



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

MATHEMATICS P2

**PAST PAPER QUESTIONS
ORGANISED BY TOPIC**

October 2019 Edition

CONTENTS

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PROGRESS TRACKER.....3	TRIG: EQUATIONS AND IDENTITIES 33	EUCLIDEAN GEOMETRY 62
STATISTICS.....5	DBE Nov 18 Q5 33	DBE Nov 18 Q9 62
DBE Nov 18 Q1 5	DBE Nov 17 Q5 34	DBE Nov 18 Q10 64
DBE Nov 18 Q2 6	DBE Nov 16 Q5 35	DBE Nov 18 Q11 66
DBE Nov 17 Q1 8	DBE Nov 15 Q5 36	DBE Nov 17 Q9 67
DBE Nov 17 Q2 9	DBE Nov 14 Q5 37	DBE Nov 17 Q10 68
DBE Nov 16 Q1 10	DBE Nov 13 Q5 38	DBE Nov 17 Q11 69
DBE Nov 16 Q2 10	DBE Exemplar 13 Q5..... 39	DBE Nov 17 Q12 70
DBE Nov 15 Q1 11		DBE Nov 16 Q9 71
DBE Nov 15 Q2 12	TRIG: FUNCTIONS 40	DBE Nov 16 Q10 72
DBE Nov 14 Q1 13	DBE Nov 18 Q6 40	DBE Nov 16 Q11 73
DBE Nov 14 Q2 14	DBE Nov 17 Q6 41	DBE Nov 16 Q12 74
DBE Nov 13 Q1 15	DBE Nov 16 Q6 42	DBE Nov 15 Q9 75
DBE Nov 13 Q2 16	DBE Nov 15 Q6 43	DBE Nov 15 Q10 76
DBE Exemplar 13 Q1..... 17	DBE Nov 14 Q6 45	DBE Nov 15 Q11 78
DBE Exemplar 13 Q2..... 18	DBE Nov 13 Q6 46	DBE Nov 14 Q8 79
	DBE Exemplar 13 Q6..... 47	DBE Nov 14 Q9 81
ANALYTIC GEOMETRY 19	TRIG: SINE, COSINE & AREA RULE 48	DBE Nov 14 Q10 82
DBE Nov 18 Q3 19	DBE Nov 18 Q7 48	DBE Nov 13 Q9 83
DBE Nov 18 Q4 20	DBE Nov 17 Q7 49	DBE Nov 13 Q10 84
DBE Nov 17 Q3 21	DBE Nov 16 Q7 50	DBE Nov 13 Q11 85
DBE Nov 17 Q4 22	DBE Nov 15 Q7 51	DBE Nov 13 Q12 86
DBE Nov 16 Q3 23	DBE Nov 14 Q7.1 52	DBE Exemplar 13 Q9..... 87
DBE Nov 16 Q4 24	DBE Nov 13 Q7 53	DBE Exemplar 13 Q10..... 88
DBE Nov 15 Q3 25	DBE Exemplar 13 Q7..... 54	DBE Exemplar 13 Q11..... 89
DBE Nov 15 Q4 26		
DBE Nov 14 Q3 27	MEASUREMENT.....55	
DBE Nov 14 Q4 28	DBE Nov 18 Q8 55	
DBE Nov 13 Q3 29	DBE Nov 17 Q8 56	
DBE Nov 13 Q4 30	DBE Nov 16 Q8 57	
DBE Exemplar 13 Q3..... 31	DBE Nov 15 Q8 58	
DBE Exemplar 13 Q4..... 32	DBE Nov 14 Q7.2 59	
	DBE Nov 13 Q8 60	
	DBE Exemplar 13 Q8..... 61	

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PROGRESS TRACKER

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Statistics	Attempts	DBE Nov 18 Q1		DBE Nov 18 Q2		DBE Nov 17 Q1		DBE Nov 17 Q2		DBE Nov 16 Q1		DBE Nov 16 Q2		DBE Nov 15 Q1		DBE Nov 15 Q2	
		[10]	%	[13]	%	[10]	%	[12]	%	[12]	%	[9]	%	[14]	%	[8]	%
	1 st																
	2 nd																
	3 rd																
	Attempts	DBE Nov 14 Q1		DBE Nov 14 Q2		DBE Nov 13 Q1		DBE Nov 13 Q2		Exemplar 13 Q1		Exemplar 13 Q2					
		[10]	%	[10]	%	[15]	%	[6]	%	[14]	%	[9]	%				
	1 st																
	2 nd																
	3 rd																

Analytic Geometry	Attempts	DBE Nov 18 Q3		DBE Nov 18 Q4		DBE Nov 17 Q3		DBE Nov 17 Q4		DBE Nov 16 Q3		DBE Nov 16 Q4		DBE Nov 15 Q3		DBE Nov 15 Q4	
		[19]	%	[12]	%	[15]	%	[12]	%	[15]	%	[13]	%	[14]	%	[16]	%
	1 st																
	2 nd																
	3 rd																
	Attempts	DBE Nov 14 Q3		DBE Nov 14 Q4		DBE Nov 13 Q3		DBE Nov 13 Q4		Exemplar 13 Q3		Exemplar 13 Q4					
		[13]	%	[17]	%	[12]	%	[17]	%	[14]	%	[15]	%				
	1 st																
	2 nd																
	3 rd																

Trigonometry: Equations and Identities	Attempts	DBE Nov 18 Q5		DBE Nov 17 Q5		DBE Nov 16 Q5		DBE Nov 15 Q5		DBE Nov 14 Q5		DBE Nov 13 Q5		Exemplar 13 Q5	
		[28]	%	[26]	%	[31]	%	[24]	%	[33]	%	[26]	%	[26]	%
	1 st														
	2 nd														
	3 rd														

Trigonometry: Functions	Attempts	DBE Nov 18 Q6	DBE Nov 17 Q6	DBE Nov 16 Q6	DBE Nov 15 Q6	DBE Nov 14 Q6	DBE Nov 13 Q6	Exemplar 13 Q6
		[11] %	[16] %	[9] %	[21] %	[11] %	[10] %	[9] %
	1 st							
	2 nd							
	3 rd							

Trigonometry: Sine, Cosine and Rule	Attempts	DBE Nov 18 Q7	DBE Nov 17 Q6	DBE Nov 16 Q6	DBE Nov 15 Q6	DBE Nov 14 Q7.1	DBE Nov 13 Q6	Exemplar 13 Q6
		[13] %	[12] %	[16] %	[7] %	[8] %	[18] %	[17] %
	1 st							
	2 nd							
	3 rd							

Measurement	Attempts	DBE Nov 18 Q8	DBE Nov 17 Q8	DBE Nov 16 Q8	DBE Nov 15 Q8	DBE Nov 14 Q7.2	DBE Nov 13 Q8	Exemplar 13 Q8
		[7] %	[6] %	[6] %	[8] %	[6] %	[5] %	[6] %
	1 st							
	2 nd							
	3 rd							

Euclidean Geometry	Attempts	DBE Nov 18 Q9	DBE Nov 18 Q10	DBE Nov 18 Q11	DBE Nov 17 Q9	DBE Nov 17 Q10	DBE Nov 17 Q11	DBE Nov 16 Q9	DBE Nov 16 Q10
		[13] %	[14] %	[10] %	[10] %	[9] %	[7] %	[15] %	[6] %
	1 st								
	2 nd								
	3 rd								
	Attempts	DBE Nov 16 Q10	DBE Nov 16 Q11	DBE Nov 16 Q12	DBE Nov 15 Q9	DBE Nov 15 Q10	DBE Nov 15 Q11	DBE Nov 14 Q8	DBE Nov 14 Q9
		[6] %	[9] %	[18] %	[6] %	[15] %	[17] %	[15] %	[12] %
	1 st								
	2 nd								
	3 rd								
	Attempts	DBE Nov 14 Q10	DBE Nov 13 Q9	DBE Nov 13 Q10	DBE Nov 13 Q11	DBE Nov 13 Q12	Exemplar 13 Q9	Exemplar 13 Q10	Exemplar 13 Q11
		[15] %	[7] %	[10] %	[19] %	[5] %	[7] %	[18] %	[15] %
	1 st								
	2 nd								
	3 rd								

QUESTION 1

A school held a sports day. One of the items on the programme was an obstacle race. Teams of 10 parents and learners participated in this race. The table below shows the time taken, in minutes, by each member of a particular team to complete the race.

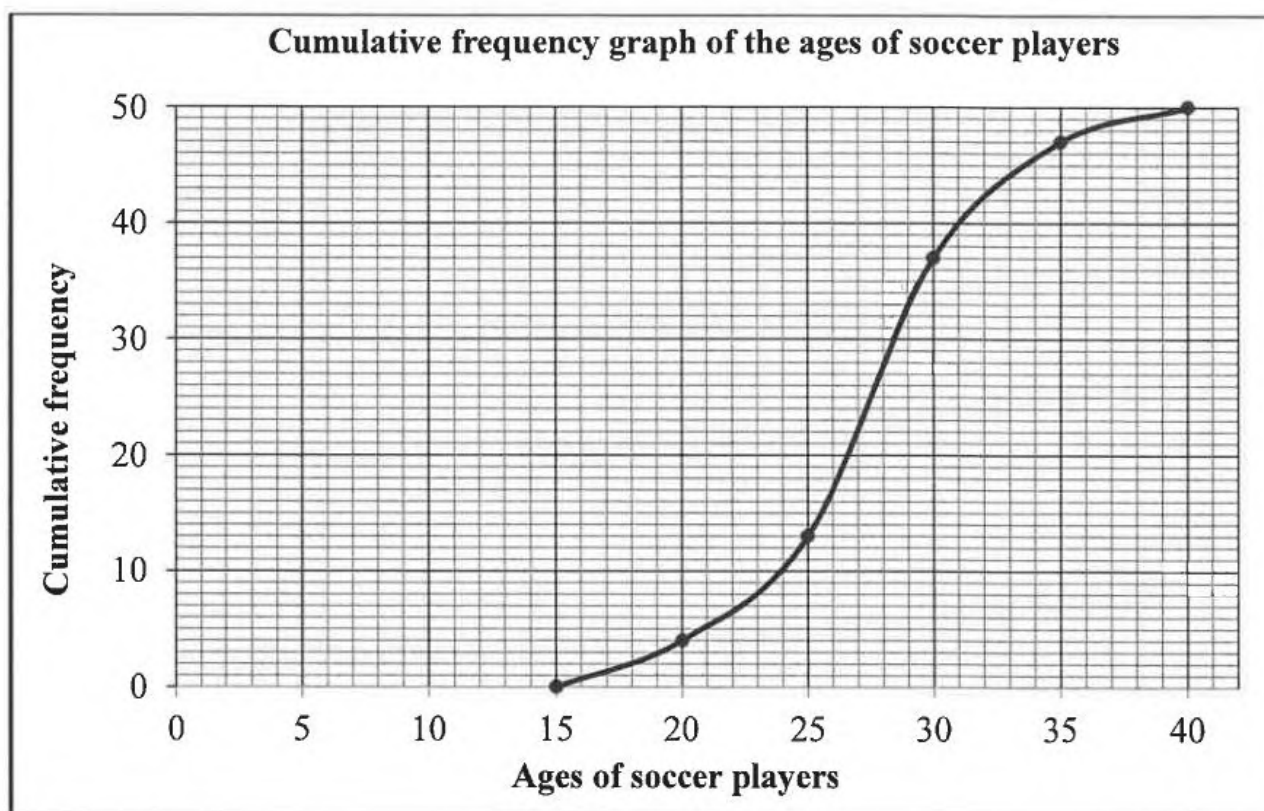
4	12	13	16	17	18	20	22	22	25
---	----	----	----	----	----	----	----	----	----

- 1.1 How long, in minutes, did it take for the fastest member of this team to complete the race? (1)
 - 1.2 Determine the mean time taken by this team. (2)
 - 1.3 Calculate the standard deviation for the data. (1)
 - 1.4 How many members of the team completed the obstacle race outside of two standard deviations of the mean? (3)
 - 1.5 It took another team a total time of $x+5$ minutes to complete the race. Calculate the value of x if the overall mean of the two teams combined was 18 minutes. (3)
- [10]**

QUESTION 2

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- 2.1 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph (ogive) below.



- 2.1.1 How many players took part in the soccer tournament? (1)
- 2.1.2 Determine the number of players between the ages of 24 and 31 years old. (2)
- 2.1.3 Complete the frequency column of the table below in the ANSWER BOOK.

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

- 2.1.4 Use the grid provided in the ANSWER BOOK to draw a frequency polygon for the data. (4)

2.2 Two Grade 11 Mathematics classes have the same number of learners. The five-number summaries of the marks obtained by these classes for a test are shown below.

CLASS A (30 ; 48 ; 65 ; 82 ; 90)

CLASS B (50 ; 58 ; 65 ; 75 ; 90)

The parents of learners in CLASS A and CLASS B observe that both classes have the same median and the same maximum mark and therefore claim that there is no difference in the performance between these classes.

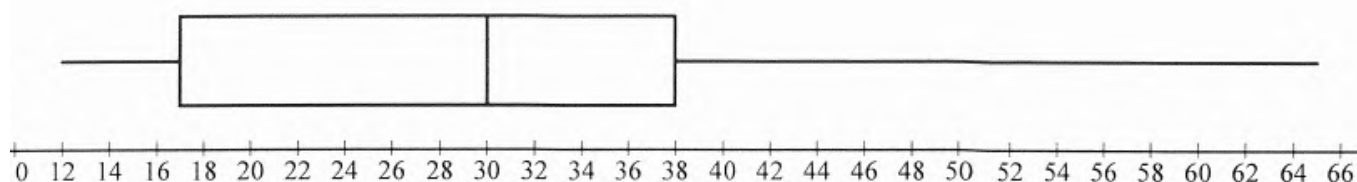
Do you agree with this claim? Use at least TWO different arguments to justify your answer.

(3)
[13]

QUESTION 1

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- 1.1 Mr Brown conducted a survey on the amount of airtime (in rands) EACH student had on his or her cellphone. He summarised the data in the box and whisker diagram below.



- 1.1.1 Write down the five-number summary of the data. (2)
- 1.1.2 Determine the interquartile range. (1)
- 1.1.3 Comment on the skewness of the data. (1)

- 1.2 A group of 13 students indicated how long it took (in hours) before their cellphone batteries required recharging. The information is given in the table below.

5	8	10	17	20	29	32	48	50	50	63	y	107
---	---	----	----	----	----	----	----	----	----	----	-----	-----

- 1.2.1 Calculate the value of y if the mean for this data set is 41. (2)
- 1.2.2 If $y = 94$, calculate the standard deviation of the data. (1)
- 1.2.3 The mean time before another group of 6 students needed to recharge the batteries of their cellphones was 18 hours. Combine these groups and calculate the overall mean time needed for these two groups to recharge the batteries of their cellphones. (3)

[10]

QUESTION 2

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A student conducted a survey among his friends and relatives to determine the relationship between the age of a person and the number of marketing phone calls he or she received within one month. The information is given in the table below.

AGE OF PERSON IN SURVEY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

- 2.1 Complete the frequency and cumulative frequency columns in the table given in the ANSWER BOOK. (4)
- 2.2 How many people participated in this survey? (1)
- 2.3 Write down the modal class. (1)
- 2.4 Draw an ogive (cumulative frequency graph) to represent the data on the grid given in the ANSWER BOOK. (3)
- 2.5 Determine the percentage of marketing calls received by people older than 54 years. (3)
- [12]**

QUESTION 1

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The table below shows the number of cans of food collected by 9 classes during a charity drive.

5	8	15	20	25	27	31	36	75
---	---	----	----	----	----	----	----	----

- 1.1 Calculate the range of the data. (1)
- 1.2 Calculate the standard deviation of the data. (2)
- 1.3 Determine the median of the data. (1)
- 1.4 Determine the interquartile range of the data. (3)
- 1.5 Use the number line provided in the ANSWER BOOK to draw a box and whisker diagram for the data above. (3)
- 1.6 Describe the skewness of the data. (1)
- 1.7 Identify outliers, if any exist, for the above data. (1)

[12]

QUESTION 2

The table below shows the time (in minutes) that 200 learners spent on their cellphones during a school day.

TIME SPENT (IN MINUTES)	FREQUENCY
$95 < x \leq 105$	15
$105 < x \leq 115$	27
$115 < x \leq 125$	43
$125 < x \leq 135$	52
$135 < x \leq 145$	28
$145 < x \leq 155$	21
$155 < x \leq 165$	10
$165 < x \leq 175$	4

- 2.1 Complete the cumulative frequency column in the table provided in the ANSWER BOOK. (2)
- 2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided. (3)
- 2.3 Use the cumulative frequency graph to determine the value of the lower quartile. (2)
- 2.4 Determine, from the cumulative frequency graph, the number of learners who used their cellphones for more than 140 minutes. (2)

[9]

QUESTION 1

The table below shows the weight (to the nearest kilogram) of each of the 27 participants in a weight-loss programme.

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

- 1.1 Calculate the range of the data. (2)
 - 1.2 Write down the mode of the data. (1)
 - 1.3 Determine the median of the data. (1)
 - 1.4 Determine the interquartile range of the data. (3)
 - 1.5 Use the number line provided in the ANSWER BOOK to draw a box and whisker diagram for the data above. (2)
 - 1.6 Determine the standard deviation of the data. (2)
 - 1.7 The person weighing 127 kg states that she weighs more than one standard deviation above the mean. Do you agree with this person? Motivate your answer with calculations. (3)
- [14]**

QUESTION 2 *Downloaded from Stanmorephysics.com*

The table below shows the weight (in grams) that each of the 27 participants in the weight-loss programme lost in total over the first 4 weeks.

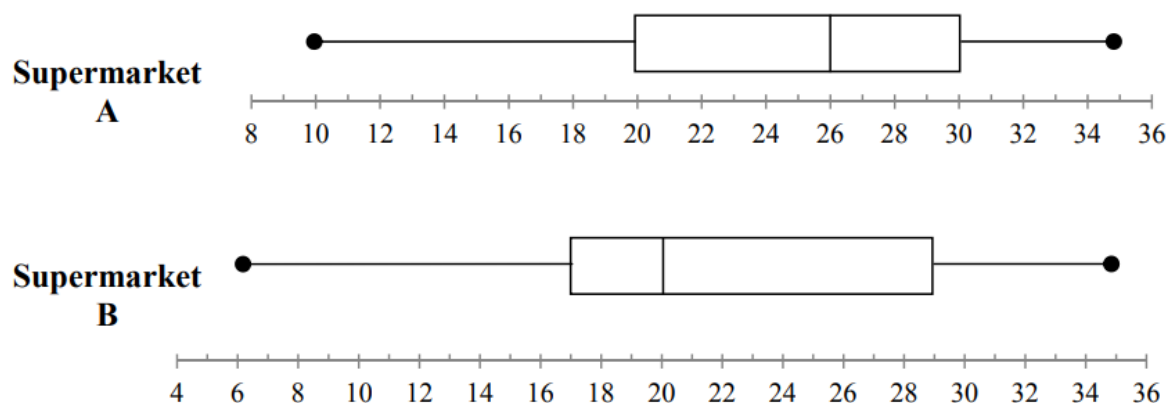
WEIGHT LOSS OVER 4 WEEKS (IN GRAMS)	FREQUENCY
$1\ 000 < x \leq 1\ 500$	2
$1\ 500 < x \leq 2\ 000$	3
$2\ 000 < x \leq 2\ 500$	3
$2\ 500 < x \leq 3\ 000$	4
$3\ 000 < x \leq 3\ 500$	5
$3\ 500 < x \leq 4\ 000$	7
$4\ 000 < x \leq 4\ 500$	2
$4\ 500 < x \leq 5\ 000$	1

- 2.1 Estimate the average weight loss, in grams, of the participants over the first 4 weeks. (2)
- 2.2 Draw an ogive (cumulative frequency graph) of the data on the grid provided. (4)
- 2.3 The weight-loss programme guarantees a loss of 800 g per week if a person follows the programme without cheating. Hence, determine how many of the participants had an average weight loss of 800 g or more per week over the first 4 weeks. (2)
- [8]**

QUESTION 1

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- 1.1 The number of delivery trucks making daily deliveries to neighbouring supermarkets, Supermarket A and Supermarket B, in a two-week period are represented in the box-and-whisker diagrams below.



- 1.1.1 Calculate the interquartile range of the data for Supermarket A. (2)
- 1.1.2 Describe the skewness in the data of Supermarket A. (1)
- 1.1.3 Calculate the range of the data for Supermarket B. (2)
- 1.1.4 During the two-week period, which supermarket receives 25 or more deliveries per day on more days? Explain your answer. (2)

- 1.2 The number of delivery trucks that made deliveries to Supermarket A each day during the two-week period was recorded. The data is shown below.

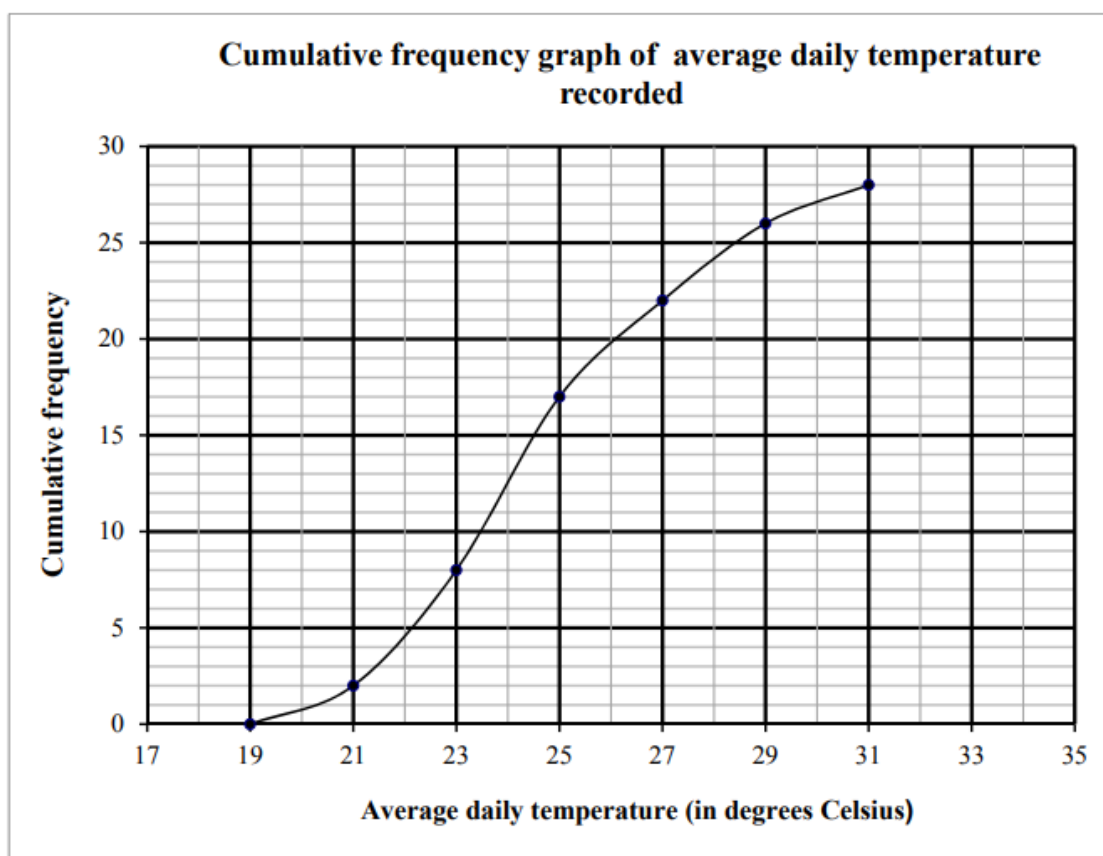
10	15	20	x	30	35	15	31	32	21	x	27	28	29
----	----	----	-----	----	----	----	----	----	----	-----	----	----	----

If the mean of the number of delivery trucks that made deliveries to supermarket A is 24,5 during these two weeks, calculate the value of x .

(3)
[10]

QUESTION 1

The 2012 Summer Olympic Games was held in London. The average daily temperature, in degrees Celsius, was recorded for the duration of the Games. A cumulative frequency graph (ogive) of this data is shown below.



- 2.1 Over how many days was the 2012 Summer Olympic Games held? (1)
 - 2.2 Estimate the percentage of days that the average daily temperature was less than 24 °C. (2)
 - 2.3 Complete the frequency table for the data on DIAGRAM SHEET 1. (3)
 - 2.4 Hence, use the grid provided on DIAGRAM SHEET 1 to draw a frequency polygon of the data. (4)
- [10]**

QUESTION 1

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The 100th Tour de France took place from 29 June 2013 to 21 July 2013. The race was made up of 21 stages of varying distances. The distance, in kilometres, covered in each stage is given in the table below:

Stage	Distance	Stage	Distance	Stage	Distance
1	213	8	195	15	247
2	156	9	168	16	168
3	145	10	197	17	32
4	25	11	33	18	172
5	228	12	218	19	204
6	176	13	173	20	125
7	205	14	191	21	133

[Source: www.letour.fr/le-tour/2013/us]

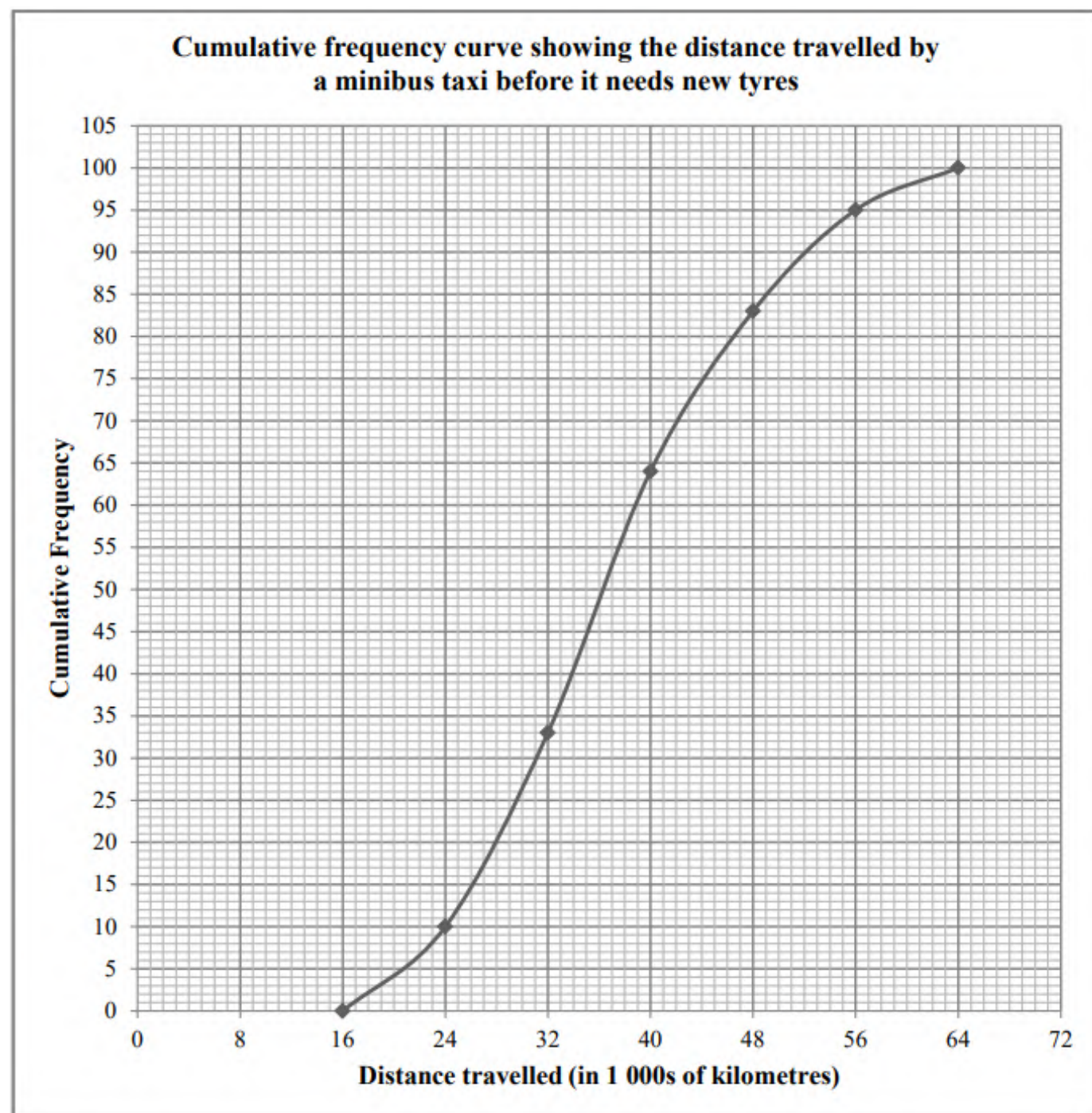
- 1.1 Calculate the mean distance. (3)
- 1.2 Calculate the standard deviation of the distances. (2)
- 1.3 Determine the number of stages that lie beyond ONE standard deviation of the mean. (2)
- 1.4 The distance covered in each stage has been rearranged in ascending order and is shown below. Determine the five-number summary of this data.
- | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 25 | 32 | 33 | 125 | 133 | 145 | 156 |
| 168 | 168 | 172 | 173 | 176 | 191 | 195 |
| 197 | 204 | 205 | 213 | 218 | 228 | 247 |
- (4)
- 1.5 Use the scaled line provided in DIAGRAM SHEET 1 to draw a box and whisker diagram to represent the distance covered in each stage. (2)
- 1.6 Are there any outliers in the data set? Explain. (2)

[15]

QUESTION 2

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A manufacturer recorded how far a minibus taxi travels before it needs new tyres. He recorded the distances, in 1 000s of kilometres, covered by a number of taxis that travelled the same route. This information is shown in the cumulative frequency graph (ogive) below.



