frac{1}{2}b.h = \frac{1}{2}(4)(2) = 4$$

$$\text{Area trapezium} = \text{area } \Delta \text{ SOC} - \text{Area } \Delta \text{ SED}$$
$$= \frac{81}{4} - 4$$
$$= \frac{65}{4} \text{ or } 16{,}25$$

✓ method

✓correct substitution

✓answer (3)

4.2.5

$$g^{-1}: \; x = \frac{1}{2}y + \frac{9}{2}$$
$$g^{-1}: \; y = 2x - 9$$

✓ changing x and y

✓answer (2)

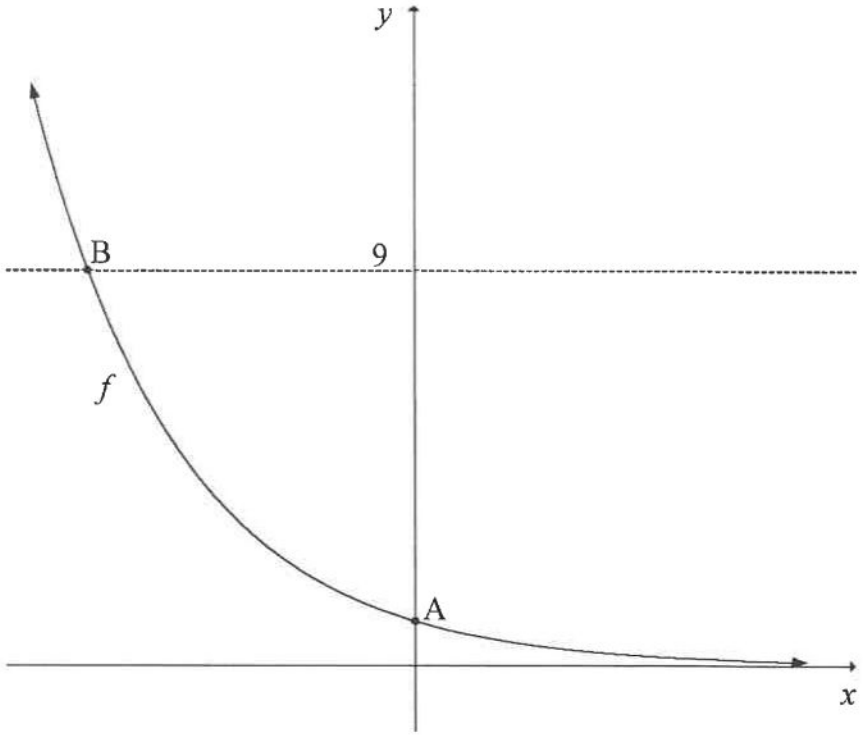
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4.2.6	$f(x)=\frac{1}{2}(x+5)^2-8$ $f(x)=\frac{1}{2}(x^2+10x+25)-8$ $f(x)=\frac{1}{2}x^2+5x+4{,}5$ $f'(x)=x+5$ $h(x)=2x-9+k$ $x+5=2$ $x=-3 \quad y=-6$ $(-3;-6)$ **OR** $f(x)=h(x)$ $\frac{1}{2}(x+5)^2-8=2x-9+k$ $\frac{1}{2}x^2+3x+\frac{27}{2}-k=0$ $x=\frac{-3}{2\left(\frac{1}{2}\right)}=-3 \qquad b^2-4ac=0$ $y=-6$ $(-3;-6)$	✓ $f'(x)$ ✓ $x+5=2$ ✓ $x=-3$ ✓ $y=-6$ (4) **OR** ✓ equating ✓ turning point / $\triangle=0$ ✓ $x=-3$ ✓ $y=-6$ (4)
		[23]

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QUESTION 5

The graph of $f(x)=3^{-x}$ is sketched below. A is the y-intercept of f.
B is the point of intersection of f and the line $y=9$.



5.1 Write down the coordinates of A. (1)

5.2 Determine the coordinates of B. (3)

5.3 Write down the domain of f^{-1}. (2)

5.4 Describe the translation from f to $h(x)=\frac{27}{3^x}$. (3)

5.5 Determine the values of x for which $h(x)<1$. (3)
[12]

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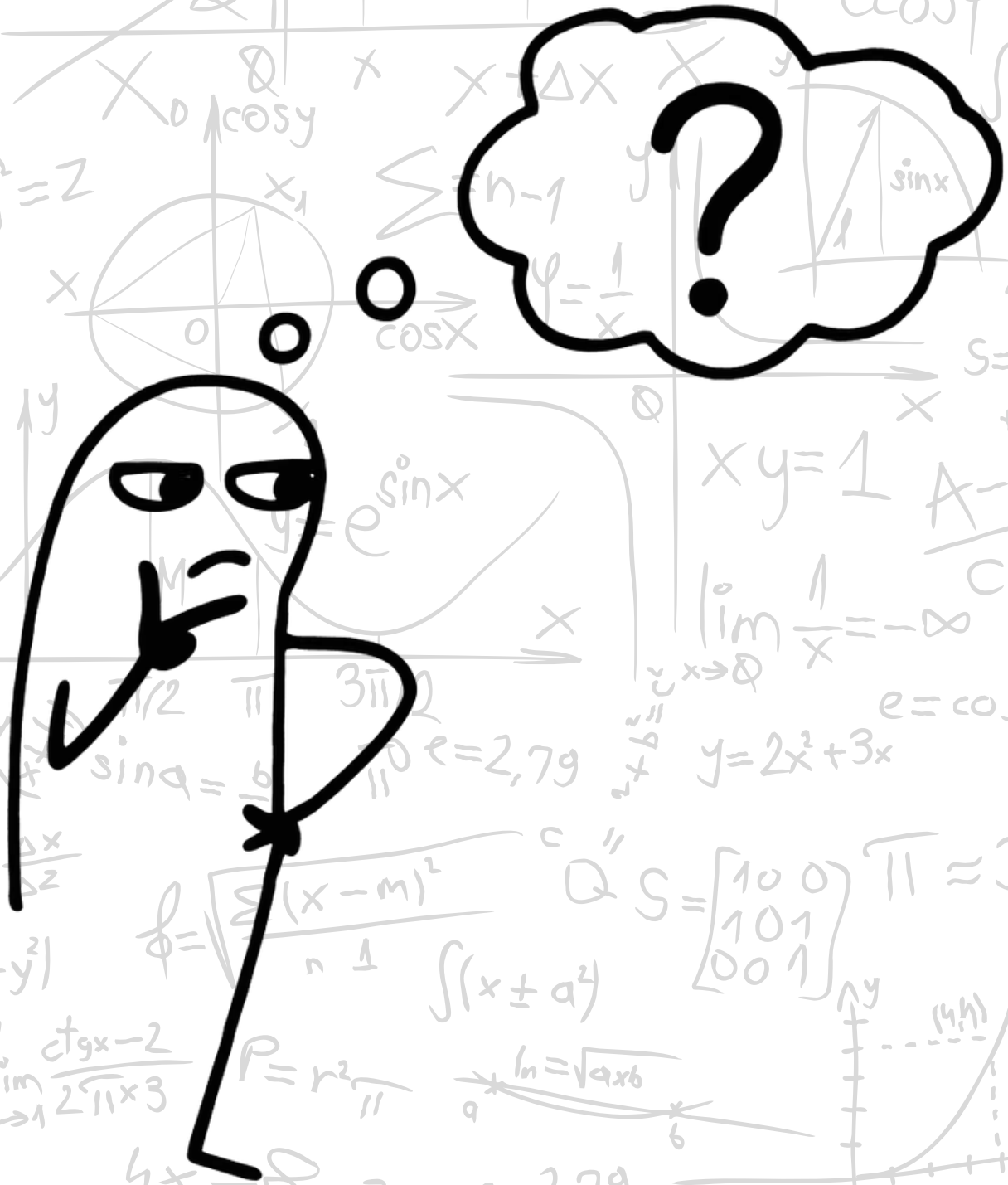
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QUESTION/*VRAAG* 5

5.1	A(0 ; 1)	✓ answer (1)
5.2	$9 = 3^{-x}$ $3^2 = 3^{-x}$ $x = -2$ B(–2 ; 9)	✓ equating ✓ $3^2 = 3^{-x}$ ✓ $x = -2$ (3)
5.3	$x \in (0;\infty)$ or $x > 0$	✓✓answer (2)
5.4	$h(x) = 27.3^{-x}$ $h(x) = 3^{-(x-3)}$ f shifted 3 units to the right	✓ $h(x) = 3^{-(x-3)}$ ✓3 units ✓ right (3)
5.5	$\frac{27}{3^x} < 1$ $\quad 3^{-x+3} < 1$ $3^x > 27$ **or** $3^{-x+3} < 3^0$ $3^x > 3^3$ $\quad -x+3 < 0$ $x > 3$ $\quad x > 3$ **OR** The graph shifts 3 units to the right Thus the y-intercept shift 3 units to the right (3 ; 1) $\therefore x > 3$	✓ $3^x > 27$ **or** $3^{-x+3} < 3^0$ ✓ $3^x > 3^3$ **or** $-x+3 < 0$ ✓ $x > 3$ (3) **OR** ✓ translation ✓ y-intercept ✓answer (3)
		[12]

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FINANCES



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QUESTION 7

7.1 Mary's grandparents deposited R5 000 into a savings account on the day that she was born. The account pays interest at a rate of 6,8% p.a., compounded quarterly. Calculate the accumulated amount in this account on Mary's 16th birthday. (3)

7.2 After 4 years, the value of a printer was half of its original value. Determine the rate at which the value of the printer depreciated over this period, if depreciation was calculated according to a straight-line method. (2)

7.3 Tshepo was granted a loan of R100 000 on 1 March 2022 at an interest rate of 13,5% p.a., compounded monthly. Tshepo agreed to repay the loan over 5 years in monthly instalments of R2 300,98, starting on 1 April 2022.

7.3.1 Calculate the total interest that he will pay over the 5 years. (2)

7.3.2 Tshepo paid R22 300,98 (his monthly instalment and an additional R20 000) on 1 March 2024 into the loan account. He continues to pay the original monthly instalment thereafter. How many months earlier will Tshepo repay the loan? (7)

[14]

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QUESTION/*VRAAG* 7

7.1	$A = P(1+i)^n$ $A = 5\,000\left(1+\frac{6,8}{400}\right)^{64}$ $= \text{R}\,14\,706,56$	✓ *i, n* ✓ substitution into correct formula ✓ answer (3)
7.2	$A = P(1-in)$ $\frac{1}{2}P = P(1-4i)$ $\frac{1}{2} = (1-4i)$ $i = \frac{1}{8}$ $\therefore r = 12,5\%$ **OR/*OF*** $r = \frac{50\%}{4}$ $r = 12,5\%$	✓substitution into correct formula ✓ answer (2) **OR/*OF*** ✓ method ✓ answer (2)
7.3.1	Already paid: $2\,300,98 \times 60 = \text{R}\,138\,058,80$ Interest: 138 058,80 – 100 000 = R 38 058,80	✓ amount already paid ✓ interest (2)

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7.3.2	Balance after 2 years: $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{2\,300,98\left[1-\left(1+\frac{13,5}{1200}\right)^{-36}\right]}{\frac{13,5}{1200}}$ $= \text{R } 67\,804,94$ R 67 804,94 – R20 000 = R 47 804,94 $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $47\,804,94 = \frac{2\,300,98\left[1-\left(1+\frac{13,5}{1200}\right)^{-n}\right]}{\frac{13,5}{1200}}$ $0,766 = (1,01125)^{-n}$ $\log_{1,01125} 0,766 = -n$ $n = 23,83$ $\therefore$ 12 months earlier **OR/*OF***	✓n= 36 ✓i ✓ substitution into correct formula ✓ answer ✓ answer ✓substitution into correct formula ✓ answer in months (7) **OR/*OF***
	$A = 100\,000\left(1+\frac{13,5}{1200}\right)^{12\times 2}$ $= \text{R}130\,799,12$ $F = \frac{2\,300,98\left[\left(1+\frac{13,5}{1200}\right)^{12\times 2}-1\right]}{\frac{13,5}{1200}}$ $= \text{R } 62\,993, 92$ Balance outstanding = A – F = 130 799,12 – 62 993,92 = R67 805,20 R 67 805,20 – R20 000 = R 47 805,20 $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $47\,805,20 = \frac{2\,300,98\left[1-\left(1+\frac{13,5}{1200}\right)^{-n}\right]}{\frac{13,5}{1200}}$ $0,766... = (1,01125)^{-n}$ $\log_{1,01125} 0,766... = -n$ $n = 23,80$ $\therefore$ 12 months' earlier	✓$n = 24$ ✓i ✓ substitution into both correct formulae ✓ answer ✓ answer ✓ substitution into correct formula ✓ answer in months (7)

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	OR/*OF*	**OR/*OF***
	$100\ 000 = \dfrac{2\ 300,98\left[1-\left(1+\dfrac{13,5}{1200}\right)^{-n}\right]}{\dfrac{13,5}{1200}} + 20\ 000\left(1+\dfrac{13,5}{1200}\right)^{-24}$	✓ i ✓ $n = 24$ ✓ $= 100\ 000$ ✓ substitution into correct P formula ✓ substitution into correct A formula
	$84\ 709,38 = \dfrac{2\ 300,98\left[1-\left(\dfrac{809}{800}\right)^{-n}\right]}{\dfrac{0,135}{12}}$	✓ simplification
	$0,4142 = 1-\left(\dfrac{809}{800}\right)^{-n}$ $\left(\dfrac{809}{800}\right)^{-n} = 0,5858$ $-n = \log_{\left(\frac{809}{800}\right)} 0,5858$	
	$n = 47,80$ months after starting payments $\therefore$ 12 months earlier	✓ answer in months (7)
		[14]

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QUESTION 7

7.1 Six years ago, Thabo bought a phone for R13 000. The value of the phone depreciated annually according to the reducing-balance method. The value of the phone is now R8 337,75. Calculate the annual rate of depreciation. (3)

7.2 Eric and Thandi need to save R80 000 each to go on a holiday at the end of December 2027.

- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
- Eric calculates that if he makes 48 deposits of R1 402,31, starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.

Calculate the difference between the total amount that Eric and Thandi will deposit into their respective savings accounts over the given period. (4)

7.3 Lesibana was granted a loan of R225 000. The rate of interest for the loan is 9% p.a., compounded monthly. Lesibana will make monthly payments of R5 500, starting exactly four months after the loan was granted. How many payments will Lesibana make to settle the loan? (6)
[13]

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QUESTION 7/*VRAAG* 7

7.1	$A = P(1-i)^n$ $8\,337{,}75 = 13\,000(1-i)^6$ $i = 7{,}14\%$	✓ substitution in correct formula ✓✓ answer (3)
7.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $80000 = \frac{x\left[\left(1+\frac{8,6}{1200}\right)^{36} - 1\right]}{\frac{8,6}{1200}}$ $x = \text{R}1\,955{,}78$ Thandi's total = 1955,78 × 36 = R 70 408,08 Eric's total = 1402,31 × 48 = R 67 310,88 Difference = 70 409,08 – 67 310,88 = R3 097,20	✓ i ✓ substitution into correct formula ✓ answer ✓ answer (4)
7.3	$A = P(1+i)^n$ $A = 225000\left(1+\frac{0,09}{12}\right)^3$ $A = R\,230100{,}5637...$ $225000\left(1+\frac{0,09}{12}\right)^3 = \frac{5500\left[1-\left(1+\frac{0,09}{12}\right)^{-n}\right]}{\frac{0,09}{12}}$ $0{,}3137734959... = 1-\left(1+\frac{0,09}{12}\right)^{-n}$ $\left(1+\frac{0,09}{12}\right)^{-n} = 0{,}6862265041...$ $-n = \log_{\left(1+\frac{0,09}{12}\right)} 0{,}6862265041...$ $n = 50{,}394375...$ $n = 51$	✓ substitution in correct formula ✓ answer ✓ substitution ✓ simplification ✓ use of logs ✓ answer (6)
		[13]

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QUESTION 6

6.1 Patrick deposited an amount of R18 500 into an account earning r% interest p.a., compounded monthly. After 6 months, his balance was R19 319,48.

6.1.1 Calculate the value of r. (3)

6.1.2 Calculate the effective interest rate. (2)

6.2 Kuda bought a laptop for R10 000 on 31 January 2019. He will replace it with a new one in 5 years' time on 31 January 2024.

6.2.1 The value of the old laptop depreciates annually at a rate of 20% p.a. according to the straight-line method. After how many years will the laptop have a value of R0? (2)

6.2.2 Kuda will buy a laptop that costs R20 000. In order to cover the cost price, he made his first monthly deposit into a savings account on 28 February 2019. He will make his 60th monthly deposit on 31 January 2024. The savings account pays interest at 8,7% p.a., compounded monthly. Calculate Kuda's monthly deposit into this account. (4)

6.3 Tino wins a jackpot of R1 600 000. He invests all of his winnings in a fund that earns interest of 11,2% p.a., compounded monthly. He withdraws R20 000 from the fund at the end of each month. His first withdrawal is exactly 1 month after his initial investment. How many withdrawals of R20 000 will Tino be able to make from this fund? (5)
[16]

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QUESTION 6/*VRAAG* 6

6.1.1	$A = P(1+i)^n$ $19\,319{,}48 = 18\,500\left(1+\frac{r}{1200}\right)^6$ $\left(1+\frac{r}{1200}\right) = \sqrt[6]{1{,}04429...}$ $\frac{r}{1200} = 0{,}00725...$ $r = 8{,}7\%$	✓ $n = 6$ ✓substitution into correct formula ✓answer (3)
6.1.2	$1+\frac{i}{100} = \left(1+\frac{8{,}7}{1200}\right)^{12}$ $r = 9{,}06\%$	✓substitution into correct formula ✓answer (2)
6.2.1	$A = P(1-in)$ $0 = 10\ 000(1-0{,}2n)$ $n = 5$	✓substitution into correct formula ✓answer (2)
6.2.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $20\,000 = \frac{x\left[\left(1+\frac{8{,}7}{1200}\right)^{60} - 1\right]}{\frac{8{,}7}{1200}}$ $x = \text{R}267{,}26$	✓ i ✓ n ✓substitution into correct formula ✓answer (4)
6.3	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $1\,600\,000 = \frac{20\,000\left[1-\left(1+\frac{0{,}112}{12}\right)^{-n}\right]}{\frac{0{,}112}{12}}$ $\frac{56}{75} = 1-\left(1+\frac{0{,}112}{12}\right)^{-n}$ $\left(1+\frac{0{,}112}{12}\right)^{-n} = \frac{19}{75}$ $-n = \log_{\left(1+\frac{0{,}112}{12}\right)}\left(\frac{19}{75}\right)$ $-n = -147{,}80$ Tino will make 147 withdrawals of R20 000	✓i ✓substitution into correct formula ✓correct use of logs ✓ $-n = -147{,}80$ ✓ $n = 147$ (5)
		[16]

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QUESTION 6

6.1 A company bought a photocopier for R150 000 on 1 July 2022. They will use the old photocopier as a trade-in when they replace it with a similar new photocopier in 5 years' time on 30 June 2027.