- 2.1 How many times did they record the distance travelled by a minibus taxi before it needed new tyres? (1)
 - 2.2 Write down the modal class of the data. (1)
 - 2.3 Estimate the median distance travelled before new tyres are needed. (1)
 - 2.4 Estimate the inter-quartile range for this data. (3)
- [6]**

QUESTION 1

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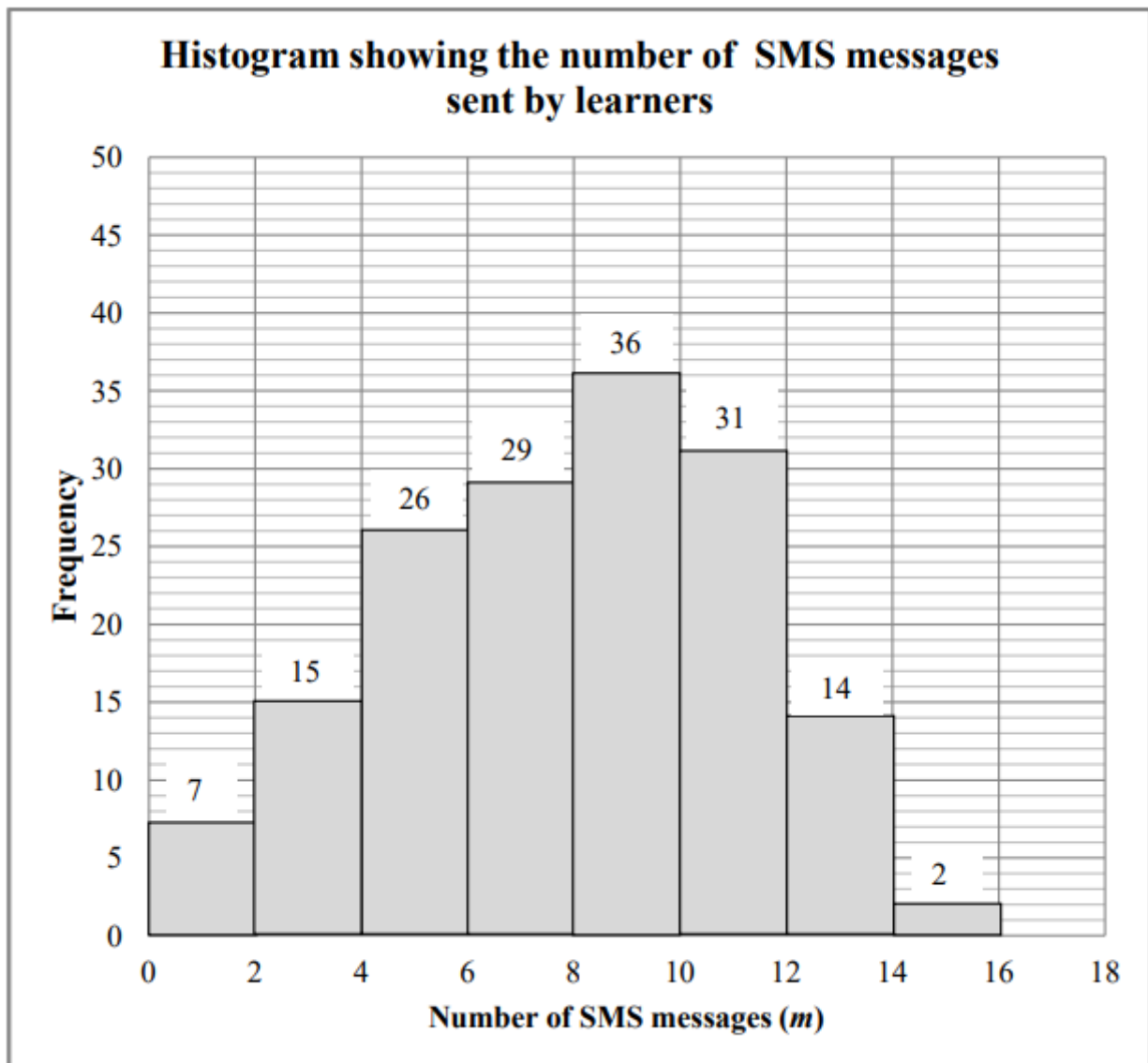
The data below shows the number of people visiting a local clinic per day to be vaccinated against measles.

5	12	19	29
35	23	15	33
37	21	26	18
23	18	13	21
18	22	20	

- 1.1 Determine the mean of the given data. (2)
 - 1.2 Calculate the standard deviation of the data. (2)
 - 1.3 Determine the number of people vaccinated against measles that lies within ONE standard deviation of the mean. (2)
 - 1.4 Determine the interquartile range for the data. (3)
 - 1.5 Draw a box and whisker diagram to represent the data. (3)
 - 1.6 Identify any outliers in the data set. Substantiate your answer. (2)
- [14]**

QUESTION 2 *Downloaded from Stanmorephysics.com*

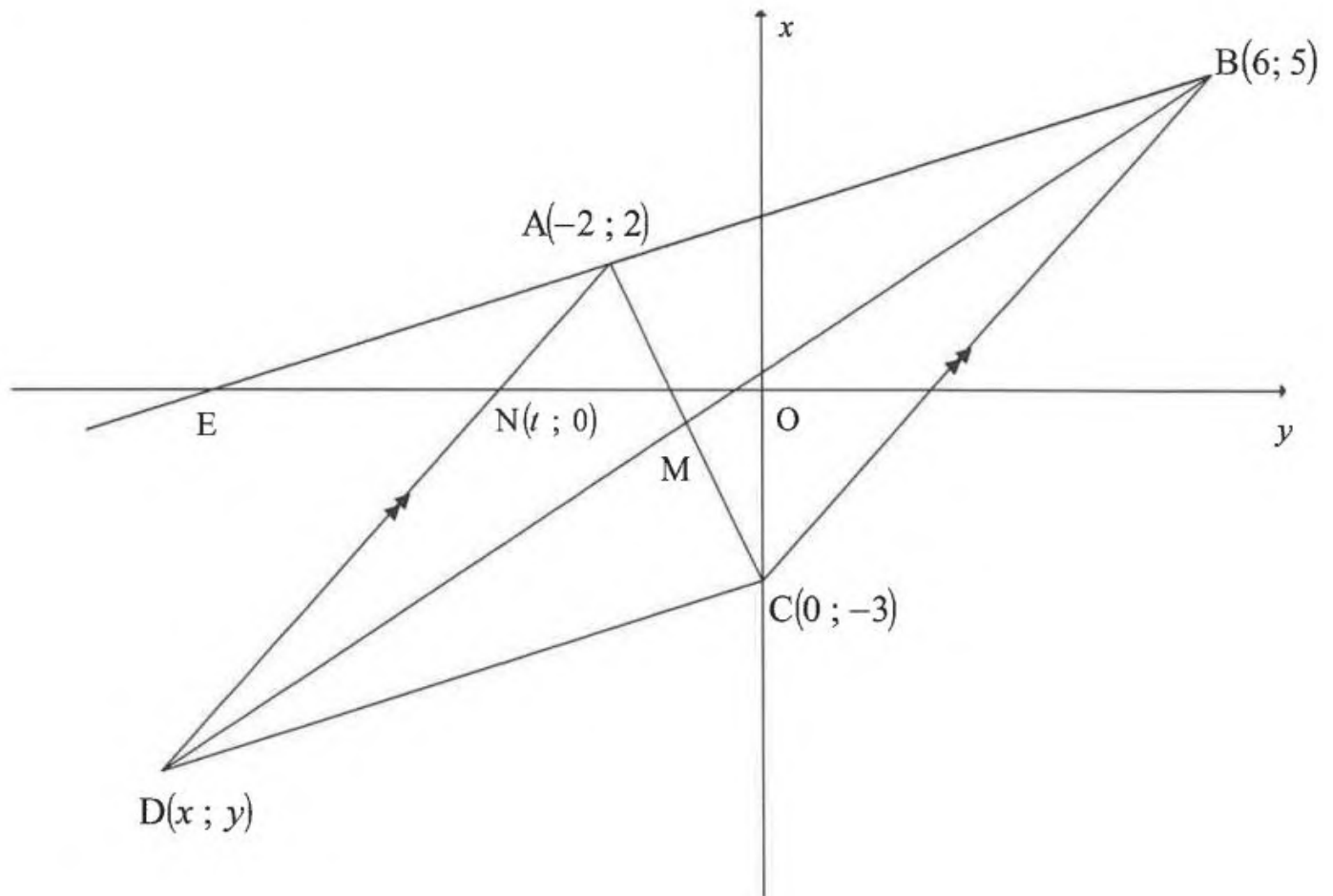
A group of Grade 11 learners were interviewed about using a certain application to send SMS messages. The number of SMS messages, m , sent by each learner was summarised in the histogram below.



- 2.1 Complete the cumulative frequency table provided in DIAGRAM SHEET 1. (2)
 - 2.2 Use the grid provided in DIAGRAM SHEET 2 to draw an ogive (cumulative frequency curve) to represent the data. (3)
 - 2.3 Use the ogive to identify the median for the data. (1)
 - 2.4 Estimate the percentage of the learners who sent more than 11 messages using this application. (2)
 - 2.5 In which direction is the data skewed? (1)
- [9]**

QUESTION 3

In the diagram, $A(-2 ; 2)$, $B(6 ; 5)$, $C(0 ; -3)$ and $D(x ; y)$ are the vertices of a quadrilateral having $AD \parallel BC$. BA produced has an x -intercept at E . BD and AC intersect at M . $N(t ; 0)$ is a point on AD .



- 3.1 Calculate the gradient of BC . (2)
- 3.2 Determine the equation of AD . (3)
- 3.3 Determine the value of t . (2)
- 3.4 Calculate the length of AN . (2)
- 3.5 If DC is defined by $y = \frac{3}{8}x - 3$, determine the coordinates of D . (4)
- 3.6 Prove that $ABCD$ is a parallelogram. (3)
- 3.7 Calculate the coordinates of M . (3)

[19]

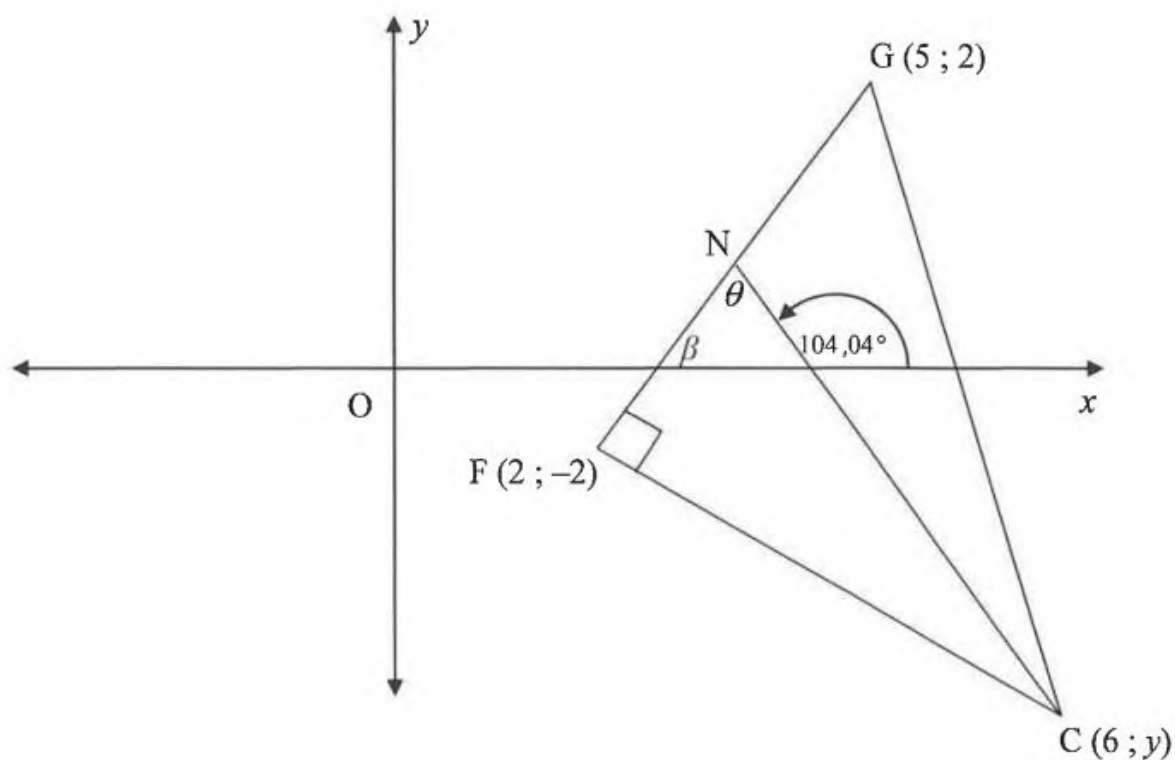
QUESTION 4

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In the diagram, $F(2; -2)$, $G(5; 2)$ and $C(6; y)$ are the vertices of $\triangle FGC$. $FG \perp FC$.

N is a point on FG such that the inclination of NC is $104,04^\circ$.

The angle of inclination of FG is β and $\hat{FNC} = \theta$.



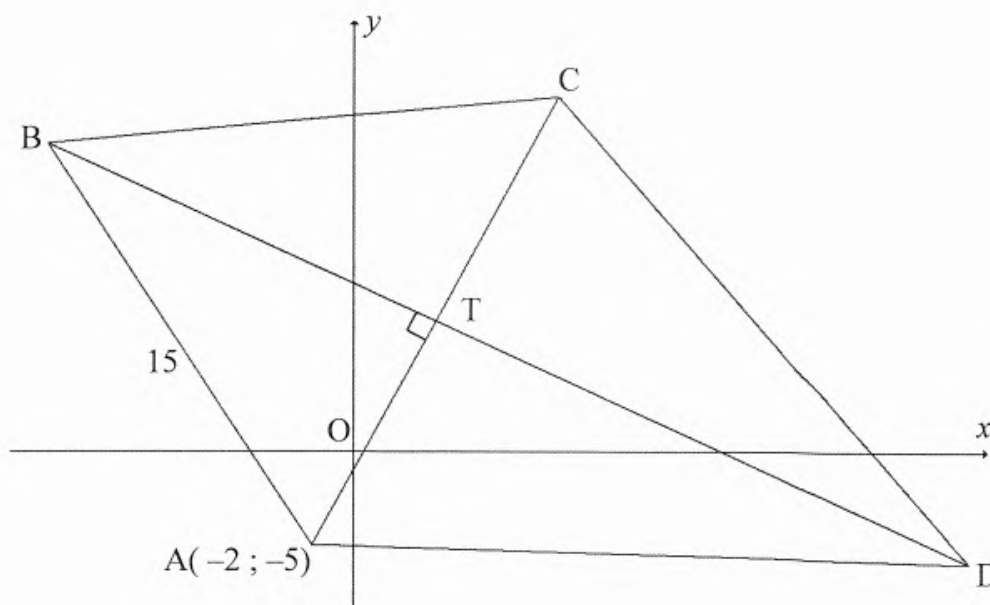
- | | | |
|-----|----------------------------------|-------------|
| 4.1 | Calculate the gradient of FG . | (2) |
| 4.2 | Calculate the value of y . | (3) |
| 4.3 | Calculate the size of θ . | (3) |
| 4.4 | Calculate the length of NC . | (4) |
| | | [12] |

QUESTION 3

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$A(-2 ; -5)$, B , C and D are the vertices of quadrilateral $ABCD$ such that diagonal AC is perpendicular to diagonal BD at T .

The equation of BD is given by $2y + x = 18$ and $AB = 15$ units.

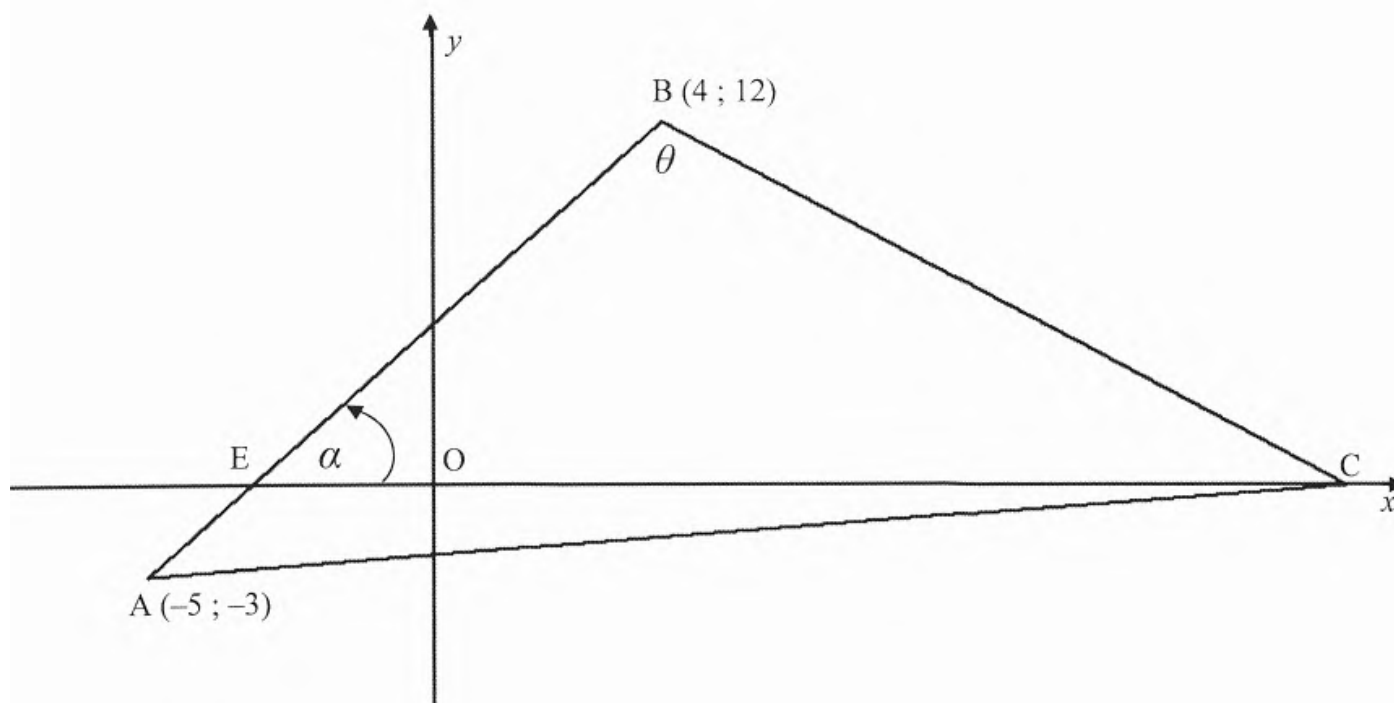


- 3.1 Determine the gradient of line AC . (2)
- 3.2 Determine the equation of AC in the form $y = mx + c$. (2)
- 3.3 If the equation of AC is $y = 2x - 1$, calculate the coordinates of T . (3)
- 3.4 If $ABCD$ is a kite with $AB = BC$:
- 3.4.1 Determine the coordinates of C . (2)
- 3.4.2 Calculate the length of BT . (4)
- 3.4.3 Write down the length of the radius of the circle passing through points B , C and T . (2)
- [15]**

QUESTION 4

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C, a point on the x -axis, $A(-5 ; -3)$ and $B(4 ; 12)$ are the vertices of a triangle. AB intersects the x -axis at E .
 $\angle ABC = \theta$ and $\angle BEC = \alpha$.

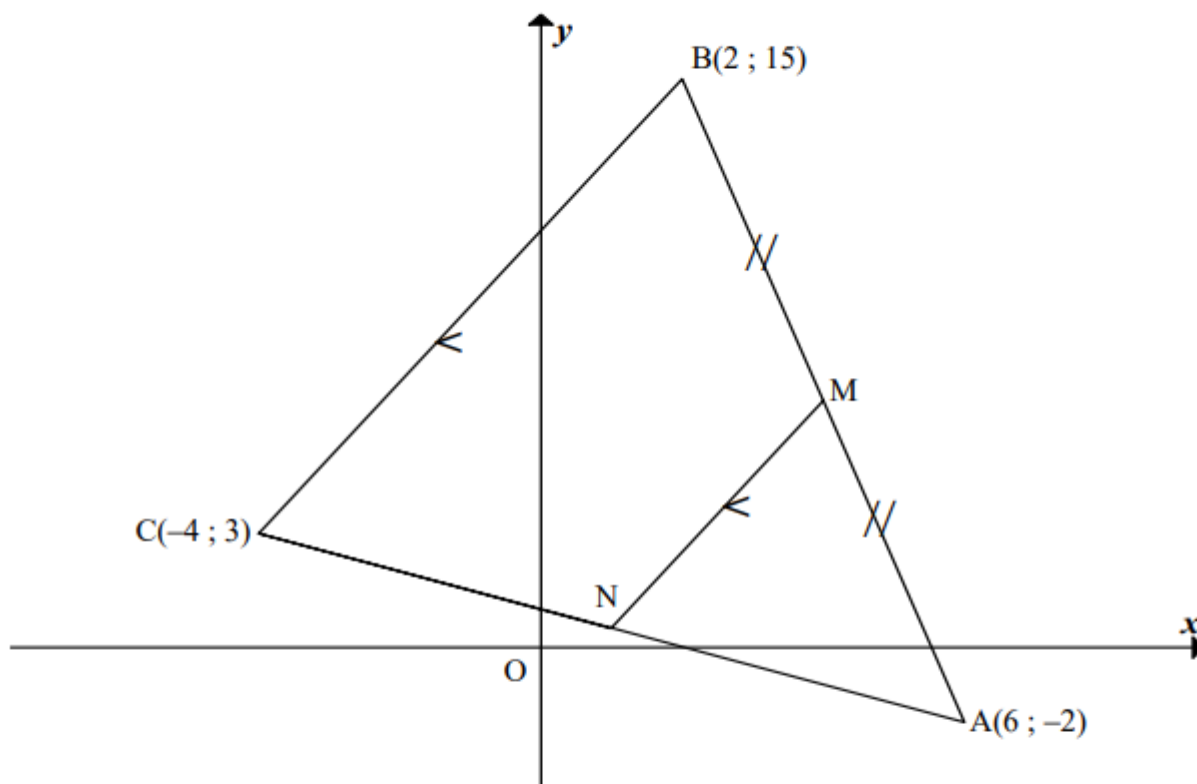


- 4.1 Calculate the gradient of AB . (2)
- 4.2 Determine the coordinates of point E . (3)
- 4.3 Determine the size of α . Round off to the nearest whole number. (2)
- 4.4 If $\theta = 76^\circ$, determine the equation of the line through A parallel to BC . (5)

[12]

QUESTION 3

In the diagram, $A(6; -2)$, $B(2; 15)$ and $C(-4; 3)$ are the vertices of $\triangle ABC$.
 M is the midpoint of AB . N is a point on CA such that $MN \parallel BC$.



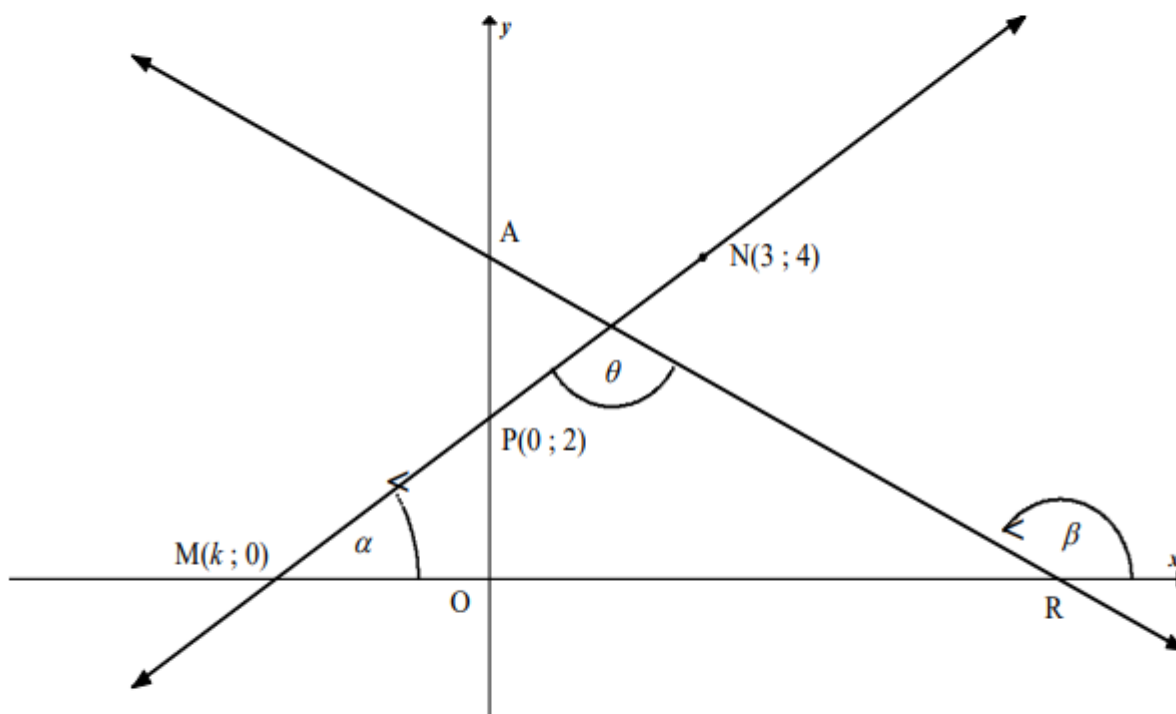
- 3.1 Determine the coordinates of M , the midpoint of AB . (2)
 - 3.2 Determine the gradient of line MN . (3)
 - 3.3 Hence, or otherwise, determine the equation of line MN , in the form $y = mx + c$. (2)
 - 3.4 Calculate, with reasons, the coordinates of point N . (4)
 - 3.5 If $ABCD$ (in that order) is a parallelogram, determine the coordinates of point D . (4)
- [15]**

QUESTION 4 *Downloaded from Stanmorephysics.com*

In the diagram, R and A are the x - and y -intercepts respectively of the straight line AR.