6.1.1 The average rate of inflation over the next 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time. (2)

6.1.2 Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a. on a straight-line method. (2)

6.1.3 The company set up a sinking fund to cover the replacement cost of the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 31 May 2027, one month prior to the new photocopier being bought. How much should be deposited at the end of each month so that the company will be able to buy the new photocopier? (4)

6.2 Today, Andrew borrowed R200 000 from a bank. The bank charges interest at 5,25% p.a., compounded quarterly. Andrew will make repayments of R6 000 at the end of every 3 months. His first repayment will be made in 3 months from now. How long, in years, will it take Andrew to settle the loan? (5)
[13]

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QUESTION 6/*VRAAG 6*

6.1.1	$A = P(1+i)^n$ $A = 150\,000(1+0{,}065)^5$ A = R205 513	✓ substitution into the correct formula ✓ answer (2)
6.1.2	$A = P(1-in)$ $A = 150\,000(1-0{,}09\times 5)$ $A = 150\,000 - 67\,000$ A = R82 500	✓ substitution into the correct formula ✓ answer (2)
6.1.3	SF = A – T = 205 513 – 82 500 = R123 013 $F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $x = \frac{F \times i}{(1+i)^n - 1}$ $x = \frac{123\,013 \times \frac{0{,}0785}{12}}{\left[\left(1+\frac{0{,}0785}{12}\right)^{59} - 1\right]\left(1+\frac{0{,}0785}{12}\right)}$ = R1 704,01	✓ answer ✓ $i = \frac{0{,}0785}{12}$ ✓ 59 and $\left(1+\frac{0{,}0785}{12}\right)$ (A) ✓ answer (A) (4)
6.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $200\,000 = \frac{6\,000\left[1-\left(1+\frac{0{,}0525}{4}\right)^{-4n}\right]}{\frac{0{,}0525}{4}}$ $\frac{7}{16} = 1-\left(1+\frac{0{,}0525}{4}\right)^{-4n}$ $\frac{9}{16} = \left(\frac{1621}{1600}\right)^{-4n}$ $-4n = \frac{\log\frac{9}{16}}{\log\left(\frac{1621}{1600}\right)}$ $-4n = -44{,}1243\ldots$ $n = 11{,}03$ years	✓ substitution into correct formula ✓ simplification ✓ use of logs ✓ $-4n = -44{,}1243\ldots$ ✓ $n = 11{,}03$ years (5)
		[13]

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QUESTION 6

6.1 R12 000 was invested in a fund that paid interest at m% p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

6.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation. (4)

6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

6.3.1 Calculate the value of the loan. (1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment. (5)
[14]

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QUESTION 6/*VRAAG 6*

6.1	$A = P(1+i)^n$ $13\,459 = 12\,000\left(1+\frac{m}{400}\right)^8$ $\left(1+\frac{m}{400}\right)^8 = 1{,}121...$ $1+\frac{m}{400} = \sqrt[8]{1{,}121..}$ $\frac{m}{400} = 0{,}0144...$ $\therefore m = 5{,}78\%$	✓ 8 ✓ subst into correct formula ✓ $1+\frac{m}{400} = \sqrt[8]{1{,}121..}$ ✓ 5,78 % (4)
6.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $F = \frac{1\,000\left[\left(1+\frac{0{,}075}{12}\right)^{12} - 1\right]}{\frac{0{,}075}{12}}$ $= R12\,421{,}22$ He won't be able to buy the computer because R13 000 – R12 421,22 = R578,78 **OR/OF** He won't be able to buy the computer because R12 421,22 < R13 000	✓ $\frac{0{,}075}{12}$ ✓ 12 ✓ answer ✓ conclusion (4)
6.3.1	Loan amount = 85% × R250 000 = R212 500 **OR/OF** Loan amount = R250 000 – (15% × R250 000) = R212 500	✓ answer (1) **OR/OF** ✓ answer (1)
6.3.2	$A = 212\,500\left(1+\frac{0{,}13}{12}\right)^5$ $A = 224\,262{,}53$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $224\,262{,}53 = \frac{x\left[1-\left(1+\frac{0{,}13}{12}\right)^{-67}\right]}{\frac{0{,}13}{12}}$ $\therefore x = R4\,724{,}96$	✓ $A = 212\,500\left(1+\frac{0{,}13}{12}\right)^5$ ✓answer ✓ substitution into correct formula ✓ -67 ✓ answer (5)
		[14]

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QUESTION 7

7.1 How many years will it take for an investment to double in value, if it earns interest at a rate of 8,5% p.a., compounded quarterly? (4)

7.2 A company purchased machinery for R500 000. After 5 years, the machinery was sold for R180 000 and new machinery was bought.

7.2.1 Calculate the rate of depreciation of the old machinery over the 5 years, using the reducing-balance method. (4)

7.2.2 The rate of inflation for the cost of the new machinery is 6,3% p.a. over the 5 years. What will the new machinery cost at the end of 5 years? (2)

7.2.3 The company set up a sinking fund and made the first payment into this fund on the day the old machinery was bought. The last payment was made three months before the new machinery was purchased at the end of the 5 years. The interest earned on the sinking fund was 10,25% p.a., compounded monthly. The money from the sinking fund and the R180 000 from the sale of the old machinery was used to pay for the new machinery.

Calculate the monthly payment into the sinking fund. (5)
[15]

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QUESTION 7

7.1	$A = P(1+i)^n$ $2 = 1\left(1+\frac{0,085}{4}\right)^{4n}$ $4n = \log_{\left(1+\frac{0,085}{4}\right)} 2$ $n = 8,24$ years	✓ 2 ✓ $\frac{0,085}{4}$ } In correct formula ✓ use of logs ✓ answer in years (4)
7.2.1	$A = P(1-i)^n$ $180\ 000 = 500\ 000(1-i)^5$ $\frac{9}{25} = (1-i)^5$ $\sqrt[5]{\frac{9}{25}} = 1-i$ $i = 0,1848068...$ $r = 18,48\%$	✓ subs into correct formula ✓ simplification ✓ $i = 0,1848\ldots$ ✓ answer (4)
7.2.2	$A = P(1+i)^n$ $A = 500\ 000(1+0,063)^5$ A = R678 635,11	✓ subs into correct formula ✓ answer (2)
7.2.3	Sinking Fund = 678 635,11 – 180 000 = R 498 635,11 $498\ 635,11 = \frac{x\left[\left(1+\frac{0,1025}{12}\right)^{58}-1\right]\left(1+\frac{0,1025}{12}\right)^{3}}{\frac{0,1025}{12}}$ $x = R6\,510,36$	✓ value of sinking fund ✓ $\frac{0,1025}{12}$ ✓ $n = 58$ (A) ✓ $\left(1+\frac{0,1025}{12}\right)^{3}$ ✓ answer (A) (5)
		[15]

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QUESTION 8

8.1 A farmer bought a tractor for R980 000. The value of the tractor depreciates annually at a rate of 9,2% p.a. on the reducing-balance method. Calculate the book value of the tractor after 7 years. (3)

8.2 How many years will it take for an amount of R75 000 to accrue to R116 253,50 in an account earning interest of 6,8% p.a., compounded quarterly? (4)

8.3 Thabo wanted to save R450 000 as a deposit to buy a house on 30 June 2018.

8.3.1 He deposited a fixed amount of money at the end of every month into an account earning interest of 8,35% p.a., compounded monthly. His first deposit was made on 31 July 2013 and his 60th deposit on 30 June 2018. Calculate the amount he deposited monthly. (3)

8.3.2 Thabo bought a house costing R1 500 000 and used his savings as the deposit. He obtained a home loan for the balance of the purchase price at an interest of 12% p.a., compounded monthly over 25 years. He made his first monthly instalment of R11 058,85 towards the loan on 31 July 2018.

(a) What will the balance outstanding on the loan be on 30 June 2039, 21 years after the loan was granted? (3)

(a) Calculate the interest Thabo will have paid over the first 21 years of the loan. (3)

[16]

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QUESTION/VRAAG 8

8.1	$A = P(1-i)^n$ $A = 980\ 000(1-0,092)^7$ $A = R498\ 685,82$	✓ correct formula ✓ substitution ✓ answer (A) (3)
8.2	$A = P(1+i)^n$ $116\ 253,50 = 75000\left(1+\frac{0,068}{4}\right)^{4n}$ $1,550\ 046\ 667 = (1,017)^{4n}$ $\log(1,550\ 046\ 667) = 4n\log(1,017)$ $4n = \frac{\log(1,550\ 046\ 667)}{\log(1,017)}$ or $4n = \log_{1,017}(1,550\ 046\ 667)$ $4n = 25,99\ \ldots$ $n =$ 6,50 years	✓ $\frac{0,068}{4}$ ✓ substitution in correct formula ✓ correct use of logs ✓ answer (4)
8.3.1	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $450\ 000 = \frac{x\left[\left(1+\frac{0,0835}{12}\right)^{60} - 1\right]}{\frac{0,0835}{12}}$ $x = R6\ 068,69$	✓ $i = \frac{0,0835}{12}$ ✓ substitution into correct formula ✓ answer (3)
8.3.2(a)	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{11\ 058,85\left[1-\left(1+\frac{0,12}{12}\right)^{-4\times12}\right]}{\frac{0,12}{12}}$ $P = R419\ 948,32$ **OR/*OF*** $\text{Balance} = A - F$ $= P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i}$ $= 1\ 050\ 000\left(1+\frac{0,12}{12}\right)^{12\times21} - \frac{11\ 058,85\left[\left(1+\frac{0,12}{12}\right)^{12\times21} - 1\right]}{\frac{0,12}{12}}$ $= R12\ 887\ 702,20 - R12\ 467\ 749,81$ $= R419\ 952,39$	✓ $n = 48$ in P-formula ✓ substitution into correct formula ✓ answer (A) (3) **OR/*OF*** ✓ $n = 252$ in both formulae ✓ subst into correct formulae ✓ answer (A) (3)

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8.3.2(b)	Total paid $= 11\,058,85 \times 21 \times 12 = 2\,786\,830,20$ Loan Paid $= 1\,050\,000 - 419\,948,32 = 630\,051,68$ Interest paid = 2 786 830,20 – 630 051,68 = R2 156 778,52 **OR/OF** Total paid $= 11\,058,85 \times 21 \times 12 = 2\,786\,830,20$ Loan Paid $= 1\,050\,000 - 419\,952,39 = 630\,047,61$ Interest paid = 2 786 830,20 – 630 047,61 = R2 156 782,59 **OR/OF** Interest paid = 11 058,85 × 21 × 12 – (1 050 000 – 419 948,32) = 2 786 830,20 – 630 051,68 = R2 156 778,52	✓ $11\,058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3) **OR/OF** ✓ $11\,058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3) **OR/OF** ✓ $11\,058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3)
		[16]

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QUESTION 7

7.1 An amount of R10 000 was invested for 4 years, earning interest at r% p.a., compounded quarterly. At the end of the 4 years, the total amount in the account was R13 080. Determine the value of r. (4)

7.2 A businesswoman deposited R9 000 into an account at the end of January 2014. She continued to make monthly deposits of R9 000 at the end of each month up to the end of December 2018. The account earned interest at a rate of 7,5% p.a., compounded monthly.

7.2.1 Calculate how much money was in the account immediately after 60 deposits had been made. (3)

7.2.2 The businesswoman left the amount calculated in QUESTION 7.2.1 for a further n months in the account. The interest rate remained unchanged and no further payments were made. The total interest earned over the entire investment period was R190 214,14. Determine the value of n. (6)
[13]

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QUESTION/*VRAAG* 7

7.1	$13\,080 = 10\,000\left(1+\frac{i}{4}\right)^{16}$ $\left(1+\frac{i}{4}\right)^{16} = \frac{13080}{10000}$ $1+\frac{i}{4} = \sqrt[16]{\frac{13080}{10000}}$ $\frac{i}{4} = 0{,}0169227...$ $i = 0{,}06769...$ $i = 6{,}77\%$	✓substitution into correct formula ✓$n = 16$ ✓ simplification ✓answer (A) (4)
7.2.1	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $F = \frac{9\,000\left[\left(1+\frac{0{,}075}{12}\right)^{60} - 1\right]}{\frac{0{,}075}{12}}$ $F = \text{R } 652\,743{,}95$	✓ $\frac{0{,}075}{12}$ ✓ substitution into correct Formula ✓ answer (3)
7.2.2	$60 \times 9000 = \text{R}540\,000$ $A = P(1+i)^n$ $652743{,}95\left(1+\frac{0{,}075}{12}\right)^n = 190\,214{,}14 + 540000$ $730\,214{,}14 = 652\,743{,}95\left(1+\frac{0{,}075}{12}\right)^n$ $1{,}1186.... = (1{,}00625)^n$ $n = \log_{1{,}00625}(1{,}1186)$ $\therefore n = 18$ months	✓ $60 \times 9000 = \text{R}540\,000$ ✓✓ equation ✓simplification ✓use of logs ✓ 18 months (6)

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	OR/OF Interest over 5 years = $652\,743{,}95 - 9\,000 \times 60$ $= 112\,743{,}95$ $\therefore$ interest on n years $= 190\,214{,}14 - 112\,743{,}95 = 77\,470{,}19$ $652\,743{,}95 + 77\,470{,}19 = 652743{,}95\left(1+\frac{0{,}075}{12}\right)^n$ $1{,}1186.... = (1{,}00625)^n$ $n = \log_{1{,}00625}(1{,}1186)$ $\therefore n = 18$ months	**OR/OF** ✓ $60 \times 9\,000$ ✓ answer ✓equating ✓simplification ✓use of logs ✓ 18 months (6)
		[13]

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QUESTION 6

6.1 On 31 January 2020, Tshepo made the first of his monthly deposits of R1 000 into a savings account. He continues to make monthly deposits of R1 000 at the end of each month up until 31 January 2032. The interest rate was fixed at 7,5% p.a., compounded monthly.

6.1.1 What will the investment be worth immediately after the last deposit? (4)

6.1.2 If he makes no further payments but leaves the money in the account, how much money will be in the account on 31 January 2033? (2)

6.2 Jim bought a new car for R250 000. The value of the car depreciated at a rate of 22% p.a. annually according to the reducing-balance method. After how many years will its book value be R92 537,64? (3)

6.3 Mpho is granted a loan under the following conditions:

- The interest rate is 11,3% p.a., compounded monthly.
- The period of the loan is 6 years.
- The monthly repayment on the loan is R1 500.
- Her first repayment is made one month after the loan is granted.

6.3.1 Calculate the value of the loan. (3)

6.3.2 How much interest will Mpho pay in total over the first 5 years? (4)
[16]

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QUESTION/*VRAAG* 6

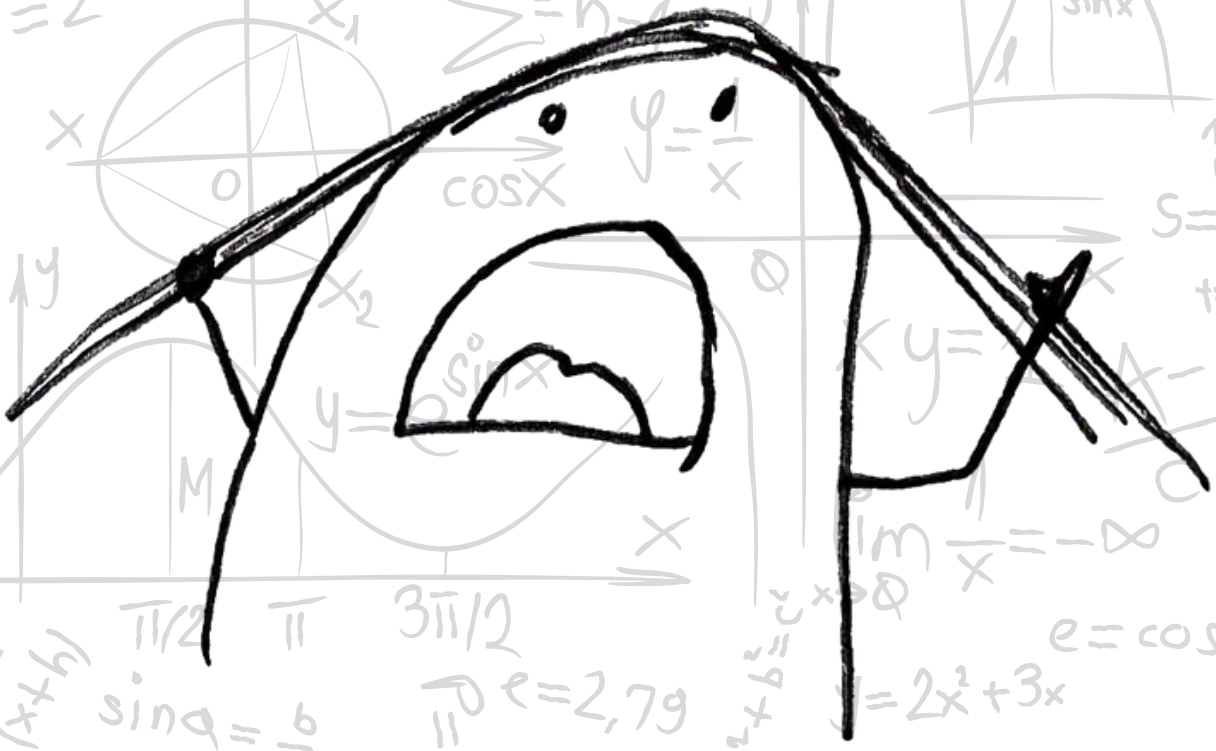
6.1.1	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $= \frac{1\,000\left[\left(1+\frac{0,075}{12}\right)^{145} - 1\right]}{\frac{0,075}{12}}$ $= \text{R}234\,888,53$	✓ $n = 145$ ✓ $i = \frac{0,075}{12}$ ✓ substitution into the correct formula ✓ answer (4)
6.1.2	$A = P(1+i)^n$ $= 234\,888,53\left(1+\frac{0,075}{12}\right)^{12}$ $= \text{R}253\,123,54$	✓ substitution into the correct formula ✓ answer (2)
6.2	$A = P(1-i)^n$ $92\,537,64 = 250\,000(1-0,22)^n$ $0,37015056 = (0,78)^n$ $n = \frac{\log 0,37015056}{\log 0,78}$ $n = 4$ years	✓substitution into the correct formula ✓correct use of logs ✓answer (3)
6.3.1	Loan amount: $= \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{1500\left[1-\left(1+\frac{0,113}{12}\right)^{-72}\right]}{\frac{0,113}{12}}$ $= \text{R}78\,173,49323$	✓ 72 ✓substitution into the correct formula ✓answer (3)
6.3.2	Balance after 5 years: P $= \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{1\,500\left[1-\left(1+\frac{0,113}{12}\right)^{-12}\right]}{\frac{0,113}{12}}$ $= \text{R}16\,945,00629$ Amount paid: R1 $500 \times 60 = \text{R}90\,000$ Interest = Amount paid – [Loan – Balance] = R90 000 – [R78 173,49323 – R16 945,00629] = R28 771,51	✓ substitution (A) ✓ R16 945,00629 (A) ✓ R90 000 – [Loan – Balance] ✓ answer (4)

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	OR Balance $= 78\,173{,}49\left(1+\frac{0{,}133}{12}\right)^{60} - \frac{1\,500\left(\left(1+\frac{0{,}113}{12}\right)^{60}-1\right)}{\frac{0{,}113}{12}}$ Balance = R16 945.00 Amount paid: R1 $500 \times 60 =$ R90 000 Interest = Amount paid – [Loan – Balance] = R90 000 – [R78 173,49323 – R16 945,00629] = R28 771,51	**OR** ✓ substitution ✓ R16 945,00629 ✓ R90 000 – [Loan – Balance] ✓ answer (4)
		[16]