The equation of AR is $y = -\frac{1}{2}x + 4$. Another straight line cuts the y -axis at $P(0; 2)$ and passes through the points $M(k; 0)$ and $N(3; 4)$.

α and β are the angles of inclination of the lines MN and AR respectively.

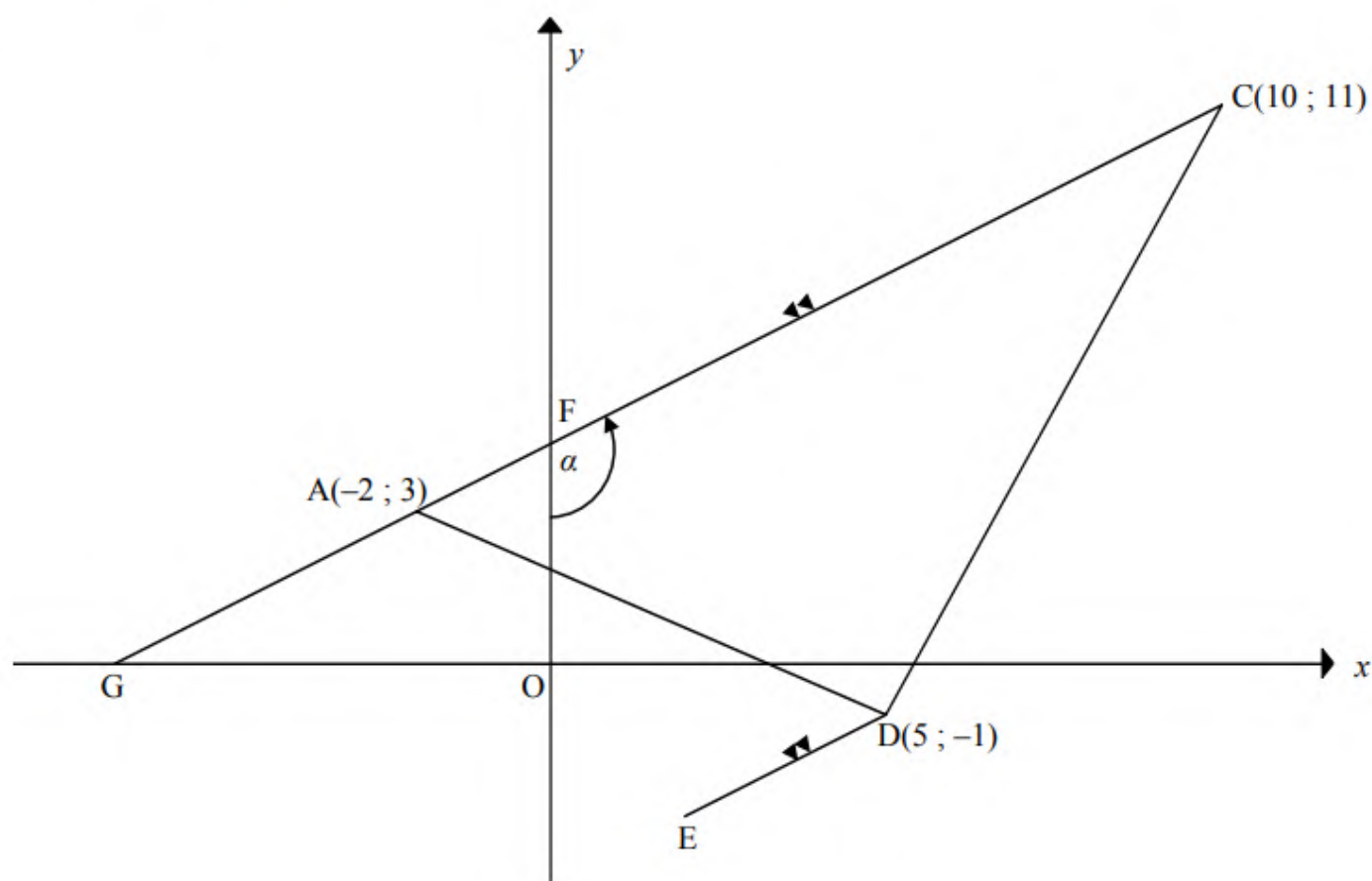


- 4.1 Given that M, P and N are collinear points, calculate the value of k . (3)
- 4.2 Determine the size of θ , the obtuse angle between the two lines. (4)
- 4.3 Calculate the length of MR. (3)
- 4.4 Calculate the area of $\triangle MNR$. (3)

[13]

QUESTION 3 *Downloaded from Stanmorephysics.com*

In the diagram, $A(-2 ; 3)$, $C(10 ; 11)$ and $D(5 ; -1)$ are the vertices of $\triangle ACD$. CA intersects the y -axis in F and CA produced cuts the x -axis in G . The straight line DE is drawn parallel to CA . $\hat{CFO} = \alpha$.



- 3.1 Calculate the gradient of the line AC . (2)
- 3.2 Determine the equation of line DE in the form $y = mx + c$. (3)
- 3.3 Calculate the size of α . (3)
- 3.4 B is a point in the first quadrant such that $ABDE$, in that order, forms a rectangle. Calculate, giving reasons, the:
 - 3.4.1 Coordinates of M , the midpoint of BE (3)
 - 3.4.2 Length of diagonal BE (3)

[14]

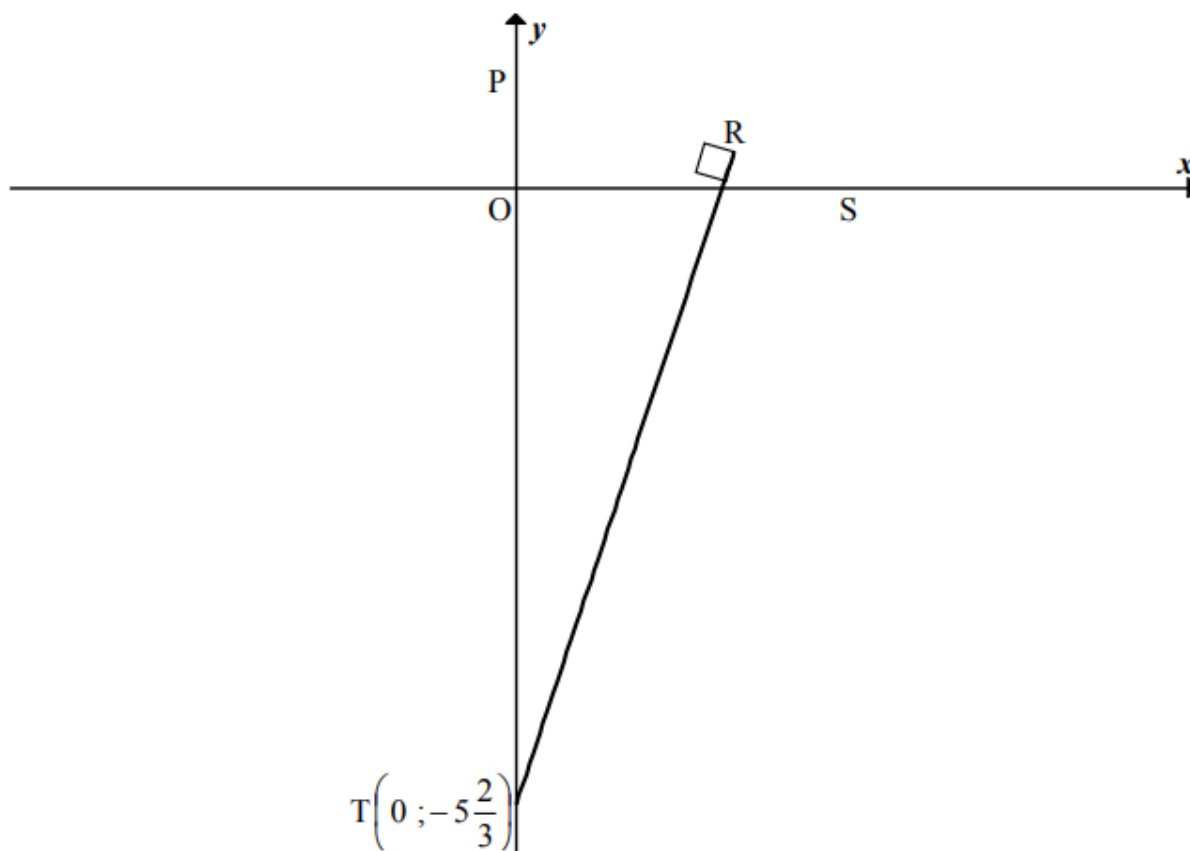
QUESTION 4

Downloaded from Stanmorephysics.com

In the diagram, the straight line SP is drawn having S and P as its x - and y -intercepts respectively. The equation of SP is $x + ay - a = 0$, $a > 0$. It is also given that $OS = 3OP$.

The straight line RT is drawn with R on SP and $RT \perp PS$. RT cuts the y -axis in

$$T\left(0; -5\frac{2}{3}\right).$$



- 4.1 Calculate the coordinates of P . (2)
- 4.2 Calculate the value of a . (2)
- 4.3 Determine the equation of RT in the form $y = mx + c$ if it is given that $a = 3$. (3)
- 4.4 Calculate the coordinates of R , the point where PS and TR meet. (4)
- 4.5 Calculate the area of $\triangle PRT$ if it is given that $R\left(2; \frac{1}{3}\right)$. (3)
- 4.6 Calculate, giving reasons, the radius of a circle passing through the points P , R and T . (2)

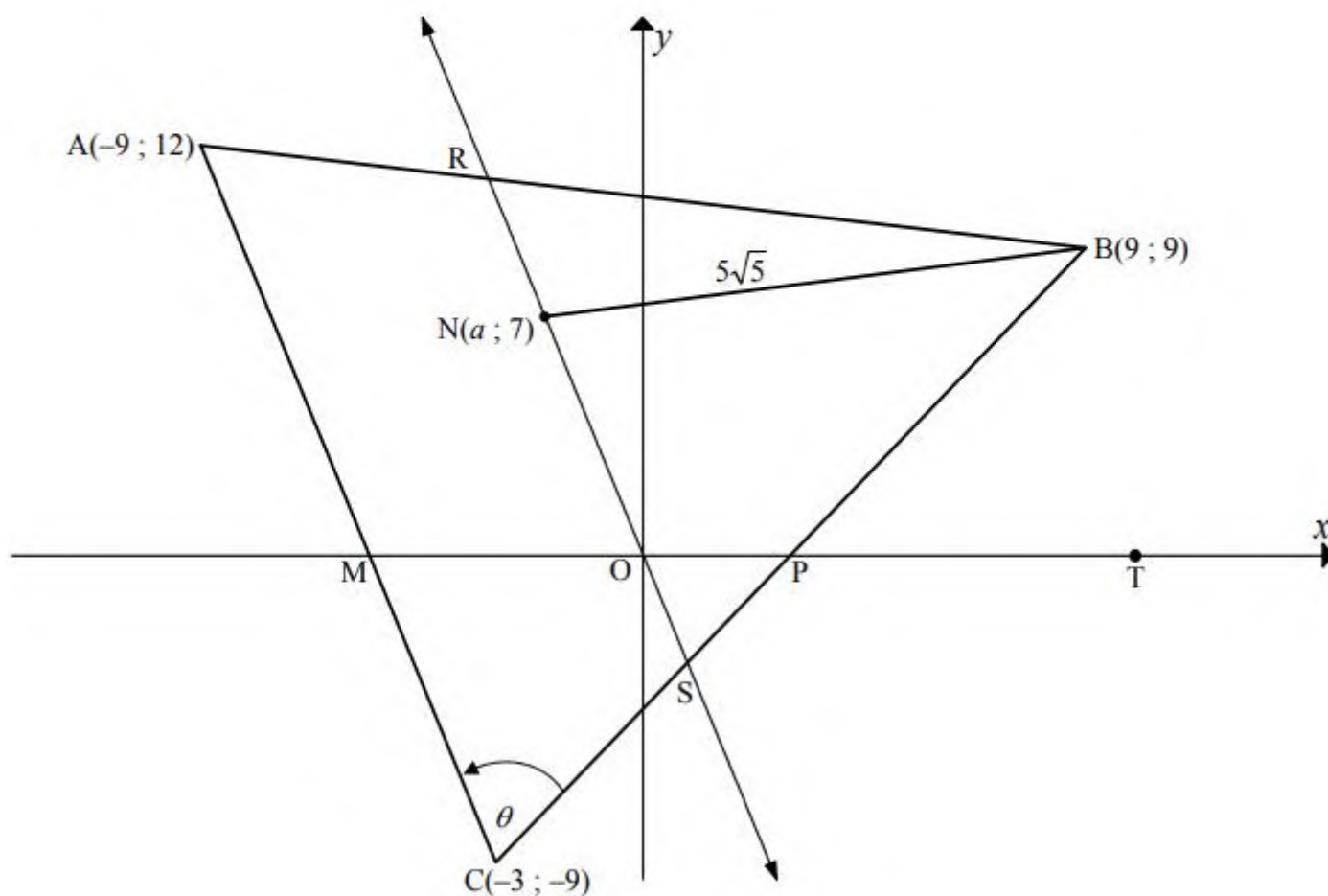
[16]

QUESTION 3

Downloaded from Stanmorephysics.com

In the diagram $A(-9 ; 12)$, $B(9 ; 9)$ and $C(-3 ; -9)$ are the vertices of $\triangle ABC$. $N(a ; 7)$ is a point such that $BN = 5\sqrt{5}$. R is a point on AB and S is a point on BC such that RNS is parallel to AC and RNS passes through the origin. T lies on the x -axis to the right of point P .

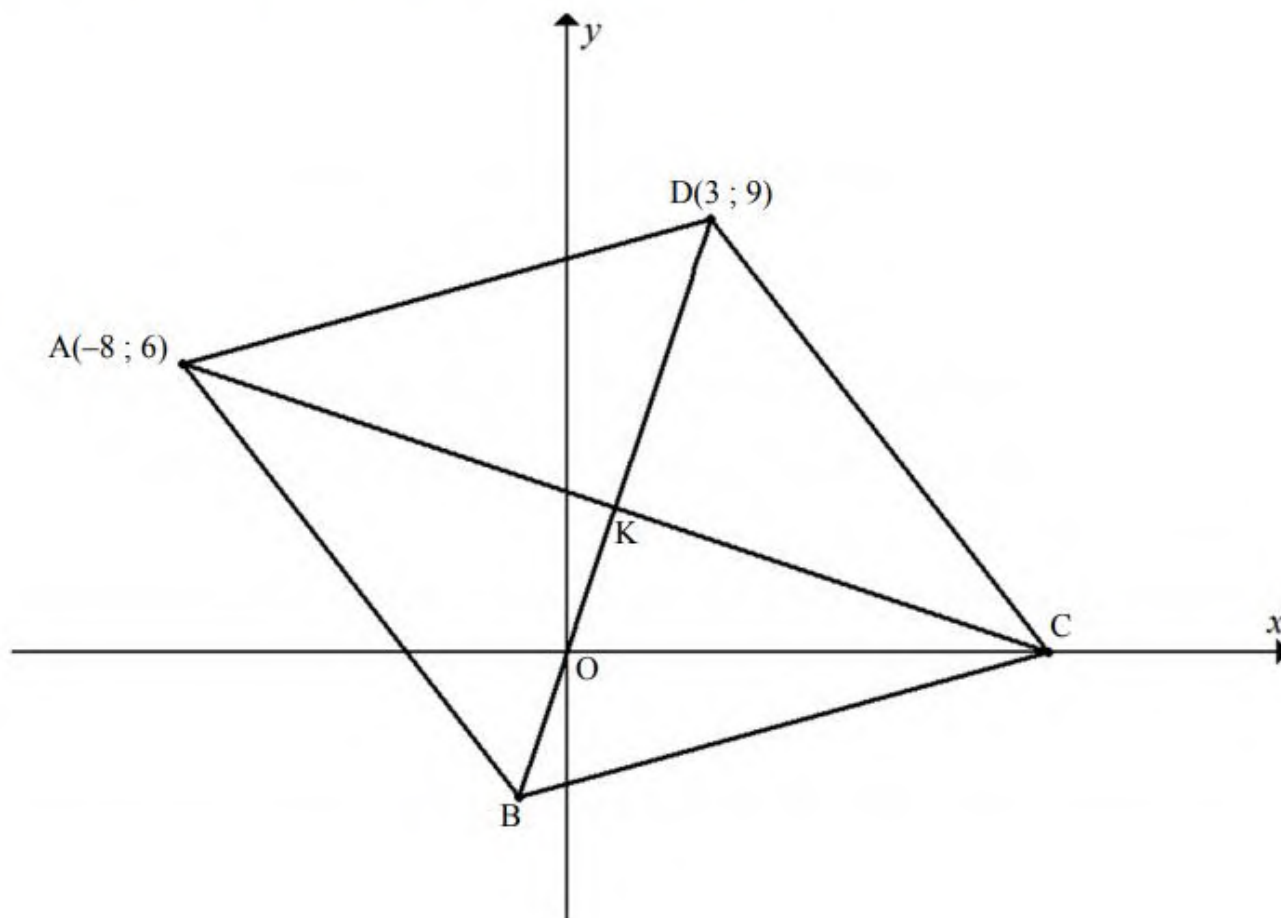
$\hat{ACB} = \theta$, $\hat{AMO} = \alpha$ and $\hat{BPT} = \beta$.



- 3.1 Calculate the gradient of the line AC . (2)
 - 3.2 Determine the equation of line RNS in the form $y = mx + c$. (2)
 - 3.3 Calculate the value of a . (4)
 - 3.4 Calculate the size of θ . (5)
- [13]**

QUESTION 4 *Downloaded from Stanmorephysics.com*

In the diagram $A(-8 ; 6)$, B , C and $D(3 ; 9)$ are the vertices of a rhombus. The equation of BD is $3x - y = 0$. The diagonals of the rhombus intersect at point K .



- 4.1 Calculate the perimeter of $ABCD$. Leave your answer in simplest surd form. (3)
 - 4.2 Determine the equation of diagonal AC in the form $y = mx + c$. (4)
 - 4.3 Calculate the coordinates of K if the equation of AC is $x + 3y = 10$. (3)
 - 4.4 Calculate the coordinates of B . (2)
 - 4.5 Determine, showing ALL your calculations, whether rhombus $ABCD$ is a square or not. (5)
- [17]**

QUESTION 3

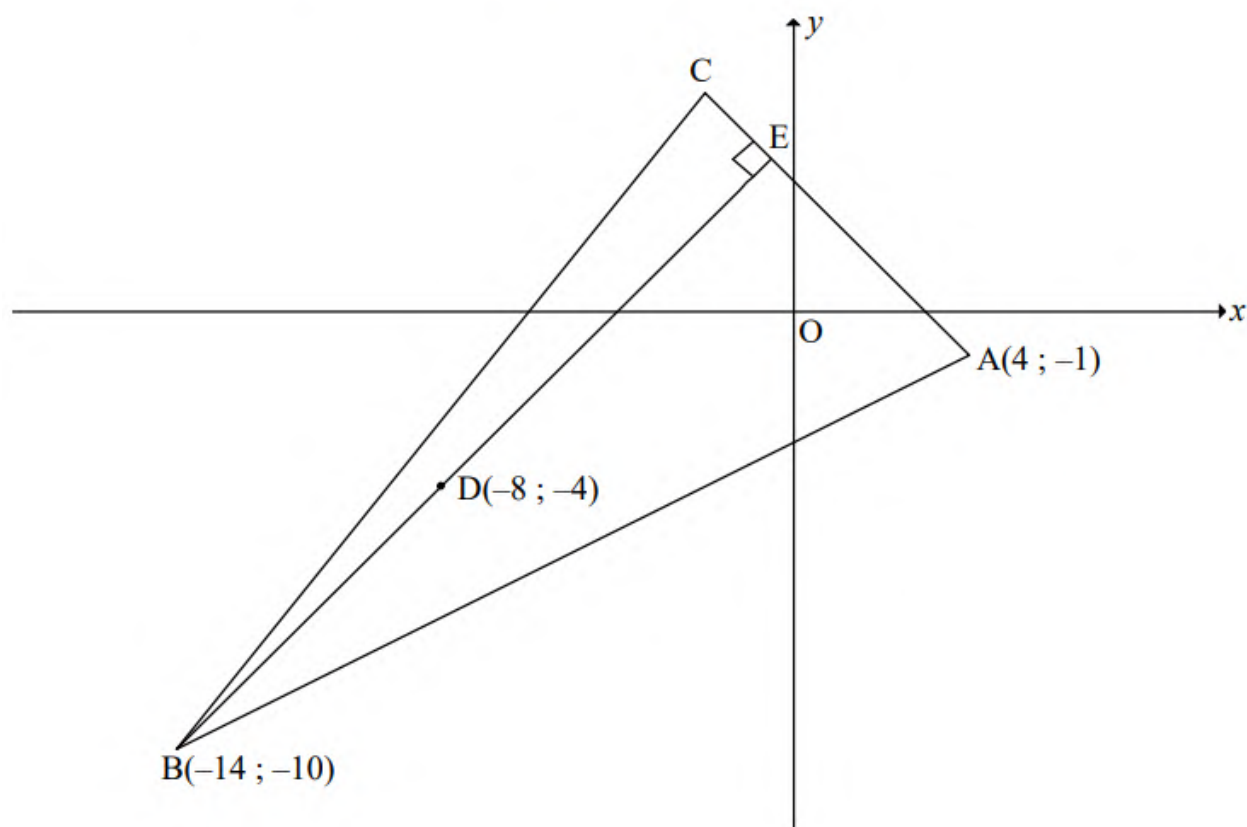
Downloaded from Stanmorephysics.com

In the diagram below, $A(4; -1)$, $B(-14; -10)$ and C are the vertices of a triangle.

E is a point on AC such that $BE \perp AC$.

The point $D(-8; -4)$ lies on BE .

The equation of the line BC is $4y - 5x - 30 = 0$.

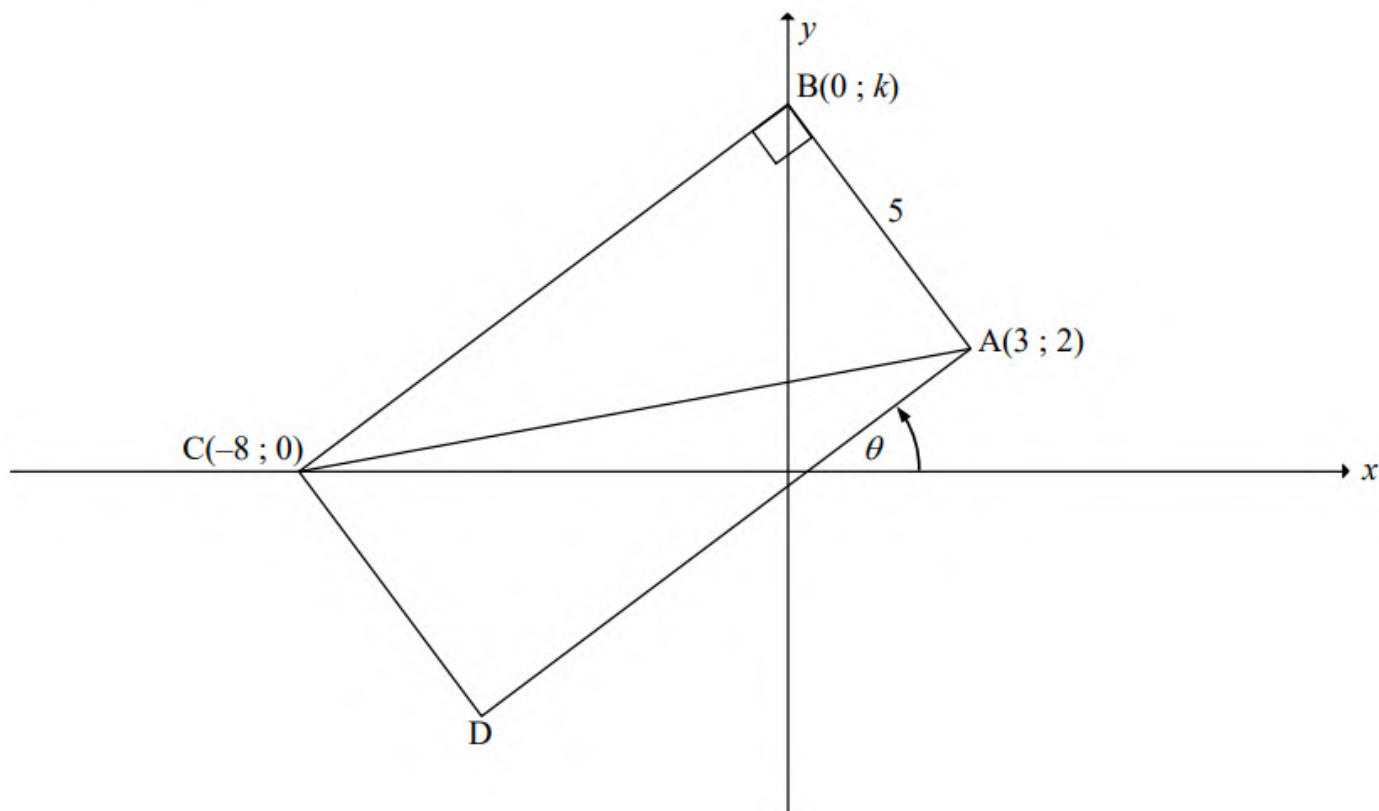


- 3.1 Calculate the gradient of BD . (2)
 - 3.2 Hence, write down the gradient of AC . (1)
 - 3.3 Determine the equation of AC in the form $y = mx + c$. (2)
 - 3.4 The point $G(p; -5)$ lies on AB . Calculate the value of p . (3)
 - 3.5 Calculate the coordinates of C . (4)
- [12]**

QUESTION 4 *Downloaded from Stanmorephysics.com*

$A(3 ; 2)$, $B(0 ; k)$, $C(-8 ; 0)$ and D are the vertices of a rectangle.
 $AB = 5$ units.

The angle of inclination of AD is θ , as shown in the diagram.