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CALCULUS



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QUESTION 8

8.1 Determine:

8.1.1 $\frac{d}{dx}\left[3x-5x^2\right]$ (2)

8.1.2 $g'(x)$ if $g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ (4)

8.2 Determine the equation of the tangent to $f(x)=x^3-4x^2+2x+3$ at $x=2$. (3)

8.3 Given: $f(x)=-6x^2$

8.3.1 Determine $f'(x)$ from first principles. (5)

8.3.2 Write down how you will restrict the domain of f such that f^{-1}, the inverse of f, is a function. (1)

8.3.3 Determine the equation of f^{-1} for $f^{-1}(x)\leq 0$. Write your answer in the form $y=\ldots$ (3)
[18]

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QUESTION/*VRAAG* 8

8.1.1	$\frac{d}{dx}\left[3x-5x^2\right]$ $=3-10x$	✓3 ✓$-10x$ (2)
8.1.2	$g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ $g(x)=2x^{-2}-x^{\frac{7}{3}}$ $g'(x)=-4x^{-3}-\frac{7}{3}x^{\frac{4}{3}}$	✓$2x^{-2}$ ✓$-x^{\frac{7}{3}}$ ✓$-4x^{-3}$ ✓$-\frac{7}{3}x^{\frac{4}{3}}$ (4)
8.2	$f(x)=x^3-4x^2+2x+3$ $f'(x)=3x^2-8x+2$ $m=f'(2)=3(2)^2-8(2)+2$ $m=-2$ $y=f(2)=(2)^3-4(2)^2+2(2)+3$ $y=-1$ $y+1=-2(x-2)$ $y=-2x+3$	✓$m=-2$ ✓$y=-1$ ✓ answer (3)
8.3.1	$f(x)=-6x^2$ $f'(x)=\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to0}\frac{-6(x+h)^2+6x^2}{h}$ $=\lim\limits_{h\to0}\frac{-6x^2-12xh-6h^2+6x^2}{h}$ $=\lim\limits_{h\to0}\frac{-12xh-6h^2}{h}$ $=\lim\limits_{h\to0}\frac{h(-12x-6h)}{h}$ $=\lim\limits_{h\to0}(-12x-6h)$ $=-12x$	✓substitution ✓ $f(x+h)=-6x^2-12xh-6h^2$ ✓simplification ✓common factor ✓answer (5)

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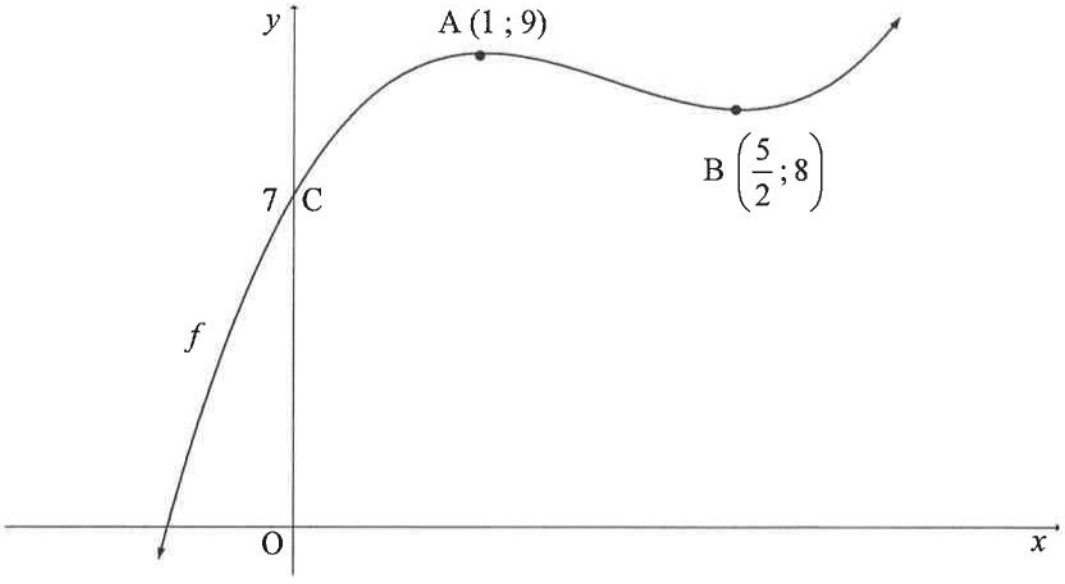
	OR/*OF*	**OR/*OF***
	$f(x)=-6x^2$ $f(x+h)=-6x^2-12xh-6h^2$ $f(x+h)-f(x)=-6x^2-12xh-6h^2+6x^2=-12xh-6h^2$	✓ $f(x+h)=-6x^2-12xh-6h^2$ ✓simplification
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-12xh-6h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-12x-6h)}{h}$ $=\lim\limits_{h\to 0}(-12x-6h)$ $=-12x$	✓substitution ✓common factor ✓answer (5)
8.3.2	$x\geq 0$ **OR/*OF*** $x\leq 0$	✓answer (1) ✓answer (1)
8.3.3	$y=-6x^2$ $x=-6y^2$ $y^2=\dfrac{-1}{6}x$ $y=\pm\sqrt{-\dfrac{1}{6}x}$ $\therefore y=-\sqrt{-\dfrac{1}{6}x}\quad ;\quad x\leq 0$	✓ swopping x and y ✓ $y=\pm\sqrt{-\dfrac{1}{6}x}$ ✓ answer (3)
		[18]

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QUESTION 9

A(1 ; 9) and $B\left(\frac{5}{2};8\right)$ are the turning points of graph f below.

C(0 ; 7) is the y-intercept of f.



9.1 For which values of x is f decreasing? (2)

9.2 Write down the x-intercepts of f', the derivative of f. (2)

9.3 For which values of x will f be concave up? (2)

9.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)
[8]

QUESTION 10

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

10.2 Calculate the distance between town P and town T. (5)
[8]

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QUESTION/*VRAAG* 9

9.1	$1 < x < \frac{5}{2}$	✓✓answer (2)
9.2	$(1\,;0)$ and $\left(\frac{5}{2};0\right)$	✓ $x=1$ ✓ $x=\frac{5}{2}$ (2)
9.3	$\frac{\frac{5}{2}+1}{2}$ $=\frac{7}{4}$ Concave up for $x > \frac{7}{4}$	✓method ✓answer (2)
9.4	$-9 < k < -8$	✓✓answer (2)
		[8]

QUESTION/*VRAAG* 10

10.1	$-6t+18=0$ $18=6t$ $3=t$ $s'(3)=-3(3)^2+18(3)=27$	✓ $=0$ ✓ value of t ✓ answer (3)
10.2	$-3t^2+18t=0$ $-3t(t-6)=0$ $t=0$ or $t=6$ $s(t)=at^3+bt^2$ $s'(t)=3at^2+2bt$ $3a=-3$ and $2b=18$ $a=-1$ $b=9$ $s(t)=-1t^3+9t^2$ $s(6)=-(6)^3+9(6)^2$ $s(6)=108$	✓ factors ✓ values ✓ a and b ✓ substitution ✓ answer (5)
		[8]

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QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=\frac{1}{x}$. (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ (3)

8.2.2 $g'(x)$ if $g(x)=\frac{3x^4-4x^2+6}{x^2}$ (3)

8.3 The equation of the tangent to $f(x)=3x^2+bx+c$ at $x=1$ is given by $y=9x-9$. Determine the values of b and c. (4)
[15]

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QUESTION8/*VRAAG* 8

8.1	$f(x)=\frac{1}{x}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$ $f'(x)=\lim_{h\to 0}\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$	✓ $f(x+h)=\frac{1}{x+h}$ ✓ $\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-h}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-1}{x(x+h)}$ ✓answer (5)
	OR/*OF* $f(x)=\frac{1}{x}$ $f(x+h)=\frac{1}{x+h}$ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$	**OR/*OF*** ✓ $f(x+h)=\frac{1}{x+h}$ ✓ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ ✓ $\frac{-h}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-1}{x(x+h)}$ ✓answer (5)

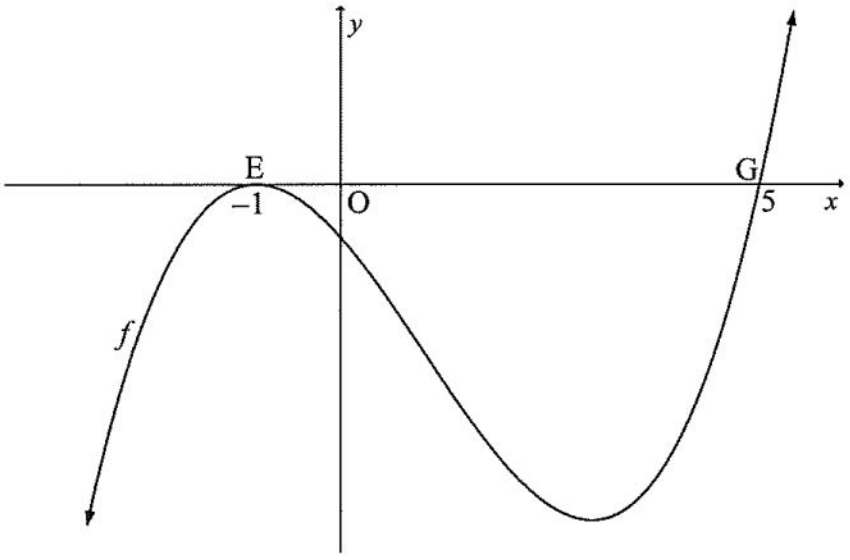
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8.2.1	$\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ $=\frac{d}{dx}\left(2x^3+\sqrt{2}.x^2\right)$ $=6x^2+2\sqrt{2}x$	✓ $2x^3$ ✓ $6x^2$ ✓ $2\sqrt{2}x$ (3)
8.2.2	$g(x)=\frac{3x^4-4x^2+6}{x^2}$ $g(x)=3x^2-4+6x^{-2}$ $g'(x)=6x-12x^{-3}$	✓ $3x^2-4+6x^{-2}$ ✓ $6x$ ✓ $-12x^{-3}$ (3)
8.3	$f(x)=3x^2+bx+c$ $f'(x)=6x+b$ $f'(1)=6+b=9$ $\therefore b=3$ $f(1)=3+3+c=0$ $c=-6$ $\therefore f(x)=3x^2+3x-6$	✓ $f'(1)=6+b=9$ ✓ $b=3$ ✓ $f(1)=3+3+c=0$ ✓ $c=-6$ (4)
		[15]

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QUESTION 9

The graph of $f(x) = ax^3 + bx^2 + cx - 5$ is drawn below. E(– 1 ; 0) and G(5 ; 0) are the x-intercepts of f.



9.1 Show that $a = 1$, $b = -3$ and $c = -9$. (3)

9.2 Calculate the value of x for which f has a local minimum value. (4)

9.3 Use the graph to determine the values of x for which $f''(x).f(x) > 0$. (3)

9.4 For which values of t will the graph of $p(x) = f(x) + t$ have two distinct positive roots and one negative root? (3)
[13]

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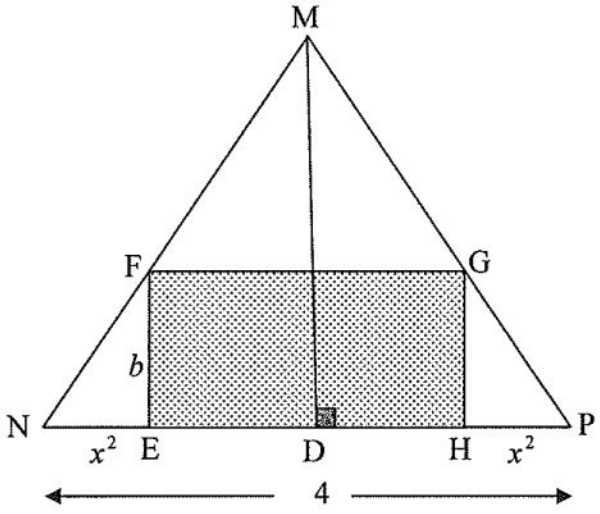
QUESTION9/*VRAAG* 9

9.1	$f(x) = ax^3 + bx^2 + cx - 5$ $-5 = a(0+1)^2(0-5)$ $-5 = -5a$ $a = 1$ $f(x) = (x+1)(x+1)(x-5)$ $f(x) = (x^2 + 2x + 1)(x-5)$ $f(x) = x^3 - 3x^2 - 9x - 5$ $\therefore\ b = -3$ and $c = -9$	✓ substitution of x-intercepts ✓ simplification ✓ simplification (3)
9.2	$f(x) = x^3 - 3x^2 - 9x - 5$ $f'(x) = 3x^2 - 6x - 9$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ Minimum value at $x = 3$	✓ $f'(x) = 3x^2 - 6x - 9$ ✓ $f'(x) = 0$ ✓factors ✓ $x = 3$ (4)
9.3	$f''(x).f(x) > 0$ Point of inflection: $x = 1$ $x < 1\ ;\ x \neq -1$ or $x > 5$	✓ $x = 1$ ✓ $x < 1\ ;\ x \neq -1$ ✓ $x > 5$ (3)
9.4	$-32 < -t < -5$ $5 < t < 32$ **OR/*OF*** Shift up more than 5 units and less than 32 units $\therefore 5 < t < 32$	✓-32 ✓$-32 < -t < -5$ ✓ $5 < t < 32$ (3) **OR/*OF*** ✓more than 5 units ✓less than 32 units ✓ $5 < t < 32$ (3)
		[13]

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QUESTION 10

EHGF is a rectangle. HE is produced x^2 cm to N and EH is produced x^2 cm to P. NF produced intersects PG produced at M to form an isosceles triangle MNP with NM = MP. D lies on NP where MD $\perp$ NP. NP = 4 cm and MD = 3 cm.

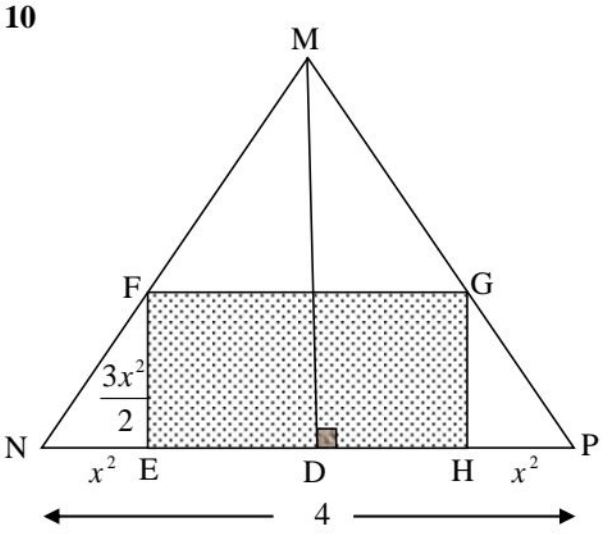


10.1 Show that the area of EFGH is given by $A(x) = 6x^2 - 3x^4$. (4)

10.2 Calculate the maximum area of rectangle EFGH. (4)
[8]

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QUESTION 10/*VRAAG* 10



10.1	$\frac{NE}{EF}=\frac{2}{3}=\frac{x^2}{b}$ $3x^2=2b$ $\therefore b=\frac{3x^2}{2}$ $EH=4-2x^2$ Area EFGH $=(4-2x^2)\left(\frac{3x^2}{2}\right)$ $A(x)=6x^2-3x^4$ **OR/OF** In ΔDMP: $\tan P=\frac{3}{2}$ In ΔHGP: $\tan P=\frac{GH}{x^2}$ $\frac{GH}{x^2}=\frac{3}{2}$ $\therefore b=\frac{3x^2}{2}$ $EH=4-2x^2$ Area EFGH $=(4-2x^2)\left(\frac{3x^2}{2}\right)$ $A(x)=6x^2-3x^4$	✓ $\frac{NE}{EF}=\frac{2}{3}=\frac{x^2}{b}$ ✓ $\therefore b=\frac{3x^2}{2}$ ✓ $EH=4-2x^2$ ✓ $(4-2x^2)\left(\frac{3x^2}{2}\right)$ (4) **OR/OF** ✓ $\frac{GH}{x^2}=\frac{2}{3}$ ✓ $\therefore b=\frac{3x^2}{2}$ ✓ $EH=4-2x^2$ ✓ $(4-2x^2)\left(\frac{3x^2}{2}\right)$ (4)
10.2	$A(x)=6x^2-3x^4$ $A'(x)=12x-12x^3=0$ $12x(1-x^2)=0$ $\therefore x\neq 0$ or $x=-1$ or $x=1$ $\therefore$ max area: $A(1)=6(1)^2-3(1)^4=3$ cm^2	✓ $12x-12x^3=0$ ✓ values of x ✓ correct substitution ✓ answer (4)
		[8]

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QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$ (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = 2x^3 - 3x$ (2)

7.2.2 $D_x\left(7\sqrt[3]{x^2} + 2x^{-5}\right)$ (3)

7.3 For which values of x will the tangent to $f(x) = -2x^3 + 8x$ have a positive gradient? (3)
[13]

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QUESTION 7/*VRAAG* 7

7.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4(x+h)^2-\left(-4x^2\right)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$ **OR/*OF*** $f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$ $f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$ $=-8xh-4h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$	✓substitution into correct formula ✓ $f(x+h)=-4x^2-8xh-4h^2$ ✓simplification ✓common factor ✓answer (5) **OR/*OF*** ✓ $f(x+h)=-4x^2-8xh-4h^2$ ✓simplification ✓substitution into correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=2x^3-3x$ $f'(x)=6x^2-3$	✓ $6x^2$ ✓ -3 (2)
7.2.2	$D_x\left[7.\sqrt[3]{x^2}+2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}}+2x^{-5}\right]$ $=\dfrac{14}{3}x^{-\frac{1}{3}}-10x^{-6}$	✓ $x^{\frac{2}{3}}$ ✓derivative with rational exp ✓ $-10x^{-6}$ (3)
7.3	$-6x^2+8>0$ $x^2<\dfrac{8}{6}$ CV's: $x=-\dfrac{2}{\sqrt{3}}$ or $x=\dfrac{2}{\sqrt{3}}$ Positive for : $-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$	✓CV's: $x=\pm\dfrac{2}{\sqrt{3}}$ ✓✓answer (3)
		[13]

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QUESTION 8

Given: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)^2(-x+4)$

8.1 Determine the coordinates of the turning points of f. (4)

8.2 Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4)

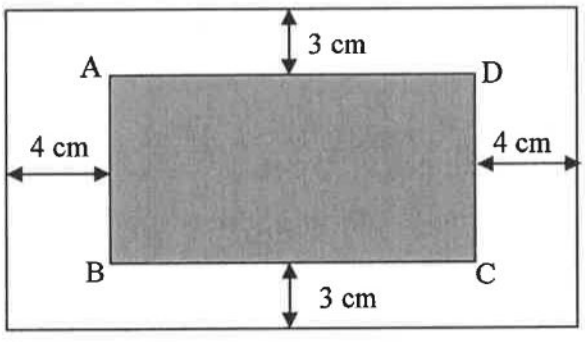
8.3 Use the graph to determine the value(s) of k for which $-x^3 + 6x^2 - 9x + 4 = k$ will have three real and unequal roots. (2)

8.4 The line $g(x) = ax + b$ is the tangent to f at the point of inflection of f. Determine the equation of g. (6)

8.5 Calculate the value of θ, the acute angle formed between g and the x-axis in the first quadrant. (2)
[18]

QUESTION 9

The diagram below represents a printed poster. Rectangle ABCD is the part on which the text is printed. This shaded area ABCD is 432 cm^2 and AD = x cm. ABCD is 4 cm from the left and right edges of the page and 3 cm from the top and bottom of the page.