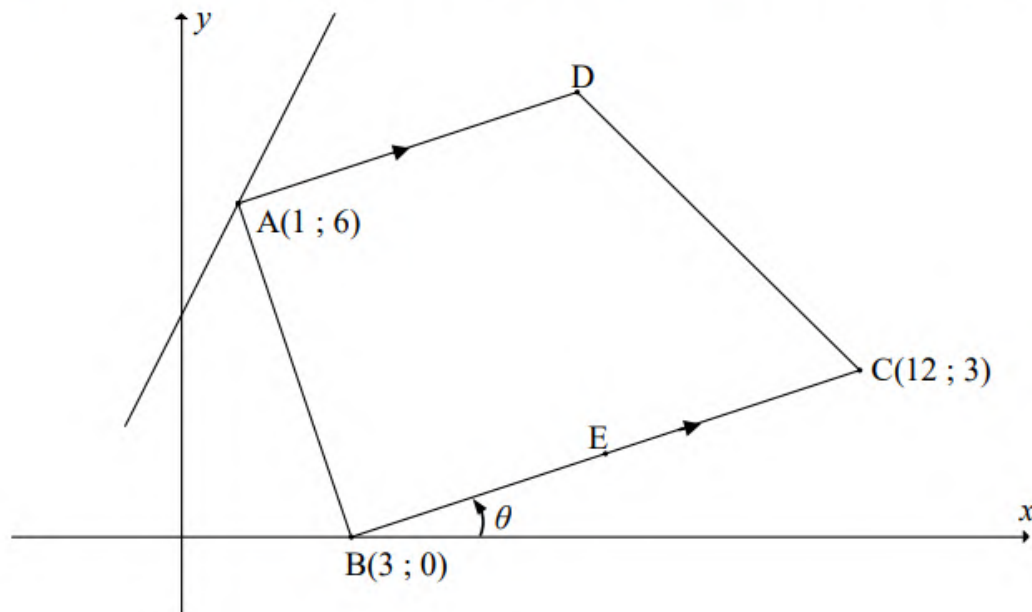


- 4.1 Calculate the length of AC . (2)
- 4.2 Calculate the value of k . (4)
- 4.3 Determine the equation of BC in the form $y = mx + c$. (3)
- 4.4 Calculate the size of θ . (3)
- 4.5 Calculate the area of $ABCD$. (3)
- 4.6 Calculate the size of $\hat{BAC}$. (2)

[17]

QUESTION 3 *Downloaded from Stanmorephysics.com*

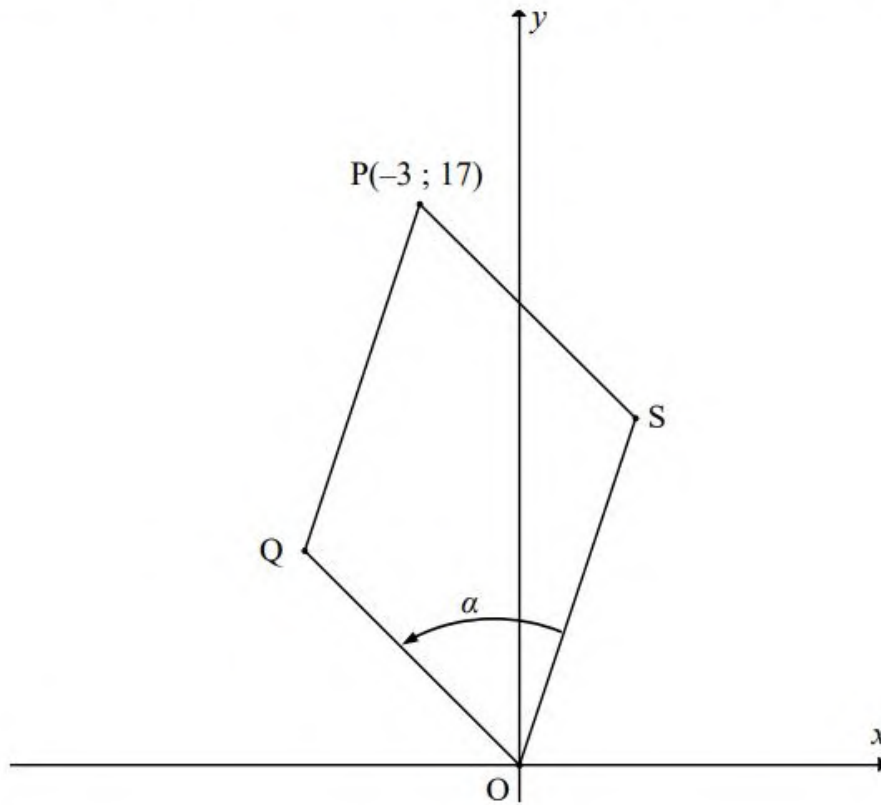
A(1 ; 6), B(3 ; 0), C(12 ; 3) and D are the vertices of a trapezium with $AD \parallel BC$. E is the midpoint of BC. The angle of inclination of the straight line BC is θ , as shown in the diagram.



- 3.1 Calculate the coordinates of E. (2)
 - 3.2 Determine the gradient of the line BC. (2)
 - 3.3 Calculate the magnitude of θ . (2)
 - 3.4 Prove that AD is perpendicular to AB. (3)
 - 3.5 A straight line passing through vertex A does not pass through any of the sides of the trapezium. This line makes an angle of 45° with side AD of the trapezium. Determine the equation of this straight line. (5)
- [14]**

QUESTION 4 *Downloaded from Stanmorephysics.com*

In the diagram below, $P(-3 ; 17)$, Q , O and S are the vertices of a parallelogram. The sides OS and OQ are defined by the equations $y = 6x$ and $y = -x$ respectively. $\hat{QOS} = \alpha$.

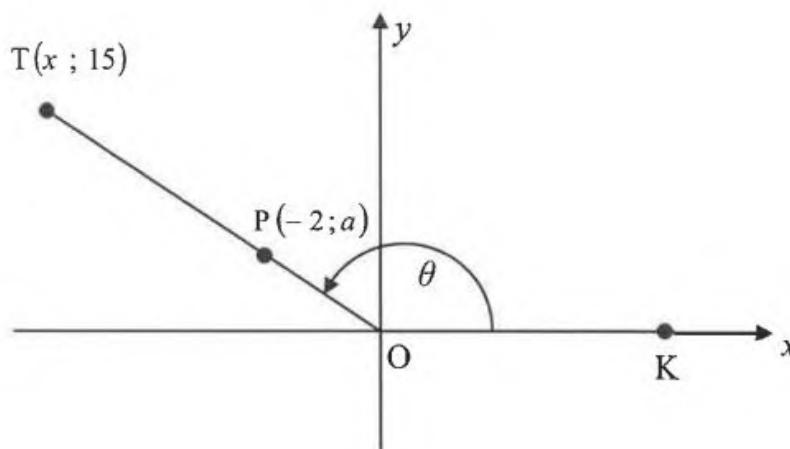


- 4.1 Determine the equation of QP in the form $y = mx + c$. (3)
- 4.2 Hence, determine the coordinates of Q . (4)
- 4.3 Calculate the length of OQ . Leave your answer in simplified surd form. (2)
- 4.4 Calculate the size of α . (3)
- 4.5 If $OS = \sqrt{148}$ units, calculate the length of QS . (3)

[15]

QUESTION 5

- 5.1 In the diagram below, $T(x; 15)$ is a point in the Cartesian plane such that $OT = 17$ units. $P(-2; a)$ lies on OT . K is a point on the positive x -axis and $\hat{TOK} = \theta$.



Determine, with the aid of the diagram, the following:

- 5.1.1 The value of x (2)
- 5.1.2 $\tan \theta$ (1)
- 5.1.3 $\cos(180^\circ - \theta)$ (2)
- 5.1.4 $\sin^2 \theta$ (2)
- 5.1.5 The value of a (3)
- 5.2 Simplify WITHOUT using a calculator:
- $$\frac{\sin 120^\circ \cdot \cos 210^\circ \cdot \tan 315^\circ \cdot \cos 27^\circ}{\sin 63^\circ \cdot \cos 540^\circ} \quad (7)$$
- 5.3 Prove the identity:
- $$\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = \tan \theta \quad (5)$$
- 5.4 Determine the general solution of $3 \sin x = 2 \tan x$ (6)
- [28]**

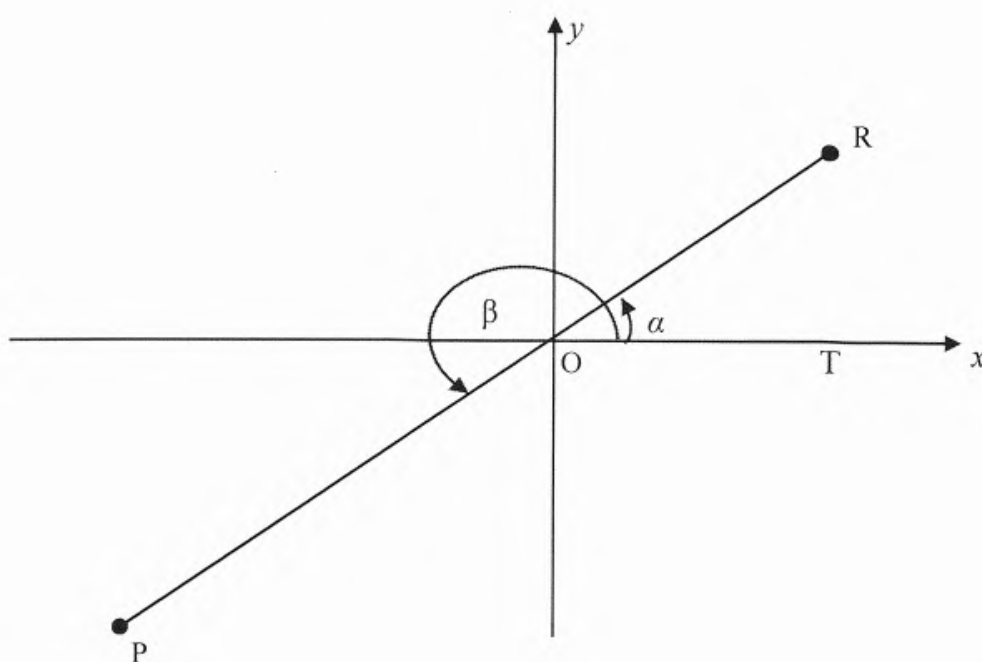
QUESTION 5 *Downloaded from Stanmorephysics.com*

5.1 Simplify fully: $\sin(90^\circ - x) \cdot \cos(180^\circ + x) + \tan x \cdot \cos x \cdot \sin(x - 180^\circ)$ (6)

5.2 Prove, WITHOUT using a calculator, that

$$\frac{\sin 315^\circ \cdot \tan 210^\circ \cdot \sin 190^\circ}{\cos 100^\circ \cdot \sin 120^\circ} = \frac{-\sqrt{2}}{3} \quad (6)$$

5.3 In the diagram below, R is a point in the first quadrant such that $\angle TOR = \alpha$.
RO is extended to P such that $OP = 2 RO$ and $\angle TOP = \beta$.
It is given that $\sin \alpha = \frac{3}{5}$.



WITHOUT using a calculator, determine:

5.3.1 The value of $\tan \alpha$ (3)

5.3.2 The value of $\sin \beta$ (3)

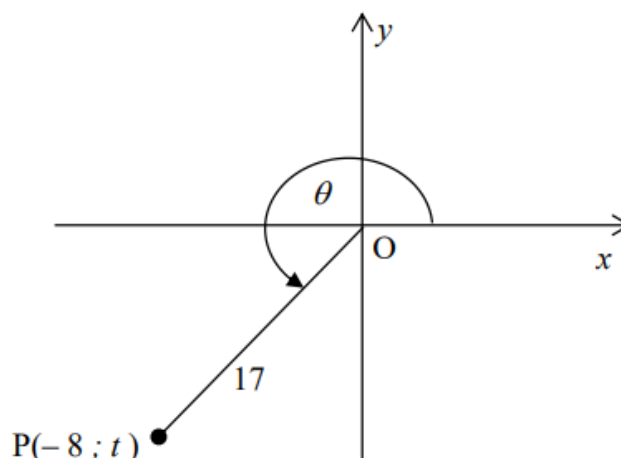
5.3.3 The coordinates of P (4)

5.4 Prove the identity: $\frac{\sin \theta - \tan \theta \cdot \cos^2 \theta}{\cos \theta - 1 + \sin^2 \theta} = \tan \theta$ (4)

[26]

QUESTION 5 *Downloaded from Stanmorephysics.com*

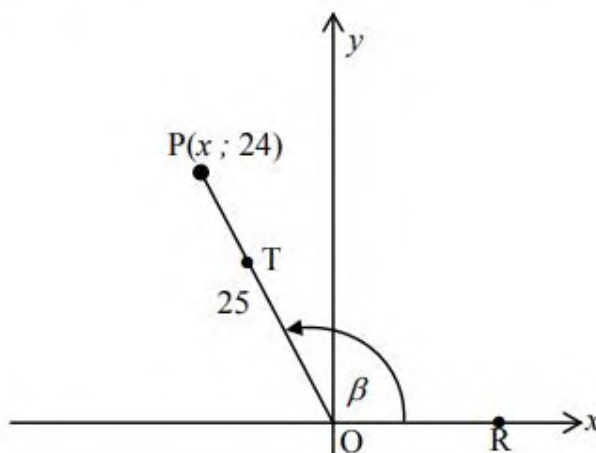
- 5.1 In the diagram below, $P(-8 ; t)$ is a point in the Cartesian plane such that $OP = 17$ units and reflex $\angle XOP = \theta$.



- 5.1.1 Calculate the value of t . (2)
- 5.1.2 Determine the value of each of the following WITHOUT using a calculator:
- (a) $\cos(-\theta)$ (2)
- (b) $1 - \sin \theta$ (2)
- 5.2 If $\sin 17^\circ = a$, WITHOUT using a calculator, express the following in terms of a :
- 5.2.1 $\tan 17^\circ$ (3)
- 5.2.2 $\sin 107^\circ$ (2)
- 5.2.3 $\cos^2 253^\circ + \sin^2 557^\circ$ (4)
- 5.3 Simplify fully, WITHOUT the use of a calculator:
- $$\frac{\cos(-225^\circ) \cdot \sin 135^\circ + \sin 330^\circ}{\tan 225^\circ} \quad (6)$$
- 5.4 Prove the identity: $\frac{1}{(\cos x + 1)(\cos x - 1)} = \frac{-1}{\tan^2 x \cdot \cos^2 x}$ (4)
- 5.5 Determine the general solution for $2\sin x \cdot \cos x = \cos x$. (6)
- [31]**

QUESTION 5 *Downloaded from Stanmorephysics.com*

- 5.1 In the diagram below, $P(x; 24)$ is a point such that $OP = 25$ and $\hat{R}OP = \beta$, where β is an obtuse angle.



- 5.1.1 Calculate the value of x . (2)

- 5.1.2 Determine the value of each of the following WITHOUT using a calculator:

(a) $\sin \beta$ (1)

(b) $\cos(180^\circ - \beta)$ (2)

(c) $\tan(-\beta)$ (2)

- 5.1.3 T is a point on OP such that $OT = 15$. Determine the coordinates of T WITHOUT using a calculator. (4)

- 5.2 Determine the value of the following expression:

$$\frac{2 \sin x \cdot \cos x (1 + \tan^2 x)}{\tan x} \quad (4)$$

- 5.3 Consider: $\frac{1 - \cos^2 A}{4 \cos(90^\circ + A)}$

- 5.3.1 Simplify the expression to a single trigonometric term. (3)

- 5.3.2 Hence, determine the general solution of $\frac{1 - \cos^2 2x}{4 \cos(90^\circ + 2x)} = 0,21$. (6)

[24]

QUESTION 5 *Downloaded from Stanmorephysics.com*

5.1 If $\cos 23^\circ = p$, express, **without the use of a calculator**, the following in terms of p :

5.1.1 $\cos 203^\circ$ (2)

5.1.2 $\sin 293^\circ$ (3)

5.2 Simplify the following expression to a single trigonometric term:

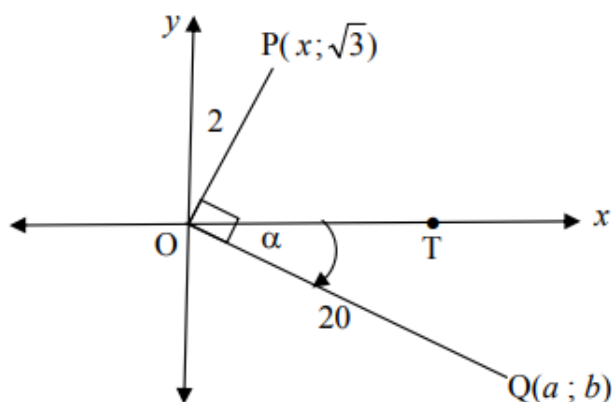
$$\frac{\sin(360^\circ - x) \cdot \tan(-x)}{\cos(180^\circ + x) \cdot (\sin^2 A + \cos^2 A)} \quad (6)$$

5.3 5.3.1 Prove the identity: $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = \frac{2}{\cos x}$ (5)

5.3.2 For which values of x in the interval $0^\circ \leq x \leq 360^\circ$ will the identity in QUESTION 5.3.1 be undefined? (2)

5.4 Determine the general solution of: $\sin 2x = 4 \cos 2x$ (5)

5.5 In the diagram below $P(x; \sqrt{3})$ is a point on the Cartesian plane such that $OP = 2$.
 $Q(a; b)$ is a point such that $\hat{TOQ} = \alpha$ and $OQ = 20$. $\hat{POQ} = 90^\circ$.



5.5.1 Calculate the value of x . (2)

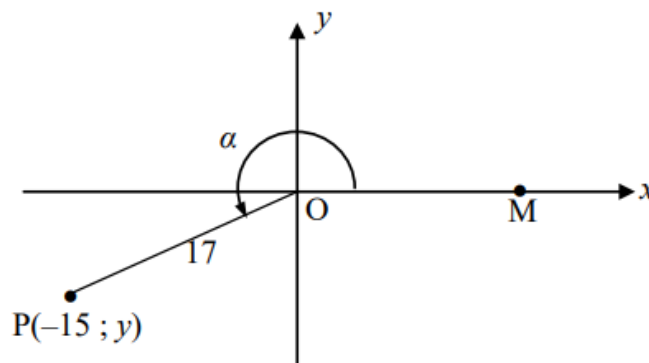
5.5.2 Hence, calculate the size of α . (3)

5.5.3 Determine the coordinates of Q . (5)

[33]

QUESTION 5

- 5.1 In the diagram, $P(-15 ; y)$ is a point in the Cartesian plane.
 $OP = 17$ units and reflex $\hat{MOP} = \alpha$.

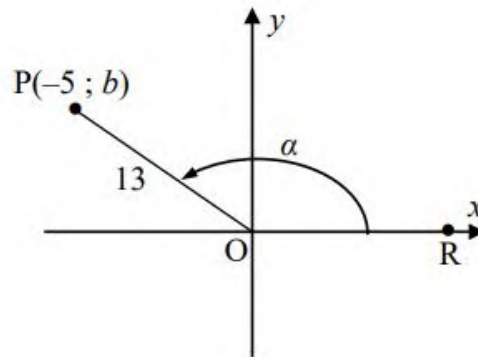


Determine the value of the following **without using a calculator**:

- 5.1.1 y (2)
- 5.1.2 $\sin(90^\circ + \alpha)$ (2)
- 5.1.3 $\tan \beta$, if $\alpha + \beta = 540^\circ$ (3)
- 5.2 Simplify the following expression to a single trigonometric ratio:
- $$\frac{\sin(180^\circ - x) - 2 \cos(90^\circ - x) \cos x}{2 \cos^2(360^\circ + x) - \cos(-x)} \quad (6)$$
- 5.3 5.3.1 Prove that $\frac{1 - \tan x}{1 + \tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$ (3)
- 5.3.2 For which value(s) of x in the interval $0^\circ \leq x \leq 180^\circ$ is the identity in QUESTION 5.3.1 undefined? (2)
- 5.4 Determine the general solution of the following equation:
- $$2 \tan x = 5 \sin x \quad (8)$$
- [26]**

QUESTION 5 *Downloaded from Stanmorephysics.com*

- 5.1 In the figure below, the point $P(-5 ; b)$ is plotted on the Cartesian plane.
 $OP = 13$ units and $\hat{ROP} = \alpha$.



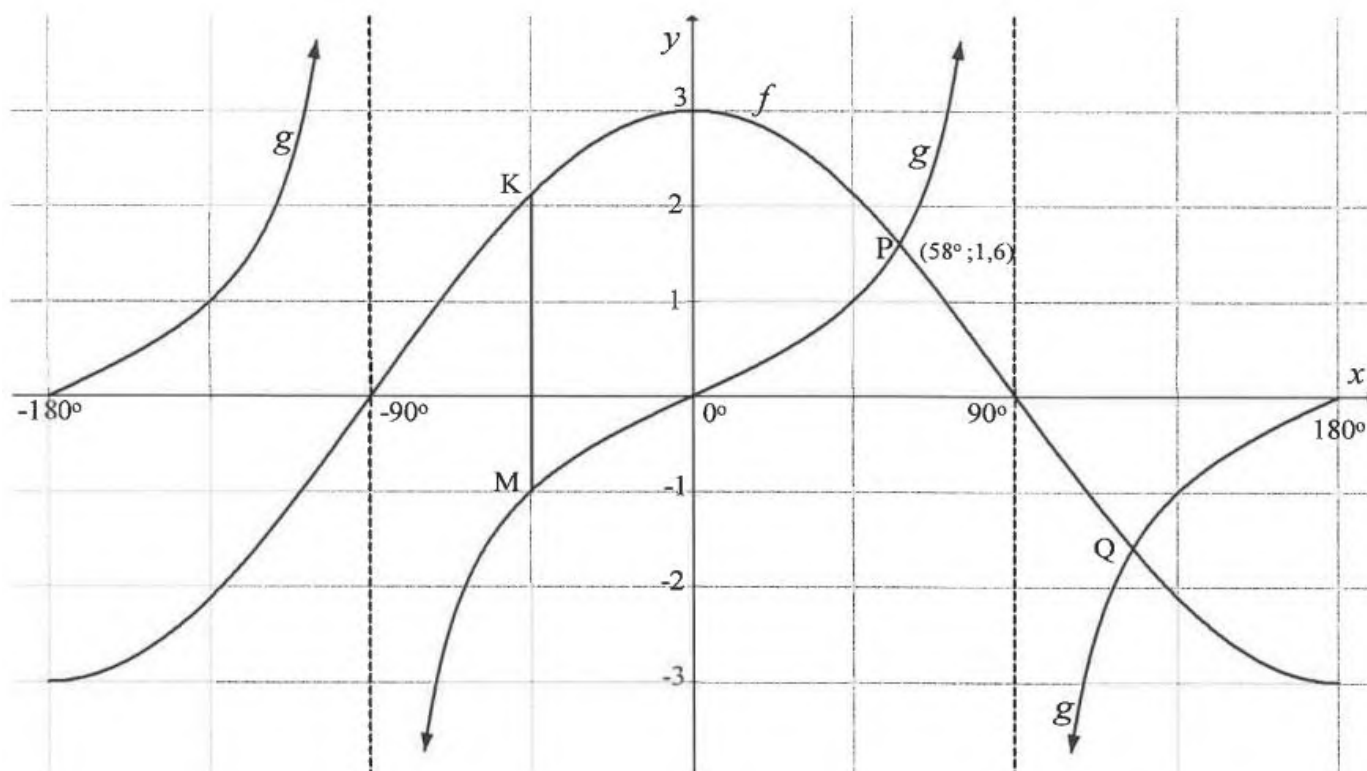
Without using a calculator, determine the value of the following:

- 5.1.1 $\cos \alpha$ (1)
- 5.1.2 $\tan(180^\circ - \alpha)$ (3)
- 5.2 Consider: $\frac{\sin(\theta - 360^\circ) \sin(90^\circ - \theta) \tan(-\theta)}{\cos(90^\circ + \theta)}$
- 5.2.1 Simplify $\frac{\sin(\theta - 360^\circ) \sin(90^\circ - \theta) \tan(-\theta)}{\cos(90^\circ + \theta)}$ to a single trigonometric ratio. (5)
- 5.2.2 Hence, or otherwise, **without using a calculator**, solve for θ if $0^\circ \leq \theta \leq 360^\circ$:
- $$\frac{\sin(\theta - 360^\circ) \sin(90^\circ - \theta) \tan(-\theta)}{\cos(90^\circ + \theta)} = 0,5 \quad (3)$$
- 5.3 5.3.1 Prove that $\frac{8}{\sin^2 A} - \frac{4}{1 + \cos A} = \frac{4}{1 - \cos A}$. (5)
- 5.3.2 For which value(s) of A in the interval $0^\circ \leq A \leq 360^\circ$ is the identity in QUESTION 5.3.1 undefined? (3)
- 5.4 Determine the general solution of $8 \cos^2 x - 2 \cos x - 1 = 0$. (6)

[26]

QUESTION 6

The graphs of the functions $f(x) = a \cos b\theta$ and $g(x) = c \tan \theta$ for $x \in [-180^\circ; 180^\circ]$ are sketched below. The graphs intersect at $P(58^\circ; 1,6)$ and Q .