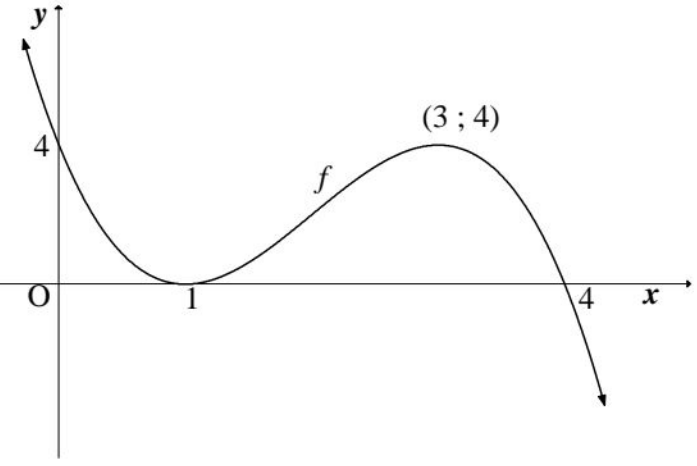
9.1 Show that the total area of the page is given by:

$$A(x) = \frac{3\,456}{x} + 6x + 480$$ (3)

9.2 Determine the value of x such that the total area of the page is a minimum. (3)
[6]

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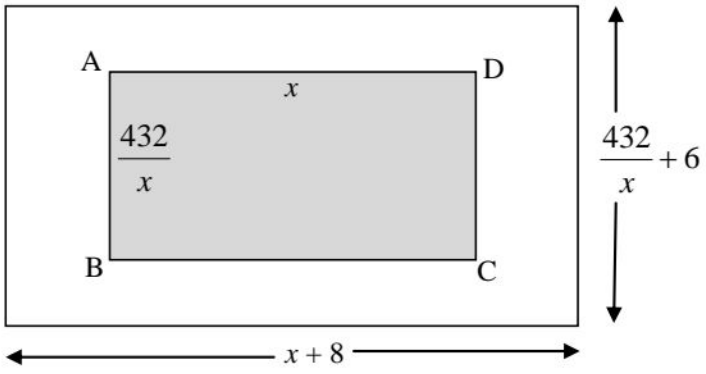
QUESTION 8/*VRAAG* 8

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ or $x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)	✓ $f'(x) = -3x^2 + 12x - 9$ ✓ $f'(x) = 0$ ✓ both x-values ✓ both y-values (4)
8.2		✓ y-intercept ✓ both x-intercepts ✓ both turning points ✓ shape (4)
8.3	$0 < k < 4$ or/*of* $k \in (0 ; 4)$	✓✓k between y-values of turning points (2)
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$ **OR/*OF*** Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$	✓ $f''(x) = -6x + 12$ ✓ $f''(x) = 0$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6) **OR/*OF*** ✓✓$\frac{3+1}{2}$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6)
8.5	$\tan\theta = 3$ $\therefore \theta = 71{,}57°$	✓ gradient of g ✓ answer (2)
		[18]

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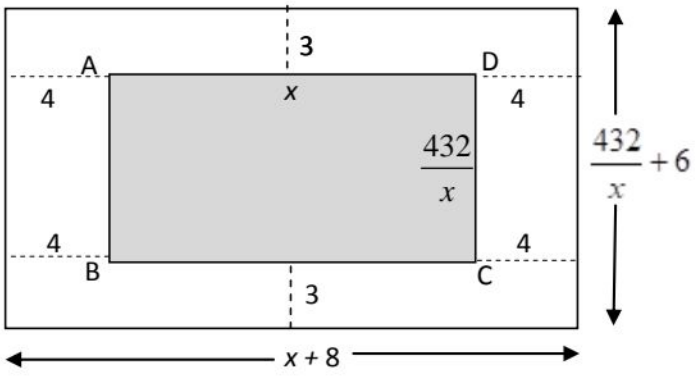
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QUESTION 9/*VRAAG* 9

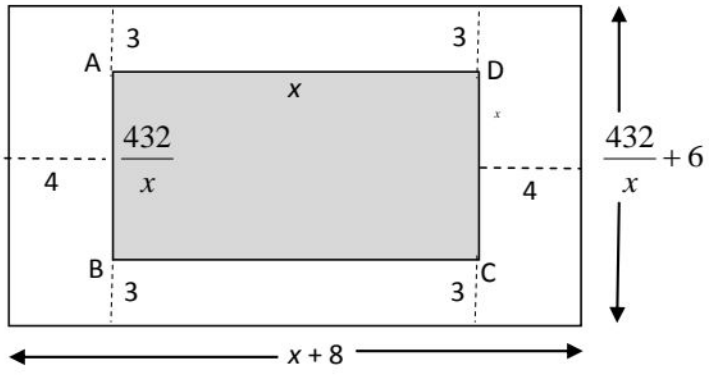


9.1	$432 = xb$ $\therefore b = \frac{432}{x}$ $A(x) = (x+8)\left(\frac{432}{x}+6\right)$ $A(x) = 432 + 6x + \frac{3456}{x} + 480$ $A(x) = \frac{3456}{x} + 6x + 480$	✓ $b = \frac{432}{x}$ ✓ $(x+8)$ ✓ $\left(\frac{432}{x}+6\right)$ (3)
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A'(x) = -\frac{3456}{x^2} + 6$ $-\frac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24\text{ cm}$	✓ $3456x^{-1} + 6x + 480$ ✓ $A'(x) = -\frac{3456}{x^2} + 6$ ✓answer (3)
		[6]

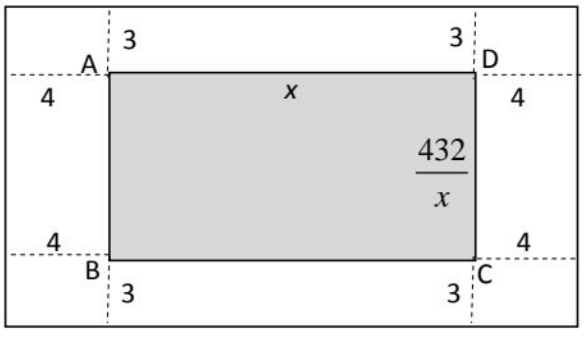
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$$\text{total area} = 2(x+8)(3)+2\left(\frac{432}{x}\right)(4)+\left(\frac{432}{x}\right)(x)$$



$$\text{total area} = 2(4)\left(\frac{432}{x}+6\right)+(x)\left(\frac{432}{x}+6\right)$$



$$\text{total area} = 4(4)(3)+2(x)(3)+\left(\frac{432}{x}\right)(x)+2\left(\frac{432}{x}\right)(4)$$

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QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -2x^2 - 1$. (5)

7.2 Determine:

7.2.1 $f'(x)$, if it is given that $f(x) = -2x^3 + 3x^2$ (2)

7.2.2 $\frac{dy}{dx}$ if $y = 2x + \frac{1}{\sqrt{4x}}$ (4)

7.3 The graph $y = f'(x)$ has a minimum turning point at $(1; -3)$.
Determine the values of x for which f is concave down. (2)
[13]

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QUESTION 7/*VRAAG* 7

7.1	$f(x)=-2x^2-1$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2(x+h)^2-1-(-2x^2-1)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2x^2-4xh-2h^2-1+2x^2+1}{h}$ $=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=-4x$ **OR/*OF*** $f(x+h)=-2(x+h)^2-1$ $f(x+h)=-2x^2-4xh-2h^2-1$ $f(x+h)-f(x)=-2x^2-4xh-2h^2-1+2x^2+1$ $f(x+h)-f(x)=-4xh-2h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=-4x$	✓substitution into the correct formula ✓$-2x^2-4xh-2h^2-1$ ✓$-4xh-2h^2$ ✓common factor ✓answer (5) **OR/*OF*** ✓$-2x^2-4xh-2h^2-1$ ✓$-4xh-2h^2$ ✓substitution into the correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=-2x^3+3x^2$ $f'(x)=-6x^2+6x$	✓$-6x^2$ ✓$+6x$ (2)
7.2.2	$y=2x+\frac{1}{\sqrt{4x}}$ $y=2x+\frac{1}{2}x^{-\frac{1}{2}}$ $\frac{dy}{dx}=2-\frac{1}{4}x^{-\frac{3}{2}}$	✓$\frac{1}{2}$ ✓$x^{-\frac{1}{2}}$ ✓2 ✓$-\frac{1}{4}x^{-\frac{3}{2}}$ (4)
7.3	$x<1$	✓✓ answer (2)
		[13]

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QUESTION 8

Given: $f(x) = x^3 + 4x^2 - 7x - 10$

8.1 Write down the y-intercept of f. (1)

8.2 Show that 2 is a root of the equation $f(x) = 0$. (2)

8.3 Hence, factorise $f(x)$ completely. (3)

8.4 If it is further given that the coordinates of the turning points are approximately at $(0{,}7\ ;-12{,}6)$ and $(-3{,}4\ ;\ 20{,}8)$, draw a sketch graph of f and label all intercepts and turning points. (3)

8.5 Use your graph to determine the values of x for which:

8.5.1 $f'(x) < 0$ (2)

8.5.2 The gradient of a tangent to f will be a minimum (2)

8.5.3 $f'(x).f''(x) \leq 0$ (3)
[16]

QUESTION 9

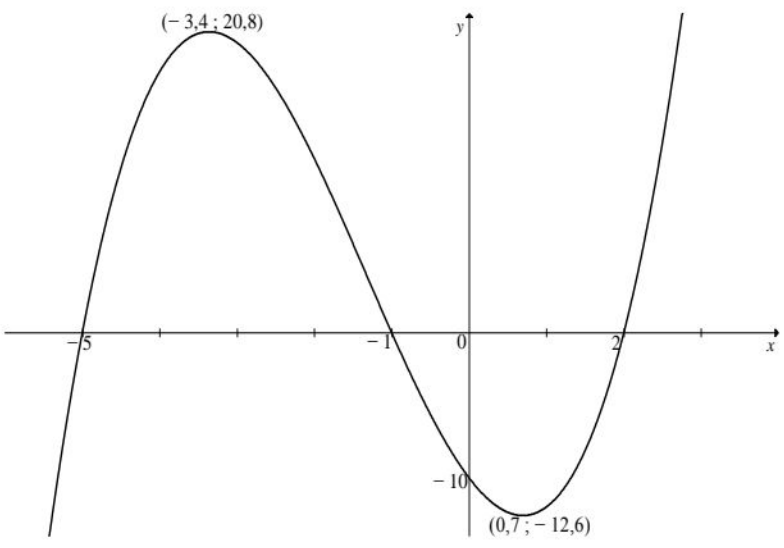
A wire, 12 metres long, is cut into two pieces. One part is bent to form an equilateral triangle and the other a square. A side of the triangle has a length of $2x$ metres.

9.1 Write down the length of a side of the square in terms of x. (2)

9.2 If this square is now used as the base of a rectangular prism with a height of $4x$ metres, determine the maximum volume of the rectangular prism. (7)
[9]

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QUESTION 8/*VRAAG* 8

8.1	$y = -10$	✓ answer (1)
8.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f(2) = 2^3 + 4(2)^2 - 7(2) - 10 = 0$	✓substitution of $x = 2$ ✓ $f(2) = 0$ (2)
8.3	$f(x) = (x-2)(x^2 + 6x + 5)$ $f(x) = (x-2)(x+5)(x+1)$	✓ $(x-2)$ ✓ $(x+5)$ ✓ $(x+1)$ (3)
8.4		✓ x- intercepts ✓ y-intercept ✓ sketching the graph with turning points in 2nd and 4th quadrant (3)
8.5.1	$x \in (-3,4\ ;\ 0,7)$ **OR/*OF*** $-3,4 < x < 0,7$	✓✓ $x \in (-3,4\ ;\ 0,7)$ (2)
8.5.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f'(x) = 3x^2 + 8x - 7$ $f''(x) = 6x + 8 = 0$ $\therefore x = -\frac{8}{6} = -\frac{4}{3} = -1,33$ **OR/*OF*** $\frac{-3,4 + 0,7}{2} = -1,35 = -1,35$	✓ $f''(x) = 6x + 8$ ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
8.5.3	$x \le -3,4$ or $-1,33 \le x \le 0,7$ **OR/*OF*** $x \in (-\infty\ ;\ -3,4] \cup [-1,33\ ;\ 0,7]$	✓ $x \le -3,4$ (A) ✓✓ $-1,33 \le x \le 0,7$ (A 0,7) (3)
		[16]

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QUESTION 9/VRAAG 9

9.1	Perimeter of the square $= 12-6x$ Side length of square $= \frac{12-6x}{4} = \frac{6-3x}{2} = 3-\frac{3}{2}x$	✓ $12-6x$ ✓ answer (2)
9.2	$V = \left(\frac{6-3x}{2}\right)^2 (4x)$ $= \left(\frac{36-36x+9x^2}{4}\right)(4x)$ $= 36x - 36x^2 + 9x^3$ $V(x) = 36x - 36x^2 + 9x^3$ $V'(x) = 36 - 72x + 27x^2$ $36 - 72x + 27x^2 = 0$ $9x^2 - 24x + 12 = 0$ $3x^2 - 8x + 4 = 0$ $(3x-2)(x-2) = 0$ $x = \frac{2}{3}$ or $x = 2$ $V\left(\frac{2}{3}\right) = 36\left(\frac{2}{3}\right) - 36\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right)^3$ $= \frac{32}{3}\text{m}^3 = 10{,}67\text{m}^3$	✓ $\left(\frac{6-3x}{2}\right)^2 (4x)$ ✓ $\left(\frac{36-36x+9x^2}{4}\right)$ ✓ $36x - 36x^2 + 9x^3$ ✓ V' ✓ $V' = 0$ ✓ values ✓ answer (7)
		[9]

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QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 + x$. (5)

7.2 Determine $f'(x)$ if $f(x) = 2x^5 - 3x^4 + 8x$. (3)

7.3 The tangent to $g(x) = ax^3 + 3x^2 + bx + c$ has a minimum gradient at the point $(-1 ; -7)$. For which values of x will g be concave up? (4)
[12]

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QUESTION 7/*VRAAG* 7

7.1	$f(x)=x^2+x$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{(x+h)^2+(x+h)-(x^2+x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2+x+h-x^2-x}{h}$ $=\lim\limits_{h\to 0}\dfrac{2xh+h^2+h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h+1)}{h}$ $\therefore f'(x)=2x+1$ **OR/*OF*** $f(x)=x^2+x$ $f(x+h)=(x+h)^2+(x+h)=x^2+2xh+h^2+x+h$ $f(x+h)-f(x)=x^2+2xh+h^2+x+h-x^2-x$ $=2xh+h^2+h$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{2xh+h^2+h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h+1)}{h}$ $\therefore f'(x)=2x+1$	✓substitution into the formula ✓ $x^2+2xh+h^2+x+h$ ✓ $2xh+h^2+h$ ✓common factor ✓answer (5) **OR/*OF*** ✓ $x^2+2xh+h^2+x+h$ ✓ $2xh+h^2+h$ ✓substitution into the formula ✓common factor ✓answer (5)
7.2	$f(x)=2x^5-3x^4+8x$ $f'(x)=10x^4-12x^3+8$	✓$10x^4$ ✓$-12x^3$ ✓8 (3)
7.3	$g(x)=ax^3+3x^2+bx+c$ $g'(x)=3ax^2+6x+b$ $g''(x)=6ax+6$ $g''(-1)=6a(-1)+6=0$ $\therefore a=1$ For concave up $g''(x)>0$ $6x+6>0$ $x>-1$	✓ $g'(x)=3ax^2+6x+b$ ✓ $g''(-1)=6a(-1)+6=0$ ✓ $a=1$ ✓ $x>-1$ (4)

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OR/OF

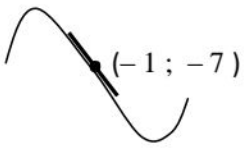
Min gradient at $(-1\ ;-7)$ implies:

at $x = -1$ - point of inflection
and g will be positive cubic
hence $a > 0$

Since g is concave up

$x > -1$

OR/OF



Since g is concave up
$x > -1$



Since g is concave up
$x > -1$

Answer only: $\frac{1}{4}$

OR/OF

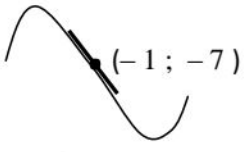
Min gradient at $(-1\ ;-7)$ implies:

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hence $a > 0$

Since g is concave up

$x > -1$

OR/OF



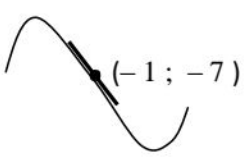
Since g is concave up
$x > -1$



Since g is concave up
$x > -1$

Answer only: $\frac{1}{4}$

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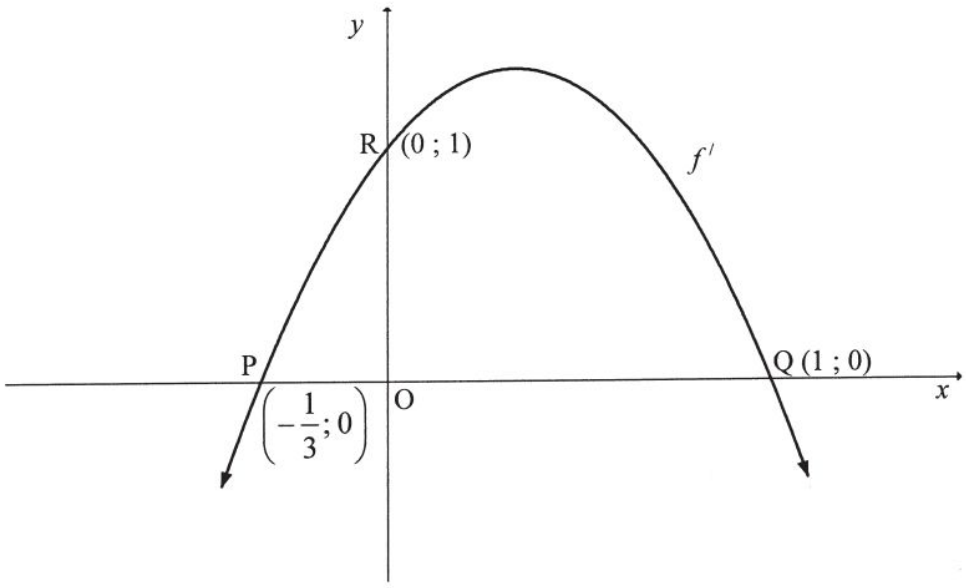
	OR/OF Min gradient at $(-1\ ;-7)$ implies: at $x=-1$ - point of inflection and g will be positive cubic hence $a>0$ Since g is concave up $x>-1$ **OR/OF**  Since g is concave up $x>-1$  $(-1\ ;\ y)$ Since g is concave up $x>-1$ Answer only: $\frac{1}{4}$	**OR/OF** ✓ point of inflection ✓✓ $a>0$ ✓ $x>-1$ (4) **OR/OF** ✓✓pos graph ✓point of inflection ✓ $x>-1$ (4)
		[12]

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QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}; 0\right)$, $Q(1; 0)$ and $R(0; 1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at D(a ; b).