- 6.1 Write down the range of f . (2)
- 6.2 If $M(-45^\circ; -1)$ lies on g , determine the value of c . (1)
- 6.3 Write down the values of a and b . (2)
- 6.4 Determine the coordinates of Q . (2)
- 6.5 K lies on f such that KM is parallel to the y -axis. Calculate the length of KM . (2)
- 6.6 If the system of axes is shifted 45° to the left and the graphs remain fixed, write down the equation that is now represented by graph f . (2)

[11]

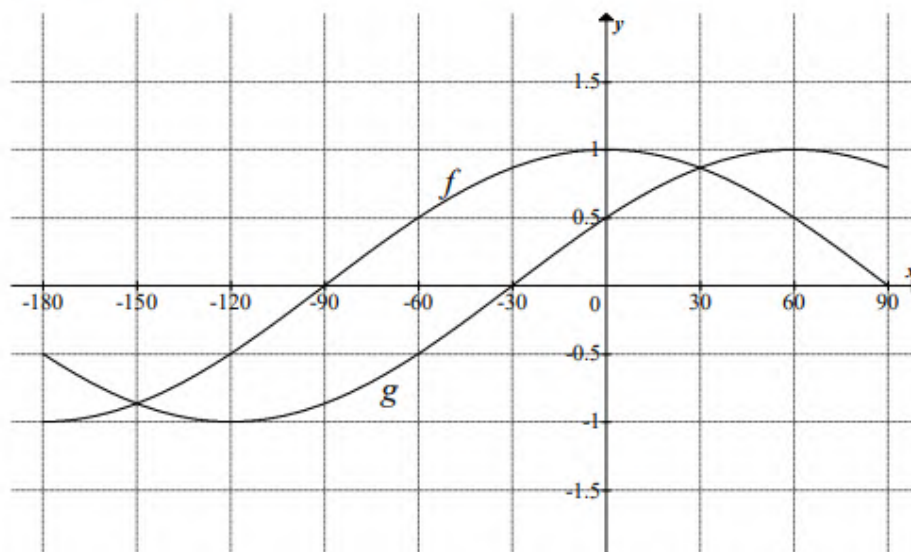
QUESTION 6

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- 6.1 Determine the general solution for $\sin(x-30^\circ) = \cos 2x$ (5)
- 6.2 Consider the functions $f(x) = \sin(x-30^\circ)$ and $g(x) = \cos 2x$
- 6.2.1 Write down the period of g . (1)
- 6.2.2 State the range of f . (2)
- 6.2.3 On the grid provided in the ANSWER BOOK, draw the graphs of f and g for $x \in [-90^\circ; 180^\circ]$.
Clearly show ALL intercepts with the axes, turning points and end points. (5)
- 6.2.4 Write down the x -coordinates of the points of intersection of f and g in the interval $x \in [-90^\circ; 180^\circ]$. (3)
- [16]

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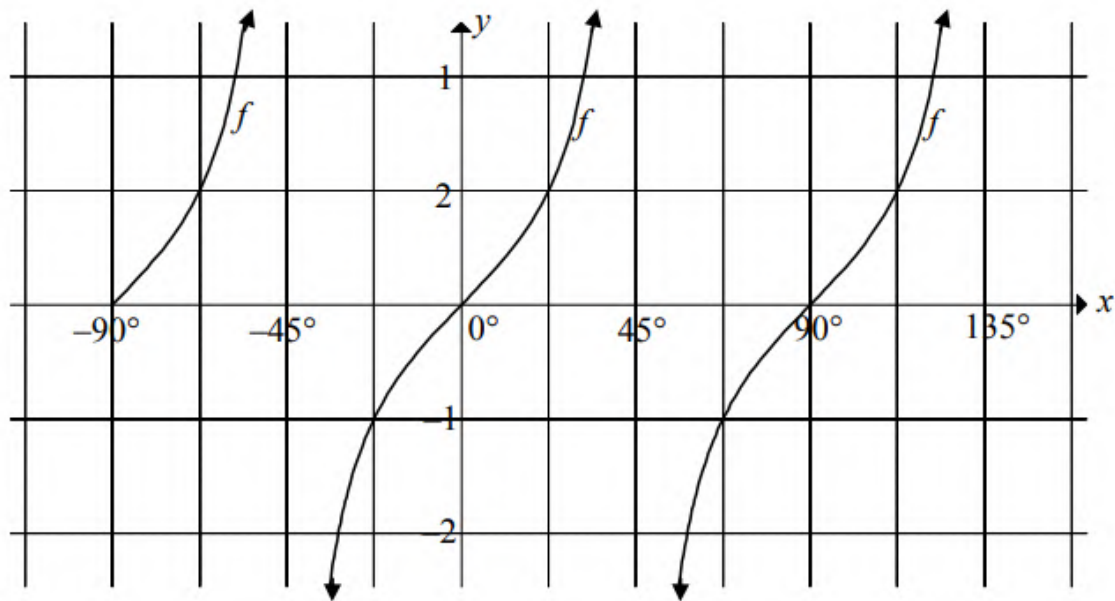
In the diagram the graphs of $f(x) = \cos x$ and $g(x) = \sin(x+b)$ are drawn for the interval $-180^\circ \leq x \leq 90^\circ$.



- 6.1 Write down the value of b . (1)
 - 6.2 Write down the period of g . (1)
 - 6.3 Write down the value(s) of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x) - g(x) = 0$. (2)
 - 6.4 For which values of x in the interval $-180^\circ \leq x \leq 90^\circ$ is $\sin(90^\circ - x) > g(x)$? (3)
 - 6.5 The graph of h is obtained by shifting f 3 units upwards. Determine the range of h . (2)
- [9]

QUESTION 6 *Downloaded from Stanmorephysics.com*

6.1 In the diagram, the graph of $f(x) = \tan bx$ is drawn for the interval $-90^\circ \leq x \leq 135^\circ$.

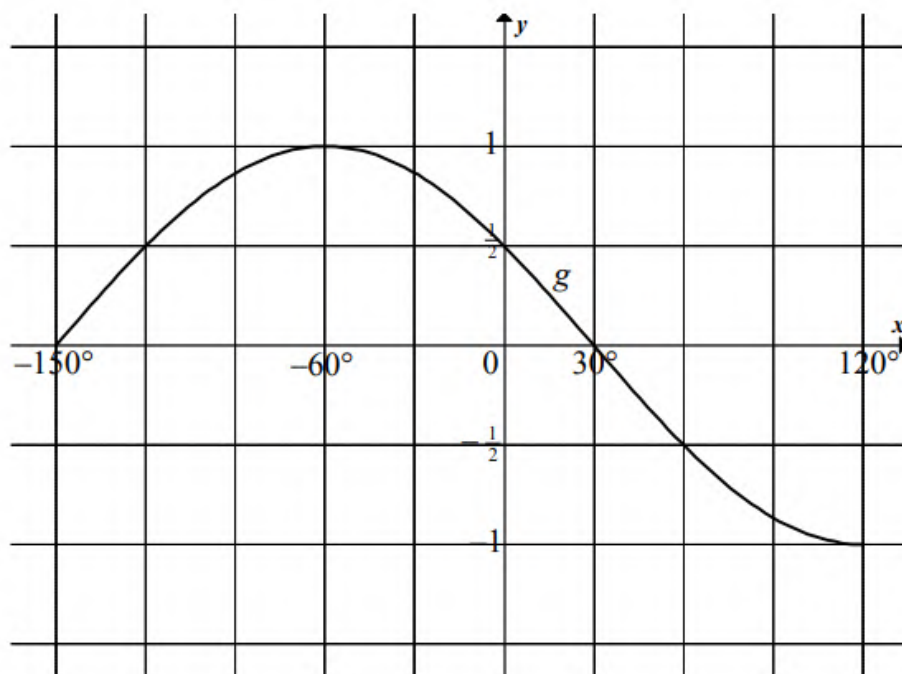


6.1.1 Determine the value of b . (1)

6.1.2 Determine the values of x in the interval $0^\circ \leq x \leq 135^\circ$ for which $f(x) \leq -1$. (2)

6.1.3 Graph h is defined as $h(x) = \tan b(x + 55^\circ)$. Write down the equations of the asymptotes of h in the interval $-90^\circ \leq x \leq 135^\circ$. (2)

6.2 In the diagram, the graph of $g(x) = \cos(x + 60^\circ)$ is drawn for the interval $-150^\circ \leq x \leq 120^\circ$.



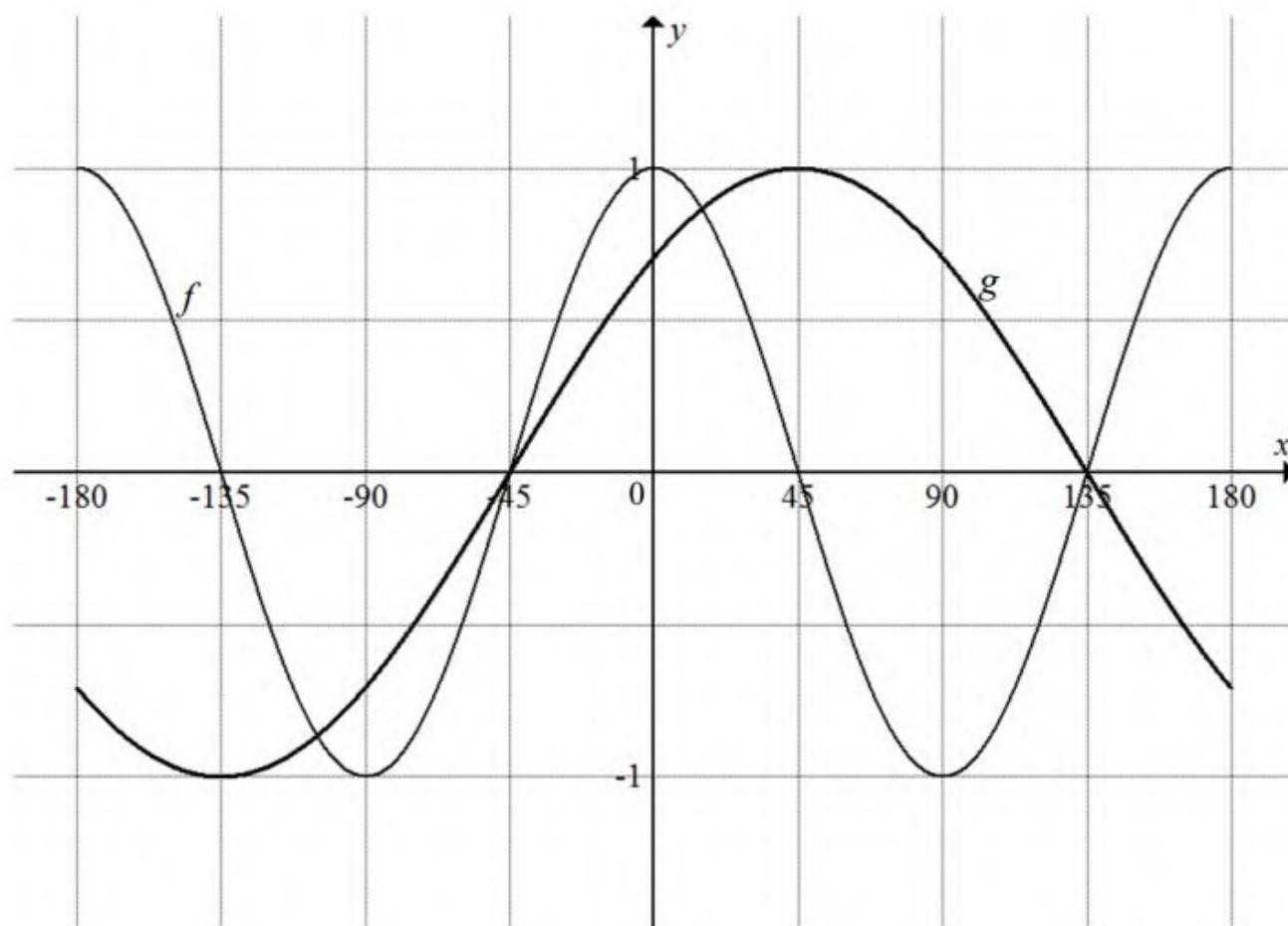
- 6.2.1 On the same system of axes, draw the graph of $k(x) = -\sin x$ for the interval $-150^\circ \leq x \leq 120^\circ$. Show ALL the intercepts with the axes as well as the coordinates of the turning points and end points of the graph. (4)
- 6.2.2 Determine the minimum value of $h(x) = \cos(x + 60^\circ) - 3$. (2)
- 6.2.3 Solve the equation $\cos(x + 60^\circ) + \sin x = 0$ for the interval $-150^\circ \leq x \leq 120^\circ$. (6)
- 6.2.4 Determine the values of x for the interval $-150^\circ \leq x \leq 120^\circ$, for which $\cos(x + 60^\circ) + \sin x > 0$. (2)
- 6.2.5 The function g can also be defined as $y = -\sin(x - \theta)$, where θ is an acute angle. Determine the value of θ . (2)

[21]

QUESTION 6

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In the diagram below the graphs of $f(x) = a \cos bx$ and $g(x) = \sin(x + p)$ are drawn for $x \in [-180^\circ; 180^\circ]$.



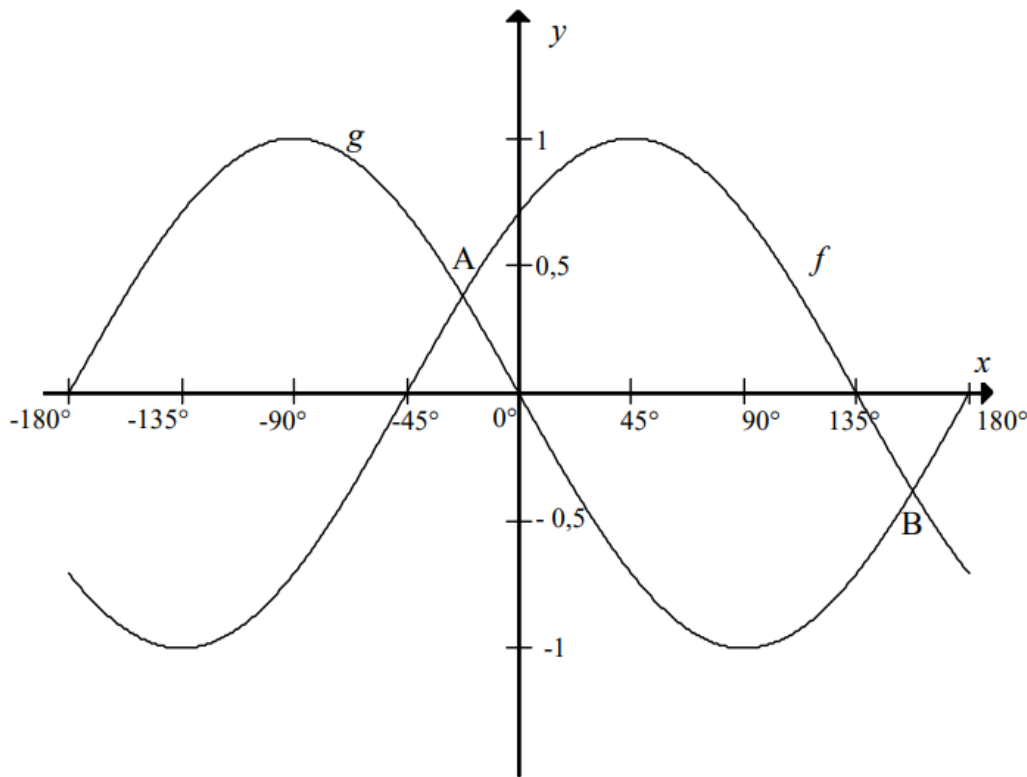
- 6.1 Write down the values of a , b and p . (3)
- 6.2 For which values of x in the given interval does the graph of f increase as the graph of g increases? (2)
- 6.3 Write down the period of $f(2x)$. (2)
- 6.4 Determine the minimum value of h if $h(x) = 3f(x) - 1$. (2)
- 6.5 Describe how the graph g must be transformed to form the graph k , where $k(x) = -\cos x$. (2)
- [11]**

QUESTION 6 *Downloaded from Stanmorephysics.com*

- 6.1 Use the system of axes provided on DIAGRAM SHEET 1 to draw the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x + 1$ for the interval $-180^\circ \leq x \leq 180^\circ$. Show clearly ALL intercepts with the axes, turning points and end points. (6)
- 6.2 Write down the period of f . (1)
- 6.3 For which value(s) of x in the interval $-180^\circ \leq x \leq 180^\circ$ will $g(x) - f(x)$ be a maximum? (1)
- 6.4 The graph f is shifted 45° to the right to obtain a new graph h . Write down the equation of h in its simplest form. (2)
- [10]**

QUESTION 6 *Downloaded from Stanmorephysics.com*

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = q \sin x$ are shown for the interval $-180^\circ \leq x \leq 180^\circ$.



- 6.1 Determine the values of p and q . (2)
 - 6.2 The graphs intersect at $A(-22,5^\circ ; 0,38)$ and B. Determine the coordinates of B. (2)
 - 6.3 Determine the value(s) of x in the interval $-180^\circ \leq x \leq 180^\circ$ for which $f(x) - g(x) < 0$. (2)
 - 6.4 The graph f is shifted 30° to the left to obtain a new graph h .
 - 6.4.1 Write down the equation of h in its simplest form. (2)
 - 6.4.2 Write down the value of x for which h has a minimum in the interval $-180^\circ \leq x \leq 180^\circ$. (1)
- [9]**

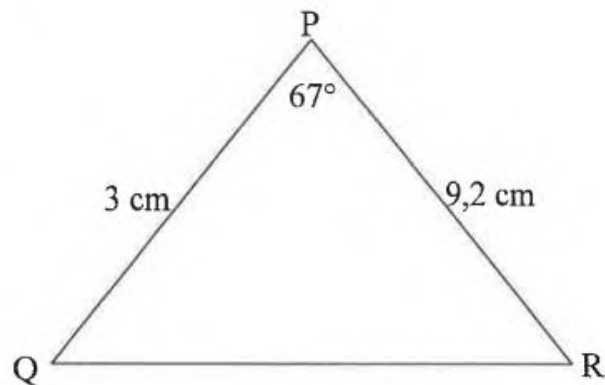
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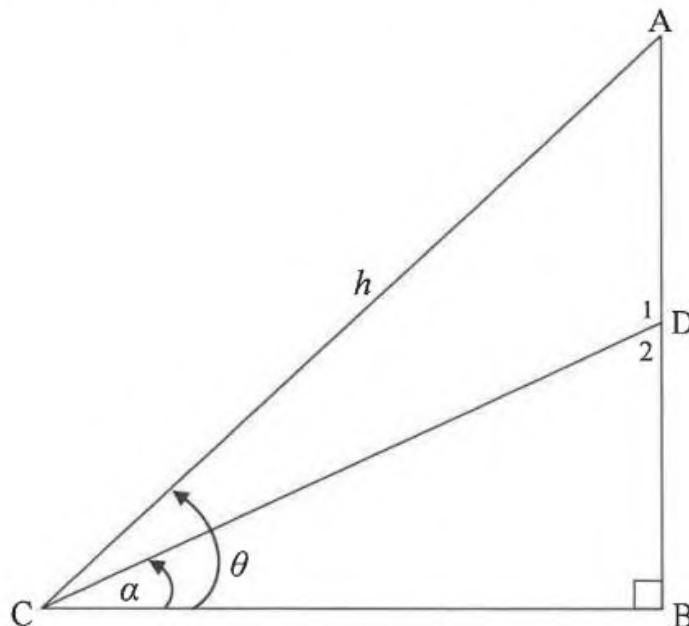
QUESTION 7

- 7.1 In the diagram, $\hat{P} = 67^\circ$, $PQ = 3$ cm and $PR = 9,2$ cm.
Determine the length of QR .



(3)

- 7.2 In the diagram below, $\hat{DCB} = \alpha$, $AC = h$ units and $\hat{ACB} = \theta$.



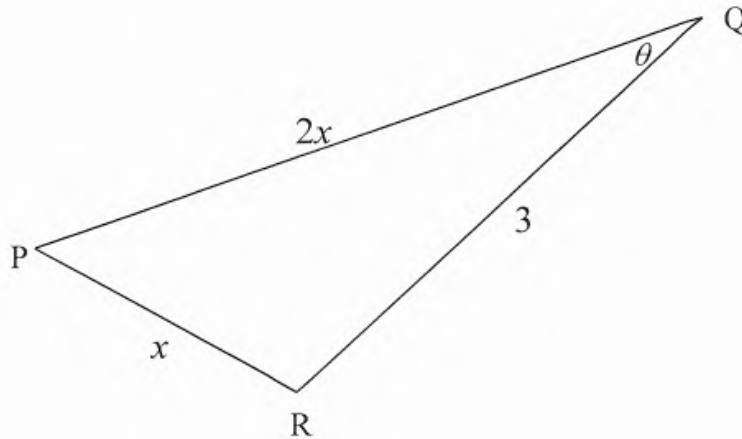
- 7.2.1 Determine size of $\hat{ACD}$ in terms of θ and α . (1)
- 7.2.2 Prove that $AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha}$ (4)
- 7.2.3 Determine the length of AD if $h = 17$ units, $\theta = 58^\circ$ and $\alpha = 23^\circ$. (2)
- 7.2.4 Calculate the area of $\triangle ADC$. (3)

[13]

QUESTION 7

Downloaded from Stanmorephysics.com

In $\triangle PQR$, $QR = 3$ units, $PR = x$ units, $PQ = 2x$ units and $\hat{PQR} = \theta$.

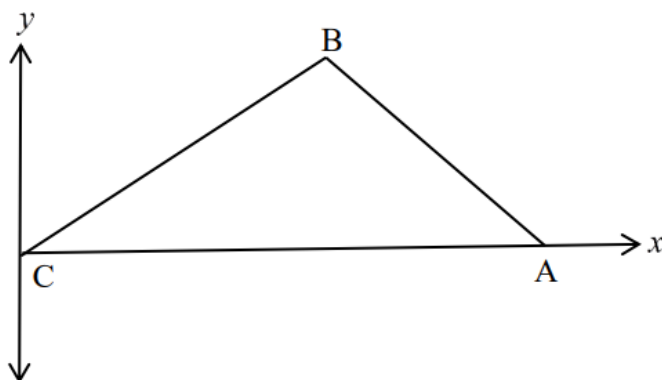


- 7.1 Show that $\cos \theta = \frac{x^2 + 3}{4x}$ (3)
- 7.2 If $x = 2,4$ units:
- 7.2.1 Calculate θ (3)
- 7.2.2 Calculate the area of $\triangle PQR$ (2)
- 7.3 Calculate the values of x for which the triangle exists. (4)
- [12]**

QUESTION 7

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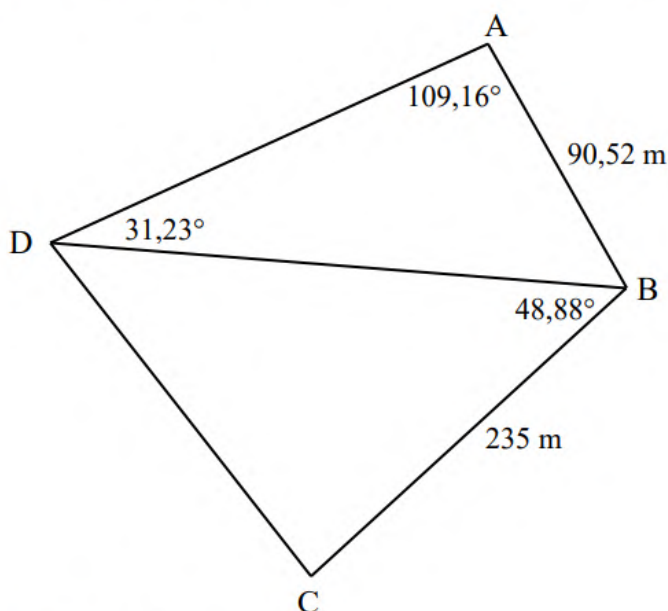
7.1 In the figure below, acute-angled $\triangle ABC$ is drawn having C at the origin.



7.1.1 Prove that $c^2 = a^2 + b^2 - 2ab \cos C$. (6)

7.1.2 Hence, deduce that $1 + \cos C = \frac{(a+b+c)(a+b-c)}{2ab}$ (4)

7.2 Quadrilateral ABCD is drawn with $BC = 235$ m and $AB = 90,52$ m. It is also given that $\hat{ADB} = 31,23^\circ$; $\hat{DAB} = 109,16^\circ$ and $\hat{CBD} = 48,88^\circ$.



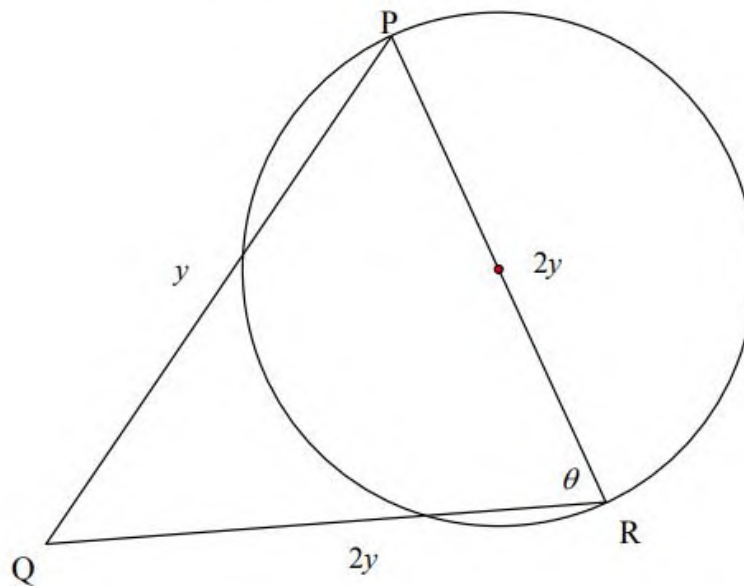
Determine the length of:

7.2.1 BD (3)

7.2.2 CD (3)
[16]

QUESTION 7 *Downloaded from Stanmorephysics.com*

In the diagram, PR is the diameter of the circle. Triangle PQR is drawn with vertex Q outside the circle. $\hat{R} = \theta$, $PR = QR = 2y$ and $PQ = y$.



- 7.1 Determine the value of $\cos \theta$. (4)
- 7.2 If QR cuts the circumference of the circle at T , determine PT in terms of y and θ . (3)
- [7]

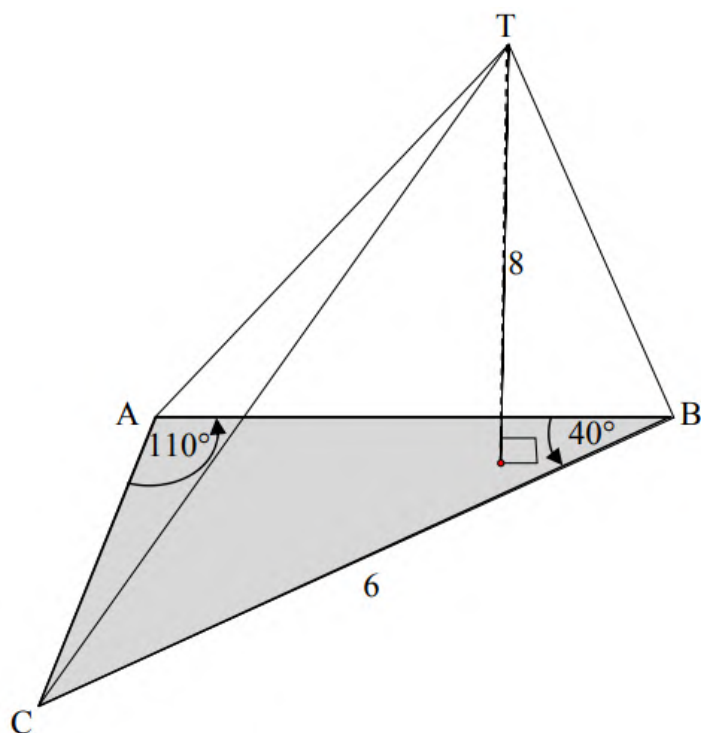
Downloaded from Stanmorephysics.com
QUESTION 7

Surface area = $\pi r^2 + \pi r S$ where S is the slant height.

Volume = $\frac{1}{3}$ area of base $\times$ perpendicular height

Volume = $\frac{1}{3} \pi r^2 h$

- 7.1 In the diagram, the base of the pyramid is an obtuse-angled $\triangle ABC$ with $\hat{A} = 110^\circ$, $\hat{B} = 40^\circ$ and $BC = 6$ metres. The perpendicular height of the pyramid is 8 metres.



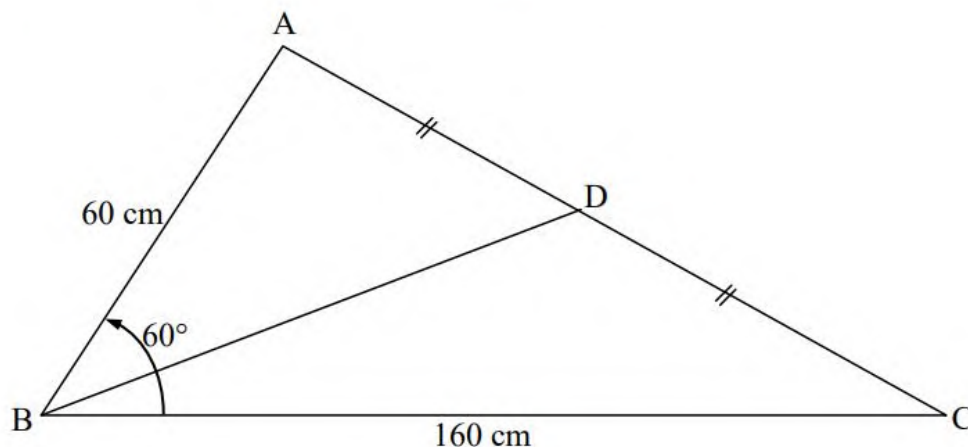
- 7.1.1 Calculate the length of AB. (3)
- 7.1.2 Calculate the area of the base, that is $\triangle ABC$. (2)
- 7.1.3 Calculate the volume of the pyramid. (3)

[8]

QUESTION 7

7.1 Prove that in any acute-angled $\triangle ABC$, $c^2 = a^2 + b^2 - 2ab \cos C$. (6)

7.2 In $\triangle ABC$, $AB = 60$ cm, $BC = 160$ cm and $\hat{A}BC = 60^\circ$.
 BD is the bisector of AC with D a point on AC .

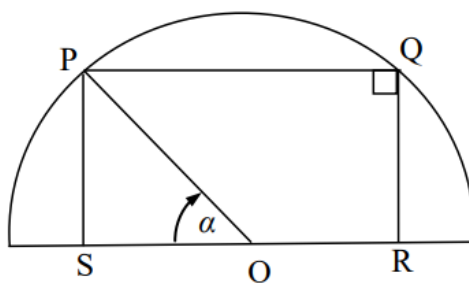


7.2.1 Calculate the length of AC . (3)

7.2.2 Determine the value of $\sin A$. Leave the answer in its simplest surd form. (3)

7.2.3 Calculate the area of $\triangle ABD$. Give your answer correct to ONE decimal place. (3)

7.3 In the diagram, O is the centre of a semi-circle.
 $PQRS$ is a rectangle drawn inside the semi-circle such that O lies on RS .
 $\hat{POS} = \alpha$.



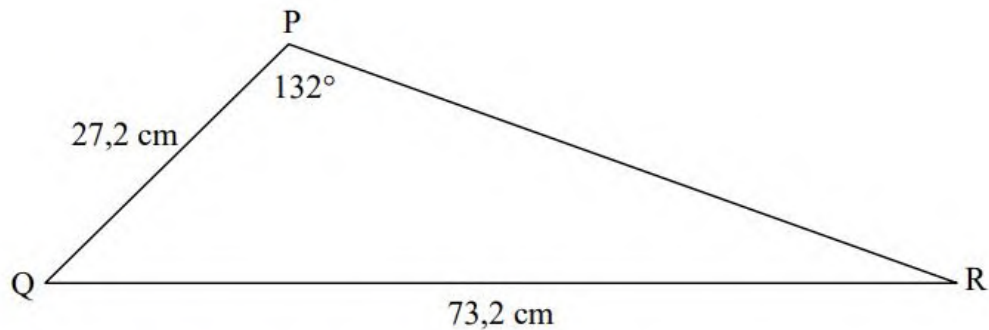
Calculate the size of α for which $PQRS$ will be a square. (3)
[18]

QUESTION 7

Downloaded from Stanmorephysics.com

7.1 Prove that in any acute-angled $\triangle ABC$, $\frac{\sin A}{a} = \frac{\sin C}{c}$. (5)

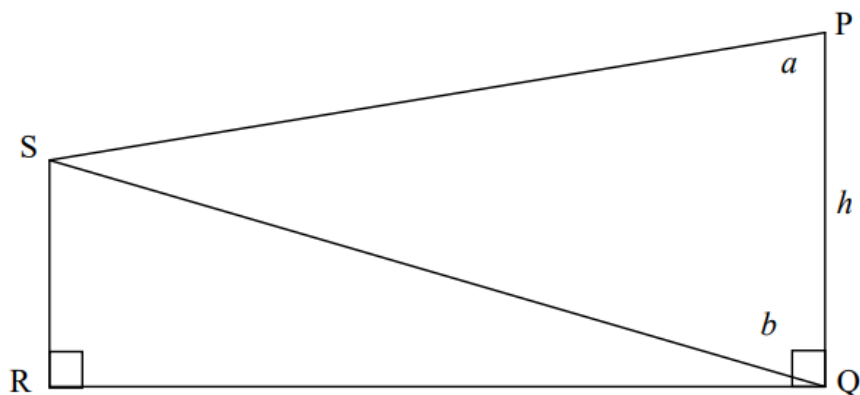
7.2 In $\triangle PQR$, $\hat{P} = 132^\circ$, $PQ = 27,2$ cm and $QR = 73,2$ cm.



7.2.1 Calculate the size of $\hat{R}$. (3)

7.2.2 Calculate the area of $\triangle PQR$. (3)

7.3 In the figure below, $\hat{SPQ} = a$, $\hat{PQS} = b$ and $PQ = h$. PQ and SR are perpendicular to RQ .



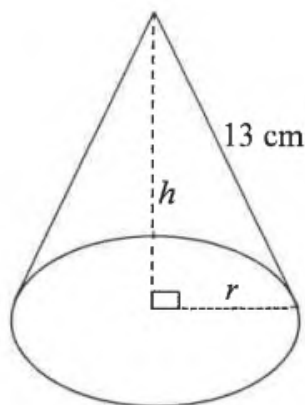
7.3.1 Determine the distance SQ in terms of a , b and h . (3)

7.3.2 Hence show that $RS = \frac{h \sin a \cos b}{\sin(a+b)}$. (3)

[17]

QUESTION 8

The diagram below shows a cone with a perpendicular height of h cm, a radius of r cm and a slant height of 13 cm.



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\begin{aligned} \text{Total surface area of the cone} \\ = \pi r^2 + \pi r s \end{aligned}$$

8.1 Show that the volume of the cone is given by $V = \frac{169\pi h - \pi h^3}{3}$ (4)

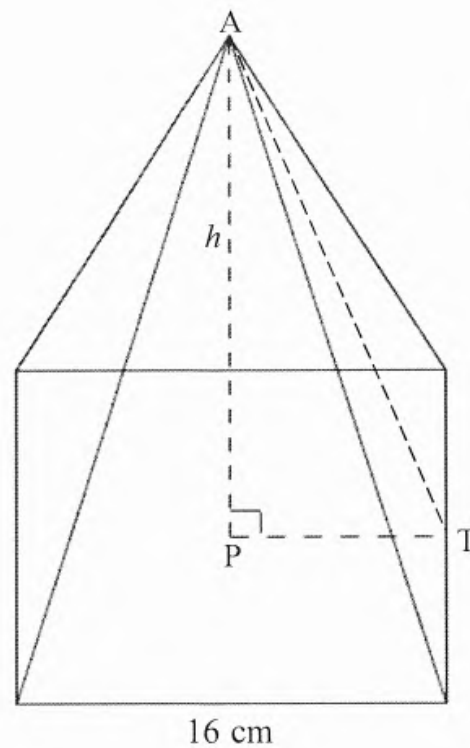
8.2 If $h = 12$ cm, determine the total surface area of the cone. (3)
[7]

QUESTION 8

Downloaded from Stanmorephysics.com

A pyramid with a square base with a side length of 16 cm is sketched below. P lies on the square base directly below A.

The volume of the pyramid is 640 cm^3 .



$$\text{Volume of pyramid} = \frac{1}{3}Ah$$

8.1 Show that the perpendicular height of the pyramid, AP, is 7,5 cm. (2)

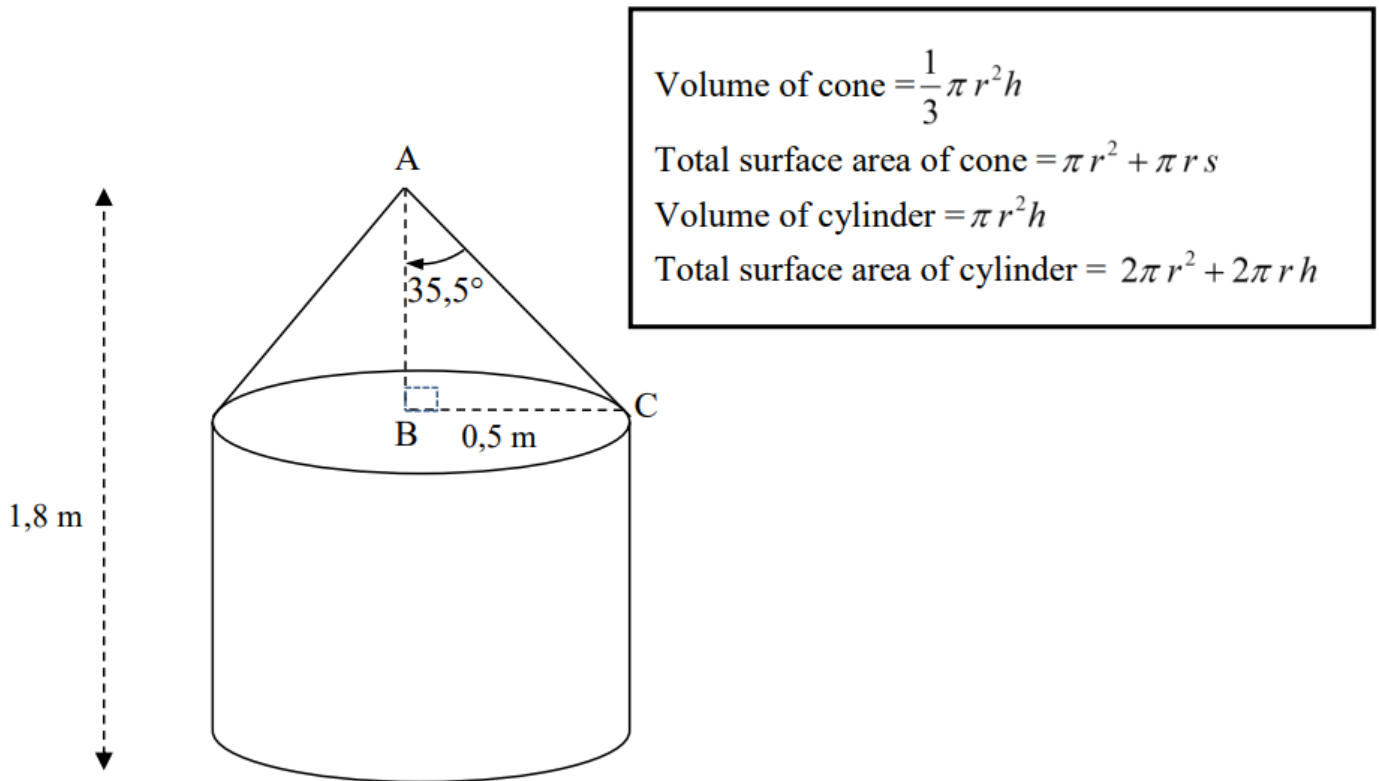
8.2 Hence, determine the total surface area of the pyramid. (4)

[6]

QUESTION 8

Downloaded from Stanmorephysics.com

The diagram below shows a water tank which is made up of a cylinder and cone having equal radii. The height of the tank is 1,8 m and the radius is 0,5 m. The angle between the perpendicular height, AB, and the slant height, AC, of the conical section is $35,5^\circ$.



- 8.1 Calculate the perpendicular height, AB, of the cone. (2)
- 8.2 When the tank is full, an electric pump switches on and pumps the water from the tank into an irrigation system at a rate of $0,52 \text{ m}^3/\text{h}$. The pump automatically switches off when the tank is $\frac{1}{4}$ full. Calculate how long, in hours, the pump feeds water into the irrigation system. (4)

[6]

QUESTION 8 Downloaded from Stanmorephysics.com

A cylindrical aerosol can has a lid in the shape of a hemisphere that fits exactly on the top of the can. The height of the can is 16 cm and the radius of the base of the can is 2,9 cm.

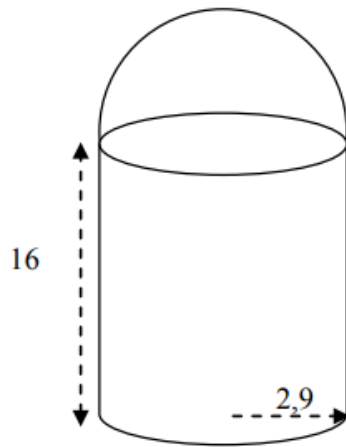


FIGURE 1



FIGURE 2

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$

- 8.1 Calculate the surface area of the can with the lid in place, as shown in FIGURE 1. (5)
- 8.2 If the lid is 80% filled with a liquid, as shown in FIGURE 2, calculate the volume of the liquid in the lid. (3)
- [8]**

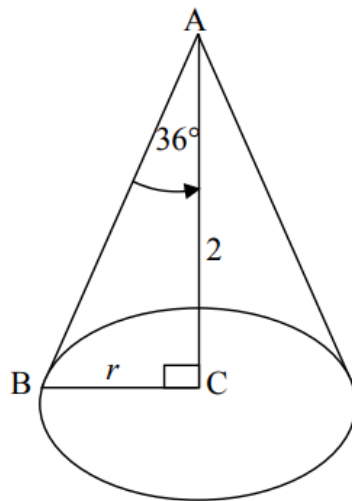
QUESTION 7 *Downloaded from Stanmorephysics.com*

Surface area = $\pi r^2 + \pi r S$ where S is the slant height.

Volume = $\frac{1}{3}$ area of base $\times$ perpendicular height

Volume = $\frac{1}{3} \pi r^2 h$

- 7.2 The perpendicular height, AC, of the cone below is 2 metres and the radius is r .
AB is the slant height.
 $\hat{BAC} = 36^\circ$



Calculate the total surface area of the cone.

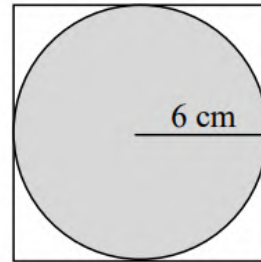
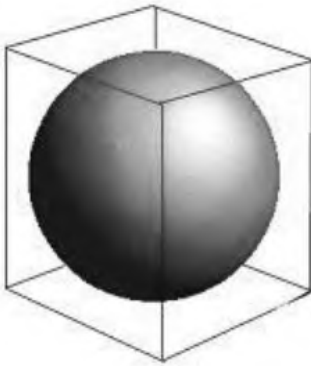
(6)

[6]

QUESTION 8

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A spherical glass ball is tightly packed in a box. The box is in the shape of a cube, as shown in the picture on the LEFT. The radius of the ball is 6 cm. The diagram on the RIGHT shows the cross-section of the glass ball placed in the box.



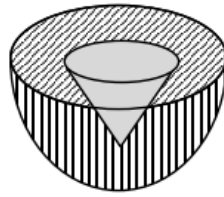
What volume of the box remains after the glass ball is placed in it?

[5]

QUESTION 8

Downloaded from Stanmorephysics.com

A solid metallic hemisphere has a radius of 3 cm. It is made of metal A. To reduce its weight a conical hole is drilled into the hemisphere (as shown in the diagram) and it is completely filled with a lighter metal B. The conical hole has a radius of 1,5 cm and a depth of $\frac{8}{9}$ cm.

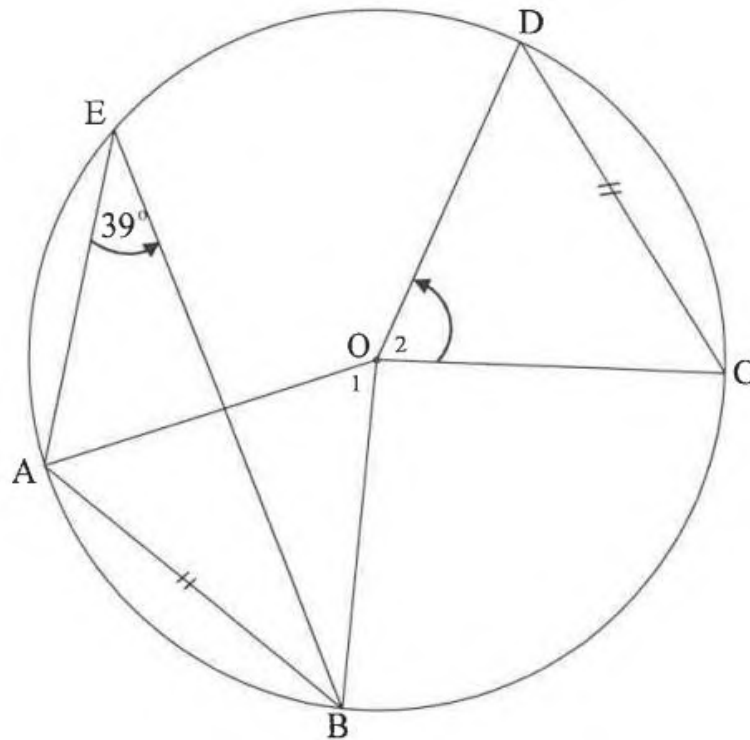


Calculate the ratio of the volume of metal A to the volume of metal B.

[6]

QUESTION 9

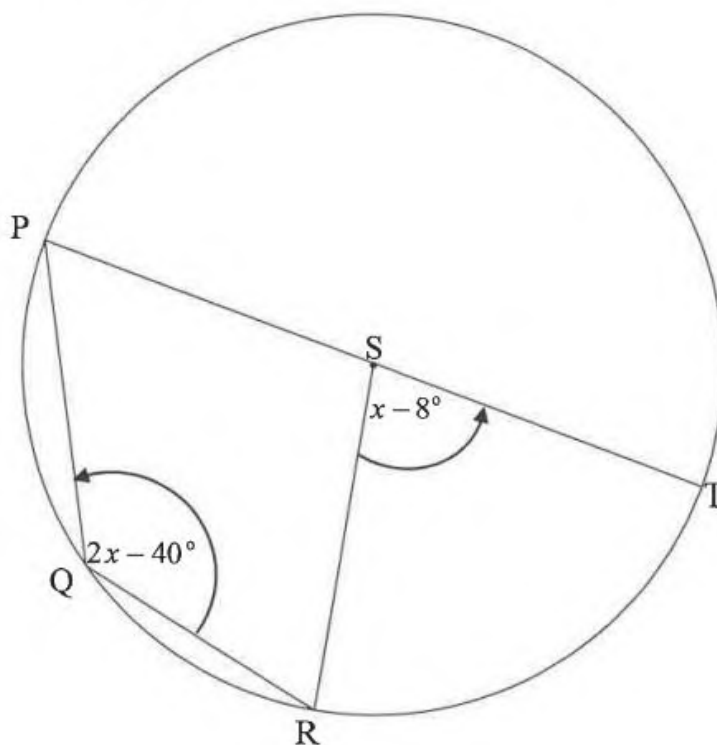
- 9.1 In the figure, O is the centre of the circle. A , B , C , D and E lie on the circle such that chord AB and chord DC are equal in length and $\angle AEB = 39^\circ$.



- 9.1.1 Determine the size of $\hat{O}_1$. (2)
- 9.1.2 Determine the size of $\hat{O}_2$. (2)

- 9.2 In the diagram, S is the centre of circle PQRT. PT is a diameter.
 $\angle PSQ = x - 8^\circ$ and $\angle PQR = 2x - 40^\circ$.

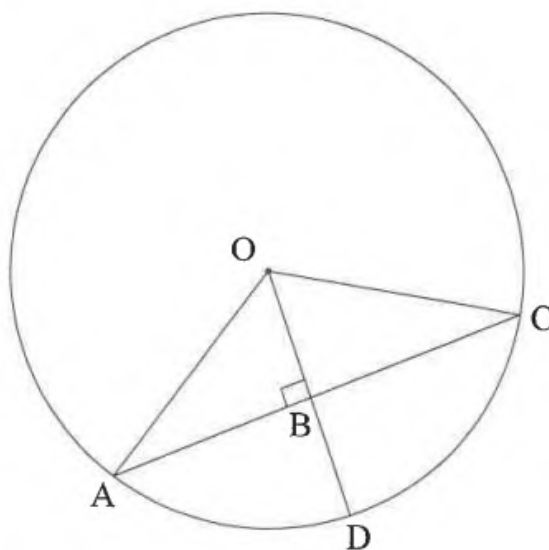
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Determine the value of x .

(4)

- 9.3 In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B. $OB = 2x$ units and $AC = 8x$ units.



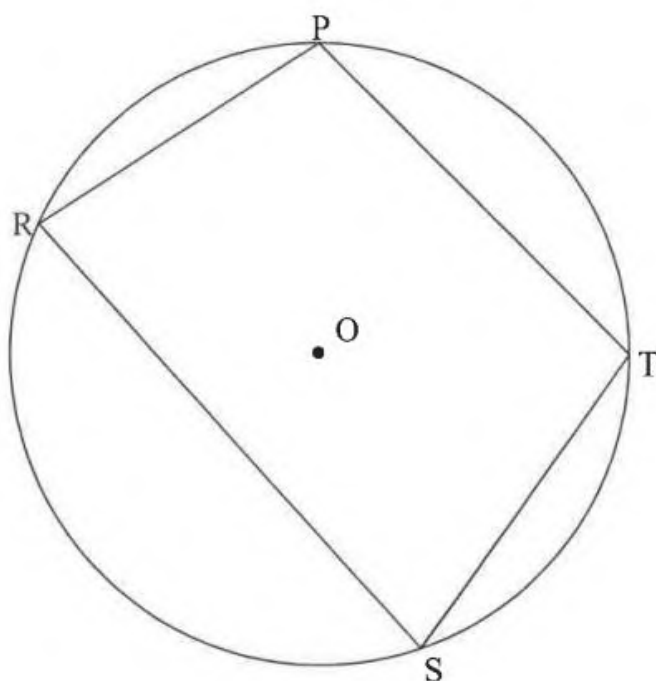
Show that the length of BD is $2x(\sqrt{5} - 1)$ units.

(5)
[13]

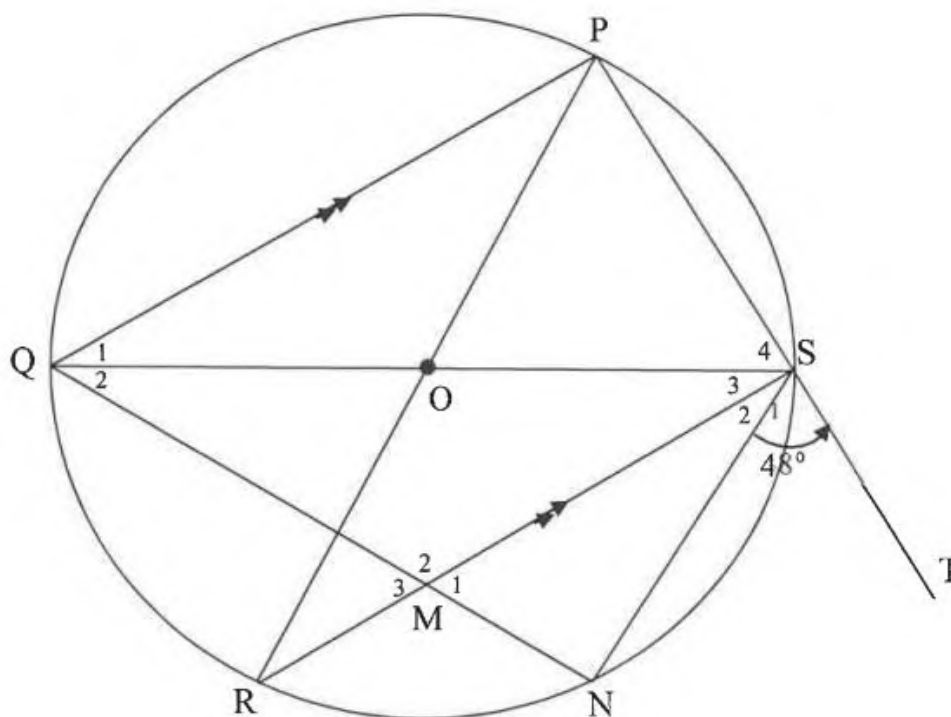
QUESTION 10

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- 10.1 In the diagram below, O is the centre of the circle and $PTSR$ is a cyclic quadrilateral.



- 10.2 In the figure, QS and PR are diameters of the circle with centre O such that $\hat{Q}_1 = \hat{Q}_2$. SN is drawn. RS intersects QN at M. $\hat{S}_1 = 48^\circ$



10.2.1 Determine, with reasons, the size of:

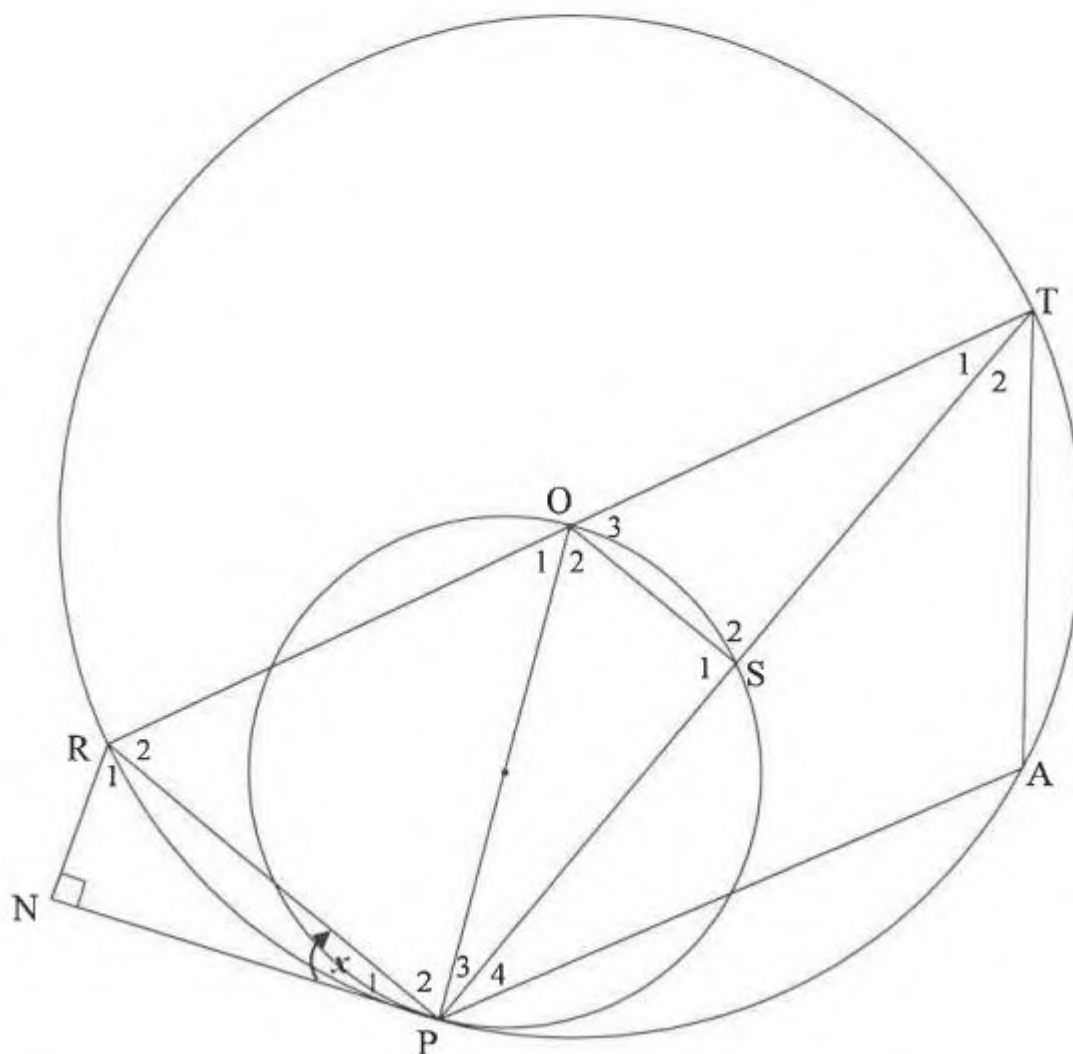
- (a) $\hat{Q}_1$ (3)
- (b) $\hat{R}$ (2)
- (c) $\hat{M}_1$ (2)

10.2.2 Prove that ST is a tangent to the circle passing through M, N and S. (2)

[14]

QUESTION 11 *Downloaded from Stanmorephysics.com*

O is the centre of the larger circle RTAP. OP is the diameter of the smaller circle PSO. NP is a tangent to both circles at P. $RN \perp NP$.
Let $\hat{P}_1 = x$.