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

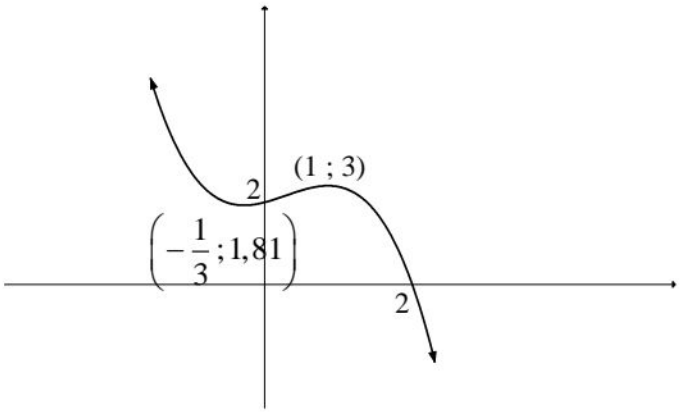
[17]

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QUESTION 8/*VRAAG* 8

8.1	$f'(x)=mx^2+nx+k$ $f'(x)=m\left(x+\frac{1}{3}\right)(x-1)$ $1=m\left(0+\frac{1}{3}\right)(0-1)$ $1=-\frac{1}{3}m$ $\therefore m=-3$ $f'(x)=-3\left(x+\frac{1}{3}\right)(x-1)$ $f'(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ $f'(x)=-3x^2+2x+1$ $\therefore n=2$ $\therefore k=1$ **OR/OF** $k=1$ $0=m+n+1$ and $\frac{1}{9}m-\frac{1}{3}n+1=0$ $m+n=-1$ (1) $m-3n=-9$ (2) (1) – (2) $4n=8$ $\therefore n=2$ $m+2=-1$ $\therefore m=-3$	✓ substitution of $\left(-\frac{1}{3};0\right)$ and $(1\ ;0)$ ✓ substitution of $(0\ ;1)$ ✓ $m=-3$ ✓ $f'(x)=-3\left(x^2-\frac{2}{3}x-\frac{1}{3}\right)$ ✓ $n=2$ ✓ $k=1$ (6) **OR/OF** ✓ $k=1$ ✓ $m+n=-1$ ✓ $m-3n=-9$ ✓ $4n=8$ ✓ $n=2$ ✓ $m=-3$ (6)
8.2.1	$f(x)=-x^3+x^2+x+2$ $f\left(-\frac{1}{3}\right)=\frac{49}{27}=1{,}81$ $\text{T.P}\left(-\frac{1}{3};\frac{49}{27}\right)$ $f(1)=3$ $\text{T.P}(1\ ;3)$	✓ x-coordinates of the TP ✓ $\text{T.P}\left(-\frac{1}{3};\frac{49}{27}\right)$ ✓ $\text{T.P}(1\ ;3)$ (3)

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8.2.2	$f(x)=-x^3+x^2+x+2$ $-x^3+x^2+x+2=0$ $(x-2)(-x^2-x-1)=0$ $x=2$ or no solution  	✓ $x=2$ ✓one x-intercept ✓ two turning points ✓y-intercept ✓ shape: neg cubic (5)
8.3.1	$a=\frac{-\frac{1}{3}+1}{2}$ $=\frac{1}{3}$ **OR/OF** $f'(x)=-3x^2+2x+1$ $f''(x)=-6x+2$ $f''(a)=-6a+2=0$ $-6a=-2$ $a=\frac{1}{3}$	✓ answer (1) **OR/OF** ✓ answer (1)
8.3.2	$b<\frac{4}{3}$ units	✓$\frac{4}{3}$ ✓ $b<\frac{4}{3}$ (2)
		[17]

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QUESTION 9

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10 ; 2)$ and a point on f. **[8]**

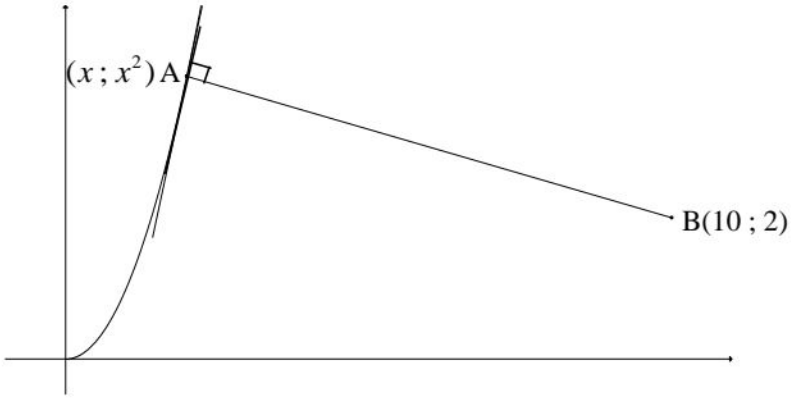
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QUESTION9/*VRAAG* 9

9.1	Any point on f : $(x\,;x^2)$ $\text{distance} = \sqrt{(x-10)^2+(x^2-2)^2}$ $= \sqrt{x^2-20x+100+x^4-4x^2+4}$ $= \sqrt{x^4-3x^2-20x+104}$ For min distance $\frac{d}{dx}\left(x^4-3x^2-20x+104\right)=0$ $4x^3-6x-20=0$ $(x-2)(4x^2+8x+10)=0$ $\Delta = 8^2-4(4)(10) = -96 \quad \therefore$ no roots $\therefore x=2$ $d=\sqrt{2^4-3(2)^2-20(2)+104}=2\sqrt{17}=8{,}25$	✓$(x\,;x^2)$ ✓ substitution ✓ simplification ✓ answer ✓$4x^3-6x-20$ ✓ derivative = 0 ✓$x=2$ ✓answer (A) (8)



9.2	$m_{AB}=\frac{y_2-y_1}{x_2-x_1}=\frac{x^2-2}{x-10}$ $\therefore m_{\tan g}=-\frac{x-10}{x^2-2}$ $f'(x)=2x$ $\therefore 2x=-\frac{x-10}{x^2-2}$ $-2x^3+4x=x-10$ $2x^3-3x-10=0$ $x=2$ $y=(2)^2=4$ $\therefore AB=\sqrt{(2-10)^2+(4-2)^2}$ $=2\sqrt{17}=8{,}25$	✓m_{AB} ✓ $m_{\tan g}=-\frac{x-10}{x^2-2}$ ✓ $f'(x)=2x$ ✓ equating ✓ standard form ✓$x=2$ ✓substitute into distance ✓answer (A) (8)
		[8]

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QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = -x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$, if it is given that $f(x) = 4x^3 - 5x^2$ (2)

8.2.2 $D_x\left[\frac{-6\sqrt[3]{x}+2}{x^4}\right]$ (4)

[11]

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QUESTION/*VRAAG* 8

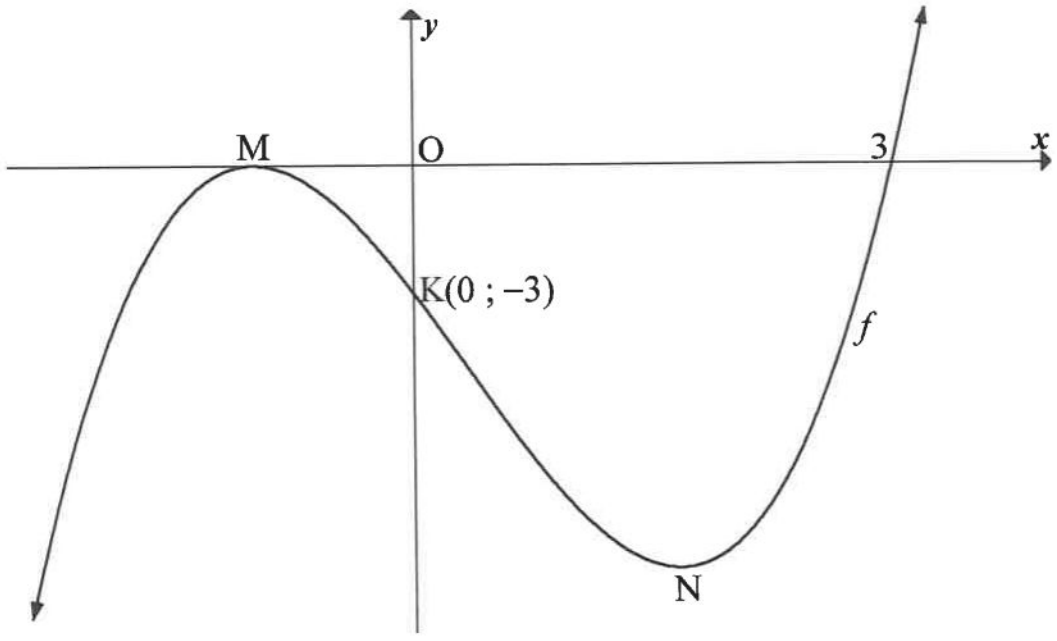
8.1	$f(x)=-x^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-(x+h)^2+x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-x^2-2xh-h^2+x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$	✓ substitution into formula ✓ $-\left(x^2+2xh+h^2\right)$ ✓ $-2xh-h^2$ ✓ $-2x-h$ ✓answer (5)
	OR/*OF* $f(x)=-x^2$ $f(x+h)=-(x+h)^2=-x^2-2xh-h^2$ $f(x+h)-f(x)=-x^2-2xh-h^2-(-x^2)=-2xh-h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$	**OR/*OF*** ✓ $-x^2-2xh-h^2$ ✓ $-2xh-h^2$ ✓substitution into the formula ✓ $-2x-h$ ✓answer (5)
8.2.1	$f(x)=4x^3-5x^2$ $f'(x)=12x^2-10x$	✓ $12x^2$ (A) ✓ $-10x$ (A) (2)
8.2.2	$D_x\left[\dfrac{-6\sqrt[3]{x}+2}{x^4}\right]$ $=D_x\left[\dfrac{-6(x)^{\frac{1}{3}}}{x^4}+\dfrac{2}{x^4}\right]$ $=D_x\left[-6x^{-\frac{11}{3}}+2x^{-4}\right]$ $=22x^{-\frac{14}{3}}-8x^{-5}$	✓ $x^{\frac{1}{3}}$ ✓ $-6x^{-\frac{11}{3}}+2x^{-4}$ ✓ $22x^{-\frac{14}{3}}$ ✓ $-8x^{-5}$ (4)
		[11]

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QUESTION 9

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.
The x-intercepts of f are at (3 ; 0) and M, where M lies on the negative x-axis.

K(0 ; –3) is the y-intercept of f.
M and N are the turning points of f.



9.1 Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$. (5)

9.2 Calculate the coordinates of N. (5)

9.3 For which values of x will:

9.3.1 $f(x) < 0$ (2)

9.3.2 f be increasing (2)

9.3.3 f be concave up (3)

9.4 Determine the maximum vertical distance between the graphs of f and f' in the interval $-1 < x < 0$. (6)
[23]

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QUESTION/*VRAAG* 9

9.1	$f(x)=(x+t)^2(x-3)$ $-3=(0+t)^2(0-3)$ $1=t^2$ $t=\pm 1$ $\therefore t=1$ $f(x)=(x+1)^2(x-3)$ $f(x)=(x^2+2x+1)(x-3)$ $f(x)=x^3-x^2-5x-3$	✓ $f(x)=(x+t)^2(x-3)$ ✓ subs (0 ; –3) ✓ t ✓ $f(x)=(x+1)^2(x-3)$ ✓ expansion (5)
9.2	$f'(x)=3x^2-2x-5$ $0=3x^2-2x-5$ $0=(x+1)(3x-5)$ $x=-1$ or $x=\frac{5}{3}$ $\text{N}\left(\frac{5}{3};-\frac{256}{27}\right)=(1,67;-9,48)$	✓ $f'(x)=3x^2-2x-5$ ✓ $=0$ ✓ factors ✓ x-value ($x>0$) ✓ y-value (A) (5)
9.3.1	$x<3$; $x\neq -1$ **OR/*OF*** $x<-1$ or $-1<x<3$ **OR/*OF*** $(-\infty;-1)$ or $(-1;3)$	✓ $x<3$ ✓ $x\neq -1$ (2) **OR/*OF*** ✓ $x<-1$ ✓ $-1<x<3$ (2) **OR/*OF*** ✓ $(-\infty;-1)$ ✓ $(-1;3)$ (2)
9.3.2	$x<-1$ or $x>\frac{5}{3}$ OR/OF $x\leq -1$ or $x\geq\frac{5}{3}$ **OR/*OF*** $(-\infty;-1)$ or $\left(\frac{5}{3};\infty\right)$ OR/OF $(-\infty;-1]$ or $\left[\frac{5}{3};\infty\right)$	✓ $x<-1$ ✓ $x>\frac{5}{3}$ (2) **OR/*OF*** ✓ $(-\infty;-1)$ ✓ $\left(\frac{5}{3};\infty\right)$ (2)
9.3.3	$f''(x)>0$ $6x-2>0$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$ **OR/OF** $\frac{\frac{5}{3}+(-1)}{2}=\frac{1}{3}$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$	ANSWER ONLY: FULL MARKS ✓ $6x-2$ ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3) **OR/OF** ✓ substitution ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3)

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9.4	Distance $= x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x + 1)(x - 3)$ $x = 3$ or $x = -\frac{1}{3}$ Max distance $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2{,}52$	✓ $x^3 - 4x^2 - 3x + 2$ ✓ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ ✓ factors ✓ x-values ✓ $x = -\frac{1}{3}$ ✓ answer (6)
		[23]

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QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

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QUESTION/VRAAG 9

9.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$ $=\lim\limits_{h\to 0}\dfrac{4xh+2h^2-3h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h-3)}{h}$ $=\lim\limits_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$ **OR/OF** $f(x)=2x^2-3x$ $f(x+h)=2(x+h)^2-3(x+h)$ $f(x+h)=2x^2+4xh+2h^2-3x-3h$ $f(x+h)-f(x)=4xh+2h^2-3h$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{4xh+2h^2-3h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h-3)}{h}$ $=\lim\limits_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5) **OR/OF** ✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
9.2.1	$y=4x^5-6x^4+3x$ $\dfrac{dy}{dx}=20x^4-24x^3+3$	✓ $20x^4$ ✓ $-24x^3$ ✓3 (3)

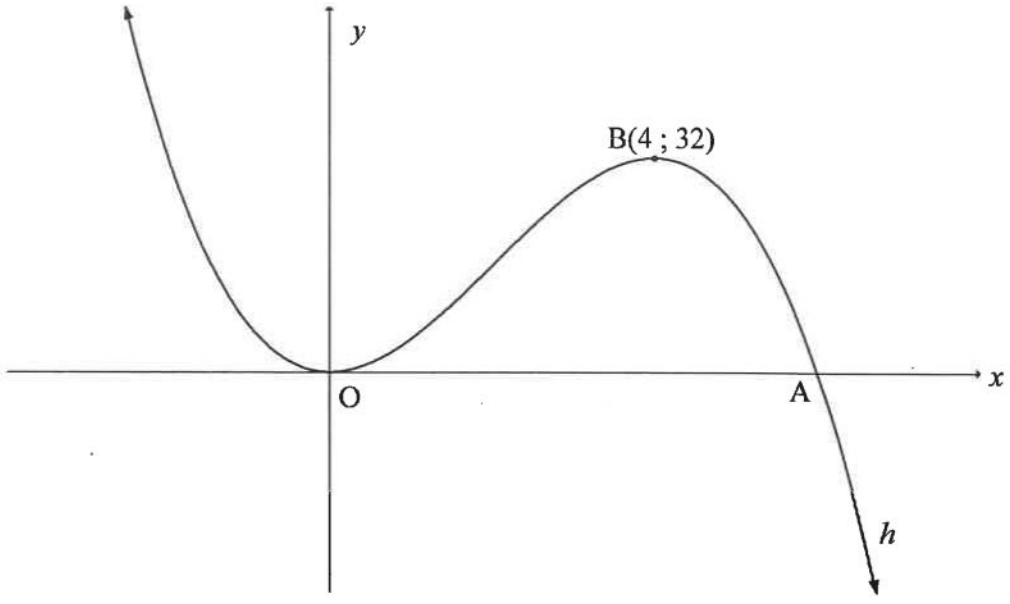
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9.2.2	$D_x\left[\frac{-\sqrt[3]{x}}{2}+\left(\frac{1}{3x}\right)^2\right]$ $D_x\left[\frac{-x^{\frac{1}{3}}}{2}+\frac{x^{-2}}{9}\right]$ $D_x\left[-\frac{1}{2}x^{\frac{1}{3}}+\frac{1}{9}x^{-2}\right]$ $=-\frac{1}{6}x^{-\frac{2}{3}}-\frac{2x^{-3}}{9}$ $=-\frac{1}{6x^{\frac{2}{3}}}-\frac{2}{9x^3}$	✓$\frac{-x^{\frac{1}{3}}}{2}$ ✓$\frac{x^{-2}}{9}$ ✓$-\frac{1}{6}x^{-\frac{2}{3}}$ ✓$-\frac{2x^{-3}}{9}$ (4)
		[12]

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QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.



10.1 Show that $a = -1$ and $b = 6$. (5)

10.2 Calculate the coordinates of A. (3)

10.3 Write down the values of x for which h is:

10.3.1 Increasing (2)

10.3.2 Concave down (2)

10.4 For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots? (3)
[15]

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QUESTION/VRAAG 10

10.1	$h(x)=ax^3+bx^2$ $h'(x)=3ax^2+2bx$ $h'(4)=0$ $48a+8b=0$ $6a+b=0$ ...(1) $h(4)=32$ $64a+16b=32$ $4a+b=2$ ...(2) (1)– (2): $6a+b=0$ $4a+b=2$ $2a=-2$ $a=-1$ $4(-1)+b=2$ $b=6$	✓ $h'(x)$ ✓ $h'(4)=0$ ✓ $48a+8b=0$ or $6a+b=0$ ✓ $h(4)=32$ ✓ $64a+16b=32$ or $4a+b=2$ (5)
10.2	$h(x)=-x^3+6x^2$ $-x^3+6x^2=0$ $x^2(-x+6)=0$ $x=0$ or $x=6$ $\therefore$ A(6 ; 0)	✓ $-x^3+6x^2=0$ ✓ factors ✓ A(6 ; 0) (3)
10.3.1	$0<x<4$ or $0\le x\le 4$ **OR/OF** $x\in(0\,;\,4)$ or $x\in[0\,;\,4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x>2$ **OR/OF** $x\in(2\,;\,\infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x)=h(x-1)=-(x-1)^3+6(x-1)^2$ $f(0)=7$ $7<k<32$ or $k\in(7\,;\,32)$	✓ $k<32$ ✓ new y-intercept = 7 ✓ $7<k<32$ (3)
		[15]

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QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.
The cost of petrol is $\left(1{,}2+\frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. [7]

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QUESTION/VRAAG 11

11	Time $= \frac{20}{x}$ Cost = (water cost per hour × time) + (kms × R/km) $C(x) = 1{,}6 \times \left(\frac{20}{x}\right) + 20\left(1{,}2 + \frac{x}{4000}\right)$ $C(x) = \frac{32}{x} + 24 + \frac{x}{200}$ $C'(x) = -\frac{32}{x^2} + \frac{1}{200} = 0$ $x^2 = 6\,400$ $x = 80\,km/h$	✓ $\frac{20}{x}$ ✓ $1{,}6 \times \left(\frac{20}{x}\right)$ ✓ $20\left(1{,}2 + \frac{x}{4000}\right)$ ✓ $C(x) = \frac{32}{x} + 24 + \frac{x}{200}$ ✓ $C'(x) = -\frac{32}{x^2} + \frac{1}{200}$ ✓ $C'(x) = 0$ ✓ answer (A) (7)
		[7]

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QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

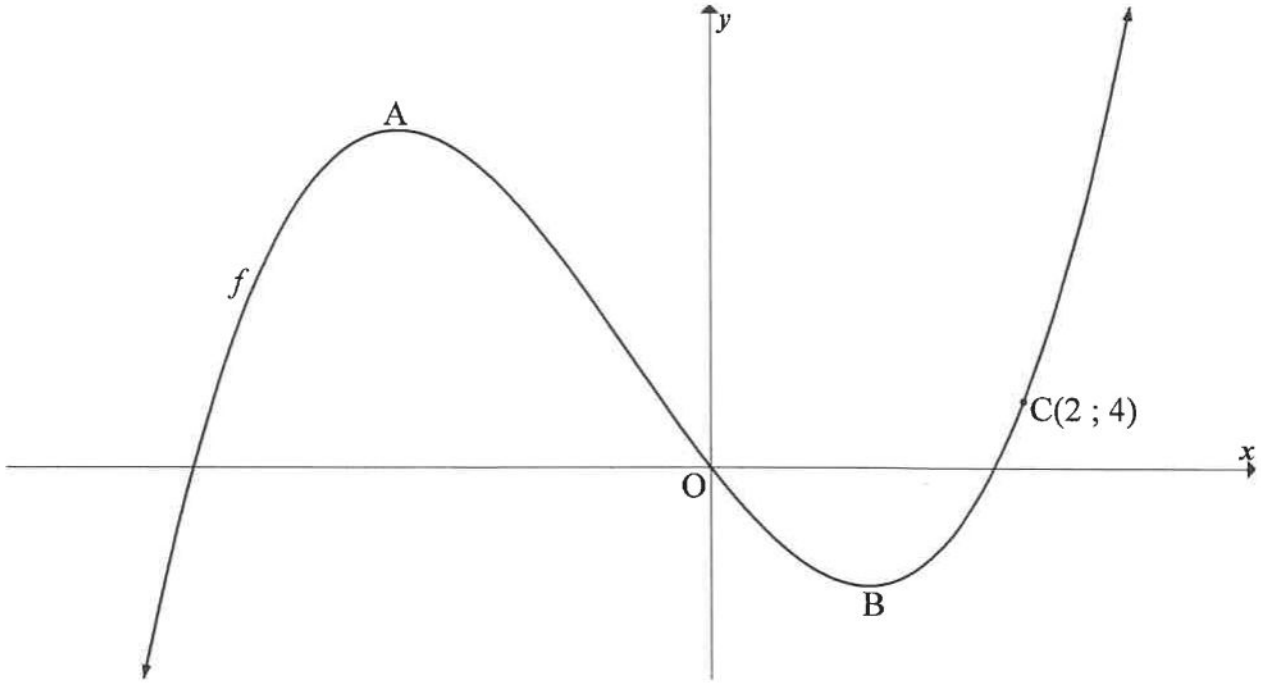
8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = \left(\sqrt{x} + 3\right)\left(\sqrt{x} - 1\right)$ (4)

[12]

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

9.3 Determine the equation of the tangent to f at C(2 ; 4). (3)

[11]

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QUESTION/*VRAAG* 8

8.1	$f(x)=3x^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{3(x+h)^2-3x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{3x^2+6xh+3h^2-3x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ $=6x$	✓ substitution ✓ expansion ✓ simplification ✓ $\lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ ✓ $6x$ (5)
8.2.1	$f(x)=x^2-3+9x^{-2}$ $f'(x)=2x-18x^{-3}$	✓ $9x^{-2}$ ✓ $2x$ ✓ $-18x^{-3}$ (3)

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8.2.2	$g(x)=(\sqrt{x}+3)(\sqrt{x}-1)$ $g(x)=x+2x^{\frac{1}{2}}-3$ $g'(x)=1+x^{-\frac{1}{2}}$	✓ x ✓ $2x^{\frac{1}{2}}$ ✓1 ✓ $x^{-\frac{1}{2}}$ (4)
		[12]

QUESTION/*VRAAG* 9

9.1	$f'(x)=6x^2+6x-12$ $6x^2+6x-12=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ or $x=1$ $y=20$ or $y=-7$ $\therefore$ A(–2 ; 20) and B(1 ; –7)	✓ $6x^2+6x-12$ ✓ $=0$ ✓factors ✓ x -values ✓ y -values (5)
9.2	$f''(x)=12x+6$ $12x+6>0$ $12x>-6$ $x>-\frac{1}{2}$ **OR/OF** $x=\frac{-2+1}{2}=-\frac{1}{2}$ $\therefore x>-\frac{1}{2}$	✓$12x+6$ ✓ $f''(x)>0$ ✓ $x>-\frac{1}{2}$ (3) **OR/OF** ✓ $x=-\frac{1}{2}$ ✓✓ $x>-\frac{1}{2}$ (3)
9.3	$f'(2)=24$ Equation of the tangent: $y-4=24(x-2)$ $y=24x-44$	✓ $f'(2)$ ✓ 24 ✓ equation (3)
		[11]

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QUESTION 10

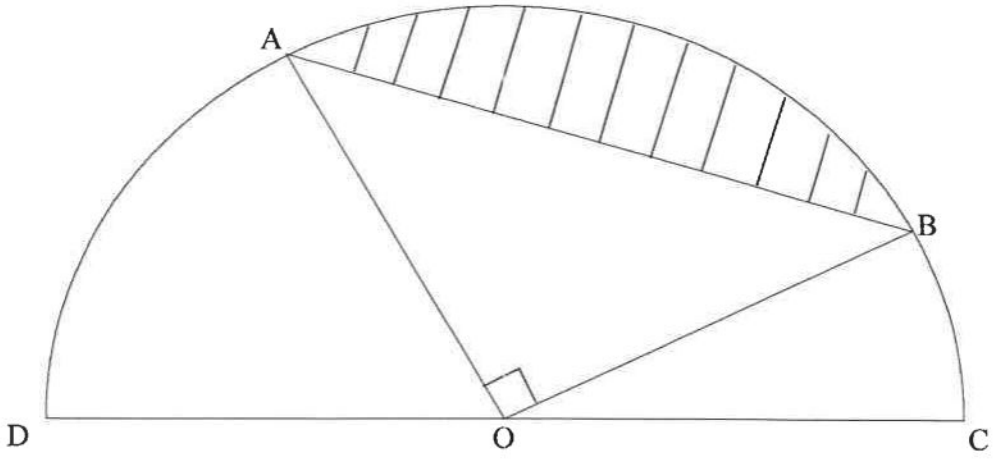
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $\left(x - x^2\right)$ units for $0 < x < 1$. ΔAOB is right-angled at O.



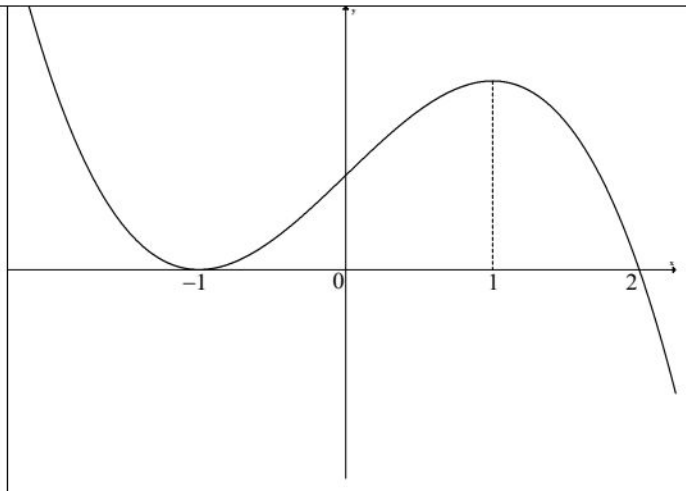
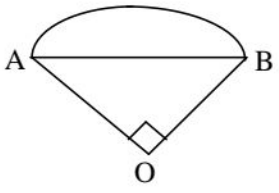
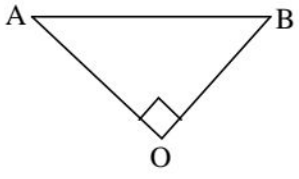
10.2.1 Show that the area of the shaded part is given by:

$$\text{Area} = \left(\frac{\pi - 2}{4}\right)\left(x^4 - 2x^3 + x^2\right)$$ (5)

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)
[13]

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QUESTION/*VRAAG* 10

10.1		✓ $x = -1$ and $x = 2$ ✓ TP at $x = -1$ ✓ TP at $x = 1$ ✓ shape (4)
10.2.1	 Area of segment = $\frac{1}{4}$ Area of big circle $=\frac{1}{4}\pi\ (x-x^2)^2$  Area triangle ABO counted = Area $\Delta = \frac{1}{2}\left(x-x^2\right)^2$ Area of shaded region $=\frac{1}{4}\pi(x-x^2)^2-\frac{1}{2}\left(x-x^2\right)^2$ $=\frac{\pi-2}{4}\left(x-x^2\right)^2$ $=\left(\frac{\pi-2}{4}\right)\left(x^2-2x^3+x^4\right)$	✓✓ $\frac{1}{4}\pi(x-x^2)^2$ ✓ Area $\Delta = \frac{1}{2}\left(x-x^2\right)^2$ ✓ subtract areas ✓ common factor (5)

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10.2.2	Area of shaded region $=\frac{(\pi-2)}{4}\left(x^4-2x^3+x^2\right)$ $\frac{dA}{dx}=\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$ $4x^3-6x^2+2x=0$ $x(2x^2-3x+1)=0$ $x(2x-1)(x-1)=0$ $x\neq 0$ or $x=\frac{1}{2}$ or $x\neq 1$	✓ $\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$ ✓ factors ✓ $x=0;x=1;x=\frac{1}{2}$ ✓ $x=\frac{1}{2}$ (4)
		[13]

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QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x)=2x^2-1$. (5)

7.2 Determine:

7.2.1 $\frac{d}{dx}\left(\sqrt[5]{x^2}+x^3\right)$ (3)

7.2.2 $f'(x)$ if $f(x)=\frac{4x^2-9}{4x+6}$; $x\neq -\frac{3}{2}$ (4)

[12]

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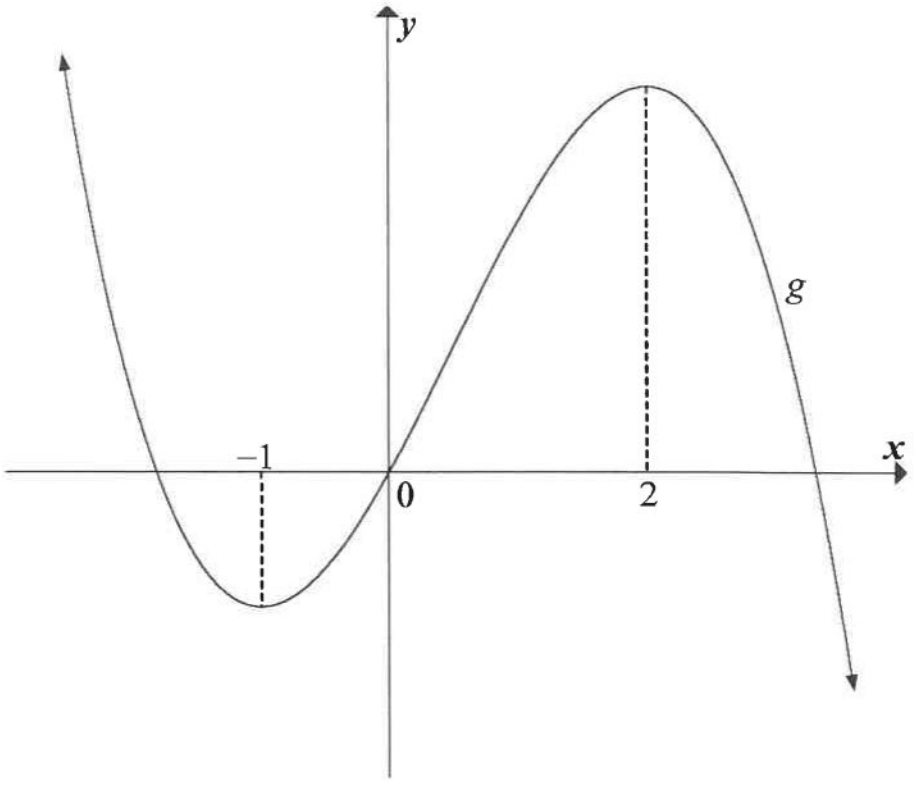
QUESTION/*VRAAG* 7
Penalty of – 1 for notation only in 7.1

7.1	$f(x)=2x^2-1$ $f(x+h)=2(x+h)^2-1$ $=2\left(x^2+2xh+h^2\right)^2-1$ $=2x^2+4xh+2h^2-1$ $f(x+h)-f(x)=2x^2+4xh+2h^2-1-\left(2x^2-1\right)$ $=2x^2+4xh+2h^2-1-2x^2+1$ $=4xh+2h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h)}{h}$ $=\lim_{h\to 0}(4x+2h)$ $=4x$	✓ $2x^2+4xh+2h^2-1$ ✓ $4xh+2h^2$ ✓ substitution ✓ simplification ✓ answer (5)
7.2.1	$\frac{d}{dx}\left(\sqrt[5]{x^2}+x^3\right)$ $=\frac{d}{dx}\left(x^{\frac{2}{5}}+x^3\right)$ $\frac{dy}{dx}=\frac{2}{5}x^{-\frac{3}{5}}+3x^2$	✓ $x^{\frac{2}{5}}$ ✓ $\frac{2}{5}x^{-\frac{3}{5}}$ ✓ $3x^2$ (3)
7.2.2	$f(x)=\frac{4x^2-9}{4x+6}$ $=\frac{(2x-3)(2x+3)}{2(2x+3)}$ $=\frac{2x-3}{2}$ $=x-\frac{3}{2}$ $f'(x)=1$	✓ $(2x-3)(2x+3)$ ✓ $2(2x+3)$ ✓ simplification to two separate terms ✓ answer (4)
		[12]

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QUESTION 8

The graph of $g(x) = ax^3 + bx^2 + cx$, a cubic function having a y-intercept of 0, is drawn below. The x-coordinates of the turning points of g are -1 and 2.



8.1 For which values of x will g increase? (2)

8.2 Write down the x-coordinate of the point of inflection of g. (2)

8.3 For which values of x will g be concave down? (2)

8.4 If $g'(x) = -6x^2 + 6x + 12$, determine the equation of g. (4)

8.5 Determine the equation of the tangent to g that has the maximum gradient. Write your answer in the form $y = mx + c$. (5)

[15]

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QUESTION/*VRAAG* 8

8.1	$-1 < x < 2$	✓✓ answer (2)
8.2	$x = \frac{-1+2}{2}$ $x = \frac{1}{2}$ **Answer Only: Full Marks**	✓ method ✓ answer (2)
8.3	From the graph $x > \frac{1}{2}$ **Answer Only: Full Marks**	✓✓ answer (2)
8.4	$g(x) = ax^3 + bx^2 + cx$ $g'(x) = 3ax^2 + 2bx + c = -6x^2 + 6x + 12$ $3a = -6.$ $2b = 6$ $c = 12$ $a = -2$ $b = 3$ $g(x) = -2x^3 + 3x^2 + 12x$	✓ $g'(x) = 3ax^2 + 2bx + c$ ✓ $a = -2$ ✓ $b = 3$ ✓ $g(x) = -2x^3 + 3x^2 + 12x$ (4)
8.5	$g'\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 12$ $m = \frac{27}{2}$ or 13,5 $y = -2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right)$ $y = \frac{13}{2}$ or 6,5 $y - y_1 = m(x - x_1)$ $y - 6{,}5 = 13{,}5(x - 0{,}5)$ $y = 13{,}5x - 0{,}25$	✓max gradient at $x = \frac{1}{2}$ ✓answer ✓ y value ✓substitution ✓answer (5)
		[15]

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QUESTION 9

A closed rectangular box has to be constructed as follows:

- Dimensions: length (l), width (w) and height (h).
- The length (l) of the base has to be 3 times its width (w).
- The volume has to be $5\ m^3$.

The material for the top and the bottom parts costs R15 per square metre and the material for the sides costs R6 per square metre.

9.1 Show that the cost to construct the box can be calculated by: $\text{Cost} = 90w^2 + 48wh$ (4)

9.2 Determine the width of the box such that the cost to build the box is a minimum. (6)
[10]

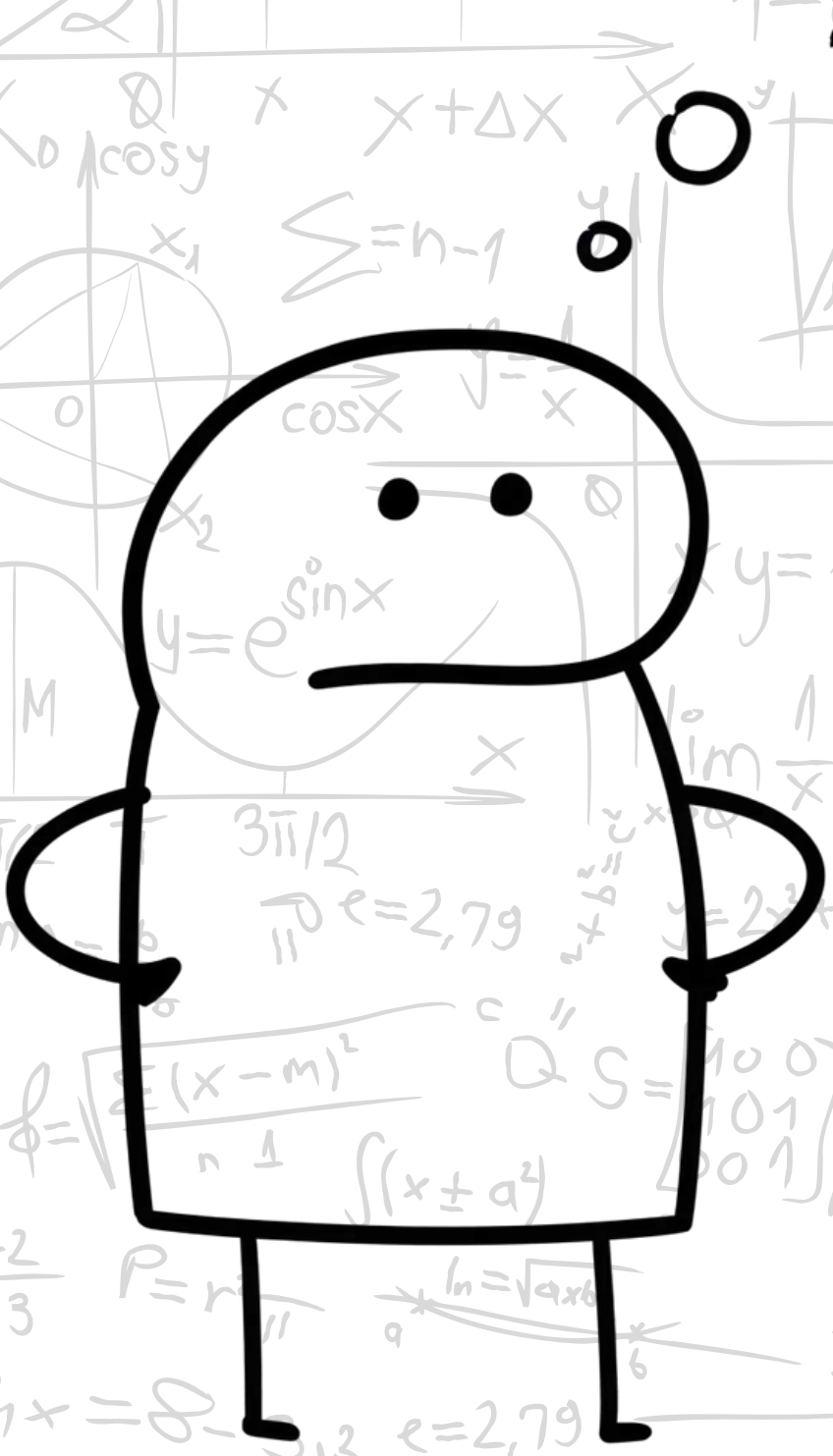
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QUESTION/*VRAAG* 9

9.1	Total surface area $= 2\ell w + 2wh + 2\ell h$ but: $\ell = 3w$ Total surface area $= 6w^2 + 2wh + 6wh$ $C = 15(6w^2) + 6(2wh + 6wh)$ $= 15(6w^2) + 6(8wh)$ $= 90w^2 + 48wh$	✓ $2\ell w + 2wh + 2\ell h$ ✓ $\ell = 3w$ ✓ $15(6w^2)$ ✓ $6(2wh + 6wh)$ (4)
9.2	$5 = 3w^2h$ $h = \frac{5}{3w^2}$ $C = 90w^2 + 48wh$ $C(w) = 90w^2 + 48w\left(\frac{5}{3w^2}\right)$ $= 90w^2 + 80w^{-1}$ $C'(w) = 180w - 80w^{-2}$ $180w - 80w^{-2} = 0$ $180w^3 - 80 = 0$ $w^3 = \frac{80}{180}$ $w = \sqrt[3]{\frac{80}{180}}$ $w = 0{,}76$	✓ $h = \frac{5}{3w^2}$ ✓ substitution ✓ $C(w) = 90w^2 + 80w^{-1}$ ✓ derivative ✓ equating derivative to zero ✓ value of w (6)
		[10]

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PROBABILITY



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QUESTION 11

A certain number of learners are sitting for examinations in Mathematics, Tourism and Geography.