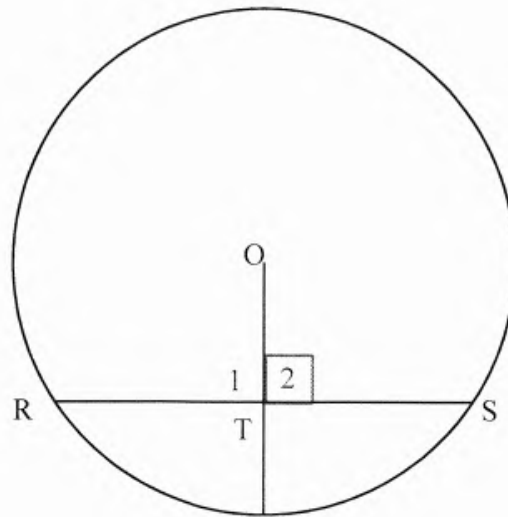
11.1 Prove that PR bisects $\hat{ORN}$. (5)

11.2 Prove that $\hat{ROS} = \hat{PAT}$. (5)

[10]

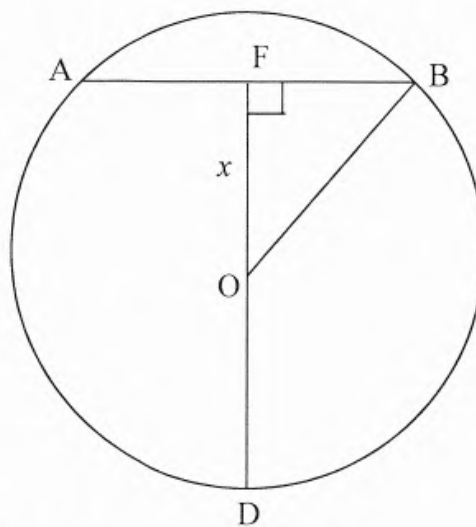
QUESTION 9 *Downloaded from Stanmorephysics.com*

- 9.1 In the diagram below, O is the centre of the circle and point T lies on chord RS . Prove the theorem which states that if $OT \perp RS$ then $RT = TS$.



(5)

- 9.2 In the diagram, O is the centre of circle ABD . F is a point on chord AB such that $DOF \perp AB$. $AB = FD = 8$ cm and $OF = x$ cm.



Determine the length of the radius of the circle.

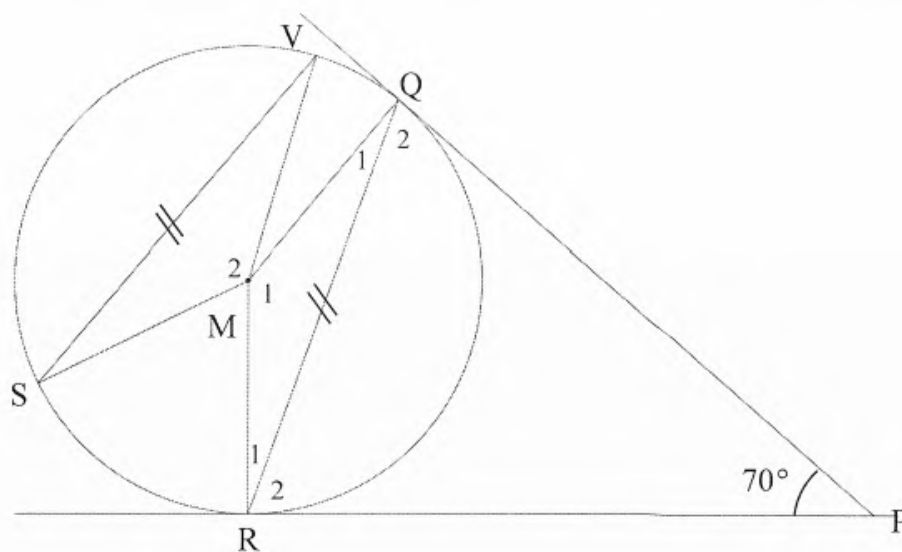
(5)
[10]

QUESTION 10

Downloaded from Stanmorephysics.com

M is the centre of the circle SVQR having equal chords SV and QR.

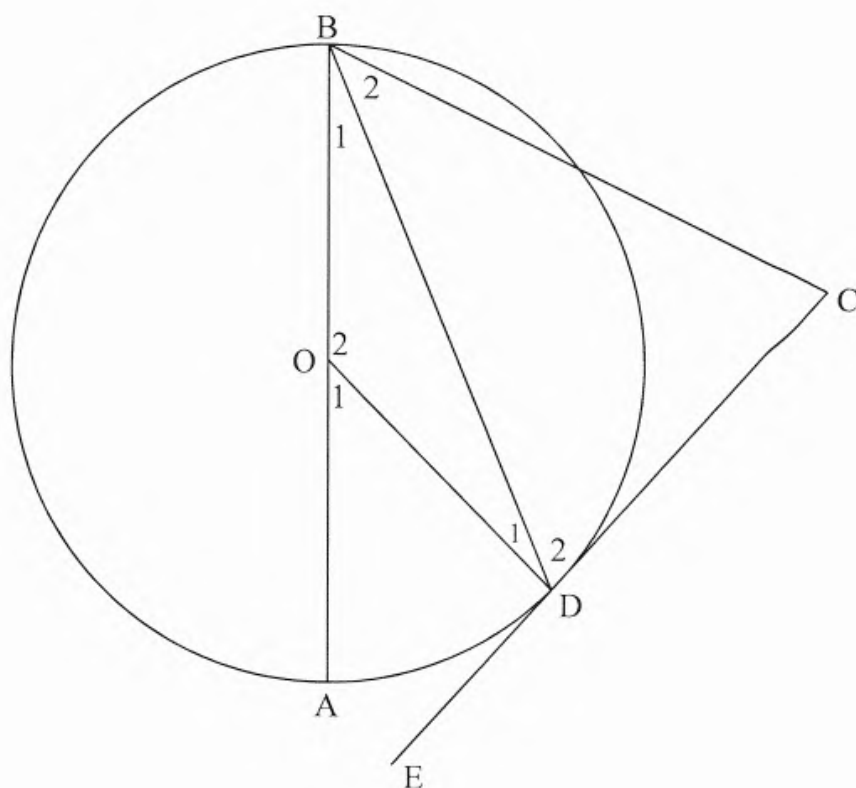
RP and QP are tangents to the circle at R and Q respectively such that $\hat{RPQ} = 70^\circ$.



- 10.1 Calculate the size of $\hat{R}_2$. (4)
- 10.2 Calculate the size of $\hat{Q}_1$. (2)
- 10.3 Determine the size of $\hat{M}_2$. (3)
- [9]

QUESTION 11
Downloaded from Stanmorephysics.com

In the diagram below, O is the centre of the circle. CDE is a tangent to the circle at D . DB bisects $\hat{A}BC$. Let $\hat{B}_1 = x$



- 11.1 Prove that $BC \parallel OD$ (4)
- 11.2 Show that $\hat{C} = 90^\circ$ (3)
- [7]

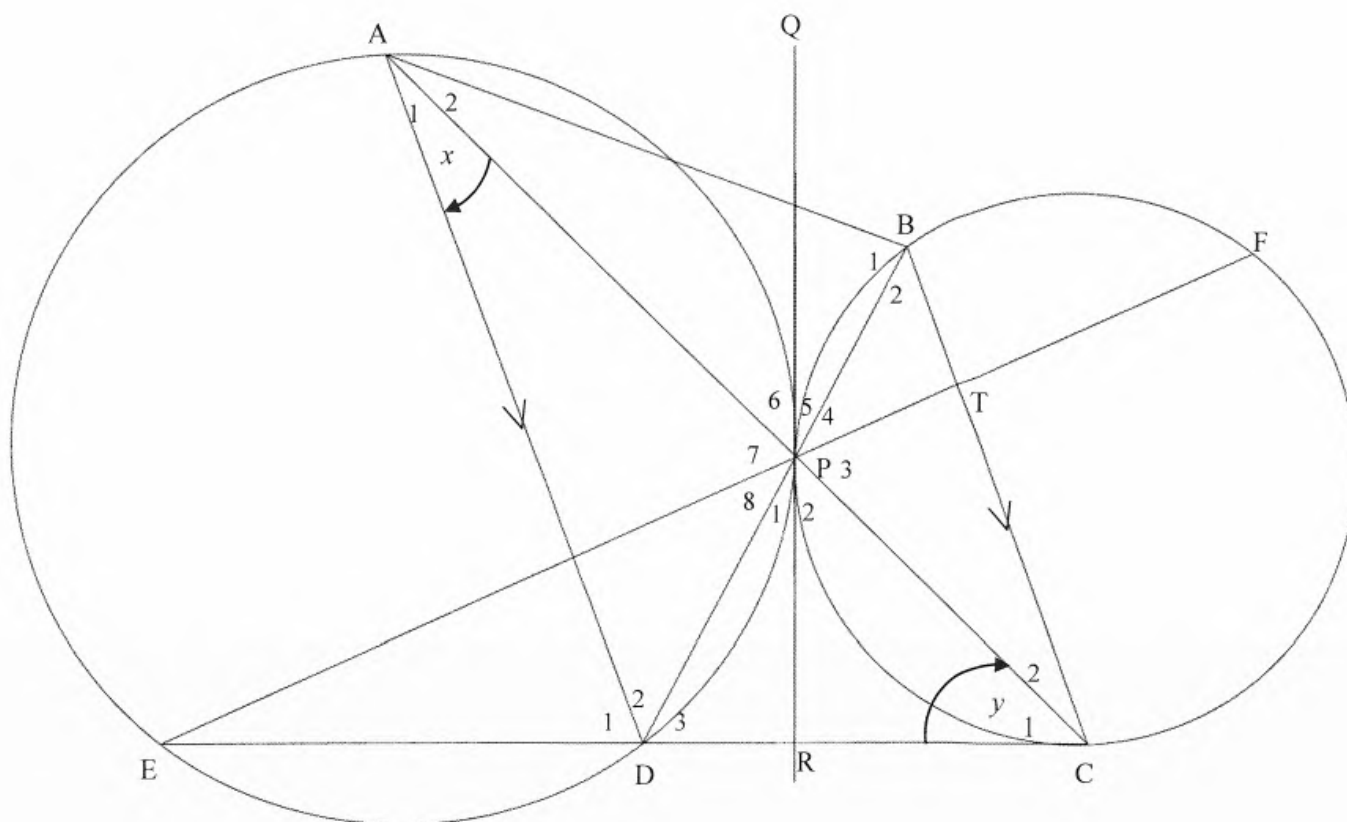
QUESTION 12

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In the diagram below, two circles touch each other externally at point P.

QPR is a common tangent to both circles at P. EDRC is a tangent to circle PBFC at C.

$\hat{RCA} = y$ and $\hat{DAC} = x$. $AD \parallel BC$.



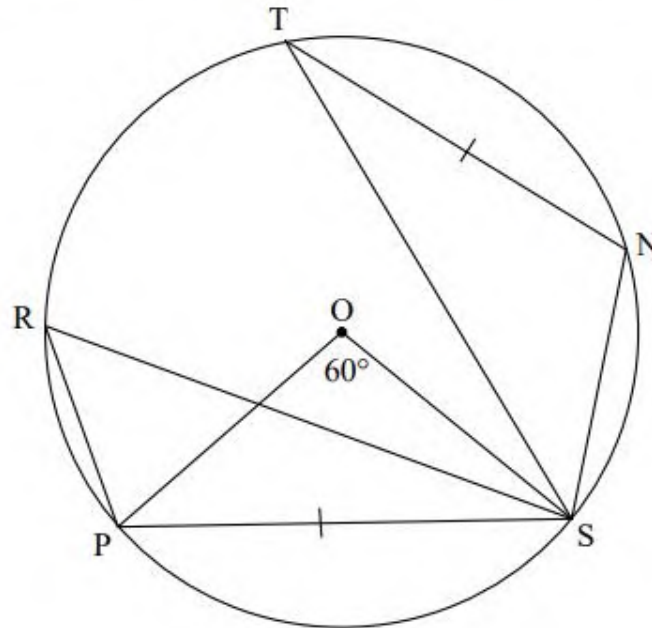
- 12.1 Name, with reasons, FOUR other angles equal to x . (7)
- 12.2 Show that $\hat{EPA} = x + y$ (4)
- 12.3 Determine the numerical value of $x + y$, if it is given that DCTP is a cyclic quadrilateral. (4)
- [15]

QUESTION 9 *Downloaded from Stanmorephysics.com*

9.1 Complete the statement so that it is TRUE:

The angle subtended by an arc at the centre of a circle is ... (2)

9.2 O is the centre of circle TNSPR. $\hat{POS} = 60^\circ$ and $PS = NT$.



Calculate the size of:

9.2.1 $\hat{PRS}$ (2)

9.2.2 $\hat{NST}$ (2)

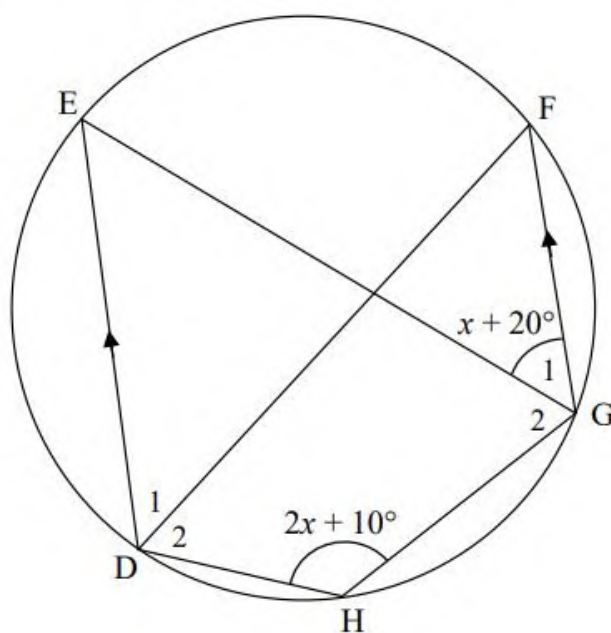
[6]

QUESTION 10

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D, E, F, G and H are points on the circumference of the circle.

$\hat{G}_1 = x + 20^\circ$ and $\hat{H} = 2x + 10^\circ$. $DE \parallel FG$.



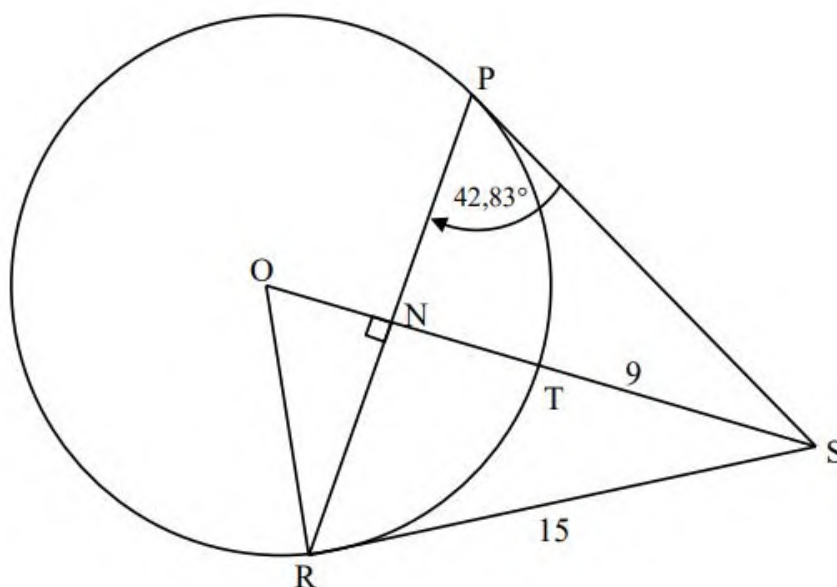
10.1 Determine the size of $\hat{DEG}$ in terms of x . (2)

10.2 Calculate the size of $\hat{DHG}$. (4)
[6]

QUESTION 11 *Downloaded from Stanmorephysics.com*

O is the centre of the circle PTR. N is a point on chord RP such that $ON \perp PR$. RS and PS are tangents to the circle at R and P respectively.

RS = 15 units; TS = 9 units; $\angle RPS = 42,83^\circ$.



11.1 Calculate the size of $\angle NOR$. (5)

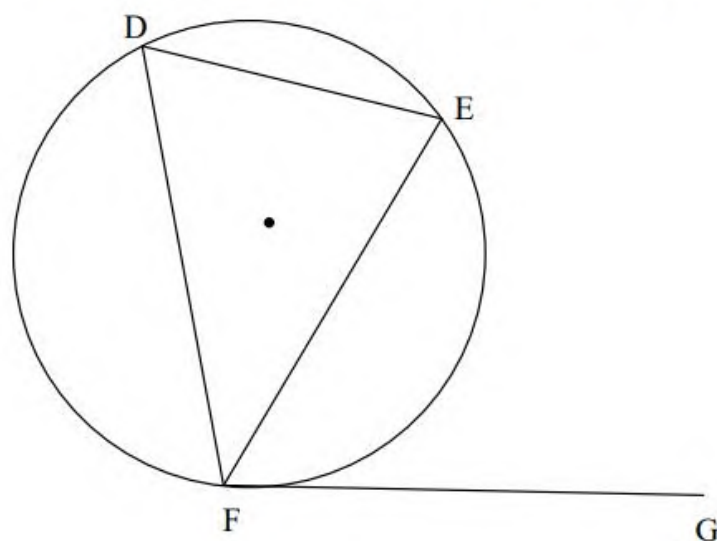
11.2 Calculate the length of the radius of the circle. (4)

[9]

QUESTION 12

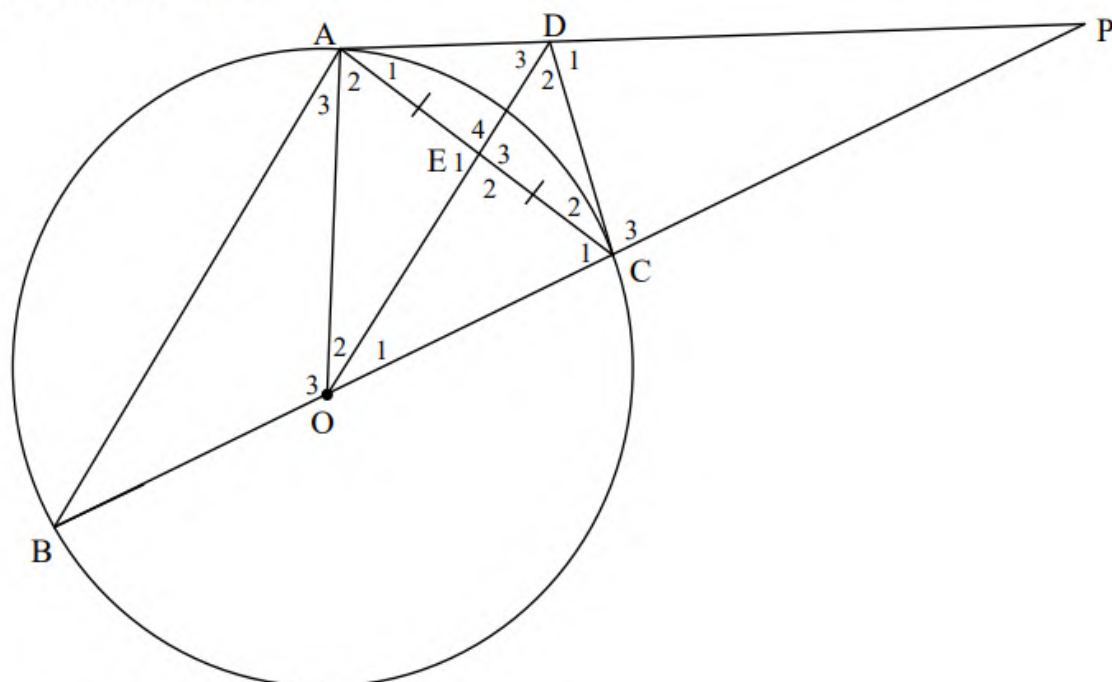
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- 12.1 Use the diagram below to prove the theorem which states that $\hat{EFG} = \hat{EDF}$.



(5)

- 12.2 In the diagram below, BOC is a diameter of the circle. AP is a tangent to the circle at A and $AE = EC$.



Prove that:

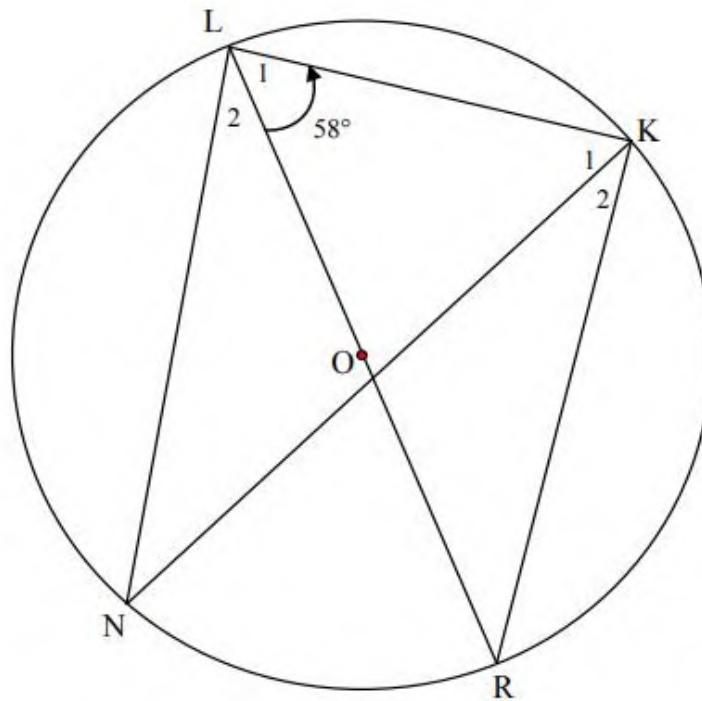
- 12.2.1 $BA \parallel OD$ (4)
- 12.2.2 $AOCD$ is a cyclic quadrilateral (5)
- 12.2.3 DC is a tangent to the circle at C (4)

[18]

QUESTION 9

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In the diagram, O is the centre of the circle. Diameter LR subtends $\hat{LKR}$ at the circumference of the circle. N is another point on the circumference and chords LN and KN are drawn. $\hat{L}_1 = 58^\circ$.



Calculate, giving reasons, the size of:

9.1 $\hat{LKR}$ (2)

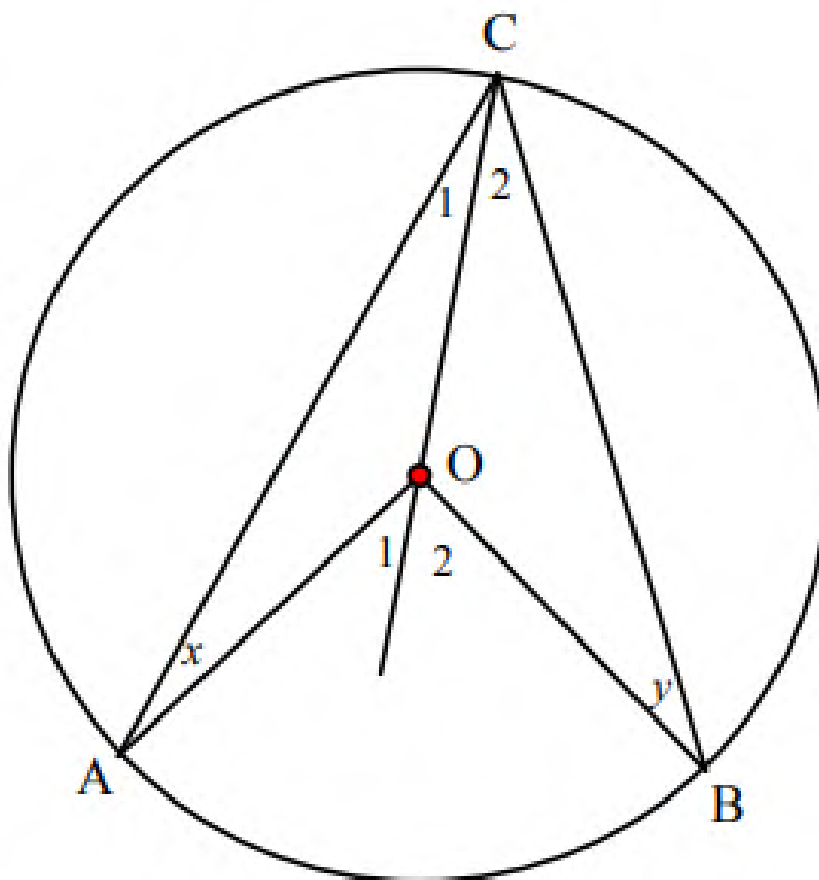
9.2 $\hat{R}$ (2)

9.3 $\hat{N}$ (2)

[6]

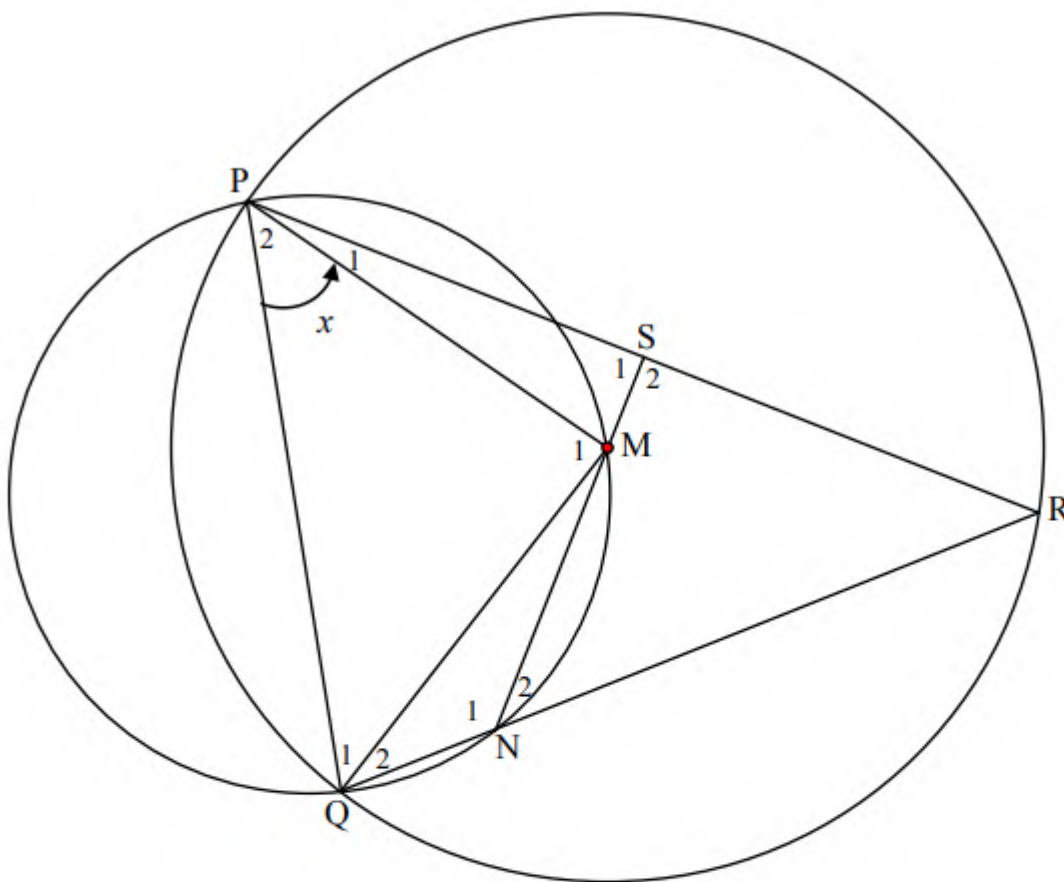
QUESTION 10 *Downloaded from Stanmorephysics.com*

- 10.1 In the diagram, O is the centre of the circle. A , B and C are points on the circumference of the circle. Chords AC and BC and radii AO , BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$.