- All these learners sit for at least one of these examinations.
- The total number of learners who sit for Mathematics (M), is 22.
- The total number of learners sitting for Tourism (T), is 16.
- The total number of learners sitting for Geography (G), is 18.
- 5 learners sit for Mathematics and Tourism, but not Geography.
- 4 learners sit for Mathematics and Geography, but not Tourism.
- 3 learners sit for Tourism and Geography, but not Mathematics.
- 6 learners sit for only Tourism.

11.1 Draw a Venn diagram to represent ALL the learners sitting for these examinations. (3)

11.2 Calculate the probability that a learner, chosen at random, will sit for examinations in at least TWO of the subjects. (2)

11.3 Determine if the events: sitting for examinations in Mathematics and sitting for examinations in Tourism are independent. Support your answer with the necessary calculations. (4)
[9]

QUESTION 12

A company generates a 4-character code using the 26 letters of the alphabet and the 10 digits, from 0 to 9.

The code is in the form:

letter	digit	letter	digit

12.1 Determine how many different codes can be formed if letters and digits may be repeated. (2)

12.2 Determine how many different codes can be formed if:

- The letters D, F, I, Q, U and V may NOT be used
- The code may NOT start with a W or a Z
- Letters or digits may NOT be repeated
- The code ends with an odd digit (4)

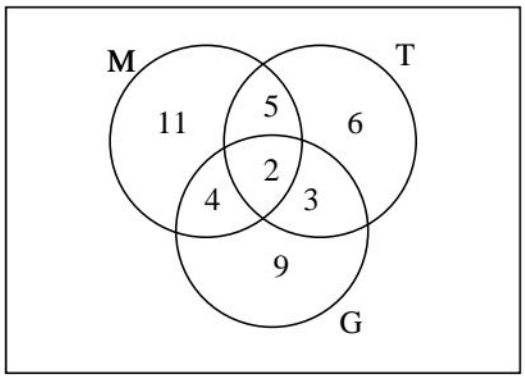
12.3 The company wishes to increase the number of 4-character codes formed in QUESTION 12.2 by allowing the letters D, F, I, Q, U and V to be used. Calculate the percentage increase in the number of different codes that can now be formed. (2)
[8]

TOTAL: 150

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QUESTION/*VRAAG* 11

11.1		✓2 ✓3 ; 4 ; 5 ✓6 ; 9 ; 11 (3)
11.2	P(at least two subjects) $= \frac{4+2+5+3}{40} = \frac{14}{40} = \frac{7}{20} = 0,35$	✓ (4 + 2 + 5 + 3) or 14 in numerator ✓ 40 in denominator (2)
11.3	$P(M) \times P(T) = \frac{22}{40} \times \frac{16}{40} = \frac{11}{50} = 0,22$ $P(M \text{ and } T) = \frac{7}{40} = 0,175$ $P(M) \times P(T) \neq P(M \text{ and } T)$ Events M and T are not independent	✓$\frac{22}{40} \times \frac{16}{40}$ ✓$\frac{11}{50}$ ✓$\frac{7}{40}$ ✓answer (4)
		[9]

QUESTION/*VRAAG* 12

12.1	$26 \times 10 \times 26 \times 10 = 67\ 600$	✓for correct letters ✓for correct digits (2)
12.2	$18 \times 9 \times 19 \times 5 = 15\ 390$	Given that its multiplication ✓ 18 ✓ 19 ✓ 9 ✓ 5 (4)
12.3	$24 \times 9 \times 25 \times 5 = 27\ 000$ $\frac{27\ 000 - 15\ 390}{15\ 390} \times 100 = 75,44\%$	✓$24 \times 9 \times 25 \times 5$ ✓answer (2)
		[8]

TOTAL/*TOTAAL*: 150

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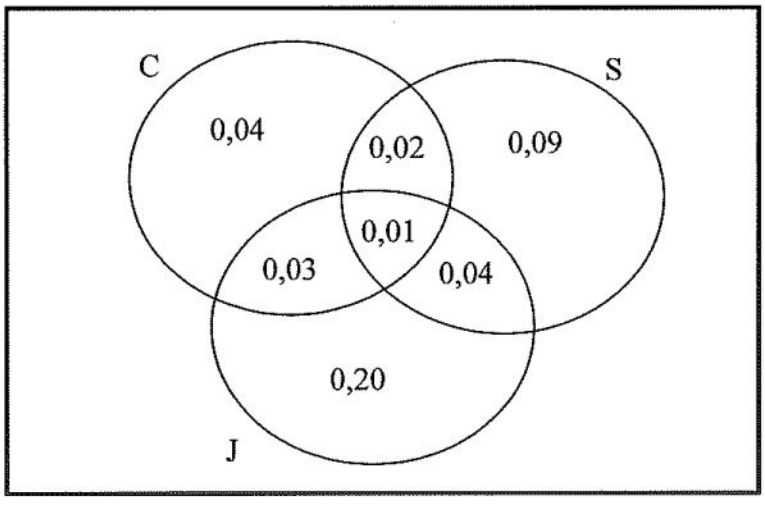
QUESTION 11

11.1 Two events, A and B, are such that:

- P(A) = 0,4
- P(A or B) = 0,52
- A and B are mutually exclusive

Calculate P(B). (2)

11.2 The items that a learner bought at a tuck shop were recorded over a period of time. The probabilities of the learner buying a sandwich (S), a chocolate (C) and a juice (J) are shown in the Venn diagram below.



11.2.1 What is the probability that the learner will buy a sandwich? (1)

11.2.2 Calculate the probability that the learner will buy at least two of the three items. (2)

11.2.3 Calculate the probability that the learner would NOT buy any of the three items. (2)

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11.3 Seven guitar players, each with a different name, participate in a concert.

11.3.1 In how many different ways can the names of the guitar players be listed, one below the other, in the programme? (1)

11.3.2 After the performance, the guitar players wait backstage. There is a bench with only room for four to sit on.

What will be the probability that the four guitar players will be sitting in alphabetical order, from left to right? (3)

11.3.3 During the performance, the seven guitar players sit in a line on stage. Four guitar players are female and three are male.

In how many different ways can they be seated if the males may not sit next to each other? (3)
[14]

TOTAL: 150

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QUESTION 11/*VRAAG* 11

11.1	$P(A) + P(B) = 0,52$ $0,4 + P(B) = 0,52$ $P(B) = 0,12$	✓ substitution ✓ answer (2)
11.2.1	$P(\text{sandwich}) = \frac{4}{25}$ **OR/*OF*** $0,02 + 0,01 + 0,04 + 0,09 = \frac{4}{25} = 0,16$	✓ answer (1) **OR/*OF*** ✓ answer (1)
11.2.2	$P(\text{at least two events}) = 0,02 + 0,01 + 0,03 + 0,04$ $= 0,1$	✓ $0,02 + 0,01 + 0,03 + 0,04$ ✓ answer (2)
11.2.3	$P(\text{not any}) = 1 - (0,1 + 0,04 + 0,09 + 0,2)$ $= 0,57$	✓ $1 - (0,1 + 0,04 + 0,09 + 0,2)$ ✓ answer (2)
11.3.1	$7! = 5040$	✓ $7!$ (1)
11.3.2	$P(\text{4 players alphabetically}) = \frac{1}{7 \times 6 \times 5 \times 4} = \frac{1}{840}$	✓ 1 ✓ 840 ✓ $\frac{1}{840}$ (3)

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11.3.3	<table><tr><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td></tr></table> F arrangements: 4! M arrangements: 5 options with 3 males = $5 \times 4 \times 3$ $4! \times 5 \times 4 \times 3$ $= 1\,440$ **OR/*OF*** 10 Options: F **M** F **M** F **M** F **M** F **M** F **M** F F F F **M** F **M** F **M** F **M** F **M** F F **M** **M** F **M** F F **M** F **M** F **M** F F F **M** F **M** F F **M** F **M** **M** F F **M** F F **M** **M** F F **M** F **M** F **M** F F F **M** F **M** Hence $10 \times 4! \times 3! = 1440$	✓4! ✓$5 \times 4 \times 3$ ✓1 440 (3) **OR/*OF*** ✓$4! \times 3!$ ✓$\times 10$ ✓1 440 (3)
		[14]
		TOTAL/*TOTAAL*: 150

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QUESTION 10

10.1 A and B are independent events. $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$

Determine:

10.1.1 P(A and B) (2)

10.1.2 P(at least ONE event occurs) (2)

10.2 The probability that it will snow on the Drakensberg Mountains in June is 5%.

- When it snows on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0 °C is 72%.
- If it does not snow on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0 °C is 35%.

10.2.1 Represent the given information on a tree diagram. Clearly indicate the probabilities associated with EACH branch. (3)

10.2.2 Calculate the probability that the temperature in Central South Africa will NOT drop below 0 °C in June 2024. (3)

10.3 Ten learners stand randomly in a line, one behind the other.

10.3.1 In how many different ways can the ten learners stand in the line? (1)

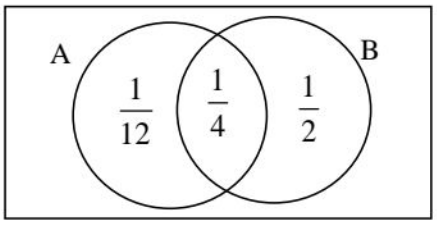
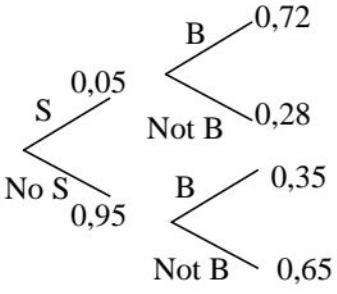
10.3.2 Calculate the probability that there will be 5 learners between the 2 youngest learners in the line. (4)
[15]

TOTAL: 150

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QUESTION 10/*VRAAG* 10

10.1.1	$P(A \text{ and } B) = P(A) \times P(B)$ $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$	✓ $\frac{1}{3} \times \frac{3}{4}$ ✓ $\frac{1}{4}$ (2)
10.1.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ **OR/*OF***  $P(A \text{ or } B) = \frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$	✓ substitution ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
10.2.1	 	✓ branch 1 with probabilities ✓ branch 2 with probabilities ✓ branch 3 with probabilities (3)
10.2.2	P(NOT below 0^o) = P(S; NOT below 0^o) + P(NS ; NOT below 0^o) = 0,05 × 0,28 + 0,95 × 0,65 = 0,6315	✓ value of P(S; NOT below 0^o) ✓ value of P(NS ; NOT below 0^o) ✓ answer (3)
10.3.1	n(S) = 10!	✓ 10! (1)

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10.3.2	4 Options; $\mathbf{2} \times 8 \times 7 \times 6 \times 5 \times 4 \times \mathbf{1} \times 3 \times 2 \times 1 = 80\,640$ $8 \times \mathbf{2} \times 7 \times 6 \times 5 \times 4 \times 3 \times \mathbf{1} \times 2 \times 1 = 80\,640$ $8 \times 7 \times \mathbf{2} \times 6 \times 5 \times 4 \times 3 \times 1 \times \mathbf{1} \times 1 = 80\,640$ $8 \times 7 \times 6 \times \mathbf{2} \times 5 \times 4 \times 3 \times 2 \times 1 \times \mathbf{1} = 80\,640$ Total number of possibilities = 322 560 P(5 learners in between) $= \frac{322\,560}{10!} = \frac{4}{45}$ **OR/*OF*** $\mathbf{2} \times 8 \times 7 \times 6 \times 5 \times 4 \times \mathbf{1} \times 3 \times 2 \times 1$ 4 possible starting positions $\therefore 4(2 \times 8! \times 1) = 322\,560$ $8(8!) = 322\,560$ P(5 learners in between) $= \frac{322\,560}{10!} = \frac{4}{45}$	✓ $(2 \times 8!)$ ✓✓$4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4) **OR/*OF*** ✓ $(2 \times 8!)$ ✓✓$4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4)
		[15]

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QUESTION 10

10.1 A group of people participated in a trial to test a new headache pill.

- 50% of the participants received the headache pill.
- 50% of the participants received a sugar pill.
- $\frac{2}{5}$ of the group receiving the headache pill were not cured.
- $\frac{3}{10}$ of the group receiving the sugar pill were cured.

10.1.1 Represent the given information on a tree diagram. Indicate on your diagram the probability associated with each branch as well as the outcomes. (3)

10.1.2 Determine the probability that a person chosen at random from the group will NOT be cured. (2)

10.2 Three events, A, B and C, are considered:
$P(A) = \frac{2}{5}$, $P(B) = \frac{1}{4}$ and $P(A \text{ or } B) = \frac{13}{20}$.

10.2.1 Are events A and B mutually exclusive? Support your answer with the necessary calculations. (2)

10.2.2 Determine P(only C), if it is further given that
$P(A \text{ or } C) = \frac{7}{10}$, $P(A \text{ and } C) = \frac{2}{5}$ and $2P(B \text{ and } C) = P(A \text{ and } C)$. (3)

10.2.3 Determine the probability that events A, B or C do NOT take place. (2)

10.3 Seven friends (4 boys and 3 girls) want to stand in a straight line next to each other to take a photo.

10.3.1 In how many ways can the 3 girls stand next to each other in the photo? (2)

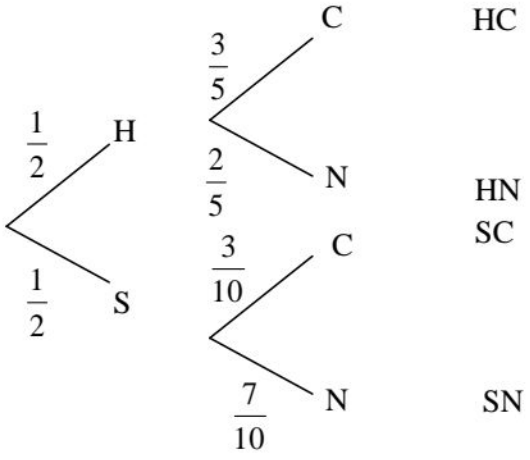
10.3.2 In the next photo, determine the probability that Selwyn (a boy) and Lindiwe (a girl) will NOT stand next to each other in the photo. (3)
[17]

TOTAL: 150

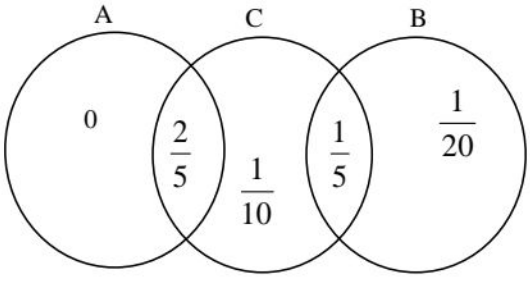
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QUESTION 10/*VRAAG* 10

10.1.1	Event A Event B  	✓ Event A ✓ Event B Medication: for P(C) = $\frac{3}{5}$ ✓ Event B sugar pill: for P(NC) = $\frac{7}{10}$ (3)
10.1.2	P(Not Cured) = P(H) × P(NC) + P(S) × P(NC) $= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)$ $= \frac{11}{20} = 0{,}55$	✓substitution ✓answer (2)
10.2.1	P(A or B) = P(A) + P(B) – P(A and B) P(A and B) = $\frac{13}{20} - \frac{2}{5} - \frac{1}{4} = 0$ Events are mutually exclusive **OR/*OF*** P(A) + P(B) = $\frac{2}{5} + \frac{1}{4}$ $= \frac{13}{20}$ P(A or B) = P(A) + P(B) P(A and B) = 0 Events are mutually exclusive	✓ substitution ✓answer (P(A and B) = 0) (2) **OR/*OF*** ✓ substitution ✓answer (P(A and B) = 0) (2)

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10.2.2	$P(B \text{ and } C) = \frac{1}{5} = 0,2$ $P(\text{ only C}) = \frac{7}{10} - \frac{2}{5} - \frac{1}{5} = \frac{1}{10} = 0,1$  	✓ $P(B \text{ and } C) = \frac{1}{5}$ (A) ✓ $\frac{7}{10} - \frac{2}{5} - \frac{1}{5}$ ✓ $\frac{1}{10}$ (3)
10.2.3	$P(\text{no event}) = 1 - \left(\frac{2}{5} + \frac{1}{10} + \frac{1}{5} + \frac{1}{20}\right) = \frac{1}{4} = 0,25$	✓1 – (P(A) or P(B) or P(C)) ✓answer (2)
10.3.1	$3! \times 5!$ $= 720$ Answer only: Full Marks	✓ $3!$ ✓ $3! \times 5!$ (A) (2)
10.3.2	$\frac{7! - 6! \times 2}{7!} = \frac{5}{7} = 0,71$ **OR/*OF*** $1 - \frac{6! \times 2}{7!}$ $= 1 - \frac{2}{7}$ $= \frac{5}{7} = 0,71$	✓ $7! - 6! \times 2$ ✓ denominator (7!) ✓ answer (3) **OR/*OF*** ✓ $6! \times 2$ ✓ denominator (7!) ✓ answer (3)
		[17]

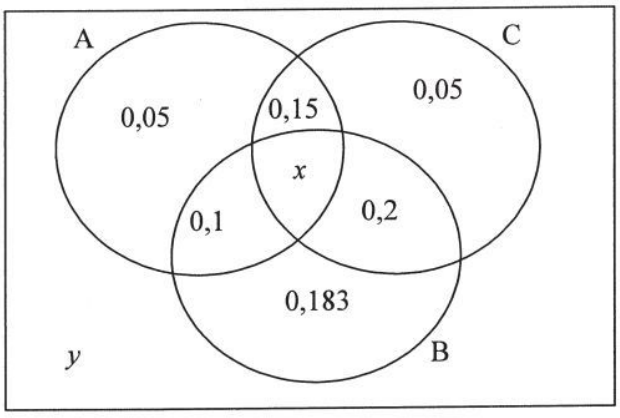
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QUESTION 10

10.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



10.1.1 If it is given that the probability that at least one of the events will occur is 0,893, calculate the value of:

(a) y, the probability that none of the events will occur. (1)

(b) x, the probability that all three events will occur. (1)

10.1.2 Determine the probability that at least two of the events will take place. (2)

10.1.3 Are events B and C independent? Justify your answer. (5)

10.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

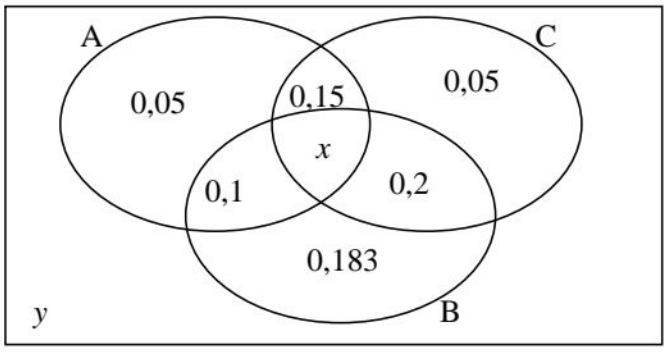
10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)
[17]

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QUESTION 10/*VRAAG* 10



10.1.1(a)	$y = 1 - 0,893 = 0,107$ (0,11)	✓ $y = 1 - 0,893$ (1)
10.1.1(b)	$x = 0,893 - 0,733$ $= 0,16$	✓ $x = 0,893 - 0,733$ (1)
10.1.2	P(at least 2 events) = 0,1 + 0,15 + 0,16 + 0,2 = 0,61 Answer only: Full Marks	✓ values ✓ answer (2)
10.1.3	P(B) = 0,643 P(C) = 0,56 P(B and C) = 0,36 P(B) × P(C) = 0,643 × 0,56 = 0,36 ∴ P(B and C) = P(B) × P(C) ∴ B and C are independent	✓ P(B) = 0,643 ✓ P(C) = 0,56 ✓ P(B and C) = 0,36 ✓ P(B) × P(C) = 0,36 ✓ independent because P(B and C) = P(B) × P(C) (5)
10.2.1	$7 \times 6 \times 5 \times 4 = 840$	✓ 4✓ 7 ✓ $7 \times 6 \times 5 \times 4 = 840$ (3)
10.2.2	start with 5, 7, 9 or start with 6 or start with 8 $(3 \times 5 \times 1 \times 3) + (1 \times 5 \times 1 \times 2) + (1 \times 5 \times 1 \times 2)$ $= 45 + 10 + 10$ $= 65$ $P = \frac{65}{840} = \frac{13}{168} = 0,08$ **OR/*OF*** ends in 4 or ends in 6 or ends in 8 $(5 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1)$ $= 25 + 20 + 20$ $= 65$ $P = \frac{65}{840} = \frac{13}{168} = 0,08$	✓ $(3 \times 5 \times 1 \times 3) = 45$ ✓ $(1 \times 5 \times 1 \times 2) = 10$ ✓ $(1 \times 5 \times 1 \times 2) = 10$ ✓65 ✓ answer (5) **OR/*OF*** ✓ $(5 \times 5 \times 1 \times 1) = 25$ ✓ $(4 \times 5 \times 1 \times 1) = 20$ ✓ $(4 \times 5 \times 1 \times 1) = 20$ ✓65 ✓ answer (5)
		[17]

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QUESTION 10

10.1 Flags from four African countries and three European countries were displayed in a row during the 2021 Olympics.

Determine:

10.1.1 The total number of possible ways in which all 7 flags from these countries could be displayed (2)

10.1.2 The probability that the flags from the African countries were displayed next to each other (3)

10.2 A and B are two independent events.

$P(A) = 0,4$ and $P(A \text{ or } B) = 0,88$

Calculate P(B). (3)

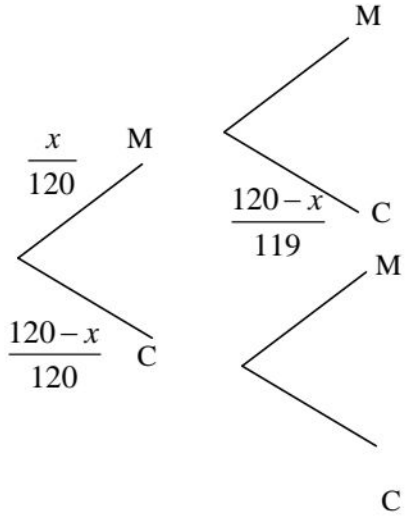
10.3 There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$. Calculate the probability that the first passenger will choose a cheese sandwich. (5)
[13]

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QUESTION/*VRAAG* 10

10.1.1	$7! = 5\,040$	✓✓ answer (2)
10.1.2	$4! \times 4!$ $= 576$ P(African flags together) $= \frac{576}{5040} \quad \left(= \frac{4}{35} = 0{,}11\right)$	✓ $4!$ ✓ $4! \times 4!$ ✓ answer (A) (3)
10.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0{,}88 = 0{,}4 + P(B) - P(A \text{ and } B)$ $0{,}88 = 0{,}4 + P(B) - 0{,}4P(B)$ $0{,}48 = 0{,}6P(B)$ $P(B) = 0{,}8$	✓subs into rule ✓P(A and B) = 0,4P(B) ✓ answer (3)
10.3	First Passenger Second Passenger  Probability of first passenger choosing meat $= \frac{x}{120}$ Probability of second passenger choosing cheese $= \frac{120-x}{119}$ $\frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$ $120x - x^2 = 3\,024$ $x^2 - 120x + 3\,024 = 0$ $(x-84)(x-36) = 0$ $x = 84 \quad \text{or} \quad x = 36$ $\therefore P(1^{st} \text{ cheese}) = \frac{36}{120} = \frac{3}{10}$	✓ $\frac{x}{120}$ ✓ $\frac{120-x}{119}$ ✓ $\frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$ ✓ $x = 84 \quad \text{or} \quad x = 36$ ✓ $\frac{3}{10}$ (5)
		[13]

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QUESTION 12

12.1 A and B are independent events. It is further given that:

P(A and B) = 0,3 and P(only B) = 0,2

12.1.1 Are A and B mutually exclusive? Motivate your answer. (1)

12.1.2 Determine:

(a) P(only A) (4)

(b) P(not A or not B) (2)

12.2 A teacher has 5 different poetry books, 4 different dramas and 3 different novels. She must arrange these 12 books from left to right on a shelf.

12.2.1 Write down the probability that a novel will be the first book placed on the shelf. (1)

12.2.2 Calculate the number of different ways these 12 books can be placed on the shelf if any book can be placed in any position. (2)

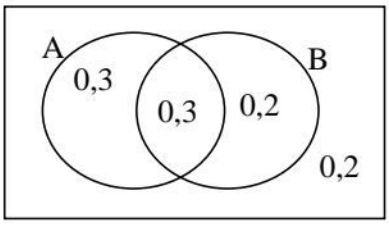
12.2.3 Calculate the probability that a poetry book is placed in the first position, the three novels are placed next to each other and a drama is placed in the last position. (4)
[14]

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QUESTION/VRAAG 12

12.1.1	No, because $P(A \text{ and } B) \neq 0$	✓ answer and reason (1)
12.1.2(a)	$P(A \text{ and } B) = 0,3 \qquad P(\text{only } B) = 0,2$ $P(A \text{ and } B) = P(A) \times P(B)$ $0,3 = P(A) \times 0,5$ $P(A) = 0,6$ $P(\text{only } A) = 0,3$	✓ $P(A \text{ and } B) = P(A) \times P(B)$ ✓ 0,5 ✓ $P(A) = 0,6$ ✓ answer (4)
12.1.2(b)	 $P(\text{not A or not B}) = 0,2 + 0,2 + 0,3 = 0,7$ **OR/OF** $P(\text{not A or not B}) = 1 - P(A \text{ and } B) = 1 - 0,3 = 0,7$ **OR/OF** $P(A' \text{ or } B') = P(A') + P(B') - P(A' \text{ and } B')$ $= 0,4 + 0,5 - 0,2 = 0,7$	✓ method ✓ answer (2) **OR/OF** ✓ method ✓ answer (2) **OR/OF** ✓ method ✓ answer (2)

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<table>
<tr><td>12.2.1</td><td>P(novel) $= \frac{3}{12} = \frac{1}{4}$</td><td>✓answer
(1)</td></tr>
<tr><td>12.2.2</td><td>12! = 479 001 600</td><td>✓✓ answer
(2)</td></tr>
<tr><td>12.2.3</td><td>
<table>
<tr><td>5</td><td>3!</td><td>8!</td><td>4</td></tr>
<tr><td>(Poetry)</td><td>(Novels all together)</td><td>(Arrangements of rest of the books including the novels)</td><td>(Drama)</td></tr>
</table>
P(start with poetry, end with drama AND all novels together)
$= \frac{5 \times 3! \times 8! \times 4}{12!}$
$= \frac{1}{99}$
</td><td>✓ 5×4
✓ $3! = 6$
✓ $8!$
✓ $\frac{5 \times 3! \times 8! \times 4}{12!} = \frac{1}{99}$ (A)
(4)</td></tr>
<tr><td></td><td></td><td>[14]</td></tr>
</table>

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QUESTION 11

11.1 Two events, A and B, are such that:

- Events A and B are independent
- P(not A) = 0,4
- P(B) = 0,3

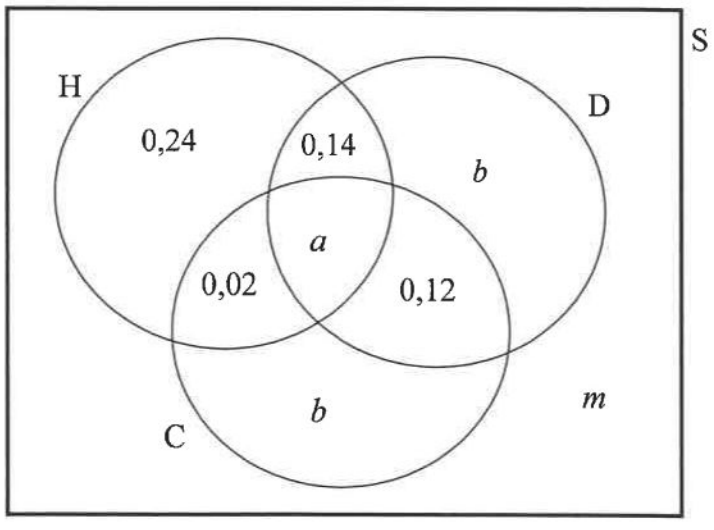
Calculate P(A and B). (3)

11.2 A survey was conducted among 150 learners at a school.

The following observations were made:

- The probability that a learner, selected at random, will take part in:
  - Only hockey (H) is 0,24
  - Hockey and debating (D), but not chess (C) is 0,14
  - Debating and chess, but not hockey is 0,12
  - Hockey and chess, but not debating is 0,02
- The probability that a learner, selected at random, participates in at least one activity is 0,7.
- 15 learners participated in all three activities.
- The number of learners that participate only in debating is the same as the number of learners who participate only in chess.

The Venn diagram below shows some of the above information.



11.2.1 Determine a, the probability that a learner, selected at random, participates in all three activities. (1)

11.2.2 Determine m, the probability that a learner, selected at random, does NOT participate in any of the three activities. (1)

11.2.3 How many learners play only chess? (4)

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11.3 A three-digit number is made up by using three randomly selected digits from 0 to 9. NO digit may be repeated.

11.3.1 Determine the total number of possible three-digit numbers, greater than 100, that can be formed. (2)

11.3.2 Determine the total number of possible three-digit numbers, both even and greater than 600, that can be formed. (4)
[15]

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QUESTION/*VRAAG* 11

11.1	P(A) = 1 – P(not A) = 0,6 P(A and B) = P(A) × P(B) $= 0{,}6 \times 0{,}3$ $= \frac{9}{50}$ $= 0{,}18$	✓ 0,6 ✓ P(A and B) = P(A) × P(B) ✓ answer (A) (3)
11.2.1	$a = \frac{15}{150} = 0{,}1$	✓ $\frac{15}{150}$ (A) (1)
11.2.2	$m = 1 - 0{,}7 = 0{,}3$	✓ 0,3 (A) (1)
11.2.3	$0{,}24 + 0{,}14 + 0{,}02 + 0{,}12 + 0{,}1 + 2b = 0{,}7$ $2b = 0{,}08$ $b = 0{,}04$ $0{,}04 \times 150 = 6$	✓ addition ✓ simplification ✓ $b = 0{,}04$ ✓ 6 (4)
11.3.1	$9 \times 9 \times 8 = 648$	✓9 ✓9×8 (2)
11.3.2	$2 \times 8 \times 4 = 64$ $2 \times 8 \times 5 = 80$ Total number = 64 + 80 = 144	✓✓$2 \times 8 \times 4$ ✓$2 \times 8 \times 5$ ✓144 (A) (4)
		[15]

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QUESTION 10

In a certain country, 10-digit telephone numbers with the following format were introduced:

Format	Area Code	Exchange Code	Number
Number of digits	3 digits	3 digits	4 digits
Example	901	544	1230

Digits may be repeated.

10.1 How many possible 10-digit telephone numbers could be formed? (2)

10.2 Certain restrictions were placed on the groups of digits:

- Area code: must be 3 digits and the first digit can NOT be 0 or 1
- Exchange code: must be 3 digits and the first and second digits can NOT be 0 or 1
- Number: must be 4 digits and the first digit MUST be a 0 or 1

10.2.1 How many valid 10-digit telephone numbers could be formed by applying the given restrictions? (3)

10.2.2 Determine the probability that any randomly chosen 10-digit telephone number would be a valid phone number. (2)
[7]

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QUESTION 11

Harry shoots arrows at a target board. He has a 50% chance of hitting the bull's eye on each shot.

11.1 Calculate the probability that Harry will hit the bull's eye in his first shot and his second shot. (2)

11.2 Calculate the probability that Harry will hit the bull's eye at least twice in his first three shots. (3)

11.3 Glenda also has a 50% chance of hitting the bull's eye on each shot. Harry and Glenda will take turns to shoot an arrow and the first person to hit the bull's eye will be the winner. Calculate the probability that the person who shoots first will be the winner of the challenge. (3)
[8]

TOTAL: 150

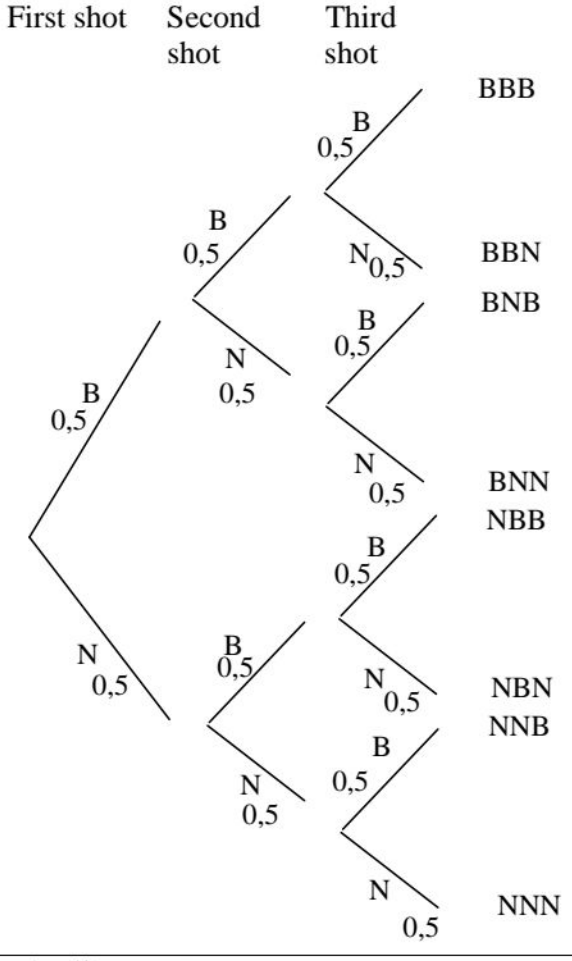
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QUESTION/*VRAAG* 10

10.1	10^{10} or 10 000 000 000	✓✓ answer (2)
10.2.1	$\underline{8} \times \underline{10} \times \underline{10}$ × $\underline{8} \times \underline{8} \times \underline{10}$ × $\underline{2} \times \underline{10} \times \underline{10} \times \underline{10}$ Area exchange number No. of valid 10-digit numbers $= (8 \times 10 \times 10) \times (8 \times 8 \times 10) \times (2 \times 10 \times 10 \times 10)$ $= 1{,}024 \times 10^9$	✓ $\underline{8} \times \underline{10} \times \underline{10}$ or $\underline{8} \times \underline{8} \times \underline{10}$ ✓ $\underline{2} \times \underline{10} \times \underline{10} \times \underline{10}$ ✓ $1{,}024 \times 10^9$ (A) (3)
10.2.2	Probability $= \dfrac{1{,}024 \times 10^9}{10^{10}}$ $= \dfrac{64}{625} = 0{,}1024 = 10{,}24\%$	✓ $\dfrac{1{,}024 \times 10^9}{10^{10}}$ ✓ answer (2)
		[7]

QUESTION/*VRAAG* 11

11		
11.1	P (Bull's eye first shot and second shot) $= 0{,}5 \times 0{,}5$ $= 0{,}25$ or $\dfrac{1}{4}$	✓two 0,5's ✓ $0{,}5 \times 0{,}5$ (2)

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11.2	P (Bull's eye at least twice in 3 shots) $= (0,5 \times 0,5 \times 0,5) + (0,5 \times 0,5 \times 0,5) + (0,5 \times 0,5 \times 0,5) + (0,5 \times 0,5 \times 0,5)$ $= 0,125 + 0,125 + 0,125 + 0,125$ $= 0,5$ or $\frac{1}{2}$	✓ $0,5 \times 0,5 \times 0,5$ ✓four events ✓ answer (A) (3)
11.3	Person shoots first: $(0,5) + (0,5)^3 + (0,5)^5 + \ldots$ $P = \frac{a}{1-r}$ $P = \frac{0,5}{1-0,25}$ $P = \frac{2}{3} = 0,67$	✓ $(0,5) + (0,5)^3$ ✓ ... + $(0,5)^5 + \ldots$ ✓ $P = \frac{0,5}{1-0,25}$ (3)
		[8]

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