- 10.1.1 Determine the size of $\hat{O}_1$ in terms of x . (3)
- 10.1.2 Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is $\hat{AOB} = 2\hat{ACB}$. (3)

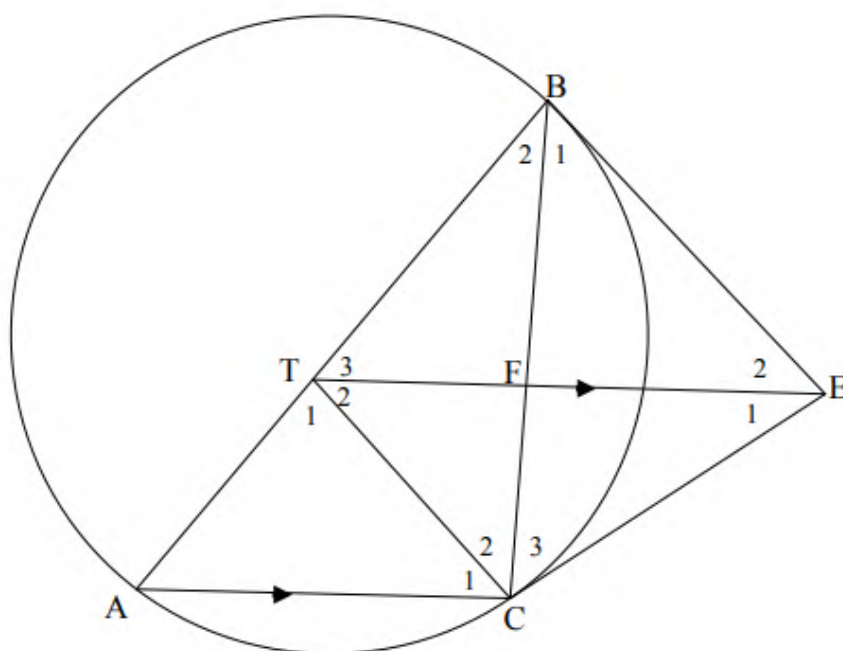
- 10.2 In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.



- 10.2.1 Give a reason why $\hat{N}_2 = x$. (1)
- 10.2.2 Write down another angle equal in size to x . Give a reason. (2)
- 10.2.3 Determine the size of $\hat{R}$ in terms of x . (3)
- 10.2.4 Prove that $PS = SR$. (3)
- [15]**

QUESTION 11 *Downloaded from Stanmorephysics.com*

In the diagram, the vertices A, B and C of $\triangle ABC$ are concyclic. EB and EC are tangents to the circle at B and C respectively. T is a point on AB such that $TE \parallel AC$. BC cuts TE in F.

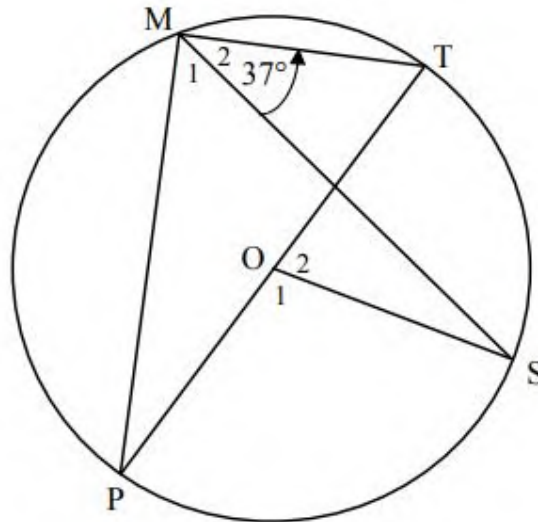


- 11.1 Prove that $\hat{B}_1 = \hat{T}_3$. (4)
 - 11.2 Prove that TBEC is a cyclic quadrilateral. (4)
 - 11.3 Prove that ET bisects $\hat{BTC}$. (2)
 - 11.4 If it is given that TB is a tangent to the circle through B, F and E, prove that $TB = TC$. (4)
 - 11.5 Hence, prove that T is the centre of the circle through A, B and C. (3)
- [17]**

QUESTION 8

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- 8.1 In the diagram below, PT is a diameter of the circle with centre O . M and S are points on the circle on either side of PT . MP , MT , MS and OS are drawn.
- $\hat{M}_2 = 37^\circ$



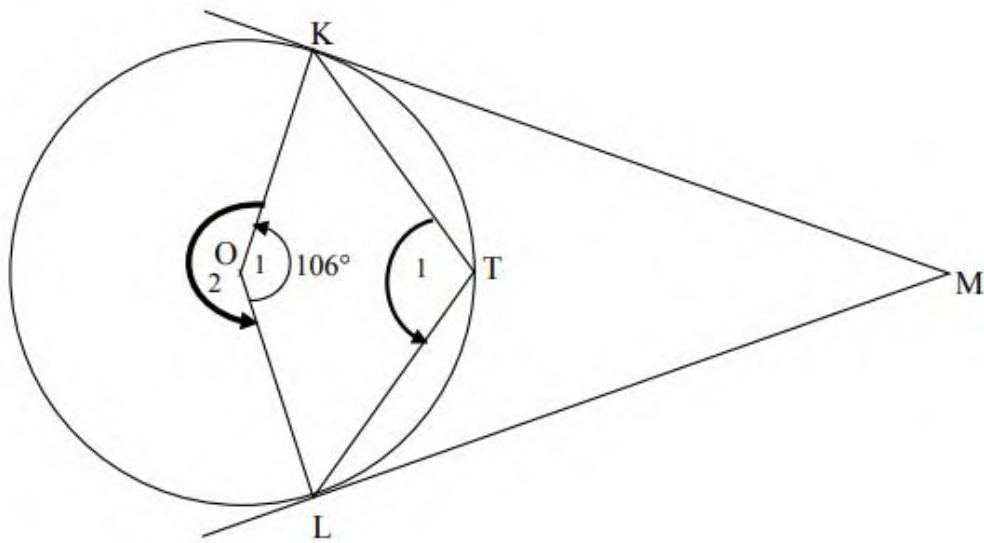
Calculate, with reasons, the size of:

8.1.1 $\hat{M}_1$ (2)

8.1.2 $\hat{O}_1$ (2)

8.2

In the diagram O is the centre of the circle. KM and LM are tangents to the circle at K and L respectively. T is a point on the circumference of the circle. KT and LT are joined. $\hat{O}_1 = 106^\circ$.

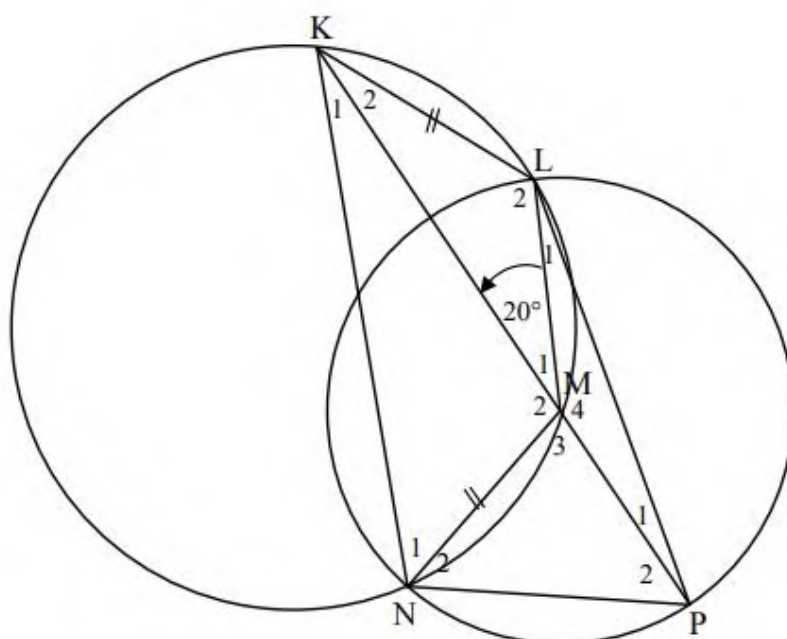


- | | | |
|-------|--|-------------|
| 8.2.1 | Calculate, with reasons, the size of $\hat{T}_1$. | (3) |
| 8.2.2 | Prove that quadrilateral $OKML$ is a kite. | (3) |
| 8.2.3 | Prove that quadrilateral $OKML$ is a cyclic quadrilateral. | (3) |
| 8.2.4 | Calculate, with reasons, the size of $\hat{M}$. | (2) |
| | | [15] |

QUESTION 9

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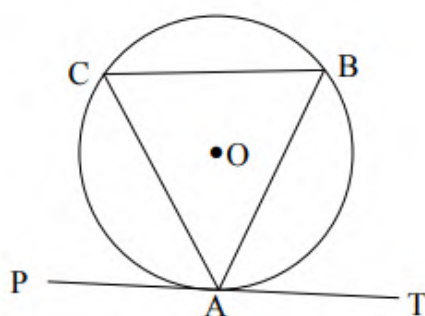
In the diagram M is the centre of the circle passing through points L , N and P . PM is produced to K . $KLMN$ is a cyclic quadrilateral in the larger circle having $KL = MN$. LP is joined. $\hat{KML} = 20^\circ$.



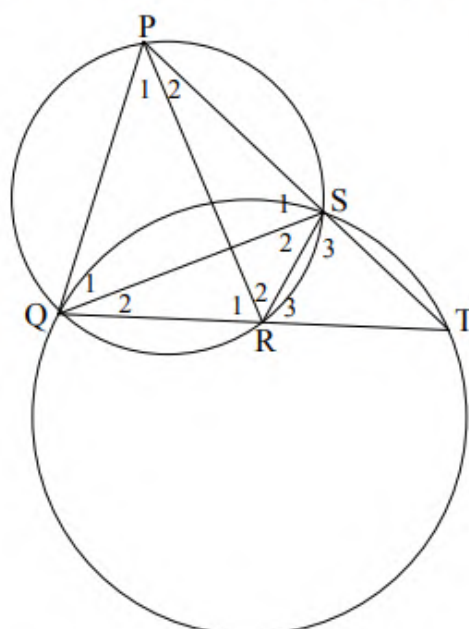
- 9.1 Write down, with a reason, the size of $\hat{NKM}$. (2)
- 9.2 Give a reason why $KN \parallel LM$. (1)
- 9.3 Prove that $KL = LM$. (2)
- 9.4 Calculate, with reasons, the size of:
- 9.4.1 $\hat{KNM}$ (4)
- 9.4.2 $\hat{LPN}$ (3)
- [12]

QUESTION 10 *Downloaded from Stanmorephysics.com*

- 10.1 Use the sketch on DIAGRAM SHEET 4 to prove the theorem which states that $\hat{BAT} = \hat{C}$. (6)



- 10.2 In the diagram PQ is a tangent to the circle QST at Q such that QT is a chord of the circle and TS produced meets the tangent at P. R is a point on QT such that PQRS is a cyclic quadrilateral in another circle. PR, QS and RS are joined.



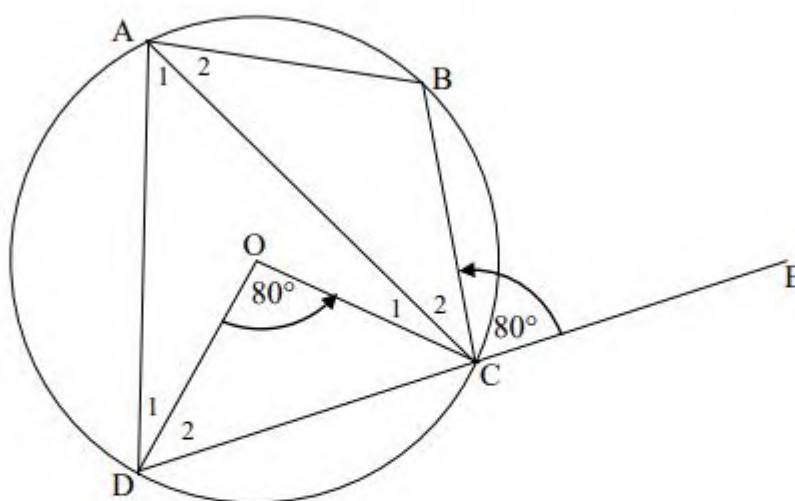
- 10.2.1 Give a reason for each statement. Write down only the reason next to the question number.

Statement	Reason
$\hat{Q}_1 = \hat{T}$	10.2.1 (a)
$\hat{Q}_2 = \hat{P}_2$	10.2.1 (b)

- 10.2.2 Prove that PQR is an isosceles triangle. (2)
- 10.2.3 Prove that PR is a tangent to the circle RST at point R. (4)
- (3)
- [15]

QUESTION 9 *Downloaded from Stanmorephysics.com*

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Chord DC is produced to E. AC is drawn. $\hat{DOC} = 80^\circ$ and $\hat{BCE} = 80^\circ$.



9.1 Calculate the size of the following angles:

9.1.1 $\hat{DAC}$ (2)

9.1.2 $\hat{DAB}$ (2)

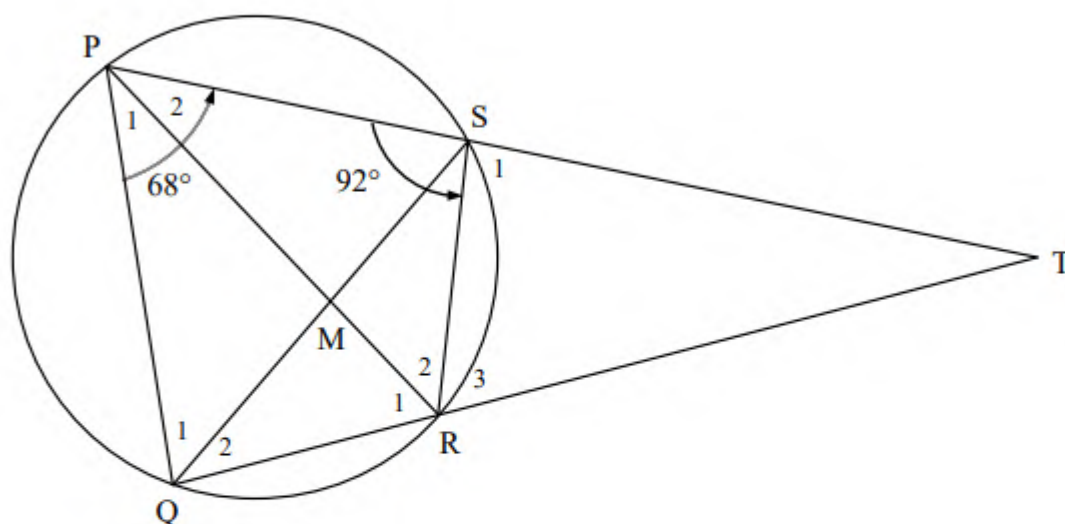
9.1.3 $\hat{BAC}$ (1)

9.2 Hence, or otherwise, prove that $DC = BC$. (2)
[7]

QUESTION 10

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In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced and meet at T. PR bisects $\angle QPS$. Also, $\angle PSR = 92^\circ$ and $\angle QPS = 68^\circ$.

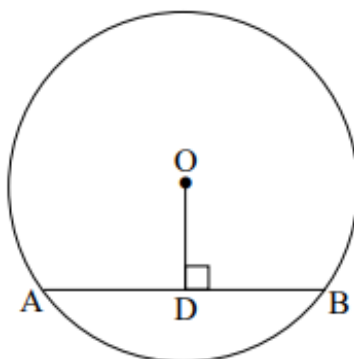


Calculate the size of the following angles:

- | | | |
|------|--------------|-------------|
| 10.1 | $\angle RPT$ | (1) |
| 10.2 | $\angle TQS$ | (2) |
| 10.3 | $\angle PQS$ | (3) |
| 10.4 | $\angle T$ | (4) |
| | | [10] |

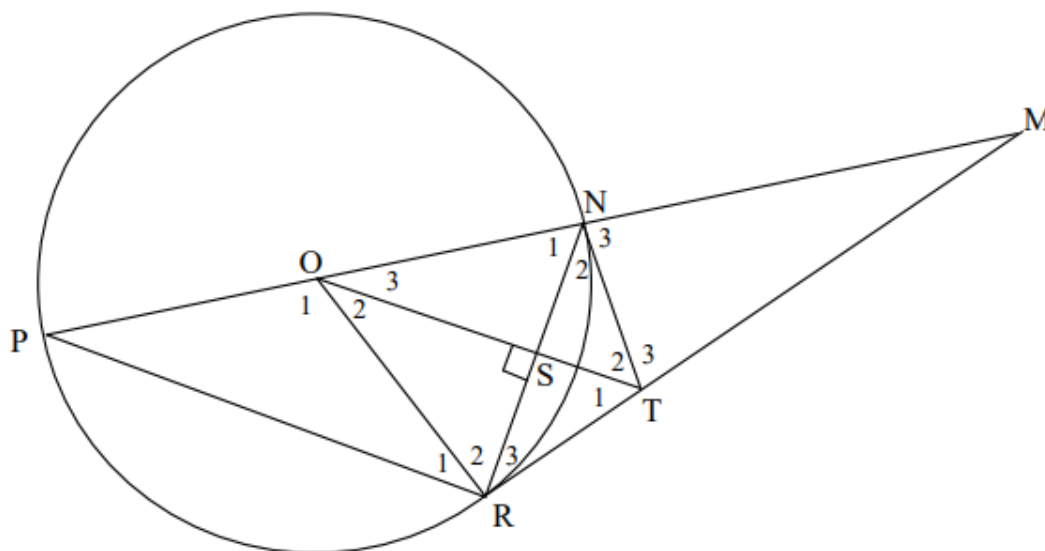
QUESTION 11

- 11.1 In the diagram, O is the centre of the circle and AB is a chord. D is a point on AB such that $OD \perp AB$. Use Euclidean geometry methods to prove the theorem which states that $AD = DB$.



(5)

- 11.2 In the diagram, PN is a diameter of the circle with centre O . RT is a tangent to the circle at R . RT produced and PN produced meet at M . OT is perpendicular to NR . NT and OR are drawn.



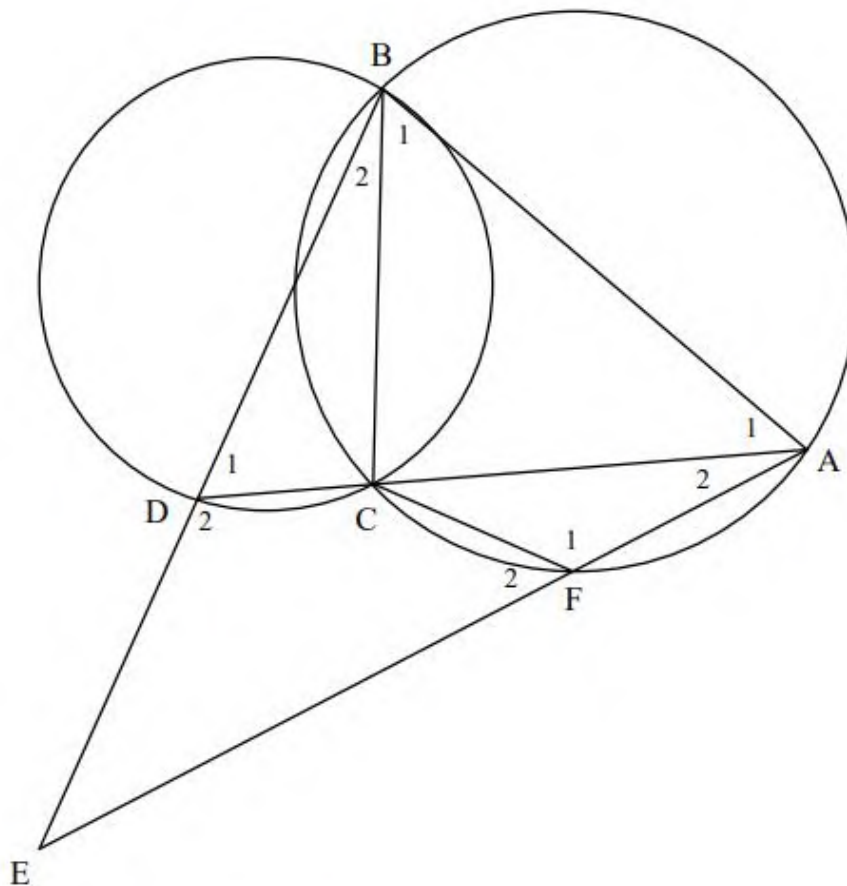
- 11.2.1 Prove that $TO \parallel RP$. (3)
- 11.2.2 It is further given that $\angle TRN = x$. Name TWO other angles each equal to x . (3)
- 11.2.3 Prove that $NTRO$ is a cyclic quadrilateral. (2)
- 11.2.4 Calculate the size of $\hat{M}$ in terms of x . (3)
- 11.2.5 Show that NT is a tangent to the circle at N . (3)

[19]

QUESTION 12

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In the diagram, $ABCF$ is a cyclic quadrilateral. AB is a tangent to circle BCD at B .



Prove that $CDEF$ is a cyclic quadrilateral.

[5]

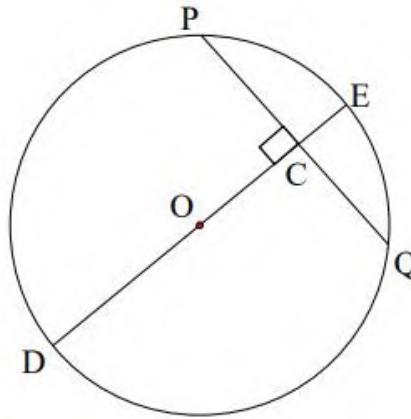
QUESTION 9

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9.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord ... (1)

9.2 In the diagram, O is the centre of the circle. The diameter DE is perpendicular to the chord PQ at C. DE = 20 cm and CE = 2 cm.



Calculate the length of the following with reasons:

9.2.1 OC (2)

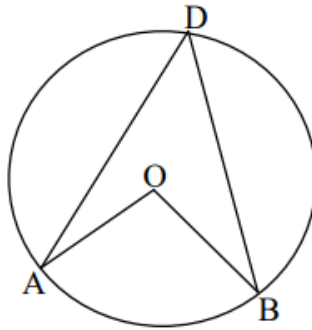
9.2.2 PQ (4)

[7]

QUESTION 10

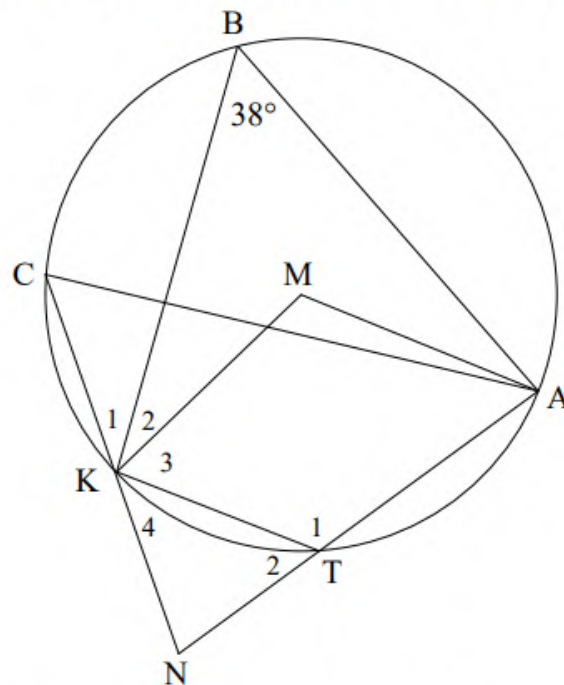
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- 10.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle. Use Euclidean geometry methods to prove the theorem which states that $\hat{AOB} = 2\hat{ADB}$.



(5)

- 10.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also $NA = NC$ and $\hat{B} = 38^\circ$.



- 10.2.1 Calculate, with reasons, the size of the following angles:

- | | | |
|-----|-------------|-----|
| (a) | $\hat{KMA}$ | (2) |
| (b) | $\hat{T}_2$ | (2) |
| (c) | $\hat{C}$ | (2) |
| (d) | $\hat{K}_4$ | (2) |

- 10.2.2 Show that $NK = NT$. (2)

- 10.2.3 Prove that AMKN is a cyclic quadrilateral. (3)

[18]

QUESTION 11

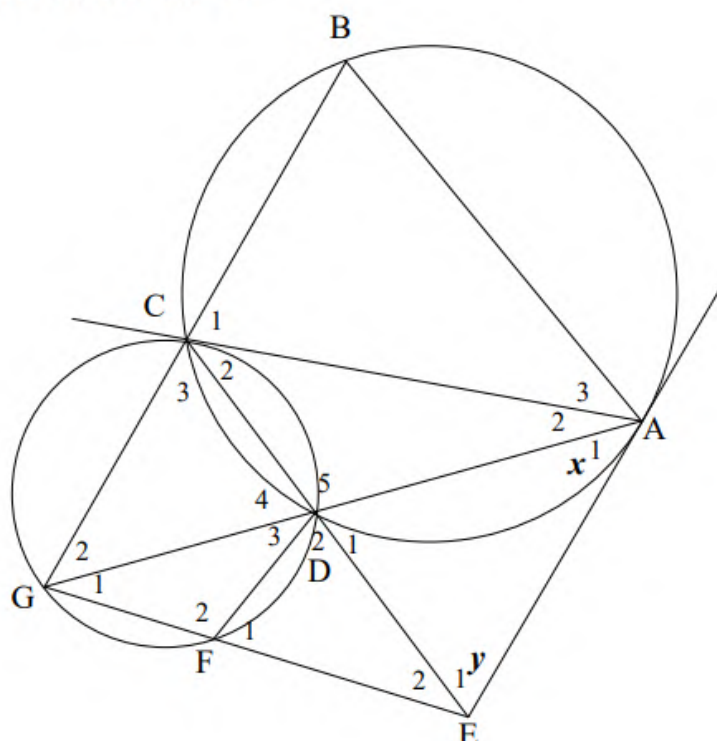
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11.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is ...

(1)

11.2 In the diagram, EA is a tangent to circle ABCD at A.
AC is a tangent to circle CDFG at C.
CE and AG intersect in D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

11.2.1 $BCG \parallel AE$

(5)

11.2.2 AE is a tangent to circle FED

(5)

11.2.3 $AB = AC$

(4)

[15]